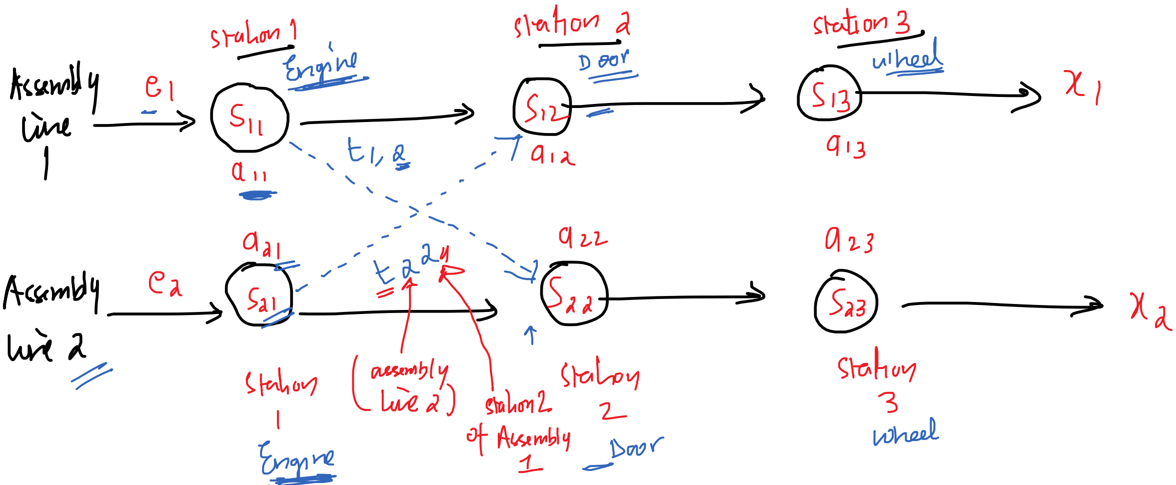
Assembly line Scheduling

* A factory has 2 assembly line with n stations each.

* A station is denoted as $\boxed{S_{i,j}}$ where i indicates assembly line and j indicate station number

Assembly line

station no



* Time Taken per station is denoted as $d_{i,j}$

station j of assembly line i

* The product must pass through each of the n stations in order before exiting the Assembly line

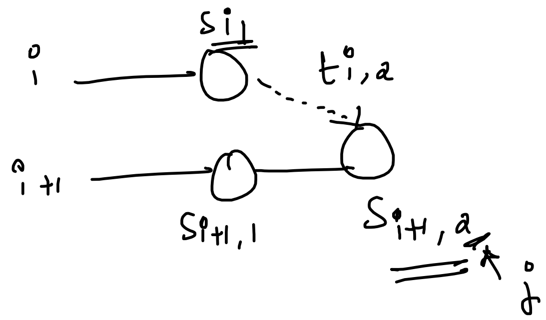
* The parallel station of the two assembly line perform the same task

* After product passes through $S_{i,j}$ then it would continue to $S_{i,j+1}$, unless it decides to shift to other Assembly line

to shift to other Assembly line

* Continuing on Same assembly line takes no extra Cost

* But transferring from line i at station $j-1$ to station j on other line will take t_{ij}



* Each assembly line takes an entry time e_i and exit time x_i, which may be different for diff assembly line.

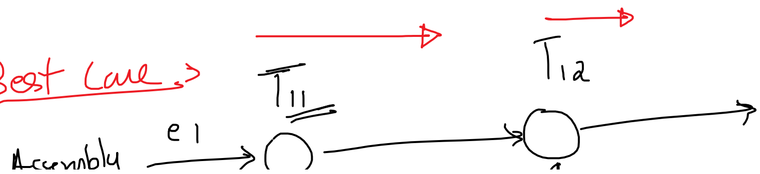
Approach ->

* The decision involves determining whether the assembly line should be switched or not

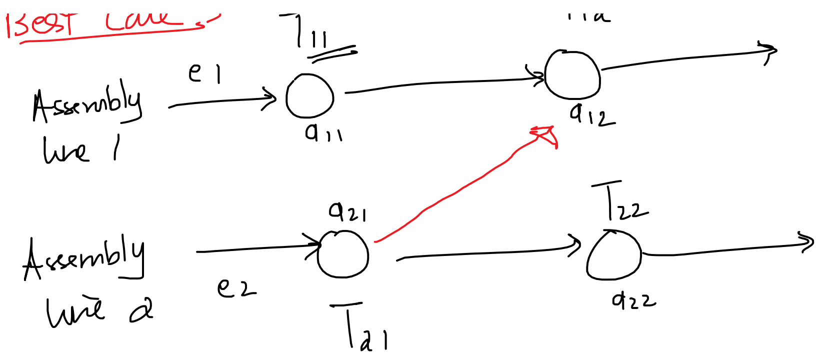
> $T_{1,j}$ ⇒ Indicates min time taken to leave station j on assembly line 1

