6/9/17.

Unit - 3

Trigonometry.

Introduction.

Trigonometry is basically a greek word which stands for 3 angle measurement i.e the angle is measured in terms of ratios of sides of the triangle.

Trigonometry has become the part of rapid growing science, engineering and technology.

Measurement of an angle.

The angles of triangle measured in many systems in older days which are mainly given as follows.

1] Sexagesimal system (British system)

2] Centesimal system (French system).

3] Radian or circular system (Greek system).

1] Sexagesimal system (British System) :-

In this system angle measure in right angle. 1 right angle is divided into 90 equal parts called degrees, each degree is divided into 60 equal parts called minutes, each minutes is divided into

60 equal parts called seconds

1 right angle = 90 degree = 90°

1 degree = 60 minutes = $60'$

1 minutes = 60 seconds = $60''$

2] Centisimal system (French system) :-

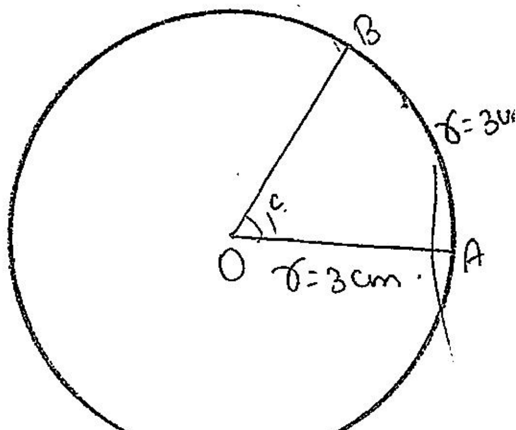
1 right angle = 100 grades = 100^g.

1 grade = 100 minutes = $100'$

1 minutes = 100 seconds = $100''$

3] Radian or circular system (Greek system) :-

Radian is a plain subtanted at a center by an arch of a circle such that radius of the circle = ~~legal~~ length of an arch of a circle.



Radian is a constant angle

$$\boxed{\begin{aligned} \pi^c &= 180^\circ \\ 1^c &= \left(\frac{180}{\pi}\right)^\circ \\ 1^\circ &= \left(\frac{\pi}{180}\right)^c \end{aligned}}$$

Convert the following degrees into radians.

30°

$1^\circ = \left(\frac{\pi}{180}\right)^c$

$30 = \frac{\pi}{\cancel{180}_{6}} \times \cancel{30}^{1}$

$30^\circ = \left(\frac{\pi}{6}\right)^c$

45°.

$1^\circ = \left(\frac{\pi}{180}\right)^c$

$45^\circ = \frac{\pi}{\cancel{180}_{4}} \times \cancel{6045}$

$45^\circ = \left(\frac{\pi}{4}\right)^c$.

60°

$1 = \left(\frac{\pi}{180}\right)^c$

$60^\circ = \frac{\pi}{\cancel{180}_{3}} \times \cancel{60}^{1}$

$60^\circ = \left(\frac{\pi}{3}\right)^c$.

90

$$1^\circ = \left(\frac{\pi}{180}\right)^c$$

$$90^\circ = \frac{\pi}{\cancel{180}_{2}} \times \cancel{90}$$

$$90^\circ = \frac{\pi}{2}$$

120°

$$1^\circ = \left(\frac{\pi}{180}\right)^c$$

$$120^\circ = \frac{\pi}{\cancel{180}_{3}} \times \cancel{120}^{2}$$

$$120^\circ = \left(\frac{2\pi}{3}\right)^c$$

150.

$$1^\circ = \left(\frac{\pi}{180}\right)^c$$

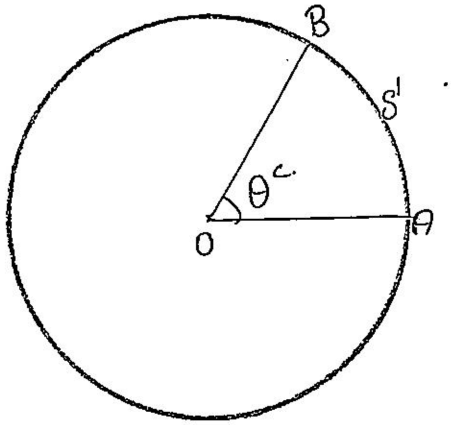
$$150 = \frac{\pi}{\cancel{180}_{3\times 6}} \times \cancel{150}^{3\times 5}$$

$$150 = \left(\frac{5\pi}{6}\right)^c$$

240°

$$1^\circ = \left(\frac{\pi}{180}\right)^c$$

Length of an arc



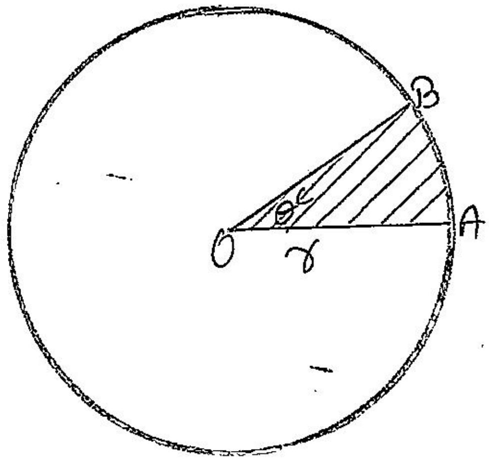
$S' = r\theta^c$.

S → length of an arc.

r = radius of the circle.

θ^c = Angle in terms of radian.

Area of a sector.



$$A = \frac{1}{2} r^2 \theta^c$$

A → Area of sector.

r = Radius of the cle.

θ^c = Angle in terms radian

Home work

1) 180°

$$1^\circ = \left(\frac{\pi}{180}\right)^\circ.$$

$$180^\circ = \frac{\pi}{\cancel{180^\circ}} \times \cancel{180^\circ}$$

$$180^\circ = \frac{\pi}{1}.$$

2) 210

$$1^\circ = \frac{\pi}{180^\circ}$$

$$210^\circ = \frac{\pi}{\cancel{210}\,\underset{3\times 6}{180}} \times \overset{3\times 7}{\cancel{210}}$$

$$300 = \frac{5\pi}{3}$$

6) 330°:

$$1^\circ = \left(\frac{\pi}{180}\right)^c$$

$$330 = \frac{\pi}{\cancel{180}_{\cancel{3}\times 6}} \times \cancel{330}^{11\times\cancel{3}}$$

$$330 = \frac{11\pi}{6}$$

7) 360°:

$$1^\circ = \left(\frac{\pi}{180}\right)^c$$

$$360 = \frac{\pi}{\cancel{180}_{\cancel{3}\times\cancel{6}}} \times \cancel{360}^{\cancel{3}\times\cancel{12}\,2}$$

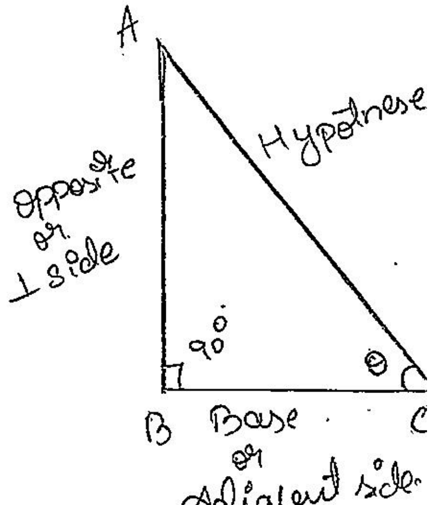
$$360 = \frac{2\pi}{1}$$

Trignometric functions.

Trignometric function is a study of measurement of angles in terms of ratio's of sides of the right triangle.

we can define 6 types of trigonometric functions which are as follows.

consider right angle triangle ABC.
in which angle $\angle ACB = \theta$, side BC is called adjacent side or base, AB is called opposite side or perpendicular side and AC is hyponese



Sine of angle of θ = $\sin\theta = \dfrac{\text{Opp side}}{\text{Hyp}} = \dfrac{AB}{AC}$.

Cosine of angle of θ = $\cos\theta = \dfrac{\text{Adj}}{\text{Hyp}} = \dfrac{BC}{AC}$.

Tangent of angle of θ = $\tan\theta = \dfrac{\text{Opp}}{\text{Adj}} = \dfrac{AB}{BC}$.

cosecant of angle of θ = $\operatorname{cosec}\theta = \dfrac{\text{Hyp}}{\text{Opp}} = \dfrac{AC}{AB}$

Secant of angle of θ = $\sec\theta = \dfrac{\text{Hyp}}{\text{Adj}} = \dfrac{AC}{BC}$

cotangent of angle of θ = $\cot\theta = \dfrac{\text{Adj}}{\text{Opp}} = \dfrac{BC}{AB}$.

Relationship b/w trigonometric function.

$\sin\theta = \dfrac{1}{\operatorname{cosec}\theta}$, $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$, $\sin\theta \cdot \operatorname{cosec}\theta = 1$.

$\cos\theta = \frac{1}{\sec\theta}$, $\sec\theta = \frac{1}{\cos\theta}$, $\cos\theta \cdot \sec\theta = 1$.

$\tan\theta = \frac{1}{\cot\theta}$, $\cot\theta = \frac{1}{\tan\theta}$, $\tan\theta \cdot \cot\theta = 1$.

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

Trigonometric identities.

1] $\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$\sin\theta = \sqrt{1-\cos^2\theta}$

$\cos^2\theta = 1 - \sin^2\theta$

$\cos\theta = \sqrt{1-\sin^2\theta}$

2] $1+\tan^2\theta = \sec^2\theta$

$\sec\theta = \sqrt{1+\tan^2\theta}$

$\tan^2\theta = \sec^2\theta - 1$

$\tan\theta = \sqrt{\sec^2\theta - 1}$

$\sec^2\theta - \tan^2\theta = 1$

3] $1+\cot^2\theta = \csc^2\theta$.

$\csc\theta = \sqrt{1+\cot^2\theta}$.

$\cot\theta = \sqrt{\csc^2\theta - 1}$

$\cot^2\theta = \csc^2\theta - 1$

$\csc^2\theta - \cot^2\theta = 1$.

