

N Queen Problem →

$N \times N$ chessboard

N Queen

[none of the queens are attacking position]

4-Queen $N=4$

← Queen

	1	2	3	4
→ 1			Q	
→ 2	Q			
→ 3				Q
→ 4		Q		

queen no	1	2	3	4
1	2	4	1	3

X

queen no

NO(4)

1X
2X
3✓

3

Here the solution array X contains the solution

Here $X[i]$ gives column position of queen i

X

2 4 1 3

1 2 3 4

← solution array X

① Every value in array X must be unique

[Every queen must be placed at diff colⁿ]

② Every value must be between 1 to 4 (inclusive)

Case 1

place(2, 1)

chk for previous

$j=1$

$X[j]$ i.e. $X[1]=1$

→ previous queen ka col

and $i=1$ ← current queen

..

queen no
NO(1)

1

x1
x2
x3

4

queen no
NO(2)

2

1X
2X
3X

4

queen no
NO(3)

3

1X
2X
3X

4

queen no
NO(4)

4

1X
2X
3X

4

queen no

NO(2)

1 → X
2 → X
3 → X

4

queen no

NO(3)

1X
2X
3X

4

queen no

NO(4)

1X
2X
3X

4

queen no

NO(3)

1X
2X
3X

4

queen no

NO(4)

1X
2X
3X

4

1 2 3 4 → Queen no

	1	2	3	4
1	Q			
2				
3				
4				

Col no

Case 1

place(2, 1)

chk for previous

1 2 3 4

X

1 2 3 4

1 2 3 4

1 2 3 4

1 2 3 4

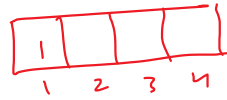
1 2 3 4

1 2 3 4

~

chk for $\Rightarrow j=1$ $x[j]$ ie $x[1]=1$ ka con
 previous queen
 and $i=1$ \Leftarrow current queen
 ka proposed col
 same hai
 waha chahoga

Case 2) chk diagonal
 $\begin{matrix} k & i \\ \text{place}(2, 2) \end{matrix}$ X



Previous queen = $j=1$
 Here $x[j]=1$ $i=2$ not same

$|x[j] - i| \stackrel{?}{=} |k - j|$ } diag attach hai
 $|1 - 2| \quad |2 - 1|$
 $\downarrow \quad \downarrow$
 1 Same hai 1

Current & previous queen no ka diff = previous and Current queen ke col ka diff (diag)



AOA17-4

BACKTRACKING

 Date _____
 Page _____

★ Backtracking :

- It is a method of solving a problem (Eg. N Queen)
- The principle idea is to construct solutions by taking one component at a time. The component is added in the solution if all the constraints in the problem definition are satisfied.
- If constraints are not satisfied then the latest component is dropped from the solution.
 i.e. while finding the solution to the problem, we find some partial solution . step 1, step 2, k
- At step k+1, the program discovers that it cannot go further with the solution due to some mistake in the previous step in such situation the program can be made to backtrack and repair the previous step solution.

★ N-QUEEN PROBLEM :

- n x n chess board and given n queen.
 We want to place all the n queen on the board in non-attacking position.

Note: Queen can attack horizontally, vertically & diagonally.

- Solution can be found using 1-D array of 'n' elements.
- kth element of array x will give column of kth queen.
- The constraints on array x are every value stored
 - ① in arr. x stored should be unique.
 - ② Every value stored in array x should be between 1 to n.

→ $x[k] = \text{col position of } k^{\text{th}} \text{ queen}$

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Algorithm :

1. Let n be number of queens. ✓
2. Let $x[n+1]$ be array that stores column of queen.
 (because array index starts at 0)

Algo for function read()

- Accept no. of queens. i.e. n
- create array x of $n+1$ elements.

