Unit - 2. (27) 18/8/17

Vectors.

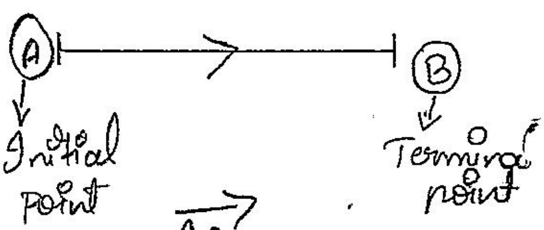
1] **Scalars**:- A physical quantity which is having only magnitude but not direction. is called scalars.

Ex:- Mass, volume, area etc.

2] **Vectors**:- A physical quantity which is having both magnitude and direction is called vector.

Ex:- Displacement, velocity, Acceleration, Force.

Representation of a vector.



Vector means a directed line segments. and it is represented by $\overrightarrow{AB}$ where A is initial point & B is terminal point

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Types of vectors.

1] **unit vector**:- A vector having magnitude 1 or unity is called unit vector. and it is denoted by

$\hat{a}$.

The formula for unit vector is given by.

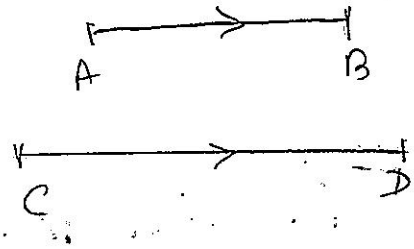
$$\boxed{\hat{a} = \frac{\vec{a}}{|\vec{a}|}}$$

(2)

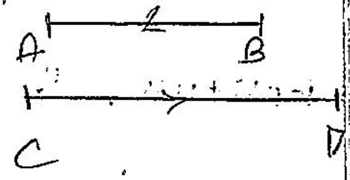
Null vector:- A vector having magnitude 0 is called null vector. And it is denoted by 0 or $\vec{0}$

In null vector initial point and terminal point coincides.

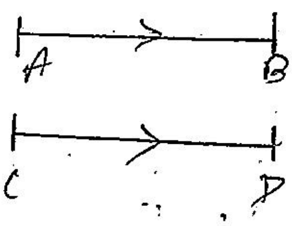
Like vectors:- 2 vectors having same direction with different magnitudes is called like vectors.



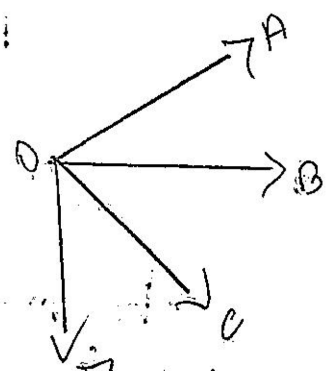
Unlike vector:- 2 vectors having oposite direction with different magnitude is called unlike vectors.



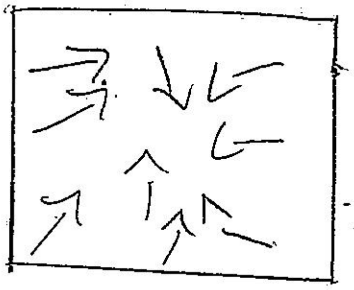
Equal vectors:- 2 vectors having same direction with same magnitude is called Equal vectors.



Co-initial vector:- Many vectors having only one initial point with different terminal points is called coinitial vector.



7] Coplaner vector:- Many vectors lying in the same plane is called coplaner vectors.



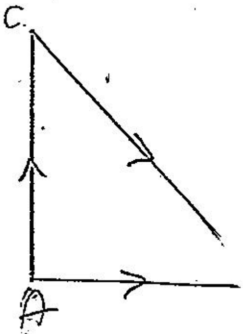
Operations on vectors.

1] Scalar multiplications:- If A is a vector and 'm' is a scalar then $m \times \vec{a}$ is also a vector.

The magnitude of $m \cdot \vec{a}$ is m times as that of ~~vector~~ $\vec{a}$.

The direction will remains same if m is positive, the direction will become opposite if m is negative.

2] Addition of 2 Vectors:-

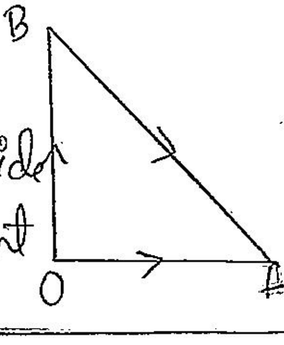


The addition of 2 vectors is the terminal point of the 1st vector. Must coincide with initial point of the 2nd vector.

The sum of the 2 vectors is obtained by

$\boxed{\vec{AB} + \vec{BC} = \vec{AC}}$

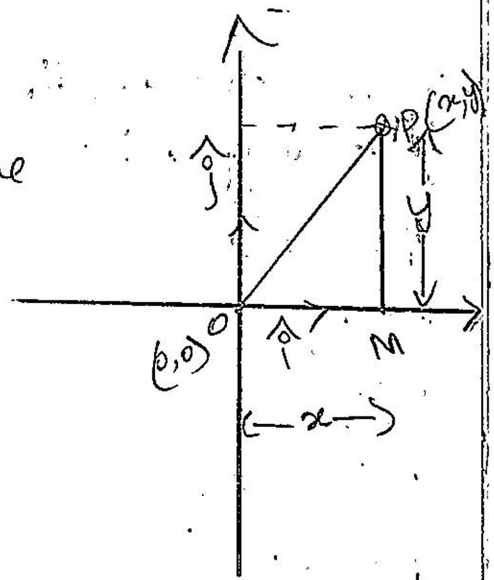
3] Subtraction of 2 Vectors:-



The Subtraction of 2 vectors is the initial point of the both vectors must coincide the difference is obtained by the terminal point of the second vector to the initial point of the first vector and it is obtained by

$$\boxed{\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}}$$

Position vector of a point in 2 dimension.



Let O be the origin in xy plane

Let $P(x, y)$ be any point on xy plane

Join ~~vector~~ $\overrightarrow{OP}$ draw $PM \perp OX$ then

The position vector of a point $P(x, y)$ is given by

$$\overrightarrow{OP} = \overrightarrow{OM} + \overrightarrow{MP}$$

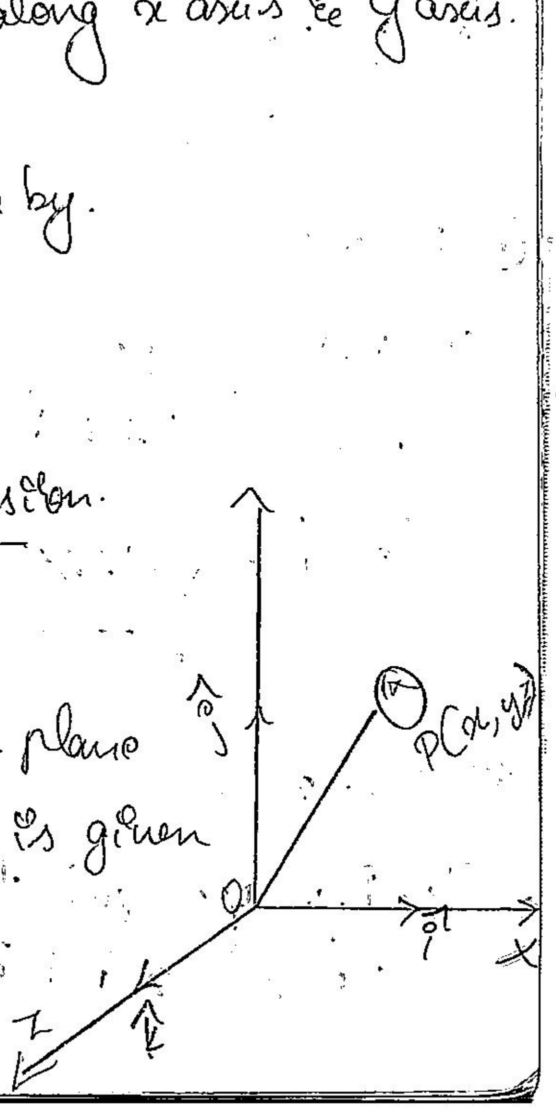
$$\boxed{\overrightarrow{OP} = x\hat{i} + y\hat{j}}$$

where $\hat{i}$ & $\hat{j}$ are the unit vectors along x axis & y axis respectively.

The magnitude of $\overrightarrow{OP}$ is given by.

$$\boxed{|\overrightarrow{OP}| = \sqrt{x^2 + y^2}}$$

Position Vector of a point in 3 dimension.



Let O be the origin

Let $P(x, y, z)$ be the point on xyz plane

The position vector of a point $P(x, y, z)$ is given by

$$\overrightarrow{OP} = x\hat{i} + y\hat{j} + z\hat{k}$$

where $\hat{i}, \hat{j}, \hat{k}$ are the unit vectors along ~~xyz~~ $x\ y\ z$ axis respectively.

The magnitude of ~~vector~~ $\overrightarrow{OP}$ is given by

$$|\overrightarrow{OP}| = \sqrt{x^2+y^2+z^2}$$

Problems

1) Find the position vector and the magnitude for the following points.

$P(3,5)$

$P(-2,3)$

$P(6,-1)$

$P(-8,-5)$

$P(1,-2,3)$

$P(-2,4,9)$

$P(6,-3,-7)$

$P(-3,7,9)$.

