1. (a) $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ (b) $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

(c) $(a+b)^2 = a^2 + b^2 + 2ab$ (d) $(a-b)^2 = a^2 + b^2 - 2ab$

(e) $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ (f) $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

(g) $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$ (h) $a^3 - b^3 = (a-b)^3 + 3ab(a-b)$

(i) $a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2}\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right]$

(j) $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$

2.

$\sin(90^\circ \pm \theta) = {}^{+}_{+}\cos\theta$	$\sin(180^\circ \pm \theta) = \mp \sin\theta$	$\sin(270^\circ \pm \theta) = \cos\theta$	$\sin(360^\circ \pm \theta) = \pm \sin\theta$
$\cos(90^\circ \pm \theta) = \mp \sin\theta$	$\cos(180^\circ \pm \theta) = \cos\theta$	$\cos(270^\circ \pm \theta) = \pm \sin\theta$	$\cos(360^\circ \pm \theta) = \pm \cos\theta$
$\tan(90^\circ \pm \theta) = \mp \cot\theta$	$\tan(180^\circ \pm \theta) = \pm \tan\theta$	$\tan(270^\circ \pm \theta) = \mp \cot\theta$	$\tan(360^\circ \pm \theta) = \pm \tan\theta$

3. Limits of the values of trigonometrical functions.

(a) $-1 \le \sin A \le 1$ (b) $-1 \le \cos A \le 1$ (c) $\sec A \le -1$ or $\sec A \ge 1$

(d) $\operatorname{cosec} A \le -1$ or $\operatorname{cosec} A \ge 1$ (e) $-\infty < \tan A < \infty$ (f) $-\infty < \cot A < \infty$

(g) Each interior angle of a regular polygon of n sides $= \frac{n-2}{n} \times 180$

4.

	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	240°	270°	300°	360°	960°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	0	$-\frac{\sqrt{3}}{2}$
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$-\frac{1}{2}$
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	∞	$-\sqrt{3}$	0	$\sqrt{3}$

5. (a) $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ (b) $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

(c) $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$ (d) $\cot(A \pm B) = \dfrac{\cot A \cot B \mp 1}{\cot B \pm \cot A}$

(e) $\tan(A + B + C) = \dfrac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$

6. (a) $\sin(A + B) + \sin(A - B) = 2\sin A \cos B$ (b) $\sin(A + B) - \sin(A - B) = 2\cos A \sin B$

(c) $\cos(A + B) + \cos(A - B) = 2\cos A \cos B$ (d) $-\cos(A + B) + \cos(A - B) = 2\sin A \sin B$

(e) $\sin C + \sin D = 2\sin\left(\dfrac{C+D)}{2}\right)\cos\left(\dfrac{C-D)}{2}\right)$ (f) $\sin C - \sin D = 2\cos\left(\dfrac{C+D)}{2}\right)\sin\left(\dfrac{C-D)}{2}\right)$

(g) $\cos C + \cos D = 2\cos\left(\dfrac{C+D}{2}\right)\cos\left(\dfrac{C-D}{2}\right)$ (h) $\cos C - \cos D = -2\sin\left(\dfrac{C+D}{2}\right)\sin\left(\dfrac{C-D}{2}\right)$

7. (a) $\sin 2A = 2\sin A \cos A = \dfrac{2\tan A}{1+\tan^2 A}$ (b) $\sin A = 2\sin\dfrac{A}{2}\cos\dfrac{A}{2} = \dfrac{2\tan\dfrac{A}{2}}{1+\tan^2\dfrac{A}{2}}$

(c) $\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A \quad = \cos^2 A - \sin^2 A \quad = \dfrac{1-\tan^2 A}{1+\tan^2 A}$

(d) $\cos A = 2\cos^2\dfrac{A}{2} - 1 \quad = 1 - 2\sin^2\dfrac{A}{2} \quad = \cos^2\dfrac{A}{2} - \sin^2\dfrac{A}{2} \quad = \dfrac{1-\tan^2\dfrac{A}{2}}{1+\tan^2\dfrac{A}{2}}$

(e) $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$; $\tan A = \dfrac{2\tan A/2}{1-\tan^2 A/2}$

8. $\sin(-\theta) = -\sin\theta$; $\cos(-\theta) = +\cos\theta$; $\tan(-\theta) = -\tan\theta$

