INTEGRATION

Integration is a reverse process of diffinentiation. It is also called as antiderivative.

Integration of $f(x)$ w.r.t 'x' is denoted by,

$$\int f(x).dx$$

Standard Results of Integration:

1) $\int 1.dx = x + C$

2) $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

3) $\int e^x.dx = e^x + C$

4) $\int a^x.dx = \frac{a^x}{\log_e a} + C$

5) $\int \frac{1}{x} dx = \log x + C$

6) $\int \sin x.dx = -\cos x + C$

7) $\int \cos x \, dx = \sin x + c$

8) $\int \sec^2 x \, dx = \tan x + c$

9) $\int \operatorname{cosec}^2 x \, dx = -\cot x + c$

10) $\int \sec x \cdot \tan x \, dx = \sec x + c$

11) $\int \operatorname{cosec} x \cdot \cot x \, dx = -\operatorname{cosec} x + c$

12) $\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1} x$

13) $\int \frac{1}{1+x^2} \, dx = \tan^{-1} x + c$

14) $\int \frac{1}{x\sqrt{x^2-1}} \, dx = \sec^{-1} x + c$

15) $\int \sqrt{x} \, dx = \frac{2}{3} x^{3/2} + c$

16) $\int \tan x \, dx = \log(\sec$

17) $\int \cot x \, dx = \log(\sin$

18) $\int \sec x \, dx = \log(\sec$ $\tan x$

19) $\int \operatorname{cosec} x \, dx = -$ $(\operatorname{cosec}$ $+ \cot x) -$

<u>Rules of Integration:</u>

1) If 'k' is any constant and $f(x)$ is function of 'x'. Then.

$$\int k(f(x)) \, dx = k \int f(x) \, dx$$

2) If $f(x)$ and $g(x)$ are the function of the

$$\int f(x) \pm g(x) \, dx = \int f(x) \, dx \pm \int g(x) \, dx$$

Problems:

1) $\int x^7 . dx$.

Soln: Integrate w.r.t 'x'.

$$\frac{x^{7+1}}{7+1} = \frac{x^8}{8} + c$$

2) $\int \frac{1}{x^5} . dx$:

$\int x^{-5} . dx$

Int. w.r.t 'x'.

$$\frac{x^{-5+1}}{-5+1} = \frac{x^{-4}}{-4} + c$$

3) $\int x^{99} . dx$

Int. w.r.t 'x'.

$$\frac{x^{99+1}}{99+1} = \frac{x^{100}}{100} + c$$

4) $\int x^{\frac{1}{101}} . dx$

Int. w.r.t 'x'.

$$\frac{x^{\frac{1}{101}+1}}{\frac{1}{101}+1} = \frac{x^{\frac{1+101}{101}}}{\frac{1+101}{101}} = \frac{x^{\frac{102}{101}}}{\frac{102}{101}} =$$

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4) $\int \frac{1}{x^{101}} \cdot dx$

$\int x^{-101} \cdot dx$

Int. w.r.t 'x'.

$$\frac{x^{-101+1}}{-101+1} = \frac{x^{-100}}{-100} + C$$

5) $\int \sqrt[3]{x^5} \cdot dx$

$\int (x^5)^{\frac{1}{3}} \cdot dx$

$\int x^{5/3} \cdot dx$

Int. w.r.t 'x'

$$\therefore \frac{x^{5/3+1}}{\frac{5}{3}+1} = \frac{x^{8/3}}{8/3} + C$$

6) $\int \frac{1}{\sqrt[4]{x^3}} \cdot dx = \int \frac{1}{x^{3/4}} \cdot dx = \int x^{-3/4} \cdot d$

Integrate w.r.t 'x'.

$$\frac{x^{-3/4+1}}{-\frac{3}{4}+1} = \frac{x^{\frac{-3+4}{4}}}{\frac{-3+4}{4}} = \frac{x^{1/4}}{\frac{1}{4}} = 4x^{\frac{1}{4}}$$

7) $\int (x^2 - 1)\, dx$

Integrate w.r.t 'x'.

$\int x^2 \cdot dx - \int 1 \cdot dx$

$\frac{x^3}{3} - x + C$

8) $\int (2x + 3) \cdot dx$

$\int 2x \cdot dx + \int 3 \cdot dx$

$2\int x \cdot dx + 3\int 1 \cdot dx$

$\cancel{2} \times \frac{x^2}{\cancel{2}} + 3x$

$\therefore x^2 + 3x + C$

9) $\int (x^2 + x + 1) \cdot dx$

Integrate w.r.t 'x'.

$\int x^2 \cdot dx + \int x \cdot dx + \int 1 \cdot dx$

$\frac{x^3}{3} + \frac{x^2}{2} + x + c$

10) $\int \left(x^3 + x + \frac{1}{x} + \frac{1}{x^3}\right) \cdot dx$

Integrate w.r.t 'x'.

$\int x^3 \cdot dx + \int x \cdot dx + \int \frac{1}{x} \cdot dx + \int \frac{1}{x^3} \cdot dx$

$\int x^3 \cdot dx + \int x \cdot dx + \int \frac{1}{x} \cdot dx + \int x^{-3} \cdot dx$

$\int \frac{x^4}{4} + \frac{x^2}{2} + \log x + \frac{x^{-3+1}}{-3+1}$

$\frac{x^4}{4} + \frac{x^2}{2} + \log x + \frac{x^{-2}}{-2} + c$

11) $\int (4x^3 + 3x^2 + 2x + 1 + \frac{5}{x}) \cdot dx$

Integrate w.r.t 'x'.

$4\int x^3 \cdot dx + 3\int x^2 \cdot dx + 2\int x \cdot dx + \int 1 \cdot dx + 5\int \frac{1}{x}$

$\frac{\cancel{4}x^4}{\cancel{4}} + \frac{\cancel{3}x^3}{\cancel{3}} + \frac{\cancel{2}x^2}{\cancel{2}} + x + 5\log x$

$x^4 + x^3 + x^2 + x + 5\log x + c$.

12) $\int (2x+1)(x+1) \cdot dx$

$\int (2x^2 + 2x + x + 1) dx$

Integrate w.r.t 'x'.

$2\int x^2 dx + 2\int x \cdot dx + \int x \cdot dx + \int 1 \cdot dx$

$\frac{2x^3}{3} + \frac{\cancel{2}x^2}{\cancel{2}} + \frac{x^2}{2} + x$

$\frac{2x^3}{3} + x^2 + \frac{x^2}{2} + x + c$

$\frac{2x^3}{3} + \frac{2x^2 + x^2}{2} + x + c$

$\frac{2x^3}{3} + \frac{3x^2}{2} + x + c$

13) $\int \left(x+\frac{1}{x}\right)^2 \cdot dx$

$\int \left(x^2+\frac{1}{x^2}+2\cdot \cancel{x}\cdot\frac{1}{\cancel{x}}\right)\cdot dx$

$\int \left(x^2+\frac{1}{x^2}+2\right)dx$

Integrate w.r.t 'x'.

$\int x^2\cdot dx + \int x^{-2}\cdot dx + 2\int 1\cdot dx$

$\int \frac{1}{x}$

$\frac{x^3}{3}+\frac{x^{-2+1}}{-2+1}+2x$

$\frac{x^3}{3}+\frac{x^{-1}}{-1}+2x+c$.

14) $\int \left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 \cdot dx$

$\int \left(x+\frac{1}{x}-2\cancel{\sqrt{x}}\times\frac{1}{\cancel{\sqrt{x}}}\right)\cdot dx$

$\int \left(x+\frac{1}{x}-2\right)\cdot dx$

Integrate w.r.t 'x'.

$\int x\cdot dx + \int \frac{1}{x}\cdot dx - 2\int 1\cdot dx$

$\frac{x^2}{2}+\log x-2x+c$

15) $\int\left(\sqrt[4]{x} + \frac{5}{x^2}\right)dx$

$\int\left(x^{\frac{1}{4}} + \frac{5}{x^2}\right)dx$

Integrate w.r.t 'x'

$\int x^{\frac{1}{4}}\cdot dx + \int \frac{5}{x^2}\cdot dx$

$\frac{x^{\frac{1}{4}+1}}{\frac{1}{4}+1} + 5\int x^{-2}\cdot dx$

$\frac{x^{\frac{1+4}{4}}}{\frac{1+4}{4}} + 5\times\frac{x^{-2+1}}{-2+1}$

$\frac{x^{5/4}}{5/4} + \frac{5x^{-1}}{-1} + c$

$\frac{x^{5/4}}{5/4} - 5x^{-1} + C$

16) $\int \frac{1}{\sqrt[5]{x^3}}\cdot dx$

$\int\left(\frac{1}{x^{3/5}}\right)dx$

$\int x^{-3/5}\cdot dx$

Integrate w.r.t 'x'.

$x^{-3/5+1}$

$$= \frac{x^{\frac{-3+5}{5}}}{\frac{-3+5}{5}}$$

$$= \frac{x^{\frac{2}{5}}}{\frac{2}{5}} = \frac{5}{2}x^{\frac{2}{5}}$$

17) $\int \left(x+\frac{1}{x}\right)^3 \cdot dx$

$$\int \left[x^3 + \frac{1}{x^3} + 3\cancel{x}\cdot\frac{1}{\cancel{x}}\left(x+\frac{1}{x}\right)\right].dx$$

$$\int \left(x^3 + \frac{1}{x^3} + 3x + \frac{3}{x}\right).dx$$

Integrate w.r.t 'x'

$$\int x^3 \cdot dx + \int \frac{1}{x^3}\cdot dx + 3\int x \cdot dx + 3\int \frac{1}{x}\cdot dx$$

$$\frac{x^4}{4} + \frac{x^{-3+1}}{-3+1} + \frac{3x^2}{2} + 3\log x$$

$$\frac{x^4}{4} + \frac{x^{-2}}{-2} + \frac{3x^2}{2} + 3\log x + c$$

18) $\int \frac{x^2+3x+5}{x} \cdot dx$

$$\int \left(\frac{x^{\cancel{2}}}{\cancel{x}} + \frac{3\cancel{x}}{\cancel{x}} + \frac{5}{x}\right) dx$$

$$\int \left(x + 3 + \frac{5}{x}\right).dx$$

Integrate w.r.t 'x'

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$\int x \cdot dx + 3\int 1 \cdot dx + 5\int \frac{1}{x} \cdot dx$

$\frac{x^2}{2} + 3x + 5\log x + C$

19) $\int \left(\frac{x^2+3x-2}{x^4}\right) dx$

$\int \left(\frac{x^2}{x^4} + \frac{3x}{x^4} - \frac{2}{x^4}\right) dx$

$\int \left(\frac{1}{x^2} + \frac{3}{x^3} - \frac{2}{x^4}\right) \cdot dx$

Integrate w.r.t 'x'.

$\int x^{-2} \cdot dx + 3\int x^{-3} \cdot dx - 2\int x^{-4} \cdot dx$

$\frac{x^{-2+1}}{-2+1} + 3\left(\frac{x^{-3+1}}{-3+1}\right) - 2\left(\frac{x^{-4+1}}{-4+1}\right)$

$\frac{x^{-1}}{-1} + \frac{3x^{-2}}{-2} - \frac{2x^{-3}}{-3}$

$-x^{-1} - \frac{3x^{-2}}{2} + \frac{2x^{-3}}{3} + C$

20) $\int (3x^2+1)(1-x) \cdot dx$

$\int (3x^2 - 3x^3 + 1 - x) dx$

Integrate w.r.t 'x'.

$3\int x^2 \cdot dx - 3\int x^3 \cdot dx + \int 1 \cdot dx - \int x \cdot dx$

$$\frac{\cancel{3}x^3}{\cancel{3}} - \frac{3x^4}{4} + x - \frac{x^2}{2} + C$$

$$x^3 - \frac{3x^4}{4} + x - \frac{x^2}{2} + C$$

21) $\int (5\cos x + 4e^x + 3^x)\,dx$

$5\int \cos x\,dx + 4\int e^x\,dx + \int 3^x\,dx \rightarrow$ Integrate w.r.t 'x'.

$$5\sin x + 4e^x + \frac{3^x}{\log_e 3} + C$$

22) $\int \left(\frac{1}{x^2} + 3\sec x \tan x + \frac{3}{1+x^2}\right) dx$

Integrate w.r.t 'x'.

$$\int \frac{1}{x^2}\,dx + 3\int \sec x \tan x\,dx + 3\int \frac{1}{1+x^2}\,dx$$

$$\int x^{-2}\,dx + 3\int \sec x \cdot \tan x \cdot dx + 3\int \frac{1}{1+x^2}\,dx$$

$$\therefore \frac{x^{-2+1}}{-2+1} + 3\sec x + 3\tan^{-1} x + C$$

23) $\int \left(5\operatorname{cosec}^2 x - \frac{5}{\sqrt{1-x^2}} + 2\sin x\right) dx$

Integrate w.r.t 'x'.

$$5\int \operatorname{cosec}^2 x\,dx - 5\int \frac{1}{\sqrt{1-x^2}}\,dx + 2\int \sin x\,dx$$

$$-5\cot x - 5\sin^{-1} x - 2\cos x + C$$

24) $\int\left(\sqrt{x} - 3\operatorname{cosec}x.\cot x + \frac{5}{x\sqrt{x^2-1}}\right)dx$

Integrate w.r.t 'x'.

$\int\sqrt{x}.dx - 3\int\operatorname{cosec}x\cot x.dx + 5\int\frac{1}{x\sqrt{x^2-1}}.dx$

$\frac{2}{3}.x^{3/2} + 3\operatorname{cosec}x + 5\sec^{-1}x + C$

25) $\int$

INTEGRATION BY SUBSTITUTION METHOD:

1) $\int (ax+b)^n \cdot dx = \frac{(ax+b)^{n+1}}{n+1} \times \frac{1}{a} + c$

2) $\int e^{ax+b} \cdot dx = \frac{e^{ax+b}}{a} + c$

3) $\int a^{bx+c} \cdot dx = \frac{a^{bx+c}}{\log_e a} \times \frac{1}{b} + c$

4) $\int \frac{1}{(ax+b)} \cdot dx = \frac{\log (ax+b)}{a} + c$

5) $\int \sqrt{ax+b} \cdot dx = \frac{2}{3} (ax+b)^{3/2} + c \times \frac{1}{a}$

6) $\int \sin(ax+b) \cdot dx = -\frac{\cos (ax+b)}{a} + c$

7) $\int \cos (ax+b) \cdot dx = \frac{\sin (ax+b)}{a} + c$

8) $\int \sec^2(ax+b) \cdot dx = \frac{\tan (ax+b)}{a} + c$

9) $\int \operatorname{cosec}^2 (ax+b) \cdot dx = -\frac{\cot (ax+b)}{a} + c$

10) $\int \operatorname{cosec}(ax+b) \cdot \cot (ax+b) = -\frac{\operatorname{cosec} (ax+b)}{a} + c$

11) $\int \sec(ax+b) \cdot \tan (ax+b) = \frac{\sec (ax+b)}{a} + c$

1) $\int e^{3x} \cdot dx$

Integrate w.r.t 'x'.

$\frac{e^{3x}}{3} + c$

2) $\int 7^{5x} \cdot dx$

Integrate w.r.t 'x'.

$\frac{7^{5x}}{\log_e 7} \times \frac{1}{5} + c$

3) $\int (7x+9)^9 \cdot dx$

Integrate w.r.t 'x'.

$\frac{(7x+9)^{10}}{10} \times \frac{1}{7} + c$

$\frac{(7x+9)^{10}}{70} + c$

4) $\int \sin(1+3x) \cdot dx$

Integrate w.r.t 'x'

$-\frac{\cos(1+3x)}{3} + c$

5) $\int \cos(3x+7) \cdot dx$

Integrate w.r.t 'x'.

$\frac{\sin(3x+7)}{3} + c$

6) $\int \sin(1-9x) \cdot dx$

Integrate w.r.t

$-\frac{\cos(1-9x)}{-9} + c$

7) $\int \cos(3-4x) \cdot dx$

Integrate w.r.t 'x'.

$\frac{\sin(3-4x)}{-4} + c$

$-\frac{\sin(3-4x)}{4} + c$

8) $\int 11^{100x} \cdot dx$

Integrate w.r.t 'x'.

$\frac{11^{100x}}{\log_e 11} \times \frac{1}{100} + c$

$\frac{11^{100x}}{100 \log_e 11} + c$

9) $\int \frac{1}{2-3x} \cdot dx$

Integrate w.r.t 'x'.

$\frac{\log(2-3x)}{-3} + c$

10) $\int \frac{1}{(2x+5)^{10}} \cdot dx = \int (2x+5)^{-10} \cdot dx$

~~Soln~~

Integrate w.r.t 'x'.

$\frac{(2x+5)^{-9}}{-9} \times \frac{1}{2}$

$\therefore \frac{(2x+5)^{-9}}{-18} + c$

11) $\int \sec^2 2x \cdot dx$

Integrate w.r.t 'x'.

$\frac{\tan 2x}{2} + c$

12) $\int \mathrm{cosec}^2 9x \cdot dx$

Integrate w.r.t 'x'.

$-\frac{\cot 9x}{9} + c$

13) $\int \sec^2 (3-5x) \cdot dx$

Integrate w.r.t 'x'.

$\frac{\tan(3-5x)}{-5} + c$

14) $\int \mathrm{cosec}\, 4x \cdot \cot 4x \cdot dx$

Integrate w.r.t 'x'.

$-\frac{\mathrm{cosec}\, 4x}{4} + c$

15) $\int \operatorname{cosec}^2(9x+7)\,.dx$

Integrate w.r.t 'x'.

$$-\frac{\cot(9x+7)}{9}+c$$

16) $\int[\sec(2-3x).\tan(2-3x)].dx$

Int.. w.r.t 'x'.

$$\frac{\sec(2-3x)}{-3}+c$$

17) $\int[\operatorname{cosec}(1-2x).\cot(1-2x)]dx$

Integrate w.r.t 'x'.

$$\frac{\cancel{-}\operatorname{cosec}(1-2x)}{\cancel{-}2}+c$$

$$\frac{\operatorname{cosec}(1-2x)}{2}+c$$

18) $\int \frac{1}{3x+7}\,.dx$

Integrate w.r.t 'x'.

$$\frac{\log(3x+7)}{3}+c$$

19) $\int \sec^2(7-8x).dx$.

Integrate w.r.t 'x'

$$\frac{\tan(7-8x)}{-8}+c$$

20) $\int \operatorname{cosec}^2(2-11x).$

Integrate . wrt 'x'.

$$\frac{\cancel{-}\cot(2-11x)}{\cancel{-}11}+c$$

$$\frac{\cot(2-11x)}{11}+c$$

21) $\int (4-5x)^3 \cdot dx$

Integrate w.r.t 'x'.

$\frac{(4-5x)^4}{4} \times \frac{1}{-5}$

$-\frac{(4-5x)^4}{20} + C$

22) $\int \sqrt{3x+7} \cdot dx$

Integrate w.r.t 'x'.

$\frac{2}{3}(3x+7)^{3/2} \times \frac{1}{3}$

$\frac{2}{9}(3x+7)^{3/2} + C$

23) $\int \frac{1}{(5-2x)^{3/2}} \cdot dx$

Int $\int (5-2x)^{-3/2} \cdot dx$

Integrate w.r.t 'x'.

$\frac{(5-2x)^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} \times \frac{1}{-2}$

$\frac{(5-2x)^{-1/2}}{-1/2} \times \frac{1}{-2}$

$(5-2x)^{-1/2} \times \frac{1}{\cancel{-2}} \times \cancel{-2}$

$(5-2x)^{-\frac{1}{2}} + C$

24) $\int (3e^{-x} + 2^{3x} + e^{5x})dx$

Int. w.r.t 'x'.

$3\int e^{-x} dx + \int 2^{3x} dx + \int e^{5x} \cdot dx$

$\frac{3e^{-x}}{-1} + \frac{2^{3x}}{\log_e 2} \times \frac{1}{3} + \frac{e^{5x}}{5}$

$-3e^{-x} + \frac{2^{3x}}{3\log_e 2} + \frac{e^{5x}}{5} + C$

25) $\int (3\sin 2x + 2\cos 3x) \cdot dx$

Integrate w.r.t 'x'.

$3\int \sin 2x \, dx + 2\int 3\cos 3x \, dx$

$-\frac{3\cos 2x}{2} + \frac{2\sin 3x}{3} + C$

26) $\int (1+\sin 2x) \cdot dx$

Integrate w.r.t 'x'.

$\int 1 \cdot dx + \int \sin 2x \cdot dx$

$x - \frac{\cos 2x}{2}$

$\frac{2x - \cos 2x}{2}$

27) $\int\left(\sqrt[3]{x^4}+\sec^2 2x+\frac{3}{1+x^2}-\cos 2x+e^{-3x}\right).dx$

$\int\left(x^{4/3}+\sec^2 2x+\frac{3}{1+x^2}-\cos 2x+e^{-3x}\right).dx$

Integrate w.r.t 'x'.

$\int x^{4/3}.dx+\int \sec^2 2x.dx+\int\frac{3}{1+x^2}.dx-\int\cos 2x.dx$
$\int e^{-}$

$\frac{x^{\frac{4}{3}+1}}{\frac{4}{3}+1}+\frac{\tan 2x}{2}+3\tan^{-1}x-\frac{\sin 2x}{2}+\frac{e^{-3x}}{-3}$

$\frac{x^{7/3}}{7/3}+\frac{\tan 2x}{2}+3\tan^{-1}x-\frac{\sin 2x}{2}-\frac{e^{-3x}}{3}$

28) $\int \mathrm{cosec}^2(1-4x).dx$

Integrate w.r.t 'x'

$\frac{-\cot(1-4x)}{-4}+c$

$\frac{\cot(1-4x)}{4}+c$

29) $\int \sec^2 3\theta \cdot d\theta$

Integrate w.r.t 'θ'.

