DFA $M = (Q, \Sigma, \delta, s, F)$ | NFA $N = (Q, \Sigma, \Delta, S, F)$

$\delta : Q \times \Sigma \to Q$ $s \in Q$. | $\Delta : Q \times \Sigma \to 2^Q$

$S \subseteq Q$.

$L(M) = \{x \in \Sigma^* \mid M \text{ accepts } x\}$

$L(N) = \{x \in \Sigma^* \mid N \text{ accepts } x\}$

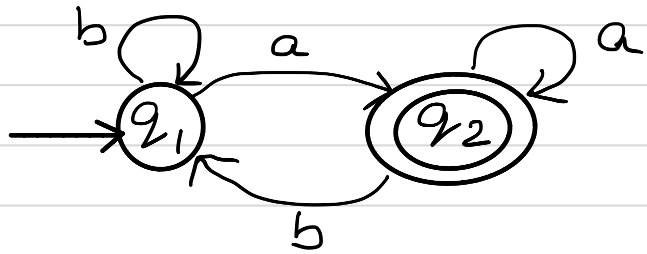
$A \subseteq \Sigma^*$ is regular if $\exists$ a DFA M s.t $L(M) = A$.

For every NFA N $\exists$ a DFA M s.t $L(M) = L(N)$.

$\to$ $A \subseteq \Sigma^*$ is regular if $\exists$ a NFA N s.t $L(N) = A$.

Examples. $\Sigma = \{a, b\}$.

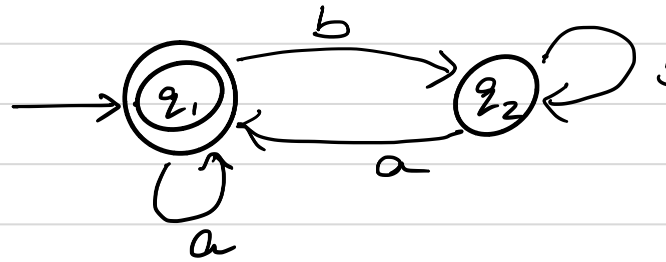
$A_1 = \{ w \in \Sigma^* \mid w \text{ ends in 'a'} \}$



– M_1

$L(M_1) = A_1$

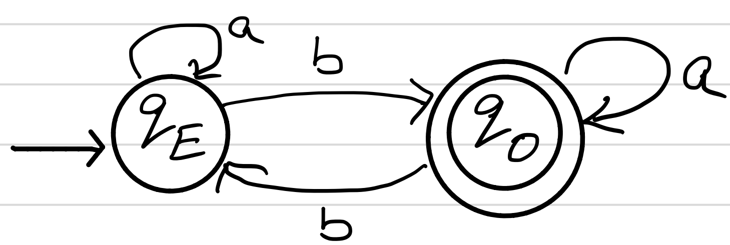
$A_2 = \{ w \in \Sigma^* \mid w \text{ is } \epsilon \text{ or } w \text{ ends in 'a'} \}$.



→ M_2

$L(M_2) = A_2$

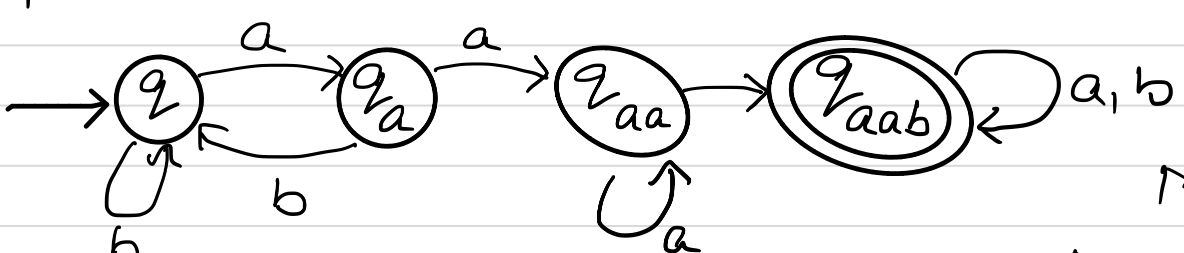
$A_3 = \{ w \in \Sigma^* \mid w \text{ contains odd number of b's} \}$.



