Trajectory and Power Control for UAVs

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Outline

Introduction

Nowadays, UAVs are using widely in many areas.

Problem Formulation

To formulate this problem mathematically, we

Main Strategies

- Two main problems
  - Large problem dimension
  - Non-convex optimization

- Two main strategies
  - Heuristic dimension-reduced method
  - Successive convex approximation (SCA)

Main Strategies

Fly-hover-fly Strategy

The fly-hover-fly strategy is described as

- UAVs fly to the hovering location.
- UAVs hover over that location.
- UAVs return to the original position from the hovering location.

Assumption 1

The whole time horizon T is much longer than the time UAVs need to fly to the hovering location.

Main Strategies

Lemma 1

Assume all UAVs return to the initial locations, and the ascending and descending speed are equal, i.e., $V_A = V_D$*. Denote one of the optimal solution of TPC problem as* $\boldsymbol{q}^*$ *and* $\boldsymbol{p}^*$*, then for all* $k \in \mathcal{K}, n \in \mathcal{N}_2^{N+2}$*,*

$$\boldsymbol{q}^*_{[:,k,n]} = \boldsymbol{q}^*_{[:,k,N+3-n]}, \ \boldsymbol{p}^*_{[k,n]} = \boldsymbol{p}^*_{[k,N+3-n]} \tag{1}$$

Main Strategies

Lemma 2

Using the same notations in Lemma 1, *for some* $M_0 \in \{2, 3, \ldots, N+1\}$ *and* $M \in \{2, 3, \ldots, M_0\}$, *we have*

$$R_{s[n]} \leq R_{s[M]} = R_{s[M+1]} = \cdots = R_{s[M_0]}, \ \forall\, n \in \mathcal{N}_2^M \tag{2}$$

where M_0 *is the time when the UAVs need to return from the hovering location, and* M *is the time when UAVs arrives the hovering location. In particular, if* $V_A = V_D$, $M_0 = N + 3 - M$.

Main Strategies

- Reformulation 1

$$
\begin{aligned}
\max_{\substack{\boldsymbol{p}_{[:,2:M]},\\ \boldsymbol{q}_{[:,:,2:M]},\\ M\in\mathcal{N}_2^{(N+1)/2}}} \quad & \sum_{n=2}^{M}\sum_{k=1}^{K} R_{[k,n]}(\boldsymbol{p},\boldsymbol{q}) \\
& \quad + \left(\frac{N+1}{2} - M\right)\sum_{k=1}^{K} R_{[k,M]}(\boldsymbol{p},\boldsymbol{q}) \\
s.t. \quad & (\textbf{??}),(\textbf{??}),(\textbf{??}),(\textbf{??}),\ \forall\, k\in\mathcal{K}, n\in\mathcal{N}_2^M \\
& (\textbf{??}),\ \forall\, k,j\in\mathcal{K}, k<j, n\in\mathcal{N}_2^M
\end{aligned} \tag{3}
$$

- Reformulation 2

$$
\begin{aligned}
\max_{\substack{\boldsymbol{p}_{[:,2:M]},\\ \boldsymbol{q}_{[:,:,2:M]}\\ M\in\mathcal{N}_2^{(N+1)/2}}} \quad & \sum_{n=2}^{M}\sum_{k=1}^{K} R_{[k,n]}(\boldsymbol{p},\boldsymbol{q}) \\
& \quad +\left(\frac{N+1}{2} - M\right)R_s^* \\
s.t. \quad & (\textbf{??}),(\textbf{??}),(\textbf{??}),(\textbf{??}),\ \forall\, k\in\mathcal{K}, n\in\mathcal{N}_2^M \\
& (\textbf{??}),\ \forall\, k,j\in\mathcal{K}, k<j, n\in\mathcal{N}_2^M \\
& \boldsymbol{q}_{[:,k,M]} = \boldsymbol{q}_{\boldsymbol{h}^*_{[:,k]}},\ \boldsymbol{p}_{[k,M]} = \boldsymbol{p}_{\boldsymbol{h}^*_{[k]}},\ \forall\, k\in\mathcal{K}
\end{aligned} \tag{4}
$$

Main Strategies

- Successive convex approximation. Find a locally tight convex surrogate function to replace the objective function or constraints.
- Trick. First-order Taylor's expansion.

$$\frac{x^2}{y} \geq \frac{2\bar{x}}{\bar{y}}x - \frac{\bar{x}^2}{\bar{y}^2}y, \text{ for fixed } \bar{y} > 0$$

$$-\log(1+x) \geq -\log(1+\bar{x}) - \frac{x-\bar{x}}{1+\bar{x}}$$

$$x^2 \geq 2x\bar{x} - \bar{x}^2$$

- Objective function.