① $P(3,5)$

The P.V of a point $P(3,5)$

$$\overrightarrow{OP} = 3\hat{i} + 5\hat{j}.$$

$$|\overrightarrow{OP}| = \sqrt{3^2+5^2} = \sqrt{9+25}$$

$$= \sqrt{34}.$$

② $P(-2,3)$.

The P.V of a point $P(-2,3)$

$$\overrightarrow{OP} = -2\hat{i} + 3\hat{j}$$

$|\overrightarrow{OP}| = \sqrt{(-2)^2+(3)^2} = \sqrt{4+9}$ $\sqrt{13}$

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③ $P(6,-1)$

$\overrightarrow{OP} = 6\hat{i} - \hat{j}$

$|\overrightarrow{OP}| = \sqrt{(6)^2+(-1)^2} = \sqrt{36+1} = \sqrt{37}$

④ The P V of P(-8, -5)

$\overrightarrow{OP} = -8\hat{i} + -5\hat{j}$

$|\overrightarrow{OP}| = \sqrt{(-8)^2+(-5)^2} = \sqrt{64+25} = \sqrt{89}$

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⑤ $P(1,-2,3)$

The P V of $P(1,-2,3)$.

$\overrightarrow{OP} = 1\hat{i} - 2\hat{j} + 3\hat{k}$.

$|\overrightarrow{OP}| = \sqrt{(1)^2+(-2)^2+(3)^2} = \sqrt{1+4+9} = \sqrt{14}$.

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⑥ $P(-2,4,9)$

The P v of $P(-2,4,9)$

$\overrightarrow{OP} = -2\hat{i} + 4\hat{j} + 9\hat{k}$.

$|\overrightarrow{OP}| = \sqrt{(-2)^2+(4)^2+(9)^2} = \sqrt{4+16+81} = \cancel{\sqrt{109}}$

$\sqrt{101}$

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⑦ $P(6,-3,-7)$

The P v of $P(6,-3,-7)$

$\overrightarrow{OP} = 6\hat{i} - 3\hat{j} - 7\hat{k}$.

$|\overrightarrow{OP}| = \sqrt{(6)^2+(-3)^2+(-7)^2} = \sqrt{36+9+49} = \sqrt{94}$.

(8) $P(-3, 7, 9)$

The P V of point $P(-3, 7, 9)$

$\vec{OP} = -3\hat{i} + 7\hat{j} + 9\hat{k}$

$|\vec{OP}| = \sqrt{(-3)^2 + (7)^2 + (9)^2} = \sqrt{9 + 49 + 81} = \sqrt{139}$

Product of 2 Vectors

1] If $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$

$\vec{b} = 3\hat{i} + 2\hat{j} + 4\hat{k}$

$\vec{c} = 6\hat{i} - \hat{j} + 2\hat{k}$

Find 1) $\vec{a} + \vec{b} + \vec{c}$

2) $2\vec{a} + 3\vec{b}$

3) $\vec{a} - 3\vec{c}$

4) $4\vec{b} - 3\vec{a}$ also find its magnitude

5) $2\vec{b} + 3\vec{c}$

1) $\vec{a} + \vec{b} + \vec{c} = 2\hat{i} + \hat{j} + 3\hat{k} + 3\hat{i} + 2\hat{j} + 4\hat{k} + 6\hat{i} - \hat{j} + 2\hat{k}$

$\vec{a} + \vec{b} + \vec{c} = 11\hat{i} + 2\hat{j} + 9\hat{k}$

$|\vec{a} + \vec{b} + \vec{c}| = \sqrt{(11)^2 + (2)^2 + (9)^2}$

$= \sqrt{121 + 4 + 81}$

$= \sqrt{206}$

(2) $2\vec{a} + 3\vec{b}$

$2(2\hat{i} + \hat{j} + 3\hat{k}) + 3(3\hat{i} + 2\hat{j} + 4\hat{k})$

$4\hat{i} + 2\hat{j} + 6\hat{k} + 9\hat{i} + 6\hat{j} + 12\hat{k}$

$13\hat{i} + 8\hat{j} + 18\hat{k}$

$\therefore |2\vec{a} + 3\vec{b}| = \sqrt{(13)^2 + (8)^2 + (18)^2} = \sqrt{169 + 64 + 324}$

$= \sqrt{557}$

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(3) $\vec{a} - 3\vec{c}$.

$2\hat{i} + \hat{j} + 3\hat{k} - 3(6\hat{i} - \hat{j} + 2\hat{k})$

$2\hat{i} + \hat{j} + 3\hat{k} - 18\hat{i} + 3\hat{j} - 6\hat{k}$

$-16\hat{i} + 4\hat{j} - 3\hat{k}$. 256

$|\vec{a} - 3\vec{c}| = \sqrt{(-16)^2 + (4)^2 + (-3)^2}$

$= \sqrt{256 + 16 + 9}$

$= \sqrt{281}$

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(4) $4\vec{b} - 3\vec{a}$.

$4(3\hat{i} + 2\hat{j} + 4\hat{k}) - 3(2\hat{i} + \hat{j} + 3\hat{k})$

$= 12\hat{i} + 8\hat{j} + 16\hat{k} - 6\hat{i} - 3\hat{j} - 9\hat{k}$

$4\vec{b} - 3\vec{a} = 6\hat{i} + 5\hat{j} + 7\hat{k}$.

$\therefore |4\vec{b} - 3\vec{a}| = \sqrt{(6)^2 + (5)^2 + (7)^2} = \sqrt{36 + 25 + 49} = \sqrt{110}$

(5) $2\vec{b} + 3\vec{c}$.

$2(3\hat{i} + 2\hat{j} + 4\hat{k}) + 3(6\hat{i} - \hat{j} + 2\hat{k})$

$6\vec{i} + 4\hat{j} + 8\hat{k} + 18\vec{i} - 3\hat{j} + 6\hat{k}$.

$2\vec{b} + 3\vec{c} = 24\vec{i} + \hat{j} + 14\hat{k}$.

$$|2\vec{b} + 3\vec{c}| = \sqrt{(24)^2 + (1)^2 + (14)^2} = \sqrt{576 + 1 + 196}.$$

$$\sqrt{773}.$$

Product of 2 Vectors.

1] DOT Product (Scalar product).

Let

$\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\hat{k}$

$\vec{b} = b_1\vec{i} + b_2\hat{j} + b_3\vec{k}$.

$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$.

Ex 1:- $\vec{a} = 2\vec{i} + \hat{j} - 3\vec{k}$.

$\vec{b} = 6\vec{i} - 2\vec{j} + 5\vec{k}$.

$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$.

$a_1 = 2 \quad a_2 = 1 \quad a_3 = -3$

$b_1 = 6 \quad b_2 = -2 \quad b_3 = 5$

$\vec{a} \cdot \vec{b} = (2 \times 6) + (1 \times -2) + (-3 \times 5)$.

$12 - 2 - 15 = -5$ //

Properties of DOT product

1. ~~Pro~~ $\vec{a}\cdot\vec{b}$ is scalar quantity.

2. $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$

3. If $\vec{a}$ & $\vec{b}$ are non zero vectors.

$$\boxed{\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|}}$$

4. If $\theta = 90^0$.

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|}$$

$$\cos 90 = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|}$$

$$0 = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|}$$

$$\boxed{\vec{a}\cdot\vec{b} = 0}$$

Then the given 2 vectors are Perpendicular or.

ORTHOGONAL.

5. If $\theta = 0$.

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|}$$

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|}$$

$$1 = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|}$$

$\sin 0 = 0$
$\sin 90 = 1$
$\cos 0 = 1$
$\cos 90 = 0$.

$$\boxed{\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|}$$

then the given 2 vectors are parallel.

6) Projection of 1 vector on another vector.

(i) Projection of $\vec{a}$ on $\vec{b}$ $= \dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

(ii) Projection of $\vec{b}$ on $\vec{a}$ $= \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$

7) $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 1$

$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 0$

Problems

I) Find the DOT product for the following pair of vectors.

i) $\vec{a} = 8\hat{i} + 2\hat{j} - 5\hat{k}$, $\vec{b} = 5\hat{i} + 6\hat{j} - \hat{k}$

$\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$.

$\vec{a}\cdot\vec{b} = (8\times5) + (2\times6) + (-5\times-1)$

$= 40 + 12 + 5$

$\vec{a}\cdot\vec{b} = 57$.

(2) $\vec{a} = 3\hat{i} + \hat{j} - 5\hat{k}$. $\vec{b} = 5\hat{i} - 3\hat{j} + 2\hat{k}$

$\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$.

$\vec{a}\cdot\vec{b} = (3\times5) + (1\times-3) + (-5\times2)$

$\vec{a}.\vec{b} = 15 - 3 - 10$

$\vec{a}.\vec{b} = 2.$

(3) $\vec{a} = \hat{i} + 2\hat{j}$ · $\vec{b} = 2\hat{j} - \hat{k}$

$\vec{a} = 0\hat{i} + 2\hat{j} + 0\hat{k}$ $\quad b = 0\hat{i} + 2\hat{j} - \hat{k}$.

$\vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

$\vec{a}.\vec{b} = (1\times 0) + (2\times 2) + (0\times -1)$

$= 0 + 4 + 0$

$= 4$

(4) $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ $\quad \vec{b} = -\hat{i} + 3\hat{j} + 2\hat{k}$.

$\vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$.

$\vec{a}.\vec{b} = (1\times -1) + (2\times 3) + (-3\times 2)$

$\vec{a}.\vec{b} = -1 + 6 - 6$.