X				
0	1	2	3	4

 ↑
 zero index
not used

Algo for function place(k, i)

It indicates whether k th queen can be placed in i th column.

```

int place(int k, int i)
{
    int j;
    for(j=1; j<=k-1; j++)
    {
        if((x[j]==i) || (abs(x[j]-i)==(k-j)))
            return 0;
    }
    return 1;
}
  
```

← j refers to previous queens.
 ← k current queen no
 ← i col no
 ← j refers to previous queens.

chke posⁿ of all previous $k-1$ queens

returns 1 ⇒ queen k can be placed at col i
 returns 0 ⇒ queen k cannot be placed at col i

Algo for function nqueen(int k)

void nqueen(int k)

```

{
    for(i=1; i<=n; i++)
    {
        if(place(k, i))
        {
            x[k]=i;
            if(k==n)
            {
                for(j=1; j<=n; j++)
                {
                    print x[j];
                }
            }
        }
    }
}
  
```

← queen no to set
 ← for every queen k we have possible col from 1 to n
 ← kya queen k ko col i pe rakh sakte hai
 ← agar yes queen k ko col i pe rakhao
 ← agar ye last queen hai

// To print solution.
 } toh print solⁿ

col 1 pe return
 agar ye last
 queen hai

```
for(j=1; j<=n; j++)  
    print x[j];
```

toh print soln

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else

nqueen(k+1);

} // end of if

} // for ends

} // end of nqueen.

→ agar queen no n nahin hai (last queen nahin hai)

→ call for next queen.

NOTE:

9n place function.

• j refers to previous queens.

• x[j] : column of previous queen j.

• i : purposed column of current queen k.

• $|x[j] - i|$: difference in column of previous & current queen.

• $(k - j)$: difference in queen number.

if((x[j] == i) || ($|x[j] - i| == (k - j)$))

↑	↑	↑	↑
previous jth queen ka column number.	current kth queen ka column number. (proposed)	Previous jth queen ka column are current kth queen ka col.	kth queen ka are previous jth queen ka difference.

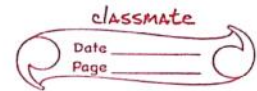
Agar column no.
same hai to

Nahinn chalega !!

ka difference.

