

# A Novel Adaptive Real-time Tracking Scheme for Underwater Networks<sup>\*</sup>

Jinwang Yi, Wei Su, Fei Yuan, Lan Zhang, En Cheng<sup>\*</sup>

*Key Laboratory of Underwater Acoustic Communication and Marine Information Technology (Ministry of Education), Xiamen University, Xiamen 361005, China*

---

## Abstract

Location information of mobile submersibles is critical to underwater networks. A considerable amount of researches have been carried out on non-real-time tracking to suffice many scenarios, while the lack of real-time tracking motivates more efforts to be addressed in Real-time underwater applications. In this paper a novel adaptive Real-time tracking scheme is presented, which can adaptively adjust the length of factor graph and then construct the resulting partial factor graph to track the positions of mobile submersibles. Since the proposed scheme tailors the basic factor graph into a partial factor graph with much shorter length in time, it makes the Real-time tracking feasible. Meanwhile, adaptively constructing the appropriate partial factor graph according to accuracy requirement of localization, can obtain high accuracy for localization and tracking. Simulation results using real sea-trail data show that our proposed scheme can adaptively trade off Real-time requirement and localization accuracy, achieving 12% to 61% improvement for tracking performance in the corresponding scenarios, when comparing with the classic TDoA (Time Difference of Arrival) localization method.

*Keywords:* Underwater Networks; Real-time Tracking; Factor Graph; Localization

---

## 1 Introduction

In recent years, underwater networks have attracted a growing attention [1] [2]. Among the services provided by underwater networks, location information of mobile underwater submersibles is a crucial requirement, because of the necessity for many applications in underwater networked systems [3]. Since GPS is not available underwater, the position of submersibles is typically tracked from acoustic communication with reference beacons that have access to GPS. Most existing localization techniques are designed for non-Real-time applications, where position estimate is calculated with all available information after the task is completed. Whereas a tracking system that operates in Real-time is in desperate need for a wide range of oceanographic applications, which include early warning systems, vehicle navigation and control, adaptive sampling

---

<sup>\*</sup>Project supported by the Natural Science Foundation of Fujian Province of China (No. 2010J05139).

<sup>\*</sup>Corresponding author.

Email address: [chengen@xmu.edu.cn](mailto:chengen@xmu.edu.cn) (En Cheng).

and formation control [4]. These have necessitated the design of Real-time tracking systems for submersibles' position.

In this paper, we propose an adaptive Real-time solution of localization and tracking for mobile submersibles in underwater acoustic networks. Based on tailoring the basic factor graph into a partial factor graph, this method adaptively trades off Real-time demand and tracking accuracy. Simulations with real sea-trial data indicate that our proposed scheme performs significantly better than the classic TDoA method in the simulated scenarios.

The rest of this paper is organized as follows. We first formulate the problem in Section 2. The principle of factor graph and sum-product algorithm is subsequently delineated in Section 3. Then we describe the adaptive Real-time tracking scheme in Section 4. Simulation results and conclusions are given in Section 5 and Section 6 respectively.

## 2 Problem Formulation

In order to achieve a better description, the problem formulation for tracking estimate is addressed at first. Without loss of generality, a network which consists of  $M$  mobile nodes with unknown positions and  $N$  beacons with known positions, is considered here. The tracking problem can be seen as a complex multi-dimensional estimation problem, which is solved in this paper by obtaining the maximum likelihood estimate of unknown mobile nodes' position during the tracking interval at a certain time granularity, given all distance estimates between reference beacons and mobile nodes and motion measurements of mobile nodes. As a solution, factor graph is applied to the frame of maximum likelihood estimate, which will be elaborates in next section. The estimate of an unknown position  $p(t_k)$  at time  $t_k$  is first constrained by Eq. (1).

$$d_k = \|p(t_k) - p_R(t_k)\|_2 + \varepsilon_k^R \quad (1)$$

where  $d_k$  is a distance measurement obtained at time  $t_k$ ,  $p_R(t_k)$  is the position of a known reference beacon, and  $\varepsilon_k^R$  is the error of distance estimate modeled as Gaussian distribution,  $\varepsilon_k^R \sim G(0, \sigma_R)$ . Further, a motion measurement  $v_i$  supplies the other constraint of the unknown position, which is given by Eq. (2).

$$v_i = \frac{1}{\Delta t} (p((i+1) \cdot \Delta t) - p(i \cdot \Delta t)) + \varepsilon_i^v \quad (2)$$

where  $\varepsilon_i^v$  is the error in velocity measurement. Then the maximum likelihood estimate of the position at the time granularity of  $\Delta t$ ,  $p_{mle}^*(k \cdot \Delta t)$ , is derived from Eq. (3).

$$p_{mle}^*(k \cdot \Delta t) = \operatorname{argmax} L(\mathbf{P}_*(k \cdot \Delta t) | \mathbf{P}_R, \mathbf{V}) \quad (3)$$

where  $L$  is the likelihood function.  $\mathbf{P}_R$  and  $\mathbf{V}$  are known as the set of reference beacons' positions and the set of motion measurements respectively, while  $\mathbf{P}^*(k \cdot \Delta t)$  indicates the set of possible position estimates.

