CSCI 2112 ASSIGNEMNT 4

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1 a: Prove: Base Case $n=1$ $3^1 = 3 = \frac{3^{1+1}-3}{2} = 3$

Assume for $n = K$, $3^1 + 3^2 + \dots + 3^K = \frac{3^{K+1}-3}{2}$.

We want to prove $3^1 + 3^2 + \dots + 3^K + 3^{K+1} = \frac{3^{K+2}-3}{2}$.

I.H. $3^1 + 3^2 + \dots + 3^K + 3^{K+1}$

$= \frac{3^{K+1}-3}{2} + 3^{K+1}$.

$= \frac{3^{K+1}-3}{2} + \frac{2 \times 3^{K+1}}{2}$

$= \frac{3 \times 3^{K+1} - 3}{2} = \frac{3^{K+2}-3}{2}$

Now we prove $3^1 + 3^2 + \dots + 3^K + 3^{K+1} = \frac{3^{K+2}-3}{2}$. is true.

1 b: prove Base Case: $n=4$ $2^4 = 16$ $4! = 24$. $2^4 < 4!$

Assume for $n = K$ $2^K < K!$

We want to prove $2^{K+1} < (K+1)!$

I.H. $2^{K+1} = 2 \times 2^K < 2 \times K!$

$\because K \geq 4 \quad \therefore K+1 \geq 5$

$2 \times K! < (K+1) \times K! = (K+1)!$

Now we prove $2^{K+1} < (K+1)!$

1c: prove Base Case: for $n=1$. $F_2=1$. $F_3-1=1$, $F_2=F_3-1$.

Assume for $n=K$. $F_2+F_4+\dots\dots+F_{2K}=F_{2K+1}-1$.

we want to prove: $F_2+F_4+\dots+F_{2K}+F_{2K+2}=F_{2K+3}-1$.

I.H. $F_2+F_4+\dots\dots+F_{2K}+F_{2K+2}$

$=F_{2K+1}-1+F_{2K+2}=(F_{2K+1}+F_{2K+2})-1=F_{2K+3}-1$

Now we prove. $F_2+F_4+\dots+F_{2K}+F_{2K+2}=F_{2K+3}-1$.

1d. Prove. Base Case: $n\in\mathbb{N}$. $n=0$. $10^{0+1}-2\times10^0+1=9$. $9/9=1$

Assume for $n=K$. $10^{K+1}-2\times10^K+1=9a$. for $a\in\mathbb{N}$.

We want to prove: $10^{K+2}-2\times10^{K+1}+1=9b$. for $b\in\mathbb{N}$.

I.H. $10^{K+2}-2\times10^{K+1}+1$

$=10\times10^{K+1}-20\times10^K+1$

$=10(10^{K+1}-2\times10^K+1)-9$

$90a-9=9(10a-1)$ where $10a-1$ is an integer.

Now, we prove. $9\mid10^{K+2}-2\times10^{K+1}+1$

2a: Plove. for $n=1$, $a_1 = 3(2^{1-1}) + 2(-1)^1 = 1$

for $n=2$. $a_2 = 3(2^{2-1}) + 2(-1)^2 = 8$.

Assume for $n = 1, 2, 3 \dots K$

we want to plove, $a_{K+1} = 3(2^K) + 2(-1)^{K+1}$

I.H. $a_{K+1} = a_K + 2a_{K-1}$.

$\Rightarrow 3(2^{K-1}) + 2(-1)^K + 2(3(2^{K-1-1}) + 2(-1)^{K-1})$

$= 3(2^{K-1}) + 2(-1)^K + 3(2^{K-1}) + 4(-1)^{K-1}$

$= 2 \times 3(2^{K-1}) + 2(-1)^K + 4(-1)^{K-1}$

$= 3 \times 2^K + 2(-1)^K + 4(-1)^{K-1}$

$\because 4(-1)^{K-1} = 4(-1)^{K+1}$, $(-1)^K = -(-1)^{K+1}$

$\therefore 2(-1)^K + 4(-1)^{K-1} = 4(-1)^{K+1} - 2(-1)^{K+1} = 2(-1)^{K+1}$

$a_{K+1} = 3(2^K) + 2(-1)^{K+1}$.

Now, we plove $a_{K+1} = 3(2^K) + 2(-1)^{K+1}$

2b: Because 31 cents cannot be represented by $5a+9b$ (a, b non-neg integer).

Prove. Base Case: $n=32$. $32 = 5\times1 + 9\times3$

$n=33$. $33 = 5\times3 + 9\times2$

$n=34$ $34 = 5\times5 + 9\times1$

$n=35$ $35 = 5\times7 + 9\times0$.

$n=36$ $36 = 5\times0 + 9\times4$.

