Circulation Flux Exact Differentials

Circulation

- $\text{Circulation} = \oint_C \mathbf{F}.d\mathbf{r}$
- **Circulation** is the **flow** of F along C, When C is **simple** and **closed**
- It's simply the Vector Line Integral
- Flow?
  - $\int_C \mathbf{F}.\mathbf{T}ds$
  - $\text{Flow} = \int_C \mathbf{F}.d\mathbf{r} = \int_C Mdx + Ndy + Pdz.$
- Some Syntax ?
  - Let F be the velocity vector of a flowing fluid of constant density 1
  - F(x,y,z).T ds = **the amount of fluid** flowing along the **arc element** ds per unit time
  - \int F.T ds = the net rate of fluid flowing along C = **The Flow**
- if circulation = k != 0
  - The field is swirling around the curve C

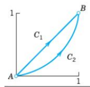
Flux

- $\text{Flux} = \int_C \mathbf{F}.\mathbf{n}ds = \oint_C Mdy - Ndx.$
- a normal vector to ds is simply **(y, -x)**
- Flux is the net rate of fluid flowing across a curve C
- Syntax
  - Let F be the velocity vector of a flowing fluid of density 1
  - F(x,y,z).n ds = **the amount of fluid** flowing across the **arc element** ds per unit time
  - \int F.n ds = the net rate of fluid flowing across C = **The Flow**

Exact Differentials

- Equivalent Statments (**Loop Property**)
  - F = Mi + nj + Pk is conservative over some region R
  - Mdx + Ndy + Pdz is exact over R
  - \int F.dr = 0 around **every** simple closed curve in R
  - \int Mdx + Ndy + Pdz = 0 around **every** siple closed curve in R
- What if we used **green's Thm**. here with that **loop property** ?
  - over **simply connected** regions R
    - The first two statements hold the same
    - curl F =0 over R
    - $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}, \ \frac{\partial M}{\partial z} = \frac{\partial P}{\partial x} \ \text{and} \ \frac{\partial N}{\partial z} = \frac{\partial P}{\partial y} \ \text{over} \ \mathcal{R}$
  - 

The Fundamental Theorem of Line Integral

- $\int_C \mathbf{F}.d\mathbf{r} = \int_A^B \nabla f.d\mathbf{r} = f(B) - f(A).$
  - So the integration depends merely on the start and end points of the curve and doesn't care about the path or the curve
  - This Thm. holds only if the field is conservative
  - To prove this thm. in any problem you just find each term of the equality
    - The good old line integral
    - and the potential difference
- A note for non-conservative Fields
  - 

  - $\int_{C_1} \mathbf{F}.d\mathbf{r} = \int_0^1 t^2 dt = 1/3$
  - $\int_{C_2} \mathbf{F}.d\mathbf{r} = 2\int_0^1 t^4 dt = 2/5$
- The differential form is said to be **exact** if it can be obtained as a differential of some function f
  - $Mdx + Ndy + Pdz = df, \quad f \text{ defined in } \mathcal{R}$
- A vector field is **conservative** over some region D with **potential function** f. If and only if the differential form is **exact**
- The fundamental thm. of line integrals can be restated in terms of exact differenials
  - $\int_C Mdx + Ndy + Pdz = \int_A^B df = f(B) - f(A).$
- if F is conservative
  - $\oint_C \mathbf{F}.d\mathbf{r} = 0.$
- OORR if df is exact
  - $\oint_C Mdx + Ndy + Pdz = 0.$

A method to prove conservative fields

- Let F be the vector field F = Mi + Nj + Pk
- 1) Try to get the potential function f using integration
  - if f exists, Conservative over R =D of f
  - else, NON
- 2) Curl
  - if Curl F = 0
    - and Region is simply connected
      - Conservative
    - not simply connected
      - get f (potential)
        - exists
          - Conservative over R=D of f
        - else
          - NON
  - Curl !=0
    - NON
- 3) Exactness test
  - $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}, \ \frac{\partial M}{\partial z} = \frac{\partial P}{\partial x} \ \text{and} \ \frac{\partial N}{\partial z} = \frac{\partial P}{\partial y} \ \text{over} \ \mathcal{R}$
  - and R is simply Connected
    - Conservative
  - else
    - NON
- 4) using the fundamental thm.
  - $\int_C \mathbf{F}.d\mathbf{r} = \int_A^B \nabla f.d\mathbf{r} = f(B) - f(A).$
  - NOTE that if the path from a to b passes through a point which is outside the domain
    - **you can't use that parametrization**