Agar same hai to diagonal
problem hai, Nahinn! chalega

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PROGRAM: math.h

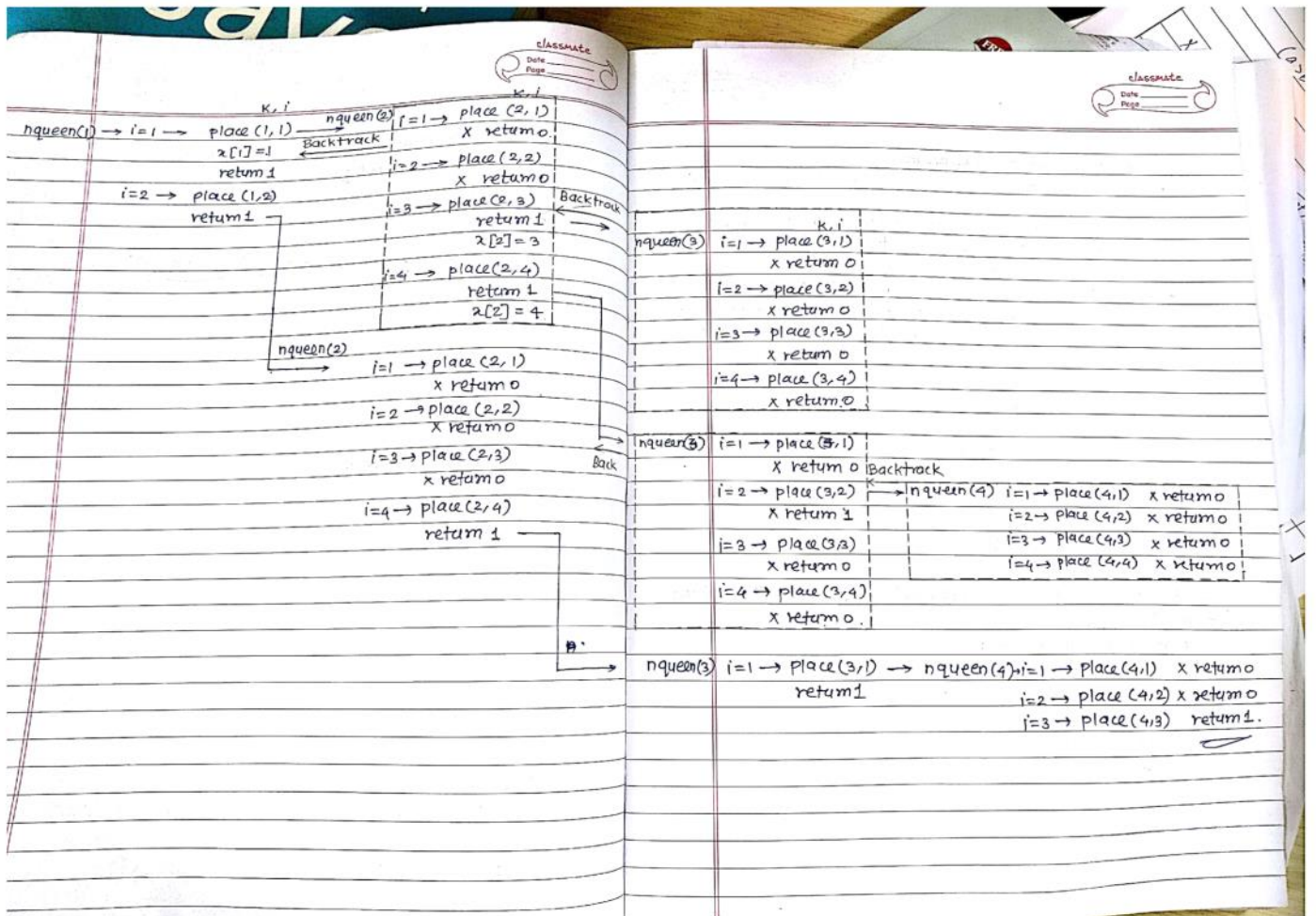
```

#include <stdio.h>
int n, x[16] = {0};
int place (int k, int j)
{
    int i;
    for (j=1; j<=k-1; j++)
        if ((x[j]==i) || (abs(x[j]-i) == (k-j)))
            return 0;
    return 1;
}

void nqueen (int k)
{
    int i, j;
    for (i=1; i<=n; i++)
        if (place (k, i))
        {
            x[k] = i;
            if (k == n)
            {
                for (j=1; j<=n; j++)
                    printf ("%d ", x[j]);
                printf ("\n");
            }
            else nqueen (k+1);
        }
}

void main ()
{
    printf ("Enter the number of Queens\n");
    scanf ("%d", &n);
    printf ("All possible solutions are :\n");
    nqueen (1);
}

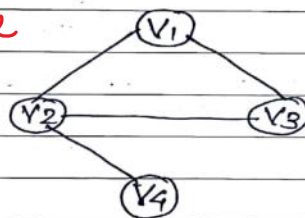
```

* GRAPH COLOURING PROBLEM :

- Given a graph of ' v ' vertices and ' e ' edges and ' n ' different colours.
- We want to colour the graph with n colours in such a way that the adjacent vertices must not be of same colour.
- A graph is said to be ' m ' colourable graph, if ' m ' is the minimum no. of colour necessarily to colour all the vertices ensuring adjacent vertices are not of same colour.

- Consider a graph :



Here if we take 4 colours
It's feasible as all vertex
will be of diff color but
not optimal because
4 is minimal value.

