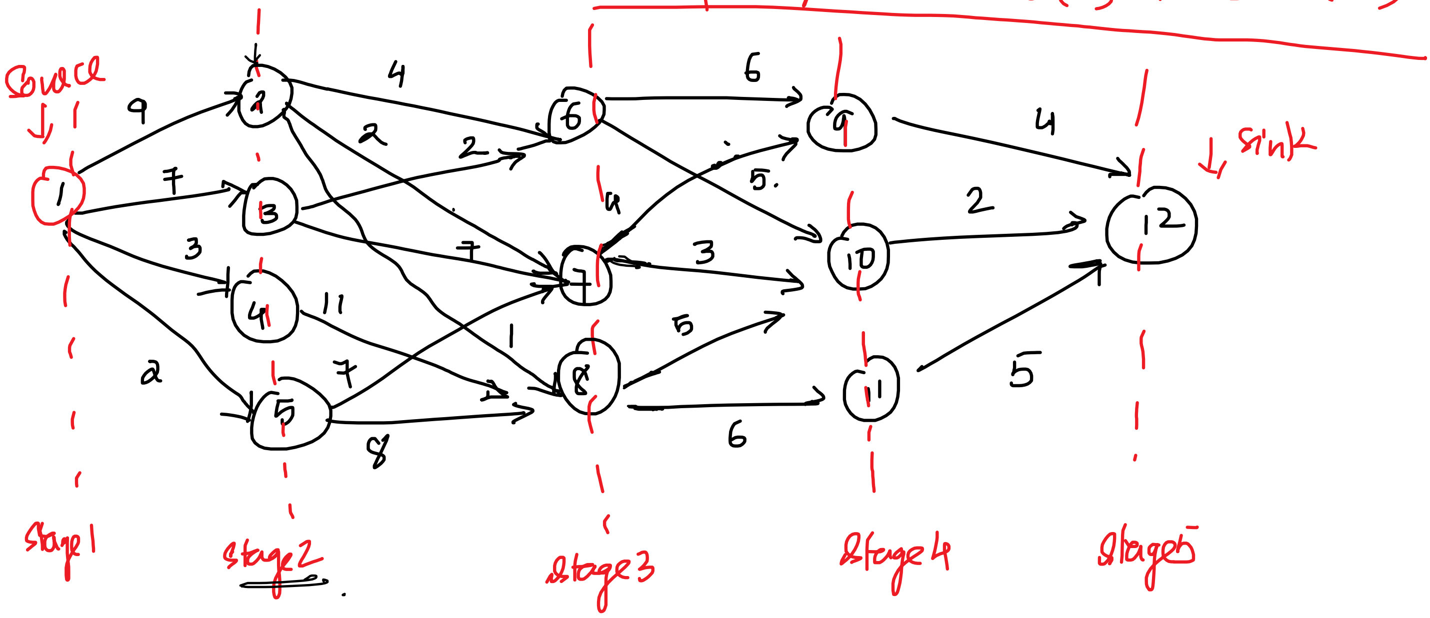
Dynamic Programming

1 Multistage Graph →

Example > Consider >

Shortest path from Source (1) to Sink (12)



Observations > All the Edges are directed

→ The Edge has some direction from source toward Sink

* The Edge are from $i-1^{th}$ stage to i^{th} stage

* We divided the overall graph into diff stages and collected nodes in form of stages

Ex In stage 1 = {1}

stage 2 = {2, 3, 4, 5}

stage 3 = {6, 7, 8}

stage 4 = {9, 10, 11}

stage 5 = {12}

Solution → [We will start from sink and approach is forward approach]

Step 1 →

Stage 5

$Cost(5, 12) = 0$

stage NO node no

[stage 5 pe node 12 se aage ka cost]

Step 2 > stage 4

$cost(4, 9) = C_{9,12} = 4$, $D(4,9) = 12$

$cost(4, 10) = C_{10,12} = 2$, $\boxed{D(4,10) = 12}$

$cost(4, 11) = C_{11,12} = 5$, $D(4,11) = 12$

Step 3 > Stage 3 >

$cost(3, 6) = \min\{ \underbrace{C_{6,9}}_{6 \text{ to } 9} + \underbrace{cost(4,9)}_{9 \text{ se aage}}, \underbrace{C_{6,10}}_{6 \text{ to } 10} + \underbrace{cost(4,10)}_{10 \text{ se aage}} \}$

$= \min\{6 + 4, 5 + 2\}$

$= \min\{10, 7\}$

$= 7$

$D(3, 6) = 10$ (from node 6 in stage 3 we have taken decision to go to node 10).