$\frac{\tan(3\theta)}{3}+c$

30) $\int\sqrt[3]{(1-2x)^4}.dx=\int(1-2x)^{4/3}$

Integrate w.r.t 'x'

$\frac{(1-2x)^{4/3+1}}{4/3+1}\times\frac{1}{-2}$

$\frac{(1-2x)^{7/3}}{7/3}\times\frac{1}{-2}+c$

30) $\int \frac{1}{(5-2x)^{3/2}} \cdot dx$, $\int e^{5-7x} \cdot dx$, $\int \sqrt{3x-2} \cdot dx$,

$\int \frac{1}{5-2x} \cdot dx$, $\int (\sqrt{x} + \frac{1}{x\sqrt{x^2-1}} + \frac{1}{2x+a} + e^{-2x} + 7)\, dx$,

$\int (e^{ax} - e^{-bx}) \cdot dx$

$\cdot dx$

$\int e^{-3}$

$\frac{e^{-3x}}{-3}$

$\frac{e^{-3x}}{-3} +$

$(5-2x)^{4/2}$

31) $\int \frac{1}{(5-2x)^{3/2}} \cdot dx$

$\int (5-2x)^{-3/2} \cdot dx$

Integrate wrt 'x'.

$$\frac{(5-2x)^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} \times \frac{1}{-2}$$

$$\frac{(5-2x)^{-\frac{1}{2}}}{\frac{-1}{\cancel{2}}} \times \frac{1}{-\cancel{2}}$$

$$(5-2x)^{-\frac{1}{2}}$$

32) $\int e^{5-7x} \cdot dx$

Integrate w.r.t 'x'.

$$\frac{e^{5-7x}}{-7}$$

$$-\frac{1}{7} e^{5-7x}$$

34) $\int \frac{1}{5-2x} \cdot dx$

Int. wrt 'x'

$$\frac{\log(5-2x)}{-2}$$

33) $\int \sqrt{3x-2} \cdot dx$

Integrate w.r.t 'x'

$$\frac{\cancel{2}}{3}(3x-2)^{3/2} \times \frac{-1}{\cancel{2}}$$

$$\frac{-1}{3}(3x-2)^{3/2}$$

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35) $\int\left(\sqrt{x} + \frac{1}{x\sqrt{x^2-1}} + \frac{1}{2x+9} + e^{-2x} + 7\right).dx$

Integrate wrt 'x'.

$\int \sqrt{x}.dx + \int \frac{1}{x\sqrt{x^2-1}}.dx + \int \frac{1}{(2x+9)}.dx + \int e^{-2x} + 7\int 1$

$\frac{2}{3}x^{3/2} + \sec^{-1}x + \frac{\log(2x+9)}{2} + \frac{e^{-2x}}{-2} + 7x$

$\frac{2}{3}x^{3/2} + \sec^{-1}x + \frac{\log(2x+9)}{2} - \frac{e^{-2x}}{2} + 7x$

36) $\int (e^{ax} - e^{-bx}).dx$

Integrate wrt 'x'.

$\int e^{ax} - \int e^{-bx}$

$\frac{e^{ax}}{a} - \frac{e^{-bx}}{(-b)}$

$\frac{e^{ax}}{a} + \frac{e^{-bx}}{b}$

INTEGRATION BY USING TRIGONOMETRIC STANDARD RESULTS:

Relations:

1) $\sin x = \dfrac{1}{\operatorname{cosec} x}$, $\operatorname{cosec} x = \dfrac{1}{\sin x}$

2) $\cos x = \dfrac{1}{\sec x}$, $\sec x = \dfrac{1}{\cos x}$

3) $\tan x = \dfrac{1}{\cot x}$ or $\dfrac{\sin x}{\cos x}$, $\cot x = \dfrac{1}{\tan x}$ or $\dfrac{\cos x}{\sin x}$

Identities:

1) $\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$\sin x = \sqrt{1-\cos^2 x}$

$\cos^2 x = 1 - \sin^2 x$

$\cos x = \sqrt{1-\sin^2 x}$

2) $1 + \tan^2 x = \sec^2 x$

$\sec x = \sqrt{1-\tan^2 x}$

$\tan^2 x = \sec^2 x - 1$

$\tan x = \sqrt{\sec^2 x - 1}$

$\tan^2 x - \sec^2 x = -1$

$\sec^2 x - \tan^2 x = 1$

3) $1 + \cot^2 x = \operatorname{cosec}^2 x$

$\cot^2 x = \operatorname{cosec}^2 x - 1$

$\cot x = \sqrt{\operatorname{cosec}^2 x - 1}$

$\cot^2 x + 1 = \operatorname{cosec}^2 x$

$\operatorname{cosec} x = \sqrt{1+\cot^2 x}$

$\operatorname{cosec}^2 x - \cot^2 x = 1$

Compound Angles:

1) $\sin(A+B) = \sin A \cdot \cos B + \cos A \sin B$

2) $\sin(A-B) = \sin A \cdot \cos B - \cos A \cdot \sin B$.

3) $\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$

4) $\cos(A-B) = \cos A \cdot \cos B + \sin A \cdot \sin B$

5) $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

6) $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$

Multiple Angles:

1) $\sin 2A = 2\sin A \cdot \cos A$

$= \dfrac{2\tan A}{1+\tan^2 A}$

2) $\cos 2A = \cos^2 A - \sin^2 A$

$= 2\cos^2 A - 1$

$= 1 - 2\sin^2 A$

$= \dfrac{1-\tan^2 A}{1+\tan^2 A}$

3) $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

4) $\sin 3A = 3\sin A - 4\sin^3 A$

5) $\cos 3A = 4\cos^3 A - 3\cos A$

* $\sin 3x = 3\sin x - 4\sin$
$4\sin^3 x = 3\sin x - \sin$
$\sin^3 x = \frac{1}{4}[3\sin x - \sin$

* $\cos 3x = 4\cos^3 x -$
$\cos 3x + 3\cos x = 4c$
$\cos^3 x = \frac{1}{4}[\cos 3x +$

6) $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

7) $\cos 2A = 2\cos^2 A - 1$

$1 + \cos 2A = 2\cos^2 A$

$\cos^2 A = \cancel{2\cos^2 A}\ \dfrac{1 + \cos 2A}{2}$

8) $\cos 2A = 1 - 2\sin^2 A$

$2\sin^2 A = 1 - \cos 2A$

$\sin^2 A = \dfrac{1 - \cos 2A}{2}$

Transformation Formulae:

1) $\sin A \cdot \cos B = \dfrac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$

2) $\cos A \cdot \sin B = \dfrac{1}{2}\left[\cancel{\cos} \sin(A+B) - \sin(A-B)\right]$

3) $\sin A \cdot \sin B = \dfrac{-1}{2}\left[\cos(A+B) - \cos(A-B)\right]$

4) $\cos A \cdot \cos B = \dfrac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$

Sum or Difference:

1) $\sin C + \sin D = 2\sin\left(\dfrac{C+D}{2}\right) \cdot \cos\left(\dfrac{C-D}{2}\right)$

2) $\sin C - \sin D = 2\cos\left(\dfrac{C+D}{2}\right) \cdot \sin\left(\dfrac{C-D}{2}\right)$

3) $\cos C + \cos D = 2\cos\left(\dfrac{C+D}{2}\right) \cdot \cos\left(\dfrac{C-D}{2}\right)$

4) $\cos C - \cos D = -2\sin\left(\dfrac{C+D}{2}\right) \cdot \sin\left(\dfrac{C-D}{2}\right)$

Problems:

1) $\int \sin x \cdot \sec^2 x \cdot dx$

$\int \frac{\sin x}{\cos^2 x} \cdot dx$

$\int \frac{\sin x}{\cos x} \times \frac{1}{\cos x} \cdot dx$

$\int \sec x \cdot \tan x \cdot dx$

Integrate w.r.t 'x'.

$\sec x + c$

2) $\int \cos x \cdot \operatorname{cosec}^2 x \cdot dx$

$\int \frac{\cos x}{\sin^2 x} \cdot dx$

$\int \frac{\cos x}{\sin x} \times \frac{1}{\sin x} \cdot dx$

$\int \cot x \cdot \operatorname{cosec} x \cdot dx$

Integrate w.r.t 'x'.

$-\operatorname{cosec} x + c$

3) $\int \tan^2 x \cdot dx$

$\int (\sec^2 x - 1) \cdot dx$

$\int \sec^2 x \cdot dx - \int 1 \cdot dx$

$\tan x - x + c$

Integrate w.r.t 'x'.

4) $\int \cot^2 x \, dx$

$\int (\operatorname{cosec}^2 x - 1) \, dx$

Integrate w.r.t 'x'.

$\int \operatorname{cosec}^2 x \cdot dx - \int 1 \cdot dx$

$-\cot x - x + c$

5) $\int (\tan x + \cot x)^2 \cdot dx$

$\int (\tan^2 x + \cot^2 x + 2\tan x \cdot$

$\int (\tan^2 x + \cot^2 x + 2) \, dx$

$\int \tan^2 x \cdot dx + \int \cot^2 x \cdot dx + 2\int 1 \cdot$

Integrate w.r.t. 'x'.

$\int (\sec^2 x - 1) \cdot dx + \int (\operatorname{cosec}^2 x - 1) \cdot$
$+ 2x$

$\int \sec^2 x \cdot dx - \int 1 \cdot dx + \int \operatorname{cosec}^2 x$
$- \int 1 \cdot dx + 2x$

$\tan x - x - \cot x - x$

$\tan x - \cancel{2x} + \cancel{2x} - \cot x$

$\tan x - \cot x + c$

6) $\int (\tan x - \cot x)^2 \cdot dx$

$\int (\tan^2 x + \cot^2 x - 2\tan x \cot x).dx$

Integrate w.r.t 'x'.

$\int \tan^2 x.dx + \int \cot^2 x.dx - 2\int 1.dx$

$\int (\sec^2 - 1)dx + \int (\mathrm{cosec}^2 x - 1).dx - 2x$

$\int \sec^2 x\, dx - \int 1.dx + \int \mathrm{cosec}^2 x - \int 1.dx - 2x$

$\tan x - x - \cot x - x - 2x$

$\tan x - \cot x - 4x + c$

7) $\int \frac{\cos^2 x}{1+\sin x} .dx$

$\int \frac{1-\sin^2 x}{1+\sin x} .dx$

$\int \frac{\cancel{(1+\sin x)}(1-\sin x)}{\cancel{(1+\sin x)}} .dx$

$\int (1-\sin x).dx$

Integrate wrt 'x'

$\int 1.dx - \int \sin x.dx$

$x + \cos x + c$

8) $\int \frac{\sin^2 x}{1-\cos x} .dx$

$\int \frac{1-\cos^2 x}{1-\cos x} .dx$

$\int \frac{\cancel{(1-\cos x)}(1+\cos x)}{\cancel{(1-\cos x)}} .dx$

$\int (1+\cos x).du$

Integrate wrt 'x'.

$\int 1.dx + \int \cos x.dx$

$x + \sin x + c$

9) $\int \frac{\cos^2 x}{1-\sin x} \cdot dx$

$\int \frac{1-\sin^2 x}{1-\sin x} \cdot dx$

$\int \frac{(1+\sin x)\cancel{(1-\sin x)}}{\cancel{(1-\sin x)}} \cdot dx$

$\int (1+\sin x) \cdot dx$

Int. wrt 'x'.

$\int 1 \cdot dx + \int \sin x \cdot dx$

$x - \cos x + C$

11) $\int \frac{1}{1+\sin t} \cdot dx$

$\int \frac{1}{1+\sin x} \times \frac{1-\sin x}{1-\sin x} \cdot dx$

$\int \frac{1-\sin x}{1-\sin^2 x} \cdot dx$

$\int \frac{1-\sin x}{\cos^2 x} \cdot dx$

$\int \frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x} \cdot dx$

$\int \sec^2 x - \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} \cdot dx$

$\int (\sec^2 x - \tan x \cdot \sec x) \cdot dx$

$\int \sec^2 x \cdot dx - \int \sec x \cdot \tan x \cdot dx$

$\tan x - \sec x + c \rightarrow$ Int. wrt 'x'.

10) $\int \frac{\sin^2 x}{1+\cos x} \cdot dx$

$\int \frac{1-\cos^2 x}{1+\cos x} \cdot dx$

$\int \frac{(1-\cos x)\cancel{(1+\cos x)}}{\cancel{(1+\cos x)}}$

$\int (1-\cos x) \cdot dx$

Integrate . wrt :

$\int 1 \cdot dx - \int \cos x \cdot dx$

$x - \sin x + C$

12) $\int \frac{1}{1-\cos x} \cdot dx$

$\int \frac{1}{1-\cos x} \times \frac{1}{1+}$

$\int \frac{1+\cos x}{1-\cos^2 x} \cdot dx$

$\int \frac{1+\cos x}{\sin^2 x} \cdot dx$.

$\int \frac{1}{\sin^2 x} + \frac{\cos x}{\sin x} \times \frac{1}{\sin x}$

$\int (\operatorname{cosec}^2 x + \operatorname{cosec} x \cdot co$

Int. wrt 'x'.

$\int \operatorname{cosec}^2 x \, dx + \int \operatorname{cosec} x \cdot \cot$

$-\cot x - \operatorname{cosec} x + C$

13) $\int \frac{1}{1-\sin x} \cdot dx$

$\int \frac{1}{1-\sin x} \times \frac{1+\sin x}{1+\sin x} \cdot dx$

$\int \frac{1+\sin x}{1-\sin^2 x} \cdot dx$

$\int \frac{1+\sin x}{\cos^2 x} \cdot dx$

$\int \left[\frac{1}{\cos^2 x} + \frac{\sin x}{\cos x} \times \frac{1}{\cos x}\right] \cdot dx$

$\int (\sec^2 x + \tan x \cdot \sec x) \cdot dx$

$\cancel{\int \tan x + \sec x}$

Integrate wrt 'x'.

$\cancel{\tan x}$.

$\int \sec^2 x \cdot dx + \int \tan x \cdot \sec x \, dx$

$\tan x + \sec x + c$

15) $\int \frac{1}{1+\cos 2x} \cdot dx$

$\int \frac{1}{2\cos^2 x} \cdot dx$

$\frac{1}{2} \int \frac{1}{\cos^2 x} \cdot dx$

$\frac{1}{2} \int \cancel{\tan} \sec^2 x \cdot dx$

Integrate wrt 'x'

$\frac{1}{2} \tan x + c$

14) $\int \frac{1}{1+\cos x} \cdot dx$

$\int \frac{1}{1+\cos x} \times \frac{1-\cos x}{1-\cos x} \cdot dx$

$\int \frac{1-\cos x}{1-\cos^2 x} \cdot dx$

$\int \frac{1-\cos x}{\sin^2 x} \cdot dx$

$\int \left[\frac{1}{\sin^2 x} - \frac{\cos x}{\sin x} \times \frac{1}{\sin x}\right]$

$\int (\mathrm{cosec}^2 x - \mathrm{cosec}\, x \cdot \cot x)$

Integrate wrt 'x'.

~~tant~~

$\int \mathrm{cosec}^2 x \cdot dx - \int \mathrm{cosec}\, x \cdot \cot x$

$-\cot x + \mathrm{cosec}\, x + c$

16) $\int \frac{1}{1-\cos 2x} \cdot dx$

$\int \frac{1}{2\sin^2 x} \cdot dx$

$\int \frac{1}{2} \mathrm{cosec}^2 x \cdot dx$

Integrate wrt 'x'.

$\frac{1}{2} \int \mathrm{cosec}^2 x \cdot dx$

$-\frac{1}{2} \cot x + c$

17) $\int \frac{\sin x}{1+\cos x} \cdot dx$

$\int \frac{\sin x}{1+\cos x} \times \frac{1-\cos x}{1-\cos x} \cdot dx$

$\int \frac{\sin x(1-\cos x)}{1-\cos^2 x} \cdot dx$

$\int \frac{\cancel{\sin x}(1-\cos x)}{\sin^2 x} \cdot dx$

$\int \frac{1-\cos x}{\sin x} \cdot dx$

$\int (\operatorname{cosec} x - \cot x) \cdot dx$

Integrate wrt 'x'.

$\int \operatorname{cosec} x \cdot dx - \int \cot \cdot dx$

$-\log(\operatorname{cosec} x + \cot x) - \log(\sin x) + C$

19) $\int \frac{\sin x}{1-\cos x} \cdot dx$

$\int \frac{\sin x}{1-\cos x} \times \frac{1+\cos x}{1+\cos x} \cdot dx$

$\int \frac{\sin x(1+\cos x)}{1-\cos^2 x} \cdot dx$

$\int \frac{\cancel{\sin x}(1+\cos x)}{\sin^2 x} \cdot dx$

$\int \frac{1+\cos x}{\sin x} \cdot dx$

$\int (\operatorname{cosec} x + \cot x)$

Integrate wrt 'x

$\int \operatorname{cosec} x \, dx + \int \cot$

$-\log(\operatorname{cosec} x + \cot x) +$
$\log(s$

18) $\int \frac{\cos}{1-\sin x} \cdot dx$

$\int \frac{\cos}{1-\sin x} + \frac{(1+\sin x)}{(1+\sin x)} \cdot dx$

$\int \frac{\cos(1+\sin x)}{1-\sin^2 x} \cdot dx$

$\int \frac{\cancel{\cos} x(1+\sin x)}{\cos^2 x} \cdot dx$

$\int (\sec x + \tan x) \cdot dx$

Integrate wrt 'x'.

$\int \sec x \, dx + \int \tan x \cdot dx$

$\log(\sec x + \tan x) + \log(s$
$+ C$

33) $\int \sin^3 x \cdot dx$

$$\left[\begin{aligned} \sin 3x &= \cancel{4\cos}\ 3\sin x - 4\sin^3 x \\ 4\sin^3 x &= 3\sin x - \sin 3x \\ \sin^3 x &= \frac{1}{4}[3\sin x - \sin 3x] \end{aligned}\right.$$

$\int \frac{1}{4}[3\sin x - \sin 3x] \cdot dx$

$\frac{1}{4}\int (3\sin x - \sin 3x) \cdot dx$

Integrate wrt 'x'.

$\frac{1}{4}\left[3\int \sin x \cdot dx - \int \sin 3x \cdot dx\right]$

$\frac{1}{4}\left(-3\cos x + \frac{\cos 3x}{3}\right) + c$

34) $\int \cos^3 x \cdot dx$

$$\left[\begin{aligned} \cos 3x &= 4\cos^3 x - 3\cos x \\ 4\cos^3 x &= \cos 3x + 3\cos x \\ \cos^3 x &= \frac{1}{4}(\cos 3x + 3\cos x) \end{aligned}\right.$$

$\int \frac{1}{4}[\cos 3x + 3\cos x] \cdot dx$

$\frac{1}{4}\int (\cos 3x + 3\cos x) \cdot dx$

Integrate wrt 'x'.

$\frac{1}{4}\left[\int \cos 3x \cdot dx + 3\int \cos x \cdot dx\right]$

$\frac{1}{4}\left(\frac{\sin 3x}{3} + 3\sin x\right) + C$

35) $\int \sin 5x \cos 2x \cdot dx$

$\int \frac{1}{2}[\sin 7x + \sin 3x] \cdot dx$ $\left[\sin A \cdot \cos B = \frac{1}{2}\sin(A+B) + \sin(A-B)\right.$

Integrate wrt 'x'

$\frac{1}{2}\left[\int \sin 7x \cdot dx + \int \sin 3x \cdot dx\right]$

$\frac{1}{2}\left[\frac{-\cos 7x}{7} - \frac{\cos 3x}{3}\right] + c$

3) $\int \cos 7x \cdot \sin 3x \cdot dx$

$\int \left[\frac{1}{2} \sin 10x - \sin 4x\right] \cdot dx$ $\left[\cos A \cdot \sin B = \frac{1}{2} \sin(A+B) - \cancel{\cos} \sin(A-B)\right]$

Integrate wrt 'x'.

$\frac{1}{2}\left[\int \sin 10x \cdot dx - \int \sin 4x \cdot dx\right]$

$\frac{1}{2}\left(\frac{-\cos 10x}{10} + \frac{\cos 4x}{4}\right) + c$

4) $\int \cos 8x \cdot \cos 3x \cdot dx$

$\int \left[\frac{1}{2} \cos 11x + \cos 5x\right] \cdot dx$ $\left[\cos A \cdot \cos B = \frac{1}{2} \cos(A+B) - \cos(A-B)\right]$

Integrate wrt 'x'.

$\frac{1}{2}\left[\int \cos 11x \cdot dx + \int \cos 5x \cdot dx\right]$

$\frac{1}{2}\left(\frac{\sin 11x}{11} + \cancel{\cos} \frac{\sin 5x}{5}\right) + c$

5) $\int \sin 9x \cdot \sin 2x \cdot dx$ $\left[\sin A \cdot \sin B = \frac{-1}{2} \cos(A+B) - \cos(A-B)\right]$

$\int \left[\frac{-1}{2}(\cos 11x - \cos 7x)\right] \cdot dx \longrightarrow$ Integrate wrt 'x'

$\frac{-1}{2}\left[\int \cos 11x \, dx - \int \cos 7x \cdot dx\right]$

$$\frac{-1}{2}\left[\frac{\sin 11x}{11} - \frac{\sin 7x}{7}\right] + c$$

Integration by Substitution Method Formulae:

1) $\int \tan x \cdot dx$

$\int \frac{\sin x}{\cos x} \cdot dx$

Put $t = \cos x$, then,

Diff^r wrt 'x'

$\frac{dt}{dx} = -\sin x$

$dt = -\sin x \cdot dx$

- Multiply B.S by $(-)$

$-dt = \sin x \cdot dx$

Now,

$\int \frac{1}{t}(-dt)$

$-\int \frac{1}{t} dt$

Int. wrt 'x'

$-\log t$

$-\log(\cos x)$

$\log(\cos)^{-1}$

$\log\left(\frac{1}{\cos}\right)$

$\log \sec x + c$

2) $\int \cot x \cdot dx$

$\int \frac{\cos x}{\sin x} \cdot dx$

Put $t = \sin x$

Diff^n wrt 'x'

$\frac{dt}{dx} = \cos x$

$dt = \cos x \cdot dx$

Now, $\int \frac{1}{t} \cdot dt$

Int wrt 'x'

$\log t$

$\log(\sin x) + c$

3) $\int \sec x \cdot dx$

$\int \sec x \frac{(\sec x + \tan x)}{(\sec x + \tan x)} \cdot dx$

$\int \left(\frac{\sec^2 x + \sec x \cdot \tan x}{\sec x + \tan x)}\right) \cdot dx$

Put $t = (\sec x + \tan x)$

& Diff^n wrt 'x'

$\frac{dt}{dx} = \frac{d(\sec x + \tan x)}{dx}$

$\frac{dt}{dx} = \sec x \tan x + \sec^2 x$

$dt = (\sec x \tan x + \sec^2 x) \cdot dx$

Now,

$\int \frac{1}{t} \cdot dt$

Int wrt 't'

$\log t$

$\log(\sec x + \tan x) + c$

4) $\int \operatorname{cosec} x \cdot dx$

$\int \operatorname{cosec} x \frac{(\operatorname{cosec} x + \cot x)}{(\operatorname{cosec} x + \cot x)} \cdot dx$

$\int \frac{\operatorname{cosec}^2 x + \operatorname{cosec} x \cot x}{(\operatorname{cosec} x + \cot x)} \cdot dx$

Put $t = (\operatorname{cosec} x + \cot x)$

$\frac{dt}{dx} = -\operatorname{cosec} x \cot x - \operatorname{cose}$

$\frac{dt}{dx} = -[\operatorname{cosec} x \cot x + \operatorname{cosec}$

$-dt = [\operatorname{cosec}^2 x + \operatorname{cosec}^2 x]$

$\int \frac{1}{t} \cdot -dt$

$-\int \frac{1}{t} \cdot dt$

Int wrt 'x'

$-\log t$

$-\log(\operatorname{cosec} x + \cot x)$

39) $\int \sin^6 x \cdot \cos x \cdot dx$

Put $t = \sin x$

Diff^n wrt 'x'

$\frac{dt}{dx} = \cos x$

$dt = \cos x \cdot dx$

Now,

$\int t^6 \cdot dt$

Int wrt 'x'.