$n=37$ $37 = 5\times2 + 9\times3$

$\vdots$

Assume for $n = k$, like $n = 32..33, 34 .... k$.

We want to prove $n = k+1$ $n+1 = 5a+9b$. Where $a.b \in \mathbb{N}$.

I.H. $n+1 = (n-4)+5 \Rightarrow 5a+9b+5$ where $32 \leq n-4 \leq k$.

$\Rightarrow 5(a+1)+9b$, $a+1, b \in \mathbb{N}$.

Now, we prove $n+1$ can be represent by $5a+9b$.

3a): worst case: before the last pieces of fruit. I have 11 apples. 11 bananas 11 oranges and 11 pears so far. Since

$11+11+11+11+1 = 45$ min

3b): There are 7 remainders. 0.1.2.3.4.5.6 only. By pigeonhole principle among 8 integers. at least 2 integers share the same remainder.

3c: A person can shake hands with 0, 1, 2.... n-1 people if one shakes hands with everyone else, no one shakes 0 hands, vice versa. So, 0 and n-1 are ~~not exist~~. exclusive, Only n-1 possibilities. But there are n guests. By pigeonhole principle At least two guests have shaken the same number of hands.

4. The LHS counts the number of pairs (X, S). where S is a subset of $\{1, 2, \dots, n\}$ and $x \in S$. The RHS counts the same thing in a diffeent manner. Dceide x first and build S with X in it. There are 2^n subsets in total. but only half of them has no x in it. which is 2^{n-1}

5a: $A = \{0, 1, 2, 3, 4, 5\}$

$R = \{(0,0), (1,1), (2,2), (3,3), (4,4)\ (5,5),$
$(0,4), (4,0), (1,3), (3,1), (1,5), (5,1),$
$(2,4), (4,2)\}$.

5b: Reflexive: $(0,0), (1,1), (2,2)\ (3,3), (4,4), (5,5)$ True.

Symmetric: $(0,0) \to (0,0)$, $(1,1) \to (1,1)$, $(2,2) \to (2,2)$ $(3,3) \to (3,3)$
$(4,4) \to (4,4)$, $(5,5) \to (5,5)$ $(0,4) \to (4,0)$ $(4,0) \to (0,4)$.
$(1,3) \to (3,1)$, $(3,1) \to (1,3)$. $(1,5) \to (5,1)$. $(5,1) \to (1,5)$
$(2,4) \to (4,2)$. $(4,2) \to (2,4)$.

True

Transitive: $(0,4), (4,2)$. but not $(0,2)$.

False

6a: Reflexive: if x is odd, x^2 is odd. x^2+x^2 is even. true.

if x is even, x^2 is even. x^2+x^2 is even. true.

Symmetric: if x^2+y^2 is even, then y^2+x^2 is even

$x^2+y^2 = y^2+x^2$. true.

Transitive: if x^2+y^2 is even, y^2+z^2 is even, then x^2+z^2 is even.

$x^2+y^2+y^2+z^2$ is even. $2y^2$ is even.

$x^2+z^2 = x^2+y^2+y^2+z^2-2y^2$ is even.

Since R is equivalence relation on A.

equivalence classes: All odd number and all even number.

6b: $R = \{ (a,d), (b,c), (e,d), (a,a), (b,b),$

$(c,c), (d,d), (e,e), (d,a), (c,b),$

$(d,e), (a,e), (e,a), (a,d), (d,a) \}$.

7a: Assume (a,b) is any pair $\in$ devides and $a \neq b$.

$a|b$ and $a \neq b \Rightarrow b = ka$. $k \in \mathbb{Z}$ and $k \neq 1$.

There is no integer such that $a = kb$.

$\therefore (b,a) \notin$ divides

Since devides is an anti-symmetric relation on A

7b: Anti-Symmetric: True, we are prove above.

Reflexive: True. For any $x \in A$, we have $x | x$.

Transitive: for any $x|y$, $y|z$, $x|z$.

$y = k_1 x$, $z = k_2 y$ $k_1, k_2 \in A$.

$z = k_2 y = k_2(k_1 x) = k_1 k_2 x$. $k_1, k_2 \in A$.

$x | z$. True.

Since '|' is an order

8. Assume the set of all equivalent classes is NOT a partition for A.

There must exist an element $x \in A$, and not belong to any equivalent class or belongs to more than one equivalent class.

Case 1: X doesn't belong to any equivalent class by definition, X must belong to an equivalence class, So can't be the case.

Case 2: X belongs to more than one equivalence classes. by definition. if x belong to more than one equivalence classes, the two equivalent classes should be in one equivalent class. This can't be the case.

Controdiction. Therefore the set of all equivalence classes is a partion of A

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