Introduction to Algorithms

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IIIT, Delhi
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About Myself

Name: Subhabrata Samajder

Research Interests:

- Lattice based cryptography
- Statistical aspects of symmetric key cryptanalysis
- Searchable Symmetric Encryption (SSE)
- e-Voting
- Broadcast Encryption
- Blockchain
- Random graphs

About Myself

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Office Hours: By appointment

Course Webpage: On Google classroom

- Class Code: l4o5nrw

Academic Integrity Policy

- Anyone caught cheating or copying will be penalised.
- Plagiarism cases will be *dealt strictly*.
- Take this opportunity to stay away from plagiarism forever.

Grading plan: Tentative Grading Components

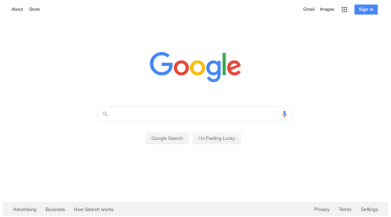
Components	Number	Weightage
MidSem Theory	1	30%
EndSem Theory	1	35%
Weekly Coding Assignments	≥ 6	20%
Quiz and/or Homework	≥ 4	15%
Bonus (Lab attendance + Lab Tests)	NA	5%

Introduction to Algorithms

Algorithms: In Our Daily Lives

Cooking

Traffic Lights



Google Search

Sorting Vinyl Records

Work Commute

Online Shopping

Algorithms: Rubik's Cube

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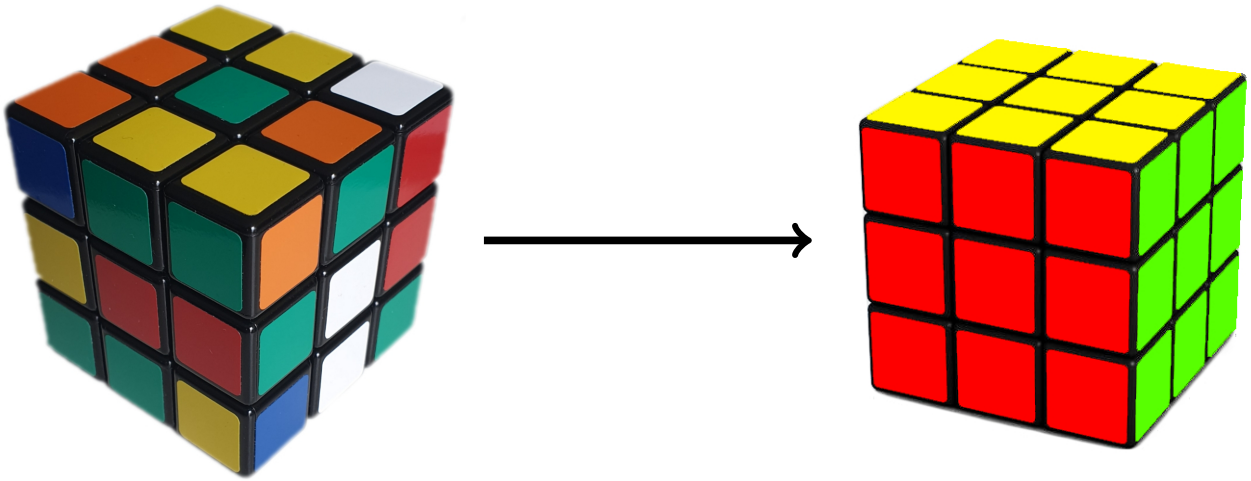

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Intuitive answer: It is a finite sequence of elementary operations with the objective of performing some (computational) task.

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Elementary Operations

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The elementary operations that we will consider will be at a higher level and include arithmetic and logical operations.

Finiteness

Algorithms: Finiteness $\Rightarrow$ an algorithm must stop.

- **Example:** Compute $(a + b) * (c + d)$

$$t_1 = a + b; \quad t_2 = c + d; \quad t_3 = t_1 * t_2.$$

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Computational Method: A procedure that has all of the characteristics of an algorithm except that it possibly *lacks finiteness*.

- **Example:** $\text{while}(1)\{\}$

Sequence

$$t_1 = a + b; \quad t_2 = c + d; \quad t_3 = t_1 * t_2.$$

- We emphasise on the sequential nature of the procedure.

- Any permutation of this sequence *does not* give the same desired output.
  - **Example:** $t_3 = t_1 * t_2; \quad t_2 = c + d; \quad t_1 = a + b$, is not the same as the algorithm above.

Finite Sequence

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Note: Sometimes different orderings of the operations may give rise to the *same* result.

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- **Note:** $t_1 = a + b$ and $t_2 = c + d$ can be executed independently of each other.
- **Single computing unit (a processor):** *Sequential execution*.
- **Two computing unit:** Can be executed simultaneously!

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We would only be concentrating on sequential algorithms!!

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- *Example:* a, b, c, d.

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  Example: Searching Problem
  - **I/P:** A list L of integer values and another value v.
  - **Question:** Does $v \in L$?
  - **O/P:** '*yes*' if $v \in L$; else it returns '*no*'.

Note: *Decision problems appear rather simple but much of the sophistication of the area of algorithms can be discovered by studying such algorithms*!!

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- Here, resources $\equiv$ the time of execution and the space required by the algorithm.

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- **Other resources:** For example, power consumption is important for battery operated devices.

Books Consulted

1. Chapter 2 of *A Course on Cooperative Game Theory* by Satya R. Chakravarty, Palash Sarkar and Manipushpak Mitra.

2. *Introduction to Algorithms: A Creative Approach* by Udi Manber.

Thank You for your kind attention!