Forward and Inverse kinematics, Dynamics, Trajectory Analysis for an RRR 3 DOF Manipulator

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RRR 3DOF Manipulator

Abstract:

In this report, forward and inverse kinematics analysis was done on the RRR manipulator. First forwards kinematics was done analytically then simulations were run on the MATLAB. Similarly for inverse kinematics, first equations in the form x, y and z were derived from DH table- Then these equations were solved for 3 joint angles. Then results were put in the MATLAB to run the simulations. In MATLAB we got two configurations for the single end effector position. Also the trajectory is also determined using joint space trajectory. Then simulations were run to determine the torques at different joints.

Keywords: RRR manipulator, Forward kinematics analysis, Inverse kinematics analysis, MATLAB, DH table.

Introduction:

Robots play a huge role in this era of technology. Many industries utilize these manipulators to reduce cost and time of production. These manipulators can also be utilized in hazardous environment where human can not reach safely. Now a days, robot is not only utilized in industries but also in homes too. Robots are also important in terms of cost and ROI (Return on investment). Robots have higher and accurate production so we can get higher and accurate production in less time as compared to workers production.

Robots can also be helpful in performing medical procedures that a human cannot perform accurately. In medical field there are robots that can perform surgeries, but these robots are controlled by a surgeon so that everything keeps under control. Robots can also help in reduction of accidents in industry because the risk of accidents in industry is greater when workers work in the risky places.

A robotic arm with three revolute joints (RRR kinematics), implemented as waist, shoulder and elbow, is our test bed for modeling and identification. The kinematics of this arm is very common in industry: the first three degrees-of-freedom of the PUMA 560 or the KUKA IR 361 robots are implemented in this fashion. Therefore, the results presented in this paper can be directly employed in industry. To stress the influence of nonlinear dynamic couplings between the robot axes, we consider a direct-drive actuation for the robot. It is well-known that with a transmission between joint actuators and links, nonlinear couplings can become small. [1]

Main objective of the RRR-robot design project is to realize a manipulator-like system to test these advanced nonlinear control strategies. To achieve a resemblance with conventional industrial robots (e .g., Puma-type robots), the system should have a chain structure and at least three rotational degrees of freedom. Although several other nonlinear effects (e .g., friction) may also reduce the performance, in this project the focus is on highlighting the Coriolis and centrifugal torques . Therefore, the main requirement of the robot is the ability to maintain these torques at a significant level. [2]

A number of modeling methods are available, meeting various requirements. As for robot kinematics, the model is a mapping between the task space (in general a 6-dimensional space of robot-tip coordinates) and the joint space (dimension equal to the number of robot degrees of freedom). The mapping from the joint to the task space is called the forward kinematics. The opposite mapping is the inverse kinematics. These mappings can be represented as recursive or closed-form algebraic models. The algebraic closed-form representation facilitates manipulation of the models and enables a straightforward mathematical analysis. [1]

Following is the Forward and inverse Kinematic analysis of RRR 3DOF robot.

Forward Kinematics:

First DH table is made for the below figure.

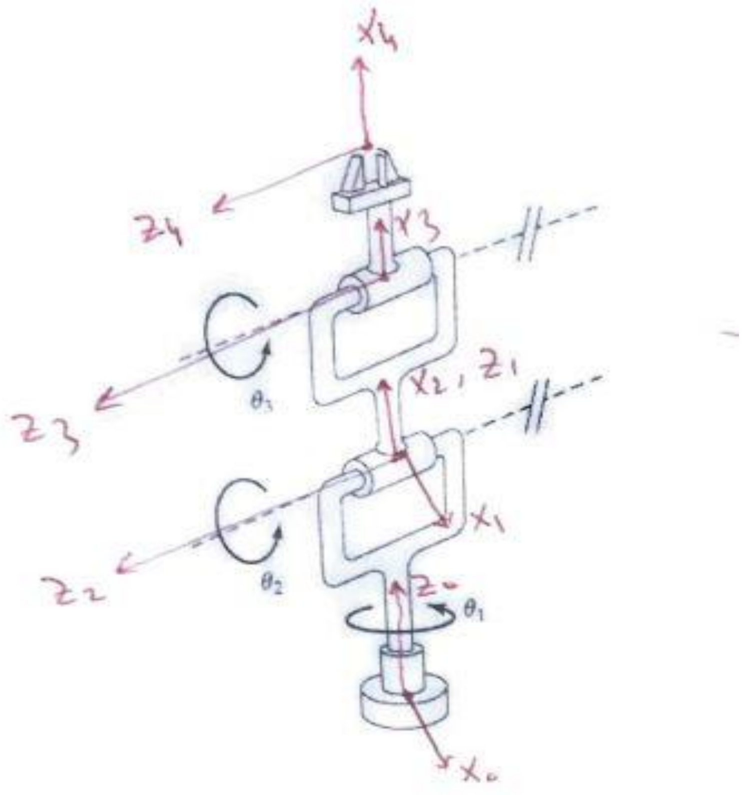


Figure 1: Axis Definition for RRR manipulator

DH Table:

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	L_1	θ_1
2	$\pi/2$	0	0	$\theta_2 + \pi/2$
3	0	L_2	0	θ_3
4	0	L_3	0	0

Table 1: DH table of RRR Manipulator

$$
{}^{0}_{1}T = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad {}^{1}_{2}T = \begin{bmatrix} -s_2 & -c_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ c_2 & -s_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad {}^{2}_{3}T = \begin{bmatrix} c_3 & -s_3 & 0 & l_2 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{3}_{4}T = \begin{bmatrix} 1 & 0 & 0 & l_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$

Now to find the ${}^{0}_{4}T$.

$$ {}^{0}_{4}T = {}^{0}_{1}T\ {}^{1}_{2}T\ {}^{2}_{3}T\ {}^{3}_{4}T = \begin{bmatrix} -c_1c_3s_2 - c_1c_2s_3 & c_1s_3s_2 - c_1c_2c_3 & s_1 & -l_3(c_1c_3s_2 + c_1c_2s_3) - c_1s_2l_2 \\ -s_1c_3s_2 - c_1c_2s_3 & s_1s_2s_3 - s_1c_2c_3 & -c_1 & -l_3(s_1s_2c_3 - s_1c_2s_3) - s_1s_2l_2 \\ c_2c_3 - s_2s_3 & -c_2s_3 - s_2c_3 & 0 & l_3(c_2c_3 - s_2s_3) - c_2l_2 + l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

So, from the above matrix we get that

x = $-L_3(c_1c_3s_2 + c_1c_2s_3) - c_1s_2L_2$

$y = -L_3(s_1s_2c_3 - s_1c_2s_3) - s_1s_2L_2$

$z = L_3(c_2c_3 - s_2s_3) - c_2L_2 + L_1$

Inverse Kinematics:

From the final transformation matrix, we get the following equations for the end effector position.

Equations:

$$x = -L_3 * [(c_1 * c_3 * s_2 + c_1 * c_2 * s_3)] - L_2 * c_1 * s_2$$

$$y = -L_3 * [(s_1 * c_3 * s_2 + s_1 * c_2 * s_3)] - L_2 * s_1 * s_2$$

$$z = L_3 * [(c_2 * c_3 - s_2 * s_3)] + L_2 * c_2 + L_1$$

As,

$$c_3 * s_2 + c_2 * s_3 = s_{23}$$

$$c_2 * c_3 - s_2 * s_3 = c_{23}$$

So, equation take the form.

$$x = -L_3 * s_{23} * c_1 - L_2 * c_1 * s_2$$

$$y = -L_3 * s_{23} * s_1 - L_2 * s_1 * s_2$$

$$z = L_3 * c_{23} + L_2 * c_2 + L_1$$

For Θ_1:

$$x = (-L_3 * s_{23} - L_2 * s_2) * c_1$$

$$c_1 = \frac{x}{(-L_3 * s_{23} - L_2 * s_2)}$$

$$y = (-L_3 * s_{23} - L_2 * s_2) * s_1$$

$$s_1 = \frac{y}{(-L_3 * s_{23} - L_2 * s_2)}$$

$$\Theta_1 = Atan2(s_2, c_2)$$

For Θ_3:

$$\frac{x}{c_1} = \dot{x} = (-L_3 * s_{23} - L_2 * s_2)$$

$$\frac{y}{s_1} = \dot{y} = (-L_3 * s_{23} - L_2 * s_2)$$

$$z - L_1 = \dot{z} = L_3 * c_{23} + L_2 * c_2$$

$$\dot{x}^2 = {L_3}^2 * {s_{23}}^2 + {L_2}^2 * {s_2}^2 + 2 * L_3 * L_2 * s_2 * s_{23}$$

$$\dot{y}^2 = {L_3}^2 * {s_{23}}^2 + {L_2}^2 * {s_2}^2 + 2 * L_3 * L_2 * s_2 * s_{23}$$

$$\dot{z}^2 = {L_3}^2 * {c_{23}}^2 + {L_2}^2 * {c_2}^2 + 2 * L_3 * L_2 * c_{23} * c_2$$

$$\dot{y}^2 + \dot{z}^2 = {L_3}^2 * ({s_{23}}^2 + {c_{23}}^2) + {L_2}^2 * ({s_2}^2 + {c_2}^2) + 2 * L_3 * L_2 * (s_2 * s_{23} + c_{23} * c_2)$$

$$\dot{y}^2 + \dot{z}^2 = {L_3}^2 + {L_2}^2 + 2 * L_3 * L_2 * c_3$$

$$c_3 = \frac{\dot{y}^2 + \dot{z}^2 - {L_3}^2 - {L_2}^2}{2 * L_3 * L_2}$$

$$s_3 = \pm\sqrt{1 - {c_3}^2}$$

$$\Theta_3 = Atan2(s_3, c_3)$$

For Θ_2:

$$x = -L_3 * c_1 * c_3 * s_2 - L_3 * c_1 * c_2 * s_3 - L_2 * c_1 * s_2$$

$$x = -(L_3 * c_1 * s_3) * c_2 + (-L_3 * c_1 * c_3 - L_2 * c_1) * s_2$$

$$y = -L_3 * s_1 * c_3 * s_2 - L_3 * s_1 * c_2 * s_3 - L_2 * s_1 * s_2$$

$$x = -(L_3 * s_1 * s_3) * c_2 + (-L_3 * s_1 * c_3 - L_2 * s_1) * s_2$$

Writing equations of x and y in matrix form

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -(L_3 * c_1 * s_3) & (-L_3 * c_1 * c_3 - L_2 * c_1) \\ -(L_3 * s_1 * s_3) & (-L_3 * s_1 * c_3 - L_2 * s_1) \end{bmatrix} * \begin{bmatrix} c_2 \\ s_2 \end{bmatrix}$$

Using cramer's rule

$$c_2 = \frac{\begin{vmatrix} x & (-L_3 * c_1 * c_3 - L_2 * c_1) \\ y & (-L_3 * s_1 * c_3 - L_2 * s_1) \end{vmatrix}}{\begin{vmatrix} -(L_3 * c_1 * s_3) & (-L_3 * c_1 * c_3 - L_2 * c_1) \\ -(L_3 * s_1 * s_3) & (-L_3 * s_1 * c_3 - L_2 * s_1) \end{vmatrix}}$$

$$s_2 = \frac{\begin{vmatrix} -(L_3 * c_1 * s_3) & x \\ -(L_3 * s_1 * s_3) & y \end{vmatrix}}{\begin{vmatrix} -(L_3 * c_1 * s_3) & (-L_3 * c_1 * c_3 - L_2 * c_1) \\ -(L_3 * s_1 * s_3) & (-L_3 * s_1 * c_3 - L_2 * s_1) \end{vmatrix}}$$

$$\Theta_2 = Atan2(s_2, c_2)$$

Joint-Space Trajectory Planning:

Joint-space trajectory generation is in common usage in robotics to provide smooth, continuous motion from one set of joint angles to another, for instance, for moving between two distinct Cartesian poses for which the inverse pose solution has yielded two distinct sets of joint angles. The joint-space trajectory generation occurs at runtime for all joints independently but simultaneously.

The general equations for Cubic polynomials are

$$\Theta_1(t) = a_{10} + a_{11}t + a_{12}t^2 + a_{13}t^3$$

$$\Theta_2(t) = a_{20} + a_{21}t + a_{22}t^2 + a_{23}t^3$$

$$\Theta_3(t) = a_{30} + a_{31}t + a_{32}t^2 + a_{33}t^3$$

$$\dot{\Theta}_1(t) = a_{11} + 2a_{12}t + 3a_{13}t^2$$

$$\dot{\Theta}_2(t) = a_{21} + 2a_{22}t + 3a_{23}t^2$$

$$\dot{\Theta}_3(t) = a_{31} + 2a_{32}t + 3a_{33}t^2$$

The given Parameters of the above equations are.

Initial position angles and joint velocities

$$\Theta_1(0), \Theta_2(0), \Theta_3(0), \dot{\Theta}_1(0), \dot{\Theta}_2(0), \dot{\Theta}_3(0)$$

Final position angles and joint velocities

$$\Theta_1(t_f), \Theta_2(t_f), \Theta_3(t_f), \dot{\Theta}_1(t_f), \dot{\Theta}_2(t_f), \dot{\Theta}_3(t_f)\ and\ t_f$$

Usually we take t_f as 1

These values can come from the inverse kinematics where x, y, z for initial position and final position is given and the joint angles for that position is can be found.

At t=0

$$\boldsymbol{\Theta_1(0) = a_{10}}$$

$$\boldsymbol{\Theta_2(0) = a_{20}}$$

$$\boldsymbol{\Theta_3(0) = a_{30}}$$

$$\boldsymbol{\dot{\Theta}_1(0) = a_{11}}$$

$$\boldsymbol{\dot{\Theta}_2(0) = a_{21}}$$

$$\boldsymbol{\dot{\Theta}_3(0) = a_{31}}$$

At t= t_f

$$\Theta_1(\,t_f) = a_{10} + a_{11}\,t_f + a_{12}\,{t_f}^2 + a_{13}\,{t_f}^3 \_\_\_\_\_\_\_\_\_\_ (1)$$

$$\Theta_2(\,t_f) = a_{20} + a_{21}\,t_f + a_{22}\,{t_f}^2 + a_{23}\,{t_f}^3 \_\_\_\_\_\_\_\_\_\_ (2)$$

$$\Theta_3(\,t_f) = a_{30} + a_{31}\,t_f + a_{32}\,{t_f}^2 + a_{33}\,{t_f}^3 \_\_\_\_\_\_\_\_\_\_ (3)$$

$$\dot{\Theta}_1(\,t_f) = a_{11} + 2a_{12}\,t_f + 3a_{13}\,{t_f}^2 \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (4)$$

$$\dot{\Theta}_2(\,t_f) = a_{21} + 2a_{22}\,t_f + 3a_{23}\,{t_f}^2 \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (5)$$

$$\dot{\Theta}_3(\,t_f) = a_{31} + 2a_{32}\,t_f + 3a_{33}\,t_f{}^2 \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (6)$$

