# Structured Sparsification with Joint Optimization of Group Convolution and Channel Shuffle

Xin-Yu Zhang, Kai Zhao, Taihong Xiao, Ming-Ming Cheng, Ming-Hsuan Yang

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- More about Structured Sparsification

## Group Convolution

Group convolution (GroupConv) is used for model compression.

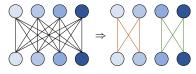


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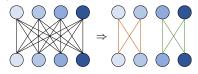


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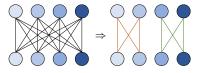


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For  $conv1x1s \Rightarrow GroupConv1x1s$ , the inter-group communication between consecutive GroupConvs?

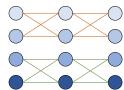


Figure: Consecutive group convs.

#### Channel Shuffle

ShuffleNet<sup>1</sup>: a *channel shuffle* operation (re-distribute channels from different groups).

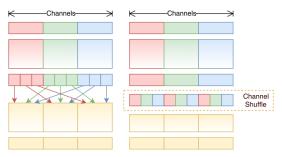


Figure: Channel shuffle in ShuffleNet.

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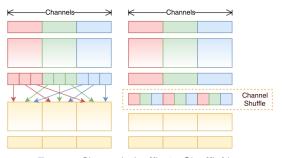


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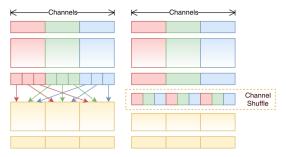


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But still a hand-crafted channel shuffle (uniformly distribute).

We propose a *learnable channel shuffle* mechanism which unifies the norm-based pruning criteria and the learning of channel permutation.

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In particular, weight norm  $\Rightarrow$  indicator of filter importance.

E.g., Network Slimming<sup>2</sup>: prune according to batch-norm scaling factor.

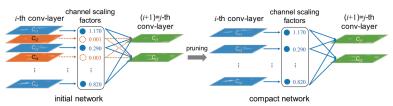


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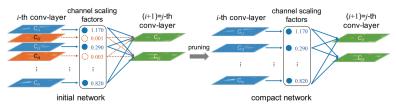


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Besides,  $L_1$  regularization (LASSO)  $\rightarrow$  batch-norm scaling factors.

However, problems of filter pruning:

(i) pruning has to deal with special network structures;

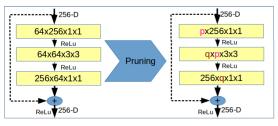


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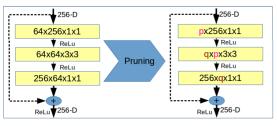


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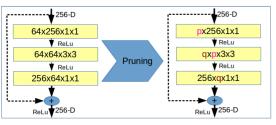


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In this work, we generalize the norm-based pruning criteria to the problem of converting vanilla convolutions into GroupConvs.

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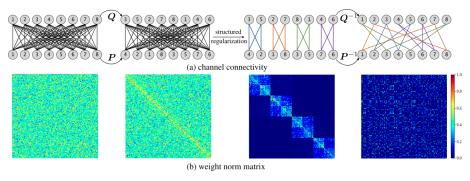


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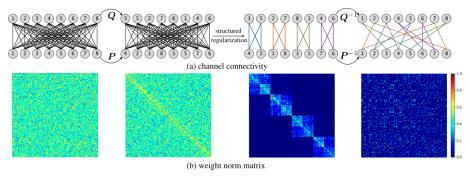


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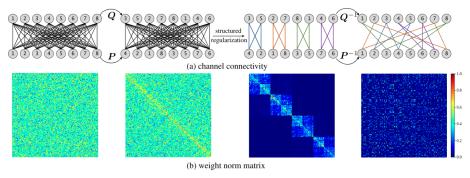


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#### Challenges.

- (i) How to define a suitable channel shuffle? (under what criteria?)
- (ii) How to *structurally* sparsify the convolutional weights?

## Learning Connectivity — Formulation

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In practice, weight norm matrix  $\rightarrow$  block-diagonal only by channel shuffle? Not impossible!

Therefore, aim of channel shuffle: permute weight norm matrix to make it "as block-diagonal as possible". Formally,

$$\begin{aligned} & \underset{P,Q}{\min} & \textit{PSQ} \otimes \textit{R} \\ & \text{s.t.} & \textit{P} \in \mathcal{P}^{\textit{C}^{\textit{out}}} & \textit{and} & \textit{Q} \in \mathcal{P}^{\textit{C}^{\textit{in}}}, \end{aligned} \tag{1}$$

where  $\mathbf{S} \in \mathbb{R}^{C^{out} \times C^{in}}$  is the weight norm matrix,  $\mathbf{R}$  is a cost matrix, and  $\mathcal{P}^N$  is the set of  $N \times N$  permutation matrices.

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$$\mathcal{B}^{N} = \{ \mathbf{X} \in \mathbb{R}_{+}^{N \times N} : \mathbf{X} \mathbf{1}_{N} = \mathbf{1}_{N}, \ \mathbf{X}^{\top} \mathbf{1}_{N} = \mathbf{1}_{N} \}.$$
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When updating P,

$$\min_{\mathbf{P}} \mathbf{P} \otimes \mathbf{R} \mathbf{Q}^{\top} \mathbf{S}^{\top} 
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In (3), the objective function is linear in  $\boldsymbol{P}$  and the feasible region  $\mathcal{B}^N$  is a simplex. Therefore, linear programming (LP), solved by network simplex method.

## Learning Connectivity — Discussion

By LP theory, one solution of a LP problem  $\rightarrow$  vertex of feasible region.