Similarly $T_{2,j}$ ⇒ Indicates min time taken to leave station j on assembly line 2

Best Case ->



Best case -



Time taken to leave station 1 →

$T_{11} \Rightarrow e_1 + a_{11}$

$T_{21} \Rightarrow e_2 + a_{21}$

$$T_{12} \Rightarrow \min\{\underbrace{T_{11} + a_{12}}_{\text{previous st}^n \text{ of Same Assembly line}},\ \underbrace{T_{21} + t_{22} + a_{12}}_{\text{Comming from previous st}^n \text{ of Assembly line 2}}\}$$

Time to leave station 1 (→ T_{11}); time to use st^n 2 (→ a_{12})

Time to leave st^n 1 of Asse 2 (→ T_{21}); time to transfer (→ t_{22}); time to use st^n 2 of Assem 1 (→ a_{12})

$$T_{22} \Rightarrow \min\{\underbrace{T_{21} + a_{22}}_{\text{comming from Same Assembly 2.}},\ \underbrace{T_{11} + t_{12} + a_{22}}_{\text{comming from Assembly 1}}\}$$

Total time taken to come out of factory →

$$T_{min} = \min\{T_{1n} + X_1,\ T_{2n} + X_2\}$$

T_{1n}: Exit time of station n Assembly 1

X_1: Exit time of Assembly 1

T_{2n}: Exit Time of station n Assembly 2

X_2: Exit time of Assembly line 2.

Q) Problem → $n=4$ (no of stations).

$a(2,4) = \{\{4,5,3,2\},\{2,10,1,4\}\}$

no of Asse, no of station

$t(2)(4) = \{(0,7,4,5),(0,9,2,8)\}$

$e(1:2) = \{10,12\}$

$X(1:2) = \{18,7\}$

Sol^n →

a — Stations — Assembly line

a	1	2	3	4
1	4	5	3	2
2	2	10	1	4

t — station — Assembly line

t	1	2	3	4
1	0	7	4	5
2	0	9	2	8

here $t_{23} = 2$

assembly 2 — to st^n 3 of Ass line 1

Sol ⇒ $j=1$ (station 1)

$T_{11} = e_1 + a_{11} \Rightarrow 10+4 = \underline{\underline{14}}$

$T_{21} = e_2 + a_{21} \Rightarrow 12+2 = \underline{\underline{14}}$

$j=2$ (station 2).

$T_{12} = \min(\underline{T_{11} + a_{12}},\ T_{21} + t_{22} + a_{12})$

$= \min(\underline{14+5},\ 14+9+5)$

$= \min(19, 28)$

$= \underline{\underline{19}}$ [when station 2 of Assembly line 1 is leaving it is comming from station 1 of Assembly line 1]

$T_{22} = \min(\underline{\underline{T_{21} + a_{22}}},\ T_{11} + t_{12} + a_{22})$

$$T_{22} = \min(\underline{T_{21} + a_{22}} \,,\; T_{11} + t_{12} + a_{22})$$

$$= \min(\underline{14 + 10} \,,\; 14 + 7 + 10)$$

$$= \min(\underline{24}, 31)$$

$= \underline{24}$ [when leaving st^n 2 of Assembly line 2, it is comming from station 1 of Assembly line 2).

③ $\underline{j=3 \text{ (station 3)}}$

$$T_{13} = \min(\underline{T_{12} + a_{13}} \,,\; T_{22} + t_{23} + a_{13})$$

$$= \min(\underline{19 + 3}, \; 24 + 2 + 3)$$

$$= \min(\underline{22}, 29)$$

$$= 22$$

$$T_{23} = \min(T_{22} + a_{23}, \; \underline{T_{12} + t_{13} + a_{23}})$$

$$= \min(24 + 1, \; 19 + \underline{4 + 1})$$

$$= \min(25, 24)$$

$$= \underline{24}$$

④ $\underline{\text{station} = 4}$ $j=4$

$$T_{14} = \min(\underline{T_{13} + a_{14}} \,,\; \underline{T_{23} + t_{24} + a_{14}})$$

$$= \min(22 + 2 \,,\; 24 + 8 + 2)$$

$$= \min(24, \; 34)$$

$= 24$

$T_{24} = \min(T_{23} + a_{24},\ T_{13} + t_{14} + a_{24})$

$= \min(24 + 4,\ 22 + 5 + 4)$

$= \min(28, 31)$

$= 28$

$T_{min} = \min(T_{14} + X_1,\ T_{24} + X_2)$

$= \min(24 + 18,\ 28 + 7)$

$= \min(42, 35)$

$= 35$

Algorithm ⇒

Function Assemblyline(a, t, n, e, X)

