Date :- 8/6/17

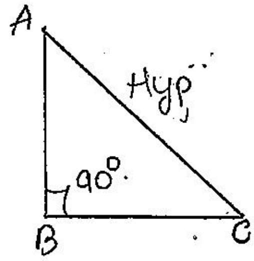
BRIDGE COURSE

Algebra standard results.

→ * $(a+b)^2 = a^2 + 2ab + b^2$
* $(a-b)^2 = a^2 - 2ab + b^2$
* $a^2 - b^2 = (a+b)(a+b)$
* $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$
* $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$
* $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
* $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
* $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca.$
* $(a+b-c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca.$
* $(a-b+c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$
* $(a-b-c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca.$

→ Phythogorus theorem

In a right angle the square of the hypotenuse is equal to the sum of the other two sides.



→ LCM.

* $\frac{8}{13} + \frac{6}{7} = \frac{56+78}{91} = \frac{132}{91}$

* $\frac{2}{4} + \frac{3}{12} = \frac{24+12}{48} = \frac{36}{48}$

Theory of Indices.

1] $a^{m^n} = a^{m \times n}$

2] $a^m \cdot a^n = a^{m+n}$

3] $\frac{a^m}{a^n} = a^{m-n}$

4] $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

5] $(ab)^m = a^m \cdot b^m$

6] $a^0 = 1$

$\frac{0}{a} = 0$

$\frac{a}{0} = \infty$ [not defined]

$\frac{\infty}{a} = \infty$ [not defined].

$\frac{a}{\infty} = 0$

$\infty + a = \infty$

$\infty - a = \infty$

$\infty \times a = \infty$.

$7x - 147 = 7$

$7x = 147 + 7$

$7x = 154$

$x = \cancel{\frac{154}{7}}\ 22$.

$7 \overline{)154}$
22

$x^2 - x + 5 = 0$

$5x^2$

* $x^2 - 6x + 5 = 0$

$x^2 - 5x - x + 5 = 0$

$x[x-5] - 1[x-5)$

$(x-1)\ (x-5) = 0$

$x - 1 = 0 \quad x - 5 = 0$

$[x = 5, 1]$

$+5$

$-1 \times -5 = +5$

$-1 - 5 = -6.$

* $x^2 - x - 12$

$x^2 - 4x + 3x - 12 = 0$

$x[x-4) + 3[x-4] = 0$

$[x+3]\ (x-4] = 0$

$x + 3 = 0 \quad x - 4 = 0.$

$[x = -3, 4.]$

-12

$-4, 3.$

* $x^2 - 3x + 2 = 0$

$x^2 - x - 2x + 2 = 0$

$x[x-1] - 2[x-1] = 0$

$(x-2]\ [x-1) = 0$

$x - 2 = 0 \quad x - 1 = 0$

$[x = 2, 1]$

$+2$

$-1 \ -2$

* $\lambda^2 + 16\lambda + 39 = 0.$

$\lambda^2 + 13\lambda + 3\lambda + 39 = 0$

$\lambda[\lambda+13] + 3[\lambda+13] = 0$

$[\lambda+3]\ [\lambda+13] = 0$

$\lambda + 3 = 0 \quad \lambda + 13 = 0$

$[\lambda = -3, -13]$

39

$13 \ 3$

Rationalization.

$\frac{\sqrt{2}+1}{\sqrt{2}-1}$

$\frac{\sqrt{2}+1}{\sqrt{2}-1} \times \frac{(\sqrt{2}+1)}{(\sqrt{2}+1)}$

$\frac{(\sqrt{2}+1)^2}{(\sqrt{2})^2 - (1)^2}$

$$\frac{(\sqrt{2})^2 + 1 + 2 \cdot \sqrt{2} \cdot 1}{2-1}$$

$$\frac{2 + 1 + 2\sqrt{2}}{1}$$

$$= 3 + 2\sqrt{2}$$

$$\frac{x}{\sqrt{1+x}+\sqrt{1-x}}$$

$$\frac{x}{\sqrt{1+x}+\sqrt{1-x}} \times \frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}$$

$$\frac{x\left(\sqrt{1+x}-\sqrt{1-x}\right)}{\left(\sqrt{1+x}\right)^2-\left(\sqrt{1-x}\right)^2}$$

$$\frac{x\left(\sqrt{1+x}-\sqrt{1-x}\right)}{\cancel{1}+x-\cancel{1}+x}$$

$$\frac{\cancel{x}\left(\sqrt{1+x}-\sqrt{1-x}\right)}{2\cancel{x}}$$

$$\frac{\sqrt{1+x}-\sqrt{1-x}}{2}$$

$$\frac{\sqrt{a+x}-\sqrt{a-x}}{x}$$

$$\frac{\sqrt{a+x}-\sqrt{a-x}}{x} \times \frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a+x}+\sqrt{a-x}}$$

$$\frac{\left(\sqrt{a+x}\right)^2-\left(\sqrt{a-x}\right)^2}{x\left(\sqrt{a+x}\right)+\left(\sqrt{a-x}\right)}$$

$$\frac{\cancel{a}+x-\cancel{a}+x}{x\left(\sqrt{a+x}\right)+\left(\sqrt{a-x}\right)}$$

$$\frac{2x}{\cancel{x}(\sqrt{a+x}+\sqrt{a-x})}$$

$$\frac{2}{\sqrt{a+x}+\sqrt{a-x}}$$

$$* \quad \frac{\sqrt{2x-1}-\sqrt{x+2}}{x^2-9}.$$

$$\frac{\sqrt{2x-1}-\sqrt{x+2}}{x^2-9} \times \frac{\sqrt{2x-1}+\sqrt{x+2}}{\sqrt{2x-1}+\sqrt{x+2}}$$

$$\frac{(\sqrt{2x-1})^2+(\sqrt{x+2})^2}{x^2-9\,(\sqrt{2x-1}+\sqrt{x+2})}$$

$$\frac{2x-1+x+2}{x^2-9\,(\sqrt{2x-1}+\sqrt{x+2})}$$

$$\frac{x-3}{x^2-3^2(\sqrt{2x-1}+\sqrt{x+2})}$$

$$\frac{\cancel{(x-3)}}{(x+3)\cancel{(x-3)}(\sqrt{2x-1}+\sqrt{x+2})}$$

$$\frac{1}{(x+3)(\sqrt{2x-1})+\sqrt{x+2})}$$

$$\frac{2x}{\cancel{x}(\sqrt{a+x}+\sqrt{a-x})}$$

$$\frac{2}{\sqrt{a+x}+\sqrt{a-x}}$$

$$* \quad \frac{\sqrt{2x-1}-\sqrt{x+2}}{x^2-9}$$

$$\frac{\sqrt{2x-1}-\sqrt{x+2}}{x^2-9} \times \frac{\sqrt{2x-1}+\sqrt{x+2}}{\sqrt{2x-1}+\sqrt{x+2}}$$

$$\frac{(\sqrt{2x-1})^2+(\sqrt{x+2})^2}{x^2-9\,(\sqrt{2x-1}+\sqrt{x+2})}$$

$$\frac{2x-1+x+2}{x^2-9\,(\sqrt{2x-1}+\sqrt{x+2})}$$

$$\frac{x-3}{x^2-3^2\,(\sqrt{2x-1}+\sqrt{x+2})}$$

$$\frac{\cancel{(x-3)}}{(x+3)\cancel{(x-3)}(\sqrt{2x-1}+\sqrt{x+2})}$$

$$\frac{1}{(x+3)(\sqrt{2x-1})+\sqrt{x+2})}$$

20/7/17

Unit - 1

Matrices and Determinants.

31 marks

Matrix :- Matrix is a rectangular arrangment of elements or numbers which are arranged in rows and columns and is denoted by [ ].

Ex :-
$$A = \begin{array}{c} \left.\begin{bmatrix} 2 & -1 & 3 \\ 6 & 5 & 9 \end{bmatrix}\begin{array}{l} \rightarrow R_1 \\ \rightarrow R_2 \end{array}\right\} \text{Rows} \\ \underbrace{\begin{array}{ccc} C_1 & C_2 & C_3 \end{array}}_{\text{Columns.}} \end{array}$$

In the above matrix horizontal elements are called rows and vertical elements are called columns

Order of the matrix :- The number of rows & columns present in the given matrixes is called order of the matrix

Ex :- $A = \begin{bmatrix} 2 & 7 \\ 6 & -1 \end{bmatrix}_{2\times 2}$ $A = \begin{bmatrix} 8 & -1 & 7 \\ 2 & -3 & 4 \end{bmatrix}_{2\times 3}$

$A = \begin{bmatrix} 1 & 4 & 8 \end{bmatrix}_{1\times 3}$ $A = \begin{bmatrix} 8 & 6 \\ 1 & 3 \\ 4 & -5 \end{bmatrix}_{3\times 2}$

$A = \begin{bmatrix} 8 \\ 2 \\ -7 \end{bmatrix}_{3\times 1}$

The no of rows and no of columns will give the number of elements present in the given matrix

Types of Matrices :-

1] Row MATRIX :- A Matrix which consists of only one row and any number of columns is called Row matrix.

Ex :- $A = \begin{bmatrix} 3 & 4 \end{bmatrix}_{1\times 2}$

$A = \begin{bmatrix} -1 & 6 & 9 \end{bmatrix}$

$A = \begin{bmatrix} 8 & 6 & -2 & 0 \end{bmatrix}_{1\times 4}$

2] COLUMN MATRIX :- A matrix consists of only one column and any no of rows is called column matrix

Ex :- $A = \begin{bmatrix} 4 \\ 5 \end{bmatrix}_{2\times 1}$

$A = \begin{bmatrix} -1 \\ 0 \\ 6 \end{bmatrix}_{3\times 1}$

$A = \begin{bmatrix} 6 \\ 8 \\ 5 \\ -7 \end{bmatrix}_{4\times 1}$

3] Null or Zero matrix :- A matrix which consists of all elements zero is called null or zero matrix

Ex :- $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

4] Square matrix :- In a matrix if no of rows is equal to no of columns then it is called as square matrix.