$cost(3, 7) = \min\{ C_{7,9} + cost(4, 9), C_{7,10} + cost(4,10) \}$

edge cost

stage 3 node 7

$= \min\{4 + 4, 3 + 2\}$

$= \min\{8, 5\}$

$= 5$

$$\boxed{D(3,7) = 10}$$

$$cost(3,8) = \min\{c_{8,10} + cost(4,10),\ c_{8,11} + cost(4,11)\}$$
$$= \min\{5 + 2,\ 6 + 5\}$$
$$= \min\{7,\ 11\}$$
$$= 7$$

$$\boxed{D(3,8) = 10}$$

Step3 > Stage 2

$$cost(2,2) = \min\{c_{2,6} + cost(3,6),\ c_{2,7} + cost(3,7),\ c_{2,8} + cost(3,8)\}$$
$$= \min\{4 + 7,\ 2 + 5,\ 1 + 7\}$$
$$= \min\{11,\ 7,\ 8\}$$

$$\boxed{D(2,2) = 7}$$

$$cost(\underset{\substack{\uparrow \\ \text{stage}}}{2},\underset{\substack{\uparrow \\ \text{node}}}{3}) = \min\{c_{3,6} + cost(3,6),\ c_{3,7} + cost(3,7)\}$$
$$= \min\{2 + 7,\ 7 + 5\}$$
$$= 9$$

$$\boxed{D(2,3) = 6}$$

$$cost(2,4) = \min\{c_{4,8} + cost(3,8)\}$$
$$= 11 + 7 = 18$$

$$\boxed{D(2,4) = 8}$$

$$cost(2,5) = \min\{c_{5,7} + cost(3,7),\ c_{5,8} + cost(3,8)\}$$
$$= \min\{7 + 5,\ 8 + 7\} = 12$$

$$\boxed{D(2,5) = 7}$$

step 5: Stage 1 →

$$cost(1,1) = \min\{c_{1,2} + cost(2,2),\ c_{1,3} + cost(2,3),\ c_{1,4} + cost(2,4),\ c_{1,5} + cost(2,5)\}$$

