Matlab Simulation of Magnetic Focusing Model

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Abstract—Similar to a lens that focuses a light beam, magnetic focusing is a charged particle beam with a small divergence angle. When they have the same velocity component in the direction of the magnetic field, the divergent particle beam converges to a point after a period. In this process, the analytical solution of the velocity and displacement vector at each moment can be obtained by solving the differential equation. However, this process is sometimes too tedious. In order to focus on understanding the physical nature of magnetic focusing dynamic process, the Matlab is used to calculate and draw the movement trajectory of charge.

Keywords—Lorentz force, Magnetic focusing, Matlab simulation.

I. INTRODUCTION

THE electric charge will be affected by the Lorentz force in the electromagnetic field. The charge will generate acceleration under the action of Lorentz force, which will produce changes in speed and displacement. In this process, the analytical solution of the velocity and displacement vector at each moment can be obtained by solving the differential equation. However, this process is sometimes too tedious and the result is not intuitive. For example, in magnetic focusing, the trajectory of particles often describes this phenomenon better than the equation of motion of particles.

Magnetic focusing is a charged particle beam with a small divergence angle. When they have the same velocity component in the direction of the magnetic field, the divergent particle beam converges to a point after a period. Magnetic focusing is widely used in fields such as electron microscopy.[1]

In order to focus on understanding the physical nature of magnetic focusing dynamic process, the movement trajectory of charge is calculated and drawn by Matlab. At the same time, the relevant parameters and properties of magnetic focusing have been verified.

II. METHOD AND ANALYSIS

A. Mathematical calculation

Charge in magnetic field will be subject to the action of Lorentz force, which is expressed as:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} \tag{1}$$

where $\vec{F}$ is vector of Lorentz force, $\vec{E}$ is the vector of electron field, $\vec{B}$ is the vector of magnetic flux density, $\vec{v}$ is the vector of charge's velocity, q is the quantity of charge.

The electromagnetic field is written as a component in the plane rectangular coordinate system, which is:

$$\vec{E}(t) = E_x(t)\vec{a}_x + E_y(t)\vec{a}_y + E_z(t)\vec{a}_z \tag{2}$$

$$\vec{B}(t) = B_x(t)\vec{a}_x + B_y(t)\vec{a}_y + B_z(t)\vec{a}_z \tag{3}$$

According to Newton's law of motion, charge will be accelerated under Lorentz force, hence the velocity and displacement will change. In the 3D Cartesian coordinate system, this process can be described by the following vector equations:

$$\vec{F}(t) = q\vec{E}(t) + q\vec{v}(t) \times \vec{B}(t) \tag{4}$$

$$\vec{A}(t) = \frac{\vec{F}(t)}{m} \tag{5}$$

where m is the mass of the charge, $\vec{A}$ is the acceleration vector.

From the kinematics formula, the analytical formulas of particle velocity and position can be obtained as:

$$\vec{v}(t) = \vec{v}(1) + \int_0^t \vec{A}(t)\mathrm{d}t \tag{6}$$

$$\vec{r}(t) = \vec{r}(1) + \int_0^t \vec{v}(t)\mathrm{d}t \tag{7}$$

It can be seen that this is a process that develops over time. In some cases, this process can be solved analytically by solving differential equations to get the velocity vector, position vector at every moment. This experiment is aimed to analyze this dynamic process without tedious mathematical derivation. The key point is to understand the physical essence of this dynamic process. For Matlab simulation, we discretize time and introduce small time-step Δt. And we assume at each time slot, the acceleration vector will not change. In the way, equation (6) and (7) can be rewritten as:

$$\vec{v}(t + \Delta t) = \vec{v}(t) + \vec{A}(t)\Delta(t) \tag{8}$$

$$\vec{r}(t + \Delta t) = \vec{r}(t) + \vec{v}(t)\Delta(t) \tag{9}$$

Thus, we could set the Δt and use "for loop" in Matlab to calculated the 3-D dimensional component of velocity and position at all the discrete time. Meanwhile, the movement trajectory of charges during a period of time can be plotted.