Ex $A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$ $A = \begin{bmatrix} 1 & 8 & -1 \\ 2 & 0 & 7 \\ 6 & 5 & 3 \end{bmatrix}$

5] Principle diagonal elements. (PDE)

In a square matrix left top to right bottom elements are called principle diagonal elements.

Ex $\begin{bmatrix} 6 & -1 \\ 2 & 1 \end{bmatrix}$ $\begin{bmatrix} 2 & -1 & 8 \\ 6 & 7 & 9 \\ 5 & 0 & 1 \end{bmatrix}$

6] Secondary diagonal Elements.

In a square matrix right top to left bottom elements are called ~~a~~ secondary diagonal elements

Ex $A = \begin{bmatrix} 7 & 5 \\ -1 & 0 \end{bmatrix}$ $A = \begin{bmatrix} 2 & 3 & 5 \\ 6 & 1 & 0 \\ 9 & -2 & -1 \end{bmatrix}$

7] Diagonal matrix

A square matrix is said to be a diagonal matrix if the principle diagonal elements are non zeros i.e. above the PDE & below the PDE the elements must be zeros

Ex $A = \begin{bmatrix} 2 & 0 \\ 0 & 7 \end{bmatrix}$ $A = \begin{bmatrix} 8 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

8] Scalar Matrix :- A diagnol matrix is said to be a scalar Matrix if the P D E must be same

Ex :-

$$\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \quad \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$$

9]. Identity or unit Matrix :

A scalar matrix is said to be identity matrix if the principle diagonial elements must be 1 or unity.

Ex

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Transpose of a matrix

If A is a matrix of order $m \times n$ then the transpose of A of the given matrix is obtained by interchanging rows into colomns or coloms into rows of the order $n \times m$ and it is denoted by A^T or A'

Ex $A = \begin{bmatrix} 2 & -1 & 7 \\ 6 & 5 & 4 \end{bmatrix}_{2\times3}$ $\quad A = \begin{bmatrix} 1 & 4 & 8 \end{bmatrix}_{1\times3}$

$A' = \begin{bmatrix} 2 & 6 \\ -1 & 5 \\ 7 & 4 \end{bmatrix}_{3\times2}$ $\quad A' = \begin{bmatrix} 1 \\ 4 \\ 8 \end{bmatrix}_{3\times1}$

Operation on Matrices.

Scalar multiplication: If 'A' is any matrix & 'k' is any scalar (constant) then kA can be obtained by multipling k to each element of the given matrix A

Ex:- 1) $A = \begin{bmatrix} 1 & 4 & 7 \\ 2 & -3 & 0 \end{bmatrix}_{2\times 3}$

$3A = \begin{bmatrix} 3 & 12 & 21 \\ 6 & -9 & 0 \end{bmatrix}$

2) $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$

$5A = \begin{bmatrix} 20 & 10 \\ 5 & 15 \end{bmatrix}$

Addition & Subtraction of 2 matrices.

2 matrices A & B of the same order can be added and subtracted & it is obtained by adding & subtracting the corresponding elements of A & B

$A = \begin{bmatrix} 1 & 4 & 7 \\ 2 & -3 & 0 \end{bmatrix}_{2\times 3}$ $B = \begin{bmatrix} 2 & 6 & -2 \\ 9 & 7 & 5 \end{bmatrix}_{2\times 3}$

$$A+B = \begin{bmatrix} 1+2 & 4+6 & 7-2 \\ 2+9 & -3+7 & 0+5 \end{bmatrix} = \begin{bmatrix} 3 & 10 & 5 \\ 11 & 4 & 5 \end{bmatrix}$$

$$A-B = \begin{bmatrix} 1-2 & 4-6 & 7+2 \\ 2-9 & -3-7 & 0-5 \end{bmatrix} \begin{bmatrix} -1 & -2 & 9 \\ -7 & +10 & -5 \end{bmatrix}$$

Problems:

$$A = \begin{bmatrix} -2 & 6 & 7 \\ -7 & 9 & 8 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 4 & -8 \\ 6 & 5 & 3 \end{bmatrix}$$

$$C = \begin{bmatrix} 7 & -1 & 0 \\ 4 & 2 & 5 \end{bmatrix}$$

① $6C + 7B$

$$6C = \begin{bmatrix} 42 & -6 & 0 \\ 24 & 12 & 30 \end{bmatrix} \quad 7B = \begin{bmatrix} 7 & 28 & -56 \\ 42 & 35 & 21 \end{bmatrix}$$

$$6C+7B = \begin{bmatrix} 42+7 & -6+28 & 0-56 \\ 24+42 & 12+35 & 30+21 \end{bmatrix} = \begin{bmatrix} 49 & 22 & -56 \\ 66 & 47 & 51 \end{bmatrix}$$

② $A+B-C$.

$$A+B-C \begin{bmatrix} -2+1-7 & 6+4+1 & 7-8-0 \\ -7+6-4 & 9+5-2 & 8+3-5 \end{bmatrix}$$

$$A+B-C = \begin{bmatrix} -8 & 11 & -1 \\ -5 & 12 & 6 \end{bmatrix}$$

③ $3B - 2C$.

$$3B = \begin{bmatrix} 3 & 12 & -24 \\ 18 & 15 & 9 \end{bmatrix}$$

$$2C = \begin{bmatrix} 14 & -2 & 0 \\ 8 & 4 & 10 \end{bmatrix}$$

$$3B - 2C = \begin{bmatrix} -11 & 14 & -24 \\ 10 & 11 & -1 \end{bmatrix}$$

④ $4A - B$

$$4A = \begin{bmatrix} -8 & 24 & 28 \\ -28 & 36 & 32 \end{bmatrix}$$

$$4A - B = \begin{bmatrix} -9 & 20 & 36 \\ -3\cancel{4} & 31 & 29 \end{bmatrix}$$

9-10

Multiplication of 2 Matrices :- If 'A' is the Matrix of order $M \times p$ & 'B' is the matrix of order $p \times n$ then product of 'AB' is defined of order $m \times n$

$A = m \times p$
$B = p \times n$
$AB = m \times n$

i.e. the no of columns of the first matrix should be equal to no of rows of the second matrix.

Ex

$$A = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 5 & 6 \end{bmatrix}$$

$$B = \begin{bmatrix} 7 & 1 \\ 9 & 0 \\ 8 & 4 \end{bmatrix}$$

Find AB.

$$AB = \begin{bmatrix} (4\times 7)+(2\times 9)+(3\times 8) & (4\times 1)+(2\times 0)+(3\times 4) \\ (1\times 7)+(5\times 9)+(6\times 8) & (\times 1)+(5\times 0)+(6\times 4) \end{bmatrix}$$

$$AB = \begin{bmatrix} 28+18+24 & 4+12 \\ 7+45+42 & 1+24 \end{bmatrix} =$$

$$AB = \begin{bmatrix} 70 & 16 \\ 100 & 25 \end{bmatrix}$$

If $A = \begin{bmatrix} 3 \\ 1 \\ 6 \end{bmatrix}$ find (i) AA^T (ii) $A^T A$.

$$A = \begin{bmatrix} 3 \\ 1 \\ 6 \end{bmatrix}_{3\times 1}$$

$$A^T = \cancel{3 \times 6} \begin{bmatrix} 3 & 1 & 6 \end{bmatrix}_{1\times 3}$$

$$AA^T = \begin{bmatrix} (3\times 3) & (3\times 1) & (3\times 6) \\ (1\times 3) & (1\times 1) & (1\times 6) \\ (6\times 3) & (6\times 1) & (6\times 6) \end{bmatrix}$$

$$AA^T = \begin{bmatrix} 9 & 3 & 18 \\ 3 & 1 & 6 \\ 18 & 6 & 36 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 3 & 1 & 6 \end{bmatrix} \quad A = \begin{bmatrix} 3 \\ 1 \\ 6 \end{bmatrix}_{(3\times 1)}$$

$$A^T A = \begin{bmatrix} (3\times 3) + (1\times 1) + (6\times 6) \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 9 + 1 + 36 \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 46 \end{bmatrix}$$

② $A = \begin{bmatrix} 1 & 4 & 8 \end{bmatrix}_{1\times 3}$ $B = \begin{bmatrix} 2 \\ 5 \\ 9 \end{bmatrix}_{3\times 1}$

$AB = \begin{bmatrix} (1\times 2) + (4\times 5) + (8\times 9) \end{bmatrix}$

$AB = \begin{bmatrix} 2 + 20 + 72 \end{bmatrix}$

$AB = \begin{bmatrix} 94 \end{bmatrix}_{1\times 1}.$

$B = \begin{bmatrix} 2 \\ 5 \\ 9 \end{bmatrix}_{3\times 1}$ $A = \begin{bmatrix} 1 & 4 & 8 \end{bmatrix}_{1\times 3}$

$BA = \begin{bmatrix} 2\times 1 & 2\times 4 & 2\times 8 \\ 5\times 1 & 5\times 4 & 5\times 8 \\ 9\times 1 & 9\times 4 & 9\times 8 \end{bmatrix} = \begin{bmatrix} 2 & 8 & 16 \\ 5 & 20 & 40 \\ 9 & 36 & 72 \end{bmatrix}$

9x

③ $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}_{2\times 2}$ $B = \begin{bmatrix} 6 & 0 \\ 5 & 8 \end{bmatrix}_{2\times 2}.$

$AB = \begin{bmatrix} (4\times 6)+(2\times 5) & (4\times 0)+(2\times 8) \\ (1\times 6)+(3\times 5) & (1\times 0)+(3\times 8) \end{bmatrix}$

$AB = \begin{bmatrix} 34 & 16 \\ 21 & 24 \end{bmatrix}_{2\times 2.}$

$$BA = \begin{bmatrix} (6\times4)+(0\times1) & (6\times2)+(0\times3) \\ (5\times4)+(8\times1) & (5\times2)+(8\times3) \end{bmatrix}$$

$$B = \begin{bmatrix} 6 & 0 \\ 5 & 8 \end{bmatrix} \qquad A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$$

$$BA = \begin{bmatrix} 24+0 & 12+0 \\ 20+8 & 10+24 \end{bmatrix}$$

$$BA = \begin{bmatrix} 24 & 12 \\ 28 & 34 \end{bmatrix}_{2\times2}$$

Sum:-

$A = 3\times2 \qquad B = 2\times1$

$\therefore A = 1\times2 \qquad B = 2\times3.$

$\therefore A = 3\times2 \qquad B = 2\times1$

$$A = \begin{bmatrix} -9 & 8 \\ 6 & -4 \\ 2 & 5 \end{bmatrix}_{3\times2} \qquad B = \begin{bmatrix} 3 \\ 5 \end{bmatrix}_{2\times1}$$

$$\therefore B = \begin{bmatrix} (-9\times3) + (8\times5) \\ (6\times3) + (-4\times5) \\ (2\times3) + (5\times5) \end{bmatrix}$$

$$A - B = \begin{bmatrix} (-2+7) \\ (-5-4) \\ (8+1) \end{bmatrix}_{3\times1} \quad A - B = \begin{bmatrix} 5 \\ -9 \\ 9 \end{bmatrix}_{3\times1}$$

DETERMINANTS.