$\frac{t^{6+1}}{6+1} = \frac{t^7}{7}$

$= \frac{\sin^7 x}{7} + c$

41) $\int \tan^3 x \cdot \sec^2 x \cdot dx$

Put $t = \tan x$.

Diff^n w.r.t 'x'

$\frac{dt}{dx} = \sec^2 x$

$dt = \sec^2 x \cdot dx$

Now,

$\int t^3 \cdot dt$

Int. w.r.t 't'

$\frac{t^4}{4} = \frac{\tan^4 x}{4} + c$

40) $\int \cos^4 x \cdot \sin x \cdot dx$

Put $t = \cos x$

Diff^n wrt 'x'.

$\frac{dt}{dx} = -\sin x$

$dt = -\sin x \cdot dx$

$-dt = \sin x \cdot dx$

Now, $\int t^4 \times -dt$

$-\int t^4 \cdot dt$

Int wrt 'x'

$-\frac{t^5}{5} = -\frac{\cos^5 x}{5}$

42) $\int \sqrt{\cot x} \cdot \operatorname{cosec}^2 x \cdot dx$

Put $t = \cot x$

Diff^n w.r.t 'x'

$\frac{dt}{dx} = -\operatorname{cosec}^2 x$

$-dt = \operatorname{cosec}^2 x \cdot dx$

Now, $\int \sqrt{t} \times -dt$

$-\int \sqrt{t} \cdot dt$

$-\frac{2}{3} t^{3/2}$

$-\frac{2}{3}(\cot)^{3/2} + c$

43) $\int (x^2+7)^9 \cdot 2x \cdot dx$

Put $t = (x^2+7)$.

Diff^n w.r.t 'x'.

$\frac{dt}{dx} = 2x + 0$

$\frac{dt}{2} = x \cdot dx$

Now,

$\int t^9 \cdot \frac{dt}{2}$

$\frac{1}{2} \int t^9 \cdot dt$

Int. w.r.t 't'

$\frac{t^{10}}{10} \times \frac{1}{2}$

$\frac{(x^2+7)^{10}}{10} \times \frac{1}{2}$

$\frac{(x^2+7)^{10}}{20} + c$

44) $\int (x^2+2x+6)^7 (x+1) \cdot dx$

Put $t = (x^2+2x+6)$.

Diff^n w.r.t 'x'

$\frac{dt}{dx} = 2x + 2$

$\frac{dt}{dx} = 2(x+1)$

$\frac{dt}{2} = (x+1) \cdot dx$

Now,

$\int t^7 \cdot \frac{dt}{2}$

$\frac{1}{2} \int t^7 \cdot dt$

Int. wrt 't'

$\frac{1}{2} \times \frac{t^8}{8}$

$\frac{t^8}{16} = \frac{(x^2+2x+6)^8}{16} + c$

45) $\int \frac{(\tan^{-1} x)^3}{1+x^2} \cdot dx$

$\int (\tan^{-1} x)^3 \cdot \frac{1}{1+x^2} \cdot dx$

Put $t = \tan^{-1} x$

Diff^n w.r.t 'x'.

$\frac{dt}{dx} = \frac{1}{1+x^2}$

$dt = \frac{1}{1+x^2} \cdot dx$

$\int t^3 \cdot dt$

Int. wrt 't'

$\frac{t^4}{4} = \frac{(\tan^{-1} x)^4}{4} + c$

46) $\int \sqrt{\frac{\sin^{-1}x}{1-x^2}} \cdot dx$

$\int \frac{\sqrt{\sin^{-1}x}}{\sqrt{1-x^2}} \cdot dx$

$\int \sqrt{\sin^{-1}x} \cdot \frac{1}{\sqrt{1-x^2}} \cdot dx$

Put $t = \sin^{-1}x$

Diff^n w.r.t 'x'

$\frac{dt}{dx} = \frac{1}{\sqrt{1-x^2}}$

$dt = \frac{1}{\sqrt{1-x^2}} \cdot dx$

Now,

$\int \sqrt{t} \cdot dt$

$\frac{2}{3} t^{3/2}$

$\frac{2}{3} (\sin^{-1}x)^{3/2} + C$

48) $\int \frac{1}{x \cdot \log x} \cdot dx$

$\int \frac{1}{\log x} \cdot \frac{1}{x} \cdot dx$

Put $t = \log x$.

$\cancel{\int \frac{1}{t} \cdot}$ Diff^n w.r.t 'x'

$\frac{dt}{dx} = \frac{1}{x}$

$dt = \frac{1}{x} \cdot dx$

Now,

$\int \frac{1}{t} \cdot dt$

Int. w.r.t 't'

$\log t = \log(\log x$

47) $\int \frac{\log x}{x} \cdot dx$

$\int \log x \cdot \frac{1}{x} \cdot dx$

Put $t = \log x$.

Diff^n w.r.t 'x'

$\frac{dt}{dx} = \frac{1}{x}$

$dt = \frac{1}{x} \cdot dx$

Now,

$\int t \cdot dt$

Int. w.r.t 't'

$\frac{t^2}{2} = \frac{(\log x)^2}{2} +$

49) $\int \sec^5 x \cdot \tan x \cdot dx$

$\int \sec^4 x \cdot \sec x \cdot \tan x \cdot dx$

Put $t = \sec x$

Diff^n w.r.t 'x'

$\frac{dt}{dx} = \sec x \cdot \tan x$

$dt = (\sec x \cdot \tan x) \cdot dx$

Now,

$\int t^4 \cdot dt$

Int. w.r.t 't'

$\frac{t^5}{5} = \frac{(\sec x)^5}{5} + c$

50) $\int \operatorname{cosec}^3 x \cdot \cot x \cdot dx$

$\int \operatorname{cosec}^2 x \cdot \operatorname{cosec} x \cot x \cdot dx$

Put $t = \operatorname{cosec} x$.

Diff^n w.r.t 'x'

$\frac{dt}{dx} = -\operatorname{cosec} x \cot x$

$-dt = \operatorname{cosec} x \cdot \cot x \cdot dx$

Now, $-\int t^2 \cdot dt$

Int. w.r.t 't'.

$-\frac{t^3}{3} + c$

$-\frac{\operatorname{cosec}^3 x}{3} + c$

51) $\int \frac{x+3}{\sqrt{x^2+6x+7}} \cdot dx$

Put $t = x^2 + 6x + 7$

Diff^n w.r.t 'x'

$\frac{dt}{dx} = 2x + 6 + 0$

$\frac{dt}{dx} = 2(x+3)$

$\frac{dt}{2} = (x+3) \cdot dx$

Now,

$\int \frac{1}{\sqrt{t}} \cdot \frac{dt}{2}$

$\frac{1}{2} \int t^{-1/2} \cdot dt$

Int. w.r.t 't'

$\frac{1}{2} \frac{t^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}$

$\frac{1}{\cancel{2}} \times \frac{t^{1/2}}{1/\cancel{2}}$

$(x^2+6x+7)^{1/2} + c$

52) $\int \frac{(5+7\log x)^{10}}{x} \cdot dx$

Put $t = 5+7\log x$

Diff^n w.r.t 'x'

$\frac{dt}{dx} = 0 + \frac{7}{x}$

$\frac{dt}{7} = \frac{1}{x} \cdot dx$

Now, $\int t^{10} \cdot \frac{dt}{7}$

$\frac{1}{7} \int t^{10} \cdot dt$

$\frac{1}{7} \times \frac{t^{11}}{11}$

$\frac{1}{7} \times \frac{(5+7\log x)^{11}}{11}$

$\frac{(5+7\log x)^{11}}{77} + C$

54) $\int \frac{\cos x}{a+b\sin x} \cdot dx$

Put $t = (a+b\sin x)$

Diff^n wrt 'x'

$\frac{dt}{dx} = 0 + b\cos x$

$\frac{dt}{dx} = +b\cos x$

$\frac{dt}{b} = \cos x \cdot dx$

53) $\int \frac{\sec^2 x}{(2+3\tan x)^{10}} \cdot dx$

$\int \frac{1}{(2+3\tan x)^{10}} \cdot \sec^2$

Put $t = (2+3\tan$

diff^n w.r.t 'x'

$\frac{dt}{dx} = 0 + 3 \times$

$\frac{dt}{3} = \sec^2 x \cdot d$

Now, $\int \frac{1}{t^{10}} \cdot \frac{dt}{3}$

$\frac{1}{3} \int \frac{1}{t^{10}} \cdot dt$

$\frac{1}{3} \times \frac{t^{-10+1}}{-10+1}$

$\frac{1}{3} \times \frac{t^{-9}}{-9}$

$\frac{1}{3} \times \frac{(2+3\tan}{-9}$

$-\frac{(2+3\tan x)}{27}$

Now, $\int \frac{1}{t} \times \frac{dt}{b}$

$\frac{1}{b} \int \frac{1}{t} \cdot dt$

$\frac{1}{b} \times \log t$

$\frac{1}{b} \log(a+b\sin x) +$

$\frac{\log(a+b\sin x)}{b} + C$

55) $\int e^{\tan^{-1}x} \times \frac{1}{1+x^2} \cdot dx$

Put $t = \tan^{-1}x$

Diff^n. w.r.t 'x'

$\frac{dt}{dx} = \frac{1}{1+x^2}$

$dt = \frac{1}{1+x^2} \cdot dx$

Now,

$\int e^t \cdot dt$

Int w.r.t 't'

$e^t = e^{\tan^{-1}x} + C$

57) $\int e^x \cdot \tan(e^x) \cdot dx$

$\int \tan(e^x) \cdot e^x \cdot dx$

Put $t = e^x$

Diff^n w.r.t 'x'

$\frac{dt}{dx} = e^x$

$dt = e^x \cdot dx$

Now, $\int \tan(t) \cdot dt$

Int w.r.t 't'

$\log(\sec t) + C$

$\log(\sec e^x) + C$

56) $\int e^{\tan x} \cdot \sec^2 x \cdot dx$

Put $t = \tan x$

Diff^n w.r.t 'x'

$\frac{dt}{dx} = \sec^2 x$.

$dt = \sec^2 x \cdot dx$

Now,

$\int e^t \cdot dt$

Int w.r.t 't'

$e^t = e^{\tan x} + c$

58) $\int \tan\sqrt{x} \cdot \frac{1}{\sqrt{x}} \cdot dx$

Put $t = \sqrt{x}$.

Diff^n w.r.t 'x'

$\frac{dt}{dx} = \frac{1}{2\sqrt{x}}$

$2 \cdot dt = \frac{1}{\sqrt{x}} \cdot dx$

Now, $\int \tan(t) \cdot 2dt$

$2\int \tan t \cdot dt$

$2\log(\sec t)$.

$2\log(\sec\sqrt{x}) + c$

63) $\int e^{x^2} \cdot x \cdot dx$

Put $t = x^2$

Diff^n w.r.t 'x'.

$\frac{dt}{dx} = 2x$

$\frac{dt}{2} = x \cdot dx$

Now,

$\int e^t \cdot \frac{dt}{2}$

$\frac{1}{2} \int e^t \cdot dt$

$\frac{1}{2} e^t$

$\frac{1}{2} e^{x^2} + c$

64) $\int \frac{1}{\cos^2 x (4 + \tan x)} \cdot dx$

$\int \frac{\sec^2 x}{(4 + \tan x)} \cdot dx$

Put $\cdot t = (4 + \tan x)$

Diff^n w.r.t 'x'

$\frac{dt}{dx} = \sec^2 x$

$dt = \sec^2 x \cdot dx$

Now, $\int \frac{1}{t^4} \cdot dt$

$\int t^{-4} \cdot dt$

Int w.r.t 't'.

$\frac{t^{-3}}{-3} = \frac{(4 + \tan x)^{-3}}{-3} + c$

65) $\int \sqrt{a + b\cot x} \cdot \operatorname{cosec}^2 x \cdot dx$

Put $t = (a + b\cot x)$

Diff^n w.r.t 'x'

$\frac{dt}{dx} = -b \operatorname{cosec}^2 x$

$-\frac{dt}{b} = \operatorname{cosec}^2 x \cdot dx$

Now, $-\int \sqrt{t} \cdot \frac{dt}{b}$

$-\frac{1}{b} \int \sqrt{t} \cdot dt$

$\frac{-1}{b} \times \frac{2}{3} t^{3/2}$

$\frac{-2}{3b} (a + b\cot x)^{3/2} + c$

66) $\int (2 - 5\cos x)^7 \cdot \sin x \cdot dx$

Put $t = (2 - 5\cos x)$

Diff^n w.r.t 'x'

$\frac{dt}{dx} = 5\sin x$

$\frac{dt}{5} = \sin x \cdot dx$

Now, $\int t^7 \cdot \frac{dt}{5}$

$\frac{1}{5} \int t^7 \cdot dt$

$\frac{1}{5} \times \frac{t^8}{8}$

$\frac{t^8}{40} = \frac{(2 - 5\cos x)^8}{40} + c$

71) $\int \frac{2x+7}{x^2+7x+5} \cdot dx$

Put $t = x^2+7x+5$

Diff^n w.r.t 'x'.

$\frac{dt}{dx} = 2x+7$

$dt = (2x+7) \cdot dx$

Now, $\int \frac{1}{t} \cdot dt$

$\log t$

$\log(x^2+7x+5)+c$

72) $\int \sqrt{x^2-4x+7}\,(x-2) \cdot dx$

Put $t = x^2-4x+7$

Diff^n w.r.t 'x'.

$\frac{dt}{dx} = 2x-4$

$\frac{dt}{dx} = 2(x-2)$

$\frac{dt}{2} = (x-2) \cdot dx$

Now, $\int \sqrt{t} \cdot \frac{dt}{2}$.

Int w.r.t 't'.

$\frac{1}{2} \times \frac{2}{3} t^{3/2}$

$\frac{\cancel{2}}{\cancel{6}} (x^2-4x+7)^{3/2}+c$

$\frac{1}{3} (x^2-4x+7)^{3/2}+c$

$\int \frac{1-\tan x}{1+\tan x} \cdot dx$

$\int \frac{1 - \frac{\sin x}{\cos x}}{1+\frac{\sin x}{\cos x}} \cdot dx$

$\int \frac{\frac{(\cos x - \sin x)}{\cos x}}{\frac{(\cos x + \sin x)}{\cos x}} \cdot dx$

$\int \frac{\cos x - \sin x}{\sin x + \cos x} \cdot dx$

Put $t = \sin x + \cos x$

$\frac{dt}{dx} = \cos x - \sin x$

$dt = (\cos x - \sin x) \cdot dx$

$\int \frac{1}{t} \cdot dt$

$\log t$

$\log(\sin x + \cos x)+c$

H/W.

74) $\int \frac{e^x}{1+e^x} \cdot dx$

$\int e^x \times \frac{1}{1+e^x} \cdot dx$

Put $t = 1+e^x$

Diff^n w.r.t 'x'

$\frac{dt}{dx} = e^x$

$dt = e^x . dx$

Now, $\int \frac{1}{t} \cdot dt$

Int w.r.t 't'

$\log t = \log(1+e^x) + c$

75) $\int \frac{\operatorname{cosec}^2 (\tan^{-1} x)}{1+x^2}$

Put $t = \tan^{-1} x$

Diff^n . w.r.t 'x'

$\frac{dt}{dx} = \frac{1}{1+x^2}$

$dt = \frac{1}{1+x^2}$

Now, $\int \operatorname{cosec}^2 t$

$-\cot(t)$

$-\cot(\tan^{-1} x$

76) $\int (1+m\tan x)^5 . \sec^2 x . dx$

Put $t = (1+m\tan x)$

Diff^n w.r.t 'x'

$\frac{dt}{dx} = m\sec^2 x .$

$\frac{dt}{m} = \sec^2 x . dx$

Now, $\int t^5 . \frac{dt}{m}$

$\frac{1}{m}\int t^5 . dt$

$\frac{1}{m} \frac{t^6}{6}$

$\frac{1}{m} \times \frac{(1+m\tan x}{6}$

$\frac{(1+m\tan x)^6}{6m} + c$

INTEGRATION STANDARD RESU

$$\int \frac{1}{a^2+x^2}\cdot dx = \frac{1}{a}\tan^{-1}(x/a) + c$$

$$\int \frac{1}{x^2-a^2}\cdot dx = \frac{1}{2a}\log\left(\frac{x-a}{x+a}\right) + c$$

$$\int \frac{1}{a^2-x^2}\cdot dx = \frac{1}{2a}\log\left(\frac{a+x}{a-x}\right) + c$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\cdot dx = \sin^{-1}(x/a) + c$$

$$\int \frac{1}{x\sqrt{x^2-a^2}}\cdot dx = \frac{1}{a}\sec^{-1}(x/a) + c$$

Proofs:

$$\int \frac{1}{a^2+x^2}\cdot dx = \frac{1}{a}\tan^{-1}(x/a) + c$$

Consider LHS,

$$\int \frac{1}{a^2+x^2}\cdot dx$$

Put $x = a\tan\theta$

Diff^n wrt 'θ'

$$\frac{dx}{d\theta} = a\sec^2\theta$$

$dx = a\sec^2\theta \cdot d\theta$

$\int \frac{1}{a^2+x^2} \cdot dx$

$\int \frac{1}{a^2+a^2\tan^2\theta} \cdot a\sec^2\theta \cdot d\theta$

$\int \frac{1}{a^{\cancel{2}}(1+\tan^2\theta)} \cdot \cancel{a}\sec^2\theta \cdot d\theta$

$\frac{1}{a}\int \frac{\sec^2\theta}{\sec^2\theta} \cdot d\theta$

$\frac{1}{a}\int 1 \cdot d\theta$

Int wrt 'θ'.

$\frac{1}{a}\theta$.

$\frac{1}{a}\tan^{-1}(x/a) + c$ $\left[\begin{array}{l} x = a\tan\theta \\ \frac{x}{a} = \tan\theta \\ \tan^{-1}(x/a) = \theta \end{array}\right]$

$\int \frac{1}{\sqrt{a^2-x^2}} \cdot dx = \sin^{-1}(x/a) + c$

Consider LHS,

$\int \frac{1}{\sqrt{a^2-x^2}} \cdot dx$

Put $x = a\sin\theta$

Diffⁿ wrt 'θ'

$\frac{dx}{d\theta} = a\cos\theta$

$dx = a\cos\theta \cdot d\theta$

Now, $\int \frac{1}{\sqrt{a^2 - a^2\sin^2\theta}} \times a\cos\theta \cdot d\theta$

$\int \frac{1}{\sqrt{a^2(1-\sin^2\theta)}} \times a\cos\theta\, d\theta$

$\int \frac{1}{\cancel{a}\sqrt{1-\sin^2\theta}} \times \cancel{a}\cos\theta \cdot d\theta$

$\int \frac{\cos\theta}{\cos\theta} \cdot d\theta$

$\int 1 \cdot d\theta$

Int wrt 'θ'

θ

$\sin^{-1}(x/a) + c$

$$\left[\begin{array}{l} x = a\sin\theta \\ \frac{x}{a} = \sin\theta \\ \sin^{-1}(x/a) = \theta \end{array}\right.$$

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problems:

1) $\int \frac{1}{4+x^2} \cdot dx$

$\int \frac{1}{2^2+x^2} \cdot dx$

Int wrt 'x'

$\frac{1}{2} \tan^{-1}(x/2) + c$

2) $\int \frac{1}{3+x^2} \cdot dx$

$\int \frac{1}{(\sqrt{3})^2+x^2} \cdot dx$

Int wrt 'x'.

$\frac{1}{\sqrt{3}} \cdot \tan^{-1}(x/\sqrt{3}) + c$

3) $\int \frac{1}{9+25x^2} \cdot dx$

$\int \frac{1}{25(9/25+x^2)} \cdot dx$

$\frac{1}{25} \int \frac{1}{\frac{9}{25}+x^2} \cdot dx$

$\frac{1}{25} \int \frac{1}{(3/5)^2+x^2} \cdot dx$

Int. wrt 'x'

$\frac{1}{25} \frac{1}{3/5} \tan^{-1}(x/3/5)$

$\frac{1}{\cancel{25}_5} \times \frac{\cancel{5}}{3} \tan^{-1}\left(\frac{5x}{3}\right)$

$\frac{1}{15} \tan^{-1}(5x/3) + c$

4) $\int \frac{1}{x^2-49} \cdot dx$

$\int \frac{1}{x^2-7^2} \cdot dx$

Int. wrt 'x'.

$\frac{1}{2 \times 7} \log\left(\frac{x-7}{x+7}\right)$

$\frac{1}{14} \log\left(\frac{x-7}{x+7}\right) + c$

5) $\int \frac{1}{x^2-5} \cdot dx$

$\int \frac{1}{x^2-(\sqrt{5})^2} \cdot dx$

Int wrt 'x'.

$\frac{1}{2\sqrt{5}} \log\left(\frac{x-\sqrt{5}}{x+\sqrt{5}}\right) + c$

6) $\int \frac{1}{144x^2 - 169} \cdot dx$

$\frac{1}{144} \int \frac{1}{x^2 - \frac{169}{144}} \cdot dx$

$\frac{1}{144} \int \frac{1}{x^2\ (13/12)^2} \cdot dx$

Int w.r.t 'x'.

$\cancel{\frac{1}{144} \times \frac{1}{2 \times \frac{13}{12}} \log \left[\frac{x - \frac{13}{12}}{x + 13/12} \right]}$

$\frac{1}{\cancel{144}_{12}} \times \frac{1}{2 \times \frac{13}{\cancel{12}}} \log \left(\frac{x - 13/12}{x + 13/12} \right)$

$\frac{1}{312} \log \left(\frac{\frac{12x - 13}{\cancel{12}}}{\frac{12x + 13}{\cancel{12}}} \right)$

$\frac{1}{312} \log \left(\frac{12x - 13}{12x + 13} \right) + c$

7) $\int \frac{1}{225 - x^2} \cdot dx$

$\int \frac{1}{(15)^2 - x^2} \cdot dx$

Int wrt 'x'

$\frac{1}{2 \times 15} \log \left(\frac{15 + x}{15 - x} \right)$

$\frac{1}{30} \log \left(\frac{15 + x}{15 - x} \right) + c$

8) $\int \frac{1}{11 - x^2} \cdot dx$

$\int \frac{1}{(\sqrt{11})^2 - x^2} \cdot dx$

Int wrt 'x'

$\frac{1}{2 \times \sqrt{11}} \log \left(\frac{\sqrt{11} + x}{\sqrt{11} - x} \right)$

9) $\int \frac{1}{81-16x^2} \cdot dx$

$\frac{1}{16}\int \frac{1}{\frac{81}{16}-x^2} \cdot dx$

$\frac{1}{16}\int \frac{1}{(9/4)^2-x^2} \cdot dx$

Int wrt 'x'

$\frac{1}{\cancel{16}_{8}} \times \frac{1}{\cancel{2}\times 9/\cancel{4}} \log\left(\frac{9/4+x}{9/4-x}\right)$

$\frac{1}{72} \log\left(\frac{\frac{9+4x}{\cancel{4}}}{\frac{9-4x}{\cancel{4}}}\right)$

$\frac{1}{72} \log\left(\frac{9+4x}{9-4x}\right) + c$

11) $\int \frac{1}{\sqrt{81-25x^2}} \cdot dx$

$\int \frac{1}{\sqrt{25\left(\frac{81}{25}-x^2\right)}} \cdot dx$

$\frac{1}{5}\int \frac{1}{(9/5)^2-x^2} \cdot dx$

$\frac{1}{5} \sin^{-1}\left(\frac{5x}{9}\right) + c$

10) $\int \frac{1}{\sqrt{64-x^2}} \cdot dx$

$\int \frac{1}{\sqrt{8^2-x^2}} \cdot dx$

Int wrt 'x'.

