MATLAB Simulation of Electric Field Distribution of Finite Line Charge

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***Abstract*—At present, there are many studies on the electric field and potential distribution of infinitely long line charges. However, because of the fact that the length of the line charge under investigation cannot be infinitely long, it is more important to study the electric field potential distribution of the line charge of finite length. Therefore, this paper uses calculus and microelement method to calculate the electric field of finite length line charge, and uses Matlab to visualize the equipotential line and electric field line.**

***Keywords*—Electric field distribution, Finite line charge, Matlab simulation, Infinitesimal Method.**

I. Introduction

THE electric field distribution of the finite linear charge is a basic model other than the electric field distribution of the point charge. However, the charged conductor in actual life and production cannot be infinitely long. As long as the length of the charged conductor is much larger than its width, it can be reduced to a finite length linear charge model for research.

This problem was solved long ago. Ernst Weber treated it in the 1965 revision[**?**] of his book, referencing earlier work of Abraham and Becker[**?**] among others. The latter introduced elliptic coordinates to show that the equipotentials are prolate ellipsoids. There are potential gradient method, Gauss's theorem and arc equivalent method to calculate the electric field distribution of line charge. Due to the continuous characteristics of line charges, complex integration operations are often performed when solving electric fields, and the algebraic results obtained are not particularly intuitive.

Therefore, this article will use the infinitesimal method and the integration method for comparative calculations, and simulate on Matlab to draw the electric field image. Considering that it is very easy to calculate the potential in a two-dimensional plane, and it is also very convenient to use Matlab to find the gradient, so this paper uses the potential gradient method to calculate the electric field distribution. And quantitative analysis of the approximate effect of infinitesimal method and integration method.

II. Method and Analysis

A. Calculus Method

The electric field strength E generated by the point charge in vacuum is:

$$\mathbf{E} = k\frac{Q}{R^2}\mathbf{a_R} \tag{1}$$

The coefficient $k = 9 \times 10^9\ F/m$, which is a measure of electrostatic force, Q is the amount of charge of a point charge, and R is the distance from the point charge to the field point.

If the zero potential point is taken at infinity, the potential generated by the point charge in vacuum is:

$$V = k\frac{Q}{R} \tag{2}$$

Suppose that a linear charge with a density of $\rho = 1 \times 10^{-9}\ C/m$ is distributed on the straight line between point A (-1,0) and point B (1,0) under two-dimensional rectangular coordinates. (The coordinate unit is meter m). Let the coordinates of a point on the plane be (X_0, Y_0),then by integration, the potential at that point is:

$$\begin{aligned} V &= k\int_{-1}^{1}\frac{\rho \mathrm{d}x}{R} \\ &= k\int_{-1}^{1}\frac{\rho \mathrm{d}x}{\sqrt{(x-X_0)^2+Y_0^2}} \\ &= k\rho\int_{-1}^{1}\frac{1}{\sqrt{(x-X_0)^2+Y_0^2}}\mathrm{d}x \\ &= k\rho\cdot\ln|(x-X_0)+\sqrt{(x-X_0)^2+Y_0^2}|\Big|_{-1}^{1} \\ &= k\rho\ln\Big(\frac{1-X_0+\sqrt{(1-X_0)^2+Y_0^2}}{-1-X_0+\sqrt{(-1-X_0^2)+Y_0^2}}\Big) \end{aligned} \tag{3}$$

The electric field strength can be expressed as a negative gradient of potential, namely:

$$\begin{aligned} \mathbf{E} &= -\nabla V \\ &= -(\frac{\partial V}{\partial x}\mathbf{a_x} + \frac{\partial V}{\partial y}\mathbf{a_y}) \end{aligned} \tag{4}$$

Since the analytical formula of the calculation result is too complicated, it is omitted here. This is a very easy way to get the electric field at point (X_0, Y_0) in Matlab, only need to use the Matlab function `[Ex,Ey]=Gradient(-V)` to get the electric field strength .