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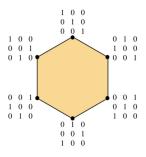


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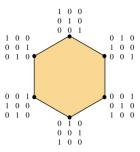


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Therefore, relaxed feasible region  $\mathcal{B}^N$  naturally reduced to  $\mathcal{P}^N$ .

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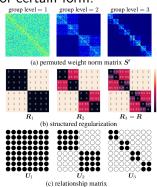
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Structured  $L_1$  regularization.

$$\mathcal{L}_{\mathsf{reg}} = \mathbf{S}' \otimes \mathbf{R}_{\mathsf{g}},$$
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where S' = PSQ permuted weight norm matrix, and  $R_g$  shown on the right.

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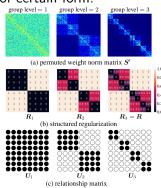
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#### **Grouping Criteria.**

$$g = \max\{g : \mathbf{S}' \otimes \mathbf{U}_g \ge p \sum_{i,j} S_{i,j}, \ g = 1, 2, \cdots\}, \tag{5}$$

where  $U_g$  is the relationship matrix.

## Training Pipeline

#### **Algorithm 1** Training Pipeline.

- 1: Initially update P and Q.
- 2: **for** t := 1 to #epochs **do**
- 3: Train with structured regularization;
- 4: Update P and Q;
- 5: Determine the current group level g by the grouping criteria;
- 6: Update the structured sparsification matrices  $(R_g)$ ;
- 7: Adjust regularization coefficient (refer to paper).
- 8: end for

#### **Experiments**

Performance on ImageNet against two prior works, *i.e.*, Slimming<sup>5</sup> and Taylor<sup>6</sup> (refer to paper for full comparison).

Methods	#Params. $(10^6) \downarrow$	GFLOPs ↓	Acc.(%) ↑				
ResNet-50							
Baseline	25.6	4.14	77.10				
Slimming-20%	17.8	2.81	75.12				
Taylor-19%	17.9	2.66	75.48				
StrucSpars-35%	17.2	3.12	76.82				
Taylor-28%	14.2	2.25	74.50				
StrucSpars-65%	10.3	1.67	75.10				
Taylor-44%	7.9	1.34	71.69				
Slimming-50%	11.1	1.87	71.99				
StrucSpars-85%	5.6	0.90	72.47				
ResNet-101							
Baseline	44.5	7.87	78.64				
Taylor-25%	31.2	4.70	77.35				
StrucSpars-40%	26.7	5.05	78.16				
Taylor-45%	20.7	2.85	75.95				
Slimming-50%	20.9	3.16	75.97				
StrucSpars-65%	16.5	2.98	77.62				
Taylor-60%	13.6	1.76	74.16				
StrucSpars-80%	10.6	1.70	75.73				
DenseNet-201							
Baseline	20.0	4.39	77.88				
Taylor-40%	12.5	3.02	76.51				
StrucSpars-38%	13.1	3.53	77.43				
Taylor-64%	9.0	2.21	75.28				
StrucSpars-60%	9.2	2.10	75.86				

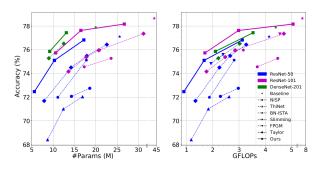
<sup>&</sup>lt;sup>5</sup>Liu et al., Learning Efficient Convolutional Networks through Network Slimming.



<sup>&</sup>lt;sup>6</sup> Molchanov *et al.*, Importance Estimation for Neural Network Pruning.

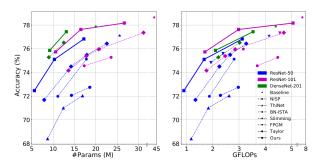
#### **Ablation Studies**

#### Accuracy vs.Complexity.



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#### Wall-time acceleration.

Model	GFLOPs	Avg. Runtime (ms)	FPS
ResNet-50	4.14	80.2	12.4
StrucSpars-35%	3.12	68.2	14.7
StrucSpars-65%	1.67	61.3	16.3
StrucSpars-85%	0.90	53.5	18.7
		7 1 7 7	

#### **Ablation Studies**

**Channel shuffle mechanism.** We empirically compare the following five settings:

- (i) FINETUNE: train  $\rightarrow$  compress  $\rightarrow$  finetune pipeline;
- (ii) FROMSCRATCH: learned channel shuffle, but train from scratch;
- (iii) SHUFFLENET: hand-crafted channel shuffle as in ShuffleNet;
- (iv) RANDOM: random channel shuffle (i.e., random permutation);
- (v) NoShuffle: no channel shuffle.

Config.	<b>ResNet-50</b> -65%		<b>ResNet-101</b> -65%	
Acc.	Top-1	Top-5	Top-1	Top-5
FINETUNE	75.10	92.52	77.62	93.72
FROMSCRATCH	75.02	92.46	77.14	93.53
SHUFFLENET	74.97	92.41	76.91	93.38
RANDOM	69.45	89.45	73.16	91.44
NoShuffle	73.30	91.39	75.31	92.64

#### Further information

Refer to our paper<sup>7</sup> for limitations and future perspectives:

- (i) Data-Driven Structured Sparsification;
- (ii) Progressive Sparsification Solution;
- (iii) Combination with Filter Pruning.

The source codes are available:

https://github.com/Sakura03/StrucSpars.



<sup>&</sup>lt;sup>7</sup>https://arxiv.org/abs/2002.08127

## Thanks!