CF969-7-SP: Machine Learning for Finance

Assignment 1

PORTFOLIO OPTIMISATION WITH LINEAR REGRESSION

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Portfolio Optimisation Report

INTRODUCTION

In this analysis, we applied the Capital Asset Pricing Model (CAPM) to estimate the expected returns, risk factors, and portfolio optimisation for a selection of 10 stocks:
Apple (AAPL), Intel (INTC), Google (GOOGL), Amazon (AMZN), Tesla (TSLA), NVIDIA (NVDA), IBM (IBM), Meta (META), Qualcomm (QCOM), and Toyota (TM).

We performed a linear regression to compute the alpha (α) and beta (β) for each stock to see how they relate to the market. Based on that output, we derived the optimised portfolio minimising risk for different target returns. Finally, we created the Efficient Frontier which displays the optimal risk-return trade-offs.

The analysis also describes the principles of diversification, risk management, and investment decision-making so investors can know how to best allocate their investments among different alternatives.

Data collection:

We collected historical stock price data for 10 stocks (AAPL, INTC, GOOGL, AMZN, TSLA, NVDA, IBM, META, QCOM, TM) and the S&P 500 index (^GSPC) from Yahoo Finance over a 5-year period, spanning from March 2, 2020, to March 2, 2025.

- Daily returns were calculated as the percentage change in adjusted closing prices.
- The S&P 500 index (^GSPC) was used as the market benchmark to compare individual stock movements.
- The risk-free rate was set to 2% **(0.02).**

This dataset serves as the foundation for estimating expected returns, betas, and portfolio risk, which will be used in the next steps for regression analysis and optimisation.

Linear regression for expected returns and risk:

We performed linear regression for each asset using the CAPM equation: $R_i - R_f = \alpha_i + \beta_i(R_m - R_f) + \epsilon_i$
The estimated parameters are:

Alpha (α): Measures asset-specific return deviation.
Beta (β): Measures sensitivity to market movements.
Idiosyncratic risk: Estimated from regression residuals.

The estimated parameters for each asset are summarised in the table below.

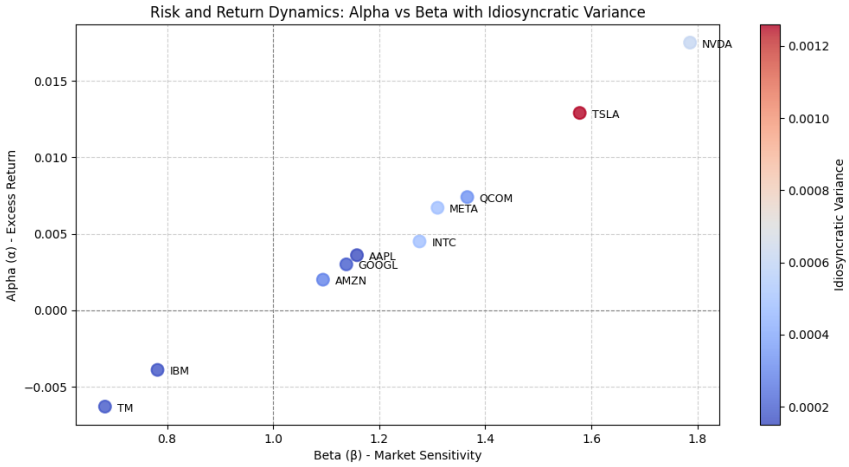
Risk and Return Analysis: Evaluating Stocks Based on Alpha, Beta, and Idiosyncratic Variance

Stock	Alpha (α)	Beta (β)	Idiosyncratic Variance	Interpretation
NVDA	0.0175	1.786	0.00061	Very high return potential but extremely volatile, significantly influenced by the market. Suitable for aggressive investors.
TSLA	0.0129	1.578	0.00126	High return potential, but very risky. Market-driven and highly affected by company-specific factors (high residual variance).
AAPL	0.0036	1.158	0.00015	Steady performer with moderate risk and return, closely follows market trends. Good for balanced portfolios.
META	0.0067	1.310	0.00049	Strong growth stock with slightly high volatility but good return expectations.
GOOGL	0.0030	1.138	0.00019	Stable growth stock with low risk and moderate return. A good balance between risk and reward.

Stock	Alpha (α)	Beta (β)	Idiosyncratic Variance	Interpretation
AMZN	0.0020	1.094	0.00029	Market-aligned return expectations with manageable risk. Suitable for diversification.
QCOM	0.0074	1.366	0.00034	Higher-than-average return, but comes with increased volatility. Good for tech-heavy portfolios.
INTC	0.0045	1.276	0.00049	Stable but slightly volatile. Moderate return with exposure to tech sector growth.
IBM	-0.0039	0.782	0.00017	Underperforming stock with low risk but negative alpha. Likely a defensive stock.
TM	-0.0063	0.683	0.00018	Low volatility but has historically underperformed expectations. Good for risk-averse investors.

