CS315: Database Systems
Indexing

Arnab Bhattacharya
arnabb@cse.iitk.ac.in

Computer Science and Engineering,
Indian Institute of Technology, Kanpur
http://web.cse.iitk.ac.in/~cs315/

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Basics

- Indexing is used to speed up search
- A search key is used
- An index file consists of records or index entries which has two fields
  1. Search key: Attribute that is used for searching
  2. Pointer to the entire object or tuple
- Index files should be smaller than data files

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- Index evaluation metrics
  - Search time
  - Modification overhead
  - Space overhead { extra space needed }

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- Index evaluation metrics
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- Two basic types of indices
  1. Ordered index: search keys are organized according to some order
  2. Hash index: search keys are organized according to a hash function

no order

Static Hashing

hash fn:

key k → h(k)

- A hash function maps a key to a bucket
- A bucket is a unit of storage
- It is typically a disk block
- A key may need to be searched sequentially inside a bucket
- Results in hash file organization
- Example: mod n where n is the number of buckets

m hash locations

h(k) → k mod m

Hash Function

- Two important qualities of an ideal hash function
- Uniform: Total number of keys from the domain is spread uniformly over all the buckets
- Random: Number of keys in each bucket is same irrespective of the actual distribution of keys

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  - Non-uniformity of hash function
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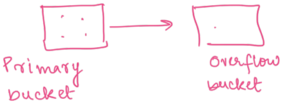

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  - Linear probing:

* Probes for another bucket

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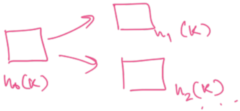
- Changing size of a database is a problem
- Periodic re-hashing is the only solution
- Dynamic hashing: h changes dynamically but deterministically

Dynamic Hashing

→ changes the hash function itself.

- Organize overflow buckets as binary trees
- m binary trees for m primary pages
- $h_0(k)$ produces index of primary page
- Particular access structure for binary trees
- Family of functions $g(k) = \{h_1(k), \ldots, h_i(k), \ldots\}$
- Each $h_i(k)$ produces a bit
- At level i, if $h_i(k) = 0$, take left branch, otherwise right branch
- Example: bit representation



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- Sparse index requires less space and less maintenance but more searching time
- Multilevel index: primary index does not fit in memory
  - Outer index: Sparse primary index $\} \rightarrow$ less space
  - Inner index: Dense primary index file

B-Tree

* Disk aware structures

- Balanced hierarchical data structure
- Keys (and associated objects) are in secondary storage, i.e., disk
- A B-tree of order Θ has the following properties:
  1. Leaf nodes are in same level, i.e., the tree is balanced
  2. Root has at least 1 key
  3. Other internal nodes have between Θ and 2Θ keys
  4. An internal node with k keys have $k+1$ children
  5. Child pointers in leaf nodes are null
- Branching factor (fan-out) is between $\Theta + 1$ and $2\Theta + 1$ [Branching factor: no. of children of a node]
- Pointer to the object corresponding to a key is stored alongside

B-tree internal node with k keys:

cp0	key1	rp1	cp1	key2	rp2	cp2	. . .	keyk	rpk	cpk

child pointer: $\leq$ key1

record pointer

cp1: { > key1 , <= key2 }

cpk: > keyk

B+-Tree

- Most important variety of B-tree
- Internal nodes do *not* contain pointers to objects [No record pointer]
- Often siblings are connected by pointers to avoid parent traversal

cp0 key1 cp1 key2

Not an actual key.

These are key to index.

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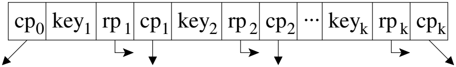
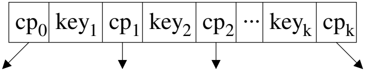


B-tree node



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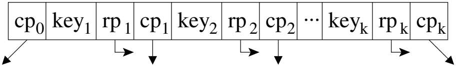

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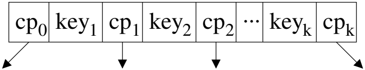

B+-tree node

- More keys can fit in a B+-tree
- Height may be less

Order

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$$\Rightarrow \Theta_{\text{B+-tree}} = \left\lfloor \frac{C - \eta}{2(\gamma + \eta)} \right\rfloor$$

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- Order mainly determined by disk page size
- Disk page capacity is C bytes
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* Reducing disk access time [Random I/O access]

Order: maximum no. of keys that can fit in a single disk page.

No of keys possible

child ptr

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$$\Rightarrow \Theta_{\text{B+-tree}} = \left\lfloor \frac{C - \eta}{2(\gamma + \eta)} \right\rfloor$$

- For B-tree

record ptr

$$C \geq 2.\Theta.\gamma + 2.\Theta.\eta + (2.\Theta + 1).\eta$$

$$\Rightarrow \Theta_{\text{B-tree}} = \left\lfloor \frac{C - \eta}{2(\gamma + 2\eta)} \right\rfloor$$

Example

- What is the order of a B+-tree and a B-tree with a page size of 4 KB indexing keys of 8 bytes each, and having pointers of size 4 bytes?

γ η

$\theta_{b+ \text{trees}} \Rightarrow \left\lfloor \frac{C-\eta}{2(\gamma+\eta)} \right\rfloor = \frac{4*1024-4}{2(4+8)}$

$C = 4KB$ (capacity)
$\gamma = 8B$ (key)
$\eta = 4B$ (ptr to key)

$\theta_{b\text{-tree}} = \frac{C-\eta}{2(\gamma+2\eta)} = \left\lfloor \frac{4*1024-4}{2(8+8)} \right\rfloor$

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  - Height of B+-tree is $\lceil \log_{2*170}(3 \times 10^7) \rceil = 3$

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  - Height of B+-tree is $\lceil \log_{2*170}(3 \times 10^7) \rceil = 3$
  - Height of B-tree is $\lceil \log_{2*127}(3 \times 10^7) \rceil = 4$

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- Separate indices may be used
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  - Quadtree, KD-tree, K-d-B-tree: Extension of BST
  - Data-partitioning
  - R-tree: Extension of B+-tree
  - Uses minimum bounding rectangles (MBRs)

Bitmap Index

- Attribute domain consists of a small number of distinct values
- A bitmap or a bit vector is an array of bits
- Each distinct value has an array of the size of the number of tuples
  - If the *i*-th bit is 1, tuple *i* has that value

Gender	Grade
Male	C
Female	A
Female	C
Male	D
Male	A

- Two sets of bit vectors
  - Male = (10011), Female = (01100)
  - A = (01001), B = (00000), C = (10100), D = (00010)

Find male students who got 'D'

→ Bitmap (M) & Bitmap (D) = 10011 & 00010 → 00010

Bitmap Operations

- Queries are answered using bitmap operations
- Example: Find the male student who got ‘D’
    - Bitmap(Male) AND Bitmap(D)
- Null values require a special bitmap for null
- O/S allows efficient bitmap operations when they are packed in word sizes

Create index i on r(a) a: attribute of relation r

⇒ create index bindex on branch(bname)