Lecture 26

Ambiguous Grammar.

Consider the grammar $S \to S+S \mid S*S \mid (S) \mid A$
$A \to a \mid b$

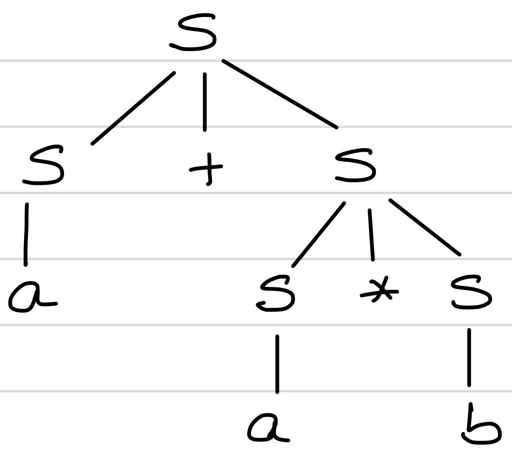
Consider the string $a+a*b$

Derivation 1. $S \to S+S \to a+S \to a+S*S \to a+a*b$

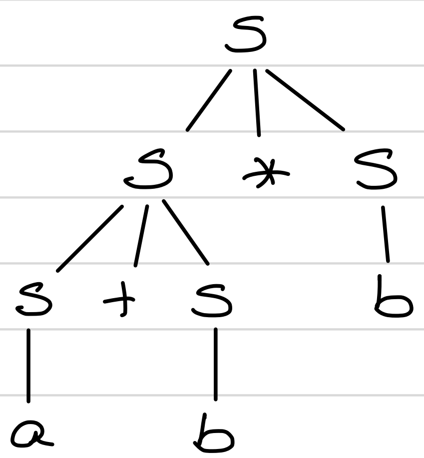
Derivation 2. $S \to S*S \to S+S*S \to a+S*S \to a+a*b$

Parse Tree

Derivation 1



Derivation 2.



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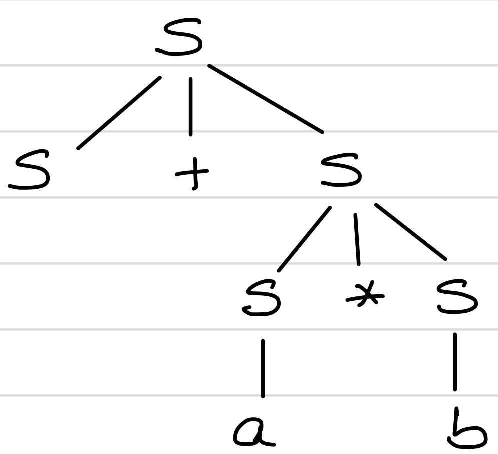
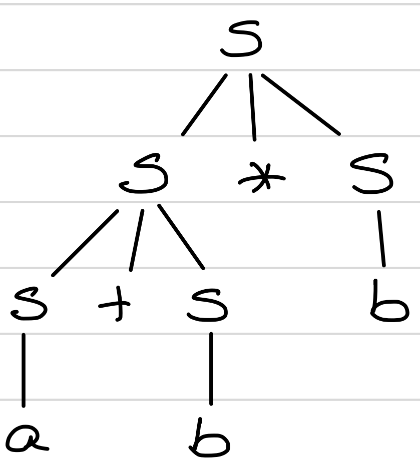


Derivation 2.



A CFG G is ambiguous if $\exists x \in L(G)$ for which there are two different parse trees.

→ Not two different derivations.

Definition

A string x is derived ambiguously in a CFG G if it has two different leftmost derivation.

Grammar G is ambiguous if it generates some string ambiguously.

G is unambiguous if G is not ambiguous.

A CFL $A \subseteq \Sigma^*$ is inherently ambiguous if $\forall$ CFG G s.t $L(G) = A$, G is ambiguous.

Note. There are inherently ambiguous CFLs.

DCFLs – CFLs that can be accepted by a DPDA.

DCFLs always admit an unambiguous grammar.

DCFLs $\subsetneq$ unambiguous CFLs.

Linear Grammar.

A CFG G is right linear if all productions are of the form

$$A \to xB, \quad A \to x \quad \text{for } A, B \in N, \; x \in \Sigma^*.$$

At most one nonterminal appears on the RHS. That nonterminal must be the rightmost symbol.

A CFG G is left linear if all productions are of the form

$$A \to Bx \quad A \to x \quad \text{for } A, B \in N, \; x \in \Sigma^*.$$

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A regular grammar is one that is either right linear or left linear.

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A regular grammar is one that is either right linear or left linear.

Example 1. $G_1 = (\{S\}, \{a,b\}, P_1, S)$ with $\underbrace{P_1 : S \to abS \mid a}_{\text{Right linear.}}$

$L(G_1) = (ab)^* a$

Example 2. $G_2 = (\{S, S_1, S_2\}, \{a,b\}, P_2, S)$ with

$P_2 : S \to S_1 ab, \; S_1 \to S_1 ab \mid S_2, \; S_2 \to a$

left linear. $\quad L(G_2) = a(ab)^*$

Both G_1 & G_2 are regular grammars.

Example 3. $G_3 = (\{S, A, B\}, \{a, b\}, P_3, S)$ where

P_3: $\underbrace{S \to A, \quad A \to aB \mid \epsilon, \quad B \to Ab}_{\text{not a regular grammar.}}$

Every production is left or right linear but the the grammar is neither left linear nor right linear.

A linear grammar is a grammar in which at most one nonterminal can occur on the RHS of any production irrespective of the position.

Note. A regular grammar is linear. Not all linear grammars are regular.

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Theorem 1. Let G be a right linear grammar, then $L(G)$ is regular.

Theorem 2. Let $A \subseteq \Sigma^*$ be a regular set then there exists a right linear grammar G s.t $A = L(G)$.

Theorem 3. $A \subseteq \Sigma^*$ is regular iff there exists a regular grammar G s.t $L(G) = A$.