



Compiler Design

Parse Table Construction

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Constructing parse table

Augment the grammar

- G is a grammar with start symbol S
 - The augmented grammar G' for G has a new start symbol S' and an additional production $S' \rightarrow S$
 - When the parser reduces by this rule it will stop with accept
- * Coming to start state \Rightarrow doesn't mean parsing is done
- * Here we can be sure at $S'.$ (stop, accept)

Production to Use for Reduction

- How do we know which production to apply in a given configuration
- We can guess!
 - May require backtracking
- Keep track of "ALL" possible rules that can apply at a given point in the input string
 - But in general, there is no upper bound on the length of the input string
 - Is there a bound on the number of applicable rules?



Some hands on!

- | | |
|--------------------------|-----------------------|
| 1. $E' \rightarrow E$ | Strings to Parse |
| 2. $E \rightarrow E + T$ | • id + id + id + id |
| 3. $E \rightarrow T$ | • id * id * id * id |
| 4. $T \rightarrow T * F$ | • id * id + id * id |
| 5. $T \rightarrow F$ | • id * (id + id) * id |
| 6. $F \rightarrow (E)$ | |
| 7. $F \rightarrow id$ | |

Parser states

- Goal is to know the valid reductions at any given point
- Summarize all possible stack prefixes α as a parser state
- Parser state is defined by a DFA state that reads in the stack α
- Accept states of DFA are unique reductions

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Viable prefixes

- α is a viable prefix of the grammar if
 - $\exists w$ such that αw is a right sentential form
 - $\langle \alpha, w \rangle$ is a configuration of the parser
- As long as the parser has viable prefixes on the stack no parser error has been seen
- The set of viable prefixes is a regular language. DFA, NFA, regular expressions
- We can construct an automaton that accepts viable prefixes

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LR(0) items

- An LR(0) item of a grammar G is a production of G with a special symbol “.” at some position of the right side
- Thus production $A \rightarrow XYZ$ gives four LR(0) items

$A \rightarrow .XYZ$ *m items on RHS*
 $A \rightarrow X.YZ$ $\Rightarrow n+1$ LR(0) items
 $A \rightarrow XY.Z$
 $A \rightarrow XYZ.$

$A \rightarrow \epsilon$ (*this $n=0$*)
($m=1$ LR(0) items)

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LR(0) items

- An item indicates how much of a production has been seen at a point in the process of parsing
 - Symbols on the left of “.” are already on the stacks
 - Symbols on the right of “.” are expected in the input *directly or indirectly*

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Start state

- Start state of DFA is an empty stack corresponding to $S' \rightarrow S$ item
- This means no input has been seen
- The parser expects to see a string derived from S

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Closure of a state s_k

- Closure** of a state adds items for all productions whose LHS occurs in an item in the state, just after “.”
- Set of possible productions to be reduced next
- Added items have “.” located at the beginning
- No symbol of these items is on the stack as yet

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* we have Y left to be derived
 ⇒ we are keeping track of all rules with Y .
 ⇒ These rules get added to the closure of state s_k .

Example

For the grammar

$$\begin{aligned} E' &\rightarrow E \\ E &\rightarrow E + T \mid T \\ T &\rightarrow T * F \mid F \\ F &\rightarrow (E) \mid id \end{aligned}$$

If I is $\{ E' \rightarrow .E \}$ then closure(I) is

$$\begin{aligned} E' &\rightarrow .E \\ E &\rightarrow .E + T \\ E &\rightarrow .E + T \\ T &\rightarrow .T * F \\ T &\rightarrow .T * F \\ T &\rightarrow .F \\ F &\rightarrow .id \\ F &\rightarrow .(E) \end{aligned}$$

Rules of T are not added now or of F
 All rules of T also to be added
 All rules of F also to be added

No. of rules are finite.

⇒ closure is finite

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Closure operation

- Let I be a set of items for a grammar G
- $\text{closure}(I)$ is a set constructed as follows:
 - Every item in I is in closure (I)
 - If $A \rightarrow \alpha.B\beta$ is in closure(I) and $B \rightarrow \gamma$ is a production then $B \rightarrow .\gamma$ is in closure(I)
- Intuitively $A \rightarrow \alpha.B\beta$ indicates that we expect a string derivable from $B\beta$ in input
- If $B \rightarrow \gamma$ is a production then we might see a string derivable from γ at this point

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I : has been
computed from
some closure

(I : can be any set)

Goto operation of items

- $\text{Goto}(I, X)$, where I is a set of items and X is a grammar symbol,
 - is closure of set of item $A \rightarrow \alpha X \beta$
 - such that $A \rightarrow \alpha X \beta$ is in I

