Regular sets - Finite state automata

$A_1 = \{a^n b^n \mid n \geq 0\}$ is not regular

Context free languages - Push down automata

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Program to check if a string $x \in A_2$?

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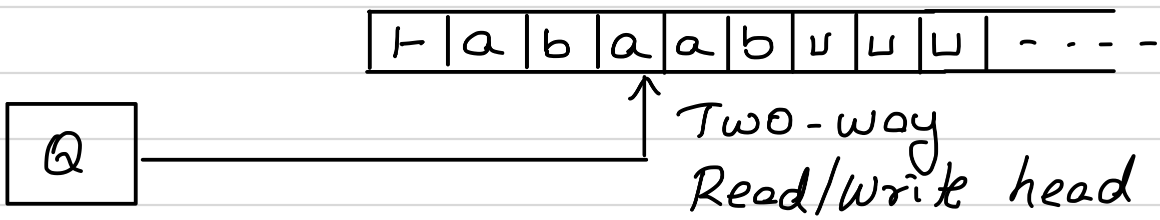
Formal model of Computation.

Finite automata + infinite tape with Read/Write functionality - Turing Machine.

Definition of computable - Computable by a Turing machine

Turing Machine - Deterministic



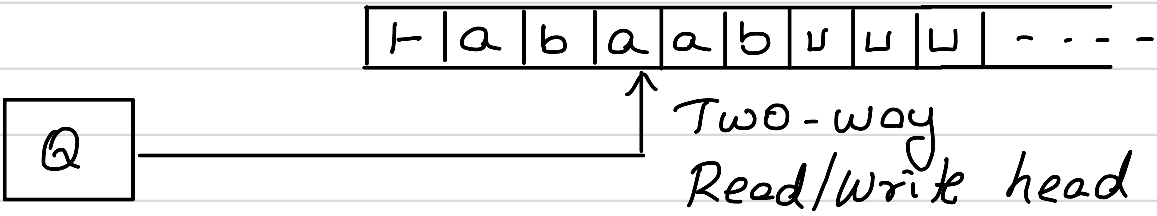
Finite set of States + semi-infinite tape

- infinite to the right
- Delimited on the left with an end marker ⊢

Head - Can move left or right over the tape, can read and write symbols on the tape.

Infinitely many cells to the right of the input string contains a special blank symbol ⊔

Deterministic Turing Machine



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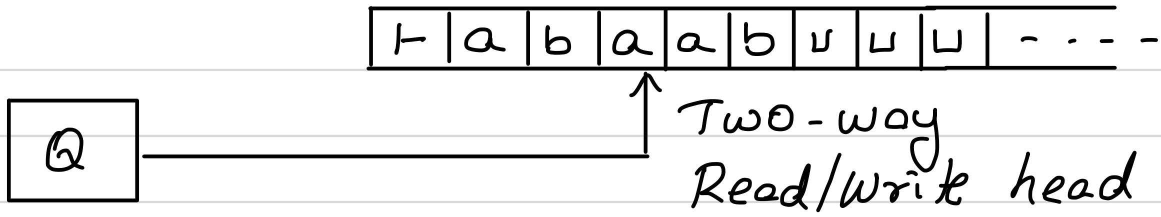
Operation of a Turing machine.

- Start in the start state s with head scanning the left end marker ⊢
- Each step, read the symbol on the tape under its head
- Depending on the symbol and current state, write a new symbol on that tape cell.
- Move the head left or right and change to a new state
- Accept input by entering a special accept state.
- Reject input by entering a special reject state.



Turing Machine - Formal definition

$$M = (Q, \Sigma, \Gamma, \vdash, \sqcup, s, \delta, t, r)$$

Σ : finite input alphabet.

Γ : finite tape alphabet [$\Sigma \subseteq \Gamma$]

$\sqcup \in \Gamma - \Sigma$: blank symbol
$\vdash \in \Gamma - \Sigma$: left end marker
$s \in Q$: start state.

$t \in Q$: Accept state , $r \in Q$: Reject state

$$\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$$

$$\delta(p, a) = (q, b, d)$$

$\hookrightarrow \in \{L, R\}$

Assumptions:

- $\forall p \in Q, \exists q \in Q$ such that $\delta(p, \vdash) = \delta(q, \vdash, R)$

- $\forall b \in \Gamma, \exists c, c' \in \Gamma$ and $\exists d, d' \in \{L, R\}$ such that
  $\delta(t, b) = (t, c, d)$
  $\delta(r, b) = (r, c', d')$

Once M enters accept/reject state it never leaves it.

Example. A TM that accepts $\{\overbrace{a^n b^n c^n \mid n \geq 0}^{\text{not a CFL}}\}$

1. Start in s, scans to the right over the input once to ensure the input is of the form $a^* b^* c^*$

It writes to the tape the same symbol it reads

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Example: $a^4 b^4 c^4$

$\vdash$	a	a	a	a	b	b	b	b	c	c	c	c	$\sqcup$	$\sqcup$	- - -

Configuration of a Turing machine

Tape of M consists of a string $y\sqcup^{\omega}$ where $y \in \Gamma^*$.

A configuration is $\in Q \times \{\underbrace{y\sqcup^{\omega}}_{} \mid y \in \Gamma^*\} \times \mathbb{N}$

(p, z, n)

infinite string with a finite presentation.

$p \to Q$

$n \to$ Current position of head on tape

$z \to$ Current content of the tape.

Start configuration - $(s, \vdash x\sqcup^{\omega}, 0)$

$x \to$ input string

Configuration of a Turing machine

Tape of M consists of a string $y\sqcup^\omega$ where $y \in \Gamma^*$.

A configuration is $\in Q \times \{\underbrace{y\sqcup^\omega}_{} \mid y \in \Gamma^*\} \times \mathbb{N}$ → infinite string with a finite presentation.

(p, z, n)

p → Q

z → Current content of the tape.

n → $n \geq 0$ Current position of head on tape

Start configuration – $(s, \vdash x \sqcup^\omega, 0)$

x → input string

Next Configuration relation $\xrightarrow[M]{1}$

For an infinite string z, let z_n be the n^{th} symbol in z

$S_b^n(z)$ – string obtained by substituting b for z_n.

Example $S_b^3(\vdash abaa---) = \vdash abba---$

$$(p, z, n) \xrightarrow[M]{1} \begin{cases} (q, S_b^n(z), n-1) & \text{if } \delta(p, z_n) = (q, b, L) \\ (q, S_b^n(z), n+1) & \text{if } \delta(p, z_n) = (q, b, R) \end{cases}$$

$\alpha \xrightarrow[M]{*} \beta$: Reflexive transitive closure of $\xrightarrow[M]{1}$

M accepts $x \in \Sigma^*$ if $(s, \vdash x \sqcup^\omega, 0) \xrightarrow[M]{*} (t, y, n)$

$L(M) = \{x \in \Sigma^* \mid M \text{ accepts } x\}$

M rejects $x \in \Sigma^*$ if $(s, \vdash x \sqcup^\omega, 0) \xrightarrow[M]{*} (r, y, n)$

M halts on input $x \in \Sigma^*$ if it either accepts x or rejects x.

M loops on input $x \in \Sigma^*$ if M neither accepts nor rejects x.