Using equation (1) and (4) at $t_f = 1$ to find a_{12} and a_{13}.

$$\Theta_1(\,t_f) = a_{10} + a_{11} + a_{12} + a_{13} \_\_\_\_\_\_ (a)$$

$$\dot{\Theta}_1(\,t_f) = a_{11} + 2a_{12} + 3a_{13} \_\_\_\_\_\_\_\_ (b)$$

From eq (a)

$$\boldsymbol{a_{12} = \Theta_1(\,t_f) - a_{10} - a_{11} - a_{13}}$$

Put in eq (b)

$$\dot{\Theta}_1(\,t_f) = a_{11} + 2(\Theta_1(\,t_f) - a_{10} - a_{11} - a_{13}) + 3a_{13}$$

$$\dot{\Theta}_1(\,t_f) = a_{11} + 2\Theta_1(\,t_f) - 2a_{10} - 2a_{11} - 2a_{13} + 3a_{13}$$

$$\dot{\Theta}_1(\,t_f) = 2\Theta_1(\,t_f) - 2a_{10} - a_{11} + a_{13}$$

$$\boldsymbol{a_{13} = \dot{\Theta}_1(\,t_f) - 2\Theta_1(\,t_f) + 2a_{10} + a_{11}}$$

Similarly solving eq (2) and eq(5) simultaneously,

$$\boldsymbol{a_{22} = \Theta_2(\,t_f) - a_{20} - a_{21} - a_{23}}$$

$$\boldsymbol{a_{23} = \dot{\Theta}_2(\,t_f) - 2\Theta_2(\,t_f) + 2a_{20} + a_{21}}$$

Similarly solving eq (3) and eq(6) simultaneously,

$$\boldsymbol{a_{32} = \Theta_3(\,t_f) - a_{30} - a_{31} - a_{33}}$$

$$\boldsymbol{a_{33} = \dot{\Theta}_3(\,t_f) - 2\Theta_3(\,t_f) + 2a_{30} + a_{31}}$$

We put these polynomials in the general equation of cubic polynomials to get the final equations of $\Theta_1(t), \Theta_2(t), \Theta_3(t), \dot{\Theta}_1(t), \dot{\Theta}_2(t), \dot{\Theta}_3(t)$

From position **(1.3 , 1.3 , 1.3) to (2, 2.5, 2.5)** we get the following values of constants using MATLAB code.

a10 = 0.7854 **a32 = -1.8867**

a20 = 3.1416

a30 = 2.5102

a11 = 0

a21 = 0

a31 = 0

a13 = -0.2213

a12 = 0.3320

a23 = 9.4248

a22 = -14.1372

a33 = 1.2578

So, the final equations become

$$\theta_1(t) = 0.7854 + 0.3320t^2 - 0.2213t^3$$

$$\theta_2(t) = 3.1416 - 14.1372t^2 + 9.4248t^3$$

$$\theta_3(t) = 2.5102 - 1.8867t^2 + 1.2578t^3$$

MATLAB code is written in appendix through which graphs are generated.

Following are the graphs that are generated for the equations q1, q2, q3, dq1, dq2, dq3.