Assume the specific cases for an example, the parameter are as follows[**?**]:

TABLE I
THE PARAMETER OF THE MAGNETIC FOCUSING

Parameter name	symbol	value
Mass of charge	m	$0.02\ kg$
Quantity of charge	q	$0.016\ C$
Initial speed	$\vec{v}_x(1)$	$0.1\sin(k\pi/8)\ m/s$
	$\vec{v}_y(1)$	$0.1\cos(k\pi/8)\ m/s$
	$\vec{v}_z(1)$	$10\ m/s \quad (k = 0, 1, 2, \cdots, 15)$
Initial position	$\vec{r}(1)$	0 (at the origin)
Electric field	$\vec{E}$	0
Magnetic flux density	$\vec{B}$	$8\vec{a}_z\ Wb/m^2$

In this case, we could rewrite the formula of Lorentz force (4) in components calculation form:

$$F_x(t) = qE_x + q(v_y(t) \cdot B_z - v_z(t) \cdot B_y) = qv_y(t)B_z$$
$$F_y(t) = qE_y + q(v_z(t) \cdot B_x - v_x(t) \cdot B_z) = -qv_x(t)B_z$$
$$F_z(t) = qE_z + q(v_x(t) \cdot B_y - v_y(t) \cdot B_x) = 0$$

Here, we use dt to represent Δt and the accelerate velocity and position are calculated as follows:

$$a_x(t) = \frac{F_x(t)}{m} \quad a_y(t) = \frac{F_y(t)}{m} \quad a_z(t) = \frac{F_z(t)}{m}$$
$$v_x(t + \Delta t) = v_x(t) + a_x(t) \cdot dt$$
$$v_y(t + \Delta t) = v_y(t) + a_y(t) \cdot dt$$
$$v_z(t + \Delta t) = v_z(t) + a_z(t) \cdot dt$$
$$r_x(t + \Delta t) = r_x(t) + v_x(t) \cdot dt$$
$$r_y(t + \Delta t) = r_y(t) + v_y(t) \cdot dt$$
$$r_z(t + \Delta t) = r_z(t) + v_z(t) \cdot dt$$

Also, it should be noted that the time-step dt should be set appropriately. If it is too long, then the error will be large; if it is too small, the computation will be time-consuming.

B. Magnetic focusing analysis

For a beam of charged particles with small angle of divergence, given the same velocity component at the direction of the magnetic field $\vec{B}$, their trajectory will have the same screw pitch. After a period, they will converge at another point. The phenomenon that the diverged charged particles focus at one point is similar to the phenomenon that lens can let the light beam focus at one point. Therefore, it is called as magnetic focusing. There are two conditions for magnetic focusing:

1. The charged particles have similar initial velocity $\vec{v}$;

2. The angle between $\vec{v}$ and $\vec{B}$ is sufficiently small so that each particle will do helical motion.

In this example, due to the zero electric field and $\vec{a}_z$ direction magnetic field, the direction of Lorentz force is parallel to xoy plane. Therefore, the x and y components of the moving charge velocity keep changing, while the z component remains unchanged. So the motion of the charge is decomposed into the combined motion of the xoy plane and the z direction. At the same time, the electric field is zero, and the direction of the Lorentz force is always perpendicular to the direction of velocity, so it only changes the direction of the velocity of the particles moving in the xoy plane without affecting the velocity. The magnitude of the velocity component in xoy plane for given situation is easy to calculate:

$$|v_{xy}| = \sqrt{(v_x)^2 + (v_y)^2} \tag{10}$$
$$= \sqrt{(0.1\sin{(k\pi/8)})^2 + (0.1\cos{(k\pi/8)})^2}$$
$$= 0.1 \ m/s$$

What's more, the direction is the counterclockwise tangent direction on the sixteenth bisector of the unit circle. Therefore, the motion of the charge in this magnetic field can be decomposed into a uniform circular motion in the xoy plane and a uniform linear motion in the z-axis direction.