B. Infinitesimal Method

Due to the huge amount of calculation of the calculus method, and the computer often processes discrete rather than continuous information, in fact, when using the calculus method to calculate the electric field distribution in the plane, Matlab also divides the plane into several grid points to calculate one by one. Therefore, using infinitesimal method to divide appropriate infinitesimaali is a better choice to reduce the amount of computation and improve the efficiency of calculation.

First, we divide the line charge into several small charges (usually using the method of even division). Suppose it is divided into n segments, the amount of charge in each segment is $\dfrac{\rho L}{n}$.

Then treat each small piece of charge as a point charge, solve the potential generated in space. The potential expression of the i-th charge $((-1+\dfrac{i-1}{n-1}),0)$ at point (X_0,Y_0) is:

$$\begin{aligned} V_i &= k\frac{Q_i}{R_i} \\ &= k\frac{\rho L/n}{\sqrt{(-1+\dfrac{i-1}{n-1}-X_0)^2+Y_0^2}} \end{aligned} \tag{5}$$

And then, sum the potential of each piece of charge to obtain the point

$$V=\sum_{i=1}^{n}V_i \tag{6}$$

where the entire line charge is generated.

Finally, the gradient formula is used as continuous charges before to solve the electric field strength generated by the entire line charge.

C. Matlab Function

Matlab has very convenient functions in studying potential electric fields. Here are some main functions for Matlab simulation.

```
xi=linspace(-1,1,n)
%Line charge n is equally divided on the interval
[Ex,Ey]=-Gradient(V)
%Calculating the electric field strength component
contour(X,Y,V,Veq)
%Draw the equipotential lines
streamline(X,Y,Ex,Ey,x_start,y_start)
%Draw the electric field lines
```

D. Potential Line and Electric Field Line Cluster Function

By formula (3), (4) can be obtained x, y direction of the electric field strength component:

$$\begin{aligned} E_x &= -\frac{\partial V}{\partial x} \\ &= k\rho\Big[\frac{1}{\sqrt{(-1-X_0)^2+Y_0^2}}-\frac{1}{\sqrt{(1-X_0)^2+Y_0^2}}\Big] \\ &= k\rho\frac{r_1-r_2}{r_1r_2} \end{aligned} \tag{7}$$

$$\begin{aligned} E_y &= -\frac{\partial V}{\partial y} \\ &= k\rho\Big[\frac{1}{\dfrac{(-1-X_0)r_2}{Y_0^2}+\dfrac{r_2^2}{Y_0^2}}-\frac{1}{\dfrac{(1-X_0)r_1}{Y_0^2}+\dfrac{r_1^2}{Y_0^2}}\Big] \end{aligned} \tag{8}$$

where $r_1=\sqrt{(1-X_0)^2+Y_0^2}$, $r_2=\sqrt{(-1-X_0)^2+Y_0^2}$.

Differential equations that can be satisfied by the power line of the (7), (8) subtype:

$$\frac{\mathbf{d}y}{\mathbf{d}x}=\frac{E_y}{E_x}=\frac{r_2(1-X_0)-r_1(-1-X_0)}{Y_0(r_1-r_2)} \tag{9}$$

Simplicity can be obtained: $\dfrac{\mathrm{d}r_2^2}{r_2}=\dfrac{\mathrm{d}r_1^2}{r_1}$, after integration:

$$\sqrt{(1-X_0)^2+Y_0^2}-\sqrt{(-1-X_0)^2+Y_0^2}=C \tag{10}$$

Equation (10) is the finite line charge power line cluster function.

From equation (3), since k and ρ are constants, the equipotential cluster function is

$$\frac{1-X_0+\sqrt{(1-X_0)^2+Y_0^2}}{-1-X_0+\sqrt{(-1-X_0)^2+Y_0^2}}=C \tag{11}$$

Simplify and organize as follows:

$$\begin{gathered} 4C(C-1)^2x^2+(C^2-1)^2y^2=4C(C+1)^2 \\ \frac{x^2}{(\dfrac{C+1}{C-1})^2}+\frac{y^2}{\dfrac{4C(C+1)^2}{(C^2-1)^2}}=1 \end{gathered} \tag{12}$$

It can be seen from the equation (12) that the equipotential lines are elliptically distributed. For ellipses $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$, the expression of eccentricity is $e=\dfrac{c}{a}=\sqrt{1-\dfrac{b^2}{a^2}}$. The result is as follows:

$$\begin{aligned} e &= \sqrt{1-\frac{4C(C+1)^2}{(C^2-1)^2}\cdot\frac{(C-1)^2}{(C+1)^2)}} \\ &= \sqrt{1-\frac{4C(C-1)^2}{(C^2-1)^2}} \end{aligned} \tag{13}$$

The relationship between the elliptical eccentricity and the constant C is shown in the Fig. 1.