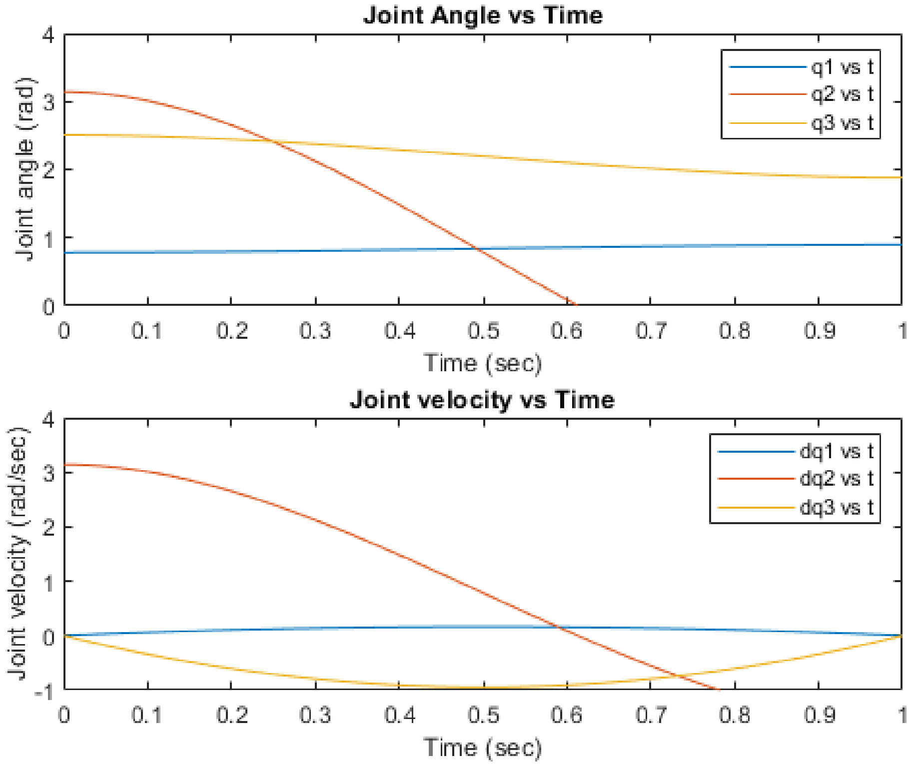


Figure 2: graph for joint velocity and joint angle vs time

Dynamic Analysis:

Lagrange method:

Detailed dynamics analysis is done in appendix A , given in the end of the report. Only main equations are shown here. The lagrangian method was chosen because the newton-euler method is to complicated. As we have to find the torques at joints so Lagrangian method was best and easy to apply.

For link1:

$$k_1 = \frac{1}{2} m_1 {v_{c1}}^T v_{c1} + \frac{1}{2} {}^1_1w^T \, {}^{c1}_1I \, {}^1_1w$$

First for velocity of link 1 center can be found as follows.

i	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	0	$L_1/2$	θ_1

Now from the transformation matrix of ${}^{0}_{1}T$, we get the center of mass position of link 1.

$$X = 0$$

$$Y = 0$$

$$Z = \frac{L_1}{2}$$

$${}^{1}_{1}w^T = [0 \quad 0 \quad \dot{\theta}_1]$$

$${}^{c1}_{1}I = \begin{bmatrix} \frac{m_1 {L_1}^2}{12} & 0 & 0 \\ 0 & \frac{m_1 {L_1}^2}{12} & 0 \\ 0 & 0 & \frac{m_1 {L_1}^2}{12} \end{bmatrix}$$

$$k.E_1 = \frac{m_1 {L_1}^2 {\dot{\theta}_1}^2}{24}$$

And for P.E of link 1 is

$$U_1 = -m_1 {g_0}^T \, {}^{0}_{c1}P$$

$$U_1 = \frac{m_1 g L_1}{2}$$

For Link 2:

$$k_2 = \frac{1}{2} m_2 {v_{c2}}^T \, v_{c2} + \frac{1}{2} \, {}^{2}_{2}w^T \, {}^{c2}_{2}I \, {}^{2}_{2}w \underline{\qquad\qquad\qquad} (B)$$

Now for v_{c2} first we have to find the position of link 2 center with respect to frame 0. For that frame was taken at center of link 2 and DH table was obtained.

i	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	0	L_1	θ_1
2	pi/2	0	0	$\theta_1 + pi/2$
3	0	$L_2/2$	0	0

Then ${}^{0}_{3}T$ was obtained from which we get link 2 center of mass positions.

$${}^{0}_{3}P = \begin{bmatrix} -\frac{L_2 c_1 s_2}{2} \\ -\frac{L_2 s_1 s_2}{2} \\ L_1 + \frac{L_2 c_2}{2} \end{bmatrix}$$

Now taking derivative of the above matrix, we get the velocity of link2 center.

$$V_{c2} = \begin{bmatrix} \frac{L_2 s_1 s_2 \dot{\theta}_1}{2} - \frac{L_2 c_1 c_2 \dot{\theta}_2}{2} \\ -\frac{L_2 s_1 c_2 \dot{\theta}_2}{2} - \frac{L_2 c_1 s_2 \dot{\theta}_1}{2} \\ -\frac{L_2 s_2 \dot{\theta}_2}{2} \end{bmatrix}$$

Also,

$$V_{c2} {V_{c2}}^T = \frac{{L_2}^2 {s_2}^2 {\dot{\theta}_2}^2}{4} + \frac{{L_2}^2 {\dot{\theta}_2}^2}{4}$$

Now the inertia matrix for link 2 is given by;

$$ {}^{c2}_{2}I = \begin{bmatrix} \frac{m_2 {L_2}^2}{12} & 0 & 0 \\ 0 & \frac{m_2 {L_2}^2}{12} & 0 \\ 0 & 0 & \frac{m_2 {L_2}^2}{12} \end{bmatrix}$$

And the angular velocity for link 2 is

$${}^{2}_{2}w^T = [c_2 \dot{\theta}_1 \quad -s_2 \dot{\theta}_1 \quad \dot{\theta}_2 + \dot{\theta}_3]$$

Now putting all these terms in eq 'B' we get the following equation given below:

$$k_2 = \frac{m_2 {L_2}^2 {s_2}^2 {\dot{\theta}_1}^2}{8} + \frac{m_2 {L_2}^2 {\dot{\theta}_2}^2}{6} + \frac{m_2 {L_2}^2 {\dot{\theta}_1}^2}{24}$$

