Introduction to Quantum Field Theory

Notes re-written from lessons' attendance, 2022
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Corso di Laurea Magistrale in Fisica - Sapienza

Chapter 1

Ottobre

Appunti delle lezioni del Prof. Polosa relative al mese di ottobre 2022.

1.1 The rod

Given a 1-dimensional rod composed by N-particles, linked each others with a "spring", the hamiltonian density is

$$\mathcal{H} = \frac{1}{2}\sum_{n=1}^{N}\Big[P_n^2 + \Omega^2(q_n - q_{n+1})^2 + \Omega_0^2 q_n^2\Big]$$

where the last term $\Omega_0^2 q_n^2$ is relative to the equilibrium position of the n-particle. The *periodic boundaries conditions* to $N \to \infty$ and $a \to 0$.

On the other side we can write the Newtonian equation as

$$\begin{aligned}
H &= \frac{1}{2}\int_0^L dx\Big[p^2(x) + v^2\Big(\frac{\partial q(x)}{\partial x}\Big)\Big] \\
p(x) &= \dot{q}(x) \\
\ddot{q}(x) &= v^2\frac{\partial^2 q(x)}{\partial x^2}
\end{aligned}$$

the solution inside the boundaries is

$$\ddot{q}_n = \Omega^2\Big(q_{n+1} + q_{n-1} - 2q_n\Big)$$

Normal modes or normal frequencies

$$\begin{aligned}
q_n &= \sum_j e^{ijn}\frac{Q_j}{\sqrt{N}} \\
q(x) &= \frac{1}{\sqrt{a}}\sum_n e^{\frac{2\pi l}{Na}(na)}\frac{Q_j}{\sqrt{N}} = \frac{1}{\sqrt{a}}\sum_k e^{ikx}\frac{Q_k}{\sqrt{N}} \\
k &= \frac{2\pi l}{L} \\
\Rightarrow \quad q(x) &= \sum_k e^{ikx}\frac{Q_k}{\sqrt{Na}} = \sum_k e^{ikx}\frac{Q_k}{\sqrt{L}}
\end{aligned}$$

Considering now the Newtonian equation, $p^2(x) = \dot{q}^2(x)$, $\sum_{n=1}^{N} e^{in(j-j')} = \delta_{j,j'}$ where $j = \frac{2\pi l}{N}$, we can move from the sum to the integral using the following relation $\sum_{n=1}^{N} \to \frac{1}{a}\int_0^L dx$ and this leads to $\int_0^L dx e^{i(k-k')x} = L\delta_{k,k'}$.

Somehow we may land on this following expression:

$$\frac{1}{L}\sum_{k,k'} L\delta_{k,k'}Q_k\dot{Q}_{k'} = \sum_k Q_k\dot{Q}_k = \sum_k |\dot{Q}_k|^2$$

To finally get a total classical description, a discrete sum on a numerable set, as follow

$$H = \frac{1}{2} \sum_k |\dot{Q}_k|^2 + k^2 v^2 |Q_k|^2$$

Chapter 2

Novembre

Appunti delle lezioni del Prof. Polosa relative al mese di novembre 2022.

Chapter 3

Dicembre

Appunti delle lezioni del Prof. Polosa relative al mese di dicembre 2022.