

PROJECT 3 ON RANDOMIZATION TESTS

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1.1: Problem Statement:

We are given 3 treatment of size respectively 2,3 and 4. Our goal is to compare the treatments by using randomization tests. We will be performing both systematic and random permutation to compare the treatments.

1.2: Test Description and Methodology:

For both systematic and random permutation, our alternatives are:

$$H_0: \mu_A = \mu_B = \mu_C \text{ (Treatments are equal)}$$

$$H_a: \text{At least One of the } \mu \text{ is not the same}$$

Given treatment values are below:

Treatments	Treatment Values
A	6,8
B	9,11,9
C	17,15,16,16

We now denote n_A, n_B and n_C to be the number of treatment values of treatment A, B and C respectively. Let r =total number of treatments. In this case, $n_A=2$, $n_B=3$ and $n_C=4$ and $r=3$. Our test statistic is given below:

$$t^* = \frac{\sum_{i=1}^2 T_{Ai}^2}{n_A} + \frac{\sum_{i=1}^3 T_{Bi}^2}{n_B} + \frac{\sum_{i=1}^4 T_{Ci}^2}{n_C} \quad (1)$$

Where T_{Ai}, T_{Bi}, T_{Ci} are treatment values for Treatment A, B and C respectively for case $i = 1, 2, \dots$ etc. After performing systematic and random permutation, our probability value for the test will be:

$$\text{p-value} = \frac{\text{Total number of test statistic values which are at least as large as the original test statistic}}{N}$$

Original test statistic is calculated from (1) keeping the same order as the original treatment values. N or total Number of permutations are different for systematic and random permutations.

We select $\alpha=0.05$ to be our significance value. Here are the decision rules:

If p-value < 0.05, Assume H_a . If p-value ≥ 0.05 , Assume H_0 .

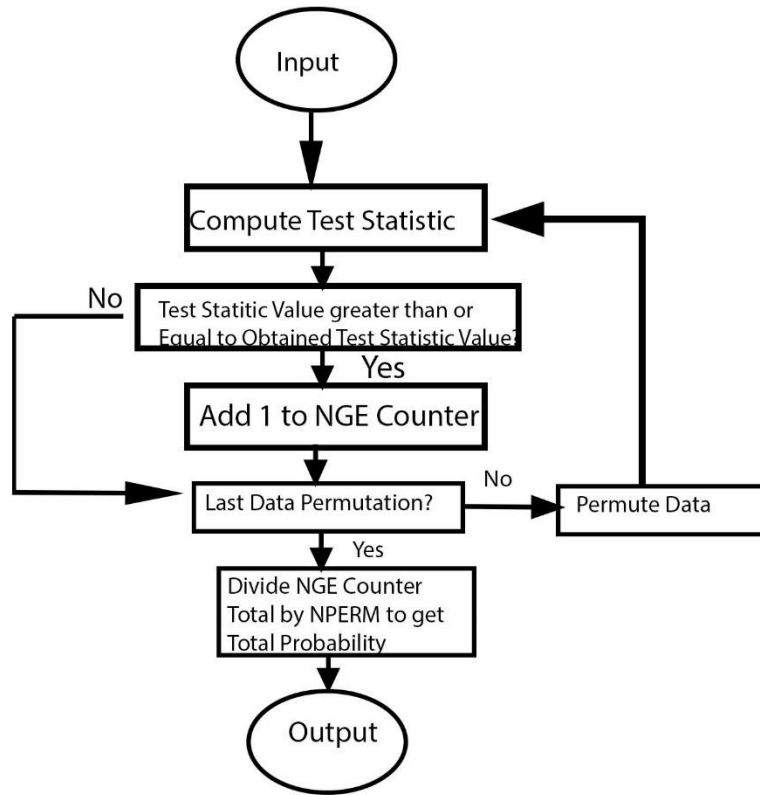


Figure: Flowchart for Randomization tests

We will conduct our tests with both systematic and random permutation of data. Here are the steps for systematic permutation:

1. Arrange the data in a tabular form and group the measurements according to the treatments and assign index numbers 1 to N to the measurements, beginning with treatment A.
2. Generate a systematic sequence of permutations of the index numbers associated with measurements, so that there are n_A index numbers for A, n_B index numbers for B and n_C index numbers for C. For each of these permutations we perform steps 3 and 4. The 1st permutation is the obtained test result, so steps 3 and 4 associated with the 1st permutation is the original test statistic value. There will be $\frac{N!}{n_A!n_B!n_C!}$ total permutations of the N measurements for k groups. .
3. Compute T_A, T_B, T_C , the totals for all treatment groups.
4. Sum $\frac{T_A^2}{n_A}, \frac{T_B^2}{n_B}, \frac{T_C^2}{n_C}$ to obtain test statistic value

5. Compute the probability for F as the proportion of the $\frac{N!}{n_A!n_B!n_C!}$ (1260 total in this case) permutations that provide as large a value of the obtained test statistic, the value computed from the 1st permutation.