$\vec{a}.\vec{b} = -1$.

Home work

(5) $\vec{a} = \hat{i} - 4\hat{j} + 3\hat{k}$ $\quad \vec{b} = 5\hat{i} + 2\hat{j} + \hat{k}$

$\vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$.

$\vec{a}.\vec{b} = (\times 5) + (-4\times 2) + (3\times 1)$

$= 5 - 8 + 3$

$\vec{a}.\vec{b} = 0$

⑥ $\vec{a} = 6\hat{i} - \hat{j} + \hat{k}$ $\vec{b} = 7\hat{i} - \hat{k}$.

$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$

$\vec{a} \cdot \vec{b} = (6 \times 7) + (-1 \times 0) + (1 \times -1)$

$42 - 1$

$\vec{a} \cdot \vec{b} = 41$

Find the cosine of angle b/w following pair of vectors.

1] $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$.

2] $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$.

3] $\vec{a} = \hat{i} + \hat{j} + \hat{k}$. $\vec{b} = \hat{i} - \hat{j} + \hat{k}$.

4] $\vec{a} = \hat{i} + 2\hat{j} + 5\hat{k}$ $\vec{b} = \hat{i} - 7\hat{k}$.

5] $\vec{a} = \hat{i} + 2\hat{j}$. $\vec{b} = 2\hat{i} - 3\hat{k}$.

6] $\vec{a} = 6\hat{i} + 7\hat{j} - k$ $\vec{b} = \hat{i} - 7\hat{j} + 9\hat{k}$.

① $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$.

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} \quad \text{--- ①}$$

$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$.

$\vec{a} \cdot \vec{b} = (2 \times 1) + (-1 \times 2) + (1 \times 1)$

$= \cancel{2} - \cancel{2} + 1$

$\vec{a} \cdot \vec{b} = 1$

$\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$

$|\vec{a}| = \sqrt{(2)^2 + (-1)^2 + (1)^2}$

$|\vec{a}| = \sqrt{4+1+1}$

$|\vec{a}| = \sqrt{6}.$

$\vec{b} = \hat{i} + 2\hat{j} + \hat{k}.$

$|\vec{b}| = \sqrt{(1)^2 + (2)^2 + (1)^2}$

$|\vec{b}| = \sqrt{1+4+1}$

$|\vec{b}| = \sqrt{6}.$

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4. $\vec{a} = \hat{i} + 2\hat{j} + 5\hat{k}.$ $\quad \vec{b} = \hat{i} - 7\hat{k}.$

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|\,|\vec{b}|}$$

$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$

$\vec{a} \cdot \vec{b} = (1 \times 1) + (2 \times 0) + (5 \times -7)$

$1 + 0 - 35$

$= -34.$

$\vec{a} = \hat{i} + 2\hat{j} + 5\hat{k}$.

$|\vec{a}| = \sqrt{(1)^2 + (2)^2 + (5)^2}$.

$|\vec{a}| = \sqrt{1+4+25}$

$|\vec{a}| = \sqrt{30}$.

$\vec{b} = \hat{i} - 7\hat{k}$.

$|\vec{b}| = \sqrt{(1)^2 + (-7)^2}$

$|\vec{b}| = \sqrt{1+49}$

$|\vec{b}| = \sqrt{50}$.

From (1)

$$\cos\theta = \frac{-34}{\sqrt{30}\sqrt{50}}$$

$$\theta = \frac{-34}{\sqrt{1500}}$$

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(2) $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$.

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} \text{ ------ } (1).$$

$\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$.

$\vec{a}\cdot\vec{b} = (2\times1) + (3\times2) + (-1\times3)$

$\vec{a}\cdot\vec{b} = 2+6-3.$

$\vec{a}\cdot\vec{b} = 5.$

$\vec{a} = 2\vec{i}+3\vec{j}-\vec{k}.$

$|\vec{a}| = \sqrt{(2)^2+(3)^2+(-1)^2}$

$\sqrt{4+9+1}$

$\sqrt{14}.$

$\vec{b} = \vec{i}+2\vec{j}+3\vec{k}.$

$|\vec{b}| = \sqrt{(1)^2+(2)^2+(3)^2}$

$|\vec{b}| = \sqrt{1+4+9}.$

$\vec{b} = \sqrt{14}$

$\cos\theta = \dfrac{5}{\sqrt{14}\sqrt{14}},$

$\cos\theta = \dfrac{5}{(\sqrt{14})^2}$

$\cos\theta = \dfrac{5}{14}.$

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(3) $\vec{a} = \vec{i}+\vec{j}+\vec{k}$ $\quad \vec{b} = \vec{i}-\vec{j}+\vec{k}.$

$\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

$\vec{a}\cdot\vec{b} = a_1b_1+a_2b_2+a_3b_3$

$(1\times1)+(1\times-1)+(1\times1)$

$1 - 1 + 1$

$= 1$

$\vec{a} = \hat{i} + \hat{j} + \hat{k}$

$|\vec{a}| = \sqrt{(1)^2 + (1)^2 + (1)^2}$

$\sqrt{1+1+1}$

$|\vec{a}| = \sqrt{3}$

$\vec{b} = \hat{i} - \hat{j} + \hat{k}$

$|\vec{b}| \sqrt{(1)^2 - (1)^2 + (1)^2}$

$= \sqrt{1+1+1}$

$|\vec{b}| \sqrt{3}$

$\cos\theta = \dfrac{1}{\sqrt{3}\sqrt{3}}$

$= \dfrac{1}{(\sqrt{3})^2}$

$\cos\theta = \dfrac{1}{3}$

(5) $\vec{a} = \hat{i} + 2\hat{j} \cdot \vec{b} = 2\hat{i} - 3\hat{k}$

(6) $= \vec{a} = 6\hat{i} + 7\hat{j} - \hat{k}$ $\quad \vec{b} = \hat{i} - 7\hat{j} + 9\hat{k}$

Show that the following pair of vectors are orthogonal. (18)

i) $\vec{a} = 2\hat{i} - 5\hat{j} + \hat{k}$. $\vec{b} = 3\hat{i} + \hat{j} - \hat{k}$

$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$

$\vec{a} \cdot \vec{b} = (2 \times 3) + (-5 \times 1) + (1 \times -1)$

$= 6 - 5 - 1$

$= 0$ ∴ Thus given 2 vectors are orthogonal.

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(4) $(2, -3, 4)$ & $(-3, 2, 3)$

P.V of a point $(2, -3, 4)$

$\vec{OP} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

P.V of a point $(-3, 2, 3)$

$\vec{OQ} = -3\hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{OP} \cdot \vec{OQ} = (2 \times -3) + (-3 \times 2) + (4 \times 3)$

$-6 - 6 + 12$

$= 0$

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(2) $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ $\vec{b} = 2\hat{i} - 16\hat{j} + 5\hat{k}$.

$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$

$\vec{a} \cdot \vec{b} = (3 \times 2) + (1 \times -16) + (2 \times 5)$

$\vec{a} \cdot \vec{b} = 6 \cancel{+} - 16 + 10$

$= 0$

∴ They given 2 vectors are orthogonal.

(5) $(5,-1,2)$ & $(3,1,-7)$

P.V of a point $(5,-1,2)$.

$\vec{OP} = 5\hat{i} - 1\hat{j} + 2\hat{k}$

P.V of a point $(3,1,-7)$

$\vec{OQ} = 3\hat{i} + 1\hat{j} - 7\hat{k}$.

$\vec{OP} \cdot \vec{OQ} = (5\times3) + (-1\times1) + (2\times-7)$

$15 - 1 - 14$

$\vec{OP} \cdot \vec{OQ} = 0$.

$\therefore$ Thus the given 2 vectors are orthogonal.

Home work

cosine angle

(6) $\vec{a} = \hat{i} + 2\hat{j}$ $\quad \vec{b} = 2\hat{i} - 3k$.

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} \quad \text{--- (1)}$$

$\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$.

$(1\times2) + (2\times0) + (0\times-3)$

$2 + 0 - 0$

$\vec{a}\cdot\vec{b} = 2$

$\vec{a} = 2\hat{i} + \hat{j} - 3\hat{k}$

$|\vec{a}| = \sqrt{(2)^2 + (1)^2 + (-3)^2}$

$\sqrt{4+1+9}$

$\sqrt{14}$

From ①

Projection of $\vec{b}$ on $\vec{a} = \boxed{\dfrac{-5}{\sqrt{14}}}$

ICqN

CROSS PRODUCT (VECTOR PRODUCT)

Let

$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then

$$\vec{a} \times \vec{b} = \begin{array}{ccc} + & - & + \\ \end{array}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$= \hat{i}\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \hat{j}\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \hat{k}\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

Ex: $\vec{a} \times \vec{b}$

$\vec{a} = \hat{i} + 3\hat{j} + 2\hat{k}$

$\vec{b} = 5\hat{i} - \hat{j} + 3\hat{k}$

$$\begin{array}{ccc} + & - & + \\ \end{array}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 2 \\ 5 & -1 & 3 \end{vmatrix} = \hat{i}(9+2) - \hat{j}(3-10) + \hat{k}(-1-15)$$

$\vec{a} \times \vec{b} = 11\hat{i} + 7\hat{j} - 16\hat{k}$

③ $\vec{a} = \hat{i} + 2\hat{j} - 4\hat{k}$ $\vec{b} = 4\hat{i} + 2\hat{j} - \hat{k}$

~~Since the given 2 vectors are orthogonal~~ the

$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ $\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$ --- ①

$(1 \times 4) + (2 \times 2) + (-4 \times -1)$

$4 + 4 + 4$

$\vec{a} \cdot \vec{b} = 12$

$|\vec{b}| = \sqrt{(4)^2 + (2)^2 + (-1)^2}$

$|\vec{b}| = \sqrt{16 + 4 + 1}$

$|\vec{b}| = \sqrt{21}$

From ①

$$\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{12}{\sqrt{21}}$$

Projection of $\vec{b}$ on $\vec{a}$ $= \frac{\vec{a} \cdot \vec{b}}{\vec{a}}$

$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$

$(1 \times 4) + (2 \times 2) + (-4 \times 1)$

$4 + 4 + 4$

$\vec{a} \cdot \vec{b} = 12$

$|\vec{a}| = \sqrt{(1)^2+(2)^2+(-4)^2}$

$|\vec{a}| = \sqrt{1+4+16}$

$|\vec{a}| = \sqrt{5+16}$

$|\vec{a}| = \sqrt{21}$

From ①

$$\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|} = \frac{12}{\sqrt{21}}$$

PROPERTIES OF CROSS PRODUCT

1] Cross product is a vector.