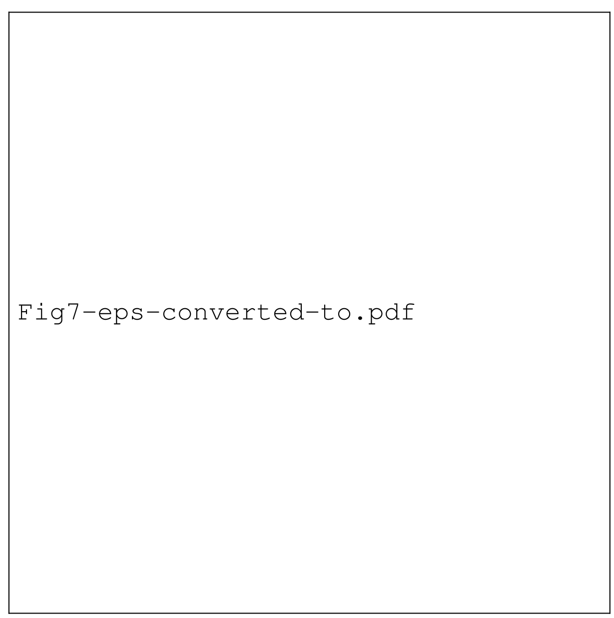


Fig. 1. The relationship between eccentricity e and constant C.

Due to the line charge is positive, so the potential is not negative. So $C > 1$ for $\ln C > 0$. It can be seen from the monotonically increasing relationship of the logarithmic function that the larger the constant C, the larger the potential on the equipotential line. Therefore, we found that when the electric potential is small (where the area far away from the line charge), the eccentricity of the ellipse approaches 0, and the shape of the equipotential line tends to be circle ($e = 0$). At the same time, the higher the potential, the closer the eccentricity is to 1 and the flatter the ellipse. And also, this result is consistent with Rowley's conclusion at polar coordinates.[**?**]

E. Error Analysis

There is a certain error between the result obtained by the infinitesimal method and the true value. The error mainly depends on the number of segments divided by the line charge. Generally speaking, the more segments, the smaller the error.

First we introduce the concept of squared residuals v^2.Let

$$v^2 = (V - V_i)^2$$

V is the real potential calculated by the integral method, V_i is the electric potential approximated by the infinitesimal method.

In this way, we can infer the relative magnitude of the error between the results of the infinitesimal method and the true value through the value of the squares of the residuals.

However, due to the potentials at different points in the plane are different, in order to eliminate this effect, the normalized residual square is used to quantitatively describe the error.

We Define the normalized residual square is v':

$$v' = (\frac{V - V_i}{V})^2$$

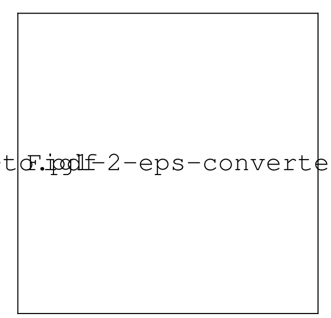
The smaller v' is, the closer the infinitesimal method result is to the true value.

III. Results and Discussion

A. Calculus Method Result

Using the results of calculus, in Matlab simulation, the potential, the potential and the electric field distribution of the finite line chargeas are shown in Fig. 2.