$$\begin{aligned} \overline{R}[k,n](\boldsymbol{a},\boldsymbol{q}) \triangleq \quad & \log\left(1+\sum_{j=1}^{K} \frac{G_0(\boldsymbol{a}[k,j])^2}{BN_0\boldsymbol{d}[j,k,n]}\right) \\ & -\log\left(1+\sum_{j=1,j\neq k}^{K} \frac{G_0(\boldsymbol{a}[k,j])^2}{BN_0\boldsymbol{d}[j,k,n]}\right) \end{aligned} \tag{6}$$

Main Strategies

- Surrogate function.

$$\widetilde{R}_{[k,n]}(\boldsymbol{a},\boldsymbol{q};\boldsymbol{a}^r,\boldsymbol{q}^r) \triangleq \log\left(1+\frac{G_0}{BN_0}\sum_{j=1}^{K}\left[\frac{2\boldsymbol{a}^r_{[j,n]}}{\boldsymbol{d}^r_{[j,k,n]}}\boldsymbol{a}_{[j,n]} -\frac{(\boldsymbol{a}^r_{[k,j]})^2}{(\boldsymbol{d}^r_{[j,k,n]})^2}\|\boldsymbol{q}_{[:,j,n]}-\boldsymbol{s}_{[:,k]}\|^2\right]\right) - \log(1+\boldsymbol{I}^r_{[k,n]}) + \frac{\boldsymbol{I}^r_{[k,n]}}{1+\boldsymbol{I}^r_{[k,n]}} - \frac{G_0}{BN_0(1+\boldsymbol{I}^r_{[k,n]})}\sum_{j=1,j\neq k}^{K}\frac{(\boldsymbol{a}_{[j,n]})^2}{\boldsymbol{d}^r_{[j,k,n]}+2(\boldsymbol{q}^r_{[:,j,n]}-\boldsymbol{s}_{[:,k]})^{\mathrm{T}}(\boldsymbol{q}_{[:,j,n]}-\boldsymbol{q}^r_{[:,j,n]})} \quad (7)$$

- Collision avoidance constraint

$$2(\boldsymbol{q}^r_{[:,k,n]}-\boldsymbol{q}^r_{[:,j,n]})^{\mathrm{T}}(\boldsymbol{q}_{[:,k,n]}-\boldsymbol{q}_{[:,j,n]}) \geq \left\|\boldsymbol{q}^r_{[:,k,n]}-\boldsymbol{q}^r_{[:,j,n]}\right\|^2 + d^2_{\min} \quad (8)$$

Main Strategies

Therefore, the convex approximation problem for (**??**) given M is

$$
\begin{aligned}
\max_{\substack{\boldsymbol{a}_{[:,2:M]},\\ \boldsymbol{q}_{[:,:,2:M]}}} \quad & \sum_{n=2}^{M} \sum_{k=1}^{K} \widetilde{R}_{[k,n]}(\boldsymbol{a}, \boldsymbol{q}; \boldsymbol{a}^r, \boldsymbol{q}^r) \\
s.t. \quad & (\textbf{??}), (\textbf{??}), (\textbf{??}), (\textbf{??}),\ \forall\, k \in \mathcal{K}, n \in \mathcal{N}_2^M \\
& (8),\ \forall\, k, j \in \mathcal{K}, k < j, n \in \mathcal{N}_2^M \\
& \boldsymbol{q}_{[:,k,M]} = \boldsymbol{q}_{\boldsymbol{h}^*_{[:,k]}},\ \boldsymbol{p}_{[k,M]} = \boldsymbol{p}_{\boldsymbol{h}^*_{[k]}},\ \forall\, k \in \mathcal{K}
\end{aligned}
\tag{9}
$$

Main Strategies

Algorithm 1 SCA algorithm for solving problem (4)

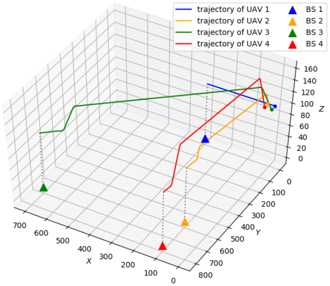
Set iteration index $r = 0$, tolerance $\epsilon > 0$.
Initialize $\boldsymbol{q}^0_{[:,k,n]}$ and $\boldsymbol{p}^0_{[k,n]}$ for $k \in \mathcal{K}, n \in \mathcal{N}_2^M$.
Calculate $R^0 = \sum_{k=1}^{K} \sum_{n=2}^{M} R_{[k,n]}(\boldsymbol{p}^0, \boldsymbol{q}^0)$.
repeat
    Calculate $\left\{\boldsymbol{a}^r_{[k,n]}, \boldsymbol{d}^r_{[j,k,n]}, \boldsymbol{I}^r_{[k,n]}\right\}$ by using $\boldsymbol{p}^r_{[k,n]}$ and $\boldsymbol{q}^r_{[:,k,n]}$.
    Update $\left\{\boldsymbol{a}^{r+1}_{[k,n]}, \boldsymbol{q}^{r+1}_{[:,k,n]}\right\}$ by solving problem (9) with parameters $\boldsymbol{a}^r_{[k,n]}$, $\boldsymbol{d}^r_{[j,k,n]}$, and $\boldsymbol{I}^r_{[k,n]}$.
    Calculate $\boldsymbol{p}^{r+1}_{[k,n]}$ by using $\boldsymbol{a}^{r+1}_{[k,n]}$.
    Set $r = r + 1$.
until $\frac{|R^r - R^{r-1}|}{R^{r-1}} \leq \epsilon$
return $\left\{\boldsymbol{p}^r_{[k,n]}, \boldsymbol{q}^r_{[:,k,n]}\right\}$