### 3 Factor Graph and Sum-product Algorithm

#### 3.1 Description of Factor Graph

As mentioned in our previous work [5], the solution of tracking problem is to find the maximum likelihood estimate for the overall probability distribution function given all constraints (derived from the measurements). Factor graph, as a bipartite graph that expresses the structure of the factorization, is an effective way to represent any global function in terms of simpler local functions that depends only on a subset of variables. For example, some function  $f(u, w, x, y, z)$  can be factored as

$$f(u, w, x, y, z) = f_1(u, w, x)f_2(x, y, z)f_3(z) \quad (4)$$

In Eq. (4), the global function is  $f$ , and  $f_1, f_2, f_3$  are the local functions.

In general, factor graph consists of nodes, edges and half edges which are connected only to one node [6] [7]. The key point is to set up the appropriate factor graph for tracking problem. Since only distance and motion measures are given in estimate, the factor graph description of these measurements between the unknown positions of mobile nodes over time is determined in Fig. 1. Subsequently, the joint distribution of nodes' positions is derived as a whole graph and can be viewed in terms of a number of basic sub-graphs. Fig. 1 shows the trajectory chain of an unknown mobile node moving over time. The circles stand for the positions at fixed time instances that have to be estimated, namely position state variable. Function nodes indicated by square blocks, not only show which state variables are related but also how they are related. For example, in this figure the state variables  $x$  are linked together by the function nodes  $f$  and the constraints between state variables are then defined. Function node  $f_1$  describes the spatial constraint between reference beacons and mobile submersibles by distance measurements, while function node  $f_2$  offers the other constraint how the velocity of each mobile node is related to its position over time. Next, the sum-product algorithm, which operates by iterative message passing in the factor graph, exploits these simple relations to estimate the global function.

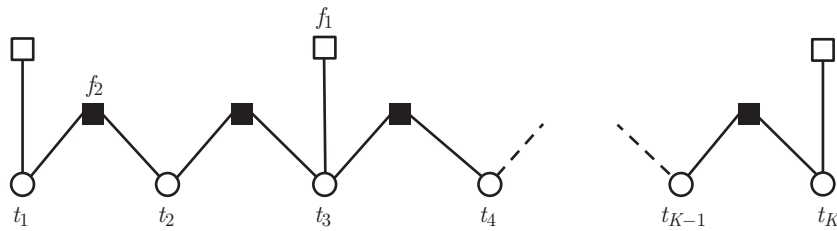


Fig. 1: Factor graph of tracking for a mobile submersible

#### 3.2 Sum-product Algorithm

The sum-product algorithm operates by computing various sums and product [8], where the summary operation is defined by Eq. (5).

$$\sum_{\sim x_1} f(x_1, x_2, x_3) = \sum_{x_2 \in A_2} \sum_{x_3 \in A_3} f(x_1, x_2, x_3), \quad (5)$$

where  $x_1, x_2, x_3$  is a collection of variables and for each  $i$ ,  $x_i$  takes on values in some domain  $A_i$ . Then a following simple rule takes effect: the message out of a node  $g(l, \dots)$  along the edge  $l$  is the product of  $g(l, \dots)$  and all messages towards  $g$  along all edges except  $l$ , summarized over all variables except  $l$ . A factor graph fragment is illustrated in Fig. 2 to show the rules of sum-product algorithm. And the messages are computed by Eq. (6) and Eq. (7) as follows.

$$\mu_{f \rightarrow x}(x) = \sum_{\sim(x)} (f(\mathbf{X}) \prod_{y \in n(f) \setminus \{x\}} \mu_{y \rightarrow f}(y)) \quad (6)$$

$$\mu_{x \rightarrow f}(x) = \prod_{h \in n(x) \setminus \{f\}} \mu_{h \rightarrow x}(x) \quad (7)$$

where  $\mu_{x \rightarrow f}(x)$  indicates messages sent from state variable  $x$  to function node  $f$  in the operation of the sum-product algorithm and  $\mu_{f \rightarrow x}(x)$  denotes messages sent from function node  $f$  to state variable  $x$ ;  $\mathbf{X}$  is the set of arguments  $f$ ;  $n(w)$  stands for the set of neighbors of a given node  $w$ .