$$= \min\{9+7,\ 7+9,\ 3+18,\ 2+12\}$$
$$= \min\{16,\ 16,\ 21,\ 14\}$$

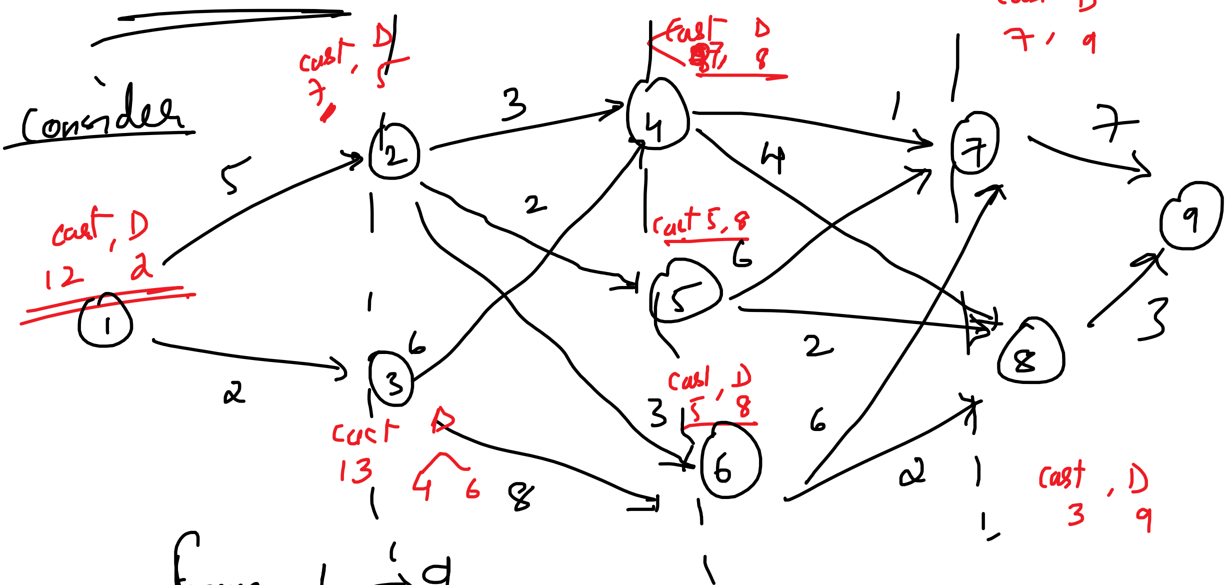
$cost(1,1) = 14$

$D(1,1) = 5$

Path →

$1 \rightarrow 5 \rightarrow 7 \rightarrow 10 \rightarrow 12$ Total cost = 14

Consider



from $1 \rightarrow 9$

$1 - 2 - 5 - 8 - 9$ cost = 12

Multistage graph → It is directed graph where the vertices are partitioned into $\underline{K}$ disjointe Subset where $\underline{K \geq 2}$

Stages are represented as $V_1, V_2, V_3 \ldots V_K$

If Edge $\langle u, v \rangle \in E(G)$
then $\underline{u \in V_i}$ [i^{th} stage]
$\underline{v \in V_{i+1}}$ [next stage]

Here V_1 and V_K are Source and Sink stages
Such that $|V_1| = 1$ & $|V_K| = 1$
[no of vertex in stage 1] [No of vertex in stage K is 1]

The cost of the path from Source(s) and to Sink(t) is sum of cost of all the edges on the path from $\underline{s \text{ to } t}$

Approach →
Here we implement forward approach where the decision involves which vertex from the next stage is to be taken along the path

formula -
$$\text{Cost}(\underset{\substack{\uparrow \\ \text{stage no}}}{i}, \underset{\substack{\uparrow \\ \text{vertex no}}}{j}) = \min\{ \underline{C_{j,l}} + \text{cost}(\underline{i+1}, \underline{l}) \}$$

j se l ka cost [l is in next stage]

[Next stage main l se aage ka cost]

where $\underline{<j, l>} \in E(G)$ [means there must be edge from j to l]

$\underline{l \in V_{i+1}}$ } l vertex belongs to next stage

$D(i,j)$ = Decision taken on stage i and vertex j to select which particular vertex l from $\underline{i+1}$ stage

[Minimum cost]

Algo → function M-graph (c, k, n, p)

{ n = no of vertices

c = cost of adjacency matrix

k = no of stages

P = path array

Integer cost [1 : n], D[1 : n], j, r, min.

1 start

2 cost (n) = 0

(last ~~stage~~ second vertex to first)

6 for j = $\underline{n-1 \text{ to } 1}$ do

{ min = ∞

for (r = $\underline{1 \text{ to } n}$) do

[we know that rth vertex is in which stage]

rth vertex

{ if ($\underline{c(j, r) \neq 0}$ and c(j, r) + cost(r) < min)

edge has

then

{ min = $\underline{c(j,r) + cost(r)}$

Pmin = r

}

cost (j) = min

$D(j) = r_{min}$

}

}

stage 1 vertex 1   At stage k vertex n

$P(1) = 1 \quad P(k) = n$

for $j = 2$ to $k-1$ do → from stage 2 to second last stage.

$P(j) = D(P(j-1))$

Decision taken a stage before

return

— j^{th} stage ka kaunsa vertex

}