Lecture 6.

DFA $M = (Q, \Sigma, \delta, s, F)$

$L(M) = \{ x \in \Sigma^* \mid \hat{\delta}(s, x) \in F \}$

$A \subseteq \Sigma^*$ is regular if $\exists M$ s.t $L(M) = A$.

Closure Properties.

Nondeterminism. Next state of the computation is not uniquely defined.

Combinatorial problems with "efficient" nondeterministic algorithm.

but no known "efficient" deterministic algorithm.

SAT - Given $\alpha \in Bool(P)$, is α satisfiable?

↳ does $\exists v$ s.t $v \models \alpha$?

$$P \stackrel{?}{=} NP$$

Clique - subset of vertices where all vertices are adjacent to each other.

Decision Problem. Input: Undirected graph G + k - a number.

Output $\begin{cases} \text{Yes} - \text{if } G \text{ has a clique of size } k. \\ \text{No} - \text{otherwise}. \end{cases}$

Nondeterministic Finite State automata - NFA.

NFA - next state is not uniquely defined

- Set of start states.

DFA

$M = (Q, \Sigma, \delta, s, F)$

$\delta : Q \times \Sigma \to Q$

Input to an NFA - $x \in \Sigma^*$.

Run of NFA on x ?

x is accepted by an NFA.

Language of an NFA.

$x = a \quad b \quad a \quad a \quad b$

$s \; q^1 q^2 \ldots \; .$

NFA - N, Input - x.

N on x

$N = (Q, \Sigma, \Delta, S, F)$

Q - Finite set of states, Σ - alphabet set.

$F \subseteq Q$ - Set of final states. $\quad S \subseteq Q$ - Set of start states.

DFA: $\delta : Q \times \Sigma \to Q$. / $\Delta : Q \times \Sigma \to 2^Q$

$2^Q = \{A \mid A \subseteq Q\}$.

$\Delta(p,a)$ - Set of all states that N can possibly move to from p under input symbol a.

$\Delta(p,a) = \phi$ - possible.

$\Delta \mapsto \hat{\Delta}$ $\qquad \Delta : Q \times \Sigma \to 2^Q$.

$\hat{\Delta} : 2^Q \times \Sigma^* \to 2^Q$

$$\hat{\Delta}(A, \epsilon) = A$$

$$\hat{\Delta}(A, xa) = \bigcup_{q \in \hat{\Delta}(A,x)} \Delta(q,a).$$

$\hat{\Delta}(A,x)$ - All states reachable under input x from some state in A.

$q \in \hat{\Delta}(A, xa)$ if $\exists\, p' \in \hat{\Delta}(A,x)$ s.t $q \in \Delta(p', a)$.

$$p \xrightarrow{x} p' \xrightarrow{a} q$$

N accepts $x \in \Sigma^*$ if $\hat{\Delta}(S,x) \cap F \neq \phi$

if $\exists\, q \in F$ s.t q is reachable from a start state under string x $(q \in \hat{\Delta}(S,x))$.

$L(N) = \{x \in \Sigma^* \mid N \text{ accepts } x\}$.