2nd order determinants.

The expression $a_1 b_2 - a_2 b_1$ is denoted by $\begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix}$ is called 2nd order determinant.

Eg :- $\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix} = (4\times3) - (2\times1)$
$= 12 - 2$
$= 10$ //

$\begin{bmatrix} 5 & -1 \\ 2 & 3 \end{bmatrix} = (5\times3) - (-1\times2)$
$= 15 + 2$
$= 17$ //

3rd order determinent.

The expression $a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1 \ldots$ is denoted by $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

eg $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 1 \begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} - 2 \begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix} 3 \begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix}$

$= 1(45-48) - 2(36-42) + 3(32-35)$
$= 1(-3) - 2(-6) + 3(-3)$
$= -3 + 12 - 9 = 0$ //

$$\begin{vmatrix} 3 & 2 & -9 \\ 5 & 4 & 1 \\ 6 & 6 & 2 \end{vmatrix} = 3\begin{vmatrix} 4 & 1 \\ 0 & 2 \end{vmatrix} - 2\begin{vmatrix} 5 & 1 \\ 6 & 2 \end{vmatrix} - 9\begin{vmatrix} 5 & 4 \\ 6 & 0 \end{vmatrix}.$$

$$3|8-0| - 2|10-6| - 9|0-24|$$

$$24 - 8 + 216 = 232$$

~~Home work~~.

(1)
$$\begin{vmatrix} \overset{+}{5} & \overset{-}{-2} & \overset{+}{3} \\ 3 & 1 & 6 \\ 7 & 3 & -9 \end{vmatrix}$$

$$= 5\begin{vmatrix} 1 & 6 \\ 3 & -9 \end{vmatrix} + 2\begin{vmatrix} 3 & 6 \\ 7 & -9 \end{vmatrix}\ 3\begin{vmatrix} 3 & 1 \\ 7 & 3 \end{vmatrix}$$

$$= 5|-9 - 18| + 2|-27 - 42|\ 3|9-7|$$

$$= 5|-27| + 2|-69|\ 3|2|$$

$$= -135 - 138 + 6$$

$$= -267$$

Home work.

1)
$$\begin{vmatrix} 5 & 4 \\ -7 & 6 \end{vmatrix} = (5\times 6) - (4\times -7)$$

$$= (30 - (-28))$$

$$= (30 + 28) = 58$$

2)
$$\begin{vmatrix} 3 & -3 \\ 9 & -7 \end{vmatrix} = (3\times -7) - (-3\times 9)$$

$$= -21 - (-27)$$

$$= (-21 + 27) = 6.$$

3)
$$\begin{vmatrix} \overset{+}{7} & \overset{-}{4} & \overset{+}{-2} \\ 9 & 8 & 5 \\ -3 & 1 & 6 \end{vmatrix}$$

$$= 7\begin{vmatrix} 8 & 5 \\ 1 & 6 \end{vmatrix} - 4\begin{vmatrix} 9 & 5 \\ -3 & 6 \end{vmatrix} - 2\begin{vmatrix} 9 & 8 \\ -3 & 1 \end{vmatrix}$$

$= 7|48-5| - 4|54+15| - 2|9+24|$

$= 7|43| - 4|69| - 2|33|$

$= -41$

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④ $\begin{vmatrix} \overset{+}{6} & \overset{-}{2} & \overset{+}{-5} \\ 4 & 0 & 1 \\ 3 & -2 & 5 \end{vmatrix}$

$= 6\begin{vmatrix} 0 & 1 \\ -2 & 5 \end{vmatrix} - 2\begin{vmatrix} 4 & 1 \\ 3 & 5 \end{vmatrix} - 5\begin{vmatrix} 4 & 0 \\ 3 & -2 \end{vmatrix}$

$= 6|0+2| - 2|20-3| - 5|8-0|$

$= 6|2| - 2|17| - 5|-8|$

$= 18$

---

⑤ $\begin{vmatrix} \overset{+}{-3} & \overset{-}{2} & \overset{+}{1} \\ 1 & -2 & 3 \\ 5 & 5 & -5 \end{vmatrix}$

$= -3\begin{vmatrix} -2 & 3 \\ 5 & -5 \end{vmatrix} - 2\begin{vmatrix} 1 & 3 \\ 5 & -5 \end{vmatrix} + 1\begin{vmatrix} 1 & -2 \\ 5 & 5 \end{vmatrix}$

$= -3|10-15| - 2|-5-15| + |5+10|$

$= 3|-5| - 2|-20| + 1|15|$

$= 15 + 40 + 15$

$= 70$ //

Solve for x.

1. $\begin{vmatrix} x & -7 \\ 8 & 5 \end{vmatrix} = 0$

$5x + 56 = 0$

$5x = -56$

$\boxed{x = \frac{-56}{5}}$

2. $\begin{vmatrix} 8 & -3 \\ x+2 & 1 \end{vmatrix} = 7x + 3$

$8 + 3(x+2) = 7x + 3$

$8 + 3x + 6 = 7x + 3$

$14 + 3x = 7x + 3$

$14 - 3 = 7x - 3x$

$11 = 4x$

$\boxed{11/4 = x}$ //

(7)

$$\begin{vmatrix} 3 & -1 & -2 \\ 0 & 5 & 3 \\ -1 & 2 & x \end{vmatrix} = 0.$$

$$3\begin{vmatrix} 5 & 3 \\ 2 & x \end{vmatrix} + 1\begin{vmatrix} 0 & 3 \\ -1 & x \end{vmatrix} - 2\begin{vmatrix} 0 & 5 \\ -1 & 2 \end{vmatrix} = 0$$

$3(5x-6)+1(0+3)-2(0+5)=0$

$15x-18+3-10=0$

$15x-25=0$

$15x=25$

$x=\frac{\cancel{25}^{5}}{\cancel{15}_{3}}$

$\boxed{x=5/3}$

(8)

$$\begin{vmatrix} 5 & -3 & +2 \\ 4 & x+1 & -1 \\ 2 & x+2 & -3 \end{vmatrix} = x+5$$

$$+5\begin{vmatrix} x+1 & -1 \\ x+2 & -3 \end{vmatrix} + 3\begin{vmatrix} 4 & -1 \\ 2 & -3 \end{vmatrix} + 2\begin{vmatrix} 4 & x+1 \\ 2 & x+2 \end{vmatrix} = x+5$$

$$5\left[-3(x+1)+1(x+2)\right]+3\left[-12+2\right]+2\left[4(x+2)-2(x+1)\right]=x+5$$

$$5\left[-3x-3+x+2\right]-30+2\left[4x+8-2x-2\right]=x+5$$

$$5(-2x-1)-30+2(2x+6)=x+5$$

$$-10x-5-30+4x+12=x+5$$

$$-6x-23=x+5$$

$$-5-23=x+6x$$

$$-28=7x$$

$$x=\frac{\cancel{-28}\,-4}{\cancel{7}\,1}$$

$$x=\frac{-4}{1} \quad x=-4$$

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(9)
$$\begin{array}{c} \\ \\ \\ \end{array}\begin{array}{ccc} + & - & + \end{array}$$
$$\begin{vmatrix} 1 & -1 & 2 \\ 5 & -2 & 3 \\ 4 & x & 7 \end{vmatrix}=0$$

$$+1\begin{vmatrix} -2 & 3 \\ x & 7 \end{vmatrix}+1\begin{vmatrix} 5 & 3 \\ 4 & 7 \end{vmatrix}+2\begin{vmatrix} 5 & -2 \\ 4 & x \end{vmatrix}=0$$

$$+1\left[14-3x\right]+1\left[35-12\right]+2\left[5x+8\right]=0$$

$$\left[14-3x\right]+35-12+10x+16=0$$

$14 + 12$

$\cancel{14 - 25 - 12 + 16 - 3x + 10x}$

$25 + 7x = 0$

$25 = -7x$

$-\frac{25}{+7} = x$

$\cancel{x = \frac{25}{7}}$

---

(10) $\begin{vmatrix} 1 & 5 & x \\ 2 & 0 & 4 \\ 3 & -1 & 2x \end{vmatrix} = -x + 9$

$1 \begin{vmatrix} 0 & 4 \\ -1 & 2x \end{vmatrix} - 5 \begin{vmatrix} 2 & 4 \\ 3 & 2x \end{vmatrix} x \begin{vmatrix} 2 & 0 \\ 3 & -1 \end{vmatrix} = -x +$

$1(0 + 4) - 5(4x - 12)\ x(-2 - 0) = -x + 9$

$4 - 20x + 60 - 2x = -x + 9$

$64 - 22x = -x + 9$

$x - 22x = 64 - 9$

$21x = 55$

$\boxed{x = \frac{55}{21}}$

Cramer's Rule (Determinant Method)

