DFA NFA

$A \subseteq \Sigma^*$ is regular if $\exists$ a DFA M s.t $L(M) = A$.

For every NFA N, $\exists$ DFA M s.t $L(N) = L(M)$.
[Subset construction]

$A \subseteq \Sigma^*$ is regular if $\exists$ a NFA N s.t $L(N) = A$.

NFA $\Longrightarrow$ ϵ-transitions.

ls $\underbrace{*.\text{pdf}}_{\text{pattern } \gamma}$ - list all files if pdf extension.

$$L(\gamma) = \{x \mid x \text{ matches } \gamma\}.$$

Atomic patterns. + Compound patterns.

ls a   ls $\overline{abc}$ ls $\overline{*.\text{pdf}}$

Σ - alphabet set.

Σ-alphabet.

Atomic Pattern

Syntax.	Semantics.
	$L(\gamma)$
- $a \in \Sigma$	$L(a) = \{a\}$
- ϵ	$L(\epsilon) = \{\epsilon\}$.
- ϕ	$L(\phi) = \phi$ - matches nothing.
· $\#$	$L(\#) = \Sigma$
· $@$	$L(@) = \Sigma^*$

Compound patterns.

Syntax	Semantics.
α, β - patterns.	
$\alpha + \beta$	$L(\alpha+\beta) = L(\alpha) \cup L(\beta)$
$\alpha \cap \beta$	$L(\alpha \cap \beta) = L(\alpha) \cap L(\beta)$
$\alpha\beta$	$L(\alpha\beta) = \{yz \mid y \in L(\alpha) \text{ and } z \in L(\beta)\}$ $\underbrace{L(\alpha)L(\beta)}_{} \rightarrow \subseteq \Sigma^*$
α^*	$L(\alpha^*) = L(\alpha)^0 \cup L(\alpha)^1 \cup L(\alpha)^2 \cup \cdots$ $L(\alpha^*) = \{x_1 x_2 \cdots x_n \mid n \geq 0, x_i \in L(\alpha), 1 \leq i \leq n\}$
α^+	$L(\alpha^+) = L(\alpha)^1 \cup L(\alpha)^2 \cup \cdots$ $= L(\alpha)^+$
$\neg\alpha$	$L(\neg\alpha) = \overline{L(\alpha)} = \Sigma^* - L(\alpha)$.

Examples.

$$\Sigma^* = L(@) = L(\underbrace{\#^*}_{})$$

$\hookrightarrow$ atomic $\hookrightarrow$ Compound.

$$a \in \Sigma = L(a) = \{a\}.$$

- $\alpha = @a@a@a@$ $L(\alpha) = \{w \in \Sigma^* \mid w$ has at least 3 occurences of $a\}$.
- $\alpha = @a@b@$ - $xaybz$ $x, y, z \in \Sigma^*$.
- Strings with no occurence of the symbol a.

$$(\# \cap \neg a)^*$$

Questions.

- Given string x and α, how hard is it to determine if $x \in L(\alpha)$.

- Can you represent every set by some pattern.

- α and β are equivalent if $L(\alpha) = L(\beta)$. Check for equivalence of patterns.

- $\epsilon \equiv \neg(\# @)$
  $@ \equiv \#^*$
  $\alpha^+ \equiv \alpha\alpha^*$
  $\# \equiv a_1 + a_2 + \cdots + a_n \quad : \Sigma = \{a_1, a_2, \ldots a_n\}$

De Morgan laws: $\alpha \cap \beta \equiv \neg(\neg\alpha + \neg\beta)$

Regular Expression. ~~Regut~~

Atomic Pattern

$a \in \Sigma$

ϵ

ϕ

Compound Pattern.

$+$

$\cdot$

$*$

Pattern using the above symbols are called Regular Expressions.

Theorem. Let $A \subseteq \Sigma^*$. The following statements are equivalent.

1) A is regular. $\exists$ a finite automaton M s.t $L(M) = A$.

2) $A = L(\alpha)$ for some pattern α.

3) $A = L(\alpha)$ for some regular expression α.

Proof. $3 \Rightarrow 2$ is trivial

$2 \Rightarrow 1$

$1 \Rightarrow 3$ $\quad M$ to regular expression α.