Prefix $\alpha \rightarrow I$

Prefix $\alpha X \rightarrow \text{goto}(I, X)$

- Intuitively if I is a set of items for some valid prefix α then $\text{goto}(I, X)$ is set of valid items for prefix αX

$\text{Goto}(I, X)$: I can be any set (General defn)

But parsers will use with $I = \text{closure of state}$ ¹³

Closure $\rightarrow \text{Goto} \rightarrow \text{Closure} \dots$

Sets of items

No. of productions = m
Max symbols on RHS = K

C : Collection of sets of LR(0) items for grammar G'

$C = \{\text{closure}(\{S' \rightarrow .S\})\}$

repeat

for each set of items I in C

for each grammar symbol X

if $\text{goto}(I, X)$ is not empty and not in C

ADD $\text{goto}(I, X)$ to C

until no more additions to C

Finite repetitions
 \Rightarrow Terminates

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Closure: lists what we can see next

Goto operation

If I is $\{E' \rightarrow E, E \rightarrow E, + T\}$ then
 $\text{goto}(I, +)$ is $.T \beta = T$

$E \rightarrow E + .T \rightarrow$ Rules of T added
 $T \rightarrow .T^* F$
 $T \rightarrow .F \rightarrow$ Rules of F added
 $F \rightarrow .(E)$
 $F \rightarrow .id$

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Example

Grammar:
 $E' \rightarrow E$
 $E \rightarrow E + T \mid T$
 $T \rightarrow T^* F \mid F$
 $F \rightarrow .(E) \mid id$

I_0 : closure($E' \rightarrow .E$)
 $E' \rightarrow .E$
 $E \rightarrow .E + T$
 $E \rightarrow .T$
 $T \rightarrow .T^* F$
 $T \rightarrow .F$
 $F \rightarrow .(E)$
 $F \rightarrow .id$

I_1 : $\text{goto}(I_0, E) \rightarrow .E, \beta, S$
 $E' \rightarrow .E$
 $E \rightarrow .E + T$

I_2 : $\text{goto}(I_0, T) \rightarrow .T \beta$
 $E \rightarrow .T$
 $T \rightarrow .T^* F$

I_3 : $\text{goto}(I_0, F) \rightarrow .F$

I_4 : $\text{goto}(I_0, .) \rightarrow .F$
 $F \rightarrow .(E)$
 $E \rightarrow .E + T$
 $E \rightarrow .T$
 $T \rightarrow .T^* F$
 $T \rightarrow .F$
 $F \rightarrow .(E)$
 $F \rightarrow .id$

I_5 : $\text{goto}(I_0, id) \rightarrow .id$

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$\Rightarrow E \rightarrow T \rightarrow T^* F = I_2$
 $\Rightarrow T \rightarrow F = I_3$
 $\Rightarrow E \rightarrow T \rightarrow T^* F \rightarrow T \rightarrow F = I_4$
 $\Rightarrow T \rightarrow F = I_5$

$I_6: \text{goto}(I_1, +)$
 $E \rightarrow E + .T$
 $T \rightarrow .T * F$
 $T \rightarrow .F$
 $F \rightarrow .(E)$
 $F \rightarrow .id$

$I_7: \text{goto}(I_2, *)$
 $T \rightarrow T * .E$
 $F \rightarrow .(E)$
 $F \rightarrow .id$

$I_8: \text{goto}(I_4, E)$
 $F \rightarrow (E.)$
 $E \rightarrow E. + T$

$\text{goto}(I_4, T) \text{ is } I_2$
 $\text{goto}(I_4, F) \text{ is } I_3$
 $\text{goto}(I_4, ()) \text{ is } I_4$
 $\text{goto}(I_4, id) \text{ is } I_5$

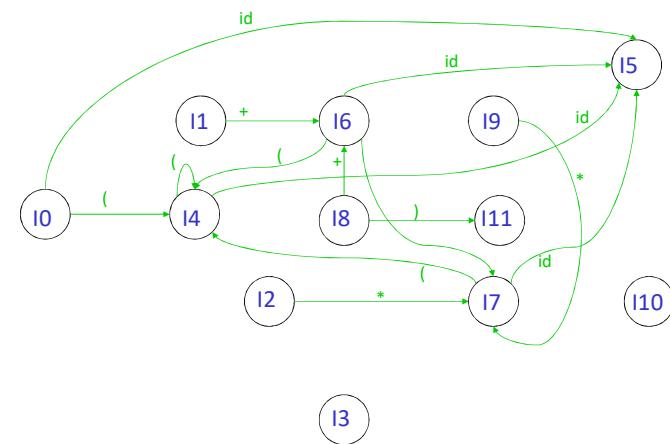
$I_9: \text{goto}(I_6, T)$
 $E \rightarrow E + T.$
 $T \rightarrow T. * F$
 $\text{goto}(I_6, F) \text{ is } I_3$
 $\text{goto}(I_6, ()) \text{ is } I_4$
 $\text{goto}(I_6, id) \text{ is } I_5$