Let $a_1 x + b_1 y = c_1$

Let $a_2 x + b_2 y = c_2$

are the simultaneous 2 equations, to solve x & y

$$A = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

$$A_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$$

$$x = A_x / A$$

$$A_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$$

$$y = A_y / A$$

Ex:- $5x - 3y = 1$

$2x - 5y = -11$

$$A = \begin{vmatrix} 5 & -3 \\ 2 & -5 \end{vmatrix}$$

$$A_x = \begin{vmatrix} 1 & -3 \\ -11 & -5 \end{vmatrix}$$

$$A_x = -5 - 33$$

$$A_x = -38$$

$$x = \frac{\Delta x}{\Delta}$$

$$x = \frac{\cancel{-}38}{\cancel{-}19}$$

$$\boxed{x = 2}$$

$$\Delta_y = \begin{vmatrix} 5 & 1 \\ 2 & -11 \end{vmatrix}$$

$$\Delta_y = -55 - 2$$

$$\Delta_y = -57$$

$$y = \frac{\Delta_y}{\Delta}$$

$$y = \frac{-57}{-19}$$

$$\boxed{y = 3.}$$

---

(2) $x + 9 = 6y$

$11 - y = 3x.$

$x - 6y = -9$

$-3x - y = -11$

$$\Delta = \begin{vmatrix} 1 & -6 \\ -3 & -1 \end{vmatrix}$$

$$\Delta = -1 - 18$$

$$\Delta = -19.$$

$$\Delta_x = \begin{vmatrix} -9 & -6 \\ -11 & -1 \end{vmatrix}$$

$\Delta x = 9 - 66$

$\Delta_x = -57$

$x = \dfrac{\Delta x}{\Delta} = \dfrac{-57}{-19} = 3$

$x = 3$

$\Delta_y = \begin{vmatrix} 1 & -9 \\ -3 & -11 \end{vmatrix}$

$\Delta_y = -11 - 27$

$\Delta_y = -38$

$y = \dfrac{\Delta y}{\Delta} = \dfrac{-38}{-19} = 2$

$y = 2.$

---

(3) $5x + 7y = -4$

$4x - 3y = 14.$

$\Delta = \begin{vmatrix} 5 & 7 \\ 4 & -3 \end{vmatrix}$

$\Delta = -15 - 28$

$\Delta = -43$

$\Delta_x = \begin{vmatrix} \cancel{7}\ -4 & 7\ \cancel{-4} \\ \cancel{-3}\ 14 & 3\ \cancel{14} \end{vmatrix}$

$\Delta_x = \cancel{98 - 12}. -12 - 98.$

$$\Delta_x = \begin{vmatrix} 5 & 3 \\ 1 & -1 \end{vmatrix}$$

$$A_x = -5 - 3$$

$$\Delta_x = -8.$$

$$x = \frac{\Delta_x}{A}.$$

$$x = \frac{-8}{-4}.$$

$$\boxed{x = 2}$$

$$\Delta_y = \begin{vmatrix} 1 & 5 \\ 1 & 1 \end{vmatrix}.$$

$$\Delta_y = +1 - 5$$

$$A_y = -4.$$

$$y = \frac{\Delta_y}{A}$$

$$y = \frac{-4}{-4}$$

$$\boxed{y = 1.}$$

---

(4) $2x - y + 5z = 12$

$x + y - z = 4.$

$3x + 7y - 2z = 17$

$$A = \begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{vmatrix} 2 & -1 & 5 \\ 1 & 1 & -1 \\ 3 & 7 & -2 \end{vmatrix} \end{array}$$

$$2\begin{vmatrix} 1 & -1 \\ 7 & -2 \end{vmatrix} + 1\begin{vmatrix} 1 & -1 \\ 3 & -2 \end{vmatrix} + 5\begin{vmatrix} 1 & 1 \\ 3 & 7 \end{vmatrix}.$$

$$= 2(-2+7) + 1(-2+3) + 5(7-3)$$

$$= 10 + 1 + 20 = 31.$$

$$\begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \Delta_x = \begin{vmatrix} 12 & -1 & 5 \\ 4 & 1 & -1 \\ 17 & 7 & -2 \end{vmatrix} \end{array}$$

$$= 12\begin{vmatrix} 1 & -1 \\ 7 & -2 \end{vmatrix} + 1\begin{vmatrix} 4 & -1 \\ 17 & -2 \end{vmatrix} + 5\begin{vmatrix} 4 & 7 \\ 17 & 7 \end{vmatrix}$$

$$12(-2+7) + 1(-8+17) + 5(28-17)$$

$$60 + 9 + 55$$

$$= 124.$$

$$x = \frac{\Delta_x}{A} = \frac{124}{31}$$

$$x = 4.$$

$$\begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \Delta_y = \begin{vmatrix} 2 & 12 & 5 \\ 1 & 4 & -1 \\ 3 & 17 & -2 \end{vmatrix} \end{array}$$

$$\Delta_y = 2\begin{vmatrix} 4 & -1 \\ 17 & -2 \end{vmatrix} - 12\begin{vmatrix} 1 & -1 \\ 3 & -2 \end{vmatrix} \; 5\begin{vmatrix} 1 & 4 \\ 3 & 17 \end{vmatrix}.$$

$$= 2(-8+17) - 12(-2+3) + 5(17-12).$$

$$= 18 - 12 + 25 = 31$$

$$y = \frac{\Delta y}{\Delta} = \frac{31}{31} = 1 \quad y = 1$$

$$\Delta_z = \begin{vmatrix} \overset{+}{2} & \overset{-}{-1} & \overset{+}{12} \\ 1 & 1 & 4 \\ 3 & 7 & 17 \end{vmatrix}$$

$$\Delta_z = 2 \begin{vmatrix} 1 & 4 \\ \cancel{7} & 17 \end{vmatrix} + 1 \begin{vmatrix} 1 & 4 \\ 3 & 17 \end{vmatrix} + 12 \begin{vmatrix} 1 & 1 \\ 3 & 7 \end{vmatrix}.$$

$$2(17 - 28) + 1(17 - 12) + 12(7 - 3).$$

$$2(-11) + 1(5) + 12(4).$$

$$-22 + 5 + 48$$

$$= 31$$

$$z = \frac{A_3}{A} = \frac{31}{31}$$

$$z = 1.$$

---

(5) $2x - y + 3z = 2$

$5x + 2y + 0z = 8$

$0x + 3y + 2z = -5$

$$\Delta = \begin{vmatrix} \overset{+}{2} & \overset{-}{-1} & \overset{+}{3} \\ 5 & 2 & 0 \\ 0 & 3 & 2 \end{vmatrix}$$

$$A = 2\begin{vmatrix}2 & 0\\ 3 & 2\end{vmatrix} + 1\begin{vmatrix}5 & 0\\ 0 & 2\end{vmatrix} + 3\begin{vmatrix}5 & 2\\ 0 & 3\end{vmatrix}.$$

$$A = 2(4-0) + 1(10-0) + 3(15-0)$$

$$A = 8 + 10 + 45 = 63.$$

$$A_x = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{vmatrix} 2 & -1 & 3 \\ 8 & 2 & 0 \\ -5 & 3 & 2 \end{vmatrix} \end{array}$$

$$A_x = 2\begin{vmatrix}2 & 0\\ 3 & 2\end{vmatrix} + 1\begin{vmatrix}8 & 0\\ -5 & 2\end{vmatrix} \; 3\begin{vmatrix}8 & 2\\ -5 & 3\end{vmatrix}.$$

$$2(4-0) + 1(16+0)\; 3(24+10).$$

$$8 + 16 + 102$$

$$= 126$$

$$x = \frac{A_x}{A} = \frac{126}{63}$$

$$x = 2.$$

$$A_y = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{vmatrix} 2 & 2 & 3 \\ 5 & 8 & 0 \\ 0 & -5 & 2 \end{vmatrix} \end{array}$$

$$+2\begin{vmatrix}8 & 0\\ -5 & 2\end{vmatrix} - 2\begin{vmatrix}5 & 0\\ 0 & 2\end{vmatrix} + 3\begin{vmatrix}5 & 8\\ 0 & -5\end{vmatrix}$$

$$+2(16+\cancel{0}) - 2(10-0) + 3(-25-0)$$

$$32 \mp 20 - 75$$

$$= -63$$

$$y = \frac{A_y}{A}$$

$$y = \frac{-63}{63}$$

$$y = -1$$

$$A_z = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{vmatrix} 2 & -1 & 2 \\ 5 & 2 & 8 \\ 0 & 3 & -5 \end{vmatrix} \end{array}$$

$$+2\begin{vmatrix} 2 & 8 \\ 3 & -5 \end{vmatrix} + 1\begin{vmatrix} 5 & 8 \\ 0 & -5 \end{vmatrix} + 2\begin{vmatrix} 5 & 2 \\ 0 & 3 \end{vmatrix}$$

$$+2(-10-24) + 1(-25-0) + 2(15-0)$$

$$+2(-34) \cancel{+1}(-25+30)$$

$$-68 - 25 + 30$$

$$= -63$$

$$z = \frac{A_3}{A} = \frac{-63}{63} \quad z = -1$$

Singular matrix

A matrix is said to be singular ma[trix]
then $|A| = 0$.

$$A = \begin{bmatrix} 6 & 2 \\ 3 & 1 \end{bmatrix} \quad |A| = \begin{vmatrix} 6 & 2 \\ 3 & 1 \end{vmatrix}$$

$$= (6 \times 1) - (2 \times 3)$$

$$= 6 \times 6 = 0.$$

Ex :- $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 0.$$

---

2) If $A = \begin{bmatrix} x & 3 \\ 7 & 9 \end{bmatrix}$ is a singular matrix find x

$$|A| = 0$$

$$\begin{vmatrix} x & 3 \\ 7 & 9 \end{vmatrix} = 0 \quad \Rightarrow 9x - 21 = 0$$

$$9x = 21$$

$$x = \frac{21}{9}$$

$$= 7/3.$$

Minor of a Matrix.