Interpretation of Regression Results(significance of αi and βi)):

- **High-return, high-risk stocks:** NVDA (1.786 β, 0.0175 α), TSLA (1.578 β, 0.0129 α) – Very volatile but strong potential returns.
- **Balanced stocks:** AAPL (1.158 β), GOOGL (1.138 β), AMZN (1.094 β) – Moderate risk and return, stable choices.
- **Growth stocks with some risk:** META (1.310 β), QCOM (1.366 β), INTC (1.276 β) – Good return potential with moderate volatility.
- **Defensive stocks:** IBM (0.782 β, -0.0039 α), TM (0.683 β, -0.0063 α) – Low volatility but underperforming expectations.



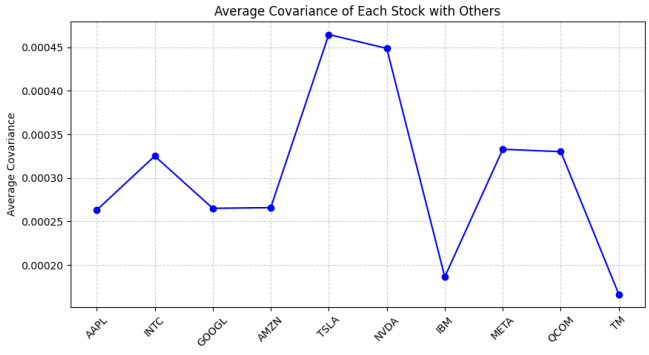
- Higher beta stocks (NVDA, TSLA) move aggressively with the market.
- AAPL, GOOGL, and AMZN provide stability with solid returns.
- IBM and TM are safer but less rewarding investments.

Portfolio optimisation:

Covariance Matrix Construction:

The covariance matrix was constructed using systematic and idiosyncratic risk.

- Systematic Risk: Risk driven by the market, affecting all stocks (computed using beta values).
- Idiosyncratic Risk: Stock-specific risk that is not explained by market movements (computed using CAPM residuals).



- Stocks with **higher beta values** (NVDA, TSLA) had stronger covariance with other stocks, indicating higher systematic risk.
- Stocks with **lower beta values** (IBM, TM) had weaker covariance, suggesting they provide stability in a diversified portfolio.
- The market variance was calculated as **0.00017787**, showing the overall risk level in the market.

Portfolio optimisation:

The optimisation problem was formulated as: $\min \frac{1}{2} w^T \Sigma w$

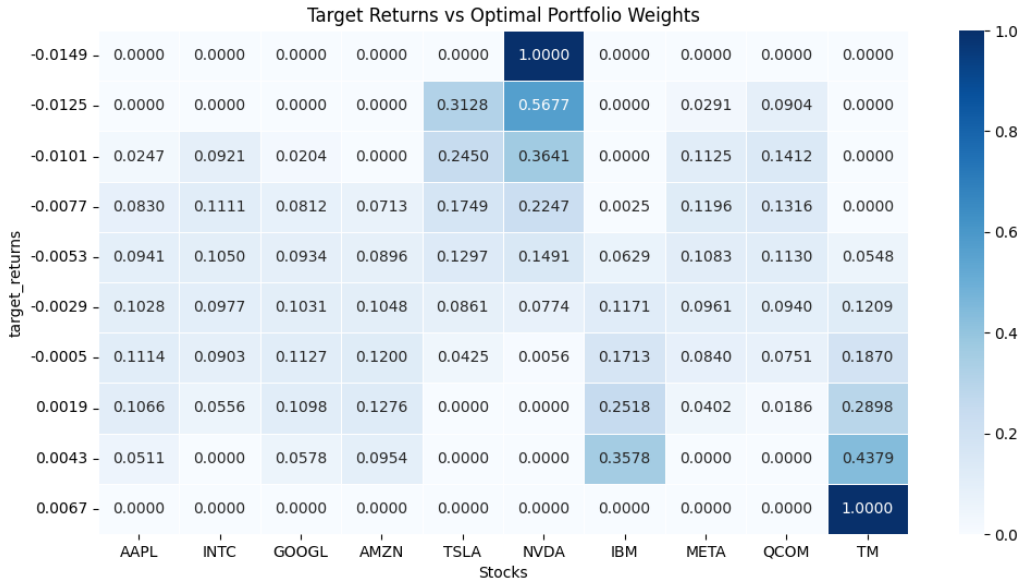
subject to: $w^T \mu = \mu_p, \quad w^T 1 = 1, \quad w \geq 0$

Optimisation Method: Portfolio optimisation was performed using **Sequential Least SQuares programming (SLSQP)** algorithm to find optimal stock weights for different return targets.

