CS685: Data Mining
Decision Trees and Rule-Based Learners

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Decision Trees

- A decision tree is a tree structure used for classification
- Each *internal node* represents a test on an attribute
- Each *branch* represents an outcome of the test
- Each *leaf* represents a class outcome
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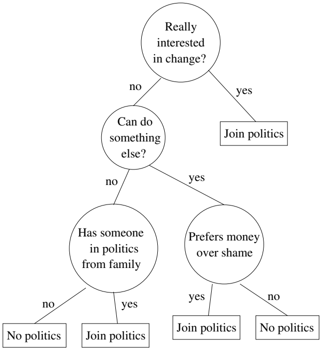

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- Decision tree building is top-down and backtracking is *not* allowed

Information Gain

- Entropy impurity or information impurity

$$info(D) = -\sum_{i=1}^{k}(p_i \log_2 p_i)$$

- For n partitions into $D_1, \ldots, D_n$, denoted by S

$$info_S(D) = \sum_{j=1}^{n}(|D_j|/|D|)info(D_j)$$

- Information gain is

$$gain_S(D) = info(D) - info_S(D)$$

- More the gain, better the split
- Choose attribute and split point that maximizes gain

Gini Index

- Variance impurity for two classes

$$var(D) = p_1.p_2$$

- For k classes, generalized to Gini index or Gini impurity

$$gini(D) = \sum_{i=1}^{k} \sum_{j=1, j \neq i}^{k} p_i.p_j = 1 - \sum_{i=1}^{k} p_i^2$$

- For n partitions into $D_1, \dots, D_n$, denoted by S

$$gini_S(D) = \sum_{j=1}^{n} (|D_j|/|D|) gini(D_j)$$

- Less the gini index, better the split
- Choose attribute and split point that minimizes gini index

Classification Error

- Classification error or misclassification index

$$class(D) = 1 - \max_i p_i$$

- This is the *probability of misclassification* when no more split is done and majority voting is used
- Find reduction in impurity by splitting

$$class(D) - class_S(D) = class(D) - \sum_{j=1}^{n} (|D_j|/|D|) class(D_j)$$

- More the reduction in impurity, better the split
- Choose attribute and split point that maximizes reduction

Gain Ratio

- Most impurity measures are biased towards multiway splits
- Higher chance that a node becomes purer
- Gain ratio counters it
- For n partitions into $D_1, \dots, D_n$, denoted by S
- Split information is defined as

$$splitinfo_S(D) = -\sum_{j=1}^{n} (|D_j|/|D|) \log_2(|D_j|/|D|)$$

- Similar to information measure, although just uses the number of objects in each partition and *not* any class information
- This is used to normalize information gain

$$gainratio_S(D) = gain_S(D)/splitinfo_S(D)$$

- Higher the gain ratio, better the split
- Choose attribute and split point that maximizes gain ratio

Example

- D with 60 C_1 and 40 C_2
    - D_1 with 50 C_1 and 5 C_2
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  - D: $40/100 = 0.40$
  - D_1: $5/55 = 0.09$ and D_2: $10/45 = 0.22$

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  - Split D': $0.09 \times (55/100) + 0.22 \times (45/100) = 0.15$

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Measure	D		D_1		D_2		D'	Gain
	C_1	C_2	C_1	C_2	C_1	C_2		
# Points	60	40	50	5	10	35		
Entropy	0.97		0.44		0.76		0.58	0.39
Gini Index	0.48		0.16		0.34		0.24	0.24
Mis-classification	0.40		0.09		0.22		0.15	0.45
Split Info	-		-		-		0.99	-
Gain Ratio	0.48		0.16		0.34		$0.24/0.99$	0.24

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- Why not polythetic trees where decisions are based on multiple attributes?
  - Theoretically possible but practically too complex

Variants of Decision Trees

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- ID3 (from Iterative Dichotomiser generation 3)
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- CART (from Classification and Regression Trees)
  - Binary split
  - Uses gini index

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- Tends to correct overfitting of decision trees

Rules

- Rules are of the form

  `if` condition `then` class

- *condition* is a conjunct (i.e., logical AND) of tests on *single* attributes
- If the condition holds, then the object is said to be from *class*
- *condition* is called antecedent or precondition
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- Example: `if` in family = yes AND can do something = no `then` politics
- Two important parameters of a rule
  - Coverage: Number of objects the rule applies to

    $$coverage = |covers|/|D|$$

  - Accuracy: Number of correctly classified objects when rule is applied

    $$accuracy = |correct|/|covers|$$

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- *Rule-based ordering*: Priority list according to some function based on coverage, accuracy and size
- For a query tuple, the rule that satisfies it is invoked
- If no such rule, then a *default* rule is invoked: `if` () `then` class i
  - Class i is the most abundant class

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- Rules are ordered according to their order of inception
- Variants are AQ, CN2 and RIPPER

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- New rule R_2 has a_2 as antecedent
- Let the number of tuples covered by a rule be denoted by D_i
- For the particular class in question, p_i is the number of tuples correctly classified, i.e., the consequent is this class
- Correspondingly, n_i is the number of negative tuples

$$D_i = p_i + n_i$$

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- Four rule quality measures

FOIL Gain

- FOIL_Gain measure proposed as part of the sequential covering algorithm First Order Inductive Learner (FOIL) used in RIPPER

$$FOIL\_Gain(R_1 \rightarrow R_2) = p_2 \times \left(\log_2 \frac{p_2}{D_2} - \log_2 \frac{p_1}{D_1} \right)$$

- Considers both coverage and accuracy

Likelihood Ratio

- Statistical test using the likelihood ratio statistic

$$LR = 2 \sum_{i=1}^{m} f_i \log \frac{f_i}{e_i}$$

  where m is the number of classes, f_i and e_i are the observed and expected frequencies of tuples in each class
- LR statistic has a chi-square distribution with $m - 1$ degrees of freedom
- The larger the statistic, the more deviated it is from the random rule, and thus, the better

Entropy

- Entropy: Rule with less entropy is better

M-Estimate

- M-estimate measure considers the number of classes as well

$$m\text{-}estimate = \frac{p_i + m.c_i}{D_i + m}$$

  where m is the number of classes and c_i is the prior probability of class C_i
- If the prior probabilities are not known, replacing it by $1/m$ yields the Laplacian estimate

$$Laplacian = \frac{p_i + 1}{D_i + m}$$

- The larger the estimate, the better is the rule

Rule Pruning

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- *Specific-to-general* strategy first learns the most specific rule and then prunes the antecedent
- This is rule pruning
- Each training instance starts as a rule
- From a rule R_1, an antecedent is removed to yield rule R_2
- Measure of rule quality is FOIL_Prune

$$FOIL\_Prune = \frac{p_i - n_i}{D_i}$$

- If this measure is higher for R_2, then pruning is applied

Frequent Pattern-Based Classifier

- Uses the idea of frequent patterns from association rule mining
- Assume a frequent pattern $\{A, B, C\}$
- Suppose C is a class label
- Suppose confidence of the rule $\{A, B\} \implies C$ is high
- Then presence of $\{A, B\}$ is a good rule for classifying into class C

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- This frequent pattern-based mining can be done for each class

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- Rules have an interpretation and can lead to descriptive models
- Can handle imbalance in class distribution very well