$I_{10}: \text{goto}(I_7, F)$
 $T \rightarrow T * F.$
 $\text{goto}(I_7, ()) \text{ is } I_4$
 $\text{goto}(I_7, id) \text{ is } I_5$

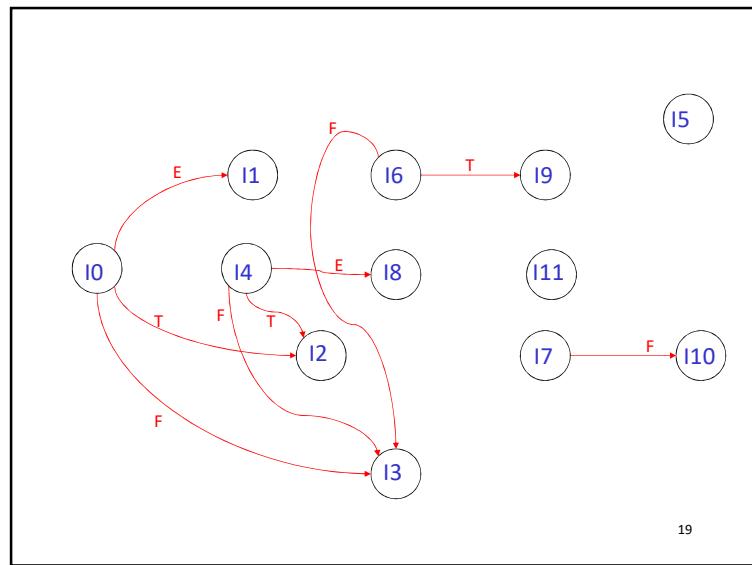
$I_{11}: \text{goto}(I_8,)$
 $F \rightarrow (E.)$
 $\text{goto}(I_8, +) \text{ is } I_6$
 $\text{goto}(I_9, *) \text{ is } I_7$

No. of I sets
 $= 12$

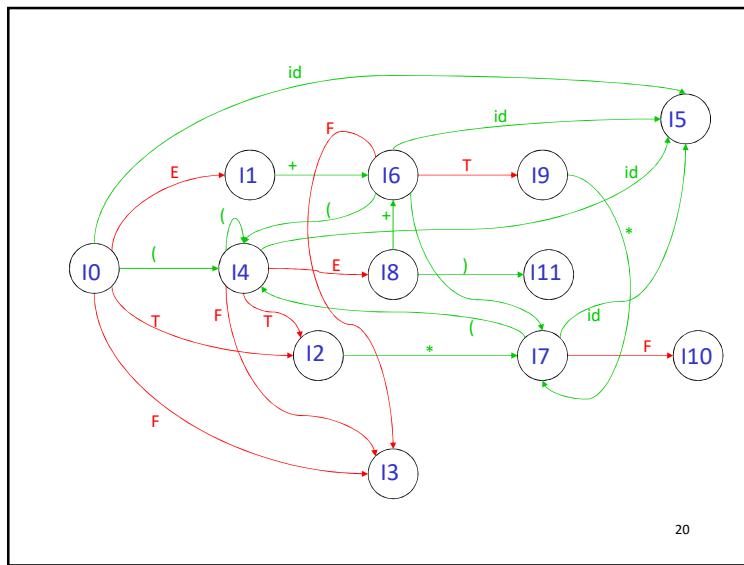
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LR(0) (?) Parse Table

- The information is still not sufficient to help us resolve shift-reduce conflict.

For example the state:

$$I_1: E' \rightarrow E.$$

$$E \rightarrow E. + T$$

- We need some more information to make decisions.