Standard Angles:-

Angles of the form 0°, 30° or $(\pi/6)^c$ or 45° or $(\pi/4)^c$, 60° or $(\pi/3)^c$, 90° or $(\pi/2)^c$ are called standard angles.

	$\sqrt{\frac{0}{4}}$ 0°	$\sqrt{\frac{1}{4}}$ 30° $(\pi/6)^c$	$\sqrt{\frac{2}{4}}$ 45° $(\pi/4)^c$	$\sqrt{\frac{3}{4}}$ 60° $(\pi/3)^c$	$\sqrt{\frac{4}{4}}$ 90° $(\pi/2)^c$
Sin	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
Cos	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
tan	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞
Cosec	∞	2	$\sqrt{2}$	$2/\sqrt{3}$	1
Sec	1	$2/\sqrt{3}$	$\sqrt{2}$	2	∞
cot	∞	$\sqrt{3}$	1	$1/\sqrt{3}$	0

Problems:-

Simplify the following.

Sin 30 x cos 60 + Cos 30 x Sin 60

$\frac{1}{2} \cdot \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} \quad = (\sqrt{3})^2 = 3$

$\frac{1}{4} + \frac{3}{4} = \frac{1+3}{4} = \frac{4}{4} = 1$

$\sin^2 30^\circ + \cos^2 45 - \tan^2 30$

$\left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^{\cancel{2}} - \left(\frac{1}{\sqrt{3}}\right)^{\cancel{2}}$

$\frac{1}{4}^{\times 3} + \frac{1}{2}^{\times 6} - \frac{1}{3}^{\times 4}$

$\frac{3+6-4}{12} = 5/12.$

$$\frac{\cos^2 \pi/4 + \sec^2 \pi/3}{\cot^2 \pi/3}$$

$$\frac{\left(\frac{1}{\sqrt{2}}\right)^2 + (2)^2}{\frac{1}{(\sqrt{3})^2}}$$

$$\frac{\frac{1}{2} + \frac{4}{1}}{\frac{1}{3}} = \frac{\frac{1+8}{2}}{\frac{1}{3}} = \frac{\frac{9}{2}}{\frac{1}{3}}$$

$\frac{9}{2} \times \frac{3}{1}$

$= 27/2$

$$\frac{4 \sin^2 60 \cos^2 60 \sec^2 30 \csc^2 45}{2 \csc^2 30 - \frac{1}{2} \sin^2 60 \tan^2 30}$$

$$\frac{4\left(\frac{\sqrt{3}}{2}\right)^2 \cdot \left(\frac{1}{2}\right)^3 \cdot \left(\frac{2}{\sqrt{3}}\right)^2 (\sqrt{2})^2}{2 \times 2^2 - \frac{1}{2}\left(\frac{\sqrt{3}}{2}\right)^2 \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$\frac{\cancel{4}\,\frac{3}{\cancel{4}}\cdot\frac{1}{\cancel{8}}\;\frac{\cancel{4}}{\cancel{3}}\cdot\cancel{2}}{\cancel{8}-\frac{1}{\cancel{2}}\times\frac{\cancel{3}}{\cancel{4}}\times\frac{1}{\cancel{3}}} = \frac{1}{8-\frac{1}{8}} \qquad \frac{1}{\frac{64-1}{8}} \qquad 1/6$$

$$\frac{1}{\frac{64-1}{8}} = \frac{1}{63/8} = \frac{8}{63}.$$

$$(\tan^2 60 \times \sin^2 45) - (4\cos 60^\circ \times \sin 30) + (\sec^2 0 \cdot \sin 90)$$

$$(\sqrt{3})^2 \cdot \left(\frac{1}{\sqrt{2}}\right)^2 - \cancel{4}\cdot\frac{1}{\cancel{2}}\times\frac{1}{\cancel{2}} + (1^2\cdot 1)$$

$$3\cdot\frac{1}{2} - \cancel{1} + \cancel{1}$$

$$= 3/2$$

$$\frac{\cos 0 \;\sin \pi/6 \;\cdot \cos^2\pi/3 \;\; \sec^2 \pi/4}{\tan \pi/3 + \cot \pi/3}$$

$$\frac{1\cdot\frac{1}{2}\left(\frac{1}{2}\right)^2\left(\sqrt{2}\right)^2}{\frac{\sqrt{3}}{1} + \frac{1}{\sqrt{3}}}$$

$$\frac{\frac{1}{\cancel{2}}\cdot\frac{1}{4}\cdot\cancel{2}}{\frac{3+1}{\sqrt{3}}}$$

$$\frac{\frac{1}{4}}{4/\sqrt{3}} = \frac{1}{4}\times\frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{16}$$

$$(\tan^2 60 \cdot \sin^2 45) - 4\cos 60 \cdot \sin 30 + \sec^2 0 \cdot \sin 90$$

$$\left((\sqrt{3})^2 \cdot \left(\frac{1}{\sqrt{2}}\right)^2\right) - \cancel{4} \times \frac{1}{\cancel{2}} \cdot \frac{1}{2} + (1)^2 \cdot 1$$

$$3 \cdot \frac{1}{2} \cancel{-1} + \cancel{1}$$

$$= 3/2$$

$$\frac{3\sin^2 60 \cdot \sin 30 \cdot \sec^2 30}{\operatorname{cosec}^2 45 + \frac{1}{2}\sin^2 60 \cdot \tan^2 30}$$

$$\frac{3 \times \left(\frac{\sqrt{3}}{2}\right)^2 \cdot \frac{1}{2} \cdot \left(\frac{2}{\sqrt{3}}\right)^2}{(\sqrt{2})^2 + \frac{1}{2} \times \left(\frac{\sqrt{3}}{2}\right)^2 \cdot \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$\frac{3 \times \frac{\cancel{3}}{\cancel{4}} \cdot \frac{1}{\cancel{2}} \cdot \frac{\cancel{4}}{\cancel{3}}}{\cancel{2} + \frac{1}{\cancel{2}} \times \frac{\cancel{3}}{4} \cdot \frac{1}{\cancel{3}}}$$

$$\frac{\frac{\cancel{9}}{\cancel{4}_2} \cdot \frac{\cancel{2}}{3}}{1 \times \frac{1}{4}}$$

$$\frac{\frac{\cancel{9}}{2} \cdot \frac{1}{3}}{\frac{4+1}{4}}$$

$$\frac{\cancel{\frac{9}{2} \cdot \frac{1}{3}}}{\cancel{\frac{5}{4}}}$$

$$\frac{\cancel{\frac{3+2}{6}}}{\cancel{\frac{5}{4}}}$$

$$= \frac{\cancel{\frac{5}{6}}}{\cancel{\frac{5}{4}}}$$

$$\cancel{\frac{5}{6} \times \frac{4}{5}}$$

$$\cancel{\frac{4}{6}} \cancel{\frac{2\times2}{2\times3}}$$

$$\cancel{\frac{2}{3}}$$

$$\frac{\frac{3}{2}}{\frac{2}{1} + \frac{1}{8}}$$

$$= \frac{\frac{3}{2}}{\frac{16+1}{8}}$$

$$\frac{3}{\cancel{2}} \times \frac{\cancel{8}\,4}{17}$$

$$= \frac{12}{17}.$$

H.W.

① $\tan^2 30 + \sin^2 45 + \cos^2 60 + \cos^2 90$

② $\cos^2 30 \cdot \tan^2 60 + 3\sin 30 \cdot \cot^2 30$.

③ $(\tan \pi/6 + \cot \pi/6)^2$

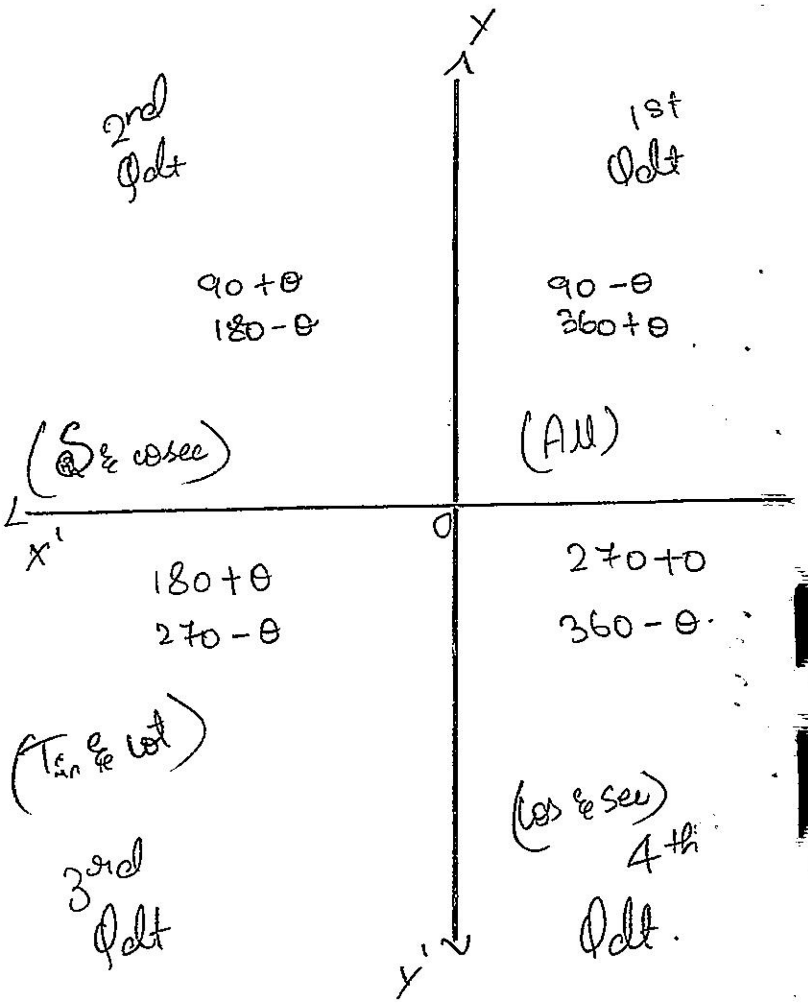
④ $\frac{1}{2}\sec^2 \pi/4 - 3/2 \cot^2 \pi/3 + 1/3 \tan^2 \pi/6 - \frac{1}{2}\operatorname{cosec}^2 \pi/3$

⑤ $(\tan \pi/4 + \cot \pi/4)(\tan 60 + \cot 60)(\sin \pi/6 + \cot \pi/6)$

V.I.I.

