Theorem. Let $A \subseteq \Sigma^*$. The following statements are equivalent.

1) A is regular. $\exists$ a finite automaton M s.t $L(M) = A$

2) $A = L(\alpha)$ for some pattern α.

3) $A = L(\alpha)$ for some regular expression α.

$1 \Rightarrow 3$ Given a finite state automaton M, we can construct a regular expression α s.t $L(\alpha) = L(M)$.

$M = (Q, \Sigma, \Delta, S, F)$ - NFA : No ϵ-transition

$Y \subseteq Q$, $u, v \in Q$, we will construct a regular expression α_{uv}^{Y}

α_{uv}^{Y} - the set of all strings x such that there is a path from state u to v in M labelled x. Formally, $v \in \hat{\Delta}(\{u\}, x)$.
and
all states along the path with possible exception of u and v lie in Y.

Induction on size of Y.

Base Case : $Y = \phi$.

$a_1, a_2, \ldots a_k \in \Sigma$ s.t $v \in \Delta(u, a_i)$

Case 1. $u \neq v$

$$\alpha_{uv}^{\phi} = \begin{cases} a_1 + a_2 + \cdots + a_k & \text{if } k \geq 1. \\ \phi & \text{if } k = 0 \end{cases}$$

Case 2. $u = v$

$$\alpha_{uv}^{\phi} = \begin{cases} a_1 + a_2 + \cdots + a_k + \epsilon & \text{if } k \geq 1 \\ \epsilon & \text{if } k = 0 \end{cases}$$

Induction step.

$$\alpha_{uv}^{Y} = \alpha_{uv}^{Y - \{q\}} + \alpha_{uq}^{Y - \{q\}} \left(\alpha_{qq}^{Y - \{q\}} \right)^{*} \alpha_{qv}^{Y - \{q\}}$$

Choose an arbitrary state $q \in Y$

α - sum of all expressions of the form.

α_{sf}^{Q} $s \in S$, $f \in F$

Floyd-Warshall Algorithm.

graph G vertices $V = \{1, \ldots n\}$.

ShortestPath (i, j, k). $\{1, \ldots k\}$ - intermediate vertices.

$= \min($ shortestPath $(i, j, k-1)$,

ShortestPath $(i, k, k-1)$ + shortestPath $(k, j, k-1))$