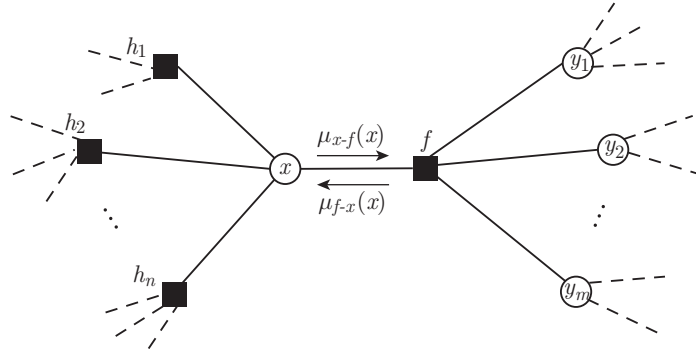


Fig. 2: Update of the sum-product algorithm in a factor graph fragment

## 4 Design of Adaptive Real-time Tracking Algorithm

The initial solution of tracking problem based on factor graph can only run in the offline and non-Real-time way due to adopting the factor graph with whole time interval. The tracking estimate performs after the mission is completed. To meet the demands of Real-time applications, we present an adaptive Real-time tracking approach in this section. Since the estimate of unknown positions is decided by factor graph, the intuition is to adaptively adjust the length of time window of factor graph, that is, the number of time instances involved for one estimate over time, according to the accuracy requirement.

Assume that the length of time window is  $\tau$  for one estimate. Then a partial factor graph with the length of  $\tau$  is constructed and only the measurements in current time window are employed for this estimate. Usually smaller  $\tau$  is applied to satisfy the Real-time requirement, while longer  $\tau$  increases the estimate accuracy due to the fact that more measurements are exploited for estimating the unknown states. The key point is how to do the tradeoff between Real-time requirement and estimate accuracy, both of which depend on the length of time window  $\tau$ , that is, it is essential to design a scheme that can not only reach the accuracy requirement but also get the needed Real-time performance. Since the relationship of Real-time requirement and estimate accuracy is changing over time during the tracking period, a novel scheme that can adaptively

adjust the length of time window to get a good tradeoff between them is in demand. Therefore, as shown in Fig. 3, an adaptive solution that the length of time window is varied by iterations till achieving the desired accuracy is proposed.

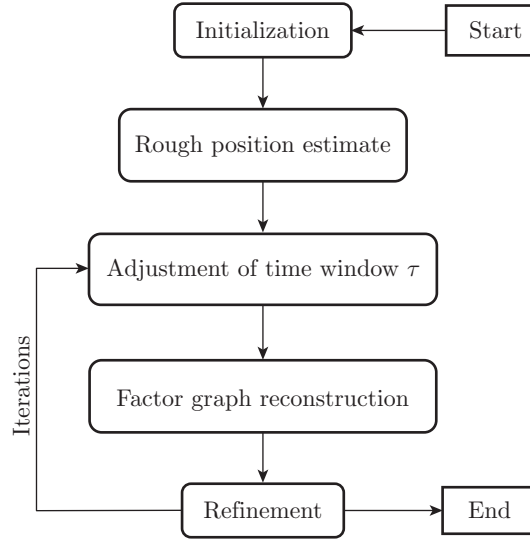


Fig. 3: The work flow of adaptive Real-time tracking algorithm

The procedure of this adaptive Real-time tracking algorithm consists of four major phases, initialization, rough position estimate, factor graph reconstruction along with adjustable time window, and refinement. In the phrase of initialization, measurements of time-of-flight and submersibles motion in the time interval with minimum length are obtained. With these inputs, a basic Factor graph is determined to estimate the rough position of this submersible. Then in the step of refinement, the length of time window,  $\tau$ , is adaptively changed to construct a new factor graph to refine the position estimate by iterations until the desired requirement is reached.

## 5 Simulation Results

To evaluate the performance of our adaptive Real-time tracking scheme, we compare it with the classic TDoA scheme in simulations with different scenarios, where the speed of submersibles' motion is varied. The deployment of simulation is shown in Fig. 4, consisting of 4 beacons and one mobile submersible. The trajectory of submersible was generated using spline-interpolated way-points and simulated for a sufficiently long time to capture the statistical variation in the RMS error due to its motion relative to the beacons. In accordance with our own experimental results, a maximum ranging error of 10 meters is selected. And the range of these simulation parameters is listed in Table 1.