Allied Angles.

Angles of the form $90^\circ \pm \theta$, $180^\circ \pm \theta$, $270 \pm \theta$, $360^\circ \pm \theta$ are called Allied angles (AA)



I The Given 8 AA are divided in 4 qdts.

The AA $90-\theta$ & $360+\theta$ lies in the 1st qdts.

The AA $90+\theta$ & $180-\theta$ lies in the 2nd qdts

The AA $180+\theta$ & $270-\theta$ lies in the 3rd qdts.

The AA $270+\theta$ & $360-\theta$ lies in the 4th qdts

ASTC Rule

A → All Trigonometric Function are (+) in the 1st qdt

S → sin & cosec are (+) ve in the 2nd qdt

T → Tan & cot are (+) ve in the 3rd qdt

C → cos & sec are (+) ve in the 4th qdt.

3] For the AA 90 ±, 270 ± θ Trigonometric function will change from

sin → cos.

cos → sin

tan → cot

cot → tan

sec → cosec

cosec → sec

(4) For the allied angles 180 ± θ 360 ± θ Trigonometric function will not change (or) remains same.

(5) 90 − θ (1st qdt)

$\sin(90-\theta) = +\cos\theta$

$\cos(90-\theta) = +\sin\theta$

$\tan(90-\theta) = +\cot\theta$

$\operatorname{cosec}(90-\theta) = +\sec\theta$

$\sec(90-\theta) = +\operatorname{cosec}\theta$

$\cot(90-\theta) = +\tan\theta.$

90 + θ (2nd qdt).

$\sin(90+\theta) = +\cos\theta$

$\cos(90+\theta) = -\sin\theta$

$\tan(90+\theta) = -\cot\theta$

$\operatorname{cosec}(90+\theta) = +\sec\theta$

$\sec(90+\theta) = -\operatorname{cosec}\theta.$

$\cot(90+\theta) = -\tan\theta.$

Home work

[1] $\tan^2 30 + \sin^2 45 + \cos^2 60 + \cos^2 90$

$$\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + 0$$

$$\frac{1}{3} + \frac{1}{2} + \frac{1}{4}$$

$$\frac{4+6+3}{12} = \frac{13}{12}$$

$$\begin{array}{r|l} 2 & 3, 2, 4 \\ \hline 3 & 3, 1, 2 \\ \hline 2 & 1\ \ 1\ \ 2 \\ \hline \end{array}$$

$3 = 12$

$9 \times$ $\cancel{2 = 12}$

$\cancel{3 = 2}$

$4 =$

[2] $\cos^2 30 \cdot \tan^2 60 + 3 \sin 30 \cdot \cot^2 30$

$$\left(\frac{\sqrt{3}}{2}\right)^2 \cdot (\sqrt{3})^2 + 3 \cdot \frac{1}{2} \cdot (\sqrt{3})^2$$

$$\frac{3}{4} \cdot 3 + 3 \cdot \frac{1}{2} \cdot 3$$

$$\frac{9}{4} + \frac{9}{2}$$

$$\frac{9}{4} + \frac{18}{4} = \frac{27}{4}$$

[3] $(\tan \pi/6 + \cot \pi/6)^2$

$$\left(\frac{1}{\sqrt{3}} + \sqrt{3}\right)^2$$

$$\left(\frac{1}{\sqrt{3}}\right)^2 + (\sqrt{3})^2$$

$$\frac{1}{3} + 3 \quad \frac{4}{(\sqrt{3})^2} = \frac{16}{3}$$

(17)

4] $\frac{1}{2}\sec^2\pi/4 - 3/2\cot^2\pi/3 + 1/3\tan^2\pi/6 - 1/2\operatorname{cosec}^2\pi/3.$

$\frac{1}{2}\cdot(\sqrt{2})^2 - \frac{3}{2}\cdot\left(\frac{1}{\sqrt{3}}\right)^2 + \frac{1}{3}\cdot\left(\frac{1}{\sqrt{3}}\right)^2 - \frac{1}{2}\left(\frac{2}{\sqrt{3}}\right)^2$

$\frac{1}{\cancel{2}}\cdot\cancel{2} - \frac{\cancel{3}}{2}\cdot\frac{1}{\cancel{3}} + \frac{1}{3}\cdot\frac{1}{3} - \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{\cancel{\sqrt{3}}}$

$1 - \frac{1}{2} + \frac{3+3}{3} - \frac{1}{3}$ $\quad 1 - \frac{1}{2} + \frac{1}{9} - \frac{4}{3}$

$1 - \frac{1}{2} + \frac{9}{3} - \frac{1}{3}$ $\quad \frac{2-1}{2} +$

$1 - \frac{1}{2} + \frac{8}{3}$ $\quad = \frac{1}{2} + \frac{1}{9} - \frac{2}{3} = \frac{1}{2} + \frac{1}{9} - \frac{6}{9} = \frac{1}{2}$

$\frac{1}{2} + \frac{5}{9} = \frac{9}{18} + \frac{10}{18} = \frac{19}{18}$

5] $\tan\pi/4 + \cot\pi/4)(\tan 60^\circ + \cot 60)(\sin\pi/6 + \cot\pi/6).$

$(1+1)\left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)\left(\frac{1}{2} + \sqrt{3}\right)$

$\cancel{2}\cdot\frac{\sqrt{3}+1}{\sqrt{3}}\cdot\frac{1}{\cancel{2}} + \sqrt{3}$

$\frac{\sqrt{3}+1}{\sqrt{3}}\cdot\sqrt{3}$ $\quad 2\times\frac{3+1}{\sqrt{3}}, \cancel{\times} + \frac{+2\sqrt{3}}{2}$

$= \sqrt{3}$

$\cancel{2}\times\cancel{\times}\,4\times\frac{3}{\cancel{2}}$

$= 12$

$180-\theta$ 2^{nd} Qudt.

$\sin(180-\theta) = +\sin\theta$

$\cos(180-\theta) = -\cos\theta$

$\tan(180-\theta) = -\tan\theta$

$\operatorname{cosec}(180-\theta) = +\operatorname{cosec}\theta$

$\sec(180-\theta) = -\sec\theta$

$\cot(180-\theta) = -\cot\theta$

$180+\theta$ 3^{rd} qdt

$\sin(180+\theta) = -\sin\theta$

$\cos(180+\theta) = -\cos\theta$

$\tan(180+\theta) = +\tan\theta$

$\operatorname{cosec}(180+\theta) = -\operatorname{cosec}\theta$

$\sec(180+\theta) = -\sec\theta$

$\cot(180+\theta) = +\cot\theta$

$270-\theta$ 3^{rd} qdt.

$\sin(270-\theta) = -\cos\theta$

$\cos(270-\theta) = -\sin\theta$

$\tan(270-\theta) = +\cot\theta$

$\operatorname{cosec}(270-\theta) = -\sec\theta$

$\sec(270-\theta) = -\operatorname{cosec}\theta$

$\cot(270-\theta) = +\tan\theta$

$270+\theta$ 4^{th} qdt

$\sin(270+\theta) = -\cos\theta$

$\cos(270+\theta) = +\sin\theta$

$\tan(270+\theta) = -\cot\theta$

$\operatorname{cosec}(270+\theta) = -\sec\theta$

$\sec(270+\theta) = +\operatorname{cosec}\theta$

$\cot(270+\theta) = -\tan\theta$

$360-\theta$ 4^{th} qdt.

$\sin(360-\theta) = -\sin\theta$

$\cos(360-\theta) = +\cos\theta$

$\tan(360-\theta) = -\tan\theta$

$\operatorname{cosec}(360-\theta) = -\operatorname{cosec}\theta$

$\sec(360-\theta) = +\sec\theta.$

$\cot(360-\theta) = -\cot\theta.$

$360+\theta$ 1^{st} qdt.

$\sin(360+\theta) = +\sin\theta$

$\cos(360+\theta) = +\cos\theta$

$\tan(360+\theta) = +\tan\theta$

$\operatorname{cosec}(360+\theta) = +\operatorname{cosec}\theta$

$\sec(360+\theta) = +\sec\theta$

~~sec~~ $\cot(360+\theta) = +\cot\theta.$

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = +\cos\theta$

$\tan(-\theta) = -\tan\theta$

$\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

$\sec(-\theta) = +\sec\theta$

$\cot(-\theta) = -\cot\theta$

Find the values for the following allied angles

120, 135, 150, 180, 210, 225, 240, 270, 300, 315, 330

360

120

$\sin 120 = \sin(90+30)$

$= \cancel{+\cos 60} = +\cos 30$

$= \cancel{1/2} \quad = \sqrt{3}/2.$

$\cos 120 = \cos(90+30)$

$= -\sin 30$

$= \cancel{-\sqrt{3}/2} = -1/2.$

$\tan 120 = \tan(90+30)$

$= -\cot 30$

$= \cancel{-1/\sqrt{3}} = -\sqrt{3}$

$\operatorname{cosec} 120 = \operatorname{cosec}(90+30)$

$= +\sec 30$

$= \cancel{\sqrt{3}}\ +2/\sqrt{3}$

$\sec 120 = \sec(90+30)$

$= -\operatorname{cosec} 30$

$= -2$

$\cot 120 = \cot(90+30)$

$= -\tan 30$

$= -1/\sqrt{3}.$

225

$\sin 225 = \sin(180+45)$

$= -\sin 45$

$= -1/\sqrt{2}$

$\cos 225 = \cancel{\sin}\, \cos(180+45)$

$= -\cos 45$

$= -1/\sqrt{2}.$

$\tan 225 = \tan(180+45)$

$= +\tan 45$

$= +1$

$\operatorname{cosec} 225 = \operatorname{cosec}(180+45)$

$= -\operatorname{cosec} 45$

$= -\sqrt{2}.$

$\sec 225 = \sec(180+45)$

$= -\sec 45$

$= -\sqrt{2}.$

$\cot 225 = \cot(180+45)$

$= +\cot 45$

$= +1$

330.