M_3.

$L(M_3) = A_3$.

$A_4 = \{ w \in \Sigma^* \mid w \text{ contains a substring aab} \}$.



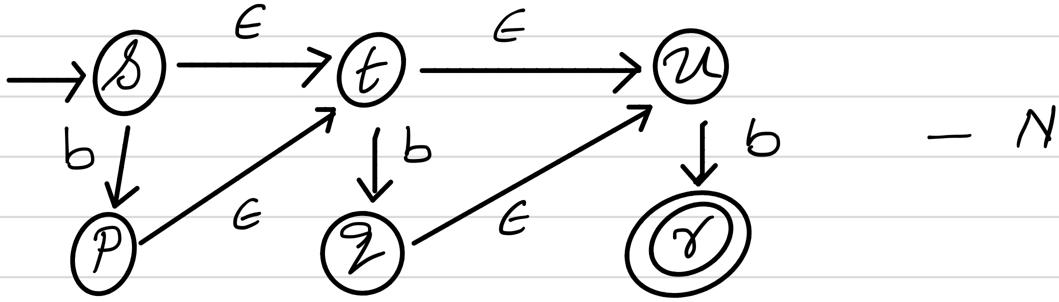
M_4

$L(M) = A_4$.

NFA : ϵ - transitions.

Transitions with ϵ - label.

$p \xrightarrow{\epsilon} q$: Automaton can take an ϵ transition any time without reading an input symbol.



if N is in state s and the next symbol is b.

- Read b and move to state p.
- move to t without reading any symbol then read b and move to q.
- Move to t (no input), move to u (no input) read b, move to r.

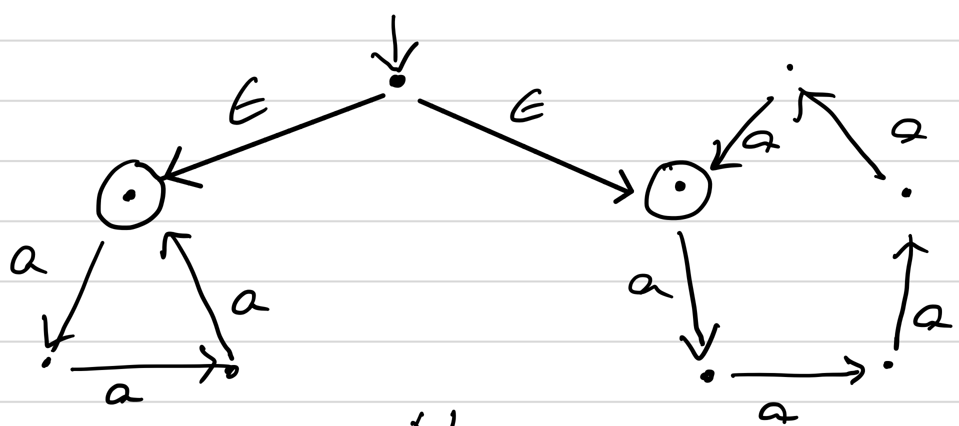
$L(N) = \{ b, bb, bbb \}$.

Example. $\Sigma = \{a\}$.

$A = \{ w \in \Sigma^* \mid |w| \text{ is divisible by } 3 \}$.

$B = \{ w \in \Sigma^* \mid |w| \text{ is divisible by } 5 \}$.

$C = \{ w \in \Sigma^* \mid |w| \text{ is divisible by } 3 \text{ or } 5 \}$.



N

$L(N) = C$.

For every ε-NFA N, $\exists$ a DFA M s.t. $L(N) = L(M)$.

ε-transitions - Convenience.

$AB = \{xy \mid x \in A \text{ and } y \in B\}$.

