MASTER'S THEOREM.

If $f(n) \in \Theta(n^d)$ or $f(n) = c * n^d$ where $d \geq 0$ in recurrence $T(n) = a\,T(n/b) + f(n)$ then

$$T(n) \in \begin{cases} \Theta(n^a), & \text{if } (a < b^d) \\ \Theta(n^d \log n), & \text{if } (a = b^d) \\ \Theta(n^{\log_b a}), & \text{if } (a > b^d) \end{cases}$$

(1) $T(n) = 8T(n/2) + 1000n^2$

$a = 8, \quad b = 2, \quad f(n) = 1000n^2$

$= c * n^d$

$\therefore d = 2$

$b^d = 2^2 = 4$ $\qquad \log_b a = \log_2 8 = 3.$

Hence, $a > b^d$

$\therefore T(n) \in \Theta(n^{\log_b a})$

$\therefore T(n) \in \Theta(n^3)$

2) $T(n) = 2T(n/2) + n^2$

$a = 2, \quad b = 2, \quad d = 2$

$b^d = 2^2 = 4$

As, $a < b^d$

$\therefore T(n) \in \Theta(n^2)$

(3) $T(n) = 2T(n/2) + 10n$

$a = 2, \quad b = 2, \quad d = 1$

$b^d = 2^1 = 2$

As, $a = b^d$

$\therefore T(n) = \Theta(n^d \log n)$

$\Rightarrow T(n) = \Theta(n \log n)$