Ex 1 - 1] $A = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$

$$|A| = \begin{vmatrix} 4 & 3 \\ 1 & 2 \end{vmatrix}$$

minor of 4 = 2
minor of 3 = 1
minor of 1 = 3
minor of 2 = 4.

minor of matrix $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$

2] $A = \begin{bmatrix} -3 & -7 \\ 6 & 5 \end{bmatrix}$

$$|A| = \begin{vmatrix} -3 & -7 \\ 6 & 5 \end{vmatrix}$$

minor of -3 = 5
minor of -7 = 6
minor of 6 = -7
minor of 5 = -3.

Minor of Matrix $A = \begin{bmatrix} 5 & 6 \\ -7 & -3 \end{bmatrix}$

3) $A = \begin{bmatrix} 9 & -1 \\ -5 & 6 \end{bmatrix}$

$|A| = \begin{vmatrix} 9 & -1 \\ -5 & 6 \end{vmatrix}$.

~~Matrix~~ minor of 9 = 6

~~matrix~~ minor of -1 = -5

~~matrix~~ minor of -5 = -1

~~matrix~~ minor. of 6 = 9.

~~Matrix~~ Minor of matrix $A = \begin{bmatrix} 6 & -5 \\ -1 & 9 \end{bmatrix}$

---

4) $A = \begin{bmatrix} 4 & 1 & 6 \\ -2 & 0 & 3 \\ -1 & 5 & 7 \end{bmatrix}$

$|A| = \begin{vmatrix} 4 & 1 & 6 \\ -2 & 0 & 3 \\ -1 & 5 & 7 \end{vmatrix}$

minor of 4 $= \begin{vmatrix} 0 & 3 \\ 5 & 7 \end{vmatrix} = 0 - 15 = -15$

minor of 1 $= \begin{bmatrix} -2 & 3 \\ -1 & 7 \end{bmatrix} = -14 + 3 = -11$

minor of 6 = $\begin{vmatrix} -2 & 0 \\ -1 & 5 \end{vmatrix} = -10+0 = -10$

minor of -2 = $\begin{vmatrix} 1 & 6 \\ 5 & 7 \end{vmatrix} = 7-30 = -23$

minor of 0 = $\begin{vmatrix} 4 & 6 \\ -1 & 7 \end{vmatrix} = 28+6 = 34.$

Minor of 3 = $\begin{vmatrix} 4 & 1 \\ -1 & 5 \end{vmatrix} = 20+1 = 21$

Minor of -1 = $\begin{vmatrix} 1 & 6 \\ 0 & 3 \end{vmatrix} = 3-0 = 3.$

Minor of 5 = $\begin{vmatrix} 4 & 6 \\ -2 & 3 \end{vmatrix} = 12+12 = 24.$

minor of 7 = $\begin{vmatrix} 4 & 1 \\ -2 & 0 \end{vmatrix} = 0+2 = 2.$

Minor of matrix A = $\begin{bmatrix} -15 & -11 & -10 \\ -23 & 34 & 21 \\ 3 & 24 & 2 \end{bmatrix}$

Home work.

(1) $2y - z = 0$

$x + 3y = -4$

$3x + 4y = 44.$

$0x + 2y - z = 0$

$x + 3y + 0z = -4.$

$3x + 4y + 0z = 44.$

$$A = \begin{vmatrix} \overset{+}{0} & \overset{-}{2} & \overset{+}{-1} \\ 1 & 3 & 0 \\ 3 & 4 & 0 \end{vmatrix}$$

$$0\begin{vmatrix} & \end{vmatrix} - 2\begin{vmatrix} 1 & 0 \\ 3 & 0 \end{vmatrix} - 1\begin{vmatrix} 1 & 3 \\ 3 & 4 \end{vmatrix}$$

$0() - 2(0 - 0) - 1(4 - 9)$

$= -1(-5)$

$= 5$

$$A_x = \begin{vmatrix} \overset{+}{0} & \overset{-}{2} & \overset{+}{-1} \\ -4 & 3 & 0 \\ 44 & 4 & 0 \end{vmatrix}$$

$$0\,|1| - 2\begin{vmatrix} -4 & 0 \\ 44 & 0 \end{vmatrix} - 1\begin{vmatrix} -4 & 3 \\ 44 & 4 \end{vmatrix}.$$

$$0\,|1| - 2\,|-0 - 0| - 1\,|-16 - 132|.$$

$$-1\,(-148)$$

$$= +148.$$

$$x = \frac{\Delta x}{A} = \frac{\cancel{148}}{\cancel{5}}\ 35 \qquad x = 35$$

$$
\begin{array}{cccc}
 & + & - & + \\
A_y = & \begin{vmatrix} 0 & 0 & -1 \\ 1 & -4 & 0 \\ 3 & 44 & 0 \end{vmatrix}
\end{array}
$$

$$A_y = 0\,|\ 1\ | - 0\,|\ |\ + -1\begin{vmatrix} 1 & -4 \\ 3 & 44 \end{vmatrix}.$$

$$-1\,|44 + 16|.$$

$$-1\,(60)$$

$$= -60.$$

$$y = \frac{A_y}{A} = \frac{-60}{5} = -12.$$

$$A_z = \begin{vmatrix} 0^{+} & \bar{2} & 0^{+} \\ 1 & 3 & -4 \\ 3 & 4 & 44 \end{vmatrix}$$

$$A_z = 0 \left| \right| - 2 \begin{vmatrix} 1 & -4 \\ 3 & 44 \end{vmatrix} 0 \left| \right|$$

$$A_z = -2(44+16)$$

$$A_z = -2(60)$$

$$A_z = -120$$

$$Z = \frac{A_z}{A} = \frac{-120}{5} = -24.$$

---

(2) $A = \begin{bmatrix} 2 & 10 & 9 \\ 3 & -7 & -6 \\ -2 & 8 & 4 \end{bmatrix}$

$$|A| = \begin{vmatrix} 2 & 10 & 9 \\ 3 & -7 & -6 \\ -2 & 8 & 4 \end{vmatrix}$$

Minor of 2 $= \begin{vmatrix} -7 & -6 \\ 8 & 4 \end{vmatrix} = -28 + 48 = 20.$

Minor of 10 = $\begin{vmatrix} 3 & -6 \\ -2 & 4 \end{vmatrix} = 12 - 12 = 0.$

Minor of 9 = $\begin{vmatrix} 3 & -7 \\ -2 & 8 \end{vmatrix} = 24 - 14 = 10$

Minor of 3 = $\begin{vmatrix} 10 & 9 \\ 8 & 4 \end{vmatrix} = 40 - 72 = -32$

Minor of -7 = $\begin{vmatrix} 2 & 9 \\ -2 & 4 \end{vmatrix} = 8 + 18 = 26.$

Minor of -6 = $\begin{vmatrix} 2 & 10 \\ -2 & 8 \end{vmatrix} = 16 + 20 = 36.$

Minor of -2 = $\begin{vmatrix} 10 & 9 \\ -7 & -6 \end{vmatrix} = -60 + 63 = 3.$

Minor of 8 = $\begin{vmatrix} 2 & 9 \\ 3 & -6 \end{vmatrix} = -12 - 27 = -39.$

Minor of 4 = $\begin{vmatrix} 2 & 10 \\ 3 & -7 \end{vmatrix} = -14 - 30 = -44.$

Minor of matrix A = $\begin{bmatrix} 20 & 0 & 10 \\ -32 & 26 & 36 \\ 3 & -39 & -44 \end{bmatrix}$

Cofactor of a matrix

1] $A = \begin{vmatrix} 4 & 3 \\ 1 & 2 \end{vmatrix}$

$|A| = \begin{vmatrix} 4 & 3 \\ 1 & 2 \end{vmatrix} \begin{vmatrix} + & - \\ - & + \end{vmatrix}$

Cofactor of 4 = +2
Cofactor of 3 = −1
Cofactor of 1 = −3
Cofactor of 2 = +4.

Cofactor of matrix $A = \begin{bmatrix} +2 & -1 \\ -3 & +4 \end{bmatrix}$

2] $A = \begin{bmatrix} -7 & 6 \\ -4 & 1 \end{bmatrix}$

$|A| = \begin{vmatrix} -7 & 6 \\ -4 & 1 \end{vmatrix} \begin{vmatrix} + & - \\ - & + \end{vmatrix}$

CF of −7 = +1
CF of 6 = −(−4) = 4
CF of −4 = −6
CF of 1 = +(−7) = −7.

CF of matrix $A = \begin{bmatrix} 1 & 4 \\ -6 & -7 \end{bmatrix}$

3] $A = \begin{bmatrix} 6 & -4 \\ 2 & -8 \end{bmatrix}$

$|A| = \begin{vmatrix} 6 & -4 \\ 2 & -8 \end{vmatrix} \quad \begin{vmatrix} + & - \\ - & + \end{vmatrix}$

CF of $6 = +(-8) = -8$

CF of $-4 = -(+2) = -2$

CF of $2 = -(-4) = +4$

CF of $-8 = +6$

CF of matrix $A = \begin{bmatrix} -8 & -2 \\ +4 & +6 \end{bmatrix}$

$A = \begin{bmatrix} 1 & -2 & 4 \\ 0 & 5 & 3 \\ 6 & -1 & 2 \end{bmatrix}$ $\quad 5 \quad + \begin{vmatrix} 1 & 4 \\ 6 & 2 \end{vmatrix} \; 2 - 24 = -22$

$|A| = \begin{vmatrix} 1 & -2 & 4 \\ 0 & 5 & 3 \\ 6 & -1 & 2 \end{vmatrix} \quad \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$