2] $\vec{a}\times\vec{b} \neq \vec{b}\times\vec{a}$

3] $\vec{a}\times\vec{b} = -(\vec{b}\times\vec{a})$

4] If $\vec{a}$ & $\vec{b}$ are non zero vectors, then

$$\sin\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}$$

5] If $\theta = 0$.

$|\vec{a}\times\vec{b}| = 0$ or $\vec{a}\times\vec{b} = 0$

then the given vectors are parallel.

6] If $\theta = 90°$

then $|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|$.

the given 2 vectors are perpendicular.

7] unit vector for 2 vector.

$$\hat{n} = \frac{\vec{a}\times\vec{b}}{|\vec{a}||\vec{b}|}$$

⑧ The area of the triangle whose any two sides given then $A = \frac{1}{2}|\vec{a} \times \vec{b}|$

⑨ The Area of parallelogram whose Adjacent sides gives then $A = |\vec{a} \times \vec{b}|$

Find the cross product for the following pair of Vectors and also find their magnitude.

② $\vec{a} = \hat{i} + 2\hat{j} + 5\hat{k} \quad \vec{b} = 2\hat{i} + 4\hat{j} + 5\hat{k}$.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 5 \\ 2 & 4 & 5 \end{vmatrix} = \hat{i}\begin{vmatrix} 2 & 5 \\ 4 & 5 \end{vmatrix} - \hat{j}\begin{vmatrix} 1 & 5 \\ 2 & 5 \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix}$$

$$\vec{a} \times \vec{b} = \hat{i}(10-20) - \hat{j}(5-10) + \hat{k}(4-4)$$

$$\vec{a} \times \vec{b} = -10\hat{i} + 5\hat{j} + 0$$

$$|\vec{a} \times \vec{b}| = \sqrt{(-10)^2 + (5)^2} = \sqrt{100+25} = \sqrt{125}$$

① $\vec{a} = \hat{i} + \hat{j} - 7\hat{k} \quad \vec{b} = 3\hat{i} + 4\hat{j} - \hat{k}$.

③ $\vec{a} = 3\hat{i} - 6\hat{j} + 2\hat{k} \quad \vec{b} = \hat{i} - \hat{j} - \hat{k}$.

④ $\vec{a} = 6\hat{i} + 4\hat{j} + \hat{k} \quad \vec{b} = 7\hat{i} - \hat{j} + 3\hat{k}$.

⑤ $\vec{a} = 2\hat{i} - \hat{j} \quad \vec{b} = 4\hat{j} + \hat{k}$

⑥ $\vec{a} = 8\hat{i} + \hat{j} + 2\hat{k} \quad \vec{b} = 6\hat{i} + \hat{j} - 2\hat{k}$.

$\vec{a} = \hat{i} + \hat{j} - 7\hat{k} \qquad \vec{b} = 3\hat{i} + 4\hat{j} - \hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -7 \\ 3 & 4 & -1 \end{vmatrix} = \hat{i}\begin{vmatrix} 1 & -7 \\ 4 & -1 \end{vmatrix} - \hat{j}\begin{vmatrix} 1 & -7 \\ 3 & -1 \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix}$$

$$\vec{a} \times \vec{b} = \hat{i}(-1+28) - \hat{j}(-1+21) + \hat{k}(4-3)$$

$$27\hat{i} - 20\hat{j} + \hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{(27)^2 + (-20)^2 + (1)^2}$$

$$= \sqrt{729 + 400 + 1} = \sqrt{1130}$$

③ $\vec{a} = 3\hat{i} - 6\hat{j} + 2\hat{k} \qquad \vec{b} = \hat{i} - \hat{j} - \hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -6 & 2 \\ 1 & -1 & -1 \end{vmatrix} = \hat{i}\begin{vmatrix} -6 & 2 \\ -1 & -1 \end{vmatrix} - \hat{j}\begin{vmatrix} 3 & 2 \\ 1 & -1 \end{vmatrix} \hat{k}\begin{vmatrix} 3 & -6 \\ 1 & -1 \end{vmatrix}$$

$$\hat{i}(6+2) - \hat{j}(-3-2)\,\hat{k}(-3+6)$$

$$8\hat{i} + 5\hat{j} + 3\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{(8)^2 + (5)^2 + (3)^2}$$

$$= \sqrt{64 + 25 + 9} = \sqrt{98}$$

④ $\vec{a} = 6\hat{i} + 4\hat{j} + \hat{k} \qquad \vec{b} = 7\hat{i} - \hat{j} + 3\hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 4 & 1 \\ 7 & -1 & 3 \end{vmatrix} = \hat{i}\begin{vmatrix} 4 & 1 \\ -1 & 3 \end{vmatrix} - \hat{j}\begin{vmatrix} 6 & 1 \\ 7 & 3 \end{vmatrix} \hat{k}\begin{vmatrix} 6 & 4 \\ 7 & -1 \end{vmatrix}$$

$$\hat{i}(12+1)-\hat{j}(18-7)\,k\,(-6-28)$$
$$13\hat{i}-11\hat{j}-34k$$

$$|\vec{a}\times\vec{b}|=\sqrt{(13)^2+(-11)^2+(34)^2}$$

$$|\vec{a}\times\vec{b}|=\sqrt{169+121+1156}=\sqrt{1446}$$

(5) $\vec{a}=2\hat{i}-\hat{j}$ $\quad\vec{b}=4\hat{j}+\hat{k}$

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&-1&0\\0&4&1\end{vmatrix}=\hat{i}\begin{vmatrix}-1&0\\4&1\end{vmatrix}-\hat{j}\begin{vmatrix}2&0\\0&1\end{vmatrix}\,k\begin{vmatrix}2&-1\\0&4\end{vmatrix}$$

$$=\hat{i}(-1-0)-\hat{j}(2-0)\,k\,(8+0)$$
$$-\hat{i}-2\hat{j}+8k$$

$$|\vec{a}\times\vec{b}|=\sqrt{(-1)^2+(-2)^2+(8)^2}$$
$$=\sqrt{1+4+64}=\sqrt{69}$$

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(6) $\vec{a}=8\hat{i}+\hat{j}+2\hat{k}$ $\quad\vec{b}=6\hat{i}+\hat{j}-2\hat{k}$

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\8&1&2\\6&1&-2\end{vmatrix}=\hat{i}\begin{vmatrix}1&2\\1&-2\end{vmatrix}-\hat{j}\begin{vmatrix}8&2\\6&-2\end{vmatrix}\,k\begin{vmatrix}8&1\\6&1\end{vmatrix}$$

$$\hat{i}(-2-2)-\hat{j}(-16-12)\,k\,(8-6)$$
$$-4\hat{i}-28\hat{j}+k$$

$$|\vec{a} \times \vec{b}| = \sqrt{(-4)^2 + (-28)^2 + (2)^2}$$

$$\sqrt{16 + 784 + 4}$$

$$= \sqrt{804}$$

Find sin of the angle b/w for the following pair of vectors.