$\sin 330 = \sin(270+60)$

$= -\cos 60$

$= -1/2$

$\cos 330 = \cos(270+60)$

$= +\sin 60$

$= +\sqrt{3}/2$

$\tan 330 = \tan(270+60)$

$= -\cot 60$

$= -1/\sqrt{3}$

$\operatorname{cosec} 330 = \operatorname{cosec}(270+60)$

$= -\sec 60$

$= -2$

$\sec 330 = \sec(270+60)$

$= +\operatorname{cosec} 60$

$= +2/\sqrt{3}$

$\cot 330 = \cot(270+60)$

$= -\tan 60$

$= -\sqrt{3}$

300

$\sin 300 = \sin(270 + 30)$

$= -\cos 30$ (~~sec~~)

$= -\sqrt{3}/2.$

$\cos 300 = \cos(270 + 30)$

$= +\sin 30$ (~~cos~~)

$= +\tfrac{1}{2}.$

$\tan 300 = \tan(270 + 30)$

$= +\cot 30$ (~~tan~~)

$= \pm\sqrt{3}$

$\operatorname{cosec} 300 = \operatorname{cosec}(270 + 30)$

$= -\sec 30$ (~~cosec~~)

$= -2/\sqrt{3}$

$\sec 300 = \sec(270 + 30)$

$= -\operatorname{cosec} 30$ (~~sec~~)

$= +2$

$\cot 300 = \cot(270 + 30)$

$= \pm\tan 30.$ (~~cot~~)

$= -1/\sqrt{3}.$

315 } IGN

Simplify the following

1) $\sin 480 \cdot \cos 510 - \sin(-150)\cos(-120)$

2) $\operatorname{cosec}(-1110)\sec 210 + \cot(-240)\cdot\cos(210)$

3) $\tan^2 \pi/6 + \tan^2(5\pi/6) + \tan^2 7\pi/6 + \tan^2 \frac{11\pi}{6}$

4) $\sin^2 \pi/4 + \sin^2(3\pi/4) + \sin^2 5\pi/4 + \sin^2 \frac{7\pi}{4}$

5) $\sin 420 \cos(-390) + \sin(-330)\cos 300$

6) $\tan 225 \cot 405 + \tan 765 \times \cot 675$

1] $\sin 480 \cdot \cos 510 - \sin(-150)\cos(-120)$ --- ①

$\sin 480$
$\sin(360+120)$
$\sin 120$
$\sin(180-60)$
$\sin 60$
$\sqrt{3}/2$

$\cos 510$
$\cos(360+150)$
$\cos 150$
$\cos(90+60)$
$-\sin 60$
$= -\sqrt{3}/2$

$\sin(-150)$
$-\sin 150$
$-\sin(180-30)$
$-\sin 30$
$-\frac{1}{2}$

$\cos(-120)$
$\cos 120$
$\cos(180-60)$
$-\cos 60$
$= -1/2$

From ①

$\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - (-1/2)(-1/2)$

$-3/4 - 1/4 = \frac{-3-1}{4} = -\frac{4}{4} = -1$

② $\tan^2 \pi/6 + \tan^2 \frac{5\pi}{6} + \tan^2 \frac{7\pi}{6} + \tan^2 \frac{11\pi}{6}$ --- C

$\tan 150 = \tan(180-30) = -\tan 30 = -1/\sqrt{3}$

$\tan 210 = \tan(180+30) = +\tan 30 = 1/\sqrt{3}$.

$\tan 330 = \tan(360-30) = -\tan 30 = -1/\sqrt{3}$

From ©

$\left(\frac{1}{\sqrt{3}}\right)^2 + (-1/\sqrt{3})^2 + (1/\sqrt{3})^2 + (-1/\sqrt{3})^2$

$= \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{1+1+1+1}{3} = \frac{4}{3}$

2] $\operatorname{cosec}(-1110) \sec 210 + \cot(-240) \cos 210$

$\operatorname{cosec}(-1110)$

$-\operatorname{cosec} 1110$

$-\operatorname{cosec}(3 \times 360 + 30)$

$-\operatorname{cosec} 30$

$= -2$

$\sec 210$

$\sec(180+30)$

$= -\sec 30$

$= -2/\sqrt{3}$

$\cot(-240)$

$-\cot 240^\circ$

$-\cot(360+120)$

$= -\cot 120^\circ$

$= -\cot(180+60)$

$= -\cot 60$

$= -1/\sqrt{3}$

$\cos 210^\circ$

$\cos(270-60)$

$= \cancel{ta}\sin 60$

$= -\sqrt{3}/2$

$-2 \times \frac{-2}{\cancel{\sqrt{3}}} + -\frac{1}{\cancel{\sqrt{3}}} \times \frac{-\cancel{\sqrt{3}}}{2}$

$\frac{4\times 2}{\sqrt{3}} + \frac{1\times\sqrt{3}}{2} = \frac{8+\sqrt{3}}{2\sqrt{3}}$

(4) $\sin^2 \pi/4 + \sin^2(3\pi/4) + \sin^2 5\pi/4 + \sin\frac{7\pi}{4}$

$\sin \pi/4 = \frac{1}{\sqrt{2}}$

$\cancel{\sin}\ \sin 3\pi/4$

$=$

$\frac{2}{2} + \frac{-1}{2} - \frac{1}{2} = \frac{2}{2} + \frac{2}{2} = \frac{\cancel{4}}{\cancel{2}} = 2$

P.T.O

(5) $\sin 420 \cos(-390) + \sin(-330)\cos 300$ --- (1)

$\sin 420$

$\sin(360+60)$

$\sin 60$

$\sin(90-30)$

$= \cos 30$

$= \sqrt{3}/2$

$\cos(-390)$

$+\cos 390$

$= +\cos(360+30)$

$= +\cos 30$

$= +\sqrt{3}/2$

$\sin(-330)$

$= -\sin 330$

$= -\sin(360-30)$

$= + \cancel{\cos} \sin 30$

$= 1/2$.

$\cos 300$

$= \cos(360+60)$

$= \cos 60$

$= 1/2$

From ①

$\sqrt{3}/2 \times \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{1}{2}$

$\frac{3}{4} + \frac{1}{4} = \frac{\cancel{4}}{\cancel{4}} = 1$

(6) $\tan 225 \cdot \cot 405 - \sin(-150) \cos(-120)$

$\tan 225$

$\tan(270-45)$

$= \cancel{\tan}\ 45$

$= 1$

$\cot 405$

$\cot(360+45)$

$\cot 45$

$= 1$

$\sin(-150°)$

$- \sin 150$

$= -\sin(180-30)$

$= -\sin 30$

$= 1/2$

$\cos(-120)$

$\cos 120$

$\cos(180 - 60)$

$= \cos 60$

$= 1/2$

[illegible]
[illegible]
[illegible]
[illegible]
[illegible]

From ①

$|X| - \frac{1}{2} \cdot \frac{1}{2}$

$\frac{1}{4} - \frac{1}{4}$

$= 0$

Simplify the following

1] $\dfrac{\sin(180-\theta) \cdot \cos(360-\theta)\ \tan(180+\theta)}{\cos(270+\theta)\ \sin(90+\theta)\ \cot(270-\theta)}$

$\dfrac{\cancel{\sin\theta} \cdot \cancel{\cos\theta} \cdot (\cancel{-\tan\theta})}{\cancel{\sin\theta} \cdot \cancel{\cos\theta} \times \cancel{-\tan\theta}} = +1$

2] $\dfrac{\sin(-\theta)}{\sin(\pi-\theta)} + \dfrac{\tan(\pi/2-\theta)}{\cot(\pi-\theta)} + \dfrac{\cos(\pi/2+\theta)}{\cos(3\pi/2-\theta)}$

$\dfrac{-\cancel{\sin\theta}}{\cancel{\sin\theta}} + \dfrac{\cancel{\cot\theta}}{-\cancel{\cot\theta}} + \dfrac{-\cancel{\sin\theta}}{-\cancel{\sin\theta}}$

$-1 - 1 + 1 = -1$

3] $\dfrac{\sin(-\alpha)\ \operatorname{cosec}(270-\alpha)\ \tan(180-\alpha)}{\cot(270-\alpha)\ \cos(-\alpha)\ \sec(270-\alpha)}$

27 (28)

$$\frac{-\sin\alpha \times -\sec\alpha \times -\cancel{\tan\alpha}}{-\cancel{\tan\alpha} \times \cos\alpha \times -\mathrm{cosec}\,\alpha}$$

$$= \frac{-\sin\alpha \cdot \sec\alpha}{\cos\alpha \cdot \mathrm{cosec}\,\alpha}$$

$$= -\tan\alpha \cdot \frac{\frac{1}{\cos\alpha}}{\frac{1}{\sin\alpha}}$$

$$= -\tan\alpha \frac{\sin\alpha}{\cos\alpha} = -\tan^2\alpha.$$

4] $$\frac{\sin(180-A)\cot(90-A)\cos(360-A)}{\tan(180-A)\tan(90+A)\sin(-A)}$$

$$\frac{\cancel{\sin A}\cdot\cancel{\tan A}\cdot\cos A}{\cancel{-\tan A}\cdot -\cot A\cdot\cancel{-\sin A}}$$

$$\frac{\cos A}{-\cot A} = -\cos A\cdot\tan A$$

$$= -\cancel{\cos A}\cdot\frac{\sin A}{\cancel{\cos A}} = -\sin A$$

5] $$\frac{\sin(180-A)}{\cos(\pi/2-A)} + \frac{\tan(180+A)}{\cot(3\pi/2+A)} + \frac{\sin(-A)}{\cos(\pi/2+A)}$$

$$\frac{\sin A}{\cos(90-A)} + \frac{\tan A}{\cot(270+A)} + \frac{(-\sin A)}{\cos(90+A)}$$

$$\frac{\cancel{\sin A}}{\cancel{\sin A}} + \frac{\cancel{\tan A}}{-\cancel{\tan A}} + \frac{(-\cancel{\sin A})}{-\cancel{\sin A}} = \frac{1+1-1}{1-1-1} \quad \frac{-1}{1}$$

$$-1-1+1 = +1.$$

(2010)