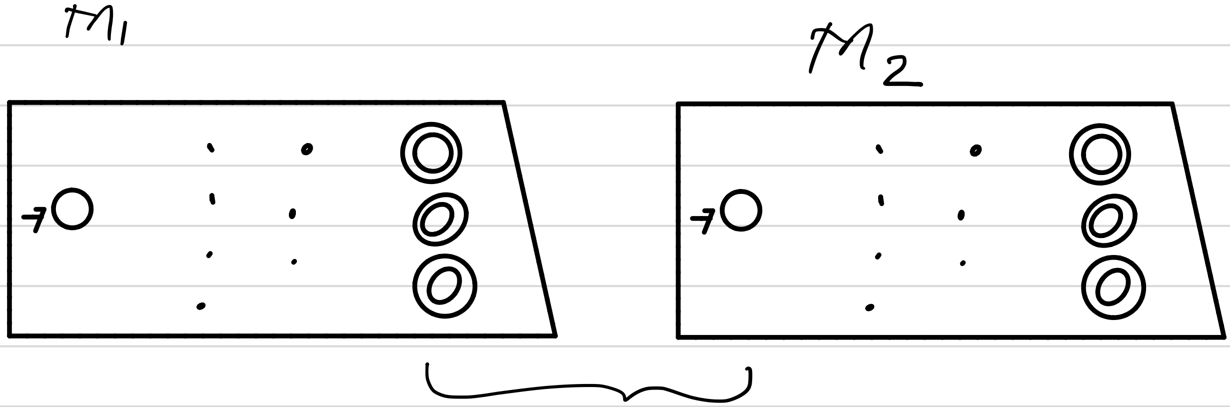
if $A, B \subseteq \Sigma^*$ are regular then AB is regular.

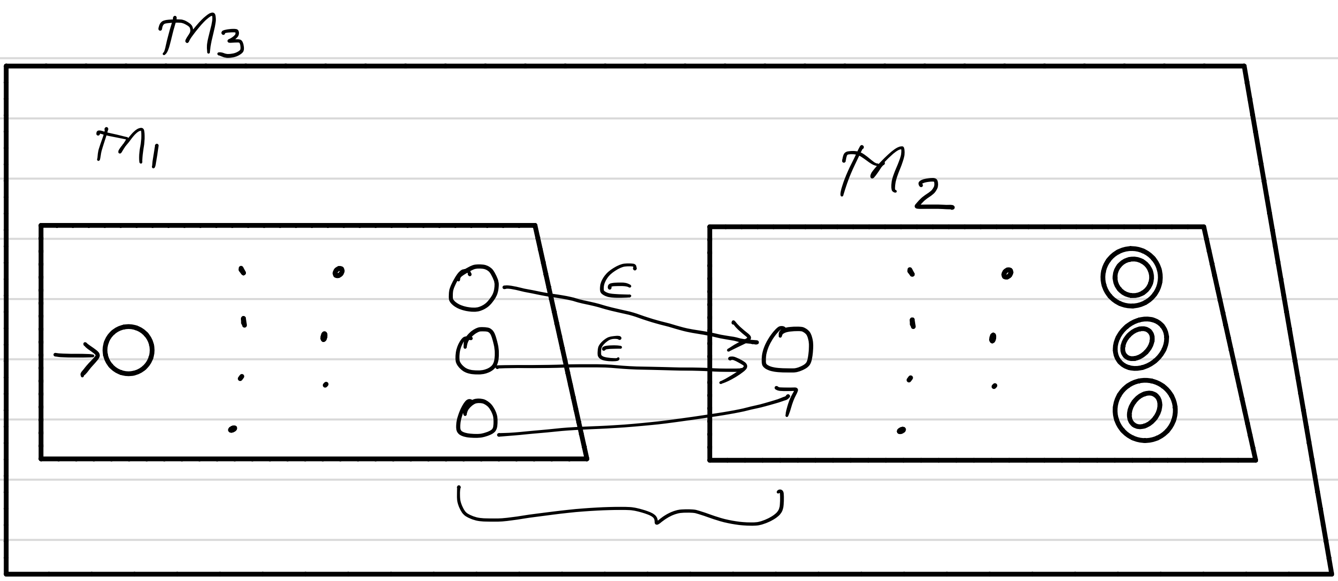
$A \longrightarrow \exists \overset{\text{DFA}}{M_1}$ s.t $L(M_1) = A$; $B \longrightarrow \exists \overset{\text{DFA}}{M_2}$ s.t $L(M_2) = B$

To Construct M_3 s.t $L(M_3) = AB$

$\downarrow$

ϵ-NFA





$L(M_3) = AB$.