1. $\vec{a} = \hat{i} + \hat{j} - \hat{k}$. $\vec{b} = \hat{i} - \hat{j} - \hat{k}$

2. $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$. $\vec{b} = 2\hat{i} + 4\hat{j} + \hat{k}$

3. $\vec{a} = 6\hat{i} - 5\hat{j} + \hat{k}$ $\vec{b} = 3\hat{i} - \hat{k}$

$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ $\vec{b} = \cancel{\hat{k} - 3\hat{i}}$ $3\hat{i} - \hat{j} + 4\hat{k}$

$\vec{a} = 2\hat{i} + 2\hat{k}$ $\cancel{\hat{k} - 3\hat{i}}$

$\vec{a} = 4\hat{i} - 2\hat{j} + \hat{k}$ $\vec{b} = \hat{i} + 5\hat{j}$

① $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ $\vec{b} = \hat{i} - \hat{j} - \hat{k}$

$$\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \overset{+}{\hat{i}} & \overset{-}{\hat{j}} & \overset{+}{\hat{k}} \\ 1 & 1 & -1 \\ 1 & -1 & -1 \end{vmatrix} = +\hat{i} \begin{vmatrix} 1 & -1 \\ -1 & -1 \end{vmatrix} - \hat{j} \begin{vmatrix} 1 & -1 \\ 1 & -1 \end{vmatrix} + \hat{k} \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}$$

$$\hat{i}(-1-1) - \hat{j}(-1+1) + \hat{k}(-1-1)$$

$$\vec{a} \times \vec{b} = -2\hat{i} - 2\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{(-2)^2 + (-2)^2}$$

$$\sqrt{4+4}$$

$$\sqrt{8}$$

$= \sqrt{8}$

$\vec{a} = \hat{i} + \hat{j} - \hat{k}$

$|\vec{a}| = \sqrt{1^2 + 1^2 + (-1)^2}$

$= \sqrt{3}$

$\vec{b} = \hat{i} - \hat{j} - \hat{k}$

$|\vec{b}| = \sqrt{1^2 + (-1)^2 + (-1)^2}$

$|\vec{b}| = \sqrt{3}.$

From (1)

$$\sin\theta = \frac{\sqrt{8}}{\sqrt{3}\sqrt{3}} = \frac{\sqrt{8}}{3}$$

---

(6) $\vec{a} = 4\hat{i} - 2\hat{j} + \hat{k}$ $\qquad$ $\vec{b} = \hat{i} + 5\hat{j}.$

$$\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}|\,|\vec{b}|}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -2 & 1 \\ 1 & 5 & 0 \end{vmatrix}$$

$$= \hat{i}\begin{vmatrix} -2 & 1 \\ 5 & 0 \end{vmatrix} - \hat{j}\begin{vmatrix} 4 & 1 \\ 1 & 0 \end{vmatrix} + \hat{k}\begin{vmatrix} 4 & -2 \\ 1 & 5 \end{vmatrix}.$$

$$= \hat{i}(0-5) - \hat{j}(0-1) + \hat{k}(20+2)$$

$$\vec{a} \times \vec{b} = -5\hat{i} + \hat{j} + 22\hat{k}.$$

$$|\vec{a} \times \vec{b}| = \sqrt{(5)^2 + (1)^2 + (22)^2} = \sqrt{25 + 1 + 484} = \sqrt{510}.$$

$$\vec{a} = 4\hat{i} - 2\hat{j} + \hat{k}$$

$$|\vec{a}| = \sqrt{4^2 + (-2)^2 + (1)^2}$$

$$= \sqrt{16 + 4 + 1}$$

$$= \sqrt{21}$$

$$\vec{b} = \hat{i} + 5\hat{j}$$

$$|\vec{b}| = \sqrt{1^2 + 5^2}$$

$$\sqrt{26}$$

From ①

$$\sin\theta = \frac{\sqrt{510}}{\sqrt{21}\sqrt{26}}.$$

$$\frac{\sqrt{510}}{\sqrt{546}}$$

---

③ $\vec{a} = 6\hat{i} - 5\hat{j} + \hat{k}$ $\qquad \vec{b} = 3\hat{i} + \hat{k}.$

$$\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}|\ |\vec{b}|}.$$

$$\vec{a} \times \vec{b} = (\cancel{6 \times 3}) \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & -5 & 1 \\ 3 & 0 & k \end{vmatrix}$$

$$= \hat{i} \begin{vmatrix} -5 & 1 \\ 0 & 1 \end{vmatrix} - \hat{j} \begin{vmatrix} 6 & 1 \\ 3 & 1 \end{vmatrix} + k \begin{vmatrix} 6 & -5 \\ 3 & 0 \end{vmatrix}$$

$\hat{i}(-5-0)+\hat{j}(-6-3)+k(0+15)$

$\hat{i}$

$-5\hat{i}-9\hat{j}+15\hat{k}$

$|\vec{a}\times\vec{b}| = \sqrt{(-5)^2+(-9)^2+(15)^2}$

$\sqrt{25+81+225}$

$\vec{a}\times\vec{b} = \sqrt{\cancel{259}}\quad\sqrt{331}$

$\vec{a} = 6\hat{i}-5\hat{j}+\hat{k}$

$|\vec{a}| = \sqrt{(6)^2+(-5)^2+(1)^2}$

$|\vec{a}| = \sqrt{36+25+1}$

$|\vec{a}| = \sqrt{62}$

$\vec{b} = 3\hat{i}-\hat{k}$

$|\vec{b}| = \sqrt{(3)^2+(-1)^2}$

$\sqrt{9+1}$

$|\vec{b}| = \sqrt{10}$

From (1) $\sin\theta = \dfrac{\sqrt{\cancel{259}\ 331}}{\sqrt{62}\sqrt{10}}$

$= \dfrac{\sqrt{\cancel{259}\ 331}}{\sqrt{620}}$

Find the unit vector for the following vectors

1. $\vec{a} = 6\vec{i} - \vec{j} + 2\vec{k}$.

$|\vec{a}| = \sqrt{(6)^2 + (-1)^2 + (2)^2}$

$|\vec{a}| = \sqrt{36+1+4}$

$|\vec{a}| = \sqrt{41}$

$\hat{a} = \dfrac{\vec{a}}{|\vec{a}|}$

$\hat{a} = \dfrac{6\vec{i} - \vec{j} + 2\vec{k}}{\sqrt{41}}$

2. $\vec{a} = 7\vec{i} + 2\vec{j} - 3\vec{k}$

$|\vec{a}| = \sqrt{(7)^2 + (2)^2 + (-3)^2}$

$|\vec{a}| = \sqrt{49+4+9}$

$|\vec{a}| = \sqrt{62}$

$\hat{a} = \dfrac{\vec{a}}{|\vec{a}|}$

$\hat{a} = \dfrac{7\vec{i} + 2\vec{j} - 3\vec{k}}{\sqrt{62}}$.

3. $\vec{a} = 5\vec{i} - \vec{j} - 2\vec{k}$.

$|\vec{a}| = \sqrt{(5)^2 + (-1)^2 + (-2)^2}$

$|\vec{a}| = \sqrt{25+1+4}$.

$|\vec{a}| = \sqrt{30}$

$\hat{a} = \dfrac{\vec{a}}{|\vec{a}|}$

$$\vec{a} = \frac{5\vec{i} - \vec{j} + -2k}{\sqrt{30}}$$

Find the unit vectors for the following pair of vectors.

1] $\vec{a} = \vec{i} - 2\vec{j} + 2\vec{k}$. $\vec{b} = 6\vec{i} - \vec{j} + 3\vec{k}$.

2] $\vec{a} = 7\vec{i} - 2\vec{j} + 3\vec{k}$ $\vec{b} = 5\vec{i} + 2\vec{j} + \vec{k}$ — Hw

3] $\vec{a} = 7\vec{k} - 2\vec{i}$ $\vec{b} = \vec{i} + \vec{j} - \vec{k}$.

4] $\vec{a} = 2\vec{j} - \vec{i}$. $\vec{b} = 2\vec{i} + 9\vec{k} + 2\vec{j}$

5] $\vec{a} = 4\vec{i} + 2\vec{j} - \vec{k}$ $\vec{b} = \vec{i} - 3\vec{j}$ — Hw

6] $\vec{a} = 2\vec{i} + \vec{j} + 8\vec{k}$ $\vec{b} = 7\vec{i} + 2\vec{j} - \vec{k}$. — Hw

① $\vec{a} = 2\vec{j} - \vec{i}$ $\vec{b} = 2\vec{i} + 9\vec{k} + 2\vec{j}$.

$$\vec{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a}||\vec{b}|}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 2 & 0 \\ 2 & 2 & 9 \end{vmatrix}$$

$$= \vec{i}\begin{vmatrix} 2 & 0 \\ 2 & 9 \end{vmatrix} - \vec{j}\begin{vmatrix} -1 & 0 \\ 2 & 9 \end{vmatrix} + \vec{k}\begin{vmatrix} -1 & 2 \\ 2 & 2 \end{vmatrix}$$

$$\vec{a} \times \vec{b} = 18\vec{i} + 9\vec{j} - 6\vec{k}.$$

$\vec{a} = 2\vec{j} - \vec{i}$

$|\vec{a}| = \sqrt{(-1)^2 + 2^2}$

$\sqrt{5}$

$\vec{b} = 2\vec{i} + 9\vec{k} + 2\vec{j}$

$|\vec{b}| = \sqrt{2^2 + 2^2 + 9^2}$

$\sqrt{4+4+81}$

$\sqrt{89}.$

From Q

$$\vec{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a}||\vec{b}|}$$

$$\vec{n} = \frac{18\hat{i} + 9\hat{j} - 6\hat{k}}{\sqrt{5} \times \sqrt{89}} = \frac{18\hat{i} + 9\hat{j} - 6\hat{k}}{\sqrt{445}}$$

② ~~$\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$~~ ~~$\vec{b} = 5\hat{i} + 2\hat{j} - \hat{k}$.~~

① $\vec{a} = \hat{i} - 2\hat{j} + 2\hat{k}$ $\quad \vec{b} = 6\hat{i} - \hat{j} + 3\hat{k}$.