- Beta values were incorporated to adjust for systematic risk.
- The optimiser allocated weights to stocks based on return targets.
- The target returns were **generated as 10 evenly spaced values** between the minimum and maximum expected returns of the stocks, ensuring a diverse range of portfolio allocations from conservative (low risk) to aggressive strategies (high risk). The target returns **(μ_p)** are

[-0.0146, -0.0122, -0.0098, -0.0075, -0.0051, -0.0027, -0.0003, 0.0020, 0.0044, 0.0068].

The optimal weights for different target returns were calculated



Analysis of Optimal Portfolio Weights and Sensitivity to Expected Returns & Risks:

- At low target returns, the optimiser allocates more weight to low-beta stocks (IBM, TM) to minimize volatility.
- At moderate target returns, a mix of AAPL, GOOGL, META, and AMZN is included for diversification and balanced risk.
- At high target returns, the portfolio heavily invests in high-beta stocks (NVDA, TSLA) due to their strong return potential but higher risk.

❖ Sensitivity to Expected Returns:

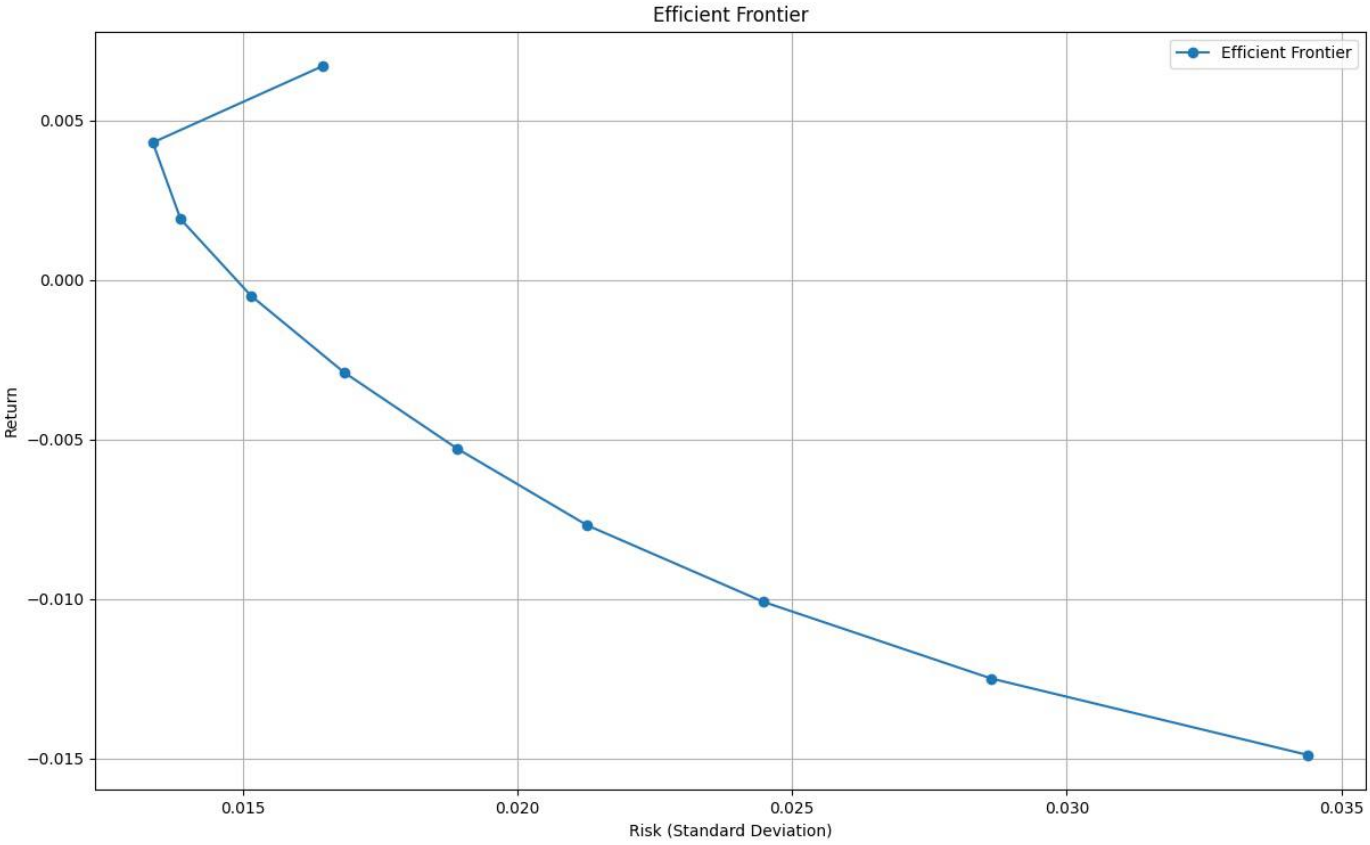
- Higher expected returns shift allocations toward riskier stocks (TSLA, NVDA).
- Lower expected returns favor safer, low-volatility assets (IBM, TM).
- Overestimated expected returns may cause excessive exposure to market risk.