(a)



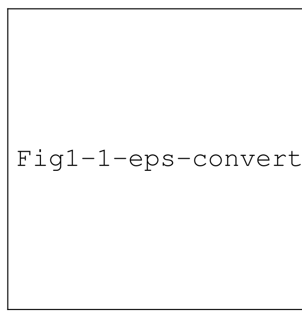
(b)

Fig. 2. Calculus Method Calculation results: (a) The potential of the finite line chargeas, (b) The electric field distribution of the finite line chargeas

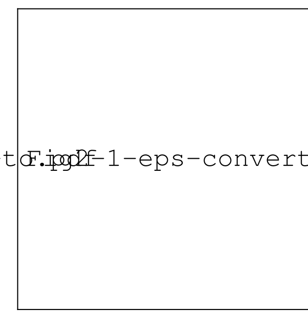
From Fig. 2 (a), we can see the characteristics of finite line charge potential distribution: potential is a long bar, close to the position of the wire charge potential is high, far away from the line charge where the potential is low. At line charge intermediate accessory, the potential changes gently. At both ends of the line, the potential drops rapidly. From Fig. 2 (b), it can be seen that the distribution of finite line charge electric field is indeed elliptical, as mentioned above. The area close to the line charge, the more flat the ellipse formed. The area away from the line charge, the more elliptical closer to the circle. At the same time, the electric field line is perpendicular to the isopotential face, like the electric field line of the point charge, dispersing in all directions.

B. Infinitesimal Method Result

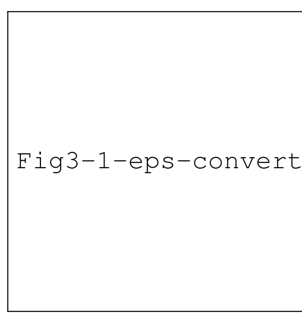
Using the results of infinitesimal method, in Matlab simulation, the potential, the potential and the electric field distribution of the finite line chargeas are shown in Fig. 3 and Fig. 4 as follows.



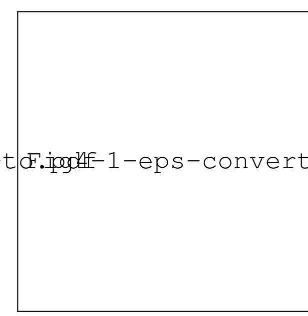
(a) Actuality



(b) n=20



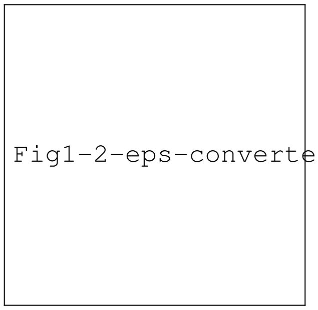
(c) n=50



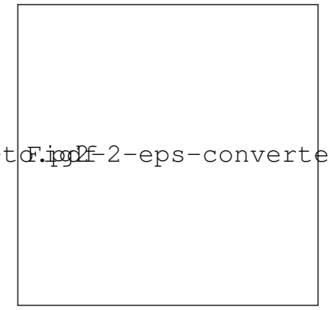
(d) n=100

Fig. 3. The potential distribution of the finite line chargeas (a) The actually result calculated by Calculus Method (b) The approximate result calculated by Infinitesimal Method when n=20 (c) n=50 (d) n=100

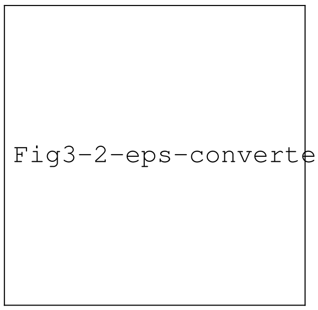
As can be seen from Fig. 3 (b), when the number of equal segments $n$is small, the potential distribution on the line charge is jagged. When n is increased to 50 shown as Fig. 3 (c), the jagged becomes dense. When n is increased to 100 shown as Fig. 3 (d), the jagged edge become almost invisible. The infinitesimal method and the calculus method get almost the same graph.