Also, potential energy for link 2 is:

$$U_2 = -m_2 {g_0}^T \, {}^{0}_{c2}P$$

$$U_2 = m_2 g L_1 + \frac{m_2 g L_2 c_2}{2}$$

For Link 3:

$$k_3 = \frac{1}{2} m_3 {v_{c3}}^T v_{c3} + \frac{1}{2} \, {}^{3}_{3}w^T \, {}^{c3}_{3}I \, {}^{3}_{3}w$$

Same approach applied here to find the center of mass velocity for link 3.

i	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	0	L_1	θ_1
2	pi/2	0	0	$\theta_1 + pi/2$
3	L_2	0	0	θ_3
4	$L_3/2$	0	0	0

$$x_{c3} = -L_2c_1s_2 - \frac{L_3}{2}(c_1s_3c_2 + c_1c_3s_2)$$

$$y_{c3} = -L_2s_1s_2 - \frac{L_3}{2}(s_1s_3c_2 + s_1c_3s_2)$$

$$x_{c3} = L_1 + \frac{L_3}{2}(c_3c_2 + s_3s_2) + L_2c_2$$

$$V_{c3} = L_2s_1s_2\dot{\Theta}_1 + L_2c_1c_2\dot{\Theta}_2 + \frac{L_3}{2}s_1s_{23}\dot{\Theta}_1 - \frac{L_3}{2}c_1c_{23}\dot{\Theta}_{23} - L_2s_1c_2\dot{\Theta}_2 - L_2c_1s_2\dot{\Theta}_1 - \frac{L_3}{2}c_1s_{32}\dot{\Theta}_1 - \frac{L_3}{2}s_1c_{32}\dot{\Theta}_{32} - \frac{L_3}{2}s_{32}\dot{\Theta}_{32} - L_2s_2\dot{\Theta}_2$$

$${}^3_3w^T = [c_{23}\dot{\Theta}_1 \quad -s_{23}\dot{\Theta}_1 \quad \dot{\Theta}_2 + \dot{\Theta}_3]$$

$${}^{c3}_3I = \begin{bmatrix} \frac{m_3{L_3}^2}{12} & 0 & 0 \\ 0 & \frac{m_3{L_3}^2}{12} & 0 \\ 0 & 0 & \frac{m_3{L_3}^2}{12} \end{bmatrix}$$

After putting above equations in eq 'C' we get the following equation:

$$k_3 = \frac{m_3{L_2}^2{\dot{\Theta}_1}^2}{2} + \frac{m_3{L_3}^2\dot{\Theta}_{32}}{8} + \frac{m_3{L_2}^2{s_2}^2{\dot{\Theta}_1}^2}{2} + \frac{m_3{L_3}^2{s_{23}}^2{\dot{\Theta}_1}^2}{8} + \frac{m_3L_2L_3s_2s_{23}{\dot{\Theta}_1}^2}{2} + \frac{m_3L_2L_3c_3\dot{\Theta}_2\dot{\Theta}_{23}}{2} + \frac{m_3{L_3}^2{\dot{\Theta}_1}^2}{24} + \frac{m_3{L_3}^2{\dot{\Theta}_{23}}^2}{24}$$

And potential energy equation for link 2 is:

$$U_3 = m_3gL_1 + \frac{m_3gL_3c_{32}}{2} + m_3gL_2c_2$$

Now adding all the K.E's and P.E's and then applying the Lagrangian formulation. Finally, we get the following torques at joints

$$T_1 = \frac{m_1{L_1}^2\ddot{\theta}_1}{12} + \frac{m_2{L_2}^2(2s_2c_2\dot{\theta}_1\dot{\theta}_2 + s_2^2\ddot{\theta}_1)}{12} + \frac{m_1{L_2}^2\ddot{\theta}_1}{12} + m_3{L_2}^2\ddot{\theta}_1 + m_3L_2(2s_2c_2\dot{\theta}_1\dot{\theta}_2 + s_2^2\ddot{\theta}_1) + \frac{m_3{L_3}^2(2s_{23}c_{23}\dot{\theta}_{23}\dot{\theta}_1 + s_{23}^2\ddot{\theta}_1)}{4} + \frac{m_3{L_3}^2\ddot{\theta}_1}{12}$$

$$T_2 = m_2{L_2}^2\ddot{\theta}_2 + m_3L_2L_3(-s_3\dot{\theta}_3\dot{\theta}_2 + c_3\ddot{\theta}_2) + m_3L_2L_3\left(-s_3{\dot{\theta}_3}^2 + c_3\ddot{\theta}_3\right) + \frac{m_3{L_3}^2(\ddot{\theta}_2 + \ddot{\theta}_3)}{12} - m_3{L_2}^2{\ddot{\theta}_1}^2s_2c_2 - \frac{m_3{L_3}^2s_{23}c_{23}{\dot{\theta}_1}^2}{4} - m_3L_2L_3(c_2s_{23} + s_2c_{23}) - \frac{m_2L_2s_2g}{2} - \frac{m_3L_3s_{32}g}{2} - m_3L_2s_2g$$

$$T_3 = \frac{m_3 {L_3}^2(\ddot{\theta}_2 + \ddot{\theta}_3)}{3} - \frac{m_3 {L_3}^2 {\dot{\theta}_1}^2 s_{23} c_{23}}{4} - \frac{m_3 L_2 L_3 {\dot{\theta}_1}^2 s_2 c_{23}}{2} + \frac{m_3 L_2 L_3 \dot{\theta}_2 s_3 c_{23} \dot{\theta}_{23}}{2} - \frac{m_3 L_3 s_{23} g}{2}$$

