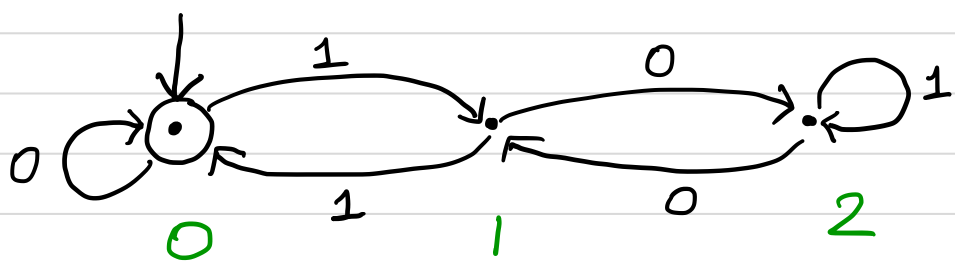
$A = \{ x \in \{0,1\}^* \mid x$ represents a multiple of three in binary $\}$.

- leading zeros permitted.
- ϵ - represents 0.

A is regular. Construct a DFA M s.t $L(M) = A$.

$$\in A \begin{cases} 0 & 0 \\ 11 & 3 \\ 110 & 6 \\ 1001 & 9 \\ 1100 & 12 \end{cases}$$



$\Rightarrow M$

Theorem: $L(M) = A$.

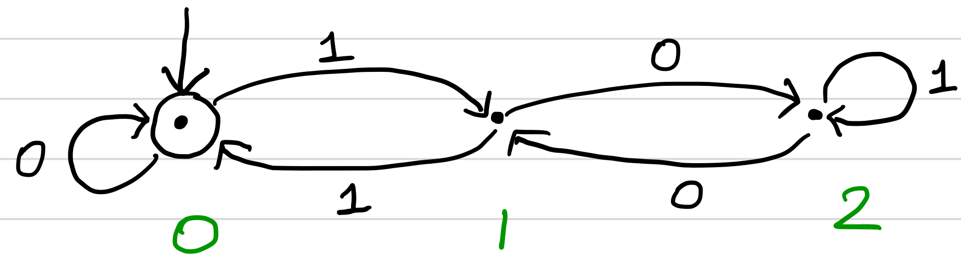
Let $\# x$ - the number represented by string x in binary.

$\# \epsilon = 0$

$\# 0 = 0$

$\# 11 = 3$

$A = \{x \in \{0,1\}^* \mid x \text{ represents a multiple of three in binary}\}.$



$\rightarrow M$

For any $x \in \{0,1\}^*$

$\hat{\delta}(0, x) = 0$ iff $\# x \equiv 0 \bmod 3$

$\hat{\delta}(0, x) = 1$ iff $\# x \equiv 1 \bmod 3$

$\hat{\delta}(0, x) = 2$ iff $\# x \equiv 2 \bmod 3$

$$\hat{\delta}(0, x) = \# x \bmod 3. \leftarrow \text{To prove}$$

$$\left.\begin{aligned} \#(x0) &= 2(\# x) + 0 \\ \#(x1) &= 2(\# x) + 1 \end{aligned}\right\} \overbrace{\#(xc) = 2(\# x) + c}^{B} \quad \forall c \in \{0,1\}.$$

For all $q \in \{0,1,2\}$ and symbol $c \in \{0,1\}$.

$$\delta(q, c) = (2q + c) \bmod 3. \quad \text{— } A$$

Induction on $|x|$.

Base case. $x = \epsilon$.

$\hat{\delta}(0, \epsilon) = 0$ [def. of $\hat{\delta}$]

$= \# \epsilon$

$= \# \epsilon \bmod 3.$

$\hat{\delta}(0, xc) = \delta(\hat{\delta}(0, x), c)$ [Def of $\hat{\delta}$]

$= \delta(\# x \bmod 3, c)$ [IH]

$= (2(\# x \bmod 3) + c) \bmod 3$ [From A].

$= (2(\# x) + c) \bmod 3.$

$= \# xc \bmod 3.$ [From B]