For the steps above $N=9$, $n_A = 2$, $n_B = 3$ and $n_C = 4$, $k=3$.

So according to systematic permutation, our 1st permutation will look like below:

Permutation Sequence	1	2	3	4	5	6	7	8	9
Data	6	8	9	11	9	17	15	16	16

Random permutations can be used in place of systematic permutation if the number of systematic permutations is too large. In this project, we are going to use 800 random permutations. Here are the steps for random permutation:

1. Arrange the data in the form of a table, where the measurements are grouped by treatment and assign 1 to N to the measurements, beginning with the measurements for treatment 1.
2. Perform step 5 and 6 for the observed data to compute the test statistic value.
3. Using a random number generation algorithm, select n_A ($n_A = 2$ in this case) index numbers without replacement from N ($N=9$ in this case) index. From the remaining $N-n_A$ (7) index numbers, randomly select $n_B = 3$ index numbers to treatment B the remaining $N-n_A-n_C$ (4) treatments to treatment C. Each assignment of index numbers to every treatment constitutes a permutation of the data. We will perform 799 permutations (Considering the original statistic as 1 permutation)
4. Compute T_A, T_B, T_C , the totals for all treatment groups.
5. Sum $\frac{T_A^2}{n_A}, \frac{T_B^2}{n_B}, \frac{T_C^2}{n_C}$ to obtain test statistic value.
6. Compute the probability for F as the proportion of 800 permutations which gives at least as large test statistic as the original test statistic.

1.3 Test Results

Systematic Permutation

There are total 1260 permutations for systematic permutations. The p-value obtained is 1/1260 or 0.0007936507936507937. Here are all the test statistics calculated from python (1st one is the original test statistic):

Permutation Sequence									Test Statistic(rounded)
1	2	3	4	5	6	7	8	9	1402
1	2	3	4	6	5	7	8	9	1338
1	2	3	4	7	5	6	8	9	1347
1	2	3	4	8	5	6	7	9	1342
1	2	3	4	9	5	6	7	8	1342
1	2	3	5	6	4	7	8	9	1347
1	2	3	5	7	4	6	8	9	1361
1	2	3	5	8	4	6	7	9	1353
1	2	3	5	9	4	6	7	8	1353
1	2	3	6	7	4	5	8	9	1334
1	2	3	6	8	4	5	7	9	1336
1	2	3	6	9	4	5	7	8	1336
1	2	3	7	8	4	5	6	9	1333
1	2	3	7	9	4	5	6	8	1333
1	2	3	8	9	4	5	6	7	1334
1	2	4	5	6	3	7	8	9	1338
1	2	4	5	7	3	6	8	9	1347
1	2	4	5	8	3	6	7	9	1342
1	2	4	5	9	3	6	7	8	1342
1	2	4	6	7	3	5	8	9	1339
1	2	4	6	8	3	5	7	9	1343

1	2	4	6	9	3	5	7	8	1343
1	2	4	7	8	3	5	6	9	1336
1	2	4	7	9	3	5	6	8	1336
1	2	4	8	9	3	5	6	7	1339
1	2	5	6	7	3	4	8	9	1334
1	2	5	6	8	3	4	7	9	1336
1	2	5	6	9	3	4	7	8	1336
1	2	5	7	8	3	4	6	9	1333
1	2	5	7	9	3	4	6	8	1333
1	2	5	8	9	3	4	6	7	1334
1	2	6	7	8	3	4	5	9	1372
1	2	6	7	9	3	4	5	8	1372
1	2	6	8	9	3	4	5	7	1382
1	2	7	8	9	3	4	5	6	1363
1	3	2	4	5	6	7	8	9	1397
1	3	2	4	6	5	7	8	9	1328
1	3	2	4	7	5	6	8	9	1338
1	3	2	4	8	5	6	7	9	1333
1	3	2	4	9	5	6	7	8	1333
1	3	2	5	6	4	7	8	9	1338
1	3	2	5	7	4	6	8	9	1353
1	3	2	5	8	4	6	7	9	1345
1	3	2	5	9	4	6	7	8	1345
1	3	2	6	7	4	5	8	9	1321
1	3	2	6	8	4	5	7	9	1323
1	3	2	6	9	4	5	7	8	1323
1	3	2	7	8	4	5	6	9	1321

1	3	2	7	9	4	5	6	8	1321
1	3	2	8	9	4	5	6	7	1321
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1	3	5	8	9	2	4	6	7	1323
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1	3	6	7	9	2	4	5	8	1364
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8	9	3	4	7	1	2	5	6	1320
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8	9	3	5	7	1	2	4	6	1316
8	9	3	6	7	1	2	4	5	1361
8	9	4	5	6	1	2	3	7	1329
8	9	4	5	7	1	2	3	6	1320
8	9	4	6	7	1	2	3	5	1384
8	9	5	6	7	1	2	3	4	1361

As p-value obtained is less than the significance value, we reject the null hypothesis. So, we conclude that at least 1 of the treatments are different (H_a).

