Finite State Automata

Idle   Preperation   Dispense

Tea
Coffee
⋮

Scanner
copier

Finite state transition system

↓

Finite State Automaton.

Deterministic Finite Automata. (DFA)

$$M = (Q, \Sigma, \delta, s, F)$$

Q - finite set of states.

Σ - input alphabet - Finite set

$s \in Q$: Start state of M.

$F \subseteq Q$: set of final states / accept states.

$\delta : Q \times \Sigma \to Q$: transition function.

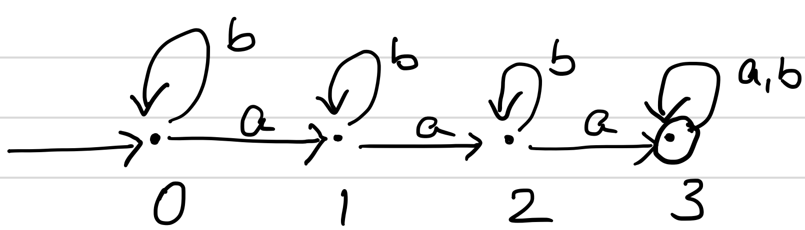
M is in some state q and it sees input a

$\delta(q,a) = q'$ M moves to state q' : $\delta(q,a)$.

Example. $Q = \{0,1,2,3\}$ $\Sigma = \{a,b\}$ $s = 0$
$F = \{3\}$

$\delta(0,a) = 1$ | $\delta(2,a) = \delta(3,a) = 3$

$\delta(1,a) = 2$ | $\delta(q,b) = q$ $q \in \{0,1,2,3\}$.



$\longrightarrow\cdot$: start state

$\odot$: Final State.

$$M = (Q, \Sigma, \delta, s, F)$$

Input : $x \in \Sigma^*$

M runs on input x. / Decides whether to accept x or reject x.

Scan x from left to right one symbol at a time.

Start state is s. - $\underbrace{\text{first symbol}}_{a \in \Sigma}$ in x.

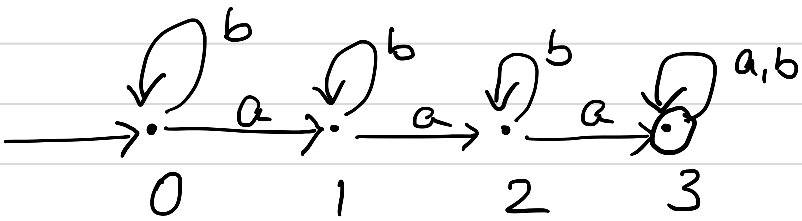
$$\delta(s, a)$$

After reading all the symbols in x, M is in some state $q \in Q$.

if $q \in F$ then M accepts x

if $q \notin F$ then M rejects x.

M.



$s = 0$
$F = \{3\}$.

Input $x^1 = aabba$

M accepts x : $3 \in F$

Input $x^2 = abbab$ M is in state 2.

$2 \notin F$: x^2 is rejected by M.

$L(M) = \{x \in \{a,b\}^* \mid x$ contains at least three a's$\}$

$M = (Q, \Sigma, \delta, s, F)$

$\delta : Q \times \Sigma \to Q.$

$\hat{\delta} : Q \times \Sigma^* \to Q.$

Definition of $\hat{\delta}$:

$$\hat{\delta}(q, \epsilon) = q$$

$$\hat{\delta}(q, xa) = \delta(\underbrace{\hat{\delta}(q, x)}_{\text{IH}}, a)$$

To show $\hat{\delta}(q,a) = \delta(q,a)$

$$\hat{\delta}(q, a) = \hat{\delta}(q, \epsilon a) \quad [\text{Since } a = \epsilon a]$$

$$= \delta(\hat{\delta}(q, \epsilon), a)$$

$$= \delta(q, a)$$

x is accepted by M if $\hat{\delta}(s, x) \in F$

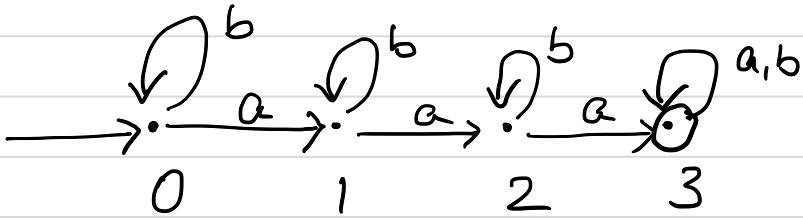
x is rejected by M if $\hat{\delta}(s, x) \notin F$

Language accepted by M

$$L(M) = \{x \in \Sigma^* \mid \hat{\delta}(s, x) \in F\}$$

$A \subseteq \Sigma^*$ is regular if $A = L(M)$ for some DFA M.

M.



$F = \{3\}$.

Input $x^1 =$ a a b b a

M accept x : $3 \in F$.

$x^2 =$ a b b a b   Resulting state of M 2.

$2 \notin F$: x^2 is rejected by M.

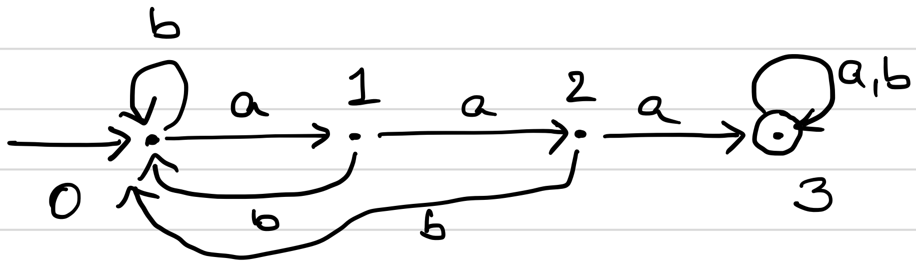
$L(M) = \{ x \in \{a,b\}^* \mid x$ contains at least three a's $\}$.

$A = \{x \in \{a,b\}^* \mid x$ contains a substring of three consecutive a's$\}$.

b aabb aaabb $\in A$

b aabbabb $\notin A$

A is regular. - Construct a DFA M s.t $L(M) = A$.



- M.