Problem?
Agar ek graph hain
to minimum kitne color
use karne padenge to ensure
no two vertex are of same color
adjacent

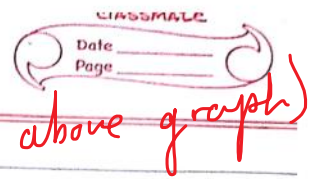
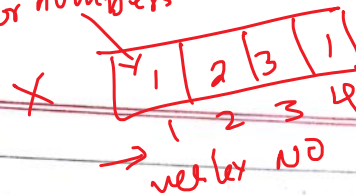
The above graph is 3 colourable graph. Let, the colours be 1, 2 & 3.

- possible solutions are :

V_1	V_2	V_3	V_4
1	2	3	1
1	2	3	3
3	2	1	3
3	2	1	1

...

color numbers



Solution :

Consider 1-D array X that has V elements where every element in X store color number.

Constraint on array X :

- (1) If i and j are adjacent vertices then $x[i] \neq x[j]$
- (2) Every value in X must be between 1 to n

Algorithm :

Let g represents adjacency matrix of graph with V vertices and E edges

Let $\text{int } x[V]$ stores solution.

Algo for color()

[k th vertex pe i th color chalega kya ?]

$\text{int color}(\text{int } k, \text{int } i)$ \leftarrow i is current color no

$\{$ $\text{int } j;$

$\text{for } (j=1; j \leq k-1; j++)$

$\{$ $\text{if } (g[k][j] \neq 0 \ \&\& \ x[j] == i)$

$\text{return } 0;$

$\}$

$\text{return } 1;$

$\}$

k : vertex number.

i : colour number (current proposed)

$g[k][j] \neq 0$: k th vertex aur j th vertex adjacent hai

$x[j] == i$

colour of vertex j

proposed colour of k th vertex

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Yaha

Algo. for graphcolor()
 void graphcolor (int k)

```

{
  int i;
  for (i=1; i<=n; i++) {
    if (color (k,i)) {
      // kth vertex ko color karo
      // possible n colors from 1 to n.
      z[k] = i;
      if (k==v) {
        for (j=1 to v) {
          print // solution.
          z[j]
        }
      }
      else graphcolor (k+1);
      // color next vertex
    }
  }
}

```

if yes → Chk agar last vertex hai
 agar last vertex nahi hai

* SUM OF SUBSET :

Problem statement :

We are given 'n' positive integers and a positive integer number 's'.

We have to find all possible subset of 'n' such that their sum is 's'.

$N = \{1, 2, 3, 10, 12, 13, 15\}$ ← Set of +ve integers

$s = 15$ ← Sum

$\{1, 2, 12\} \Rightarrow 15$
 $\{3, 12\} \Rightarrow 15$
 $\{15\} \Rightarrow 15$
 $\{2, 3, 10\} \Rightarrow 15$

} Possible solutions

Possible solutions are:

{ 15 }

⇒ X

15

{ 2, 13 }

⇒ X

2 13

{ 3, 12 }

⇒ X

3 12

{ 1, 2, 12 }

⇒ X

1 2 12

{ 2, 3, 10 }

⇒ X

2 3 10

⋮

⋮

1 2 3

$$\sum_{i=1}^n x[i] = \text{sum}$$

The solution for the problem can be found using Backtracking approach. Let there be 1-D array X that has n locations. The array X will store those integers whose sum will be S.

Constraints on array X[] are :

1. Sum of elements in array X should not exceed S.
2. The values stored in array X should be unique as we do not want to repeat the subset.

Program :

```
#include <stdio.h>
```

```
#include <conio.h>
```

```
void sumset(int);
```

```
void proper(int, int);
```

```
int sum(int)
```

```
int v[10];
```

// To store elements

```
int x[10];
```

// To store solution.

```
int n, sum_req;
```


sumreq is global (outside fn)

Assumption.



```
void main()
```

```
{
```

```
int i;
```

```
clrscr();
```

```
printf("Enter number of elements\n");
```

```
scanf("%d", &n); ✓
```

```
printf("Enter unique elements in increasing order\n");
```

```
for(i=1; i<=n; i++)
```

```
scanf("%d", &v[i]); ✓
```

```
printf("Enter sum required\n");
```

```
scanf("%d", &sumreq);
```

```
sumset(1);
```

```
getch();
```

```
}
```

