CS685: Data Mining
Basics of Classification

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Classification

- A dataset of n objects $O_i, i = 1, \dots, n$
- A total of k classes $C_j, j = 1, \dots, k$
- Each object belongs to a *single* class
- If object O_i belongs to class C_j, then $C(O_i) = j$
- Given a new object O_q, classification is the problem of determining its class, i.e., $C(O_q)$ out of possible k choices

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- Given a new object O_q, classification is the problem of determining its class, i.e., $C(O_q)$ out of possible k choices
- If, instead of k discrete classes, there is a continuum of values, the problem of determining the value $V(O_q)$ of a new object O_q is called prediction

Sets

- Total available data is divided *randomly* into two parts: training set and testing set
- Classification algorithm or model is built using *only* the training set
- Testing set should not be used *at all*
- Quality of method is measured using testing set
- Sometimes validation set is separated from training set to evalute method

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  - Training is repeated k times with a new validation set each time
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- Classification is called supervised learning
  - Algorithm or model is "supervised" by class information

Over-Fitting and Under-Fitting

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  - It is too complex or uses too many parameters
  - Generally performs poorly with testing set
  - Ends up modeling noise rather than data characteristics

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- Bias-variance tradeoff
  - Bias measures errors in the model learnt (under-fitting)
  - Variance measures errors when training set is perturbed (over-fitting)
  - Low bias generally implies higher variance and vice versa

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- In statistics,
  - Type I error: FP
  - Type II error: FN

Confusion Matrix

- Confusion matrix visually represents the information
- Rows indicate "true" answers: P and N
- Columns indicate those returned by $\mathcal{A}$: P' and N'
- Shows which error is more

Sets		Returned by $\mathcal{A}$	
		Positives P'	Negatives N'
True answers	Positives P	TP	FN
	Negatives N	FP	TN

Confusion Matrix for Multiple Classes

- Is more useful when extended for multiple classes
- Shows which classes are confused more against which other classes

Sets		Predicted by $\mathcal{A}$		
		Class C'_1	Class C'_2	Class C'_3
True answers	Class C_1	5	3	0
	Class C_2	2	3	1
	Class C_3	0	2	9

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False negative rate	Proportion of positives not returned by $\mathcal{A}$	$\frac{\lvert FN \rvert}{\lvert TP \cup FN \rvert} = \frac{\lvert FN \rvert}{\lvert P \rvert}$
Accuracy	Proportion of positives returned and negatives not returned by $\mathcal{A}$	

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Accuracy	Proportion of positives returned and negatives not returned by $\mathcal{A}$	$\frac{\lvert TP \cup TN\rvert}{\lvert D\rvert}$
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- EER or Equal Error Rate is when FP rate is equal to FN rate

Weighting Precision versus Recall

- Suppose recall and precision are weighted at a ratio $\alpha : (1 - \alpha)$
- F-score is the *weighted harmonic mean*

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  - $\alpha \in [0,1]$ while $\beta \in [0,\infty]$
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  - $\beta > 1$ emphasizes precision, while $\beta < 1$ emphasizes recall
- Using β^2, weighted F-score is

$$F = \frac{(\beta^2 + 1).P.R}{\beta^2.P + R}$$

- When $\beta = 1$, precision and recall are equally weighted ($\alpha = 1/2$)
- F1-score is the harmonic mean

Example

$$\begin{aligned} D =& \{O_1, O_2, O_3, O_4, O_5, O_6, O_7, O_8\} \\ \text{Correct answer set} =& \{O_1, O_5, O_7\} \\ \text{Algorithm returns} =& \{O_1, O_3, O_5, O_6\} \end{aligned}$$

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\text{Algorithm returns} =& \{O_1, O_3, O_5, O_6\} \\
\therefore \qquad P &= \{O_1, O_5, O_7\} \\
N &= \{O_2, O_3, O_4, O_6, O_8\} \\
TP &= \{O_1, O_5\} \\
TN &= \{O_2, O_4, O_8\} \\
FP &= \{O_3, O_6\} \\
FN &= \{O_7\}
\end{aligned}
$$

$$
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\therefore \qquad \text{Recall} = \text{Sensitivity} &= 2/3 = 0.67 \\
\text{Precision} &= 2/4 = 0.5 \\
\text{Specificity} &= 3/5 = 0.6 \\
\text{F-score} &= \\
\text{Accuracy} &= \\
\text{Error rate} &=
\end{aligned}
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\text{Specificity} &= 3/5 = 0.6 \\
\text{F-score} &= 4/7 = 0.571 \\
\text{Accuracy} &= 5/8 = 0.625 \\
\text{Error rate} &=
\end{aligned}
$$

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\therefore \qquad \text{Recall} = \text{Sensitivity} &= 2/3 = 0.67 \\
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\text{Specificity} &= 3/5 = 0.6 \\
\text{F-score} &= 4/7 = 0.571 \\
\text{Accuracy} &= 5/8 = 0.625 \\
\text{Error rate} &= 3/8 = 0.375
\end{aligned}
$$

ROC Curve

- Performance of an algorithm depends on parameters
- To assess over a range of parameters, ROC curve is used
  - 1 - Specificity (x-axis) versus Sensitivity (y-axis)
  - False positive rate (x-axis) versus True positive rate (y-axis)

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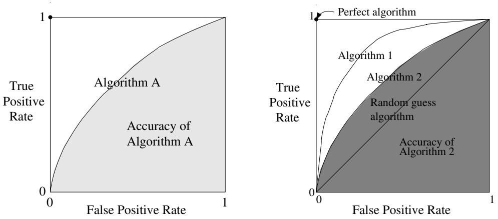
- A random guess algorithm is a 45° line



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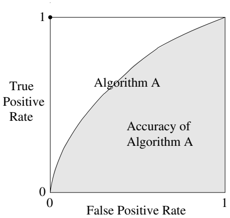
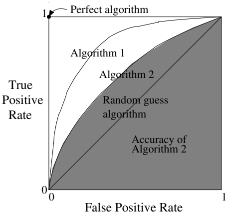
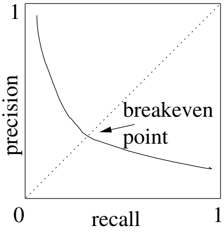




- Area under the ROC curve (AUC or AUROC) measures accuracy (or discrimination)
- What AUC is good?
  - 0.9+: excellent; 0.8+: good; 0.7+: fair; 0.6+: poor; 0.6-: fail
- EER denotes the point in ROC where FP rate is equal to FN rate

Precision-Recall Curve

- Precision versus recall



- Breakeven point where precision is the same as recall