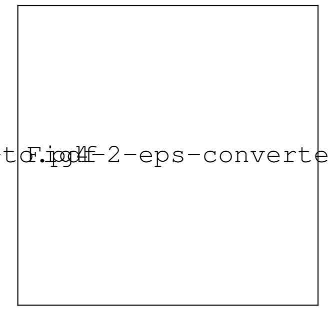
(a) Actuality



(b) n=20



(c) n=50



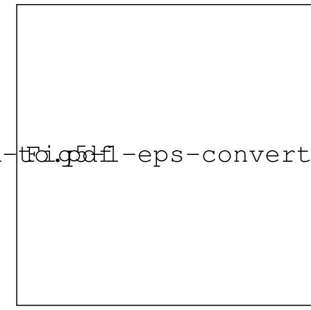
(d) n=100

Fig. 4. The potential and the electric field distribution of the finite line chargeas (a) The actually result calculated by Calculus Method (b) The approximate result calculated by Infinitesimal Method when n=20 (c) n=50 (d) n=100

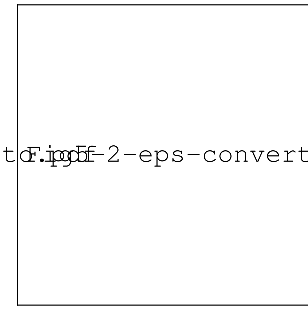
Fig. 4 is the potential and the electric field distribution of the finite line chargeas in a two-dimensional plane. As n increases, the subdivision of point charge increases, and the potential surface of the ellipse becomes smoother.

C. Error Analysis Results

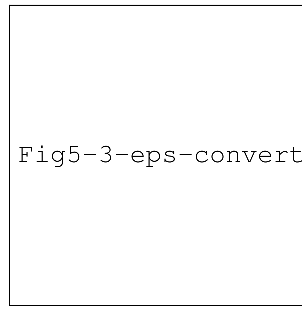
Using the square residual method, the potential obtained by the infinitesimal was compared with the potential obtained by the calculus, the figures are as follows:



(a) n=20



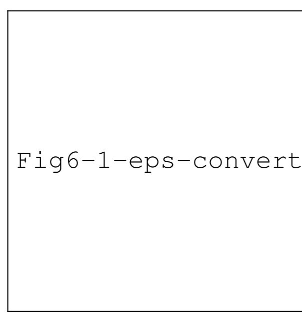
(b) n=50



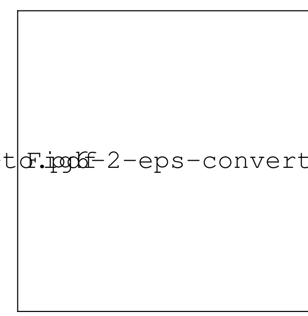
(c) n=100

Fig. 5. The square residual between infinitesimal and calculus (a) n=20 (b) n=50 (c) n=100

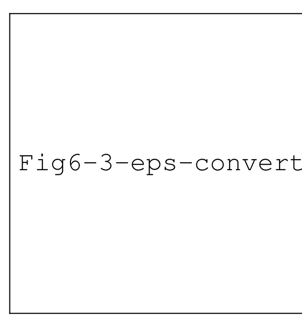
Since the actual potentials are different in different locations, and the z-axis ratioof of the three graphs is also different. In order to compare the results more directly and accurately, the residuals are normalized and squared, and the z-axis scale is set the same as the graph, the results are as follows.



(a) n=20



(b) n=50



(c) n=100

Fig. 6. The square of normalized residual between infinitesimal and calculus (a) n=20 (b) n=50 (c) n=100

As can be seen from Fig. 5 (a), when $n = 20$, the residual square is quite large, indicating that the results of the infinitesimal method calculation are very different from the actual result. The gap between infinitesimal method result and actual result is significantly smaller when n is 50 as shown in Fig. 5 (b). When $n = 100$, only the two ends of the line charge have a little significantly difference as shown in Fig. 5 (c).

From Fig. 6, it can be seen more clearly that with the increase of n, the result of infinitesimal method and the actual result of the generalization residual smaller, the less error. That is, the more micro-meta-segments, the smaller the error with the real result.

IV. Conclusion

In this article, we use infinitesimal method and calculus method, respectively, to calculate the potential and electric field distribution of finite-length line charge, and to simulate and visualize in Matlab. Through calculation and simulation, we find that the isopotential plane of the finite length line charge is elliptically distributed, and the power line is perpendicular to the isopotential face from the line charge to the surrounding.