The centripetal force is provided by the Lorentz force, and the radius R of the uniform circular motion of the charge can be calculated:

$$F_{xy} = qv_{xy}B = m\frac{v_{xy}^2}{R} \tag{11}$$
$$R = \frac{mv_{xy}}{qB} \tag{12}$$
$$T = \frac{2\pi R}{v_{xy}} \tag{13}$$
$$L = v_z \cdot T \tag{14}$$

where T is a cycle of circular motion in the plane and a cycle of spiral motion in space, L is the distance between two adjacent focus points along the z-axis.

III. RESULTS AND DISCUSSION

After the theoretical analysis in the previous section, a trajectory of a charge from the origin along all directions was drawn in Matlab. At the same time, some cross-sectional figures are displayed, and the results of theoretical calculations are verified.

A. Three-dimensional figure

The charge movement trajectory is shown in Fig. 1. Since k is from 0 to 15, there are sixteen charges, so sixteen spiral curves are shown in the figure.
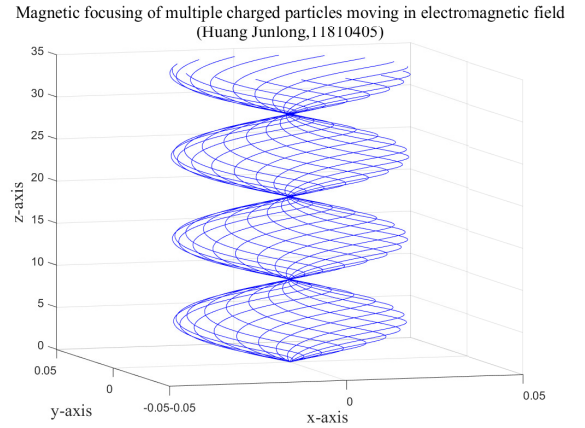

Fig. 1. Magnetic focusing of multiple charged particles moving in electromagnetic field

It can be seen from the Fig. 1 that the trajectory of the overall charge is symmetrical about the z-axis and is of a spindle shape. In the z-axis direction, it will focus on a point every other distance,

B. The cross section figure of xoy

Fig. 2 shows the motion curve of the xoy section during magnetic focusing. At the same time, since the radius of motion of each charge is R, then the envelope surface of their trajectory is centered on the origin and $2R$ is the envelope circle of radius. (As the red circle in Fig. 2)
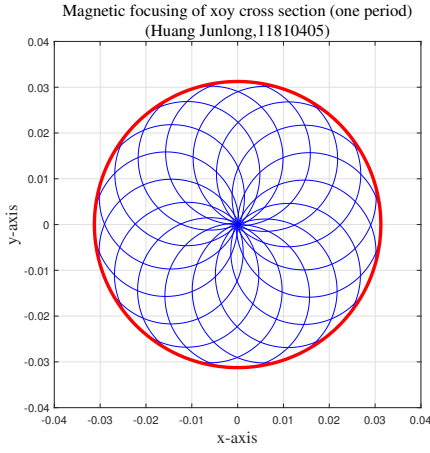
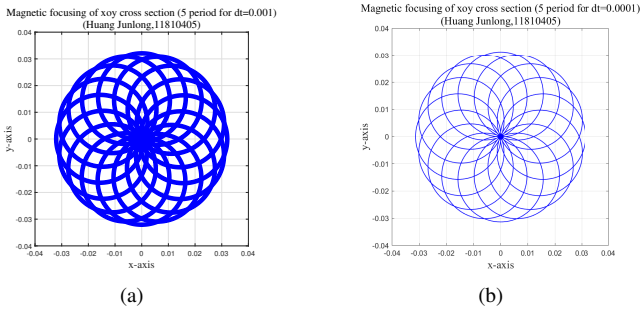Fig. 2. Magnetic focusing of xoy cross section (one period)

It should be noted that only one cycle is used here. This is because the dt is used as an approximation. There is a small error between the simulated trajectory and the real trajectory. When t is large, the error will accumulate in the xoy plane, resulting in not completely overlap.