⑥ $\dfrac{\sin(\pi - A)\cot(\pi/2 - A)\cdot\cos(2\pi - A)}{\tan(\pi + A)\tan(\pi/2 + A)\sin(-A)}$

$\dfrac{\sin(180 - A)\cot(\pi/2 - A)\cdot\cos(2\pi - A)}{\tan(180 + A)\tan(90 + A)\cdot -\sin A}$

$\dfrac{\cancel{+\sin A}\cdot\cancel{+\tan A}\cdot +\cos A}{+\cancel{\tan A}\cdot + -\cot A\cdot\cancel{-\sin}}$

$\dfrac{+\cos A}{+\cos A}$ $\cos A\cdot\tan A$

$\cancel{\cos A}\cdot\dfrac{\sin A}{\cancel{\cos A}}$

$= \sin A$

⑦ $\dfrac{\cos(-\theta)}{\sin(270+\theta)} - \dfrac{\tan(90-\theta)}{\cot(180-\theta)} + \dfrac{\cos(90+\theta)}{\cos(270-\theta)}$

$+\dfrac{\cancel{\cos\theta}}{\cancel{-\cos\theta}} - \dfrac{(+\cancel{\cot}\,\theta)}{\cancel{-\cot\theta}} + \dfrac{(\cancel{-\sin\theta})}{\cancel{-\sin\theta}}$

$\cancel{-1}\ \cancel{+1} + 1 \quad = +1$

⑧ $\dfrac{\sin(2\pi - A)}{\sin(\pi - A)} - \dfrac{\tan(\pi/2 + A)}{\cot(2\pi + A)} + \dfrac{\operatorname{cosec}(-A)}{\sec(\pi/2 + A)}$

$\dfrac{\cancel{-\sin}A}{\cancel{+\sin A}} - \dfrac{(\cancel{-\cot A})}{\cancel{\cot A}} + \dfrac{(\cancel{-\operatorname{cosec} A})}{\cancel{-\operatorname{cosec} A}}$

$= \cancel{1} + \cancel{1} + 1$

$= 1$

(9) $\dfrac{\sin(180-A)\,(\cos 270-A)\,(\operatorname{cosec} 90+A)}{\sec(270+A)\,(\cot 90+A)\,\tan(360+A)}$ (2013)

$\dfrac{\sin A \cdot -\sec A \cdot \sec A}{+\operatorname{cosec} A \cdot -\tan A \cdot -\tan A}$

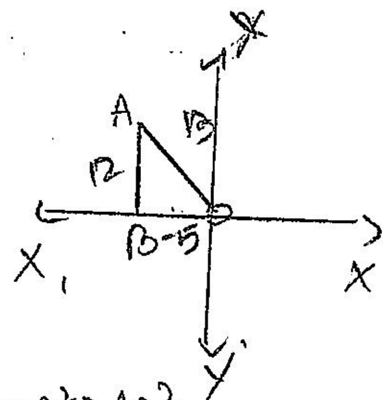
$\dfrac{-\sin^2 A \cdot \frac{1}{\cos A}}{\frac{1}{\sin\theta} \cdot \tan^2 A}$

$= \dfrac{-\sin^2 A \;\tan A}{\tan^2 A}$

$-\sin^2 A \cdot \cot A$

$= -\cancel{\sin^2} A \cdot \dfrac{\cos A}{\cancel{\sin A}} = -\sin A \cdot \cos A$

(10) If $\cos\theta = -5/13$ & $90 < \theta < 180$ then find the value of $\dfrac{13\sin\theta + 5\sec\theta}{5\tan\theta - 13\cos\theta}$



$\sin\theta = \dfrac{12}{13}$

$\sec = \dfrac{13}{-5}$

$\tan\theta = \dfrac{12}{-5}$

$OA^2 = OB^2 + AB^2$

$13^2 = (-5)^2 + AB^2$

$169 = 25 + AB^2$

$169 - 25 = AB^2$

$144 = AB^2$

$AB = 12.$

$$\frac{13\sin\theta + 5\sec\theta}{5\tan\theta - 13\cos\theta}$$

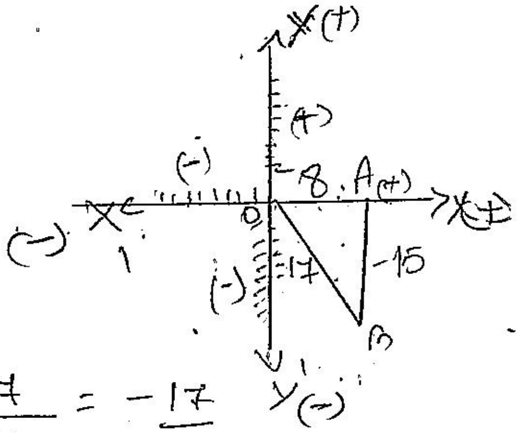
$$\frac{\cancel{13}\cdot\frac{12}{\cancel{13}} + \cancel{5}\,\frac{13}{\cancel{5}}}{\cancel{5}\times\frac{-12}{\cancel{5}} - \cancel{13}\times\frac{-5}{\cancel{13}}}$$

$$\frac{12-13}{-12+5}$$

$$= \frac{-1}{-7} = 1/7$$

11] If $\sec\theta = 17/8$ & $3\pi/2 < \theta < 2\pi$ then find the value of

$$\frac{15\operatorname{cosec}\theta - 8\tan\theta}{17\cos\theta + 15\operatorname{cosec}\theta}$$



$OB^2 = OA^2 + AB^2$

$17^2 = 8^2 + AB^2$

$289 = 64 + AB^2$

$289 - 64 = AB^2$

$\sqrt{225} = \sqrt{AB^2}$

$AB = \pm 15$

$\operatorname{cosec}\theta = \frac{17}{-15} = -\frac{17}{15}$

$\tan\theta = \frac{-15}{8} = -\frac{15}{8}$

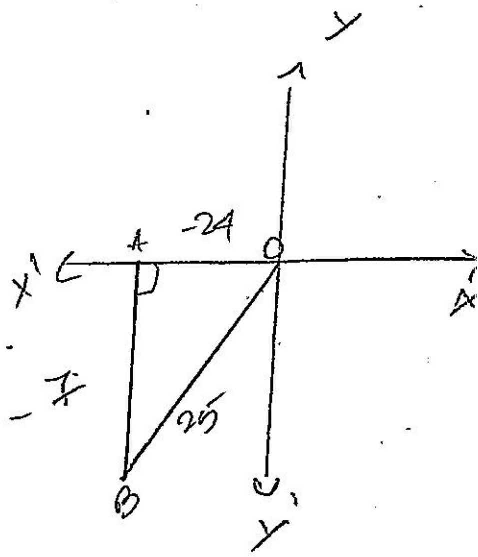
$\cos\theta = \frac{8}{17}$

$$\frac{\cancel{15}\times\frac{-17}{\cancel{15}} - \cancel{8}\times\frac{-15}{\cancel{8}}}{\cancel{17}\cdot\frac{8}{\cancel{17}} + \cancel{15}\times\frac{-17}{\cancel{15}}}$$

$$\frac{-17+15}{8-17} = \frac{\cancel{-}2}{\cancel{-}9} = \frac{2}{9}$$

1. If $\tan\beta = \frac{7}{24}$ where β in the 3rd qdt then find the value of.

$$\frac{\sin\beta - 2\cos\beta}{3\sin\beta + 4\cos\beta}$$



$OB^2 = OA^2 + AB^2$

$OB^2 = (-24)^2 + (-7)^2$

$OB^2 = 576 + 49 = 625$

$OB^2 = 625$

$OB = 25$

$\tan\beta = \frac{7}{24} = \frac{-7}{-24}$

$\sin\beta = \frac{-7}{25}$

$\cos\beta = \frac{-24}{25}$

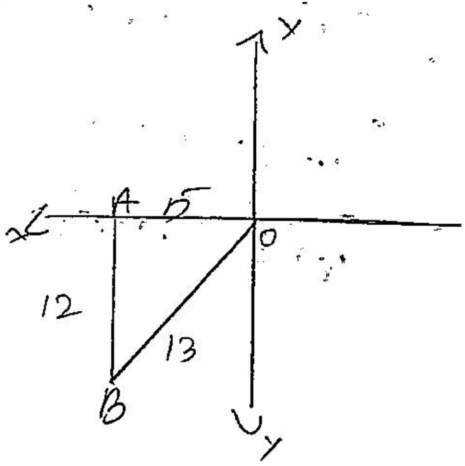
$$\frac{\frac{-7}{25} - 2\times\frac{-24}{25}}{3\times\frac{-7}{25} + 4\times\frac{-24}{25}}$$

$$= \frac{\frac{-7}{25} + \frac{48}{25}}{\frac{-21}{25} - \frac{96}{25}}$$

$$= \frac{\frac{-7+48}{\cancel{25}}}{\frac{-21-96}{\cancel{25}}} = \frac{41}{-117}$$

2] $\tan\theta = 12/5$ & θ lies in 3rd qdt.
find the value of

$$\frac{3\sin\theta + 4\cos\theta}{3\sin\theta - 4\cos\theta}$$



$OB^2 = OA^2 + AB^2$

$OB^2 = (5)^2 + (12)^2$

$OB^2 = 25 + 144$

$OB^2 = 169$

$OB = \sqrt{169} = 13$

$\sin\theta = \frac{-12}{13}$

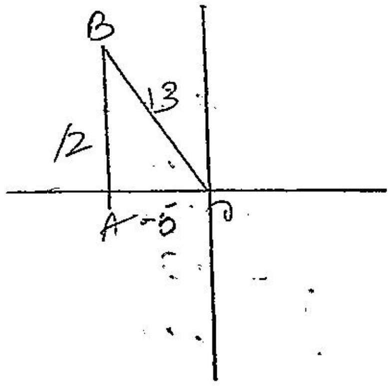
$\cos\theta = \frac{-5}{13}$

$$\frac{3 \times \frac{-12}{13} + 4 \times \frac{-5}{13}}{3 \times \frac{-12}{13} - 4 \times \frac{-5}{13}}$$

$$\frac{\frac{-36}{13} + \frac{(-20)}{13}}{\frac{-36}{13} - \frac{(-20)}{13}} = \frac{\frac{-36-20}{\cancel{13}}}{\frac{-36+20}{\cancel{13}}} = \frac{+56}{+16} = \frac{\cancel{56}\,7}{\cancel{16}\,2} = \frac{7}{2}$$