$\sin^{-1}(x/8) + c$

12) $\int \frac{1}{\sqrt{11-x^2}} \cdot dx$

$\int \frac{1}{\sqrt{(\sqrt{11})^2-x^2}} \cdot dx$

Int wrt 'x'.

$\sin^{-1}(x/\sqrt{11}) + c$

13) $\int \frac{1}{x\sqrt{x^2-121}} \cdot dx$

$\int \frac{1}{x\sqrt{x^2-11^2}} \cdot dx$

Int° wrt 'x'

$\frac{1}{a} \sec^{-1}(x/11) + c$

14) $\int \frac{1}{x\sqrt{x^2-13}} \cdot dx$

$\int \frac{1}{x\sqrt{x^2-(\sqrt{13})^2}} \cdot dx$

$\int \frac{1}{x\sqrt{x^2-(\sqrt{13})^2}} \cdot dx$

Int. wrt 'x'.

$\frac{1}{\sqrt{13}} \sec^{-1}(x/\sqrt{13}) + c$

16) $\int \frac{\sec^2 x}{289 - \tan^2 x} \cdot dx$

Put $t = \tan x$

Diff^ wrt 'x'.

$\frac{dt}{dx} = \sec^2 x$.

$dt = \sec^2 x \cdot dx$

15) $\int \frac{x}{4 + x^4} \cdot dx$

$\int \frac{x}{2^2 + (x^2)^2} \cdot dx$

Put $t = x^2$

Diff^ wrt 'x'.

$\frac{dt}{dx} = 2x$

$\frac{dt}{2} = x \cdot dx$

Now, $\int \frac{1}{2^2 + t^2} \cdot \frac{dt}{2}$

$\frac{1}{2} \int \frac{1}{2^2 + t^2} \cdot dt$

Int wrt 't'.

$\frac{1}{2} \times \frac{1}{2} \tan^{-1}(t/2$

$\frac{1}{4} \tan^{-1}(x^2/2) + c$

Now, $\int \frac{1}{17^2 - t^2} \cdot dt$

$\frac{1}{2 \times 17} \log\left(\frac{17+t}{17-t}\right)$

$\frac{1}{34} \log\left(\frac{17+\tan x}{17-\tan x}\right) +$

17) $\int \frac{\sin x}{\cos^2 x - 9} \cdot dx$

$\int \frac{1}{\cos^2 x - 9} \cdot \sin x \cdot dx$

Put $t = \cos x$

Diff^n w.r.t 'x'

$\frac{dt}{dx} = -\sin x$

$-dt = \sin x \cdot dx$

Now $-\int \frac{1}{t^2 - (3)^2} \cdot dt$

Int w.r.t 't'.

$-\frac{1}{2\times 3} \log\left(\frac{t-3}{t+3}\right)$

$\frac{1}{-6} \log\left(\frac{\cos x - 3}{\cos x + 3}\right) + c$

18) $\int \frac{\cos x}{\sqrt{16 - \sin^2 x}} \cdot dx$

$\int \frac{1 \cancel{\cos}}{\sqrt{4 - \sin^2 x}} \times \cos x \, dx.$

Put $t = \sin x$

Diff^n w.r.t 'x'.

$\frac{dt}{dx} = \cos x$

$dt = \cos x \cdot dx$

Now, $\int \frac{1}{\sqrt{(4)^2 - t^2}} \cdot dt$

Int w.r.t 't'.

$\sin^{-1}\left(t/4\right)$

$\sin^{-1}\left(\frac{\sin x}{4}\right) + c$

19) $\int \frac{1}{x\left[36 + (\log x)^2\right]} \cdot dx$

$\int \frac{1}{36 + (\log x)^2} \times \frac{1}{x} \cdot dx$

Put $t = \log x$

Diff^n w.r.t 'x'.

$\frac{dt}{dx} = \frac{1}{x}$

$dt = \frac{1}{x} \cdot dx$

Now, $\int \frac{1}{6^2 + t^2} \, dt$

Int w.r.t 't'.

$\frac{1}{6} \tan^{-1}\left(t/6\right)$

$\frac{1}{6} \tan^{-1}\left(\frac{\log x}{6}\right) + c$

INTEGRATION BY PARTS

If u & v are the function of 'x'.
Then;

$$\int uv\,dx = u\int v\,dx - \int\left(\int v\,dx\right)\frac{du}{dx}\,dx$$

I – Inverse Trigonometric
L – Logarithmic
A – Algebraic
T – Trigonometric
E – Exponential.

Problems:

1) $\int x \cdot e^x \cdot dx$

Sol:

Formula,

$$\int uv\,dx = u\int v\,dx - \int\int v\,dx \cdot \frac{du}{dx}\,dx$$

Now,

$$\int x\cdot e^x\,dx = x\int e^x\,dx - \int\int e^x\,dx \cdot \frac{dx}{dx}\cdot dx$$

$$\int x\cdot e^x\,dx = xe^x - \int e^x \cdot 1\cdot dx$$

$$\int x\cdot e^x\,dx = xe^x - \int e^x\,dx$$

$$\int x\cdot e^x\,dx = xe^x - e^x$$

$$\int x\cdot e^x\,dx = e^x(x-1)+c$$

2) $\int x \cdot \sin x \cdot dx$

Formula,

$$\int u.v\,dx = u\int v.dx - \iint v.dx \cdot \frac{du}{dx}.dx$$

Now,

$$\int x.\sin x.dx = x\int \sin x.dx - \iint \sin x.dx \cdot \frac{d(x)}{dx}.$$

$$\int x.\sin x.dx = -x\cos x + \int \cos x.1.dx$$

$$\int x.\sin x.dx = -x\cos x + \int \cos x.dx$$

$$\int x.\sin x.dx = -x\cos x + \sin x + c$$

3) $\int \log x.dx = \int \log x \cdot 1 \cdot dx$

Formula,

$$\int u v\,dx = u\int v.dx - \iint v.dx \cdot \frac{du}{dx}.dx$$

Now,

$$\int \log x.1.dx = \log x\int 1.dx - \iint 1.dx \cdot \frac{d(\log x)}{dx}$$

$$\int \log x.1.dx = \log x.x - \int \cancel{x} \cdot \frac{1}{\cancel{x}}.dx$$

$$\int \log x.dx = x\log x - \int 1.dx$$

$$\int \log x.dx = x\log x - x$$

$$\int \log x.dx = x(\log x - 1) + c$$

4) $\int x \cdot \log x \cdot dx = \int \log x \cdot x \cdot dx$

Formula,

$$\int UV \cdot dx = U\int V \cdot dx - \int\int V \cdot dx \cdot \frac{dU}{dx} \cdot dx$$

Now,

$$\int x \cdot \log x \cdot dx = \log x \int x \cdot dx - \int\int x \cdot dx \, \frac{d(\log x)}{dx} \cdot dx$$

$$\int \log x \cdot x \cdot dx = \log x \cdot \frac{x^2}{2} - \int \frac{x^{\cancel{2}}}{2} \times \frac{1}{\cancel{x}} \cdot dx$$

$$\int \log x \cdot x \cdot dx = \frac{x^2}{2}\log x - \frac{1}{2}\int x \cdot dx$$

$$\int \log x \cdot x \cdot dx = \frac{x^2}{2}\log x - \frac{1}{2} \times \frac{x^2}{2}$$

$$\int \log x \cdot x \cdot dx = \frac{x^2}{2}\log x - \frac{x^2}{4}$$

$$\int \log x \cdot x dx = \frac{x^2}{2}\left[\log x - \frac{x^2}{2}\right] + c$$

5) $\int x \cdot \cos x \cdot dx$

$$\int UV \cdot dx = U\int V \cdot dx - \int\int V \cdot dx \frac{dU}{dx} \cdot dx$$

$$\int x \cdot \cos x \cdot dx = x\int \cos x \cdot dx - \int\int \cos x \cdot dx \cdot \frac{d(x)}{dx} \cdot d$$

$$\int x \cdot \cos x \cdot dx = x\sin x - \int \sin x \cdot 1 \cdot dx$$

$$\int x \cdot \cos x \cdot dx = x\sin x - \int \sin x \cdot dx$$

$$\int x \cdot \cos x \cdot dx = x\sin x + \cos x \cdot$$

$$\int x \cos x \, dx = x\sin x + \cos x + c$$

6) $\int x \sec^2 x \cdot dx$

$\int uv \cdot dx = u\int v \cdot dx - \iint v \cdot dx \cdot \frac{du}{dx} \cdot dx$

$\int x \sec^2 x \cdot dx = x\int \sec^2 x \cdot dx - \iint \sec^2 x \cdot dx \cdot \frac{d1}{d}$

$\int x \sec^2 x \cdot dx = x \tan x - \int \tan x \cdot 1 \cdot dx$

$\int x \cdot \sec^2 x \cdot dx = x \tan x - \int \tan x \cdot dx$

$\int x \cdot \sec^2 x \cdot dx = x \tan x - \log(\sec x) + c$

7) $\int x \cdot \operatorname{cosec}^2 x \cdot dx$

$\int u \cdot v \cdot dx = u\int v \cdot dx - \iint v \cdot dx \cdot \frac{du}{dx} \cdot dx$

$\int x \cdot \operatorname{cosec}^2 x \cdot dx = x\int \operatorname{cosec}^2 x \cdot dx - \iint \text{cocosec} \cdot \frac{d(x)}{dx}$

$\int x \operatorname{cosec}^2 x \cdot dx = -x \cot x + \int \cot x \cdot 1 \cdot dx$

$\int x \operatorname{cosec}^2 x \cdot dx = -x \cot x + \log(\sin x) + c$

8) $\int x \cdot \sin 3x \cdot dx$

Formula,

$\int uv \cdot dx = u\int v \cdot dx - \iint v \cdot dx \cdot \frac{du}{dx} \cdot dx$

Now,

$\int x \cdot \sin 3x \cdot dx = x\int \sin 3x - \iint \sin 3x \cdot dx \frac{d(x)}{dx}$

$= -x\frac{\cos 3x}{3} + \int \frac{\cos 3x}{3} \cdot 1 \cdot dx$

$$= -\frac{x\cos 3x}{3} + \frac{1}{3}\int \cos 3x \cdot dx$$

$\frac{d(}{dx}$ $\int x.\sin 3x.dx = -\frac{x\cos 3x}{3} + \frac{\sin 3x}{9} + c$

9) $\int x \cdot e^{ax} \cdot dx$

Formula,

$\int uv.dx = u\int v.dx - \int\int v.dx \cdot \frac{du}{dx} \cdot dx$

Now,

$$\int x.e^{ax}dx = x\int e^{ax}dx - \int\int e^{ax}dx \cdot \frac{d(x)}{dx} \cdot dx$$

$$= x\frac{e^{ax}}{a} - \int \frac{e^{ax}}{a} \cdot 1 \cdot dx$$

$$= \frac{x\,e^{ax}}{a} - \frac{1}{a}\int e^{ax}.dx$$

$$= \frac{xe^{ax}}{a} - \frac{1}{a}\int e^{ax}.dx$$

$$= \frac{xe^{ax}}{a} - \frac{1}{a} \cdot \frac{e^{ax}}{a}.$$

$$\int x.e^{ax}.dx = \frac{xe^{ax}}{a} - \frac{e^{ax}}{a^2}$$

$$\int x.e^{ax}.dx = \frac{e^{ax}}{a}\left[x - \frac{1}{a}\right] + c$$

10) $\int x \cdot \cos 5x \cdot dx$

Formula,

$$\int uv \cdot dx = u\int v \cdot dx - \iint v \cdot dx \cdot \frac{du}{dx} \cdot dx$$

Now,

$$\int x \cdot \cos 5x \cdot dx = x\int \cos 5x \cdot dx - \iint \cos 5x \cdot dx \, \frac{d(x)}{dx}$$

$$\int x \cdot \cos 5x \cdot dx = \frac{x\sin 5x}{5} - \int \frac{\sin 5x}{5} \cdot 1 \cdot dx$$

$$\int x \cdot \cos 5x \cdot dx = \frac{x\sin 5x}{5} - \int \frac{\sin 5x}{5} \cdot dx$$

$$\int x \cdot \cos 5x \cdot dx = \frac{x\sin 5x}{5} - \frac{1}{5}\int \sin 5x \cdot dx$$

$$\int x \cdot \cos 5x \cdot dx = \frac{x\sin 5x}{5} + \frac{1}{5}\,\frac{\cos 5x}{5}$$

$$\int x \cdot \cos 5x \cdot dx = \frac{x\sin 5x}{5} + \frac{\cos 5x}{25} + C$$

11) $\int \tan^{-1}x \, dx = \int \tan^{-1}x \cdot 1 \cdot dx$

Formula,

$$\int uv \cdot dx = u\int v \cdot dx - \iint v \cdot dx \cdot \frac{du}{dx} \cdot dx$$

Now,

$$\int \tan^{-1}x \cdot 1 \cdot dx = \tan^{-1}x \int 1 \cdot dx - \iint 1 \cdot dx \cdot \frac{d(\tan^{-1}x)}{dx}$$

$$= \tan^{-1}x \cdot x - \int x \cdot \frac{1}{1+x^2} \cdot dx$$

$$= x\tan^{-1}x - \int \frac{1}{1+x^2} \cdot x \cdot dx$$

Put $t = 1+x^2$

Diffⁿ wrt 'x'.

$$\frac{dt}{dx} = 2x$$

$$\frac{dt}{2} = x \cdot dx$$

So,

$$\int \tan^{-1}x \cdot 1 \cdot dx = x \cdot \tan^{-1}x - \int \frac{1}{t} \cdot \frac{dt}{2}$$

$$\int \tan^{-1}x \cdot 1 \cdot dx = x\tan^{-1}x - \frac{1}{2}\int \frac{1}{t} \cdot dt$$

$$\int \tan^{-1}x \cdot 1 \cdot dx = x\tan^{-1}x - \frac{1}{2}\log t$$

$$\int \tan^{-1}x \cdot 1 \cdot dx = x\tan^{-1}x - \frac{1}{2}\log(1+x^2) + c$$

12) $\int \sin^{-1}x \cdot dx = \int \sin^{-1}x \cdot 1 \cdot dx$

Formula,

$$\int uv \cdot dx = u\int v \cdot dx - \iint v \cdot dx \cdot \frac{du}{dx} \cdot dx$$

Now,

$$\int \sin^{-1}x \cdot 1 \cdot dx = \sin^{-1}x \int 1 \cdot dx - \iint 1 \cdot dx \cdot \frac{d(\sin^{-1}x)}{dx} dx$$

$$= \sin^{-1}x \cdot x - \int x \cdot x \frac{1}{\sqrt{1-x^2}} \cdot dx$$

$$= \sin^{-1}x \cdot x - \int \frac{1}{\sqrt{1-x^2}} \cdot x \cdot dx$$

$$\int \sin^{-1}x \cdot 1 \, dx = x \cdot \sin^{-1}x - \int \frac{1}{\sqrt{1-x^2}} \cdot x \, x$$

Put $t = 1 - x^2$

Diff^n wrt 'x'

$$\frac{dt}{dx} = -2x$$

$$-\frac{dt}{2} = x \cdot dx$$

So,

$$\int \sin^{-1}x \cdot dx = x\sin^{-1}x - \int \frac{1}{\sqrt{t}} \cdot -\frac{dt}{2}$$

$$\int \sin^{-1}x \cdot dx = x\sin^{-1}x + \frac{1}{2}\int \frac{1}{\sqrt{t}} \cdot dt$$

$$\int \sin^{-1}x \cdot dx = x\sin^{-1}x + \frac{1}{2}\int t^{-1/2} \cdot dt$$

$$\int \sin^{-1}x \cdot dx = x\sin^{-1}x + \frac{1}{2} \times \frac{t^{-1/2+1}}{-1/2+1}$$

$$\int \sin^{-1}x \cdot dx = x\sin^{-1}x + \frac{1}{\cancel{2}} \frac{t^{1/2}}{1/\cancel{2}}$$

$$\int \sin^{-1}x \cdot dx = x\sin^{-1}x + (\sqrt{1-x^2}) + c$$

13) $\int x \cdot \sin^2 x \cdot dx$

$$\int uv \cdot dx = u\int v \cdot dx - \int\int v \cdot dx \, \frac{du}{dx} \cdot dx$$

Now,

$$\int x \cdot \sin^2 x \cdot dx = \cancel{x\int \sin^2 x\, dx} \int x \, \frac{(1-\cos 2x)}{2} \cdot dx$$

$$\int x \cdot \sin^2 x \, dx = \frac{1}{2}\int (x - x\cos 2x) \cdot dx$$

$$\int x \cdot \sin^2 x \cdot dx = \frac{1}{2}\int x \cdot dx - \frac{1}{2}\int x \cdot \cos 2x \cdot dx$$

$$\int x \cdot \sin^2 x \cdot dx = \frac{1}{2}\frac{x^2}{2} - \frac{1}{2}\left[x\int \cos 2x \cdot dx - \int\int \cos 2x \cdot dx \cdot \frac{d(x)}{dx} \cdot dx\right]$$

$$= \frac{x^2}{4} - \frac{1}{2}\left[x\frac{\sin 2x}{2} - \int \frac{\sin 2x}{2} \cdot 1 \cdot dx\right]$$

$$= \frac{x^2}{4} - \frac{1}{2}\left[\frac{x\sin 2x}{2} - \frac{1}{2}\int \sin 2x \cdot dx\right]$$

$$= \frac{x^2}{4} - \frac{1}{2}\left[x\frac{\sin 2x}{2} + \frac{1}{2}\frac{\cos 2x}{2}\right]$$

$$= \frac{x^2}{4} - \frac{1}{2}\left[\frac{x\sin 2x}{2} + \frac{\cos 2x}{4}\right]$$

$$= \frac{x^2}{4} - \frac{x\sin 2x}{4} - \frac{\cos 2x}{8} + c$$

14) $\int x \cdot \cos^2 x \cdot dx$

$$\int x \cdot \cos^2 x \cdot dx = \int x\left(\frac{1+\cos 2x}{2}\right) \cdot dx$$

$$\int x \cdot \cos^2 x \cdot dx = \int \frac{x + x\cos 2x}{2} \cdot dx$$

$$\int x \cdot \cos^2 x \cdot dx = \frac{1}{2}\left[\int x \cdot dx + \int x \cdot \cos 2x \cdot dx\right]$$

$$\int x \cdot \cos^2 x \cdot dx = \frac{1}{2} \times \frac{x^2}{2} + \frac{1}{2}\left[x\int \cos 2x \cdot dx - \int\int \cos 2x \cdot dx \cdot \frac{d(x)}{dx} \cdot dx\right]$$

$$\int x \cdot \cos^2 x \cdot dx = \frac{x^2}{4} + \frac{1}{2}\left[x \cdot \frac{\sin 2x}{2} - \int \frac{\sin 2x}{2} \cdot 1\right.$$

$$\int x \cdot \cos^2 x \cdot dx = \frac{x^2}{4} + \frac{1}{2}\left[\frac{x \sin 2x}{2} - \frac{1}{2}\int \sin 2x \cdot dx\right.$$

$$= \frac{x^2}{4} + \frac{1}{2}\left[\frac{x \sin 2x}{2} + \frac{\cos 2x}{4}\right]$$

$$= \frac{x^2}{4} + \frac{x \sin 2x}{4} + \frac{\cos 2x}{8} + C$$

5) $\int x^2 \cdot e^x \cdot dx$

Formula,

$$\int uv \cdot dx = u\int v \cdot dx - \iint v \cdot dx \cdot \frac{du}{dx} \cdot dx$$

$$\int x^2 \cdot e^x \cdot dx = x^2 \int e^x \cdot dx - \iint e^x \cdot dx \cdot \frac{d(x^2)}{dx} \cdot dx$$

$$\int x^2 \cdot e^x \cdot dx = x^2 e^x - \int e^x \cdot 2x \cdot dx$$

$$\int x^2 \cdot e^x \cdot dx = x^2 \cdot e^x - 2\int e^x \cdot x \cdot dx$$

$$\int x^2 \cdot e^x \cdot dx = x^2 \cdot e^x - 2\int x \cdot e^x \cdot dx$$

$$\int x^2 \cdot e^x \cdot dx = x^2 \cdot e^x - 2\left[x\int e^x \cdot dx - \iint e^x \cdot dx \cdot \frac{d}{d}\right.$$

$$\int x^2 \cdot e^x \cdot dx = x^2 \cdot e^x - 2\left[x e^x - \int e^x \cdot dx\right]$$

$$\int x^2 \cdot e^x \cdot dx = x^2 \cdot e^x - 2x e^x + 2e^x$$

$$\int x^2 \cdot e^x \cdot dx = e^x\left[x^2 - 2x + 2\right] + C$$

16) $\int x^2 \cdot \sin x \cdot dx$

Formula,

$$\int uv \, dx = u \int v \, dx - \int\int v \, dx \cdot \frac{dv}{dx} \cdot dx$$

$$\int x^2 \cdot \sin x \cdot dx = x^2 \int \sin x \cdot dx - \int\int \sin x \cdot dx \cdot \frac{d(x^2)}{dx} \cdot dx$$

$$= -x^2 \cos x - \int \cos x \cdot 2x \cdot dx$$

$$= -x^2 \cdot \cos x - 2\int \cos x \cdot x \cdot dx$$

$$\int x^2 \cdot \sin x \cdot dx = -x^2 \cdot \cos x - 2\int x \cdot \cos x \cdot dx$$

$$\int x^2 \cdot \sin x \cdot dx = -x^2 \cdot \cos x - 2\left[x \int \cos x \cdot dx - \int\int \cos x \cdot dx \cdot \frac{d(x)}{dx} \cdot dx\right]$$

$$\int x^2 \cdot \sin x \cdot dx = -x^2 \cdot \cos x - 2\left[x \sin x - \int \sin x \cdot dx\right.$$

$$= -x^2 \cdot \cos x - 2\left[x \sin x + \cos x\right]$$

$$\int x^2 \cdot \sin x \cdot dx = -x^2 \cdot \cos x - 2x \sin x + 2\cos x + C$$

17) $\int x^2 \cdot \cos x \cdot dx$

Formula,

$$\int uv \cdot dx = u \int v \cdot dx - \int\int v \cdot dx \cdot \frac{dv}{dx} \cdot dx$$

$$\int x^2 \cdot \cos x \cdot dx = x^2 \int \cos x \cdot dx - \int\int \cos x \cdot dx \cdot \frac{d(x^2)}{dx} \cdot dx$$

$$\int x^2 \cdot \cos x \cdot dx = x^2 \sin x - \int \sin x \cdot 2x \cdot dx$$

$$= x^2 \sin x - 2\int x \cdot \sin x \cdot dx$$

$$= x^2 \sin x - 2\left[x \int \sin x \, dx - \iint \sin x \cdot \frac{d(x)}{dx} \right.$$

$$\int x^2 \cdot \cos x \cdot dx = x^2 \sin x - 2\left[-x \cos x + \int \cos x \cdot d\right.$$

$$\int x^2 \cdot \cos x \cdot dx = x^2 \sin x + 2x \cos x - 2\sin x + $$

18) $\int x^2 \cdot \log x \cdot dx = \int \log x \cdot x^2 \cdot dx$.