Now in state space model:

Mass/Inertia matrix is M(θ) $\ddot{\theta}$=

$$\begin{bmatrix} \frac{m_1 {L_1}^2}{12} + \frac{m_1 {L_1}^2 s_2^2}{4} + \frac{m_2 {L_2}^2}{12} + m_3 {L_2}^2 + m_3 L_2 s_2^2 + \frac{m_3 {L_3}^2 s_{23}^2}{4} + \frac{m_3 {L_3}^2}{12} & 0 & 0 \\ 0 & \frac{m_2 {L_2}^2}{4} + m_3 L_2 L_3 c_3 + \frac{m_3 {L_3}^2}{12} & \frac{m_2 {L_2}^2}{4} + m_3 L_2 L_3 c_3 + \frac{m_3 {L_3}^2}{12} \\ 0 & \frac{m_3 {L_3}^2}{3} & \frac{m_3 {L_3}^2}{3} \end{bmatrix}$$

Now the centrifugal vector is V;

$$\begin{bmatrix} \frac{m_2 {L_2}^2 s_2 c_2 \dot{\theta}_1 \dot{\theta}_2}{2} + 2 m_3 L_2 s_2 c_2 \dot{\theta}_1 \dot{\theta}_2 + \frac{m_3 {L_3}^2 s_{23} c_{23} \dot{\theta}_{23} \dot{\theta}_1}{2} \\ -m_3 L_2 L_3 s_3 \dot{\theta}_3 \dot{\theta}_2 - m_3 L_2 L_3 s_3 \dot{\theta}_3 - m_3 {L_2}^2 s_2 c_2 {\dot{\theta}_1}^2 - \frac{m_3 {L_3}^2 s_{23} c_{23} {\dot{\theta}_1}^2}{4} - m_3 L_3 L_2 {\dot{\theta}_1}^2 (c_2 s_{23} + s_2 c_{23}) \\ \frac{-m_3 {L_3}^2 s_{23} c_{23} {\dot{\theta}_1}^2}{4} - \frac{m_3 L_2 L_3 s_2 c_{23} {\dot{\theta}_1}^2}{2} + \frac{m_3 L_2 L_3 s_3 \dot{\theta}_2 \dot{\theta}_{23}}{2} \end{bmatrix}$$

Now the gravity matrix is given by G:

$$\begin{bmatrix} 0 \\ \frac{m_2 L_2 s_2 g}{2} - \frac{m_3 L_3 s_{23} g}{2} - m_3 L_2 s_2 g \\ \frac{-m_3 L_3 s_{23} g}{2} \end{bmatrix}$$

Results and Discussion:

After running the simulation on MATLAB, the following results are obtained.

Forward Kinematics:

First joint angles(q1,q2,q3) were given and end-effector position (x,y,z) was found. The link lengths are L1=1, L2=3, L3=3