C.F of $1 = + \begin{vmatrix} 5 & 3 \\ -1 & 2 \end{vmatrix} = +(10+3) = +13$

C.F of $-2 = - \begin{vmatrix} 0 & 3 \\ 6 & 2 \end{vmatrix} = -(0-18) = +18$

CF of $4 = + \begin{vmatrix} 0 & 5 \\ 6 & -1 \end{vmatrix} = +(0-30) = -30$

CF of 0 = $-\begin{vmatrix} -2 & 4 \\ -1 & 2 \end{vmatrix} = -(-4+4) = 0$

CF of 5 = $+\begin{vmatrix} 1 & 4 \\ 6 & 2 \end{vmatrix} = +(2-24) = -22$

CF of 5 = $-\begin{vmatrix} 1 & -2 \\ 6 & -1 \end{vmatrix} = -(-1+12) = -11$

CF of 6 = $+\begin{vmatrix} -2 & 4 \\ 5 & 3 \end{vmatrix} = +(-6-20) = -26.$

CF of -1 = $-\begin{vmatrix} 1 & 4 \\ 0 & 3 \end{vmatrix} = -(3-0)$
$= -3.$

CF of 2 = $+\begin{vmatrix} 1 & -2 \\ 0 & 5 \end{vmatrix} = +(5+0)$
$= 5$

C.F of matrix A = $\begin{bmatrix} 13 & 18 & -30 \\ 0 & -22 & -11 \\ -26 & -3 & 5 \end{bmatrix}$

5] $A = \begin{bmatrix} 8 & 2 & -7 \\ 5 & -4 & 9 \\ 2 & 1 & 4 \end{bmatrix}$

$|A| = \begin{vmatrix} 8 & 2 & -7 \\ 5 & -4 & 9 \\ 2 & 1 & 4 \end{vmatrix} \quad \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$

$$CF\ of\ 8 = +\begin{vmatrix} -4 & 9 \\ 1 & 4 \end{vmatrix} = +(-16-9) = -25$$

$$CF\ of\ 2 = -\begin{vmatrix} 5 & 9 \\ 2 & 4 \end{vmatrix} = -(20-18) = -2.$$

$$CF\ of\ -7 = +\begin{vmatrix} 5 & -4 \\ 2 & 1 \end{vmatrix} = +(5+8) = +13$$

$$CF\ of\ 5 = -\begin{vmatrix} 2 & -7 \\ 1 & 4 \end{vmatrix} = -(8+7) = -15$$

$$CF\ of\ -4 = +\begin{vmatrix} 8 & -7 \\ 2 & 4 \end{vmatrix} = +(32+14) = 46.$$

$$CF\ of\ 9 = -\begin{vmatrix} 8 & 2 \\ 2 & 1 \end{vmatrix} = -(8-4) = -4.$$

$$CF\ of\ 2 = +\begin{vmatrix} 2 & -7 \\ -4 & 9 \end{vmatrix} = +(18-28) = -10$$

$$CF\ of\ 1 = -\begin{vmatrix} 8 & -7 \\ \cancel{2}\,5 & \cancel{4}\,9 \end{vmatrix} = -(\cancel{3}72 + \cancel{14}\ 35) = -107.\ \cancel{-107}\ \cancel{-46}$$

$$CF\ of\ 4 = +\begin{vmatrix} 8 & 2 \\ 5 & -4 \end{vmatrix} = +(-32-10) = -42.$$

$$CF\ of\ matrix\ A = \begin{bmatrix} -25 & -2 & 13 \\ -15 & 46 & -4 \\ -10 & \cancel{46}\ \cancel{-107}\ -107 & -42 \end{bmatrix}.$$

Adjoint of a matrix :-

Adj of a matrix A = [Co factor of a matrix A]^T

1] $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & 4 \\ 2 & 3 \end{vmatrix} \quad \begin{vmatrix} + & - \\ - & + \end{vmatrix}$

CF of 1 = +3

CF of 4 = −2

CF of 2 = −4

CF of 3 = +1

CF of matrix $A = \begin{bmatrix} 3 & -2 \\ -4 & 1 \end{bmatrix}$

Adjoint of matrix $A = \begin{bmatrix} 3 & -4 \\ -2 & 1 \end{bmatrix}$

2] $A = \begin{bmatrix} -8 & -5 \\ 7 & 6 \end{bmatrix}$

$|A| = \begin{vmatrix} -8 & -5 \\ 7 & 6 \end{vmatrix} \quad \begin{vmatrix} + & - \\ - & + \end{vmatrix}$

CF of $-8 = +6$.

CF of $-5 = -7$

CF of $7 = -(-5) = +5$

CF of $6 = +(-8) = -8$.

CF of matrix $A = \begin{bmatrix} +6 & -7 \\ +5 & -8 \end{bmatrix}$

Adjoint of a matrix $= \begin{bmatrix} +6 & +5 \\ -7 & -8 \end{bmatrix}$.

Home work.

Adjecent of Matrix.

1] $A = \begin{bmatrix} 7 & 2 \\ 5 & 6 \end{bmatrix}$

$|A| = \begin{vmatrix} -7 & -2 \\ 5 & +6 \end{vmatrix} \quad \begin{vmatrix} + & - \\ - & + \end{vmatrix}$

CF of $-7 = +6$

CF of $-2 = -5$

CF of $5 = +2$

CF of $6 = +(-7) = -7$

CF of matrix $A = \begin{bmatrix} +6 & -5 \\ 2 & -7 \end{bmatrix}$

Adj of matrix $A = \begin{bmatrix} 6 & 2 \\ -5 & -7 \end{bmatrix}$

2] $A = \begin{bmatrix} 3 & 7 \\ 2 & 1 \end{bmatrix}$

$|A| = \begin{vmatrix} -3 & 7 \\ 2 & -1 \end{vmatrix} \cdot \begin{vmatrix} + & - \\ - & + \end{vmatrix}$

CF of $-3 = -1$

CF of $7 = -2$

CF of $2 = -7$

CF of $-1 = +(-3) = -3$.

CF of matrix $A = \begin{bmatrix} -1 & -2 \\ -7 & -3 \end{bmatrix}$

Adj of matrix $A = \begin{bmatrix} -1 & -7 \\ -2 & -3 \end{bmatrix}$

II] Cofactor of Matrix.

1] $A = \begin{bmatrix} 4 & -1 & 3 \\ 5 & 7 & -6 \\ -8 & 2 & 9 \end{bmatrix}$

~~CEq~~

$$|A| = \begin{vmatrix} 4 & -1 & 3 \\ 5 & 7 & -6 \\ -8 & 2 & 9 \end{vmatrix} \quad \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

$$\text{CF of } 4 = +\begin{vmatrix} 7 & -6 \\ 2 & 9 \end{vmatrix} = +(63+12) = 75$$

$$\text{CF of } -1 = -\begin{vmatrix} 5 & -6 \\ -8 & 9 \end{vmatrix} = -(45-48) = \cancel{6}3$$

$$\text{CF of } 3 = +\begin{vmatrix} 5 & 7 \\ -8 & 2 \end{vmatrix} = +(10+56) = 66.$$

$$\text{CF of } 5 = -\begin{vmatrix} -1 & 3 \\ 2 & 9 \end{vmatrix} = -(-9-6) = 15$$

$$\text{CF of } 7 = +\begin{vmatrix} 4 & 3 \\ -8 & 9 \end{vmatrix} = +(36+24) = 60.$$

$$\text{CF of } -6 = -\begin{vmatrix} 4 & -1 \\ -8 & 2 \end{vmatrix} = -(8-8) = 0.$$

$$\text{CF of } -8 = +\begin{vmatrix} -1 & 3 \\ 7 & -6 \end{vmatrix} = +(6-21) = -15$$

$$\text{CF of } 2 = -\begin{vmatrix} 4 & 3 \\ 5 & -6 \end{vmatrix} = -(-24-15) = 39.$$

$$CF\ of\ 9 = + \begin{vmatrix} 4 & -1 \\ 5 & 7 \end{vmatrix} = +(28+5) = 33$$

$$CF\ of\ matrix\ A = \begin{bmatrix} 75 & \cancel{6}3 & 66 \\ 15 & 60 & 0 \\ -15 & 39 & 33 \end{bmatrix}$$

$$\cancel{Adj\ of\ Matrix\ A} = \begin{bmatrix} 75 & 15 & 66 \\ 3 & 60 & 0 \\ 66 & 39 & 33 \end{bmatrix}$$

2] $$A = \begin{bmatrix} -8 & 2 & -4 \\ 1 & -3 & 0 \\ 9 & 5 & -6 \end{bmatrix}$$

$$|A| = \begin{vmatrix} -8 & 2 & -4 \\ 1 & -3 & 0 \\ 9 & 5 & -6 \end{vmatrix} \quad \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

$$\text{CF of } -8 = +\begin{vmatrix} -3 & 0 \\ 5 & -6 \end{vmatrix} = +(18-0) = 18.$$

$$\text{CF of } 2 = -\begin{vmatrix} 1 & 0 \\ 9 & -6 \end{vmatrix} = -(-6-0) = 6.$$

$$\text{CF of } -4 = +\begin{vmatrix} 1 & -3 \\ 9 & 5 \end{vmatrix} = +(-27-5) = -22.$$

$$\text{CF of } 1 = -\begin{vmatrix} 2 & -4 \\ 5 & -6 \end{vmatrix} = -(-12+20) = -8$$

$$\text{CF of } -3 = +\begin{vmatrix} -8 & -4 \\ 9 & -6 \end{vmatrix} = +(48+36) = 84.$$

$$\text{CF of } 0 = -\begin{vmatrix} -8 & 2 \\ 9 & 5 \end{vmatrix} = -(-40-18) = 58.$$

$$\text{CF of } 9 = +\begin{vmatrix} 2 & -4 \\ -3 & 0 \end{vmatrix} = +(0+12) = 12.$$

$$\text{CF of } 5 = -\begin{vmatrix} -8 & -4 \\ 1 & 0 \end{vmatrix} = -(0+4) = 4$$

$$\text{CF of } -6 = +\begin{vmatrix} -8 & 2 \\ 1 & -3 \end{vmatrix} = +(24-2) = 22$$

$$\text{CF of matrix } A = \begin{bmatrix} 18 & 6 & -22 \\ -8 & 84 & 58 \\ 12 & 4 & 22 \end{bmatrix}$$

① $A = \begin{bmatrix} 7 & -2 & 0 \\ 6 & 1 & 3 \\ 4 & -5 & 8 \end{bmatrix}$

$|A| = \begin{vmatrix} 7 & -2 & 0 \\ 6 & 1 & 3 \\ 4 & -5 & 8 \end{vmatrix} \quad \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$