Input that has
Unique and Sorted
elements

```
int sum(int n)
```

```
{
```

```
int s, i;
```

```
s=0;
```

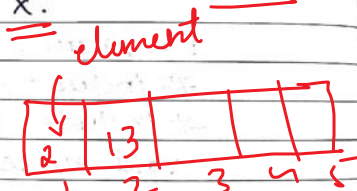
```
for(i=1; i<=n; i++)
```

```
s = s + x[i];
```

```
return s;
```

```
}
```

// gives sum of first n elements
of array x.



Solution array

```
int proper(int k, int i)
```

```
{
```

```
int j;
```

```
for(j=1; j<=k-1; j++)
```

```
{ if(x[j] == v[i])
```

```
return 0;
```

```
}
```

chk karo array x main posⁿ k par array
v ka ith posⁿ ka element chalga kya?

// If same element is present before

// Repeated toh nahi

```

// Same as previous k-1 posn ka sum in X
if ((sum(k-1) + v[i]) > sumreq) // previous kth element ka sum.
    return 0;
else return 1;

int sumset (int k)
{
    int i, s, j;
    for (i=1; i<=n; i++)
    {
        if (proper(k, i))
        {
            x[k] = v[i];
            s = sum(k);
            if (s == sumreq)
            {
                // Print solution
                for (j=1; j<=k; j++)
                    printf("%d ", x[j]);
            }
            else
                sumset(k+1);
        }
    }
}

```

Annotations:
 - sum(k-1): previous kth element ka sum.
 - v[i]: current element ka
 - sum(k): In array X at posⁿ K place some element of array V ensuring the total sum of elements of array X does not exceed sumreq
 - proper(k, i): In array X at posⁿ K any of n elements from array V
 - sumreq: sum of array X
 - fill next posⁿ of array X

NOTE:

K is position in array X.

i is position in array V.

proper() specifies whether element at ith element position can be placed at Kth position in array X.

$V = (1, 2, 3, 10, 12, 13, 14, 15)$
 1 2 3 4 5 6 7 8
 ↑
 Sumreq = 15

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proper(1, 1) ✓

x ↑ v
 proper(2, 1)
 proper(4, 4)

X =

1	2	3	1				
---	---	---	---	--	--	--	--

 1 2 3 4 5 6 7 8
 ↑ ↑
 s = 1 K
 s = 3
 s = 6

$\text{Sum}(3) \text{ of } X + v[i] \text{ i.e. } v[4]$
 $= 6 + 10 = 16 > \text{sumreq}$

$l = 12$
 $T =$ ~~0~~ 1 ~~2~~ 3 4 5 6 7 8 9 10 11 12
 $a \ b \ a \ b \ c \ b \ a \ b \ a \ b \ a \ b$
 $m = 3$
 $P =$ ~~0~~ 1 2 3
 $a \ b \ c$
 $a \ b \ c$

$a \ b \ a \ b \ c \ b \ a \ b \ a \ b \ a \ b$

Slide Size = 1

STRING MATCHING ALGORITHMS

Given a text T of length L and given pattern P of length m , we want to find index of text T where pattern P occurs.

Ex.
$$T = \begin{matrix} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ a & b & a & b & c & b & a & b & a & b & a & b \end{matrix} \quad : L = 12$$

$$P = \begin{matrix} & 1 & 2 & 3 \\ a & b & c & \\ & 0 & 1 & 2 \end{matrix} \quad : m = 3$$

```

3 for(i=0 to L-m)
4     {
5         j=0;
6         while (P[j] == T[i+j])
7             {
8                 if (j==m-1)
9                     return i;
10                else
11                    j=j+1
12            }
13     }
14 return -1; // P is not found in T.

```