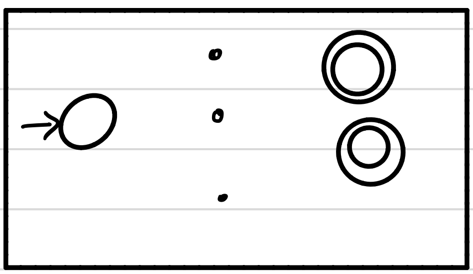
$A^* = \{x_1 x_2 \ldots x_n \mid n \geq 0 \text{ and } x_i \in A,\ 1 \leq i \leq n\}.$

$= \{\epsilon\} \cup A \cup A^2 \cup \ldots$

if $\underbrace{A \text{ is regular}}$ then A^* is regular.

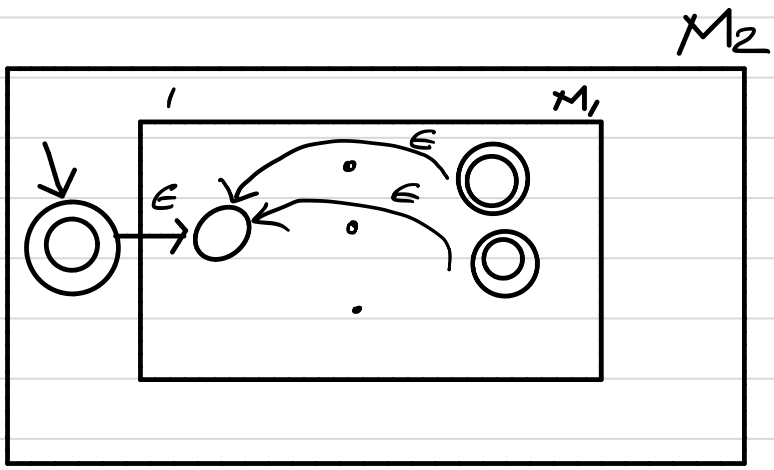
$\exists$ DFA M_1 s.t $L(M_1) = A$.

M_1



To construct M_2 s.t $L(M_2) = A^*$.

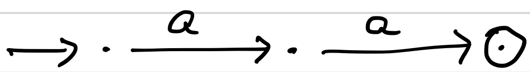
↳ NFA with ϵ-transitions.



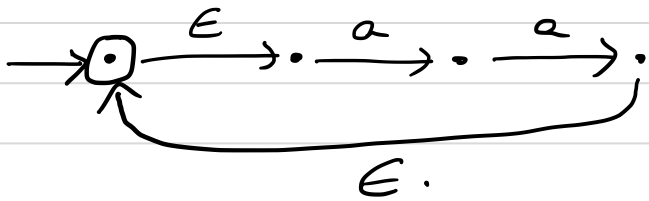
$L(M_2) = A^*$.

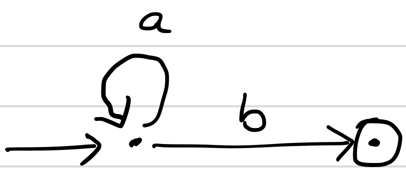
Example

$A = \{aa\}$ $\Sigma = \{a\}$.



A^*





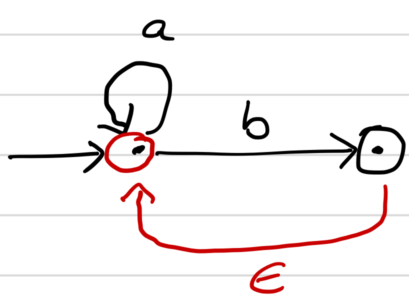
$\Sigma = \{a, b\}$

$\Rightarrow M$.

$L(M) = \{a^n b \mid n \geq 0\}. = A$.

$A^* = \{\epsilon\} \cup \{\text{strings ending with } b\}$.

M'



$L(M') = \{a, b\}^*$

$L(M') \neq A^*$