

Set membership question.

Question. Given a regular set $A \subseteq \Sigma^*$ and $x \in \Sigma^*$ is $x \in A$?

DFA M : Simulate M on the input x

$O(k)$ where $|x| = k$

NFA N : Is $x \in L(N)$?

$O(k n^2)$ $|x| = k$, $|Q| = n$.

Question. Given a CFL $A \subseteq \Sigma^*$ and $x \in \Sigma^*$ is $x \in A$?

Algorithm - Due to Cocke, Kasami and Younger

CKY - algorithm.

Runs in cubic-time.

Determines for each substring y of x the set of

all nonterminals that generate y .

Inductively on the length of y .

Assume G is in Chomsky normal form.

Example. $S \rightarrow AB \mid BA \mid SS \mid AC \mid BD$

$A \rightarrow a, B \rightarrow b, C \rightarrow SB, D \rightarrow SA$

$L(G)$ - all strings with equal number of a 's and b 's.

Let $x = aab bab$. Let $n = |x|$ (So $n=6$ here)

	a	a	b	b	a	b
0	1	2	3	4	5	6

For $0 \leq i < j \leq n$, let x_{ij} denote the substring of x between i & j . Ex. $x_{0,3} = aab, x_{2,6} = bbab$
 $x = x_{0,n}$.

0						
-	1					
-	-	2				
-	-	-	3			
-	-	-	-	4		
-	-	-	-	-	5	
-	-	-	-	-	-	6

Entry T_{ij} of T - Set of nonterminals of G that generates the substring x_{ij} of x .

Induction on length of substrings.

Example. $S \rightarrow AB \mid BA \mid SS \mid AC \mid BD$

$A \rightarrow a, B \rightarrow b, C \rightarrow SB, D \rightarrow SA$

$x \rightarrow$ a a b b a b
0 1 2 3 4 5 6

Substring of length 1.

0
A 1
— A 2
— — B 3
— — — B 4
— — — — A 5
— — — — — B 6

Example. $S \rightarrow AB \mid BA \mid SS \mid AC \mid BD$

$A \rightarrow a, B \rightarrow b, C \rightarrow SB, D \rightarrow SA$

$x \rightarrow$ a a b b a b
0 1 2 3 4 5 6

Substring of length 1.

0						
<u>A</u>	1					
—	<u>A</u>	2				
—	—	<u>B</u>	3			
—	—	—	<u>B</u>	4		
—	—	—	—	<u>A</u>	5	
—	—	—	—	—	<u>B</u>	6

Substring of length 2

0						
<u>A</u>	1					
<u>∅</u>	<u>A</u>	2				
—	<u>S</u>	<u>B</u>	3			
—	—	<u>∅</u>	<u>B</u>	4		
—	—	—	<u>S</u>	<u>A</u>	5	
—	—	—	—	<u>S</u>	<u>B</u>	6

For each $x_{i,i+2}$: split into 2 substrings $x_{i,i+1}$ and $x_{i+1,i+2}$. Select a nonterminal X from $T_{i,i+1}$ and Y from $T_{i+1,i+2}$.

Look for a production $z \rightarrow xy$ in G .

Label $T_{i,i+2}$ with Z for each such production

Example. $S \rightarrow AB \mid BA \mid SS \mid AC \mid BD$

$A \rightarrow a, B \rightarrow b, C \rightarrow SB, D \rightarrow SA$

$x \rightarrow$ a a b b a b
0 1 2 3 4 5 6

Substring of length 1.

0						
<u>A</u>	1					
—	<u>A</u>	2				
—	—	<u>B</u>	3			
—	—	—	<u>B</u>	4		
—	—	—	—	<u>A</u>	5	
—	—	—	—	—	<u>B</u>	6

Substring of length ≥ 3

0						
<u>A</u>	1					
<u>ϕ</u>	<u>A</u>	2				
<u>ϕ</u>	<u>S</u>	<u>B</u>	3			
<u>S</u>	<u>C</u>	<u>ϕ</u>	<u>B</u>	4		
<u>D</u>	<u>S</u>	<u>ϕ</u>	<u>S</u>	<u>A</u>	5	
<u>S</u>	<u>C</u>	<u>ϕ</u>	<u>C</u>	<u>S</u>	<u>B</u>	6

For each $x_{i,i+2}$,
break into 2 substring
 $x_{i,i+1}$ & $x_{i+1,i+2}$
Select a nonterminal
 X from $T_{i,i+1}$ and
 Y from $T_{i+1,i+2}$
Look for production
 $Z \rightarrow XY$ in G .
Label $T_{i,i+2}$ with Z
for each such production.

Example. $S \rightarrow AB \mid BA \mid SS \mid AC \mid BD$

$A \rightarrow a, B \rightarrow b, C \rightarrow SB, D \rightarrow SA$

$x \rightarrow$ a a b b a b
0 1 2 3 4 5 6

Substring of length 1.

0						
<u>A</u>	1					
—	<u>A</u>	2				
—	—	<u>B</u>	3			
—	—	—	<u>B</u>	4		
—	—	—	—	<u>A</u>	5	
—	—	—	—	—	<u>B</u>	6

Substring of length ≥ 3

0						
<u>A</u>	1					
<u>∅</u>	<u>A</u>	2				
<u>∅</u>	<u>S</u>	<u>B</u>	3			
<u>S</u>	<u>C</u>	<u>∅</u>	<u>B</u>	4		
<u>D</u>	<u>S</u>	<u>∅</u>	<u>S</u>	<u>A</u>	5	
<u>S</u>	<u>C</u>	<u>∅</u>	<u>C</u>	<u>S</u>	<u>B</u>	6

For each $x_{i,i+2}$,
break into 2 substring
 $x_{i,i+1}$ & $x_{i+1,i+2}$
Select a nonterminal
 X from $T_{i,i+1}$ and
 Y from $T_{i+1,i+2}$
Look for production
 $Z \rightarrow XY$ in G .
Label $T_{i,i+2}$ with Z
for each such production.

$S \in T_{0,6}$ indicates $S \xrightarrow{*}_G x_{0,6} = x$.
Therefore $x \in L(G)$.

Algorithm

```
For  $i := 0$  to  $n-1$  do
  begin
     $T_{i,i+1} = \phi$ 
    For  $A \rightarrow a$ , a production in  $G$  do
      if  $a = x_{i,i+1}$  then  $T_{i,i+1} = T_{i,i+1} \cup \{A\}$ 
  end
```

[length 1 strings].
[Initialise to ϕ]

```
For  $m := 2$  to  $n$  do
  For  $i := 0$  to  $n-m$  do
    begin
       $T_{i,i+m} := \phi$ 
      For  $j = i+1$  to  $i+m-1$  do
        For  $A \rightarrow BC$ , a production in  $G$  do
          if  $B \in T_{i,j}$  and  $C \in T_{j,i+m}$ 
            then
               $T_{i,i+m} = T_{i,i+m} \cup \{A\}$ 
      end
    end
```

[length $m \geq 2$]
[for each substring of length m]
[Initialise]
[All possible splits]

Running Time - $O(n^3 p)$ where $p = |P|$ and $n = |x|$.