At the same time, we introduce the concept of residual square, and quantitatively analyze the error between the result of infinitesimal method and the real value. It is verified that the more the number of microelements, the closer the result is to the true value. As can be seen from the Fig. 6, $n = 50$ is an approximate processing value that reduces the calculation quantity to improve the computational efficiency without affecting too much the accuracy of the result.

Other calculation methods for finite-length wire charge electric fields and the best values for n will be expanded in subsequent work.

APPENDIX A

```
%% This is the code for Lab 2

%% Basic setup
clear all
clear;                     % Empty all variables in memory
clc;                       % Clear the contents of the command window
k=9e9;                     % Set the electrostatic force measure
rho=1e-9;                  % Set the charge density
xm=2;                      % Set the range in the x direction of the field
ym=2.5;                    % Set the range in the y direction of the field
x=linspace(-xm,xm,100);    % Divide the X-axis into 100 equal parts
y=linspace(-ym,ym,100);    % Divide the Y-axis into 100 equal parts
[X,Y]=meshgrid(x,y);       % Formed the coordinates of the points in the field

%% Calculus method
% Calculate the potential at each point in the field
V=k*rho.*log((1-X+sqrt((1-X).^2+Y.^2))./(-1-X+sqrt((-1-X).^2+Y.^2)));
% Plot the distribution of the potential
mesh(X,Y,V);
title({['Potential distribution of finite linear charges'],...
['(Huang Junlong,11810405)']},'FontName','Times New Roman','fontsize',16);
xlabel('X-axis (Unit: m)','FontName','Times New Roman','fontsize',12);
ylabel('Y-axis (Unit: m)','FontName','Times New Roman','fontsize',12);
zlabel('Z-axis (Unit: V)','FontName','Times New Roman','fontsize',12);
figure
Vmin=10;               % Set the minimum potential of the family of equipotential lines
Vmax=60;               % Set the maximum potential of the family of equipotential lines
Veq=linspace(Vmin,Vmax,18);        % Set the potential value of 18 equipotential lines
contour(X,Y,V,Veq);                % Draw 18 equipotential lines
grid on                            % Form a grid
hold on                            % Keep the graphics
line([-1,1],[0,0],'LineWidth',3,'color','y');  % Draw the line charge
% Calculate two components of the electric field intensity at each point in the field
[Ex,Ey]=gradient(-V);
% Generate the X-axis coordinate of the starting point of the power line;
xs=linspace(-1.05,1.05,15);
% Generate the Y-axis coordinate of the starting point of the power line;
ys1=repmat(0.05,1,15);
ys2=repmat(-0.05,1,15);
% Generating power lines;
streamline(X,Y,Ex,Ey,xs,ys1);
streamline(X,Y,Ex,Ey,xs,ys2);
streamline(X,Y,Ex,Ey,-1.05,0);
streamline(X,Y,Ex,Ey,1.05,0);
title({['Electric potential lines and field lines of finite line charge'],...
['(Huang Junlong,11810405)']} ,'FontName','Times New Roman', 'fontsize', 16);
xlabel('X-axis (Unit: m)','FontName','Times New Roman','fontsize',12);

%% Infinitesimal method
qi=zeros(1,3);                  % Set micro-point charges
ni=[20,50,100];                 % Set the number of equal parts of the line charge
Error=zeros(100,100,3);         % Use residuals as errors
ErrorNor=zeros(100,100,3);      % Use normalized residuals as errors
xqi=zeros(1,100,3);             % Initialization the micro-point charges x coordinate
% Set thr micro-point charges x coordinate
xqi(:,:,1)=[linspace(-1,1,20),zeros(1,80)];
xqi(:,:,2)=[linspace(-1,1,50),zeros(1,50)];
xqi(:,:,3)=[linspace(-1,1,100)];