$CF\ of\ 7 = + \begin{vmatrix} 1 & 3 \\ -5 & 8 \end{vmatrix} = +(8+15) = +23.$

$CF\ of\ -2 = - \begin{vmatrix} 6 & 3 \\ 4 & 8 \end{vmatrix} = -(48-12) = -36.$

$CF\ of\ 0 = + \begin{vmatrix} 6 & 1 \\ 4 & -5 \end{vmatrix} = +(30-4) = -34.$

$CF\ of\ 6 = - \begin{vmatrix} -2 & 0 \\ -5 & 8 \end{vmatrix} = -(-16+0) = +16.$

$CF\ of\ 1 = + \begin{vmatrix} 7 & 0 \\ 4 & 8 \end{vmatrix} = +(56-0) = +56.$

$CF\ of\ 3 = - \begin{vmatrix} 7 & -2 \\ 4 & -5 \end{vmatrix} = -(-35+8) = +27$

$CF\ of\ 4 = + \begin{vmatrix} -2 & 0 \\ 1 & 3 \end{vmatrix} = +(-6-0) = -6.$

$CF\ of\ -5 = - \begin{vmatrix} 7 & 0 \\ 6 & 3 \end{vmatrix} = -(21-0) = -21$

$$CF\ of\ 8 = + \begin{vmatrix} 7 & -2 \\ 6 & 1 \end{vmatrix} = +(7+12) = 19.$$

$$CF\ of\ matrix\ A = \begin{bmatrix} 23 & -36 & -34 \\ 16 & 56 & 27 \\ -6 & -21 & 19 \end{bmatrix}$$

$$Adj\ of\ matrix\ A = \begin{bmatrix} 23 & 16 & -6 \\ -36 & 56 & -21 \\ -34 & 27 & 19 \end{bmatrix}$$

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(2)
$$A = \begin{bmatrix} 2 & 4 & -8 \\ 6 & -3 & 5 \\ -1 & 0 & 9. \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 4 & -8 \\ 6 & -3 & 5 \\ -1 & 0 & 9 \end{vmatrix} \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}.$$

$$CF\ of\ 2 = + \begin{vmatrix} -3 & 5 \\ 0 & 9 \end{vmatrix} = +(\cancel{-27} - 0) = -\cancel{27}.$$

$$CF\ of\ 4 = - \begin{vmatrix} 6 & 5 \\ -1 & 9 \end{vmatrix} = -(54+5) = -59.$$

$$CF\ of\ -8 = + \begin{vmatrix} 6 & -3 \\ -1 & 0 \end{vmatrix} = +(0-3) = \cancel{\phi}-3$$

$$CF \text{ of } 6 = -\begin{vmatrix} 4 & -8 \\ 0 & 9 \end{vmatrix} = -(36+0) = -36$$

$$CF \text{ of } -3 = +\begin{vmatrix} 2 & -8 \\ -1 & 9 \end{vmatrix} = +(18-8) = 10.$$

$$CF \text{ of } 5 = -\begin{vmatrix} 2 & 4 \\ -1 & 0 \end{vmatrix} = -(0+4) = -4.$$

$$CF \text{ of } -1 = +\begin{vmatrix} 4 & -8 \\ -3 & 5 \end{vmatrix} = +(20-24) = -4$$

$$CF \text{ of } 0 = -\begin{vmatrix} 2 & -8 \\ 6 & 5 \end{vmatrix} = -(10+48) = -58$$

$$CF \text{ of } 9 = +\begin{vmatrix} 2 & 4 \\ 6 & -3 \end{vmatrix} = +(-6-24) = -30$$

$$CF \text{ of matrix } A = \begin{bmatrix} -27 & -59 & -3 \\ -36 & 10 & -4 \\ -4 & -58 & -30 \end{bmatrix}$$

$$\text{Adj of matrix } A = \begin{bmatrix} -27 & -36 & -4 \\ -59 & 10 & -58 \\ -3 & -4 & -30 \end{bmatrix}$$

Inverse of a matrix

$$A^{-1} = \frac{\text{Adj of } A}{|A|} \quad A \neq 0.$$

$$A = \begin{bmatrix} 3 & -4 \\ 2 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj of } A}{|A|} \quad A \neq 0$$

$$|A| = \begin{vmatrix} 3 & -4 \\ 2 & 1 \end{vmatrix} = 3+8 = 11 \neq 0.$$

$$|A| = \begin{vmatrix} 3 & -4 \\ 2 & 1 \end{vmatrix} \begin{vmatrix} + & - \\ - & + \end{vmatrix}.$$

CF of $3 = +1$

CF of $-4 = -2$

CF of $2 = -(-4) = +4$

CF of $1 = +(3) = 3.$

CF of matrix $A = \begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix}$

Adj of matrix $A = \begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix}$

$$A^{-1} = \frac{\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix}}{11}$$

$$A^{-1} = \frac{1}{11} \begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix}$$

2] $A = \begin{bmatrix} 3 & -5 \\ 6 & -1 \end{bmatrix}$

$A^{-1} = \dfrac{\text{Adj of } A}{|A|}$ where $|A| \neq 0$.

$|A| = \begin{vmatrix} 3 & -5 \\ 6 & -1 \end{vmatrix} \quad \begin{bmatrix} + & - \\ - & + \end{bmatrix}$

$= -3 + 30 = 27 \neq 0$.

~~Adj of~~

CF of $3 = +(-1) = -1$

CF of $-5 = -(6) = -6$.

CF of $6 = +(-5) = +5$

CF of $-1 = -(3) = +3$.

CF of matrix $A = \begin{bmatrix} -1 & -6 \\ -5 & -3 \end{bmatrix}$.

Adj of matrix $A = \begin{bmatrix} -1 & -5 \\ -6 & -\cancel{5}\,3 \end{bmatrix}$

$A^{-1} = \dfrac{\begin{bmatrix} -1 & 5 \\ -6 & 3 \end{bmatrix}}{27} = \dfrac{1}{27}\begin{bmatrix} -1 & 5 \\ -6 & 3 \end{bmatrix}$

3) $A = \begin{bmatrix} -5 & 0 & 2 \\ 6 & 7 & -3 \\ -1 & 8 & 10 \end{bmatrix}$

$A^{-1} = \dfrac{\text{Adj of } A}{|A|} \quad |A| \neq 0.$

$|A| \begin{vmatrix} \overset{+}{-5} & \overset{-}{0} & \overset{+}{2} \\ 6 & 7 & -3 \\ -1 & 8 & 10 \end{vmatrix} = -5 \begin{vmatrix} 7 & -3 \\ 8 & 10 \end{vmatrix} - 0 \left| \quad \right| + 2 \begin{vmatrix} 6 & 7 \\ -1 & 8 \end{vmatrix}$

$= -5(70+24) + 2(48+7) = -470 + 110 = -360 \neq 0.$

$|A| = \begin{vmatrix} -5 & 0 & 2 \\ 6 & 7 & -3 \\ -1 & 8 & 10 \end{vmatrix} \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$

CF of $-5 = + \begin{vmatrix} 7 & -3 \\ 8 & 10 \end{vmatrix} = +(70+24) = +94.$

CF of $0 = - \begin{vmatrix} 6 & -3 \\ -1 & 10 \end{vmatrix} = -(60-3) = \cancel{-63}\ -57$

CF of $2 = + \begin{vmatrix} 6 & 7 \\ -1 & 8 \end{vmatrix} = +(48+7) = \cancel{61}.55$

CF of $6 = - \begin{vmatrix} 0 & 2 \\ 8 & 10 \end{vmatrix} = -(0-16) = +16.$

CF of $7 = + \begin{vmatrix} -5 & 2 \\ -1 & 10 \end{vmatrix} = +(-50+2) = -48.$

$$CF\ of\ -3 = -\begin{vmatrix} -5 & 0 \\ -1 & 8 \end{vmatrix} = -(-40+0) = +40$$

$$CF\ of\ -1 = +\begin{vmatrix} 0 & 2 \\ 7 & -3 \end{vmatrix} = +(-0-14) = -14$$

$$CF\ of\ 8 = -\begin{vmatrix} -5 & 2 \\ 6 & -3 \end{vmatrix} = -(+15-12) = -3$$

$$CF\ of\ 10 = +\begin{vmatrix} -5 & 0 \\ 6 & 7 \end{vmatrix} = +(-35-0) = -35$$

$$CF\ of\ matrix\ A = \begin{bmatrix} 94 & -57 & 55 \\ +16 & -48 & +40 \\ -14 & -3 & -35 \end{bmatrix}$$

$$Adj\ of\ matrix\ A = \begin{bmatrix} 94 & 16 & -14 \\ -57 & -48 & -3 \\ 55 & +40 & -35 \end{bmatrix}$$

$$A^{-1} = \frac{Adj\ of\ A}{|A|}$$

$$A^{-1} = \frac{\begin{bmatrix} 94 & 16 & -14 \\ -57 & -48 & -3 \\ 55 & +40 & -35 \end{bmatrix}}{-360}$$

$$A^{-1} = \frac{1}{-360} \begin{bmatrix} 94 & 16 & -14 \\ -57 & -48 & -3 \\ 55 & +40 & -35 \end{bmatrix}$$

Characteristic Equation & Characteristic Roots.

Home work

$$A = \begin{bmatrix} -1 & 4 \\ 7 & -8 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj of } A}{|A|} \quad |A| \neq 0.$$

$$|A| = \begin{vmatrix} -1 & 4 \\ 7 & -8 \end{vmatrix} = (8 - 28) = -20. \neq 0.$$

CF of $-1 = +(-8) = -8$.

CF of $4 = -(+7) = -7$

CF of $\cancel{7} \cancel{4}$ 4

CF of $-8 = +(-1) = -1$

CF of matrix $A = \begin{bmatrix} -8 & -7 \\ 4 & -1 \end{bmatrix}$

Adj of matrix $= \begin{bmatrix} -8 & -4 \\ -7 & -1 \end{bmatrix}$

$A^{-1} = \frac{\text{adj of } A}{|A|}$ $|A| \neq 0$.