$$\vec{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a}||\vec{b}|}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \overset{+}{\hat{i}} & \overset{-}{\hat{j}} & \overset{+}{\hat{k}} \\ 1 & -2 & 2 \\ 6 & -1 & 3. \end{vmatrix}$$

$$\hat{i} \begin{vmatrix} -2 & 2 \\ -1 & 3 \end{vmatrix} - \hat{j} \begin{vmatrix} 1 & 2 \\ 6 & 3 \end{vmatrix} + \hat{k} \begin{vmatrix} 1 & -2 \\ 6 & -1 \end{vmatrix}$$

$$\hat{i}(-6 + 2) - \hat{j}(3 - 12) + \hat{k}(-1 + 12)$$

$$-4\hat{i} + \overset{9}{\cancel{15}}\hat{j} + 11\hat{k}.$$

$$\vec{a} = \hat{i} - 2\hat{j} + 5\hat{k}.$$

$$|\vec{a}| = \sqrt{(1)^2 + (-2)^2 + (2)^2}$$

$$|\vec{a}| = \sqrt{1 + 4 + \cancel{25}4}$$

$$|\vec{a}| = \sqrt{\cancel{30}\,9}$$

$$\vec{b} = 6\hat{i} - \hat{j} + 3\hat{k}.$$

$$|\vec{b}| = \sqrt{(6)^2 + (-1)^2 + (3)^2}$$

$\sqrt{36+1+9}$

$|\vec{b}| \sqrt{46}$

From ①

$$\vec{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a}|\,|\vec{b}|}$$

$$\frac{-4\hat{i} + \cancel{9}\hat{j} + 11\hat{k}}{\sqrt{414}}$$

---

③ $\vec{a} = 7\vec{k} - 2\hat{i}$ ~~+3~~ $\vec{b} = \hat{i} + \hat{j} - \vec{k}$

$$\vec{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a}|\,|\vec{b}|}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \overset{+}{\hat{i}} & \overset{-}{\hat{j}} & \overset{+}{\hat{k}} \\ -2 & 0 & 7 \\ 1 & 1 & -1 \end{vmatrix}$$

$$= \hat{i}\begin{vmatrix} 0 & 7 \\ 1 & -1 \end{vmatrix} - \hat{j}\begin{vmatrix} -2 & 7 \\ 1 & -1 \end{vmatrix} + \vec{k}\begin{vmatrix} -2 & 0 \\ 1 & 1 \end{vmatrix}$$

$$\hat{i}(-0-7) - \hat{j}(+2-7) + \vec{k}(-2-0)$$

$$-7\hat{i} + 5\hat{j} \cancel{++} - 2\vec{k}$$

$\vec{a} = 7\vec{k} - 2\hat{i}$

$|\vec{a}| = \sqrt{(7)^2 + (-2)^2}$

$\sqrt{49+4}$

$|\vec{a}| = \sqrt{53}$

$\vec{b} = \hat{i} + \hat{j} - \vec{k}$

$|\vec{b}| = \sqrt{1^2 + 1^2 + (-1)^2}$

$|\vec{b}| = \sqrt{3}$

From ①

$$\vec{n} = \frac{-7\hat{i} + 5\hat{j} - 2\vec{k}}{\sqrt{159}}$$

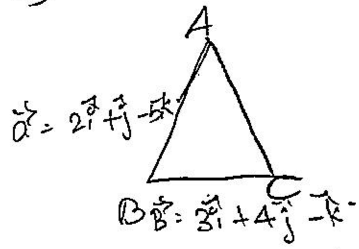
Find the area of the triangle whose sides are

$\vec{a} = 2\hat{i} + \hat{j} - 5\hat{k}$ $\quad \vec{b} = 3\hat{i} + 4\hat{j} - \hat{k}$.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -5 \\ 3 & 4 & -1 \end{vmatrix} \qquad A = \frac{1}{2} |\vec{a} \times \vec{b}| \text{ --- } (1)$$

$$= \hat{i} \begin{vmatrix} 1 & -5 \\ 4 & -1 \end{vmatrix} - \hat{j}(-2+15) + \hat{k}(8-3)$$

$$= \cancel{+8}\ 19\hat{i} - 13\hat{j} + 5\hat{k}$$



$$|\vec{a} \times \vec{b}| = \sqrt{19^2 + (-13)^2 + (5)^2}$$

$$|\vec{a} \times \vec{b}| = \sqrt{361 + 169 + 25}$$

$$= \sqrt{555}$$

From (1)

$A = \frac{1}{2}\sqrt{555}$ squnits.

Find the area of triangle whose sides are

$\vec{a} = 4\hat{i} - \hat{j} + 8\hat{k}$ $\qquad \vec{b} = \hat{i} + \hat{j} - 3\hat{k}$.

$$A = \frac{1}{2} |\vec{a} \times \vec{b}| \text{ ---- } (1).$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 8 \\ 1 & 1 & -3 \end{vmatrix}$$

$$= \hat{i} \begin{vmatrix} -1 & 8 \\ 1 & -3 \end{vmatrix} - \hat{j} \begin{vmatrix} 4 & 8 \\ 1 & -3 \end{vmatrix} + \hat{k} \begin{vmatrix} 4 & -1 \\ 1 & 1 \end{vmatrix}$$

$\vec{i}(+3-8) - \vec{j}(-12-8) + \vec{k}(4+1)$

$-5\vec{i} + 20\vec{j} + 5\vec{k}$

$|\vec{a}\times\vec{b}| = \sqrt{(-5)^2 + (-20)^2 + (5)^2}$

$= \sqrt{25 + 400 + 25}$

$\sqrt{450}$

From ①

$A = \frac{1}{2}\sqrt{450}$ sq units

Find the area of the parallelogram whose adjecent sides are $\vec{a} = \vec{i} + 3\vec{j} - 7\vec{k}$ $\vec{b} = 2\vec{i} - \vec{j} + 4\vec{k}$.

$A = |\vec{a}\times\vec{b}|$ - - - ①.

$$\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 3 & -7 \\ 2 & -1 & 4 \end{vmatrix}$$

$a = \vec{i}\begin{vmatrix} 3 & -7 \\ -1 & 4 \end{vmatrix} - \vec{j}\begin{vmatrix} 1 & -7 \\ 2 & 4 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix}$

$\vec{i}(12-7) - \vec{j}(4+14) + \vec{k}(-1-6)$.

$5\vec{i} - 18\vec{j} - 7\vec{k}$.

$|\vec{a}\times\vec{b}| = \sqrt{(5)^2 + (-18)^2 + (-7)^2}$

$\sqrt{25 + 324 + 49}$

$|\vec{a} \times \vec{b}| \sqrt{398}$

$\cancel{|\vec{a} \times \vec{b}|} A = \sqrt{398}$ sq unit

Home work

(2) $\vec{a} = -\hat{i} - 2\hat{j} + 3\hat{k}$ $\quad \vec{b} = 5\hat{i} + 2\hat{j} + \hat{k}$.

$$\hat{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a}||\vec{b}|}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \overset{+}{\hat{i}} & \overset{-}{\hat{j}} & \overset{+}{\hat{k}} \\ -1 & -2 & 3 \\ 5 & 2 & 1 \end{vmatrix}$$

$$= \hat{i} \begin{vmatrix} -2 & 3 \\ 2 & 1 \end{vmatrix} - \hat{j} \begin{vmatrix} -1 & 3 \\ 5 & 1 \end{vmatrix} + \hat{k} \begin{vmatrix} -1 & -2 \\ 5 & 2 \end{vmatrix}$$

$$\hat{i}(-2-6) - \hat{j}(-1-15) + \hat{k}(-2+10)$$

$$-8\hat{i} \cancel{+} - 16\hat{j} + 8\hat{k}.$$

$$|\vec{a}| = \sqrt{(-1)^2 + (-2)^2 + (3)^2}$$

$$|\vec{a}| = \sqrt{1+4+9}$$

$$|\vec{a}| = \sqrt{14}.$$

$$|\vec{b}| = \sqrt{(5)^2 + (2)^2 + (1)^2}$$

$$= \sqrt{25+4+1}$$

$$= \sqrt{30}.$$

From (1).

$$\vec{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a}||\vec{b}|}$$

$$\vec{n} = \frac{-8\vec{i} - 16\vec{j} + 8\vec{k}}{\sqrt{14}\ \sqrt{30}}$$

$$\vec{n} = \frac{-8\vec{i} - 16\vec{j} + 8k}{420}$$

---

(5) $\vec{a} = 4\vec{i} + 2\vec{j} - \vec{k}$. $\vec{b} = \vec{i} - 5\vec{j}$

$$\vec{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a}||\vec{b}|}.$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \overset{+}{\vec{i}} & \overset{-}{\vec{j}} & \overset{+}{\vec{k}} \\ 4 & 2 & -1 \\ 1 & -5 & 0 \end{vmatrix}$$

$$= \vec{i} \begin{vmatrix} 2 & -1 \\ -5 & 0 \end{vmatrix} - \vec{j} \begin{vmatrix} 4 & -1 \\ 1 & 0 \end{vmatrix} + \vec{k} \begin{vmatrix} 4 & 2 \\ 1 & -5 \end{vmatrix}$$

$$\vec{i}(0-5) - \vec{j}(0+1) + \vec{k}(-20-2)$$

$$-5\vec{i} - \vec{j} - 22\vec{k}.$$

$$|\vec{a}| = \sqrt{(4)^2 + (2)^2 + (-1)^2}$$

$$= \sqrt{16 + 4 + 1}$$

$$|\vec{a}| = \sqrt{21}.$$

$$|\vec{b}| = \sqrt{(1)^2 + (-5)^2}.$$

$$|\vec{b}| = \sqrt{1 + 25}$$

$$= \sqrt{26}$$

From (1).