V1=zeros(100,100);              % Initialization the electric potential
for m=1:3                       % Set for loop for 3 cases
figure
qi(m)=rho*2/ni(m);          % Calculate the amount of point charge
for i=1:ni(m)
% Calculate the potential generated by the point charge at each point
Vi=k*qi(m)./sqrt((X-xqi(1,i,m)).^2+Y.^2);
% Sum up to get the potential generated by the entire line charge
V1=V1+Vi;
end
mesh(X,Y,V1);               % Plot the distribution of the potential
title({['Potential distribution of finite linear charges (n=' num2str(ni(m)) ')'],...
['(Huang Junlong,11810405)']},'FontName','Times New Roman','fontsize',16);
xlabel('X-axis (Unit: m)','FontName','Times New Roman','fontsize',12);
ylabel('Y-axis (Unit: m)','FontName','Times New Roman','fontsize',12);
```

```
zlabel('Z-axis (Unit: V)','FontName','Times New Roman','fontsize',12);
figure
for n=1:ni(m)
plot(xqi(1,n,m),0,'o', 'MarkerSize',5)  % Draw n point charges
hold on
end
Vmin=10;              % Set the minimum potential of the family of equipotential lines
Vmax=60;              % Set the maximum potential of the family of equipotential lines
Veq=linspace(Vmin,Vmax,18);       % Set the potential value of 18 equipotential lines
contour(X,Y,V1,Veq);              % Draw 18 equipotential lines
hold on
grid on
[Ex,Ey]=gradient(-V);
xs=linspace(-1.05,1.05,15);
ys1=repmat(0.05,1,15);
ys2=repmat(-0.05,1,15);
% Generating power lines;
streamline(X,Y,Ex,Ey,xs,ys1);
streamline(X,Y,Ex,Ey,xs,ys2);
streamline(X,Y,Ex,Ey,-1.05,0);
streamline(X,Y,Ex,Ey,1.05,0);
title({['Electric potential lines and field lines of finite line charge (n=' num2str(ni(m)) ')'],...
['(Huang Junlong,11810405)']} ,'FontName','Times New Roman', 'fontsize', 15);
xlabel('X-axis (Unit: m)','FontName','Times New Roman','fontsize',12);
% Calculate the sum of squared residuals.
% between the infinitesimal and the integrated potential
Error(:,:,m)=(V1-V).^2;
% Calculate the sum of squared normalized residuals.
% between the infinitesimal and the integrated potential
ErrorNor(:,:,m)=((V1-V)./V).^2;
% Clear and reset the voltage at each point of the planar field
clear Vi;
V1=zeros(100,100);
end

%% Error Analysis
% Infinitesimal and integral electric potential residual square
for k=1:3
figure
mesh(X,Y,Error(:,:,k));
% axis([-2 2,-2.5,2.5,0 200])
title({['Infinitesimal and integral electric potential residual square (n=' num2str(ni(k)) ')'],...
['(Huang Junlong,11810405)']},'FontName','Times New Roman','fontsize',14);
xlabel('X-axis (Unit: m)','FontName','Times New Roman','fontsize',12);
ylabel('Y-axis (Unit: m)','FontName','Times New Roman','fontsize',12);
zlabel('Z-axis (Unit: V)','FontName','Times New Roman','fontsize',12);
end
% Infinitesimal and integral electric potential normalized residual square
for k=1:3
figure
mesh(X,Y,ErrorNor(:,:,k));
axis([-2 2,-2.5,2.5,0 0.1])
title({['Infinitesimal and integral electric potential normalized residual square (n=' ...
num2str(ni(k)) ')'],['(Huang Junlong,11810405)']},'FontName','Times New Roman','fontsize',12);
xlabel('X-axis (Unit: m)','FontName','Times New Roman','fontsize',12);
ylabel('Y-axis (Unit: m)','FontName','Times New Roman','fontsize',12);
zlabel('Z-axis ','FontName','Times New Roman','fontsize',12);
end

%% The calculation of elliptical eccentricity
clc,clear
xn=linspace(1,1e4,1e5);
y=sqrt(1-(4.*xn.*(xn-1).^2)./((xn.^2-1).^2));
plot(xn,y)
semilogx(xn,y)
grid on
xlabel('Constant C','FontName','Times New Roman','fontsize',12)
ylabel('Eccentricity e','FontName','Times New Roman','fontsize',12)
title('Relationship between eccentricity and constant C','FontName','Times New Roman','fontsize',16)
```