Fig. 5 shows the histograms of RMS error in the position estimates between two different schemes when the speed of submersible is equal to 0.2 m/s, 0.4m/s, 0.8 m/s and 1.6 m/s respectively. In order to obtain more evident comparison, the mean and standard deviation of RMS error for both schemes are labeled at the top of each figure. As shown in Fig. 5 (a)-(d), the performance of TDoA obviously deteriorates as the speed increases, while that of our proposed scheme only varies in a small range, which is less than 1 m. When the speed of submersibles is augmented from 0.2 m/s to 1.6 m/s, the corresponding mean for TDoA increases by 1.16, 1.75

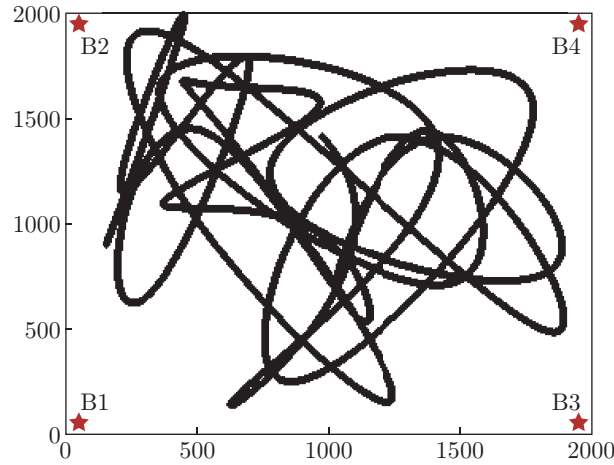


Fig. 4: Simulated deployment showing the location of 4 beacons and the trajectory of the submersible in 2D (Unit: m)

Table 1: Simulation parameters

No. of Beacons	4
Sound Speed	1500 (m/s)
Velocity $v$	0.2, 0.4, 0.8, 1.6 m/s
Std. of Velocity Error	0.2 m/s
Max Velocity Error	$v/10$
Std. of Ranging Error	3 m
Simulation Time	100 hrs

and 3.34 times respectively and standard deviation increases by 1.17, 1.89 and 3.29 times. At the same time, the maximum variation of mean and standard deviation for our proposed method are 1.26 and 1.45 times when the speed changes from 0.2 m/s to 1.6 m/s. Note that the amount of variation of both mean and standard deviation for the proposed scheme is less than 1 m, which is the up-limit of estimate accuracy. This implies that our proposed scheme is robust to speed variation when tracking the mobile submersibles in underwater networks. For each case illustrated in Fig. 5, the performance of proposed scheme surpasses TDoA and this becomes significant as the speed increases. The performance of the proposed scheme versus TDoA for mean RMS error improves by 14%, 15%, 42% and 56% and for standard deviation this is 12%, 13%, 41% and 61% respectively. Overall, simulation results with real sea-trial data imported show that our proposed scheme outperforms TDoA in each case. Especially when the speed of motion is relative large, the tracking performance can be significantly improved using our proposed scheme.

## 6 Conclusion

In this paper, we presented an adaptive Real-time tracking scheme for underwater networks. By means of adaptively tailoring the factor graph with appropriate length, the scheme efficiently balances the Real-time requirement and tracking accuracy, given all corresponding measurements of distance and motion. Simulations exploiting the real sea-trial data demonstrate that our