$$\vec{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a}||\vec{b}|}$$

$$= \frac{-5\hat{i} - \hat{j} - 22\hat{k}}{\sqrt{21}\ \sqrt{26}}$$

$$= \frac{-5\hat{i} - \hat{j} - 22\hat{k}}{546}$$

(b) $\vec{a} = 2\hat{i} + \hat{j} + 8\hat{k}$ $\quad \vec{b} = 7\hat{i} + 2\hat{j} + -\hat{k}$.

$$\vec{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a}||\vec{b}|}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 8 \\ 7 & 2 & -1 \end{vmatrix}$$

$$\hat{i} \begin{vmatrix} 1 & 8 \\ 2 & -1 \end{vmatrix} - \hat{j} \begin{vmatrix} 2 & 8 \\ 7 & -1 \end{vmatrix} + \hat{k} \begin{vmatrix} 2 & 1 \\ 7 & 2 \end{vmatrix}$$

$$\hat{i}(-1 - 16) - \hat{j}(-2 - 56) + \hat{k}(4 - 7)$$

$$-17\hat{i} - 58\hat{j} - 3\hat{k}.$$

$$|\vec{a}| = \sqrt{(2)^2 + (1)^2 + (8)^2} \qquad |\vec{b}| = \sqrt{(7)^2 + (2)^2 + (-1)^2}$$

$$= \sqrt{4 + 1 + 64} \qquad = \sqrt{49 + 4 + 1}$$

$$= \sqrt{69} \qquad = \sqrt{54}$$

From 1

$$\vec{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a}|\,|\vec{b}|}$$

$$= \frac{-17\hat{i} - 58\hat{j} - 3\vec{k}}{\sqrt{69}\ \sqrt{54}}$$

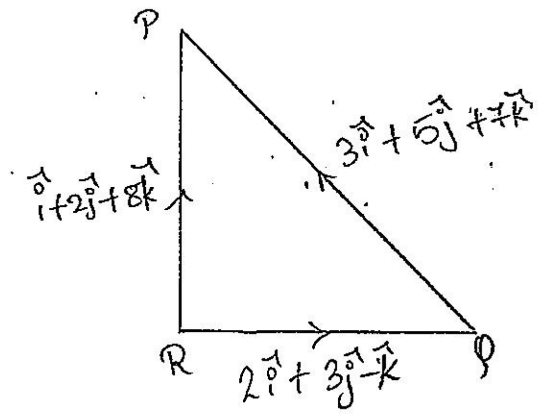
$$= \frac{-17\hat{i} - 58\hat{j} - 3\vec{k}}{\cancel{3726}}$$

Show that the vectors $\hat{i} + 2\hat{j} + 8\vec{k}$, $3\hat{i} + 5\hat{j} + 7\vec{k}$ & $2\hat{i} + 3\hat{j} - \vec{k}$ forms sides of a right angle ~~tr~~ triangle.

$$|\vec{PQ}| = \sqrt{3^2 + 5^2 + 7^2} = \sqrt{9 + 25 + 49} = \sqrt{83}$$

$$|\vec{QR}| = \sqrt{2^2 + 3^2 + (-1)^2} = \sqrt{4 + 9 + 1} = \sqrt{14}$$

$$|\vec{PR}| = \sqrt{1^2 + 2^2 + 8^2} = \sqrt{1 + 4 + 64} = \sqrt{69}$$



Phythogorus theorem.

$$|\vec{PQ}|^2 = |\vec{QR}|^2 + |\vec{RP}|^2$$

$$(\sqrt{83})^2 = (\sqrt{14})^2 + (\sqrt{69})^2$$

$$83 = 14 + 69.$$

$$83 = 83.$$

Therefore the given sides forms a right angle triangle.

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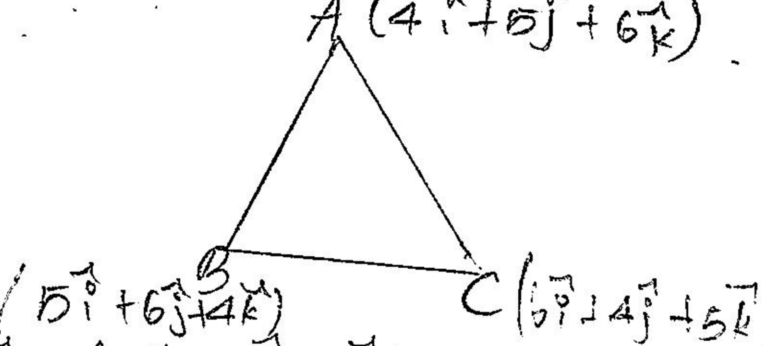
4. If the vertices (Points) of a triangle have the position vectors for $4\vec{i}+5\vec{j}+6\vec{k}$, $5\vec{i}+6\vec{j}+4\vec{k}$, $6\vec{i}+4\vec{j}+5\vec{k}$ prove that the triangle is an equilateral triangle.



$\vec{OA} = 4\vec{i}+5\vec{j}+6\vec{k}$

$\vec{OB} = 5\vec{i}+6\vec{j}+4\vec{k}$

$\vec{OC} = 6\vec{i}+4\vec{j}+5\vec{k}$

$\vec{AB} = \vec{OB}-\vec{OA}$

$= 5\vec{i}+6\vec{j}+4\vec{k} - (4\vec{i}+5\vec{j}+6\vec{k})$

$\vec{AB} = \vec{i}+\vec{j}-2\vec{k}$

$|\vec{AB}| = \sqrt{1^2+1^2+(-2)^2}$

$= \sqrt{1+1+4} = \sqrt{6}$.

$\vec{BC} = \vec{OC}-\vec{OB}$

$= 6\vec{i}+4\vec{j}+5\vec{k} - (5\vec{i}+6\vec{j}+4\vec{k})$

$= \vec{i} - 2\vec{j}+\vec{k}$

$|\vec{BC}| = \sqrt{1+(-2)^2+1^2}$

$\sqrt{1+4+1}$

$= \sqrt{6}$

$\vec{CA} = \vec{OA}-\vec{OC}$

$= 4\vec{i}+5\vec{j}+6\vec{k} - (6\vec{i}+4\vec{j}+5\vec{k})$

$\cancel{|\vec{CA}|} = \sqrt{\ }$

$\vec{CA} = -2\vec{i}+\vec{j}+\vec{k}$.

$|\vec{CA}| = \sqrt{(-2)^2+1^2+1^2} = \sqrt{6}$.

10) 45. Show that the vectors whose vertices of a triangle $4\hat{i}+\hat{j}+3\hat{k}$, $\hat{i}+3\hat{j}+2\hat{k}$, $2\hat{i}+7\hat{k}$ form right angle triangle.

Sol. $\vec{OA} = 4\hat{i}+\hat{j}+3\hat{k}$

$\vec{OB} = \hat{i}+3\hat{j}+2\hat{k}$

$\vec{OC} = 2\hat{i}+7\hat{k} = 2\hat{i}+0\hat{j}+7\hat{k}$

$\vec{AB} = \vec{OB}-\vec{OA}$

$= \hat{i}+3\hat{j}+2\hat{k} - (4\hat{i}+\hat{j}+3\hat{k})$

$= -3\hat{i}+2\hat{j}-\hat{k}$

$|\vec{AB}| = \sqrt{(-3)^2+(2)^2+(-1)^2}$

$\sqrt{9+4+1} = \sqrt{14}$

$\vec{BC} = \vec{OC}-\vec{OB}$

$2\hat{i}+0\hat{j}+7\hat{k} - (\hat{i}+3\hat{j}+2\hat{k})$

$\hat{i}-3\hat{j}+5\hat{k}$

$|\vec{BC}| = \sqrt{(1)^2+(-3)^2+(5)^2}$

$= \sqrt{1+9+25} = \sqrt{35}$

$\vec{AC} = \cancel{\vec{OC}-\vec{OB}}\ \vec{OA}-\vec{OC}$

$\cancel{2\hat{i}+0\hat{j}+7k} - (\cancel{\hat{i}+3\hat{j}+2\hat{k}})$

$\cancel{\hat{i}-3\hat{j}+5}$

$= 4\hat{i}+\hat{j}+3\hat{k} - (2\hat{i}+0\hat{j}+7\hat{k})$

K

$= 2\hat{i}+\hat{j}-4\hat{j}$

$|\vec{AC}| = \sqrt{(2)^2+1^2+(-4)^2} = \sqrt{4+1+16} = \sqrt{21}$

$\therefore |\vec{BC}|^2 = |\vec{AB}|^2 + |\vec{AC}|^2$

$(\sqrt{35})^2 = (\sqrt{14})^2 + (\sqrt{21})^2$

$35 = 14 + 21$

$35 = 35$

$\therefore$ The given vertices of triangle forms right angle triangle

Work done :-

work done = $\vec{F} \cdot \vec{S}$
or
$\vec{F} \cdot \vec{AB}$

where force $\vec{F}$ actiongon a particle & s is the displacement moving of a particle

---

Find the work done by the force $\vec{F} = 4\hat{i} - 3\hat{j} + 2\hat{k}$ in moving a particle from the point A (5, 4, 1) to the point B of (4, 1, 6)

$\vec{F} = 4\hat{i} - 3\hat{j} + 2\hat{k}$

A (5, 4, 1)

B (4, 1, 6)

PV of a pt A(5, 4, 1)

$\vec{OA} = 5\hat{i} + 4\hat{j} + \hat{k}$

PV of a pt B (4, 1, 6)

$\vec{OB} = 4\hat{i} + \hat{j} + 6\hat{k}$

$\vec{AB} = \vec{OB} - \vec{OA}$

$4\vec{i} + \vec{j} + 6\vec{k} - (5\vec{i} + 4\vec{j} + \vec{k})$

$\vec{AB} = -\vec{i} - 3\vec{j} + 5\vec{k}$

work done
$= \vec{F} \cdot \vec{AB}$

$a_1 b_1 + a_2 b_2 + a_3 b_3$

$(4 \times -1) + (-3 \times -3) + (2 \times 5)$

$= -4 + 9 + 10 = 15$ units

A force $2\vec{i} + 3\vec{j} + \vec{k}$ acting on a particle displaces it from A(1, 3, 2) to B(4, -1, 3) ~~fide~~ find the work done.

work done $= \vec{F} \cdot \vec{AB}$.