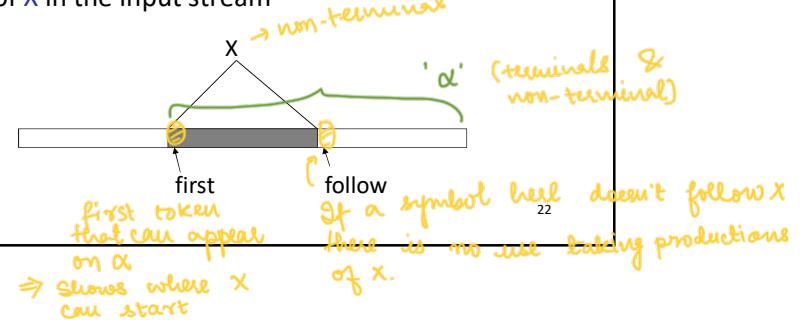
Constructing parse table

- $\text{First}(\alpha)$ for a string of terminals and non terminals α is

– Set of symbols that might begin the fully expanded (made of only tokens) version of α .

- $\text{Follow}(X)$ for a non terminal X is

– set of symbols that might follow the derivation of X in the input stream



Compute first sets

- If X is a terminal symbol then $\text{first}(X) = \{X\}$
- If $X \rightarrow \epsilon$ is a production then ϵ is in $\text{first}(X)$
- If X is a non terminal and $X \rightarrow Y_1 Y_2 \dots Y_k$ is a production, then
 - if for some i , a is in $\text{first}(Y_i)$ and ϵ is in all of $\text{first}(Y_j)$ (such that $j < i$) then a is in $\text{first}(X)$
- If ϵ is in $\text{first}(Y_1) \dots \text{first}(Y_k)$ then ϵ is in $\text{first}(X)$
- Now generalize to a string α of terminals and non-terminals.

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Set contains only tokens

Example

- For the expression grammar

$$E \rightarrow TE' \quad E' \rightarrow +TE' \mid \epsilon$$

$$T \rightarrow FT' \quad T' \rightarrow *FT' \mid \epsilon$$

$$F \rightarrow (E) \mid \text{id}$$

$$T \not\rightarrow E (\text{first } E' \text{ not included})$$

$$\text{First}(E) = \text{first}(T)$$

$$= \text{First}(F)$$

$$= \{ , , \text{id} \}$$

$$\text{First}(E') = \{ +, \epsilon \}$$

$$\text{First}(T') = \{ *, \epsilon \}$$

$$\text{First}(E) = \text{First}(T) = \text{First}(F) = \{ , \text{id} \}$$

$$\text{First}(E') = \{ +, \epsilon \}$$

$$\text{First}(T') = \{ *, \epsilon \} * \text{First}(EF) = \{ +, , \text{id} \}$$

$$\text{First}(T') = \{ *, \epsilon \}$$

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Whatever is in first(β) Compute follow sets
can follow(B).

1. Place \$ in follow(S) // S is the start symbol
 2. If there is a production $A \rightarrow \underline{\alpha}B\underline{\beta}$
then everything in first(β) (except ϵ) is in follow(B)
 3. If there is a production $A \rightarrow \alpha B \beta$ and first(β) contains ϵ
then everything in follow(A) is in follow(B)
 4. If there is a production $A \rightarrow \alpha B$
then everything in follow(A) is in follow(B)
- Last two steps have to be repeated until the follow sets converge.

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Example

- For the expression grammar

$$\begin{aligned} E &\rightarrow T E' \\ E' &\rightarrow + T E' \mid \epsilon \\ T &\rightarrow F T' \\ T' &\rightarrow * F T' \mid \epsilon \\ F &\rightarrow (E) \mid \text{id} \end{aligned}$$

Doubt

$$\begin{aligned} \text{follow}(E) &= \text{follow}(E') = ? \quad \{ , \}, \$ \} \\ \text{follow}(T) &= \text{follow}(T') = ? \quad \text{first}(E') \Rightarrow \{ +, \}, \$ \} \\ \text{follow}(F) &= ? \quad \text{first}(T') = ? \quad \{ *, * \} \end{aligned}$$

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Construct SLR parse table

- Construct $C = \{I_0, \dots, I_n\}$ the collection of sets of LR(0) items
- If $A \rightarrow \underline{\alpha}.\underline{a}\beta$ is in I_i and $\text{goto}(I_i, a) = I_j$
then $\text{action}[i, a] = \text{shift } j$
- If $A \rightarrow \alpha.$ is in I_i
then $\text{action}[i, a] = \text{reduce } A \rightarrow \alpha$ for all a in follow(A)
- If $S \rightarrow S.$ is in I_i then $\text{action}[i, \$] = \text{accept}$
- If $\text{goto}(I_i, A) = I_j$ then $\text{goto}[i, A] = j$ for all non terminals A
- All entries not defined are errors

here handle is
 $A \rightarrow \alpha$ only if
 $a \in \text{follow}(A)$
we are not
blindly reducing