3] $\pi/2 < A < \pi$, $\cos A = -5/13$
find the value of

$$\frac{\sin A - \cos A}{\sin A + \cos A}$$



$OB^2 = OA^2 + AB^2$

$(13)^2 = (-5)^2 + AB^2$

$169 = 25 + AB^2$

$AB^2 = 169 - 25$

$AB^2 = 144$

$AB = \sqrt{144} = 12$

$\sin A = \frac{12}{13}$

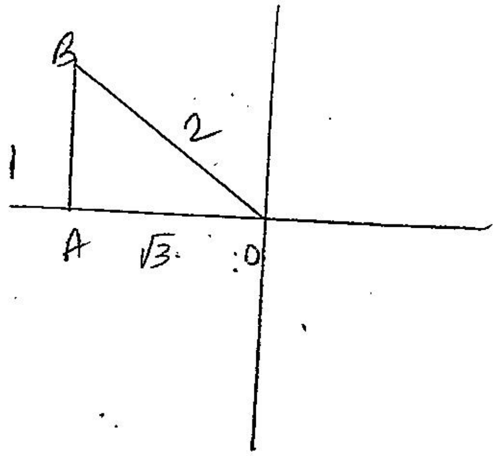
$\cos A = \frac{-5}{13}$

$$\frac{\frac{12}{13}-\frac{(-5)}{13}}{\frac{12}{13}+\frac{-5}{13}} = \frac{\frac{12}{13}+\frac{5}{13}}{\frac{12}{13}-\frac{5}{13}} = \frac{\frac{17}{13}}{\frac{7}{13}} \quad \frac{17}{\cancel{13}} \times \frac{\cancel{13}}{7}$$

$$= \frac{17}{7}$$

4] if $\sin\theta = 1/2$ & $\pi/2 < \theta < \pi$

find $\cos\theta \cdot \operatorname{cosec}\theta + \sin\theta \cdot \sec\theta$.



$OB^2 = OA^2 + AB^2$

$(2)^2 = OA^2 + (1)^2$

$4 = OA^2 + 1$

$OA^2 = 4 - 1 = 3$

$OA = \sqrt{3}$

$\cos\theta = \frac{\sqrt{3}}{2}$

$\operatorname{cosec}\theta = \frac{2}{1} = 2$

$\sin\theta = \frac{1}{2}$

$\sec\theta = \frac{2}{\sqrt{3}}$

$$\frac{\sqrt{3}}{2} \cdot \frac{2}{1} + \frac{1}{2} \cdot \frac{2}{\sqrt{3}}$$

$$\frac{\sqrt{3} \cdot \cancel{2}}{\cancel{2}} + \frac{\cancel{2}}{2\cancel{\sqrt{3}}}$$

$\sqrt{3} + \frac{1}{\sqrt{3}}$

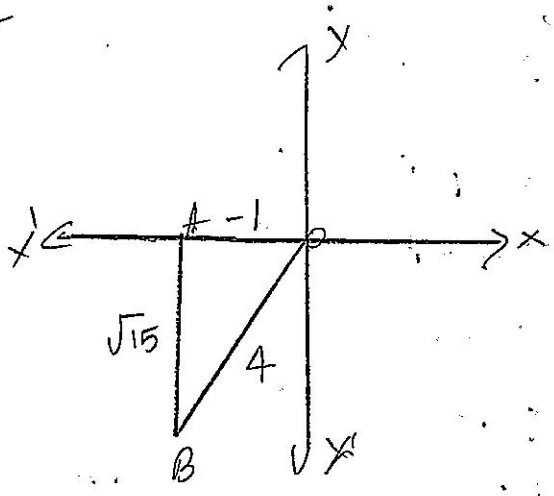
$= \frac{3+1}{\sqrt{3}}$

$= \frac{-4}{\sqrt{3}}$

5) If $\cos\theta = -1/4$ $\pi > \theta > 3\pi/2$

find

$\frac{\cos\theta + \tan\theta}{\cot\theta + \sec\theta}$



$OB^2 = AB^2 + OB^2$

$(4)^2 = AB^2 + (-1)^2$

$16 = AB^2 + 1$

$AB^2 = 16 - 1$

$AB = \sqrt{15}$

$\cos\theta = \frac{-1}{4}$

$\tan\theta = \frac{\sqrt{15}}{-1} = -\sqrt{15}$

$\cot\theta = \frac{-1}{\sqrt{15}}$

$\sec\theta = \frac{4}{-1} = -4$

$\frac{\frac{-1}{4} + -\sqrt{15}}{\frac{-1}{\sqrt{15}} + -4}$

Compound Angles.

Angles of the forms A+B, A−B, A+B+C, A+B−C etc are called as compound angles.

$\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$

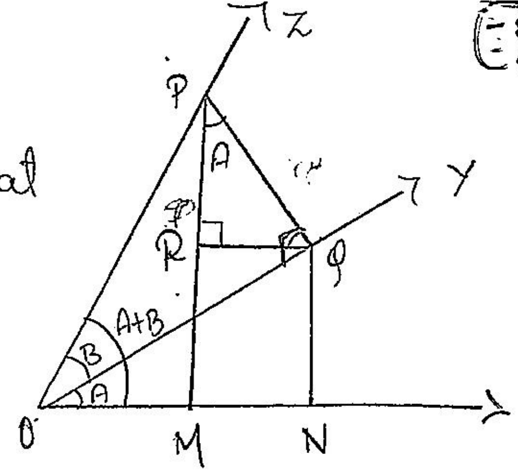
$\sin(A-B) = \sin A \cdot \cos B - \cos A \cdot \sin B$

$\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B.$

$\cos(A-B) = \cos A \cdot \cos B + \sin A \cdot \sin B.$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}.$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}.$$



Draw OX OY OZ in such a way that

$\angle XOY = A$

$\angle YOZ = B$

$\angle XOZ = A+B$

Draw $PM \perp OX$

$QN \perp OX$

$PQ \perp OY$

$RQ \perp PM$.

From the right angled Δ^{le} OMP

$$\sin(A+B) = \frac{PM}{OP}$$

$$\sin(A+B) = \frac{PR+RM}{OP}$$

$$\sin(A+B) = \frac{PR}{OP} + \frac{RM}{OP}$$

$$\sin(A+B) = \frac{PR}{OP} + \frac{QN}{OP}$$

$$\sin(A+B) = \frac{PR}{PQ} \times \frac{PQ}{OP} + \frac{QN}{OQ} \times \frac{OQ}{OP}$$

$$\sin(A+B) = \cos A \cdot \sin B + \sin A \cdot \cos B$$

$$\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B.$$

Draw OX, OY, OZ in such a way that

$\angle XOY = A$

$\angle YOZ = B$

$\angle XOZ = A+B$

-X

Draw $PM \perp OX$

$QN \perp OX$

$PQ \perp OY$

$RQ \perp PM$

From the right angled Δ^{le} OMP

$$\cos(A+B) = \frac{OM}{OP}$$

$$\cos(A+B) = \frac{ON - MN}{OP}$$

$$\cos(A+B) = \frac{ON}{OP} - \frac{MN}{OP}$$

$$\cos(A+B) = \frac{ON}{OP} - \frac{QR}{OP}$$

$$\cos(A+B) = \frac{ON}{OQ} \cdot \frac{OQ}{OP} - \frac{QR}{PQ} \cdot \frac{PQ}{OP}$$

$$\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B.$$

****

Find the values of

i) $\sin 75$

$\sin(45+30)$

$\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B.$

$\sin 75 = \sin(45+30)$

$= \sin 45 \cdot \cos 30 + \cos 45 \cdot \sin 30$

$\sin 75 = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$\sin 75 = \frac{\sqrt{3}+1}{2\sqrt{2}}$

$\underline{\cos 75}$

$= \cos(45+30)$

$\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B.$

$\cos 75 = \cos 45 \cdot \cos 30 - \sin 45 \cdot \sin 30$

$= \frac{1}{\sqrt{2}} \quad \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}.$

$\cos 75 = \frac{\sqrt{3}-1}{2\sqrt{2}}$

$\underline{\tan 75}$

$\tan(45+30)$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B.}$

$\tan 75 = \frac{\tan 45 + \tan 30}{1 - \tan 45 \tan 30}$

$\tan 75 = \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}} = \frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}}$

$$\frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$\tan 75 = \frac{\sqrt{3}+1}{\sqrt{3}-1}.$$

Find the value of

sin 15
cos 15
tan 15

sin 15

$\sin(45-30)$

$\sin(A-B) = \cancel{\cos}\ \sin A \cdot \cos B - \cos A \cdot \sin B.$

$\sin 15 = \sin(45-30)$

$= \sin 45 \cdot \cos 30 \cancel{+} - \cos 45 \cdot \sin 30.$

$$\sin 15 = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}.$$

$$\sin 15 = \frac{\sqrt{3}-1}{2\sqrt{2}}.$$

cos 15

$= \cos(45-30)$

$\cos(A-B) = \cos A \cdot \cos B + \sin A \cdot \sin B.$

$\cos(45-30) = \cos 45 \cdot \cos 30 + \sin 45 \cdot \sin 30.$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$$

$$\cos 15 = \frac{\sqrt{3}+1}{2\sqrt{2}}.$$

tan 15

tan (45 - 30)

$\tan(A \mp B) = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$

$\tan(45-30) = \frac{\tan 45 - \tan 30}{1 + \tan 45 \cdot \tan 30}$

$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \cdot \frac{1}{\sqrt{3}}} = \frac{\frac{\sqrt{3}-1}{\cancel{\sqrt{3}}}}{\frac{\sqrt{3}+1}{\cancel{\sqrt{3}}}}$

$\tan(45-30) = \frac{\sqrt{3}-1}{\sqrt{3}+1}$

H.W

Find the value of

sin 105 (60 + 45)

cos 105

tan 105

Q. No.

Prove that $\sin(A+B) \cdot \sin(A-B) = \sin^2 A - \sin^2 B$

$\sin(A+B) \cdot \sin(A-B) = \sin^2 A - \sin^2 B$

LHS

$\sin(A+B) \cdot \sin(A-B)$

$(\underbrace{\sin A \cdot \cos B}_{a} + \underbrace{\cos A \cdot \sin B}_{b})(\underbrace{\sin A \cdot \cos B}_{a} - \underbrace{\cos A \cdot \sin B}_{-b})$

$\sin^2 A \cdot \cos^2 B - \cos^2 A \cdot \sin^2 B$

$\sin^2 A(1-\sin^2 B) - (1-\sin^2 A)\sin^2 B$

$\sin^2 A - \sin^2 A \cdot \sin^2 B - \sin^2 B + \sin^2 A \cdot \sin^2 B$.