$\frac{\begin{bmatrix} -8 & -4 \\ -7 & -1 \end{bmatrix}}{-20}$

$\frac{1}{20} \begin{bmatrix} -8 & -4 \\ -7 & -1 \end{bmatrix}$

2] $A = \begin{bmatrix} -10 & 7 \\ 1 & 3 \end{bmatrix}$

$A^{-1} = \frac{\text{Adj of } A}{|A|}$ $|A| \neq 0$.

$|A| = \begin{vmatrix} -10 & 7 \\ 1 & 3 \end{vmatrix}$ $\begin{vmatrix} + & - \\ - & + \end{vmatrix}$

$= (-30 - 7) = -37$

CF of $-10 = 3$

CF of $7 = -1$

CF of $1 = -7$

CF of $3 + (-10) = -10$

CF of matrix $A = \begin{bmatrix} 3 & -1 \\ -7 & -10 \end{bmatrix}$

Adj of matrix $A = \begin{bmatrix} 3 & -7 \\ -1 & -10 \end{bmatrix}$

$A^{-1} = \dfrac{\text{adj of } A}{|A|}$ $|A| \neq 0$

$= \dfrac{\begin{bmatrix} 3 & -7 \\ -1 & -10 \end{bmatrix}}{-37}$

$= \dfrac{1}{-37} \begin{bmatrix} 3 & -7 \\ -1 & -10 \end{bmatrix}$

3] : $A = \begin{bmatrix} 4 & -2 \\ 6 & 8 \end{bmatrix}$ $|A| = \begin{vmatrix} 4 & -2 \\ 6 & 8 \end{vmatrix} = (32 + 12) = 44$

$A^{-1} = \dfrac{\text{adj of } A}{|A|}$ $|A| \neq 0$

$$|A| = \begin{vmatrix} 4 & -2 \\ 6 & 8 \end{vmatrix} \quad \begin{vmatrix} + & - \\ - & + \end{vmatrix}$$

CF of $4 = +8$

CF of $-2 = -(+6) = -6.$

CF of $6 = -(-2) = +2.$

CF of $8 = +(+4) = +4.$

CF of matrix $A = \begin{bmatrix} 8 & -6 \\ 2 & 4 \end{bmatrix}$

Adj of matrix $A = \begin{bmatrix} 8 & 2 \\ -6 & 4 \end{bmatrix}$

$$A^{-1} = \frac{\text{adj of } A}{|A|} \quad |A| \neq 0.$$

$$= \frac{\begin{bmatrix} 8 & 2 \\ -6 & 4 \end{bmatrix}}{44.}$$

$$A^{-1} = \frac{1}{44} \begin{bmatrix} 8 & 2 \\ -6 & 4 \end{bmatrix}.$$

$$A^{-1} = \frac{\text{adj of } A}{|A|} \quad |A| \neq 0.$$

$$= \frac{\begin{bmatrix} 146 & -64 & 41 \\ 64 & -4 & 22 \\ 22 & 0 & 11 \end{bmatrix}}{44}$$

$$= \frac{1}{44} \begin{bmatrix} 146 & -64 & 41 \\ 64 & -4 & 22 \\ 22 & 0 & 11 \end{bmatrix}$$

Chareteristics Equation & Chareteristics Roots

If 'A' is any square matrix, I is the identity matrix of the same order as that of matrix A & λ is any constant then $|A - \lambda I| = 0$. is called Chareteristics equation.

Ex :-

$$A = \begin{bmatrix} 3 & 4 \\ 6 & 13 \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \lambda I = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 3-\lambda & 4 \\ 6 & 13-\lambda \end{bmatrix}$$

The CE is given by.

$|A - \lambda I| = 0$.

$$\begin{vmatrix} 3-\lambda & 4 \\ 6 & 13-\lambda \end{vmatrix} = 0$$

$(3-\lambda)(13-\lambda) - 24 = 0$

$3(13-\lambda) - \lambda(13-\lambda) - 24 = 0$.

$39 - 3\lambda - 13\lambda + \lambda^2 - 24 = 0$.

$\boxed{\lambda^2 - 16\lambda + 15 = 0}$ (C.E)

$\lambda^2 - 16\lambda + 15 = 0$

$\lambda^2 - 15\lambda - \lambda + 15 = 0$.

$\lambda(\lambda - 15) - 1(\lambda - 15) = 0$.

$(\lambda - 1)(\lambda - 15) = 0$

$\lambda - 1 = 0$; $\lambda - 15 = 0$.

$\boxed{\lambda = 1, 15}$

CR.

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② $A = \begin{bmatrix} 3 & -1 \\ 0 & -2 \end{bmatrix}$ $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\lambda I = \Big[c$

$A - \lambda I \begin{bmatrix} 3-\lambda & -1 \\ 6 & -2-\lambda \end{bmatrix} = 0$

$$(3-\lambda)(-2-\lambda)+0=0$$

$$3(-2-\lambda)-\lambda(-2-\lambda)=0$$

$$-6-3\lambda+2\lambda+\lambda^2=0$$

$$\boxed{\lambda^2-\lambda-6=0}\ (C.E)$$

$$-6\lambda^2$$

$$-3\lambda\ \cancel{+2\lambda}$$

$$\lambda^2-1\lambda-6=0.$$

$$\lambda^2-3\lambda+2\lambda-6=0.$$

$$\lambda(\lambda-3)+2(\lambda-3)=0$$

$$(\lambda-3)(\lambda+2)=0$$

$$\boxed{\lambda=3,-2}\ /\ (CR)$$

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3) $A=\begin{bmatrix}3 & 6\\5 & 4\end{bmatrix}$. $I=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ $\lambda I=\begin{bmatrix}\lambda & 0\\0 & \lambda\end{bmatrix}$

$$A-\lambda I=\begin{bmatrix}3-\lambda & 6\\5 & 4-\lambda\end{bmatrix}$$

The CE is given by

$$|A-\lambda I|=0.$$

$$|A-\lambda I|=\begin{vmatrix}3-\lambda & 6\\5 & 4-\lambda\end{vmatrix}$$

$(3-\lambda)(4-\lambda)-30=0$

$3(4-\lambda)-\lambda(4-\lambda)-30=0$

$12-3\lambda-4\lambda+\lambda^2-30=0$

$-18-7\lambda+\lambda^2=0$

$\boxed{\lambda^2-7\lambda-18=0}$ (CE)

$\lambda^2-7\lambda-18$

$\lambda^2+2\lambda-9\lambda-18$

$\lambda(\lambda+2)-9(\lambda+2)$

$(\lambda+2)(\lambda-9)$

$\lambda+2=0 \quad \lambda-9=0$

$\boxed{\lambda=-2,+9}$ CR.

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(4) $A=\begin{bmatrix}7 & 4\\5 & 6\end{bmatrix}$ $I=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ $\lambda=$

$A-\lambda I=\begin{bmatrix}7-\lambda & 4\\5 & 6-\lambda\end{bmatrix}$

The CE is given by.

$|A-\lambda I|=0.$

$$= \begin{vmatrix} 7-\lambda & 4 \\ 5 & 6-\lambda \end{vmatrix}$$

$$(7-\lambda)(6-\lambda) - 20 = 0$$

$$7(6-\lambda) - \lambda(6-\lambda) - 20 = 0$$

$$42 - 7\lambda - 6\lambda + \lambda^2 - 20 = 0$$

$$22 - 13\lambda + \lambda^2 = 0$$

$$\boxed{\lambda^2 \mp 13\lambda - 22 = 0}$$ (CE)

$$\lambda^2 + 13\lambda - 22 = 0$$

$$\lambda^2 - 11\lambda \mp 2\lambda - 22 = 0$$

$$\lambda(\lambda - 11) \mp 2(\lambda - 11) = 0$$

$$(\lambda \mp 2)(\lambda - 11) = 0$$

$$\boxed{\lambda = 11, 2}$$ (CR)

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(5) $A = \begin{bmatrix} 5 & 4 \\ 9 & -14 \end{bmatrix}$ $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\lambda I = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} 5-\lambda & 4 \\ 9 & 14-\lambda \end{bmatrix}$$

The CE is given by

$$|A| = |A - \lambda I| = 0$$

$$\begin{vmatrix} 5-\lambda & 4 \\ 9 & 14-\lambda \end{vmatrix}$$

$$(5-\lambda)(14-\lambda) - 36 = 0$$

$$5(14-\lambda) - \lambda(14-\lambda) - 36 = 0$$

$$70 - 5\lambda - 14\lambda + \lambda^2 - 36 = 0$$

$$106 - 9\lambda + \lambda^2$$

$$\boxed{\lambda^2 + 9\lambda - 106 = 0} \text{ (CE)}$$

$$\lambda^2 + 9\lambda - 106 = 0$$

$$\frac{-b^2 \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1 \quad b = 9 \quad c = -106$

$$\frac{-9^2 \pm \sqrt{(9)^2 - 4 \times 1 \times (-106)}}{2 \times 1}$$

$$\lambda = \frac{-9 \pm \sqrt{81 + 424}}{2}$$

$$\lambda = \frac{-9 \pm \sqrt{505}}{2}$$

$$\lambda = \frac{-9+\sqrt{505}}{2}$$

$$\lambda = \frac{-9-\sqrt{505}}{2}.$$

Home work.

$A = \begin{bmatrix} 7 & -8 \\ -6 & 5 \end{bmatrix}$. $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\lambda I \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$.

$A - \lambda I = \begin{bmatrix} 7-\lambda & -8 \\ -6 & 5-\lambda \end{bmatrix}$

The CE is given by $|A - \lambda I| = 0$.

$\begin{vmatrix} 7-\lambda & -8 \\ -6 & 5-\lambda \end{vmatrix}$

$(7-\lambda)(5-\lambda) - 48 = 0.$

$7(5-\lambda) - \lambda(5-\lambda) - 48 = 0.$

$35 - 7\lambda - 5\lambda + \lambda^2 - 48 = 0$

$-13 - 12\lambda + \lambda^2.$