9. $\sin(A + B)\sin(A - B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$

$\cos(A + B)\cos(A - B) = \cos^2 A - \sin^2 B \quad = \cos^2 B - \sin^2 A$

10. $\sin 3A = 3\sin A - 4\sin^3 A$ $\cos 3A = 4\cos^3 A - 3\cos A$

$\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$ $\cot 3A = \dfrac{\cot^3 A - 3\cot A}{3\cot^2 A - 1}$

11. $\sin 18^\circ = \dfrac{\sqrt{5}-1}{4}$; $\cos 36^\circ = \dfrac{\sqrt{5}+1}{4}$

12. If $T = a\cos\theta + b\sin\theta$, then $T_{max} = \sqrt{a^2+b^2}$, $T_{min} = -\sqrt{a^2+b^2}$

13. (a) If $\sin\theta = 0$ or $\tan\theta = 0$ then $\theta = n\pi$ (b) If $\cos\theta = 0$ or $\cot\theta = 0$, then $\theta = (2n+1)\dfrac{\pi}{2}$

(c) If $\sin\theta = \sin\alpha$, then $\theta = n\pi + (-1)^n\alpha$ (d) If $\cos\theta = \cos\alpha$, then $\theta = 2n\pi \pm \alpha$

(e) If $\tan\theta = \tan\alpha$, then $\theta = n\pi + \alpha$ when $n = 0, \pm 1, \pm 2, \ldots\ldots\ldots$

14. In a ΔABC, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$, where R is the radius of the circumcircle of ΔABC.

15. **Tangent Rule :** In any ΔABC :

(i) $\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2}$

(ii) $\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2}$

(iii) $\tan\frac{C-A}{2} = \frac{c-a}{c+a}\cot\frac{B}{2}$

16. **Cosine Rule :** In any ΔABC :

(i) $\cos A = \frac{b^2+c^2-a^2}{2bc}$

(ii) $\cos B = \frac{c^2+a^2-b^2}{2ac}$

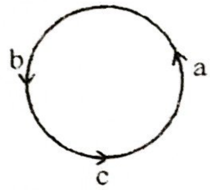
(iii) $\cos C = \frac{a^2+b^2-c^2}{2ab}$

17. **Projection Formulae :** In any ΔABC :

(i) $c = a\cos B + b\cos A$

(ii) $b = a\cos C + c\cos A$

(iii) $a = b\cos C + c\cos B$



18. (i) $\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$

(ii) $\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$

(iii) $\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$ where $2s = a+b+c$

19. If Δ be the area of ΔABC, then : $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$.

20. In any ΔABC :

(i) $\sin A = \frac{2}{bc}\sqrt{s(s-a)(s-b)(s-c)}$

(ii) $\sin B = \frac{2}{ca}\sqrt{s(s-a)(s-b)(s-c)}$

(iii) $\sin C = \frac{2}{ab}\sqrt{s(s-a)(s-b)(s-c)}$

21. **Area of triangle :** If Δ denotes the area of ΔABC, then

(i) $\Delta = \frac{1}{2}ab\sin C$ (ii) $\Delta = \frac{1}{2}bc\sin A$ (iii) $\Delta = \frac{1}{2}ca\sin B$

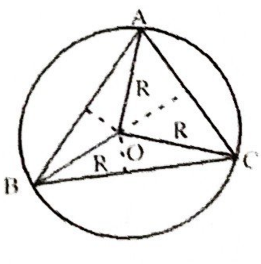
22. If Δ be the area of ΔABC and R be the radius of its circumcircle, then $\Delta = \frac{abc}{4R}$.

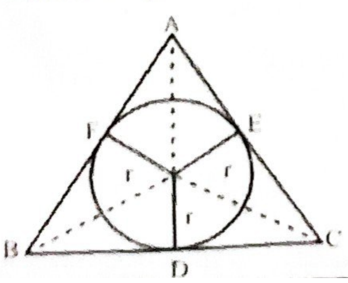
23. In any ΔABC :

(i) $\tan\frac{A}{2} = \frac{(s-b)(s-c)}{\Delta}$ (ii) $\cot\frac{A}{2} = \frac{s(s-a)}{\Delta}$

24. **Circum-radius, In-radius and Ex-radius**

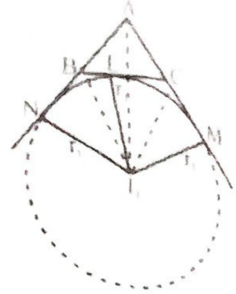
Circle ABC is known as circumcircle. Its centre O is the point where **perpendicular bisectors** of the sides of the triangle meet. Point O is known as circumcentre and its radii OA, OB, OC are denoted by R and are known as circumradius.