$\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$.

$$\left(\frac{\cos A \cdot \cos B}{a} - \frac{\sin A . \sin B}{b}\right)\left(\frac{\cos A \cdot \cos B}{a} + \frac{\sin A \cdot \sin B}{b}\right)$$

$\cos^2 A \cdot \cos^2 B - \sin^2 A \cdot \sin^2 B$

$\cos^2 A(1-\sin^2 B) - (1-\cos^2 A)\sin^2 B$

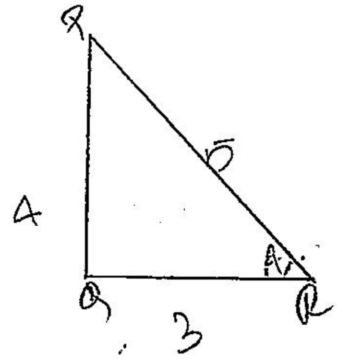
$\cos^2 A - \cancel{\cos^2 A \cdot \sin^2 B} - \sin^2 B + \cancel{\cos^2 A \cdot \sin^2 B}$

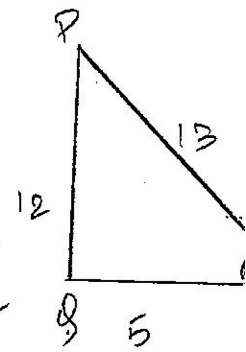
$\cos^2 A - \sin^2 B$.

LHS.

If $\cos A = \frac{3}{5}$ $\sin B = \frac{12}{13}$ Find the value of

$\sin(A \pm B)$ & $\cos(A \pm B)$





$\sin(A+B)$

$= \sin A \cdot \cos B + \cos A \cdot \sin B$

$\frac{4}{5} \cdot \frac{5}{13} + \frac{3}{5} \cdot \frac{12}{13}$

$\sin(A+B) = \frac{20+36}{65} = \frac{56}{65}$

$\sin(A-B)$

$\sin(A-B) = \sin A \cdot \cos B - \cos A \cdot \sin B$

$= \frac{4}{5} \cdot \frac{5}{13} - \frac{3}{5} \cdot \frac{12}{13}$

$\sin(A-B) = \frac{20-36}{65} = -\frac{16}{65}$

$\cos(A+B)$.

$PR^2 = QR^2 + PQ^2$

$(5)^2 = (3)^2 + PQ^2$

$25 = 9 + PQ^2$

$PQ^2 = 16$

$PQ = 4$

$PR^2 = PQ^2 +$

$(13)^2 = (12)^2 +$

$169 = 144 + Q$

$169 - 144 = QR$

$25 = QR^2$

$QR = 5$

$\cos A \cdot \cos B - \sin A \cdot \sin B$.

$\frac{3}{5} \cdot \frac{5}{13} - \frac{4}{5} \cdot \frac{12}{13}$

$= \frac{15 - 48}{65}$

$= -\frac{33}{65}$

$\cos(A - B)$.

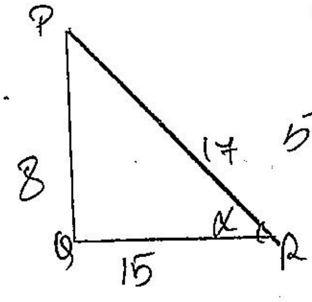
$\cos A \cdot \cos B + \sin A \cdot \sin B$

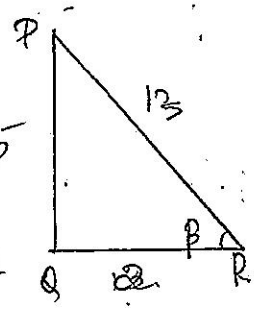
$\frac{3}{5} \times \frac{5}{13} + \frac{4}{5} \cdot \frac{12}{13}$

$= \frac{15 + 48}{65}$

$= \frac{63}{65}$.

If $\cos\alpha = \frac{15}{17}$ $\cos\beta = \frac{12}{13}$ find the value of $\cos(\alpha \pm \beta)$ & $\sin(\alpha \pm \beta)$.





$\cos(\alpha + \beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$.

$\frac{15}{17} \cdot \frac{12}{13} - \frac{8}{17} \cdot \frac{5}{13}$

$\frac{180}{221} - \frac{40}{221} = \frac{140}{221}$

$PR^2 = PQ^2 + QR^2$

$17^2 = PQ^2 + 15^2$

$289 = PQ^2 + 225$

$PQ^2 = 64$

$PQ = 8$

$PR^2 = PQ^2 + Q$

$12^2 = PQ^2 + 1$

$144 = PQ^2 + 16$

$169 - 144 = PQ^2$

$PQ^2 = 25$

$PQ = 5$

$\cos(\alpha - \beta)$.

$\cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta$.

$\frac{15}{17} \cdot \frac{12}{13} - \frac{8}{17} \cdot \frac{5}{13}$

$\frac{180}{221} - \frac{40}{221} = \frac{140}{221}$ $\frac{220}{221}$

$\sin(\alpha + \beta)$.

$\sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta$.

$\frac{8}{17} \cdot \frac{12}{13} + \frac{15}{17} \cdot \frac{5}{13}$.

$\frac{96}{221} + \frac{75}{221} = \frac{171}{221}$.

$\sin(\alpha - \beta)$.

$\sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta$.

$\frac{8}{17} \cdot \frac{12}{13} - \frac{15}{17} \cdot \frac{5}{13}$

$\frac{96}{221} - \frac{75}{221} = \frac{21}{221}$ //

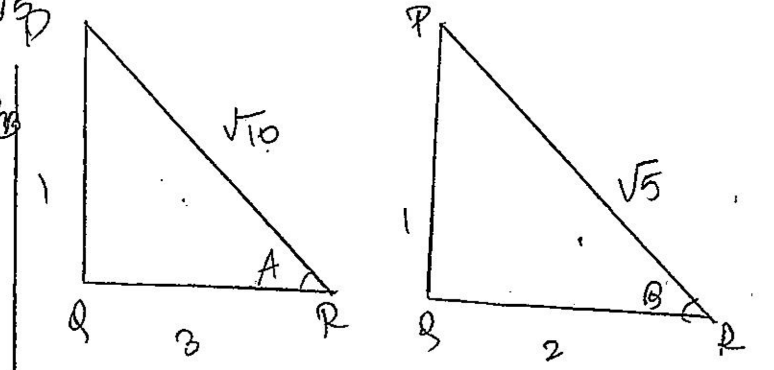
H.W

$\sin\beta = \frac{3}{5}$ $\cos A = \frac{40}{41}$ find the value

$\sin(A \pm B)$ $(\cos \pm B)$

$\frac{160}{205} + \frac{27}{205} = \frac{187}{205}$ } IGN

If $\sin A = \frac{1}{\sqrt{10}}$ & $\sin B = \frac{1}{\sqrt{5}}$ State that $A+B = \pi/4$



Consider

$\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$

$\sin(A+B) = \frac{1}{\sqrt{10}} \cdot \frac{2}{\sqrt{5}} + \frac{3}{\sqrt{10}} \cdot \frac{1}{\sqrt{5}}$

$\frac{2}{\sqrt{50}} + \frac{3}{\sqrt{50}}$

$\frac{5}{\sqrt{50}}$

$\sin(A+B) = \frac{5}{\sqrt{25 \times 2}} = \frac{\cancel{5}}{\cancel{5}\sqrt{2}}$

$\sin(A+B) = \frac{1}{\sqrt{2}}$

we know that

$\sin \pi/4 = \frac{1}{\sqrt{2}}$

Compare

$\boxed{(A+B) = \pi/4}$

$PR^2 = PQ^2 + QR^2$

$(\sqrt{10})^2 = 1^2 + QR^2$

$10 - 1 = QR^2$

$9 = QR^2$

$QR = 3.$

$PR^2 = PQ^2 + QR^2.$

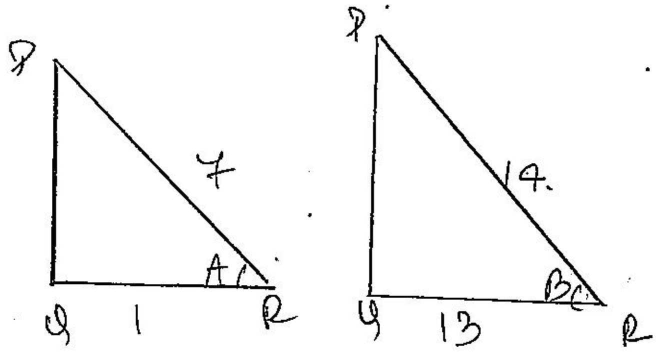
$(\sqrt{5})^2 = 1^2 + QR^2$

$5 - 1 = QR^2$

$4 = QR^2$

$QR = 2.$

If $\cos A = \frac{1}{7}$ & $\cos B = \frac{13}{14}$ than show that $(A-B) = \pi/3$