{ n = no of stations
e = entry time of each Assembly line.
X = Exit " " " " "
t(1:n) = shifting time.
a(1:n) = Processing time at each station.

step 1. start

step 2: $T_{11} = e_1 + a_{11}$

$T_{21} = e_2 + a_{21}$.

step 3: for (i=2 to n) do. ⟶ n time

step 4. { $T_{1i} = \min(T_{1,i-1} + a_{1,i},\ T_{2,i-1} + t_{2i} + a_{1i})$

step 5. $T_{2i} = \min(T_{2\,i-1} + a_{2i}\ ,\ T_{1\,i-1} + t_{1i} + a_{2i})$.

}

step 6: return $\min(T_{1n} + x_1\ ,\ T_{2n} + x_2)$.

Complexity = $O(n)$.

(4) Travelling Salesman Problem

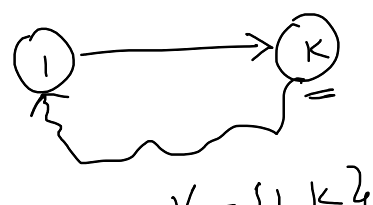
Let $G = (V, E)$ be a directed graph with edge cost C_{ij} such that

$C_{ij} > 0$ if $\langle i, j \rangle \in E(G)$.

$C_{ij} = \infty$ if $\langle i, j \rangle \notin E(G)$.

* A tour is a directed cycle that includes every vertex of graph G.
* The cost of the tour is the sum of cost of all the edges of tour
* The problem is to find the tour with minimum cost.

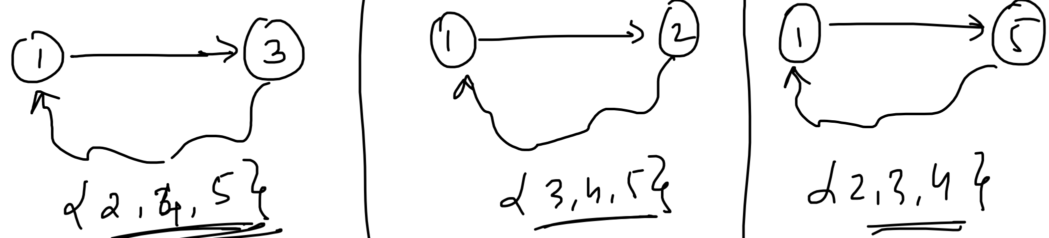
Approach :



Start from 1
Go to K
Then from K to 1
visiting all the vertex other than 1, K.

$n = 5$

ex



A tour is a simple path that starts and ends at same vertex.

"A graph tour consist of an edge $\langle 1, k \rangle$ for some $k \in V - \{1\}$ and a path from vertex k to vertex 1 which must go through all the vertex in $V - \{1, k\}$ exactly once.

* let $g(i, S)$ be the length of the shortest path from vertex i to vertex 1 which must go through each vertex in set S exactly once

$\therefore$ Hence $g(1, V - \{1\})$ represent length of optimal graph tour

(start from 1 come back to 1 visiting all the vertex in $V - \{1\}$)