Circle DEF is known as in-circle. Its centre I is the point where internal bisectors of the angles of the triangle meet. Point I is known as **In - centre** and its radii ID, IE, IF are denoted by r and are known as **In - radius**.

Circle LMN is known as escribed circle. Its centre I_1 is the point where internal bisector of angle A and external bisectors of angles B and C meet. Point I_1 is known as Ex-centre and its radii I_1L, I_1M, I_1N are denoted by r_1 and is known as ex-radius opposite to A.



Similarly we have two more radii **ex-radii**, r_2 opposite to B and r_3 opposite to C.

Escribed Circle : $I_1 = \text{Ex} - \text{centre}$

$r_1 = \text{Ex} - \text{radius opposite to A}$

25. **Some More Results**

I. (a) $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = 2R$ (b) $R = \dfrac{abc}{4\Delta}$ (c) $\Delta = \dfrac{1}{2} bc \sin A$

(d) $\tan\dfrac{A}{2} = \dfrac{\Delta}{s(s-a)} = \dfrac{(s-b)(s-c)}{\Delta}$

II. (a) $r = \dfrac{\Delta}{s} = (s-a)\tan\dfrac{A}{2} = (s-b)\tan\dfrac{B}{2} = (s-c)\tan\dfrac{C}{2} = 4R\sin\dfrac{A}{2}\sin\dfrac{B}{2}\sin\dfrac{C}{2}$

(b) $r_1 = \dfrac{\Delta}{s-a} = s\tan\dfrac{A}{2} = 4R\sin\dfrac{A}{2}\cos\dfrac{B}{2}\cos\dfrac{C}{2} = \dfrac{a\cos(B/2)\cos(C/2)}{\cos(A/2)}$

(c) $r_2 = \dfrac{\Delta}{s-b} = s\tan\dfrac{B}{2} = 4R\cos\dfrac{A}{2}\sin\dfrac{B}{2}\cos\dfrac{C}{2} = \dfrac{b\cos(C/2)\cos(A/2)}{\cos(B/2)}$

(d) $r_3 = \dfrac{\Delta}{s-c} = s\tan\dfrac{C}{2} = 4R\cos\dfrac{A}{2}\cos\dfrac{B}{2}\sin\dfrac{C}{2} = \dfrac{c\cos(A/2)\cos(B/2)}{\cos(C/2)}$

III.(a) $r_1 + r_2 + r_3 = 4R + r$ (b) $r_1r_2 + r_2r_3 + r_3r_1 = s^2$ (c) $r_1 + r_2 - r_3 + r = 4R\cos C$

Important Identities

IV.(a) $\sum \sin A = 4\cos\dfrac{A}{2}\cos\dfrac{B}{2}\cos\dfrac{C}{2}$ (b) $\sum \sin 2A = 4\sin A\sin B\sin C$

(c) $\sum \cos 2A = -1 - 4\cos A\cos B\cos C$ (d) $\sum \sin^2 A = 2(1 + \cos A\cos B\cos C)$

(e) $\sum \tan A = \tan A\tan B\tan C$ (f) $\sum \cot A\cot B = 1$

(g) $\sum \tan\dfrac{A}{2}\tan\dfrac{B}{2} = 1$ (h) $\sum \cos A = 1 + 4\sin\dfrac{A}{2}\sin\dfrac{B}{2}\sin\dfrac{C}{2}$

Inverse Circular Function

1. Summary

	Function	Domain of the function	Range (Principal Value)
1.	$y = \sin^{-1} x$	$-1 \le x \le 1$	$-\dfrac{\pi}{2} \le y \le \dfrac{\pi}{2}$
2.	$y = \cos^{-1} x$	$-1 \le x \le 1$	$0 \le y \le \pi$
3.	$y = \tan^{-1} x$	$-\infty < x < \infty$	$-\dfrac{\pi}{2} < y < \dfrac{\pi}{2}$
4.	$y = \cot^{-1} x$	$-\infty < x < \infty$	$0 < y < \pi$
5.	$y = \sec^{-1} x$	$x \le -1$ or $x \ge 1$	$0 \le y \le \pi,\ y \ne \dfrac{\pi}{2}$
6.	$y = \text{cosec}^{-1} x$	$x \le -1$ or $x \ge 1$	$-\dfrac{\pi}{2} \le y \le \dfrac{\pi}{2},\ y \ne 0$