Through the Equation (10) (12) (13) and parameter given in Table 1, we can calculate the movement period and radius of the charge:

$$R = \frac{mv_{xy}}{qB} = \frac{0.02 \times 0.1}{0.016 \times 8} = 0.015625 \ m \tag{15}$$

$$T = \frac{2\pi R}{v_{xy}} = \frac{2 \times \pi \times 0.015625}{0.1} = 0.98175 \ s \tag{16}$$

If we draw a 5-period charge trajectory, we can see from Fig. 3 that the trajectory line becomes thicker. This is because the trajectory lines do not completely overlap due to the approximation.

Fig. 3. Magnetic focusing of xoy cross section in 5 period for : (a) time interval $dt = 0.001$, (b) time interval $dt = 0.0001$

However, if we change the time interval dt from 0.001 to 0.0001, it could be found that, the smaller the time interval dt is obtained, the more accurate the simulation result is. This is in line with the actual situation.

Then, the Fig. 4 is shown that the trajectory of 16 charges dynamic changes in a cycle which projected on the xoy plane.
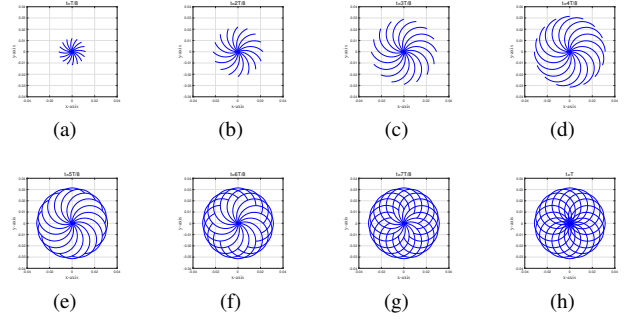
Fig. 4. Magnetic focusing of xoy cross section in one cycle: (a) $t = \frac{1}{8}T$; (b) $t = \frac{2}{8}T$; (c) $t = \frac{3}{8}T$; (d) $t = \frac{4}{8}T$; (e) $t = \frac{5}{8}T$; (f) $t = \frac{6}{8}T$; (g) $t = \frac{7}{8}T$; (h) $t = T$
.

C. The cross section figure of xoz

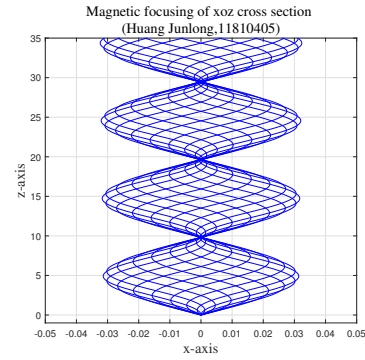Then, we draw the cross section of xoz plane as Fig. 5.

Fig. 5. Magnetic focusing of xoz cross section

It can be clearly seen from the Fig. 5 that every time, the trajectory of each charge will refocus on a point. It can be estimated that the distance between two adjacent points is about $10 \ m$. Actually, this distance could be calculated through equation (14) for the period T is already calculated in equation (16).