Random Permutation

We get different p value in each simulation. Here are the p-values we got from 100 simulations:

p-values
0.0025
0.0025
0.0013
0.0000
0.0013
0.0025
0.0013
0.0013
0.0000
0.0000
0.0025
0.0013
0.0013
0.0000
0.0000
0.0013
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
0.0013

0.0000
0.0013
0.0013
0.0000
0.0025
0.0037
0.0000
0.0000
0.0000
0.0000
0.0013
0.0000
0.0000
0.0000
0.0025
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
0.0025
0.0000
0.0025
0.0000
0.0000
0.0000

0.0025
0.0025
0.0000
0.0000
0.0025
0.0000
0.0013
0.0037
0.0000
0.0000
0.0025
0.0025
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
0.0013
0.0013
0.0000
0.0013
0.0013
0.0000
0.0000
0.0000
0.0000

0.0000
0.0000
0.0013
0.0025
0.0025
0.0013
0.0000
0.0000
0.0000
0.0013
0.0037
0.0000
0.0000
0.0000
0.0013
0.0013
0.0013
0.0000
0.0013
0.0000
0.0013
0.0000

We can see the maximum p-value we got is 0.0037 which is still less than the significance value 0.05. So according to random permutation we also reject the null hypotheses or alternatively we assume that at least 1 of the treatment is not the same.

1.4 Discussion:

Both systematic and random permutation rejects H_0 . Here is the summary of the both systematic and random permutation results:

	Systematic Permutation	Random Permutation
p-value	0.0007936507936507937	Changes in different simulations. Minimum=0, Maximum=0.00375(From 100 simulations)
Conclusion	Rejects H_0	Rejects H_0
Number of permutations	1260	800

Appendix:

```
# -*- coding: utf-8 -*-
```

```
''''''
```

Created on Tue Mar 10 22:05:39 2020

```
@author: Salehin
```

```
''''''
```

```
from itertools import combinations
```

```
import math
```

```
import numpy as np
```

```
data=[6,8,9,11,9,17,15,16,16]
```

```
indexes=list(range(1,len(data)+1))
```

```
import random
```

```
sizea=2
```

```
sizeb=3
```

```
sizec=4
```

```
def removefroma(indexes,trta):
```

```
    b=[]
```

```
    for i in range(len(indexes)):
```

```
        k=indexes[i]
```

```
        if k not in trta:
```

```
            b.append(k)
```

```
    return b
```

```
def randomselection(size,array):
```

```
    current=[]
```

```
    new=array
```

```
    for i in range(size):
```

```
        a=random.choice(new)
```

```

    if a not in current:
        current.append(a)
        new.remove(a)
    return current

```

```

def stats(index,data):
    summ=0
    for i in range(len(index)):
        summ+=data[index[i]-1]
    return summ**2/len(index)

```

```

def trta(indexes,sizea):
    return list(combinations(indexes,sizea))
removable=()
n=0
p=0
storage=np.zeros([1260,10])
original=stats((1,2),data)+stats((3,4,5),data)+stats((6,7,8,9),data)
for i in range(int(math.factorial(len(indexes))/((math.factorial(sizea))*math.factorial(len(indexes)-sizea)))):
    currenta=trta(indexes,sizea)[i]
    removable=currenta
    other=removefroma(indexes,removable)
    treatmentb=trta(other,sizeb)
    for j in range(len(treatmentb)):

```

```

currentb=treatmentb[j]
removable=removable+currentb
other=removefroma(indexes,removable)
removable=currenta
total=stats(currenta,data)+stats(currentb,data)+stats(other,data)

a=[currenta[0],currenta[1],currentb[0],currentb[1],currentb[2],other[0],other[1],other[2],other
[3],total]

storage[n][0],storage[n][1]=currenta[0],currenta[1]
storage[n][2],storage[n][3],storage[n][4]=currentb[0],currentb[1],currentb[2]
storage[n][5],storage[n][6],storage[n][7],storage[n][8]=other[0],other[1],other[2],other[3]
storage[n][9]=total
if(total>=original):
    p+=1
    n+=1

np.savetxt('result.txt',storage,delimiter=',',fmt='%1d')
print("Result with Systematic Permutation: ",p/n)
n=800
p1=0
for i in range(n-1):
    currentarray=indexes.copy()
    trta=randomselection(sizea,currentarray)

    arraytoremove=trta
    currentarray=removefroma(indexes,arraytoremove)
    trtb=randomselection(sizeb,currentarray)
    arraytoremove=trta+trtb

```

```
currentarray=removefroma(indexes,arraytoremove)
trtc=currentarray
total=stats(trta,data)+stats(trtb,data)+stats(trtc,data)
if(total>=original):
    p1+=1
currentarray.clear()
arraytoremove.clear()
np.savetxt('random.txt',pvalues,delimiter=',',fmt='%1.4f')
print('Result with random permutation: ',p1/n)
```