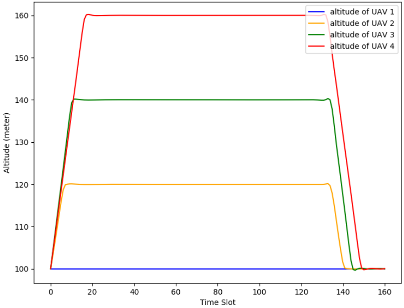
Simulation Result

- Simulation Scale
  - # of UAV-BS pairs: $K = 4$
  - # of time slots: $M = 160$
- Parameter Setup
  - $d_{\min} = 20m$
  - $V_L = 20m/s$, $V_A = V_D = 5m/s$
  - $h_{\min} = 100m$, $h_{\max} = 200m$
  - $P_{\max} = 30dbm$
  - Communication bandwidth: $B = 10MHz$
  - Power spectral density of addictive white Gaussian noise: $N_0 = -160dbm/Hz$
- Initial locations
  - UAV: (0, 0, 100), (30, 0, 100), (0, 30, 100), (30, 30, 100)
  - Base station: (300, 0, 0), (100, 600, 0), (700, 700, 0), (100, 800, 0)

Simulation Result

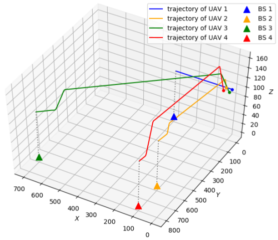
- Initial trajectory
  - Step 1: obtain the hoovering points $q^*_{[:,k,M]}$ and the transmission powers $p^*_{[k,M]}$
  - Step 2: obtain M^* and the initial trajectory $q^0_{[:,k,n]}$

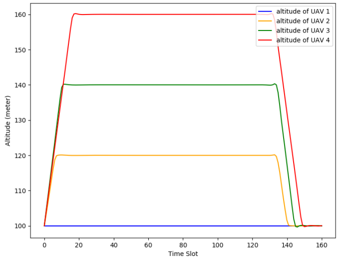




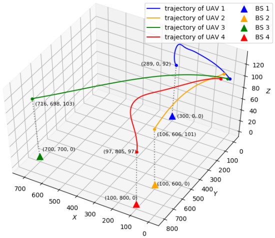
Simulation Result

- Initial trajectory

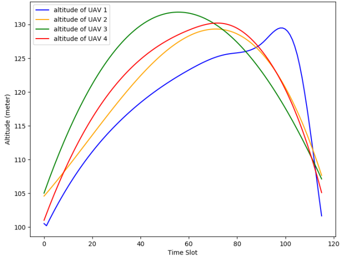




- Optimized trajectory

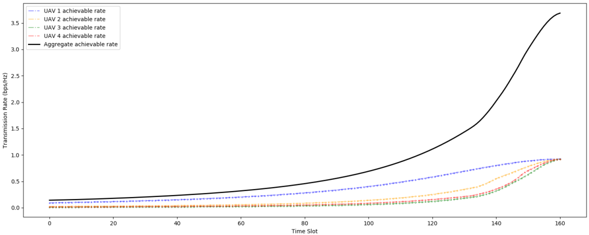


Optimized Trajectory



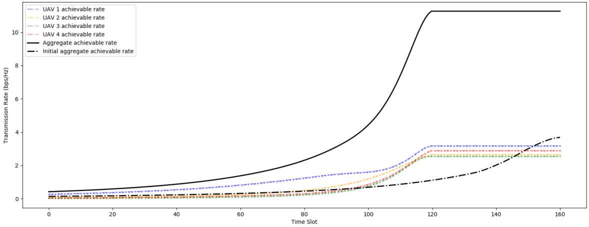
Simulation Result

- Initial achievable transmission rate



Initial Achievable Transmission Rate

- Optimized achievable transmission rate



Optimized Achievable Transmission Rate

Conclusion

In conclusion, we

References