$$L = v_z \times T = 10 \times 0.98175 = 9.8175 \ m \tag{17}$$

Therefore, after sixteen charges leave the origin, they focus on point $(0, 9.8175)$ for the first time. Since we use dt as an approximation, the graph drawn by simulation does not pass exactly this point, but fluctuates in a small interval near this point. The size of the interval depends on the size of dt. Here shows two figure for $dt = 0.001$ and $dt = 0.0001$:
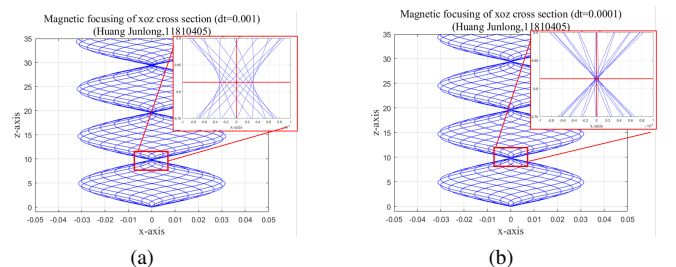
Fig. 6. Magnetic focusing of xoz cross section for: (a) $dt = 0.001$; (b) $dt = 0.0001$

As $dt = 0.001$, the maximal error is less that 4×10^{-4} m. As $dt = 0.0001$, the maximal error is less that 5×10^{-5} m. It is clear that the smaller the dt, the smaller the error. Actually, the maximal error 4×10^{-4} m compared with the radius $R = 0.015625$ m and the $L = 9.8175$ m is already negligible.

IV. CONCLUSION

In a given highly symmetrical and simple example, we calculated the movement trajectory of charges in magnetic focusing through Matlab calculations. The radius of the circular motion R, the period of motion T, and the distance between adjacent focus points L of the drawn image all agree with the theoretical calculation. This verifies the correctness of Matlab simulation. Therefore, under more complex conditions, we can use simple Matlab simulation to replace the tedious mathematical analytical solution.

Although there is a negligible error between the simulation result and the actual result, we can reduce this error by reducing dt. It should be noted that too short dt will cause a large amount of calculation leading to longer calculation time. Therefore, without affecting accuracy, this experiment uses $dt = 0.001$ s as the final time step.

In addition, we find that magnetic focusing in the given situation is actually a combination of uniform circular motion and uniform linear motion. Therefore, when the charge completes a full cycle of circling movement on the xoy plane with the same period, it will re-converge in a point on the z-axis. At the same time, since there is a velocity component in the z-axis direction, the interval between the two points of charge convergence is $L = v_z \cdot T$.

ACKNOWLEDGMENT

REFERENCES

[1] M. Knoll, "Das Elektronenmikroskop," *AMERICAN JOURNAL OF PHYSICS*, vol. 78, pp. 318–339, May 1932.
[2] Y. Jia, "Engineering Electromagneticâ Experiment 4."