Formula,

$$\int uv \cdot dx = u \int v \cdot dx - \iint v \cdot dx \cdot \frac{du}{dx} \cdot dx$$

$$\int \log x \cdot x^2 dx = \log x \int x^2 \cdot dx - \iint x^2 \cdot dx \cdot$$

$$= \log x \times \frac{x^3}{3} - \int \frac{x^{\cancel{3}\,2}}{3} \cdot \frac{1}{\cancel{x}} \cdot dx$$

$$= \log x \times \frac{x^3}{3} - \frac{1}{3}\int x^2 \cdot dx$$

$$\int \log x \cdot x^2 dx = \frac{x^3 \log x}{3} - \frac{x^3}{9} + c$$

19) $\int x \cdot \sin 5x \cdot \cos x \cdot dx$

$\int x \frac{1}{2}\left[\sin 6x + \sin 4x\right] \cdot dx$

$\frac{1}{2}\int (x\sin 6x + x\sin 4x) \cdot dx$

$\frac{1}{2}\left\{\left[x\int \sin 6x \cdot dx - \iint \sin 6x \cdot dx \cdot \frac{d(x)}{dx} \cdot dx\right] + \right.$

$\left.\left[x\int \sin 4x \cdot dx - \iint \sin 4x \cdot dx \cdot \frac{d(x)}{dx} \cdot dx\right]\right\} \cdot dx$

14) $\int x \sin 5x \cdot \cos x \cdot dx$

$\int x \frac{1}{2}\left[\sin 6x + \sin 4x\right] dx$

$\frac{1}{2}\int (x\sin 6x + \sin 4x) \cdot dx$

$\frac{1}{2}\left[x\int \sin 6x \cdot dx \iint \sin 6x \, dx \frac{d(x)}{dx} \cdot dx + x\int \sin 4x \cdot dx\right.$

$- \iint \sin 4x \, dx \cdot \frac{dx}{dx} \cdot dx$

$\frac{1}{2}\left[-\frac{x\cos 6x}{6} + \int \frac{\cos 6x}{6} - \frac{x\cos 4x}{4} + \int \frac{\cos 4x}{4} \cdot dx\right]$

$\frac{1}{2}\left[-\frac{x\cos 6x}{6} + \frac{1}{6}\int \cos 6x - \frac{x\cos 4x}{4} + \frac{1}{4}\int \cos 4x \cdot dx\right]$

$\frac{1}{2}\left[-\frac{x\cos 6x}{6} + \frac{1}{6} \times \frac{\sin 6x}{6} - \frac{x\cos 4x}{4} + \frac{1}{4} \times \frac{\sin 4x}{4}\right]$

$$-\frac{x\cos 6x}{12} + \frac{\sin 6x}{72} - \frac{x\cos 4x}{8} + \frac{\sin 4x}{32} + C$$

20) $\int x\cos 5x \sin x \cdot dx$

$\int x \frac{1}{2}[\sin 6x - \sin 4x] \cdot dx$

$\frac{1}{2}\int (x\sin 6x - x\sin 4x) \cdot dx$

$\frac{1}{2}\left[x\int \sin 6x \cdot dx - \int\int \sin 6x \cdot dx \cdot \frac{dx}{dx} \cdot dx - x\int \sin\right.$

$\left. - \int\int \sin 4x \cdot dx \frac{d(x)}{dx} \cdot dx\right.$

$\frac{1}{2}\left[-\frac{x\cos 6x}{6} + \int \frac{\cos 6x}{6} \cdot dx - \frac{x\cos 4x}{4} - \int \frac{\cos 4x}{4}\right.$

$\frac{1}{2}\left[-\frac{x\cos 6x}{6} + \frac{1}{6}\int \cos 6x \cdot dx - \frac{x\cos 6x}{4} - \frac{1}{4}\int \cos\right.$

$\frac{1}{2}\left[-\frac{x\cos 6x}{6} + \frac{\sin 6x}{36} - \frac{x\cos 4x}{4} - \frac{\sin 4x}{16}\right.$

$$-\frac{x\cos 6x}{12} + \frac{\sin 6x}{72} - \frac{x\cos 4x}{8} - \frac{\sin 4x}{32} +$$

27/03/18

DEFINITE INTEGRALS

Let

$$\int f(x)\cdot dx = g(x) + c$$

then,

$$\int_a^b f(x)\cdot dx = g(x)\Big]_a^b = g(b) - g(a)$$

Problems:

1) $\int_0^2 x^2 \cdot dx$

Int. wrt 'x'.

$$\frac{x^3}{3}\Big]_0^2$$

$$\frac{2^3}{3} - \frac{0}{3}$$

$8/3$ //.

2) $\int_1^2 (x+1)\cdot dx$

Int. wrt 'x'.

$$\frac{x^2}{2} + x\Big]_1^2$$

$$\left[\frac{2^2}{2}+2\right] - \left[\frac{1}{2}+1\right]$$

$$4 - \frac{1+2}{2}$$

$$4 - \frac{3}{2} = \frac{8-3}{2} = 5/2 //$$

3) $\int_0^1 (x^3 - 3x^2 + 2x) \cdot dx$

Int. wrt 'x'

$\left. \frac{x^4}{4} - \frac{3x^3}{3} + \frac{2x^2}{2} \right]_0^1$

$\left[\frac{1}{4} - \cancel{1} + \cancel{1}\right] - [0]$

$\frac{1}{4}$,,

4) $\int_{-1}^{1} x^2 (x-1) \cdot dx$

$\int_{-1}^{1} (x^3 - x^2) \cdot dx$

Int wrt 'x'

$\left. \frac{x^4}{4} - \frac{x^3}{3} \right]_{-1}^{1}$

$\left[\frac{1}{4} - \frac{1}{3}\right] - \left[\frac{1}{4} + \frac{1}{3}\right]$

$\left[\frac{3-4}{12}\right] - \left[\frac{3+4}{12}\right]$

$\left[\frac{-1}{12}\right] - \left[\frac{7}{12}\right] = \cancel{\frac{7}{144}}$

$\frac{-1-7}{12} = \frac{\cancel{-8}^{\,2}}{\cancel{12}_{\,3}} =$

5) $\int_0^{\pi/2} \cos x \cdot dx$

Int. wrt 'x'

$\left. \sin x \right]_0^{\pi/2}$

$\sin \pi/2 - \sin 0$

$1 - 0$

1 ,,

6) $\int_0^{\pi/4} \sec^2 x \cdot dx$

Int. wrt 'x'

$\left. \tan x \right]_0^{\pi/4}$

$\tan \pi/4 - \tan 0$

$1 - 0$

1 ,,

7) $\int_{\pi/6}^{\pi/4} \operatorname{cosec}^2 x \cdot dx$

Int wrt 'x'

$-\cot x \Big]_{\pi/6}^{\pi/4}$

$-\cot\pi/4 - [-\cot\pi/6]$

$-1 + \sqrt{3}$

$\sqrt{3} - 1$

8) $\int_0^{\pi/2} \sin 2x \cdot dx$

Int wrt 'x'.

$-\frac{\cos 2x}{2}\Big]_0^{\pi/2}$

$-\frac{\cos \cancel{2} \times \pi/\cancel{2}}{2} - \left[-\frac{\cos 0}{2}\right]$

$-\frac{\cos 180}{2} + \frac{\cos 0}{2}$

$-\frac{\cos(180-0)}{2} + \frac{1}{2}$

$\frac{\cos 0}{2} + \frac{1}{2}$

$\frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1$

9) $\int_0^1 \frac{1}{1+x^2} \cdot dx$

Int. wrt 'x'

$\tan^{-1}\Big]_0^1$

$\tan^{-1}1 - \tan^{-1}0$

$\pi/4 - 0$

$\pi/4$

10) $\int_0^1 \frac{1}{\sqrt{1-x^2}} \cdot dx$

Int. wrt 'x'

$\sin^{-1} x\Big]_0^1$

$\sin^{-1}1 - \sin^{-1}0$

$\pi/2 - 0$

$\pi/2$

$\int_0^1 (2x+1)(x-3) \cdot dx$

$\frac{\overset{1}{1}8}{\frac{18}{33}}$ $\frac{53}{4}$

11) $\int_1^{\infty} \frac{1}{x^2} \cdot dx = \int_1^{\infty} x^{-2} \cdot dx$

Int wrt 'x'

$\left.\frac{x^{-2+1}}{-2+1}\right]_1^{\infty}$

$\left.\frac{x^{-1}}{-1}\right]_1^{\infty} = \left.-\frac{1}{x}\right]_1^{\infty}$

$\left[\frac{-1}{\infty}\right] - \left[-\frac{1}{1}\right]$

$0 + 1 = 1_{||}$

12) $\int_0^1 (2x+1)(x-3) \cdot dx$

$\int_0^1 (2x^2 - 6x + x - 3)$

$\int_0^1 (2x^2 - 5x - 3) \cdot dx$

Int. wrt 'x'

$\left.\frac{2x^3}{3} - \frac{5x^2}{2} - 3x\right]_0^1$

$\left[\frac{2}{3} - \frac{5}{2} - 3\right] - [0]$

$\left[\frac{4-15-18}{6}\right] = \left[\frac{-29}{6}\right]$

13) $\int_0^{\pi/4} \tan^2 x \cdot dx$

$\int_0^{\pi/4} (\sec^2 x - 1) \cdot dx$

$\left.\tan x - x\right]_0^{\pi/4}$

$[\tan \pi/4 - \pi/4] - [\tan 0 - 0]$

$1 - \pi/4$

14) $\int_{-\pi/4}^{\pi/4} \cot^2 x \cdot dx$

$\int_{-\pi/4}^{\pi/4} (\operatorname{cosec}^2 x - 1) \cdot dx$

Int. wrt 'x'

$\left.-\cot x - x\right]_{-\pi/4}^{\pi/4}$

$(-\cot \pi/4 - \pi/4) - (-\cot$
$+ \pi/4)$

$(-1 - \pi/4) - (\cot \pi/4 - \pi/4$

$-1 - \pi/4 - 1 - \pi/4$

$-2 - 2\pi/4^{2}$ $= -2 - \pi$

16) $\int_0^2 \frac{1}{\sqrt{4-x^2}} \cdot dx = \int_0^2 \frac{1}{\sqrt{2^2-x^2}} \cdot dx$

Int. wrt 'x'.

$\left.\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2$

$\sin^{-1}\left(\frac{2}{2}\right) - \sin\left(\frac{0}{2}\right)$

$\sin^{-1} 1 - \sin^{-1} 0$

$\pi/2$ ॥

17) $\int_0^{\pi/2} (\sin x + \cos x)^2 \cdot dx$

$\int_0^{\pi/2} (\sin^2 x + \cos^2 x + 2\sin x \cos x) dx$

$\int_0^{\pi/2} (1 + \sin 2x) \cdot dx$

Int wrt. 'x'.

$\left. x - \frac{\cos 2x}{2}\right]_0^{\pi/2}$

$\left[\pi/2 - \frac{\cos 2 \times \pi/2}{2}\right] - \left[0 - \frac{\cos 2 \times 0}{2}\right]$

$\left[\pi/2 - \frac{\cos 180}{2}\right] - \left[-\frac{1}{2}\right]$

$$\left[\frac{\pi}{2} - \frac{\cos(180-0)}{2}\right] - \left[-\frac{1}{2}\right]$$

$$\left[\pi/2 + \frac{\cos 0}{2}\right] + \frac{1}{2}$$

$$\pi/2 + \frac{1}{2} + \frac{1}{2}$$

$$\frac{\pi}{2} + 1$$

18) $$\int_0^{\pi/2} \left(\sqrt{1+\sin 2x}\right).dx$$

$$\int_0^{\pi/2} \left(\sqrt{\sin^2 x + \cos^2 x + 2\sin x\cos x}\right).dx$$

$$\int_0^{\pi/2} \sqrt{(\sin x + \cos x)^2}\,.dx$$

$$\int_0^{\pi/2} (\sin x + \cos x).dx$$

Int wort 'x'.

$$-\cos x + \sin x\Big]_0^{\pi/2}$$

$$\left[-\cos\pi/2 + \sin\pi/2\right] - \left[-\cos 0 + \sin 0\right]$$

$$[0+1] - [-1+0]$$

$$1-(-1)$$

$$1+1 = 2$$

19) $\int_0^{\pi/2} \sin^2 x \cdot dx$

$\int_0^{\pi/2} \left(\frac{1-\cos 2x}{2}\right) \cdot dx$

$\frac{1}{2}\int_0^{\pi/2} (1-\cos 2x) \cdot dx$

Int wrt 'x'.

$\frac{1}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\pi/2}$

$\frac{1}{2}\left[\pi/2 - \frac{\sin \cancel{2}\times\pi/\cancel{2}}{2}\right] - \frac{1}{2}\left[0 - \frac{\sin 0}{0}\right]$

$\frac{1}{2}\left[\pi/2 - \frac{\sin 180}{2}\right] - \frac{1}{2}[0]$

$\frac{1}{2}\left[\pi/2 - \frac{\sin 0}{2}\right] - 0$

$\frac{1}{2}\left[\pi/2\right]$

$\pi/4.$

20) $\int_0^{\pi/2} \cos^2 x \cdot dx$

$\int_0^{\pi/2} \left(\frac{1+\cos 2x}{2}\right) \cdot dx$

$\frac{1}{2}\int_0^{\pi/2} (1+\cos 2x) \cdot dx$

Int wrt 'x'.

$\frac{1}{2}\left[x + \frac{\sin 2x}{2}\right]_0^{\pi/2}$

$\frac{1}{2}\left[\pi/2 + \frac{\sin \cancel{2}\times\pi/\cancel{2}}{2}\right] - \left[0 - \text{s}\right.$

$\frac{1}{2}\left[\frac{\pi}{2} + \frac{\sin 180}{2}\right]$

$\frac{1}{2}\left[\frac{\pi}{2} + \frac{\sin 0}{2}\right]$

$\frac{1}{2}\left[\frac{\pi}{2}\right]$

$\pi/4$

21) $\int_0^{\pi/3} \tan x \cdot dx$

Int. wrt 'x'.

$\log(\sec x)\Big]_0^{\pi/3}$

$\log(\sec \pi/3) - \log(\sec 0)$

$\log 2 - \log 1$

$\log 2 - 0$

$\log 2$.

22) $\int_0^{\pi/2} \sin^3 x \cdot dx$

$$\left[\begin{array}{l} \sin 3x = 3\sin x - 4\sin^3 x \\ 4\sin^3 x = 3\sin x - \sin 3x \\ \sin^3 x = \frac{1}{4}(3\sin x - \sin 3x) \end{array}\right]$$

$\frac{1}{4}\int_0^{\pi/2} (3\sin x - \sin 3x) \cdot dx$

Int. wrt 'x'.

$\frac{1}{4}\left[-3\cos x + \frac{\cos 3x}{3}\right]_0^{\pi/2}$

$\frac{1}{4}\left[\cancel{-3\cos \pi/2 + \frac{\cos 3 \times \pi/2}{3}}\right]^{0} - \frac{1}{4}\left[-3\cos 0 + \frac{\cos 0}{3}\right]$

$0 - \frac{1}{4}\left[-3 + \frac{1}{3}\right] = -\frac{1}{4}\left(\frac{-9+1}{3}\right) = \frac{-1}{\cancel{4}} \times \frac{-\cancel{8}^{2}}{3}$

$= 2$

23) $\int_0^{\pi/2} \cos^3 x \, dx$

$\cos 3x = 4\cos^3 x - 3\cos x$

$3\cos x = 4\cos^3 x - \cos 3x$

$3\cos x + \cos 3x = 4\cos^3 x$

$\cos^3 x = \frac{1}{4}(3\cos x + \cos 3x)$

$\frac{1}{4}\int_0^{\pi/2} (3\cos x + \cos 3x).dx$

Int wort 'x'.

$\frac{1}{4}\left[3\sin x + \frac{\sin 3x}{3}\right]_0^{\pi/2}$

$\frac{1}{4}\left[3\sin \pi/2 + \frac{\sin 3\times\pi/2}{3}\right] - \cancel{\frac{1}{4}\left[3\sin 0 + \frac{\sin 0}{3}\right]} \rightarrow 0$

$\frac{1}{4}\left[3 + \frac{\sin 270}{3}\right] - 0$

$\frac{1}{4}\left[3 - \frac{\cos 0}{3}\right]$

$\frac{1}{4}\left[3 - \frac{1}{3}\right] = \frac{1}{4}\left[\frac{9-1}{3}\right] = \frac{1}{4} \times \frac{8}{3} = \frac{2}{3}$

24) $\int_0^{\pi/2} \cos 5x . \cos 3x . dx$

$\frac{1}{2}\int_0^{\pi/2} (\cos 8x + \cos 2x). dx$

$\frac{1}{2}\left[\frac{\sin 8x}{8} + \frac{\sin 2x}{2}\right]_0^{\pi/2}$

$\frac{1}{2}\left[\frac{\sin 8\times\pi/2}{8} + \frac{\sin 2\times\pi/2}{2}\right] - \cancel{\frac{1}{2}\left[\frac{\sin 0}{8} + \frac{\sin}{2}\right.} \rightarrow 0$

$\frac{1}{2}\left[\frac{\sin 720}{8} + \frac{\sin 180}{2}\right]$

$$\frac{1}{2}\left[\frac{\sin(2\times 360+0)}{8}+\frac{\sin(180+0)}{2}\right]$$

$$\frac{1}{2}\left[\frac{\sin 0}{8}-\frac{\sin 0}{2}\right]$$

$$\frac{1}{2}\left[0\right]$$

$$0$$

25) $$\int_0^{\pi/2}\cos 4x\cdot\sin 2x\cdot dx$$

$$\frac{1}{2}\int_0^{\pi/2}(\sin 6x-\sin 2x)\,dx$$

Int wrt 'x'.

$$\frac{1}{2}\left[-\frac{\cos 6x}{6}+\frac{\cos 2x}{2}\right]_0^{\pi/2}$$

$$\frac{1}{2}\left[-\frac{\cos \overset{3}{\cancel{6}}\times\pi/\cancel{2}}{6}+\frac{\cos\frac{\pi}{\cancel{2}}\times\cancel{2}}{2}\right]-\frac{1}{2}\left[-\frac{\cos 0}{6}+\frac{\cos 0}{2}\right]$$

$$\frac{1}{2}\left[-\frac{\cos 540}{6}+\frac{\cos 180}{2}\right]-\frac{1}{2}\left[-\frac{1}{6}+\frac{1}{2}\right]$$

$$\frac{1}{2}\left[-\frac{\cos(360+180)}{6}+\frac{\cos(180-0)}{2}\right]-\frac{1}{2}\left[\frac{-1+3}{6}\right]$$

$$\frac{1}{2}\left[-\frac{\cos 180}{6}-\frac{\cos 0}{2}\right]-\frac{1}{\cancel{2}}\left[\frac{\cancel{2}}{6}\right]$$

$$\frac{1}{2}\left[-\frac{\cos(180-0)}{6}-\frac{1}{2}\right]-\frac{1}{6}$$

$$\frac{1}{2}\left[\frac{\cos 0}{6} - \frac{1}{2}\right] - \frac{1}{6}$$

$$\frac{1}{2}\left[\frac{1}{6} - \frac{1}{2}\right] - \frac{1}{6}$$

$$\frac{1}{2}\left[\frac{1-3}{6}\right] - \frac{1}{6}$$

$$\frac{1}{\cancel{2}} \times \frac{-\cancel{2}}{6} - \frac{1}{6}$$

$$-\frac{1}{6} - \frac{1}{6}$$

$$\frac{-1-1}{6} = \frac{-2}{6} = \frac{-1}{3}$$

26) $\int_0^{\pi/2} \sin 3x \cdot \cos x \cdot dx$

$$\frac{1}{2}\int_0^{\pi/2} (\sin 4x + \sin 2x) \, dx$$

Int wrt. x.