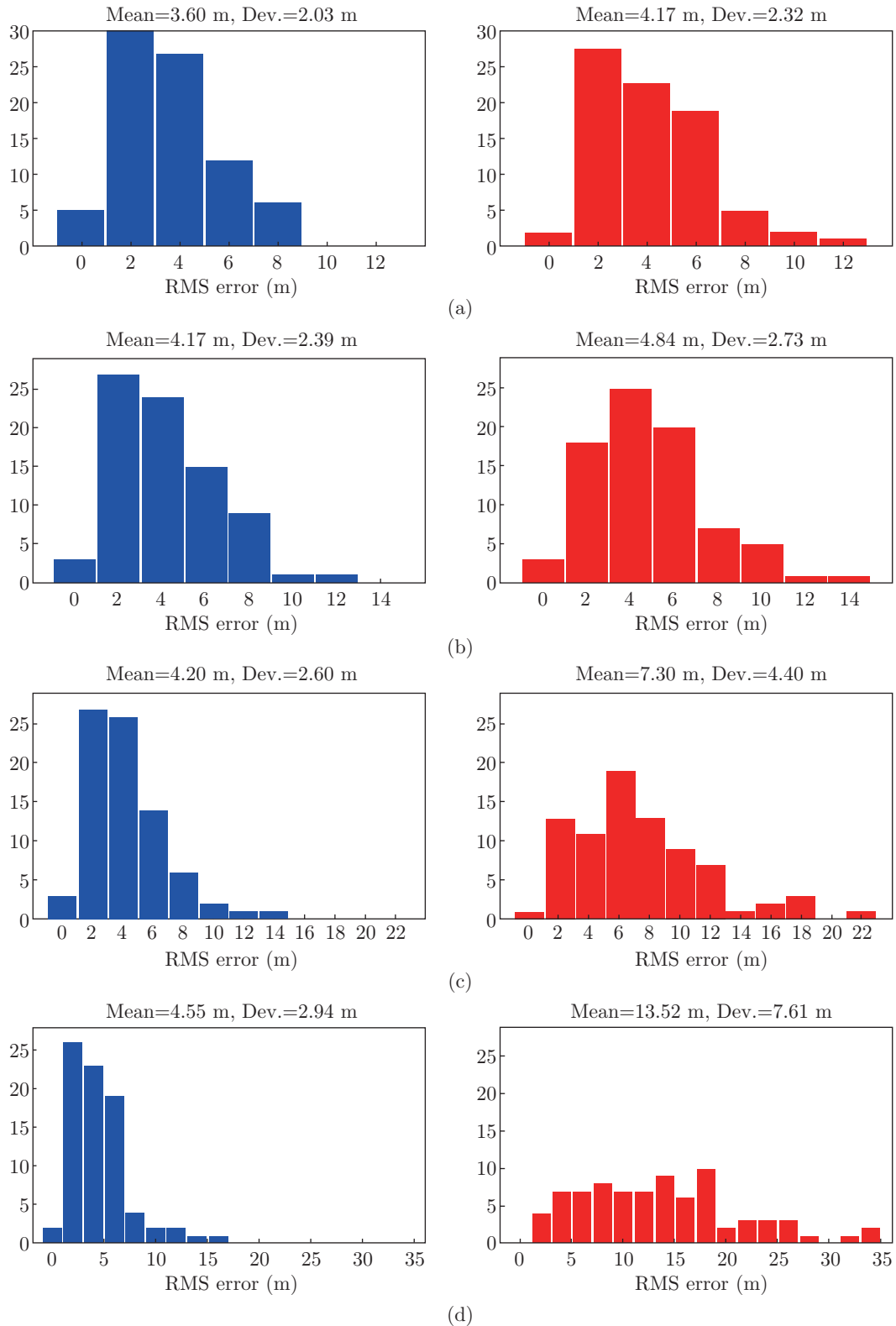


Fig. 5: Histogram of RMS error distribution for adaptive Real-time tracking (left) and TDoA method (right) when velocity is equal to (a) 0.2 m/s, (b) 0.4 m/s, (c) 0.8 m/s and (d) 1.6 m/s

proposed scheme outperforms the classic TDoA scheme and can attain 12% – 61% improvement for tracking performance.

## Acknowledgement

This work was supported by the Fundamental Research Funds for the Central Universities (2011121050, 2012121028), the National Natural Science Foundation of China (61001142, 61071150, 61301097) and the Science Technology Project of Xiamen Government (3502Z20123011).

## References

- [1] I. F. Akyildiz, D. Pompili, T. Melodia, Underwater acoustic sensor networks: Research challenges, *Ad Hoc Networks*, Vol. 3, No. 3, 2005, 257-279
- [2] J. H. Cui, J. Kong, M. Gerla, S. Zhou, Challenges: Building scalable mobile underwater wireless sensor networks for aquatic applications, *IEEE Network*, Special Issue on Wireless Sensor Networking, Vol. 20, No. 3, 2006, 12-18
- [3] J. Jaffe, C. Schurgers, Sensor networks of freely drifting autonomous underwater explorers, Special Issue on Advances in Computational Oceanography, *Oceanography Magazine*, Vol. 19, No. 1, 2006
- [4] D. Mirza, C. Schurgers, R. Kastner, Real-time collaborative tracking for underwater networked systems, *WUWNet'12*, 2012
- [5] J. Yi, D. Mirza, C. Schurgers, R. Kastner, Joint time-synchronization and tracking for mobile underwater systems, *WUWNet'13*, 2013
- [6] H. A. Loeliger, An introduction to factor graphs, *Signal Processing Magazine*, Vol. 21, No. 1, 2004
- [7] D. Mirza, C. Schurgers, Collaborative tracking in mobile underwater networks, *WUWNet'09*, 2009
- [8] F. R. Kschischang, B. J. Frey, H. A. Loeliger, Factor graphs and the sum-product algorithm, *IEEE Trans. on Information Theory*, Vol. 47, No. 2, 2001