Doubts

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Practice Assignment

Construct SLR parse table for following grammar

$$E \rightarrow E + E \mid E - E \mid E * E \mid E / E \mid (E) \mid \text{digit}$$

Show steps in parsing of string
 $9 * 5 + (2 + 3 * 7)$

- Steps to be followed
 - Augment the grammar
 - Construct set of LR(0) items
 - Construct the parse table
 - Show states of parser as the given string is parsed

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Notes

- This method of parsing is called SLR (Simple LR)
- LR parsers accept LR(k) languages
 - L stands for left to right scan of input
 - R stands for rightmost derivation
 - k stands for number of lookahead token
- SLR is the simplest of the LR parsing methods.
SLR is too weak to handle most languages!
- If an SLR parse table for a grammar does not have multiple entries in any cell then the grammar is unambiguous.
- All SLR grammars are unambiguous
- ~~• Are all unambiguous grammars in SLR?~~

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Does Conflict => Ambiguity?

No.

Example

- Consider following grammar and its SLR parse table:

$$\begin{array}{l} \text{unambiguous} \\ S' \rightarrow S \\ | \\ S \rightarrow L = R \\ S \rightarrow R \\ L \rightarrow *R \\ L \rightarrow id \\ R \rightarrow L \end{array}$$

$$\begin{array}{l} I_1: \text{goto}(I_0, S) \\ S' \rightarrow S. \quad] \text{Accept state.} \end{array}$$

$$\begin{array}{l} I_2: \text{goto}(I_0, L) \\ S \rightarrow L.=R \\ R \rightarrow L. \end{array}$$

$$\begin{array}{l} I_0: S' \rightarrow .S \\ S \rightarrow .L=R \\ S \rightarrow .R \\ L \rightarrow .*R \\ L \rightarrow .id \\ R \rightarrow .L \end{array}$$

Assignment (not to be submitted): Construct rest of the items and the parse table.

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SLR parse table for the grammar

	=	*	id	\$	S	L	R
0		s4	s5		1	2	3
1				acc			
2		s6,r6			r6		
3					r3		
4		s4	s5			8	7
5	r5				r5		
6		s4	s5			8	9
7	r4				r4		
8	r6				r6		
9					r2		

The table has multiple entries in action[2,=]

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- There is both a shift and a reduce entry in action[2,=]. Therefore state 2 has a shift-reduce conflict on symbol "=", However, the grammar is not ambiguous.
- Parse id=id assuming reduce action is taken in [2,=]

Stack	input	action
0	id=id	shift 5
0 id 5	=id	reduce by $L \rightarrow id$
0 L 2	=id	reduce by $R \rightarrow L$
0 R 3	=id	error

Reduce path is wrong

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- if shift action is taken in [2,=]

Stack	input	action
0	id=id\$	shift 5
0 id 5	=id\$	reduce by $L \rightarrow id$
0 L 2	=id\$	shift 6
0 L 2 = 6	id\$	shift 5
0 L 2 = 6 id 5	\$	reduce by $L \rightarrow id$
0 L 2 = 6 L 8	\$	reduce by $R \rightarrow L$
0 L 2 = 6 R 9	\$	reduce by $S \rightarrow L=R$
0 S 1	\$	ACCEPT

* Even though the parser is not sure whether to shift/reduce, grammar doesn't have 2 derivations for the same thing \Rightarrow Unambiguous

SLR can't do this

Another look at the grammar

- No sentential form of this grammar can start with $R=...$
 - However, the reduce action in action[2,=] generates a sentential form starting with $R=$
 - Therefore, the reduce action is incorrect
- $S' \rightarrow S$
 $S \rightarrow L = R$
 $S \rightarrow R$
 $L \rightarrow *R$
 $L \rightarrow id$
 $R \rightarrow L$

Problems in SLR parsing

- In SLR parsing method state i calls for reduction on symbol "a", by rule $A \rightarrow \alpha$ if i contains $[A \rightarrow \alpha]$ and "a" is in $\text{follow}(A)$
- However, when state i appears on the top of the stack, the viable prefix $\beta\alpha$ on the stack may be such that βA can not be followed by symbol "a" in any right sentential form.
- Thus, the reduction by the rule $A \rightarrow \alpha$ on symbol "a" is invalid
- SLR parsers cannot remember the left context

* By looking at β , we should figure out which $a \in \text{follow}(A)$ we can reduce

+ SLR parser are not powerful enough to detect that