$$\frac{1}{2}\left[-\frac{\cos 4x}{4} - \frac{\cos 2x}{2}\right]_0^{\pi/2}$$

$$\frac{1}{2}\left[\frac{-\cos \cancel{4}^{2} \times \frac{\pi}{\cancel{2}}}{4} - \frac{\cos \times \frac{\pi}{\cancel{2}} \times \cancel{2}}{2}\right] - \frac{1}{2}\left[\frac{-\cos 0}{4} - \frac{\cos 0}{2}\right]$$

$$\frac{1}{2}\left[\frac{-\cos 360}{4} - \frac{\cos 180}{2}\right] - \frac{1}{2}\left[\frac{-1}{4} - \frac{1}{2}\right]$$

$$\frac{1}{2}\left[\frac{-\cos(360+0)}{4} - \frac{\cos(180-0)}{2}\right] - \frac{1}{2}\left[\frac{-1-2}{4}\right]$$

$$\frac{1}{2}\left[-\frac{\cos 0}{4}+\frac{\cos 0}{2}\right]-\frac{1}{2}\left[\frac{-3}{4}\right]$$

$$\frac{1}{2}\left[-\frac{1}{4}+\frac{1}{2}\right]+\frac{3}{8}$$

$$\frac{1}{2}\left[\frac{-1+2}{4}\right]+\frac{3}{8}$$

$$\frac{1}{2}\left[\frac{1}{4}\right]+\frac{3}{8}$$

$$\frac{1}{8}+\frac{3}{8}=\frac{1+3}{8}=\frac{4}{8}=\frac{1}{2}$$

17) $$\int_0^{\pi/2} \sin 4x \cdot \cos 2x \cdot dx$$

$$\frac{1}{2}\int_0^{\pi/2}(\sin 6x+\sin 2x)\cdot dx$$

$$\frac{1}{2}\left[-\frac{\cos 6x}{6}-\frac{\cos 2x}{2}\right]_0^{\pi/2}$$

$$\frac{1}{2}\left[\frac{-\cos \cancel{6}^{3}x\frac{\pi}{\cancel{2}}}{6}-\frac{\cos \cancel{2}x\frac{\pi}{\cancel{2}}}{2}\right]-\frac{1}{2}\left[\frac{-\cos 0}{6}-\frac{\cos}{2}\right]$$

$$\frac{1}{2}\left[-\frac{\cos 540}{6}-\frac{\cos 180}{2}\right]-\frac{1}{2}\left[-\frac{1}{6}-\frac{1}{2}\right]$$

$$\frac{1}{2}\left[\frac{-\cos(360+180)}{6}-\frac{\cos(180-0)}{2}\right]-\frac{1}{2}\left[\frac{-1-3}{6}\right]$$

$$\frac{1}{2}\left[-\frac{\cos 180}{6}+\frac{\cos 0}{2}\right]-\frac{1}{\cancel{2}}\left[\frac{-\cancel{4}^{2}}{\cancel{6}_{3}}\right]$$

$$\frac{1}{2}\left[-\frac{\cos(180-0)}{6} + \frac{1}{2}\right] + \frac{1}{3}$$

$$\frac{1}{2}\left[\frac{\cos 0}{6}\,\frac{3}{3} + \frac{1}{2}\right] + \frac{1}{3}$$

$$\frac{1}{2}\left[\frac{1+3}{6}\right] + \frac{1}{3}$$

$$\frac{1}{\cancel{2}}\left[\frac{\cancel{4}^{2}}{\cancel{6}_{3}}\right] \times \frac{1}{3}$$

$$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$

23) $\int_0^1 x \cdot e^x \cdot dx$

Formula,

$$\int u \cdot v \cdot dx = u\int v \cdot dx - \iint v \cdot dx \cdot \frac{d(u)}{dx} \cdot dx$$

$$\int x \cdot e^x \cdot dx = x\int e^x \cdot dx - \iint e^x \cdot dx \cdot \frac{d(x)}{dx} \cdot dx$$

$$\int x \cdot e^x \cdot dx = x e^x - \int e^x \cdot dx$$

$$\int x \cdot e^x \cdot dx = x e^x - e^x$$

$$\int x \cdot e^x \cdot dx = e^x (x-1)$$

Now,

$$\int x \cdot e^x \cdot dx = e^x (x-1)\Big]_0^1$$

$$= e^1(1-1) - e^1(0-1)$$

$$= e^1(0) - e^1(-1) = 0 - 1(-1)$$

29) $\int_0^{\pi/2} x \cdot \cos x \cdot dx$

Formula,

$\int u.v.dx = u\int v.dx - \int\int v.dx \cdot \frac{d(u)}{dx} \cdot dx$

Now;

$\int x.\cos x.dx = x\int \cos x.dx - \int\int \cos x.dx \cdot \frac{d(x)}{dx} \cdot dx$

$\int x.\cos x.dx = x\sin x - \int \sin x.dx$

$\int x.\cos x.dx = x\sin x + \cos x$

$\int_0^{\pi/2} x.\cos x.dx = x\sin x + \cos x \Big]_0^{\pi/2}$

$\int_0^{\pi/2} x.\cos x.dx = \left[\frac{\pi}{2}\sin\frac{\pi}{2} + \cos\frac{\pi}{2}\right] - \left[0\sin 0 + \cdots\right.$

$= \left[\frac{\pi}{2} + 0\right] - \left[0 + 1\right]$

$= \frac{\pi}{2} - 1$

30) $\int_0^2 \log x \cdot dx = \int_0^2 \log x \cdot 1 \cdot dx$

$\int uv.dx = u\int v.dx - \int\int v.dx \cdot \frac{d(u)}{dx} \cdot dx$

$\int \log x \cdot 1.dx = \log x \int 1.dx - \int 1 \cdot dx \cdot \frac{d(\log x)}{dx} \cdot dx$

$\int \log x \cdot 1\, dx = x.\log x - \int x \times \frac{1}{x} \cdot dx$

$\int \log x \cdot 1 \cdot dx = x\log x - x$

$\int \log x \cdot 1 \cdot dx = x(\log x - 1)$

Now,

$\int_1^2 \log x \cdot 1 \cdot dx = x(\log x - 1)\Big]_1^2$

$\int_1^2 \log x \cdot 1 \cdot dx = 2(\log 2 - 1) - 1(\log 1 - 1)$

$= 2\log 2 - 2 - \log 1 + 1$

$= 2\log 2 - \log 1 - 2 + 1$

$= 2\log 2 - \log 1 - 1$

$= 2\log 2 - 0 - 1$

$\int_0^2 \log x \cdot 1 \cdot dx \quad = 2\log 2 - 1$

31) $\int_0^{\pi/4} \tan^2 x \cdot \sec x \, dx$

Put $t = \tan x$

Diff^n wrt 'x'

$\frac{dt}{dx} = \sec^2 x$

$dt = \sec^2 x \cdot dx$

Now,

Let, $t = \tan x$

Put $x = 0$

$t = \tan 0$

$t = 0$

Also, Put $x = \pi/4$

$t = \tan \pi/4$

$t = 1$

$$\int_0^1 t^2 \cdot dt$$

Int wrt 't'.

$$\left.\frac{t^3}{3}\right]_0^1 = \frac{1^3}{3} - 0 = \frac{1}{3}$$

33) $$\int_0^1 \tan^{-1}x \times \frac{1}{1+x^2} \cdot dx$$

Put $t = \tan^{-1}x$

Diffⁿ wrt 'x'.

$$\frac{dt}{dx} = \frac{1}{1+x^2}$$

$$dt = \frac{1}{1+x^2} \cdot dx$$

Now, $t = \tan^{-1}x$ Also, put $x = 1$

Put $x = 0$ $t = \tan^{-1} 1$

$t = \pi/4$

$t = \tan^{-1} 0$

$t = 0$

So, $$\int_0^{\pi/4} t \cdot dt$$

Int wrt 't',

$$\left.\frac{t^2}{2}\right]_0^{\pi/4} = \frac{\left(\frac{\pi}{4}\right)^2}{2} - 0 = \frac{\frac{\pi^2}{16}}{2} = \frac{\pi^2}{32}$$

$* \int_0^{\pi/n} \cos 4x \cdot \sin x \, dx$

$\int_0^{\pi/2} (2 - 3\sin x)^3 \cdot \cos x \, dx$

$\int_0^{\pi/4} \frac{1}{1+\sin x} \, dx$

Application of Integration.

The area bounded by the curve $y = f(x)$, on x-axis between the ordinates $x = a$ & $x = b$ is given by,

$$A = \int_a^b y \cdot dx$$

$$A = \int_a^b f(x) \cdot dx$$

The area bounded by the curve $x = g(y)$; on y-axis between the ordinates $y = a$ & $y = b$ is given by,

$$A = \int_a^b x \cdot dy$$

$$A = \int_a^b g(y) \cdot dy$$

The volume of the solid generated by the revolution about x-axis, the area bounded by the curve $y = f(x)$ between the ordinates, $x = a$ & $x = b$ is given by,

$$\text{Volume} = \pi \int_a^b y^2 . dx$$

The volume of the solid gen[...] by the revolution ~~of~~ about y-axis [...] the area bounded by the curve x= [...] between the ordinates, y = a & y [...] is given by,

$$\text{Volume} = \pi . \int_a^b x^2 . dy$$

1) Find the area bounded by the curve $y = 4x - x^2 - 3$ on x-axis between the ordinates $x = 1$, & $x = 2$.

Soln: Here, $y = 4x - x^2 - 3$

By formula, we have,

$$A = \int_a^b y . dx$$

$$A = \int_1^2 (4x - x^2 - 3) . dx$$

Int. wrt 'x'.

$$A = \left[\frac{4x^2}{2} - \frac{x^3}{3} - 3x \right]_1^2$$

$$A = \left(\frac{4\times 2^2}{2} - \frac{2^3}{3} - 3\times 2\right) - \left(\frac{4}{2} - \frac{1}{3} - 3\right)$$

$$A = \left(8 - \frac{8}{3} - 6\right) - \left(2 - \frac{1}{3} - 3\right)$$

$$A = \left(2 - \frac{8}{3}\right) - \left(-1 - \frac{1}{3}\right)$$

$$A = \left(\frac{6-8}{3}\right) - \left(\frac{-3-1}{3}\right)$$

$$A = \frac{-2}{3} + \frac{4}{3}$$

$$\therefore A = \frac{-2+4}{3} = \frac{2}{3}$$

Find the area bounded by the curve $y = x - 5$, the ordinates between $x = 0$ and $x = 5$.

Soln: Here, $y = x - 5$

So, By formula, we have,

$$A = \int_a^b y \cdot dx$$

$$A = \int_0^5 (x-5)\cdot dx$$

Int. wrt 'x'

$$A = \left.\frac{x^2}{2} - 5x\right]_0^5$$

$$A = \left[\frac{25}{2} - 25\right] - [0]$$

$$A = \left[\frac{25-50}{2}\right]$$

$$A = \left|\frac{-25}{2}\right|$$

$$A = \frac{25}{2} \text{ sq. units.}$$

3) Find the volume generated by rotati[ng] the curve $y = x+1$ above the x-a[xis] between ordinates $x=0$ and $x=2$.

Solⁿ: Here, $y = (x+1)$

By formula, we have,

$$\text{Volume} = \pi \int_a^b y^2 \, dx$$

$$\text{Volume} = \pi \int_0^2 (x+1)^2 \, dx$$

$$\cancel{V = \pi \left[\frac{x^2}{2} + x\right]_0^2}$$

$$V = \pi \int_0^2 (x^2 + 2x + 1) \, dx$$

$$V = \pi\left[\frac{x^3}{3} + \frac{\cancel{2}x^2}{\cancel{2}} + x\right]_0^2$$

$$V = \pi\left[\frac{8}{3} + 4 + 2\right] - \pi\left[0\right]$$

$$V = \pi\left[\frac{8}{3} + 6\right]$$

$$V = \pi\left[\frac{8+18}{3}\right]$$

$$V = \frac{\pi 26}{3}$$

$$V = \frac{26\pi}{3} \text{ cubic units}$$

3) Find the volume generated by rotating the curve $y = \sqrt{x^2+5x}$ between $x = 1$ and $x = 2$ about x-axis.

Soln: $\text{Volume} = \pi\int_a^b y^2 \cdot dx$

$$= \pi\int_1^2 (\sqrt{x^2+5x})^2 \cdot dx$$

$$= \pi\int_1^2 (x^2+5x) \cdot dx$$

$$= \pi\left[\frac{x^3}{3} + \frac{5x^2}{2}\right]_1^2$$

$$\text{Volume} = \pi\left[\frac{2^3}{3} + \frac{5\times 2^2}{2}\right] - \pi\left[\frac{1}{3} +\right.$$

$$= \pi\left[\frac{8}{3} + 10\right] - \pi\left[\frac{2+15}{6}\right.$$

$$= \pi\left[\frac{8+30}{3}\right] - \pi\left[\frac{17}{6}\right]$$

$$= \frac{38\pi}{3} - \frac{17\pi}{6}$$

$$= \frac{76\pi - 17\pi}{6}$$

$$= \frac{59\pi}{6} \text{ cubic units.}$$

1) Find the area bounded by the curve $y = x^2+1$, x-axis and the ordinates $x = 1$, $x = 3$.

Sol^n: Here, $y = x^2+1$, $x = 1$, $x = 3$

$$\text{Area} = \int_a^b y \cdot dx$$

$$= \int_1^3 (x^2+1) \cdot dx.$$

$$= \left.\frac{x^3}{3} + x\right]_1^3$$

$$= \left[\frac{3\cancel{2}^{2}}{\cancel{3}} + 3\right] - \left[\frac{1}{3} + 1\right]$$

$$= [9 + 3] - \left[\frac{1+3}{3}\right]$$

$$= 12 - \frac{4}{3}$$

$$= \frac{36 - 4}{3}$$

$$= \frac{32}{3} \text{ sq. units.}$$

Find the area bounded by the curve $y = 3x$, the x-axis and the ordinates between $x = 1$ & $x = 2$.

Sol^n: Here, $y = 3x$, $x = 1$, $x = 2$

$$\text{Area} = \int_a^b y \cdot dx$$

$$A = \int_1^2 3x \cdot dx$$

$$A = \left.\frac{3x^2}{2}\right]_1^2$$

$$A = \left[\frac{3 \cdot \cancel{2}^{2}}{\cancel{2}}\right] - \left[\frac{3}{2}\right] = 6 - \frac{3}{2} = \frac{12 - 3}{2}$$

$A =$ ~~6 sq. units~~ $= \frac{9}{2}$ sq. units

7) Find the volume generated by rotating curve $y = x+2$ about x-axis between x and $x = 2$.

Soln: Here, $y = x+2$, $x = 0$, $x = 2$

$$\text{volume} = \pi \int_a^b y^2 \cdot dx$$

$$V = \pi \int_0^2 (x+2)^2 \cdot dx$$

$$V = \pi \left[\frac{x^2}{2} + 2x \right]_0^2$$

$$V = \pi \left[\frac{\cancel{2^2}}{\cancel{2}} + 2\times 2 \right] - \cancel{\pi \left[\frac{0^2}{2} + 2\times 0 \right]}$$

$$V = \pi [6]$$

$V = 6\pi$ cubic units.

$$\text{Volume} = \pi \int_a^b y^2 \cdot dx$$

$$= \pi \int_0^2 (x+2)^2 \cdot dx$$

$$= \pi \int_0^2 (x^2 + 4x + 4x) \cdot dx$$

$$= \pi \left[\frac{2x^3}{3} + \frac{4x^2}{2} + 4x \right]_0^2$$

$$\text{Volume} = \pi\left[\frac{2^3}{3} + \frac{4\times 2^2}{\cancel{2}} + 4\times 2\right] - \pi\left[0\right]$$

$$\text{Volume} = \pi\left[\frac{8}{3} + \frac{16}{2} + 8\right]$$

$$= \pi\left[\frac{8+16+24}{3}\right]$$

$$= \pi\left[\frac{48}{3}\right] \qquad \frac{56\pi}{3}$$

$$= \cancel{8}\,16\pi$$

$\frac{8}{3} + 16,$ $\frac{8+48}{3}$

Find the area between the curves $y^2 = 6x$ and $x^2 = 6y$.

Solⁿ: Here, $y^2 = 6x$, $x^2 = 6y$

Consider, $x^2 = 6y$

$$y = \frac{x^2}{6}$$

Now, Subtituting $y = \frac{x^2}{6}$ in $y^2 = 6x$

$$\left(\frac{x^2}{6}\right)^2 = 6x$$

$$\frac{x^4}{36} = 6x$$

$x^4 = 216x$

$x^4 - 216x = 0$

$x(x^3 - 216) = 0$

$\cancel{x^3 - 216 = \frac{0}{x}}$ Now,

$x = 0$ and $x^3 =$

$\cancel{x^3 - 216 = 0}$ $x = 0$, $x =$

$\cancel{x^3 - 216 = 0}$. $x = 0$, , $x =$

$\cancel{x^2 = 0}$ ~~and~~

We have,

$$\text{Area} = \int_a^b y \cdot dx$$

$$A = \int_0^6 \sqrt{6x} \cdot dx$$

$$A = \sqrt{6}\int_0^6 \sqrt{x} \cdot dx$$

$$= \sqrt{6} \times \frac{2}{3} x^{3/2}\Big]_0^6$$

$$= \left[\sqrt{6} \times \frac{2}{3} 6^{3/2}\right] - [0]$$

$$= \sqrt{6} \times \frac{2}{3} \sqrt{6}^3$$

$$= \frac{2}{3} \sqrt{6}^4$$

$$= \frac{2}{3} \sqrt{(6^2)^2}$$

$A = \frac{2}{3} \times 6^2$

$A = \frac{2}{\cancel{3}} \times \cancel{36}^{12}$

$A = 24$ sq. units.

9) Find the volume of the solid generated by revolving the curve $y = \cos x$ between the ordinates $x = 0$ & $x = \pi/2$ about x-axis.

Soln: Here, $y = \cos x$, $x = 0$, $x = \pi/2$

Now,

$$\text{Volume} = \pi \int_a^b y^2 . dx$$

$$= \pi \int_0^{\pi/2} . \cos^2 x . dx$$

$$= \pi \int_0^{\pi/2} \cos^2 x . dx$$

$$= \pi \int_0^{\pi/2} \left(\frac{1 + \cos 2x}{2} \right) . dx$$

Int. wrt 'x',

$$V = \pi \left[x + \frac{\sin 2x}{2} \right]_0^{\pi/2}$$

$$V = \frac{\pi}{2}\left[\frac{\pi}{2} + \frac{\sin 2\times\pi/2}{2}\right] - [0]$$

$$V = \frac{\pi}{2}\left[\frac{\pi}{2} + \frac{\sin 180}{2}\right]$$

$$V = \frac{\pi}{2}\left[\frac{\pi}{2} - \frac{\sin 0}{2}\right]$$

$$V = \frac{\pi^2}{4} \text{ cubic units.}$$

10) Find the area between the curve $y =$ [illegible] $x^2 - 5$ and about x-axis.

Sol: If any point lies on the x-axis th[illegible] $y = 0$.

Then,

$$y = 6x - x^2 - 5$$

$$0 = 6x - x^2 - 5$$

$$x^2 - 6x + 5 = 0$$

$$x^2 - 5x - x + 5 = 0$$

$$x(x-5) - 1(x-5) = 0$$

$$(x-1)(x-5) = 0$$

$$x = 1, \quad x = 5$$

DIFFERENTIAL EQUATIONS

Differential Equations.

- Formation of DE. by eliminating arbitrary constants
- Solution of Differential Equations.
  - Variable Seperable method
  - Linear Differential Equation (LDE)

An equation that involves one or more derivatives with dependent and independent variables are called differential equation.

Order of Differential Equation:

The highest derivative present in the given differential equation is called order of the differential equation.

Degree of Differential Equation:

The maximum powers of hig order derivative is called ~~order~~ degree of Differential Equation.

Examples:

$$\left(\frac{dy}{dx}\right)^2 + 4e^x = \log y \rightarrow O \rightarrow 1, D \rightarrow 2$$

$$\left(\frac{d^3y}{dx^3}\right) + \left(\frac{d^2y}{dx^2}\right)^5 = 8y + \sin x, \; O \rightarrow 3, D -$$

$$\left(\frac{d^2y}{dx^2}\right) + 5\left(\frac{dy}{dx}\right)^2 + \tan y = \log x, \; O \rightarrow 2, D$$

$$\frac{d^4y}{dx^4} + \left(\frac{d^2y}{dx^2}\right)^5 + xy = c; \; O \rightarrow 4, D \rightarrow$$

$$\left(\frac{d^3y}{dx^3}\right)^5 + 3\left(\frac{dy}{dx}\right)^6 + e^x = \frac{d^2y}{dx^2} + \sin y,$$

$$D \rightarrow 4, O \rightarrow 3$$

Formation of Differential Equation by Eliminating Arbitrary Constants:

Examples:

1) $x^2 + y^2 = a^2$.

Diff^n wrt 'x'.

$$2x + 2y \cdot \frac{dy}{dx} = 0$$

$$\cancel{2}y \frac{dy}{dx} = -\cancel{2}x$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$\frac{dy}{dx} + \frac{x}{y} = 0$$

2) $y = x^2 + ax$

Diff^n wrt 'x'.

$$\frac{dy}{dx} = 2x + a \quad \text{—} (1)$$

We have,

$$y = x^2 + ax$$

$$y - x^2 = ax$$

$$\frac{y - x^2}{x} = a \quad \text{—} (2)$$

Substituting eqn (2) in eqn (1), we have,

$$\frac{dy}{dx} = 2x + \frac{y - x^2}{x}$$

$$\frac{dy}{dx} = \frac{2x^2 + y - x^2}{x}$$

$$\frac{dy}{dx} = \frac{x^2 + y}{x}$$

$$x\frac{dy}{dx} = x^2 + y$$

$$x\frac{dy}{dx} - x^2 - y = 0$$

3) $xy = ae^x + be^{-x} + x$

Diff^n wrt 'x'.

$$x\frac{dy}{dx} + y \cdot 1 = ae^x - be^{-x} + 1$$

$$x\frac{dy}{dx} + y = ae^x - be^{-x} + 1$$

Again, Diff^n wrt 'x'.

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{dy}{dx} = ae^x + be^{-x} + 0$$

$$x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = ae^x + be^{-x} \quad \text{---(1)}$$

Now,

$$xy = ae^x + be^{-x} + x$$

$$xy - x = ae^x + be^{-x} \quad \text{---(2)}$$

Substituting eqn (2) in eqn(1), we have,

$$x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = xy - x$$

$$x\frac{d^2y}{dx^2} + \frac{2dy}{dx} - xy + x = 0$$

$$y^2 = a^2x^3$$

Diff^n wrt 'x'.

$$2y\frac{dy}{dx} = a^2 3x^2$$

$$2y\frac{dy}{dx} = \frac{y^2}{x^{\cancel{3}}} \times 3\cancel{x^2} \quad \left[\begin{array}{l} y^2 = a^2x^3 \\ a^2 = \frac{y^2}{x^3} \end{array}\right]$$

$$2y\frac{dy}{dx} = \frac{3y^2}{x}$$

$$\frac{dy}{dx} = \frac{3y^{\cancel{2}}}{2\cancel{y}x}$$

$$\frac{dy}{dx} = \frac{3y}{2x}$$

$$\frac{dy}{dx} - \frac{3y}{2x} = 0$$

5) $y^2 = 4ax$

Diff^n w.r.t 'x'

$$2y\frac{dy}{dx} = 4a \qquad \left[\begin{array}{l} y^2 = 4ax \\ a = \dfrac{y^2}{4x} \end{array}\right]$$

$$2y\frac{dy}{dx} = \cancel{4} \times \frac{y^2}{\cancel{4}x}$$

$$2y\frac{dy}{dx} = \frac{y^2}{x}$$

$$\frac{dy}{dx} = \frac{y^{\cancel{2}}}{2\cancel{y}x}$$

$$\frac{dy}{dx} - \frac{y}{2x} = 0$$

6) Eliminate the arbitrary constants a & b from the equation $y = a\cos mx +$ $\sin mx$.