Generalizing

$$g(i, S) = \min\left(\left(C_{ij} + g(j, S - \{i, j\}) \right) \right)$$

start

To start from i → go from $i \rightarrow j \Rightarrow C_{ij}$

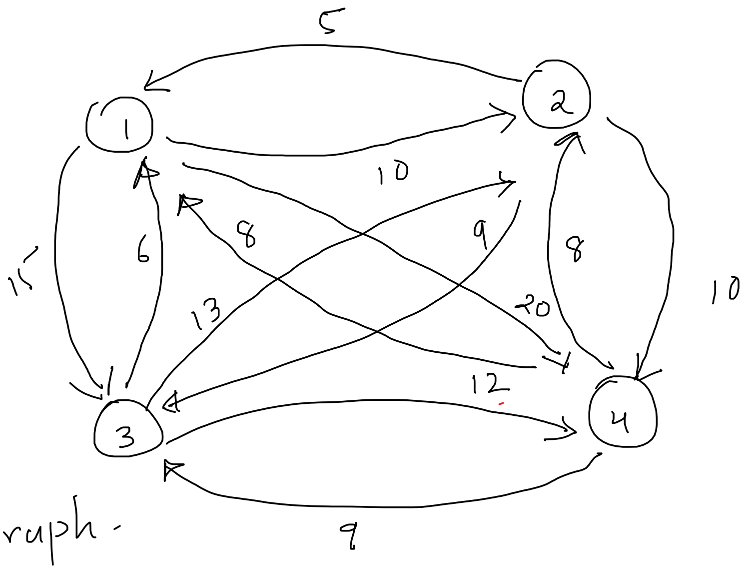
then come back from $j \Rightarrow g(j, S - \{i, j\})$

to i visiting all vertex $V - \{i, j\}$

We have $g(i, \emptyset) = C_{i1}$ } $C_i \to 1$

from vertex i to 1 visiting no other nodes since $\emptyset$

Question



Find Min Tour cost of Given Graph.

start tour from vertex = 1

Sol → $|S| = 0$

$$g(2, \emptyset) = C_{21} = 5$$
$$g(3, \emptyset) = C_{31} = 6$$
$$g(4, \emptyset) = C_{41} = 8$$

$|S| = 1$

from 2 to 1 via 3 ⇒ 2→3 then 3→1

$$g(2, \{3\}) = C_{23} + g(3, \emptyset) = 9 + 6 = 15$$
$$g(2, \{4\}) = C_{24} + g(4, \emptyset) = 10 + 8 = 18$$
$$\ldots) = C_{3\,2} + g(2, \emptyset) = 13 + 5 = 18$$

From any vertex other than 1 comming

$g(3, \{2\}) = C_{32} + g(2, \emptyset) = 13 + 5 = 18$

$g(3, \{4\}) = C_{34} + g(4, \emptyset) = 12 + 8 = 20$

$g(4, \{2\}) = C_{42} + g(2, \emptyset) = 8 + 5 = 13$

$g(4, \{3\}) = C_{43} + g(3, \emptyset) = 9 + 6 = 15$

$g(4, \{3\}) \Rightarrow 3$

than 1 comming back to one using only one intermediate Vertex

$|S| = 2$

2 se 1 via 3 & 4

2 se 3 then 3 to 1 via 4

$g(2, \{3, 4\}) = \min\left(C_{23} + g(3, \{4\}), C_{24} + g(4, \{3\})\right)$

min

$= \min(9 + 20, 10 + 15) = \min(29, 25) = 25$

2 se 4 then 4 to 1 via 3

$d(2, \{3, 4\}) = 4$

$g(3, \{2, 4\}) = \min\left(C_{32} + g(2, \{4\}), C_{34} + g(4, \{2\})\right)$

$= \min(13 + 18, 12 + 13) = 25$

$d(3, \{2, 4\}) \Rightarrow 4$

$g(4, \{2, 3\}) =$

$\min\left(C_{4,2} + g(2, \{3\}), C_{43} + g(3, \{2\})\right)$

$= \min(8 + 15, 9 + 18) = 23.$

$d(4, \{2, 3\}) = 2$

$|S| = 3$

$g(1, \{2, 3, 4\}) =$

1 sel via 2,3,4

$1 \to 2$ then 2 sel via 3,4

$1 \to 3$ then 3 sel via 2,4

$1 \to 4$ then 4 sel via 2,3.

$$= \min \{ (C_{12} + g(2,\{3,4\})), (C_{13} + g(3,\{2,4\})), (C_{14} + g(4,\{2,3\})) \}$$

$$= \min \{ 10 + 25, 15 + 25, 20 + 23 \}$$

$$= \min \{ 35, 40, 43 \}$$

$$= 35$$

$$d(1,\{2,3,4\}) \Rightarrow 2$$

Total Tour Cost = 35

Path

$$d(1,\{2,3,4\}) = 2$$
$$d(2,\{3,4\}) \Rightarrow 4$$
$$d(4,\{3\}) = 3$$
$$d(3,\{\phi\}) \Rightarrow 1$$

path

$$1 \rightarrow 2 \rightarrow 4 \rightarrow 3 \rightarrow 1$$

H/W

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 15 & 20 & 3 \\ 2 & 5 & 0 & 18 & 6 \\ 3 & 4 & 8 & 0 & 12 \\ 4 & 8 & 2 & 18 & 0 \end{array}$$