CS340 - Theory of Computation

What is Computation?
↓
Programs.

What is a program.

Binary Search - Algorithm.

What is an algorithm? Sequence of steps to implement some computation.

Algorithm - Takes an input and produces an output.

Implements a transformation

$$x \longrightarrow \boxed{\text{Algorithm}} \longrightarrow y / F(x)$$

Function $F : D \rightarrow R$

Computation $\equiv$ Functions.

Defining a function / Computing a function

prime : $\mathbb{N} \to \{Yes, No\}$.

$$\text{prime}(n) := \begin{cases} \text{Yes} & \text{if } n \text{ is a prime} \\ \text{No} & \text{if } n \text{ is not a prime.} \end{cases}$$

Which functions admit an algorithm to compute it?

Goal. Techniques to show that a function does not admit any algorithm to compute it.

Efficiency.

Propositional logic ·

P- set of propositions

$\underline{\Phi} := p \in P \mid \neg\alpha \mid \alpha \vee \beta$

$\varphi \in \Phi$ is a wff in propositional logic.

Valuation $v : P \rightarrow \{T, \perp\}$

$v \models \varphi$ [Semantics].

SAT: Given φ is there a valuation v such that $v \models \varphi$?

$Voc(\varphi)$ - Set of propositions occuring in φ.

$2^{|Voc(\varphi)|}$

"Hard problem"

Formal Models of Computation.

Finite state automata.

Deterministic Computation.
Non-deterministic Computation.

Regular Expression.

- Pushdown Automata.

- Turing Machine.

Decision Problem. Functions with simple Output - "Yes" or "No"

Set Membership Problem.

There is an underlying set S.

Question. Given an a, is $a \in S$?

$f: D \to R$

$\mathrm{graph}(f) = \{(a,b) \mid f(a) = b\}$.

Suppose there is no algorithm to solve Set membership of graph(f) then f is not computable.

Suppose $\exists$ an algorithm A_f to compute f.

$(a,b) \in \mathrm{graph}(f)$. iff $f(a) = b$

A_f with input a - check if the output is b.

Automata and Computability - Dexter Kozen

Introduction to Automata Theory, Languages and Computation - Hopcroft, Ullman, Motwani

Introduction to the Theory of Computation - Sipser