Soln: $y = a\cos mx + b\sin mx$

Diff^n w.r.t 'x'.

$$\frac{dy}{dx} = -a\sin mx \cdot \frac{d(mx)}{dx} + b\cos mx$$

$$\frac{dy}{dx} = -am\sin mx + bm\cos mx$$

Again, Diff^n w.r.t 'x'

$$\frac{d^2y}{dx^2} = -am^2\cos mx - bm^2\sin mx$$

$$\frac{d^2y}{dx^2} = -m^2 (a\cos mx + b\sin mx)$$

$$\frac{d^2y}{dx^2} = -m^2 y$$

$$\frac{d^2y}{dx^2} + m^2 y = 0$$

$-be^{-x}\,d(-x)$

7) $y = ae^{x} + be^{-x}$

Diff^n w.r.t 'x'.

$$\frac{dy}{dx} = ae^{x} - be^{-x}$$

$\cancel{\frac{d^2y}{dx^2}=}$ Again, Diff^n w.r.t 'x'.

$$\frac{d^2y}{dx^2} = ae^{x} + be^{-x}$$

$$\frac{d^2y}{dx^2} - y = 0$$

8) $y = a\cos(x+b)$

Diff^n w.r.t 'x'.

$$\frac{dy}{dx} = \cancel{-a\sin(x} - a\sin(x+b)\cdot\frac{d(x+b)}{dx}$$

$$\frac{dy}{dx} = -a\sin(x+b)(1+0)$$

$$\frac{dy}{dx} = -a\sin(x+b)$$

H/W

10) $x^2 + y^2 = 4a$

Diff^n w.r.t 'x'.

$$2x + 2y\frac{dy}{dx} = a\left(\frac{dy}{dx}\right)$$

$$2x + 2y\frac{dy}{dx} = \frac{x^2+y^2}{4}\left[\frac{dy}{dx}\right]$$

$$2x + 2y\frac{dy}{dx} - \frac{x^2+y^2}{4} \times \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\left[2y - \frac{x^2+y^2}{4}\right] + 2x = 0$$

11) $x^2 + y^2 + 2gx = 0$

Diff^n w.r.t 'x'.

$$2x + 2y\frac{dy}{dx} + 2g = 0$$

$$\left[\begin{aligned} x^2+y^2+2gx &= \\ 2gx &= -x^2-y \\ 2g &= -(x^2 \end{aligned}\right.$$

$$2x + 2y\frac{dy}{dx} + 2\left[\frac{-x^2-y^2}{x}\right] = 0$$

SOLUTION OF DIFFERENTIAL EQUATION BY VARIABLE SEPERABLE METHOD.

Examples:

1) $\frac{dy}{dx} - \frac{y}{x} = 0$

$\frac{dy}{dx} = \frac{y}{x}$

$dy = \frac{y}{x} \cdot dx$

$\frac{1}{y} \cdot dy = \frac{1}{x} \cdot dx$

Integrate w.r.t 'x'.

$\int \frac{1}{y} \cdot dy = \int \frac{1}{x} \cdot dx$

$\log y = \log x$

$\log y - \log x = c$

2) $\cos^2 y \cdot dx + \sin^2 x \cdot dy = 0$

$\cos^2 y \cdot dx = -\sin^2 x \cdot dy$

$\frac{1}{\sin^2 x} dx = -\frac{1}{\cos^2 y} \cdot dy$

Integrate b.s;

$\int \operatorname{cosec}^2 x \cdot dx = -\int \sec^2 y \cdot dy$

$-\cot x = -\tan y + c$

$-\cot x + \tan y = c$

$\tan y - \cot x = 0$

$\sin x \cdot \cos y \cdot dy + \sin y \cdot \cos x \cdot dx = 0$

$\sin x \cdot \cos y \cdot dy = -\sin y \cos x \cdot dx = 0$

$\frac{\cos y}{\sin y} dy = -\frac{\cos x}{\sin x} \cdot dx$

Integrate both sides,

$\cot y \cdot dy = -\cot x \cdot dx$

Integrate b.s,

$$\int \cot y \cdot dy = -\int \cot x \cdot dx$$

$$\log(\sin y) = -\log(\sin x) + c$$

$$\log(\sin y) + \log(\sin x) = c$$

i) $(xy^2 + x) \cdot dy + (x^2y + y) \cdot dx = 0$

$$\cancel{x(y^2+1) \cdot dy = y(x^2+1) \cdot dx}$$

$$(xy^2 + x) dy = -(x^2y + y) \cdot dx$$

$$x(y^2 + 1) \cdot dy = -y(x^2+1) \cdot dx$$

$$\frac{y^2+1}{y} \cdot dy = -\frac{(x^2+1)}{x} \cdot dx$$

Integrate both sides,

$$\int \frac{y^2+1}{y} \cdot dy = -\int \frac{x^2+1}{x} \cdot dx$$

$$\int \left(\frac{y^2}{y} + \frac{1}{y}\right) \cdot dy = -\int \left(\frac{x^2}{x} + \frac{1}{x}\right) \cdot dx$$

$$\int \left(y + \frac{1}{y}\right) dy = -\int \left(x + \frac{1}{x}\right) dx$$

$$\frac{y^2}{2} + \log y = -\frac{x^2}{2} - \log x + c$$

$$\frac{y^2}{2} + \log y + \frac{x^2}{2} + \log x = c$$

5) $(xy^2 + x) \cdot dx + (x^2y + y) \cdot dy = 0$

$$x(y^2+1) \cdot dx = -y(x^2+1) \cdot dy$$

$$\frac{x}{(x^2+1)} \cdot dx = \frac{-y}{(y^2+1)} \cdot dy$$

Integrate both sides.

$$\int \frac{x}{x^2+1} \cdot dx = \int \frac{-y}{(y^2+1)} \cdot dy$$

Put $t = x^2+1$, Put $p = y^2+1$

Diff^n w.r.t 'x'. Diff^n w.r.t 'y'.

$\frac{dt}{dx} = 2x$ $\frac{dp}{dy} = 2y$

$\frac{dt}{2} = x \cdot dx$ $\frac{dp}{2} = y \cdot dy$

Now, $\int \frac{1}{t} \cdot \frac{dt}{2} = -\int \frac{1}{p} \cdot \frac{dt}{2}$

$$\frac{1}{2}\int \frac{1}{t} \cdot dt = -\frac{1}{2}\int \frac{1}{p} \cdot dt$$

$$\frac{1}{2}\log t = -\frac{1}{2}\log p + c$$

$$\frac{1}{2}\log(x^2+1)+\frac{1}{2}\log(y^2+1) = C$$

d) $3e^x \cdot \tan y + (1+e^x)\sec^2 y \cdot \frac{dy}{dx} = 0$

$$3e^x \cdot \tan y \cdot dx = -(1+e^x)\sec^2 y \cdot dy = 0$$

$$\frac{3e^x}{(1+e^x)} \cdot dx = -\frac{\sec^2 y}{\tan y} \cdot dy$$

Integrate both sides,

$$3\int \frac{1}{(1+e^x)} \cdot e^x \cdot dx = -\int \frac{1}{\tan y} \cdot \sec^2 y \cdot dy$$

Put $p = 1+e^x$, $t = \tan y$

Diff^n w.r.t. x. Diff^n w.r.t 'y'.

$\frac{dp}{dx} = e^x$ $\qquad \frac{dt}{dy} = \sec^2 y$

$dp = e^x \cdot dx$ $\qquad dt = \sec^2 y \cdot dy$.

Now, $3\int \frac{1}{p}\,dp = -\int \frac{1}{t} \cdot dt$

$$3\log p + \log t = c$$

$$3\log(1+e^x) + \log(\cancel{\sec^2 y}\ \tan y) = c$$

7) $\frac{dy}{dx} - \frac{x}{y} = 0$

$\frac{dy}{dx} = \frac{x}{y}$

$\therefore \; y \cdot dy = x \cdot dx$

Integrate both sides.

$\int y \cdot dy = \int x \cdot dx$

$\frac{y^2}{2} = \frac{x^2}{2}$

$\frac{y^2}{2} - \frac{x^2}{2} = c$

$\tan^{-1}x \cdot dx + (1+x^2)e^{2y} \cdot dy = 0$

$\tan^{-1}x \cdot dx = -(1+x^2) \cdot e^{2y} \cdot dy$

$\frac{\tan^{-1}x}{1+x^2} \cdot dx = -e^{2y} \cdot dy$

Int. ~~w.r.t~~ both sides,

$\int \frac{\tan^{-1}x}{1+x^2} \cdot dx = -\int e^{2y} \cdot dy$

~~Put~~ $\cancel{t = \frac{1}{1+x^2}}$ $\rightarrow$ $\cancel{dt = \tan^{-1}x \cdot dx}$

Diff^n ~~w.r.t x~~

$\cancel{\frac{dt}{dx} = \tan^{-1}x}$

Now,

$$\cancel{\int \theta \frac{1}{t} \cdot dt}$$

$$\int \frac{\tan^{-1}x}{1+x^2} \cdot dx = -\int e^{2y} \cdot dy$$

Put $t = \tan^{-1}x$

Diffⁿ w.r.t 'x'.

$$\frac{dt}{dx} = \frac{1}{1+x^2}$$

$$dt = \frac{1}{1+x^2} \cdot dx$$

Now, $\int t \cdot dt = -\int e^{2y} \cdot dy$

$$\frac{t^2}{2} = -\frac{e^{2y}}{2} + c$$

$$\frac{t^2}{2} + \frac{e^{2y}}{2} = c$$

9) April/May-16

tan⁻¹ $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ .dx

$$\frac{dy}{1+y^2} = \frac{dx}{1+x^2}$$

Integrate both sides,

$$\int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2}$$

$$\int \frac{1}{1+y^2} \cdot dy = \int \frac{1}{1+x^2} \cdot dx$$

$$\tan^{-1} y = \tan^{-1} x + c$$

$$\tan^{-1} y - \tan^{-1} x = c$$

10) $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$

$$\sec^2 x \tan y \, dx = -\sec^2 y \tan x \, dx$$

$$\therefore \frac{\sec^2 x}{\tan x} \cdot dx = -\frac{\sec^2 y}{\tan y} \cdot dy$$

Integrate both sides,

$$\int \frac{\sec^2 x}{\tan x} \cdot dx = -\int \frac{\sec^2 y}{\tan y} \cdot dy$$

Put $t = \tan x$, Put $p = \tan y$

Diff^n w.r.t 'x', Diff^n w.r.t 'y'.

$$\frac{dt}{dx} = \sec^2 x, \quad \frac{dp}{dy} = \sec^2 y$$

$$\therefore dt = \sec^2 x \cdot dx, \quad dp = \sec^2 y \cdot dy$$

Now, $\int \frac{1}{t} \cdot dt = -\int \frac{1}{p} \cdot dp$

$$\log t = -\log p + c$$

$$\log t + \log p = c$$

1) $\frac{dy}{dx} = e^{3x+4y}$, that $y=0$ & $x=0$.

Sol^n:

$$\frac{dy}{dx} = e^{3x+4y}$$

$$\frac{dy}{dx} = e^{3x} \cdot e^{4y}$$

$$dy = e^{3x} \cdot e^{4y} \cdot dx$$

$$\frac{1}{e^{4y}} \cdot dy = e^{3x} \cdot dx$$

Integrate both sides,

$$\int \frac{1}{e^{4y}} \cdot dy = \int e^{3x} \cdot dx$$

$$\int e^{-4y} \cdot dy = \int e^{3x} \cdot dx$$

$$\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} + c \quad \text{——— (1)}$$

Where, $x=0$, $y=0$

$$-\frac{e^{0}}{4} = \frac{e^{0}}{3} + c$$

$$\frac{-1}{4} = \frac{1}{3} + C$$

$$\frac{-1}{4} - \frac{1}{3} = C$$

$$\frac{-3-4}{12} = C$$

$$\frac{-7}{12} = C$$

From (1),

$$-\frac{e^{-4y}}{4} = \frac{e^{3x}}{3} - \frac{7}{12}$$

(2) A-m-17

$$x(y^2+1)\,dx + y(x^2+1)\,dy = 0$$

$$x(y^2+1)\,dx = -y(x^2+1)\,dy$$

$$\frac{x}{(1+x^2)}\cdot dx = \frac{-y}{(y^2+1)}\cdot dy$$

Integrate both sides,

$$\int \frac{1}{1+x^2}\cdot x\cdot dx = -\int \frac{1}{y^2+1}\, y\, dy$$

Put $t = 1+x^2$, $p = y^2+1$

Diff^n w.r.t 'x'. Diff^n w.r.t 'y'.

$$\frac{dt}{dx} = 2x \qquad \frac{dp}{dy} = 2y$$

$dt = 2x \cdot dx$, $dp = 2y \cdot dy$

$\frac{dt}{2} = x \cdot dx$, $\frac{dp}{2} = y \cdot dy$

So, $\int \frac{1}{t} \cdot \frac{dt}{2} = -\int \frac{1}{p} \cdot \frac{dp}{2}$

$\frac{1}{2} \int \frac{1}{t} \cdot dt = -\frac{1}{2} \int \frac{1}{p} \cdot dp$

$\frac{1}{2} \log t = -\frac{1}{2} \log p + c$

$\frac{1}{2} \log t + \frac{1}{2} \log p = c$

$\frac{1}{2} \log(1+x^2) + \frac{1}{2} \log(1+y^2) = c$

(3) $\frac{dy}{dx} = 3x^2 - 2x + 5$, where $x = 1$ & $y = 2$

$dy = (3x^2 - 2x + 5) \cdot dx$

Integrate both sides,

$\int 1 \cdot dy = \int (3x^2 - 2x + 5) \cdot dx$

$y = \frac{\cancel{3}x^3}{\cancel{3}} - \frac{\cancel{2}x^2}{\cancel{2}} + 5x + c$

$y = x^3 - x^2 + 5x + c$

When, $x = 1$ & $y = 2$

$2 = \cancel{1} - \cancel{1} + 5 + c$

$2 - 5 = c = \boxed{c = -3}$

SOLUTION OF LINEAR DIFFERENTIAL EQ

The first order / LDE will be the form $\frac{dy}{dx} + Py = Q$

where P and Q are the functions

The solution of LDE is obtained by Integrating Factor Method.

The Integrating Factor of LDE is.
$I.F = e^{\int P.dx}$ where, P is co-efficient

Multiply both sides integrating factor the given LDE, then integrate both sides respect to 'x'.

$$\boxed{\log_e^e = 1, \quad e^{\log} = 1, \quad e^{\log x} = x}$$

Examples:

1) $\frac{dy}{dx} + y.\tan x = \sec x \longrightarrow (1)$

$\frac{dy}{dx} + Py = Q$

Here, $P = \tan x$, $Q = \sec x$

$I.F = e^{\int P.dx}$

$= e^{\int \tan x . dx}$

$= e^{\log(\sec x)}$

$I.F = \sec x$

Multiply I.F both sides in eqn (1),

$\sec x \left[\frac{dy}{dx} + y.\tan x\right] = \sec^2 x$

$\sec x \frac{dy}{dx} + y.\tan x \sec x = \sec^2 x$

$\frac{d(\sec x . y)}{dx} = \sec^2 x$

Integrate both sides,

$\int \frac{d(\sec x . y)}{dx} . dx = \int \sec^2 x . dx$

$\sec x . y = \tan x + c$

$y \sec x = \tan x + c$

2) $\frac{dy}{dx} + 2y = e^{-3x}$ —— (1)

$\frac{dy}{dx} + Py = Q$

Here, $P = 2, \quad Q = e^{-3x}$

$I.F = e^{\int P.dx}$

$= e^{\int 2.dx}$

$= e^{2x}$

Multiply both sides by I.F in eq

$e^{2x}\left[\frac{dy}{dx} + 2y\right] = e^{2x}.e^{-3x}$

$e^{2x}.\frac{dy}{dx} + e^{2x}.2.y = e^{-3x}.e^{2x}$

$\frac{d(e^{2x}.y)}{dx} = e^{(-3x+2x)}$

Integrate both sides w.r.t 'x'.

$\int \frac{d(e^{2x}.y)}{dx}.dx = \int e^{-x}.dx$

$e^{2x}.y = \frac{e^{-x}}{-1} + c$

$y.e^{2x} = \frac{e^{-x}}{-1} + c$

3)

3) $x \cdot \frac{dy}{dx} + y = x^2$

Divide by x throughout,

$$\frac{\cancel{x}}{\cancel{x}} \cdot \frac{dy}{dx} + \frac{1}{x} \cdot y = \frac{x^2}{x}$$

$$\frac{dy}{dx} + \frac{1}{x} y = x \longrightarrow (1)$$

$$I.F. = e^{\int P.dx}$$

$$= e^{\int \frac{1}{x} . dx}$$

$$= e^{\log x}$$

$$= x$$

Multiply I.F both sides, of eqn (1).

$$x \left[\frac{dy}{dx} + \frac{1}{x} \cdot y \right] = x^2$$

$$x \frac{dy}{dx} + y = x^2$$

$$\frac{d(xy)}{dx} = x^2$$

Integrate both sides wrt 'x'.

$$\int \frac{d(xy)}{\cancel{dx}} \cdot \cancel{dx} = \int x^2 \cdot dx$$

$$xy = \frac{x^3}{3} + c$$

$$xy - \frac{x^3}{3} = c$$

4) $x\frac{dy}{dx} + y = \log x$

~~Divide the~~ Divide x throughout,

$$\frac{x}{x}\frac{dy}{dx} + \frac{1}{x} \times y = \log x \times \frac{1}{x}$$

$$\frac{dy}{dx} + \frac{1}{x} \times y = \frac{\log x}{x} \longrightarrow (1)$$

Here, $P = \frac{1}{x}$, $Q = \frac{\log x}{x}$

$$I.F = e^{\int P \cdot dx}$$

$$= e^{\int \frac{1}{x} \cdot dx}$$

$$= e^{\log x}$$

$$I.F = x$$

Multiplying I.F both sides of eqn (1).

$$x\left[\frac{dy}{dx} + \frac{1}{x} \cdot y\right] = \frac{\log x}{\cancel{x}} \times \cancel{x}$$

$$x\frac{dy}{dx} + y = \log x$$

$$\frac{d(xy)}{dx} = \log x$$

Integrate both sides, we get,

$$\int \frac{d(xy)}{dx} \cdot dx = \int \log x \cdot dx$$

$$xy = \int \log x \, 1 \cdot dx$$

~~xy=~~

$$\int uv \cdot dx = u\int v \cdot dx - \int\int v \cdot dx \cdot \frac{d(u)}{dx} \cdot dx$$

$$xy = \log x \int 1 \cdot dx - \int\int 1 \cdot dx \cdot \frac{d(\log x)}{dx} \cdot dx$$

$$xy = x\log x - \int \cancel{x} \cdot \frac{1}{\cancel{x}} \cdot dx$$

$$xy = x\log x - \int 1 \cdot dx$$

$$xy = x\log x - x + c$$

$$xy - x\log x + x = c$$

(ii).

$$\frac{dy}{dx} + 2xy = e^{-x^2} \text{ ——— (1)}$$

Here,

$$P = 2x, \quad Q = e^{-x^2}$$

Now,

$$I.F = e^{\int P \cdot dx}$$

$$= e^{\int 2x \cdot dx}$$

$$= e^{2\int x \cdot dx}$$

$$= e^{\cancel{2} \times \frac{x^2}{\cancel{2}}}$$

$$= e^{x^2}$$

Multiply I.F both sides of eqn.(i),

$$e^{x^2}\left[\frac{dy}{dx} + 2xy\right] = e^{-x^2}\cdot e^{x^2}$$

$$e^{x^2}\frac{dy}{dx} + y\cdot e^{x^2}\cdot 2x = e^{-x+x^2}$$

$$\frac{d(e^{x^2}\cdot y)}{dx} = e^{0}$$

$$\frac{d(e^{x^2}\cdot y)}{d} = 1$$

Integrate both sides wrt x.

$$\int \frac{\cancel{d}(e^{x^2}\cdot y)}{\cancel{dx}}\cancel{dx} = \int 1\cdot dx$$

$$e^{x^2}\cdot y = x + c$$

$$e^{x^2}\cdot y - x = c$$

6) $\frac{dy}{dx} + \tan x\, y = \cos x$ ——(1)

Here, $P = \tan x$, $Q = \cos x$.

Now, $I.F = e^{\int \tan x\, dx}$

$= e^{\log(\sec x)}$

$= \sec x$

Multiply I.F both sides, we get,

$$\sec x\left[\frac{dy}{dx}+\tan x\cdot y\right]=\sec x\cdot\cos x$$

$$\sec x\frac{dy}{dx}+\sec x\cdot\tan x\cdot y=\frac{1}{\cancel{\cos x}}\times\cancel{\cos x}$$

$$\frac{d(\sec x\cdot y)}{dx}=1$$

Integrate both sides,

$$\int\cancel{d}\frac{(\sec x\cdot y)}{\cancel{dx}}\cdot\cancel{dx}=\int 1\cdot dx$$

$$\sec x\cdot y-x=c$$

7) $\frac{dy}{dx}+\frac{1}{x}\cdot y=\sin x.$

Here, $P=\frac{1}{x}$, $Q=\sin x$

$$I.F=e^{\int P\cdot dx}$$

$$=e^{\int\frac{1}{x}\cdot dx}$$

$$=e^{\log x}$$

$$=x$$

Multiply I.F both sides,

$$x\left[\frac{dy}{dx}+\frac{1}{x}\cdot y\right]=\sin x\times x$$

$$x\frac{dy}{dx}+y=x.\sin x$$

~~Integrate both sides wrt 'x'.~~

$$\frac{d(x\cdot y)}{dx}=x\sin x$$

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E

Integrate wrt 'x'.

$$\int\frac{d(xy)}{\cancel{dx}}\cdot\cancel{dx}=\int x\cdot\sin x\cdot dx$$

$$xy=x\int\sin x\,dx-\iint\sin x\cdot dx\,\frac{d(x)}{dx}$$

$$xy=-x\cos x+\int\cos x\cdot dx$$

$$xy=-x\cos x+\sin x$$

$$xy+x\cos x-\sin x=c$$

8) $x\frac{dy}{dx} + y = \cos^2 x$

~~Hence~~

~~Multiply to~~

Divide x throughout,

$$\frac{x}{x}\frac{dy}{dx} + \frac{1}{x}\cdot y = \frac{1}{x}\cdot\cos^2 x$$

$$\frac{dy}{dx} + \frac{1}{x}\cdot y = \frac{\cos^2 x}{x} \quad \text{—— (1)}$$

Here, $p = \frac{1}{x}$, $Q = \frac{\cos^2 x}{x}$

Now $I.F = e^{\int \frac{1}{x}\cdot dx}$

$$= e^{\log x}$$

$$= x$$

Multiplying both sides by x of eqn (1),

$$x\left[\frac{dy}{dx} + \frac{1}{x}\cdot y\right] = \cancel{x} \times \frac{\cos^2 x}{\cancel{x}}$$

$$x\frac{dy}{dx} + y = \cos^2 x$$

$$\frac{d(xy)}{dx} = \cos^2 x$$

Integrate b.s, w.r.t 'x'.

$$\int \frac{d(xy)}{\cancel{dx}}\cancel{dx} = \int \cos^2 x \, dx$$

$$xy = \int \frac{1+\cos 2x}{2} \cdot dx$$

$$xy = \frac{1}{2}\int (1+\cos 2x) \, dx$$

$$xy = \frac{1}{2}\int 1 \cdot dx + \frac{1}{2}\int \cos 2x \cdot dx$$

$$\cancel{xy = \frac{1}{2}\, x + \frac{\sin 2x}{2}}$$

$$\cancel{xy = \frac{x + \sin 2x}{4}}$$

$$xy = \frac{1}{2}x + \frac{1}{2}\frac{\sin 2x}{2}$$

$$xy = \frac{1}{2}x + \frac{\sin 2x}{4}$$

$$xy = \frac{2x + \sin 2x}{4} + C$$

$$xy - \frac{2x + \sin 2x}{4} = C$$

Valued

3/4/18

Applications of Differentiation

TANGENT AND NORMAL:

The equation of the tangent of the curve, $y = f(x)$ at (x_1, y_1) is given by,

$$y_1 - y_1 = \frac{dy}{dx}_{(x_1, y_1)} \times (x - x_1)$$

where, $\frac{dy}{dx}_{(x_1, y_1)}$, is called slope of the tangent.

