Assignment

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Q1. mean = θ_1 (Normal Distribution)
variance = θ_2

max likelihood estimate ?

The likelihood function is :

$$L(\theta_1, \theta_2 | x_1, x_2 \ldots x_n) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x-\theta_1)^2}{2\theta_2}}$$

Take log on both sides

$$\ln L(\theta_1, \theta_2 | x_1, x_2 \ldots x_n) = \frac{-n}{2} \ln(2\pi\theta_2) - \frac{1}{2\theta_2} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

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Differentiate wrt θ_1 & θ_2 & then equate to 0

for θ_1:

$$\frac{\partial}{\partial \theta_1} \ln L(\theta_1, \theta_2 | x_1, x_2 \ldots x_n) = \frac{1}{\theta_2} \sum_{i=1}^{n} (x_i - \theta_1)$$

equate to 0

$$\frac{1}{\theta_2} \sum_{i=1}^{n} (x_i - \hat{\theta_1}) = 0$$

$$\sum_{i=1}^{n} (x_i - \hat{\theta_1}) = 0$$

$$\hat{\theta_1} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

So MLE for θ_1 is sample mean.

for θ_2:

$$\frac{\partial}{\partial \theta_2} \ln L(\theta_1, \theta_2 | x_1, x_2, \ldots x_n) = \frac{-n}{2\theta_2} + \frac{1}{2\theta_2^2} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

equating to 0

$$\frac{-n}{2\hat{\theta_2}} + \frac{1}{2\hat{\theta}_2^2} \sum_{i=1}^{n} (x_i - \hat{\theta_1})^2 = 0$$

$$\frac{n}{2\hat{\theta_2}} = \frac{1}{2\hat{\theta}_2^2} \sum_{i=1}^{n} (x_i - \hat{\theta_1})^2$$

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$$\hat{\theta_2} = \frac{1}{n} \sum_{i=1}^{n} (x_i - \hat{\theta_1})^2$$

So MLE for θ_2 is sample variance

Q2 Bernoulli distribution

parameter $\rightarrow \theta \in \Theta = (0,1)$ unknown

$\rightarrow m$ (known +ve $\mathbb{Z}$)

The likelihood function is $\rightarrow$

$$L(\theta \mid x_1 x_2 \ldots x_n) = \prod_{i=1}^{n} P(X_i = x_i \mid \theta)$$

Since X_i follows bernoulli distribution,

$$P(X_i = x_i \mid \theta) = \theta^{x_i} (1-\theta)^{m-n_i} \text{ for each } i$$

Taking log on both sides

$$\ln L(\theta \mid x_1, x_2 \ldots x_n) = \sum_{x_i=1}^{n} \ln \left(\theta^{x_i} (1-\theta)^{m-x_i}\right)$$

$$= \sum_{i=1}^{n} \left(x_i \ln \theta + (m - x_i) \ln (1-\theta)\right)$$

Differentiate wrt θ

$$\frac{d}{d\theta} \left(\ln L (\theta \mid x_1, x_2, \ldots x_m)\right) = 0$$

$$\sum_{i=1}^{n} \left(\frac{x_i}{\theta} - \frac{m - x_i}{1-\theta} \right) = 0$$

$$\sum_{i=1}^{n} \frac{x_i}{\theta} = \frac{mn - \sum_{i=1}^{n} x_i}{1-\theta}$$

$$\therefore \quad \theta = \frac{\sum_{i=1}^{n} x_i}{n \cdot m}$$

So max likelihood estimate of θ is :

$$\hat{\theta}_{MLE} = \sum_{i=1}^{n} \frac{x_i}{n \cdot m}$$