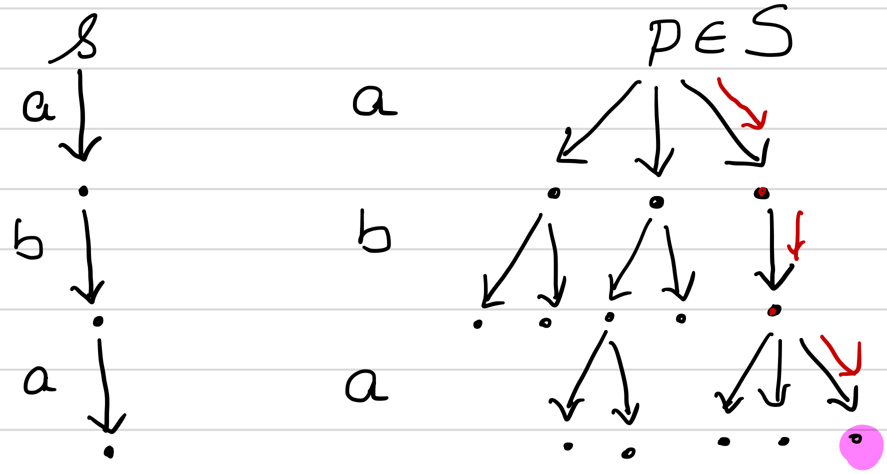
NFA $\quad N = (Q, \Sigma, \Delta, S, F)$

$\Delta : Q \times \Sigma \to 2^Q.$

Input x $\qquad$ DFA - M $\qquad$ NFA - N.

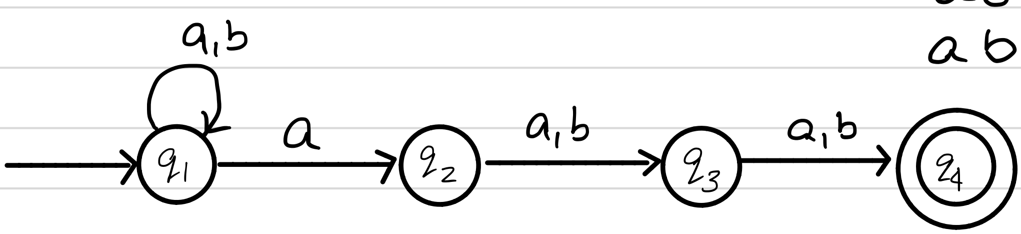
$x = aba$



$A = \{x \in \{a,b\}^* \mid$ the third symbol from right is $a\}$

$ababab b \in A$

$ababbab \notin A$.



$\leftarrow N$

$L(N) = A$.

$L(N) = A$

$M=(Q,\Sigma,\delta,s,F)$ $\quad$ $N=(Q,\Sigma,\Delta,S,F)$

Every DFA M is equivalent to the NFA

$$N=(Q,\Sigma,\Delta,\{s\},F)$$
$$\Delta(p,a)=\{\delta(p,a)\}.$$

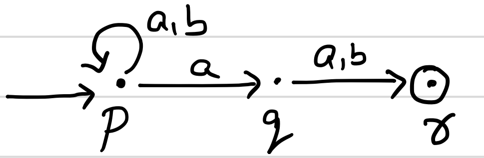
Subset Construction.

$N=(Q_N,\Sigma,\Delta_N,S_N,F_N)$

Construct $M=(Q_M,\Sigma,\delta_M,s_M,F_M)$.

$$Q_M = 2^{Q_N} \cdot \mathcal{P}(Q_N).$$
$$\delta_M(A,a)=\hat{\Delta}_N(A,a)$$
$$s_M = S_N$$
$$F_M=\{A\subseteq Q_N \mid A\cap F_N\neq\emptyset\}$$

$A=\{x\in\{a,b\}^* \mid \text{the second symbol from the right is } a\}.$



$\}N$

$Q_M=\{\emptyset,\{p\},\{q\},\{r\},$
$\{p,q\},\{p,r\},\{q,r\},$
$\{p,q,r\}\}.$

Lemma 1. For any $x, y \in \Sigma^*$, and $A \subseteq Q$.

$$\hat{\Delta}(A, xy) = \hat{\Delta}(\hat{\Delta}(A, x), y).$$

[Proof: by induction on $|y|$].

Lemma 2. For any $A \subseteq Q_N$, and $x \in \Sigma^*$

$$\hat{\delta}_M(A, x) = \hat{\Delta}_N(A, x)$$

Induction on $|x|$.

Base Case: $x = \epsilon$ $\quad \hat{\delta}_M(A, \epsilon) = A \quad \hat{\Delta}_N(A, \epsilon) = A$.

Induction Step.

$$\hat{\delta}_M(A, xa) = \delta_M(\hat{\delta}_M(A, x), a) \text{ [defn. of } \hat{\delta}_M]$$

$$= \delta_M(\hat{\Delta}_N(A, x), a) \text{ [Induction hypothesis]}$$

$$= \hat{\Delta}_N(\hat{\Delta}_N(A, x), a) \text{ [defn of } \delta_M)$$

$$= \hat{\Delta}_N(A, xa) \text{ [Lemma 1].}$$

Theorem. $L(M) = L(N)$.

For $x \in \Sigma^*$ $\quad x \in L(M) \iff \hat{\delta}_M(s_M, x) \in F_M$ [acceptance]

$$\iff \hat{\Delta}_N(S_N, x) \cap F_N \neq \emptyset \text{ [Lemma 2]}$$

$$\iff x \in L(N) \text{ [acceptance for N].}$$