Eq^n of the Normal:

The eq^n of the normal to the curve $y = f(x)$ at (x_1, y_1) is given by,

$$y - y_1 = \frac{-1}{\frac{dy}{dx}_{(x_1, y_1)}} \cdot (x - x_1)$$

where, $\frac{-1}{\frac{dy}{dx}_{(x_1, y_1)}}$ is called slope of the normal.

1) Find the equation of the tangent to curve, $y = x^2 - 3x + 4$ at $(2,2)$.

Sol^n:

Here, $y = x^2 - 3x + 4$

Diff^n w.r.t 'x'.

$$\frac{dy}{dx} = 2x - 3 + 0$$

$$\frac{dy}{dx} = 2x - 3$$

Now, $\frac{dy}{dx}$ at $(2,2)$ is,

$$\frac{dy}{dx_{(2,2)}} = 2 \times 2 - 3$$

$$= 4 - 3$$

$$\frac{dy}{dx_{(2,2)}} = 1$$

The eq^n of the tangent at $(2,2)$ is,

$$y - y_1 = \frac{dy}{dx_{(2,2)}} \cdot (x - x_1)$$

$$y - 2 = 1(x - 2)$$

$$y - 2 = x - 2$$

$$x - y = 0$$

2) Find the equation of tangent and normal to the curve $\sqrt{x} + \sqrt{y} = 5$ at $(4, 9)$.

Soln:

Here, ~~y=~~ $\sqrt{x} + \sqrt{y} = 5$

Diff^n wrt 'x'.

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \cdot \frac{dy}{dx} = 0$$

$$\frac{1}{\cancel{2}\sqrt{y}} \cdot \frac{dy}{dx} = \frac{-1}{\cancel{2}\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{-\sqrt{y}}{\sqrt{x}}$$

Now, $\frac{dy}{dx}$ at $(4, 9)$ is,

$$\frac{dy}{dx} = \frac{-\sqrt{9}}{\sqrt{4}}$$

$$\boxed{\frac{dy}{dx} = \frac{-3}{2}}$$

We have,

The eq^n of the tangent is,

$\cancel{y - y_1 = \frac{}{\frac{dy}{dx}}}$

$$y - y_1 = \frac{dy}{dx_{(x_1,y_1)}}(x - x_1)$$

$$y - 9 = -\frac{3}{2}(x - 4)$$

$$2y - 18 = -3x \cancel{8} + \cancel{8}12$$

$$2y - 18 = -3x + 12$$

$$3x - 2y - 18 \cancel{+} 12 = 0$$

$$3x - 2y - 30 = 0$$

And,

The eq^n of the normal is, given b

$$y - y_1 = \frac{-1}{\frac{dy}{dx_{(x_1,y_1)}}} \times (x - x_1)$$ 3)

$$y - 9 = \frac{\cancel{-}1}{\cancel{-}3/2}(x - 4)$$

$$y - 9 = \frac{2}{3}(x - 4)$$

$$3(y - 9) = 2x - 8$$

$$3y - 27 = 2x - 8$$

$$2x - 8 - 3y + 27 = 0$$

$$2x - 3y + 19 = 0$$

Find the equation of tangent and normal for the following curves:

1) $y = 2x^2 - 1$ at $(-1, 1)$

2) $x^2 + y^3 = 13$ at $(2, 3)$

3) $9x^2 - y = 6$ at $(-1, 3)$

4) $3y = 5x^2 + 1$ at $(2, 1)$

5) $xy^2 = 16$ at $(4, -2)$

6) $y = \cos 3x$ at $x = \pi/4$.

3) Here,

$\cancel{y = 2x^2}$

$$9x^2 - y = 6$$

Diff^n wrt 'x'.

$$18x - \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = 18x$$

Now, $\frac{dy}{dx}$ at $(-1, 3)$ is,

$$\frac{dy}{dx} = 18 \times -1$$

$$\frac{dy}{dx} = -18$$

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$$18x - \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = 18x$$

Now, $\frac{dy}{dx}$ at $(-1, 3)$ is,

$$\frac{dy}{dx} = 18 \times -1$$

$$\frac{dy}{dx} = -18$$

Eq^n of the tangent at $(-1,3)$ is,

$$y - y_1 = \frac{dy}{dx(x_1,y_1)} (x - x_1)$$

$$y - 3 = -18(x+1)$$

$$y - 3 = -18x - 18$$

$$18x + y + 15 = 0$$

Eq^n of the normal at $(-1,3)$ is,

$$y - y_1 = \frac{-1}{dy/dx(x_1,y_1)} (x - x_1)$$

$$y - 3 = \frac{-1}{-18} (x+1)$$

$$-18y + 54 = -x - 1$$

$$x - 18y + 55 = 0$$

4) $y = 2x^2 - 1$

Diff^n w.r.t 'x'.

$$\frac{dy}{dx} = 4x.$$

Now,

$\frac{dy}{dx}$ at $(-1,1)$,

$$\frac{dy}{dx} = -4$$

Eq^n of the tangent at $(-1, 1)$ is,

$$y - y_1 = \frac{dy}{dx\,(x_1 y_1)} (x - x_1)$$

$$y - y_1 = -4(x+1)$$

$$y - 1 = -4x - 4$$

$$4x + y - 1 + 4 = 0$$

$$4x + y + 3 = 0$$

Eq^n of the normal at $(-1, 1)$ is,

$$y - y_1 = \frac{-1}{dy/dx\,(x_1 y_1)} (x - x_1)$$

$$y - 1 = \frac{\cancel{-}1}{\cancel{-}4} (x+1)$$

$$4y - 4 = x + 1$$

$$x - 4y + 4 + 1 = 0$$

$$x - 4y + 5 = 0$$

5) Here, $y = \cos 3x$ at $x = \pi/4$

$y = \cos 3\pi/4$

$y = \cos 135^\circ$

$y = \cos(180 - 45)$

$y = -\cos 45$

$y = -1/\sqrt{2}$

The point are: $(\pi/4, -1/\sqrt{2})$

Now, $y = \cos 3x$

Diffⁿ wrt 'x'.

$\frac{dy}{dx} = -\sin 3x \frac{d(3x)}{dx}$

$\frac{dy}{dx} = -3\sin 3x$

We have, $\frac{dy}{dx}$ at $\pi/4 = x$

$\frac{dy}{dx} = -3\sin 3\pi/4$

$\frac{dy}{dx} = -3\sin 135$

$\frac{dy}{dx} = -3\sin(180-45)$

$\frac{dy}{dx} = -3\sin 45$

$\frac{dy}{dx} = -3/\sqrt{2}$

Eqⁿ of the tangent at $(\pi/4, -1/\sqrt{2})$,

$$y - y_1 = \frac{dy}{dx_{(\pi/4, \frac{-1}{\sqrt{2}})}} (x - x_1)$$

$$y + \frac{1}{\sqrt{2}} = \frac{-3}{\sqrt{2}} (x - \pi/4)$$

$$\sqrt{2}y + 1 = -3x + 3\pi/4$$

$$3x + \sqrt{2}y + 1 - \frac{3\pi}{4} = 0$$

Eqⁿ of the normal at $(\pi/4, -1/\sqrt{2})$.

$$y - y_1 = \frac{-1}{dy/dx_{(x_1, y_1)}} (x - x_1)$$

$$y + \frac{1}{\sqrt{2}} = \frac{\cancel{-}1}{\cancel{-}3/\sqrt{2}} (x - \pi/4)$$

$$y + \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{3} (x - \pi/4)$$

$$3y + \frac{3}{\sqrt{2}} = \sqrt{2}x - \sqrt{2}\pi/4$$

$$\sqrt{2}x - 3y - \frac{\sqrt{2}\pi}{4} - \frac{3}{\sqrt{2}} = 0$$

8) $xy^2 = 16$ at $(4, -2)$

Here, $xy^2 = 16$

Diff^n w.r.t 'x'.

$$2xy\frac{dy}{dx} + y^2 = 0$$

$$2xy\frac{dy}{dx} = -y^2$$

$$\frac{dy}{dx} = \frac{-y^2}{2xy}$$

Now,

$\frac{dy}{dx}$ at $(4, -2)$ is,

$$\frac{dy}{dx} = \frac{-(-2)^2}{2 \times 4 \times (-2)}$$

$$\frac{dy}{dx} = \frac{\cancel{-4}^{\,1}}{\cancel{-16}_{\,4}}$$

$$\frac{dy}{dx} = \frac{1}{4}$$

Eq^n of the tangent at $(4,-2)$ is,

$$y - y_1 = \frac{dy}{dx_{(x_1,y_1)}}(x - x_1)$$

$$y + 2 = \frac{1}{4}(x - 4)$$

$$4y + 8 = x - 4$$

$$x - 4y - 12 = 0$$

Eq^n of the normal at $(4,-2)$ is,

$$y - y_1 = \frac{-1}{dy/dx_{(4,-2)}}(x - x_1)$$

$$y + 2 = \frac{-1}{1/4}(x - 4)$$

$$y + 2 = -4(x - 4)$$

$$y + 2 = -4x + 16$$

$$4x + y + 2 - 16 = 0$$

$$4x + y - 14 = 0$$

9) Find the equation to the tangent to the curve $y = 2x^3 - 3$ at $(1, 3)$.

Here,

$$y = 2x^3 - 3$$

Diff^n wrt x.

$$\frac{dy}{dx} = 6x^2.$$

Now, $\frac{dy}{dx}$ at $(1, 3)$

$$\frac{dy}{dx} = 6 \times 1^2$$

$$\frac{dy}{dx} = 6$$

Eq^n of the ~~tangent~~ at $(1, 3)$

$$y - y_1 = \frac{dy}{dx}_{(1,3)} (x - x_1)$$

$$y - 3 = 6(x - 1)$$

$$y - 3 = 6x - 6$$

$$6x - y + 3 - 6 = 0$$

$$6x - y - 3 = 0$$

Eqⁿ of the normal at (1, 3)

$$y - y_1 = \frac{-1}{dy/dx_{(1,3)}} (x - x_1)$$

$$y - 3 = \frac{-1}{6} (x - 1)$$

$$6y - 18 = -x + 1$$

$$x + 6y - 18 - 1 = 0$$

$$x + 6y - 19 = 0$$

10) ~~Fi~~ $y = 3x^2 + 4x$ at $(1, 2)$

Here, $y = 3x^2 + 4x$

Diffⁿ w.r.t 'x'.

$$\frac{dy}{dx} = 6x + 4$$

$\frac{dy}{dx}$ at $(1, 2)$,

$$\frac{dy}{dx} = 6 \times 1 + 4$$

$$\frac{dy}{dx} = 6 + 4$$

$$\frac{dy}{dx} = 10$$

Eqn of the tangent at (1, 2)

$$y - y_1 = \frac{dy}{dx}_{(1,2)} (x - x_1)$$

$$y - 2 = 10(x - 1)$$

$$y - 2 = 10x - 10$$

$$10x - y + 2 - 10 = 0$$

$$10x - y - 8 = 0$$

Eqn of the normal at (1, 2)

$$y - y_1 = \frac{-1}{dy/dx_{(1,2)}} (x - x_1)$$

$$y - 2 = \frac{-1}{10} (x - 1)$$

$$10y - 20 = -x + 1$$

$$x + 10y - 20 - 1 = 0$$

$$x + 10y - 21 = 0$$

11) $y = x^2 + 1$ at $(1,2)$

Diff^n wrt 'x',

$$\frac{dy}{dx} = 2x$$

$\frac{dy}{dx}$ at $(1,2)$,

$$\frac{dy}{dx} = 2$$

Eq^n of the tangent at $(1,2)$

$$y - y_1 = \frac{dy}{dx_{(x_1,y_1)}}(x - x_1)$$

$$y - 2 = 2(x-1)$$

$$y - 2 = 2x - 2$$

$$2x - y + 2 - 2 = 0$$

$$2x - y = 0$$

Eq^n of the normal,

$$y - y_1 = \frac{-1}{dy/dx_{(1,2)}}(x - x_1)$$

$$y - 2 = \frac{-1}{2}(x-1)$$

$$2y - 4 = -x + 1$$

$$x + 2y - 4 - 1 = 0$$

$$x + 2y - 5 = 0$$

RATE MEASURE

VELOCITY:

If 's' is the displacement and 't' is the time taken then,

velocity, $v = \frac{ds}{dt}$

ACCELERATION:

If 's' is the displacement 't' is is the time taken and v is the velocity,

$$\text{Acceleration} = \frac{d\left(\frac{ds}{dt}\right)}{dt} = \frac{d^2s}{dt^2} \text{ or } \frac{dv}{dt}$$

Q) The displacement of a moving particle along a straight line is given by, $s = 4t^3 - 6t^2 + t - 3$. mts. Find the velocity and acceleration at $t = 3s$.

Solⁿ: Here,

$$s = 4t^3 - 6t^2 + t - 3$$

Diffⁿ wrt 't'.

Velocity, (v) $\frac{ds}{dt} = 12t^2 - 12t + 1$

$$\frac{ds}{dt} = 12t^2 - 12t + 1 \longrightarrow (1)$$

Velocity at 3secs.

Velocity, $\frac{ds}{dt} = 12 \times 3^2 - 12 \times 3 + 1$

$= 108 - 36 + 1$

$= 73 \, m/s$

Again Diffⁿ (1) w.r.t 't'.

Acceleration, $\frac{d^2y}{dx} = 24t - 12 + 0$

Acclⁿ at $t = 3$secs

Acclⁿ $= \frac{d^2s}{dt^2} = 24 \times 3 - 12$

$= 72 - 12$

$= 60 \, m/s^2$

2. The equation of the motion is given by $S = 2t^2 - 3t + 1$. Find the velocity and acceleration when $t = 2$secs.

Solⁿ: Here, $S = 2t^2 - 3t + 1$

Diffⁿ w.r.t 't'.

Velocity (v),

$\frac{ds}{dt} = 4t - 3$ —— (1)

Velocity at 2secs.

Velocity, $\frac{ds}{dt} = 4\times2-3$

$$\frac{ds}{dt} = 8-3$$

$$\frac{ds}{dt} = 5\,m/s$$

Again Diff^n eqn (1) wrt 't'.

Accel^n,

$$\frac{d^2s}{dt^2} = 4\cdot m/s^2$$

3. The displacement of a moving particle is given by $s = 5\cos 2t$, find the velocity and acceleration at $t = \pi/4$ secs.

Sol^n: Here,

$$s = 5\cos 2t$$

Diff^n wrt 't'.

Velocity,

$$\frac{ds}{dt} = -5\sin 2t\,\frac{d(2t)}{dt}$$

$$\frac{ds}{dt} = -10\sin 2t \quad \text{—— (1)}$$

$\frac{\cancel{ds}}{\cancel{dt}}$ ~~velocity~~ at $\pi/4$,

Velocity $t = \pi/4$

$$\frac{ds}{dt} = -10 \sin \cancel{2} \times \frac{\pi}{\cancel{4}_2}$$

$$\frac{ds}{dt} = -10 \sin 90$$

$$\frac{ds}{dt} = -10 \, m/s$$

Now, ~~Accl~~ Again Diff^n wrt to 't' for eqn (i),

Accl^n, $$\frac{ds^2}{dt} = -20 \cos t$$

Now,

Accl^n $t = \pi/4$

$$\frac{ds^2}{dt^2} = -20 \cos \pi/4$$

$$\frac{ds^2}{dt^2} = 0 \, m/s^2$$

MAXIMA AND MINIMA

1) Let $y = f(x)$

2) Find $\frac{dy}{dx}$

3) Equate $\frac{dy}{dx} = 0$ & Evaluate, let the value of $x = a$.

4) Find $\frac{d^2y}{dx^2}$, put $x = a$ in $\frac{d^2y}{dx^2}$

(1) (a) If $\frac{d^2y}{dx^2} > 0$ or +ive then the given function is MINIMA at $x = a$.

(b) If $\frac{d^2y}{dx^2} < 0$ or −ive then the given function is MINIMA at $x = a$.

(c) If $\frac{d^2y}{dx^2} = 0$ then the given function is neither MAXIMA NOR MANIMA..

but the point $\boxed{x = a}$ is called POINT OF INFLEXTION.

Example

1) Find the maximum and minimum value of the function of $y = x^3 - 3x^2 - 9x + 7$.

Sol^n:
$$y = x^3 - 3x^2 - 9x + 7$$

Diff^n w.r.t 'x'.

$$\frac{dy}{dx} = 3x^2 - 6x - 9 + 0$$

$$\frac{dy}{dx} = 3x^2 - 6x - 9 \quad \text{——— (1)}$$

Equate $\frac{dy}{dx} = 0$

$$3x^2 - 6x - 9 = 0$$

$$3(x^2 - 2x - 3) = 0$$

$$x^2 - 2x - 3 = 0$$

$$x^2 - (3-1)x - 3 = 0$$

$$x^2 - 3x + x - 3 = 0$$

$$x(x-3) + 1(x-3) = 0$$

$$(x+1)(x-3) = 0$$

$$\boxed{x = -1, 3}$$

...ue of the
...unctio

Again, diffⁿ eqⁿ (1) w.r.t 'x'.

$$\frac{d^2y}{dx^2} = 6x - 6$$

Put $x = 3$ in $\frac{d^2y}{dx^2}$

$$\frac{d^2y}{dx^2} = 6 \times 3 - 6$$

$$\frac{d^2y}{dx^2} = 12 > 0$$

∴ The given eqⁿ is MINIMA at $x = 3$.

Put $x = -1$ in $\frac{d^2y}{dx^2}$

$$\frac{d^2y}{dx^2} = 6 \times -1 - 6$$

$$\frac{d^2y}{dx^2} = -12 < 0$$

∴ The given eqⁿ is MAXIMA at $x = -1$.

MAXIMUM value at $x = 3$

$$y = x^3 - 3x^2 - 9x + 7$$

$$y = 3^3 - 3 \times 3^2 - 9 \times 3 + 7$$

$$= \cancel{27} - \cancel{27} - 27 + 7$$

$$= -20$$

MINIMUM Value at $x = -1$

$$y = x^3 - 3x^2 - 9x + 7$$

$$y = (-1)^3 - 3(-1)^2 - 9(-1) + 7$$

$$y = -1 - 3 + 9 + 7$$

$$y = -4 + 16$$

$$y = 12$$

2. Find the maximum and minimum val of the function, $y = 2x^3 - 3x^2 - 36x + 10$.

Sol^n: $y = 2x^3 - 3x^2 - 36x + 10$ —— (1)

Diff^n w.r.t 'x'.

$$\frac{dy}{dx} = 6x^2 - 6x - 36 \quad \text{—— (2)}$$

Equate $\frac{dy}{dx} = 0$

$$6x^2 - 6x - 36 = 0.$$

$$x^2 - x - 6 = 0$$

$$x^2 - (3-2)x - 6 = 0$$

$$x^2 - 3x + 2x - 6 = 0$$

$$x(x-3) + 2(x-3) = 0$$

$$(x+2)(x-3) = 0$$

$$\therefore \boxed{x = -2, 3}$$

Again, Diffⁿ eqn (2) w.r.t 'x'.

$$\frac{d^2y}{dx^2} = 12x - 6$$

Put $x = 3$ in $\frac{d^2y}{dx^2}$

$$\frac{d^2y}{dx^2} = 12 \times 3 - 6$$

$$\frac{d^2y}{dx^2} = 30 > 0$$

$\therefore$ The given eqⁿ is ~~MAX~~MINIMA at $x = 3$

Put $x = -2$ in $\frac{d^2y}{dx^2}$

$$\frac{d^2y}{dx^2} = 12 \times -2 - 6$$

$$= -24 - 6$$

$$\frac{d^2y}{dx^2} = -30 < 0$$

$\therefore$ The given eqⁿ is MAXIMA at $x = -2$

The minimum value at $x=3$

$y = 2x^3 - 3x^2 - 36x + 10$

$y = 2\times 3^3 - 3\times 3^2 - 36\times 3 + 10$

$y = 54 - 27 - 108 + 10$

$y = -71$

The maximum value at $x = -2$

$y = 2x^3 - 3x^2 - 36x + 10$

$y = 2\times(-2)^3 - 3\times(-2)^2 - 36\times(-2) + 10$

$y = -16 - 12 + 72 + 10$

$y = 54$

3. Find the maximum value of xe^{-x}

Sol^n: $y = xe^{-x}$

Diff^n wrt 'x'.

$$\frac{dy}{dx} = xe^{-x}\frac{d(-x)}{dx} + e^{-x}.1$$

$$\frac{dy}{dx} = -xe^{-x} + e^{-x}$$

$$\frac{dy}{dx} = e^{-x}[1-x] \quad \text{——} \quad (1)$$

Equate $\frac{dy}{dx} = 0$

$$e^{-x}(1-x) = 0$$

$$1 - x = 0$$

$$\boxed{x = 1}$$

Again, Diff^n eqn (1) w.r.t 'x'.

$$\frac{d^2y}{dx^2} = e^{-x}(0-1) + (1-x)e^{-x}\frac{d(-x)}{dx}$$

$$\frac{d^2y}{dx^2} = -e^{-x} - (1-x)e^{-x}$$

$$\frac{d^2y}{dx^2} = -e^{-x}[1+1-x]$$

$$= -e^{-x}[2-x]$$

Put $x = 1$ in $\frac{d^2y}{dx^2}$

$$\frac{d^2y}{dx^2} = -e^{-1}(2-1)$$

$$\frac{d^2y}{dx^2} = -e^{-1}.1$$

$$\frac{d^2y}{dx^2} = -\frac{1}{e}$$

$$\frac{d^2y}{dx^2} = \frac{-1}{e} < 0$$

The given function is MAXIMA at $x=1$

MAXIMUM value at $x=1$

$$y = xe^{-x}$$
$$y = 1 \cdot e^{-1}$$
$$y = 1/e$$

Q. Find the maximum and minimum of the functions.

$$y = x^3 + 6x^2 - 15x + 5$$
$$y = x + \frac{1}{x}$$
$$y = x^3 - 3x + 4$$

State ~~that~~ $\frac{\log x}{x}$ is maximum and find maximum value.

Prove that the function x^x is maximum at $x = 1/e$.