APPENDIX A
THE CODE OF MATLAB

```matlab
1   %% This is the code for Lab 4
2
3   clc,close all,clear all
4
5   %% Bacis parameter setting
6   m=0.02;                    % Set the mass
7   q=1.6e-2;                  % Set the quantity of charge
8   dt=0.001;                  % Set the time-step to be 0.001s
9   % alos 0.0001s for higher accuracy
10  T=0.98175;                 % The period of theoretical calculation
11  t=0:dt:100;                % Construct the array of time. (100 T 5T)
12  Ex=0; Ey=0; Ez=0;          % Set the electric field vector?
13  Bx=0; By=0; Bz=8;          % Set the magnetic flux density vector?
14  vx=zeros(16,length(t));vy=vx;vz=vx;    % Construct the velocity vector?
15  rx=zeros(16,length(t));ry=rx;rz=rx;    % Construct the position vector?
16  Fx=zeros(16,length(t));Fy=Fx;Fz=Fx;    % Construct the force vector?
17  ax=zeros(16,length(t));ay=ax;az=ax;    % Construct the acceleration vector?
18
19  %% For 16 charge trajectories
20  for k=1:16
21  % set the init value
22  rx(k,1)=0;ry(k,1)=0;rz(k,1)=0;
23  vx(k,1)=0.1*sin(k*pi/8);vy(k,1)=0.1*cos(k*pi/8);vz(k,1)=10;
24
25  for i=1:(length(t)-1)        % Calculate each position point
26  % Calculate the acted force at position i
27  Fx(k,i)=q*Ex+q*(vy(k,i)*Bz-vz(k,i)*By);
28  Fy(k,i)=q*Ey+q*(vz(k,i)*Bx-vx(k,i)*Bz);
29  Fz(k,i)=q*Ez+q*(vx(k,i)*By-vy(k,i)*Bx);
30  % Calculate the acceleration at position i
31  ax(k,i)=Fx(k,i)/m;
32  ay(k,i)=Fy(k,i)/m;
33  az(k,i)=Fz(k,i)/m;
34  % Calculate the velocity at position i+1
35  vx(k,i+1)=vx(k,i)+ax(k,i)*dt;
36  vy(k,i+1)=vy(k,i)+ay(k,i)*dt;
37  vz(k,i+1)=vz(k,i)+az(k,i)*dt;
38  %  Calculate the position at point i+1
39  rx(k,i+1)=rx(k,i)+vx(k,i)*dt;
40  ry(k,i+1)=ry(k,i)+vy(k,i)*dt;
41  rz(k,i+1)=rz(k,i)+vz(k,i)*dt;
42  end
43  plot3(rx(k,:),ry(k,:),rz(k,:),'blue');
44  hold on
45  axis([-0.05,0.05,-0.05,0.05,0,35])  % set the axis
46  end
47
48  grid;
49  title({['Magnetic focusing of multiple charged particles moving in electromagnetic field'],...
50  ['(Huang Junlong,11810405)']} ,'FontName','Times New Roman', 'fontsize', 15);
51  xlabel('x-axis','FontName','Times New Roman','fontsize',15);
52  ylabel('y-axis','FontName','Times New Roman','fontsize',15);
53  zlabel('z-axis','FontName','Times New Roman','fontsize',15);
54
55  %% The cross section of xoy plane
56  figure
57
58  for m=1:16
59  plot(rx(m,:),ry(m,:),'blue');
60  grid on
61  hold on
62  end
63
64  axis equal
65  axis([-0.04,0.04,-0.04,0.04])
66  title({['Magnetic focusing of xoy cross section (5 period for dt=0.001)'],...
67  ['(Huang Junlong,11810405)']} ,'FontName','Times New Roman', 'fontsize', 15);
68  % title({['Magnetic focusing of xoy cross section (t=T/8)'],...
69  %      ['(Huang Junlong,11810405)']} ,'FontName','Times New Roman', 'fontsize', 15);
70  % title('t=T', 'fontsize', 15);
71  xlabel('x-axis','FontName','Times New Roman','fontsize',15);
72  ylabel('y-axis','FontName','Times New Roman','fontsize',15);
```

```matlab
%% The envelope circle in xoy plane
theta = 0:0.001:2*pi;
R=0.02*0.1/0.016/8;
x=2*R.*cos(theta);
y=2*R.*sin(theta);
plot(x,y,'red','LineWidth',3)

%% The cross section of xoz plane
figure
for n=1:16
plot(rx(n,:),rz(n,:),'blue');
grid on
hold on
end
axis([-0.05,0.05,-1,35])
% title({['Magnetic focusing of xoy cross section (5 period for dt=0.001)'],...
%      ['(Huang Junlong,11810405)']} ,'FontName','Times New Roman', 'fontsize', 15);
title({['Magnetic focusing of xoz cross section (dt=0.0001)'],...
['(Huang Junlong,11810405)']} ,'FontName','Times New Roman', 'fontsize', 15);
xlabel('x-axis','FontName','Times New Roman','fontsize',15);
ylabel('z-axis','FontName','Times New Roman','fontsize',15);

%% The datum line of xoz plane
uz=linspace(-0.05,0.05,100);
z=repmat(10*T,100);
ux=zeros(1,100);
x=linspace(9.75,9.90,100);
plot(ux,x,'red','LineWidth',2)
plot(uz,z,'red','LineWidth',2)
axis([-0.001,0.001,9.75,9.90])
%%
```