(45)

consider

$\cos(A-B) = \cos A \cdot \cos B + \sin A \cdot \sin B$

$\frac{1}{7} \cdot \frac{13}{14} + \frac{\sqrt{48}}{7} \cdot \frac{\sqrt{27}}{14}$

$\frac{13 + \sqrt{1296}}{98} = \frac{13 + 36}{98}$

$PR^2 = PQ^2 + QR^2$	$PR^2 = PQ^2 + QR^2$
$7^2 = PQ^2 + 1^2$	$14^2 = PQ^2 + 13^2$
$49 = PQ^2 + 1$	$196 = PQ^2 + 169$
$49 - 1 = PQ^2$	$196 - 169 = PQ^2$
$48 = PQ^2$	$27 = PQ^2$
$PQ = \sqrt{48}$	$PQ = \sqrt{27}$

$\cos(A-B) = \frac{49}{98} = \frac{1}{2}$

$\cos(A-B) = \frac{1}{2}$

we know that,

$\cos \pi/3 = 1/2$

compaire

$\boxed{A - B = \pi/3.}$

If $\tan A = \frac{5}{6}$ & $\tan B = \frac{1}{11}$ then ST. $A+B = \pi/4$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

$\tan(A+B) = \frac{\frac{5 \times 11}{6} + \frac{1 \times 6}{11}}{\frac{1}{1} - \frac{5}{6} \cdot \frac{1}{11}}$

$\tan(A+B) = \frac{\frac{55+6}{\cancel{66}}}{\frac{66-5}{\cancel{66}}}$

$\frac{\frac{5}{6} + \frac{1}{11}}{1 - \frac{5}{66}}$

$\tan(A+B) = \frac{\cancel{61}}{\cancel{61}} = 1$

$\tan(A+B) = 1$

we know that.

$\tan \pi/4 = 1$

compare

$A+B = \pi/4$.

$\tan A = \frac{3}{4}$ & $\tan B = 1/7$ then show that $A+B = \pi/4$.

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

$$\frac{\frac{3}{4} + \frac{1}{7}}{1 - \frac{3}{4} \cdot \frac{1}{7}}$$

$$= \frac{\frac{21+4}{28}}{1 - \frac{3}{28}}$$

$$= \frac{\frac{25}{\cancel{28}}}{\frac{28-3}{\cancel{28}}}$$

$$\frac{\cancel{25}}{\cancel{25}} = 1$$

$\tan(A+B) = 1$

we know that

$\tan \pi/4 = 1$

compare

$A+B = \pi/4$.

$\cancel{x^{\circ}}\cancel{x^{\circ}}\cancel{x^{\circ}}$

If $\tan A = 1/3$, $\tan(A+B) = 2/7$ then find $\tan B$.

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

$$\frac{2}{7} = \frac{\frac{1}{3} + \frac{\tan B}{1}}{1 - \frac{1}{3}\tan B}$$

$$\frac{2}{7} = \frac{\frac{1+3\tan B}{\cancel{3}}}{\frac{3-\tan B}{\cancel{3}}}$$

$$\frac{2}{7} = \frac{1+3\tan B}{3-\tan B}$$

$$2(3 - \tan B = 7(1+3\tan B)$$

$$6 - 2\tan B = 7 + 21\tan B$$

$$6 - 7 = 21 + 2\tan B$$

$$-1 = 23\tan B$$

$$\frac{-1}{23} = \tan B$$

$\text{Tan } A = \frac{2}{5}$, $\tan(A+B) = \frac{1}{9}$ then find $\tan B$.

$$\frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

$$\frac{1}{9} = \frac{\frac{2}{5} + \tan B}{1 - \frac{2}{5} \cdot \tan B}$$

$$\frac{1}{9} = \frac{\frac{2 + 5\tan B}{\cancel{5}}}{\frac{5 - 2\tan B}{\cancel{5}}}$$

$$\frac{1}{9} = \frac{2 + 5\tan B}{5 - 2\tan B}$$

$$1(5 - 2\tan B) = 9(2 + 5\tan B)$$

$$5 - 2\tan B = 18 + 45\tan B$$

$$5 - 18 = 45\tan B + 2\tan B$$

$$-13 = 47\tan B$$

$$\frac{-13}{47} = \tan B$$

$$\tan B = \frac{4}{7} \qquad \tan(A+B) = \frac{3}{13}$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

$$\frac{3}{13} = \frac{\tan A + \frac{4}{7}}{1 - \tan A \cdot \frac{4}{7}}$$

$$\frac{3}{13} = \frac{\frac{7\tan A + 4}{\cancel{7}}}{\frac{7 - 4\tan A}{\cancel{7}}}$$

$$\frac{3}{13} = \frac{7\tan A + 4}{7 - 4\tan A}$$

~~13(7 tan A + 4) = 13(7 - 4 tan A)~~

~~91 tan A + 52 = 21 - 12 tan A~~

$3(7 - 4\tan A) = 13(7\tan A + 4)$

$21 - 12\tan A = 91\tan A + 52$

$21 - 52 = 91\tan A + 12\tan A$

$-31 = 103\tan A$.

$$\boxed{\frac{-31}{103} = \tan A}$$

H.w

(1) If $\tan A = \frac{7}{3}$, $\tan(A+B) = \frac{1}{8}$ then find $\tan B$

(2) If $\tan A = 1/6$, $\tan(A+B) = 2/11$ then find $\tan A$.

(3) $\tan A = 6/7$ & $\tan B = 7/6$ then show that $A + B = \pi/2$

(4) If $\tan A = \frac{m}{m+1}$, $\tan B = \frac{1}{2m+1}$ then show that $A + B = \pi/4$.

$\cos(A+B)$ $\cos(A-B)$

$\cos A + \cos(120 + A) + \cos(120 - A) = 0$.

$\cos A + \cos 120 \cdot \cos A - \cancel{\sin 120 \cdot \sin A} + \cos 120 \cdot \cos A + \cancel{\sin 120 \cdot \sin A}$

$\cos A + 2 \cdot \cos 120 \cdot \cos A$

$\cos A + 2\cos(180 - 60)\cos A$

$\cos A + 2 \times -\cos 60 \cdot \cos A$

Prove that ~~5A~~ $\tan 5A - \tan 3A - \tan 2A = \tan 5A \cdot \tan 3A \cdot \tan$

50

$5A = 3A + 2A$

Apply tan bothe the side.

$\tan(\underset{A}{3A} + \underset{B}{2A})$

$\tan 5A = \tan(3A + 2A)$

$$\tan 5A = \frac{\tan 3A + \tan 2A}{1 - \tan 3A \cdot \tan 2A}$$

$\tan 5A (1 - \tan 3A \cdot \tan 2A) = \tan 3A + \tan 2A$

$\tan 5A - \tan 5A \cdot \tan 3A \cdot \tan 2A = \tan 3A + \tan 2A$

$\tan 5A - \tan 3A - \tan 2A = \tan 5A \cdot \tan 3A \cdot \tan 2A$

<u>Multiples</u>. <u>Angle</u> :- Angles of the form $2A, 3A, 4A$ etc are called multiple angle.

1) $\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$

Put $B = A$.

$\sin(A+A) = \sin A \cdot \cos A + \cos A \cdot \sin A$

$\sin 2A = 2\sin A \cdot \cos A$

2) $\sin 2A = \dfrac{2\sin A \cdot \cos A}{1}$

$= \dfrac{2\sin A \cdot \cos A}{\sin^2 A + \cos^2 A}$ common

$= \dfrac{2\sin A \cdot \cancel{\cos A}}{\cancel{\cos^2} A \left(\frac{\sin^2 A}{\cos^2 A} + 1\right)}$

$$\frac{2\sin A}{\cos A\left(\frac{\sin^2 A}{\cos^2 A}+1\right)}$$

$$\frac{\sin A}{\cos A} = \tan A$$

51

$$\sin 2A = \frac{2\tan A}{1+\tan^2 A}.$$

3] $\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B.$

Put $B = A$

$\cos(A+A) = \cos A \cdot \cos A - \sin A \cdot \sin A$

$\cos 2A = \cos^2 A - \sin^2 A.$

$\cos 2A = \cos^2 A - \sin^2 A.$

4] $\cos 2A = \cos^2 A - \sin^2 A$

$\cos 2A = \cos^2 A - (1-\cos^2 A)$

$\cos 2A = \cos^2 A - 1 + \cos^2 A.$

$\cos 2A = 2\cos^2 A - 1$

5] $\cos 2A = \cos^2 A - \sin^2 A$

$= 1 - \sin^2 A - \sin^2 A.$

$\cos 2A = 1 - 2\sin^2 A.$

6) $\cos 2A = \dfrac{\cos^2 A - \sin^2 A}{1}$

$$\cos 2A = \frac{\cos^2 A - \sin^2 A}{\cos^2 A + \sin^2 A}$$

$$\cos 2A = \frac{\cancel{\cos^2 A}\left(1 - \frac{\sin^2 A}{\cos^2 A}\right)}{\cancel{\cos^2 A}\left(1 + \frac{\sin^2 A}{\cos^2 A}\right)}$$

$$\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$$

7] $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

Put $B = A$

$$\tan(A+A) = \frac{\tan A + \tan A}{1 - \tan A \cdot \tan A}$$

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

8) $\sin 3A = \sin(2A+A)$

$= \sin 2A \cdot \cos A + \cos 2A \cdot \sin A$

$= 2\sin A \cdot \cos A \cdot \cos A + (1 - 2\sin^2 A)\sin A$

$\sin 3A = 2\sin A\,(1-\sin^2 A) + \sin A - 2\sin^3 A$

$\sin 3A = 2\sin A - 2\sin^3 A + \sin A - 2\sin^3 A$

$\sin 3A = 3\sin A - 4\sin^3 A$

$\sin 3A = 3\sin A - 4\sin^3 A$

Valued

11/10/19