❖ Sensitivity to Risk (Covariance Matrix Adjustments):

- If market volatility increases, the optimiser shifts toward defensive stocks with lower beta.
- If stock-specific risks increase, stocks with high idiosyncratic variance (TSLA, NVDA) get lower weights.
- Lower overall risk levels allow for greater exposure to growth stocks.

Efficient Frontier:

The efficient frontier was plotted based on varying target returns and their respective portfolio risks.
The results are shown in the figure below:



Efficient Frontier Analysis & Portfolio Diversification:

The Efficient Frontier was plotted for 10 target returns, ranging from -0.0146 to 0.0068, with corresponding minimum risk (standard deviation) values.

❖ Results & Shape of the Efficient Frontier:

- The curve is downward-sloping for most of its range, which is unusual, as efficient frontiers typically slope upward.
- The lowest-risk portfolio occurs at 0.0134 standard deviation, corresponding to a return of -0.0003, emphasising a low-risk, defensive portfolio.
- The highest-risk portfolio (0.0344 std dev) corresponds to the lowest target return (-0.0146), showing that excessive risk does not always guarantee higher returns.

❖ Implications for Portfolio Diversification:

- **Risk Reduction:** Portfolios on the lower end of the frontier (lower standard deviation) are composed of low-beta, low-volatility stocks, offering stability.

- **Return Enhancement:** The higher end of the frontier consists of high-beta stocks (e.g., TSLA, NVDA), meaning higher risk but potential for higher returns.
- **Diversification Benefit:** The curve suggests limited diversification benefits since the increase in return comes with a relatively high increase in risk.
- **Unusual Shape:** The curve bending downward at the top suggests some portfolios have high volatility without necessarily achieving high returns, possibly due to inefficient asset combinations or constraints in optimisation.

❖ Interpretation:

- A well-diversified portfolio helps reduce risk, as seen in the lower-risk region of the frontier.

For higher returns, investors must take on significantly more risk, indicating that diversification cannot eliminate all risks.

- The slight downturn at the top suggests that adding more risk does not always lead to proportionally higher returns, emphasising the need for careful asset selection.

Investors should focus on diversified portfolios near the efficient frontier's lowest-risk point, as excessive risk does not necessarily lead to better returns.

Coding Choices:

- **Data Collection:** Used adjusted close prices for accuracy and a 5-year period for stability.
- **Regression Analysis:** Used OLS regression to estimate alpha & beta with CAPM.
- **Portfolio Optimisation:** Used **Sequential Least SQuares Programming (SLSQP)** solver for risk minimization and efficiently handles equality constraints (portfolio weights sum to 1) and inequality constraints (no short selling).
- **Target Returns (μ_p) Selection:** Used evenly spaced values between min and max expected returns to ensure diverse portfolio allocations.
- **Efficient Frontier Plotting:** Used minimum risk for each return and plotted it with clear visualisation features.
- These choices ensure an accurate, efficient, and interpretable portfolio optimisation process.

CONCLUSION

This report analysed portfolio optimisation using the Capital Asset Pricing Model (CAPM), focusing on expected returns, risk estimation, and efficient asset allocation. We applied OLS regression to estimate alpha (α) and beta (β) values for 10 selected stocks, constructed the covariance matrix, and used quadratic programming (SLSQP) to minimize portfolio risk while achieving different target returns.

The Efficient Frontier exhibited an unusual downward-sloping shape, suggesting that higher risk does not always lead to proportionally higher returns, indicating the presence of inefficient asset combinations or optimisation constraints. This reinforces that diversification alone cannot always improve risk-adjusted returns. However, the frontier still demonstrates the trade-off between risk and return, where higher returns require significantly more risk, while diversification helps reduce overall portfolio volatility in the lower-risk region. The analysis confirms that excessive risk does not guarantee higher returns, emphasising the importance of optimal asset selection in portfolio management.

Overall, this study demonstrates how quantitative finance techniques can help investors construct efficient portfolios, balancing return expectations with risk tolerance. The results reinforce the importance of diversification and data-driven decision-making in portfolio management.

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- Sharpe, W. F. (1964). Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk. Journal of Finance, 19(3), 425-442.
- Markowitz, H. (1952). Portfolio Selection. Journal of Finance, 7(1), 77-91.