P.V of a pt A(1, 3, 2)

$\vec{OA} = \vec{i} + 3\vec{j} + 2\vec{k}$

P.V of a pt B(4, -1, 3)

$\vec{OB} = 4\vec{i} - \vec{j} + 3\vec{k}$.

$\vec{AB} = \vec{OB} - \vec{OA}$.

$4\vec{i} - \vec{j} + 3\vec{k} - (\vec{i} + 3\vec{j} + 2\vec{k})$

$\vec{AB} = 3\vec{i} - 4\vec{j} + \vec{k}$.

work done $= \vec{F} \cdot \vec{AB}$.

$= a_1 b_1 + a_2 b_2 + a_3 b_3$

$(2 \times 3) + (3 \times -4) + (1 \times 1) = 6 - 12 + 1 = -5$.

Find the work done by the force $F = 4\vec{i} + 3\vec{j} - 2\vec{k}$ when it displaces the particle from the point A(5, 2, -3) to B(6, -3, 1).

work done $= \vec{F} \cdot \vec{AB}$.

P.V of a pt $A(5, 2, -3)$

$\vec{OA} = 5\vec{i} + 2\vec{j} - 3\vec{k}$.

P.V of a pt $B(6, -3, 1)$.

$\vec{OB} = 6\vec{i} - 3\vec{j} + 1\vec{k}$

$\vec{AB} = \vec{OB} - \vec{OA}$.

$6\vec{i} - 3\vec{j} + \vec{k} - (5\vec{i} + 2\vec{j} - 3\vec{k})$

$= \vec{i} - 5\vec{j} - 4\vec{k}$

work done $= \vec{F} \cdot \vec{AB}$.

$a_1b_1 + a_2b_2 + a_3b_3$

$(4 \times 1) + (3 \times -5) + (-2 \times -4)$

$4 - 15 - 8$

$= -19$ units.

A constant force $2\vec{i} - 5\vec{j} + 6\vec{k}$ acting on a partic and it displaces from $4\vec{i} - 3\vec{j} - 2\vec{k}$ to $6\vec{i} + \vec{j} - 3\vec{k}$ find the work done.

A work done $= \vec{F} \cdot \vec{AB}$.

$\vec{AB} = \vec{OB} - \vec{OA}$

$6\vec{i} + \vec{j} - 3\vec{k} - (4\vec{i} - 3\vec{j} - 2\vec{k})$

$2\vec{i} + 4\vec{j} - 1\vec{k}$

$\vec{F} \cdot \vec{AB}$.

~~$2\vec{i} - 5\vec{j} + 1$~~

$a_1b_1 + a_2b_2 + a_3b_3$

$(2 \times 2) + (-5 \times 4) + (6 \times -1)$

$4 - 20 - 6.$

$= -22$ units.

A particle acted by a constant forces $\vec{F_1} = 4\vec{i} + 3\vec{j} + \vec{k}$
$\vec{F_2} = 3\vec{i} + 2\vec{j} - 7\vec{k}$ displaces from the point $(1, 2, -1)$
$(5, 4, 0)$ find the total work done by the force.

$\vec{F} = \vec{F_1} + \vec{F_2}$

$4\vec{i} + 3\vec{j} + \vec{k} + 3\vec{i} + 2\vec{j} - 7\vec{k}.$

$\vec{F} = 7\vec{i} + 5\vec{j} - 6\vec{k}.$

PV of pt A $(1, 2, -1)$

$\vec{OA} = \vec{i} + 2\vec{j} - \vec{k}.$

PV of pt B$(5, 4, 0)$

$\vec{OB} = 5\vec{i} + 4\vec{j} + 0\vec{k}.$

$\vec{AB} = \vec{OB} - \vec{OA}$

$5\vec{i} + 4\vec{j} - (\vec{i} + 2\vec{j} - \vec{k})$

$4\vec{i} + 2\vec{j} + \vec{k}.$

$\vec{F} \cdot \vec{AB}.$

$a_1b_1 + a_2b_2 + a_3b_3.$

$(7 \times 4) + (5 \times 2) + (-6 \times 1). = 28 + 10 - 6$

$= 32$ units.

Moment of a force. $= \vec{F} \times \vec{s}$

or

$\vec{F} \times \vec{AB}$.

Magnitude moment of a force $= |F \times \vec{AB}|$.

Find the magnitude of moment of a force $\vec{F} = 4\vec{i} - 2\vec{j} + 5\vec{k}$ about the point $(2,5,-7)$ to $(4,7,0)$

Moment of force.

P.V of a pt $A(2,5,-7)$

$\vec{OA} = 2\vec{i} + 5\vec{j} - 7\vec{k}$.

P.V of a pt $B(4,7,0)$

$\vec{OB} = 4\vec{i} + 7\vec{j}$.

$\vec{AB} = \vec{OB} - \vec{OA}$.

$4\vec{i} + 7\vec{j} - (2\vec{i} + 5\vec{j} - 7\vec{k})$

$\vec{AB} = 2\vec{i} + 2\vec{j} + 7\vec{k}$.

Moment of force.

$= F \times \vec{AB}$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & -2 & 5 \\ 2 & 2 & 7 \end{vmatrix}$$

$$= \hat{i}\begin{vmatrix} -2 & 5 \\ 2 & 7 \end{vmatrix} - \hat{j}\begin{vmatrix} 4 & 5 \\ 2 & 7 \end{vmatrix} + k\begin{vmatrix} 4 & -2 \\ 2 & 2 \end{vmatrix}.$$

$$\hat{i}(-14-10) - \hat{j}(28-10) + \vec{k}(8+4)$$

moment of force.

$$= -24\hat{i} - 18\hat{j} + 12\vec{k}.$$

magnitude of M of F

$$= \sqrt{(24)^2 + (-18)^2 + 12^2}$$

$$= \sqrt{576 + 324 + 144}$$

$$= \sqrt{1044} \text{ units.}$$

Find the magnitude of moment of a force about the point $\hat{i} + 2\hat{j} + 3\hat{k}$ of a force represented by $\hat{i} + \hat{j} + \vec{k}$ acting through the point $-2\hat{i} + 3\hat{j} + \vec{k}$

$$\vec{AB} = \vec{OB} - \vec{OA}.$$

$$-2\hat{i} + 3\hat{j} + \vec{k} - (\hat{i} + 2\hat{j} + 3\vec{k}).$$

$$\vec{AB} = -3\hat{i} + \hat{j} - 2\vec{k}.$$

Moment of force.

$$\begin{array}{ccc} + & - & + \end{array}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \vec{k} \\ 1 & 1 & 1 \\ -3 & 1 & -2 \end{vmatrix}$$

$$= \hat{i}\begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} - \hat{j}\begin{vmatrix} 1 & 1 \\ -3 & -2 \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & 1 \\ -3 & 1 \end{vmatrix}$$

$$\hat{i}(-2-1) - \hat{j}(-2+3) + \hat{k}(1+3)$$

$$-3\hat{i} - \hat{j} + 4\hat{k}$$

Magnitude of moment of force.

$$= \sqrt{(-3)^2 + (-1)^2 + (4)^2}$$

$$= \sqrt{9+1+16}$$

$$= \sqrt{26} \text{ units}$$

Find the magnitude of moment of force $\hat{i} + 3\hat{j} - 5\hat{k}$ acting at a point $A(1,-1,2)$ about the point $B(2,3,-1)$

PV of pt $A(1,-1,2)$

$\vec{OA} = \hat{i} - \hat{j} + 2\hat{k}$

P.V of $B(2,3,-1)$

$\vec{OB} = 2\hat{i} + 3\hat{j} - \hat{k}$

$\vec{AB} = \vec{OB} - \vec{OA}$

$$2\hat{i} + 3\hat{j} - \hat{k} - (\hat{i} - \hat{j} + 2\hat{k})$$

$$\hat{i} + 4\hat{j} - 3\hat{k}$$

moment of force.

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & -5 \\ 1 & 4 & -3 \end{vmatrix}$$

$$\hat{i} = \begin{vmatrix} 3 & -5 \\ 4 & -3 \end{vmatrix} - \hat{j}\begin{vmatrix} 1 & -5 \\ 1 & -3 \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix}$$

$$\hat{i}\,(\cancel{-11+9}) - \hat{j}\,(-3+5) + \hat{k}\,(4-3)$$

$$\hat{i}\,(-9+20)$$

$$11\hat{i} - 2\hat{j} + \hat{k}$$

magnitude of m of F is

$$\sqrt{(11)^2 + (-2)^2 + (1)^2}$$

$$\sqrt{121 + 4 + 1}$$

$$= \sqrt{126} \text{ units.}$$

Valued

7/9/12