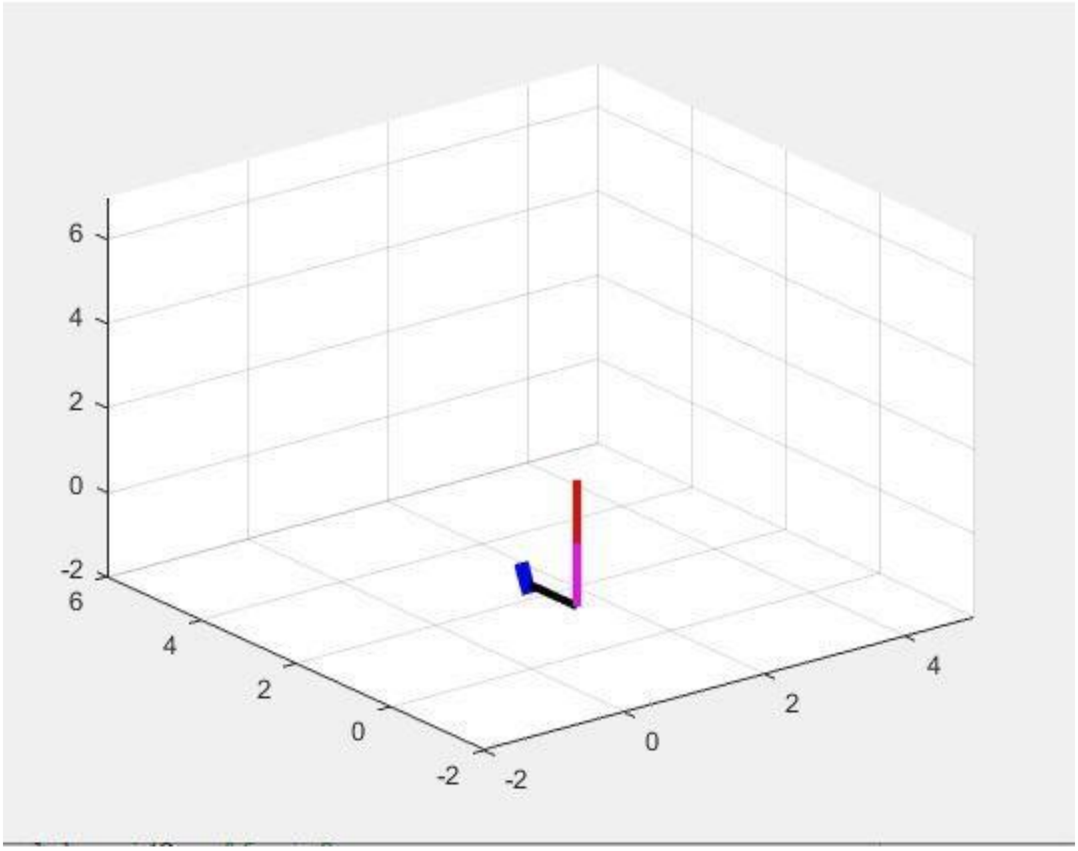


Figure 2 Forward Kinematics simulation

Inverse Kinematics:

In inverse kinematics, end effector position was given, and joint angles were found. The circle near end-effector is the path followed by the end-effector. Thus, for the given circular path, the joint angles were calculated by the algorithm given in appendix A.

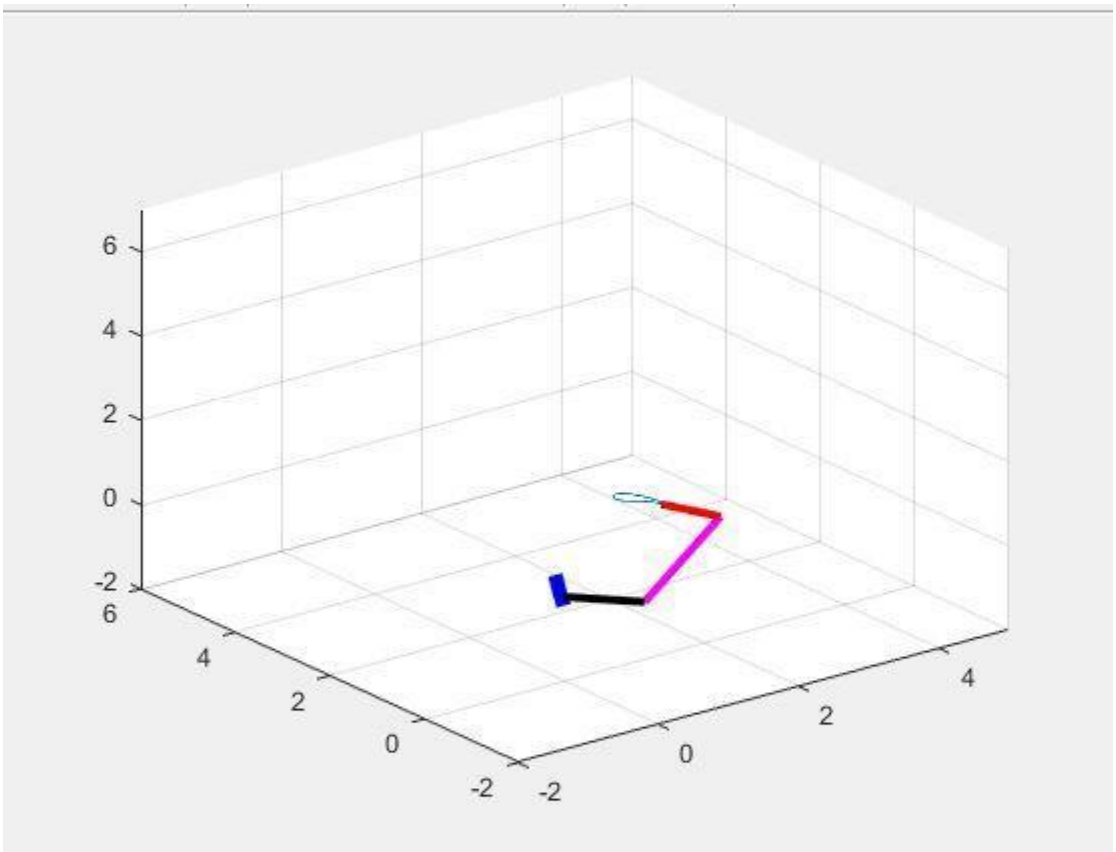


Figure 3 Inverse Kinematics with positive angle

Similarly for negative s3, the following result is obtained.

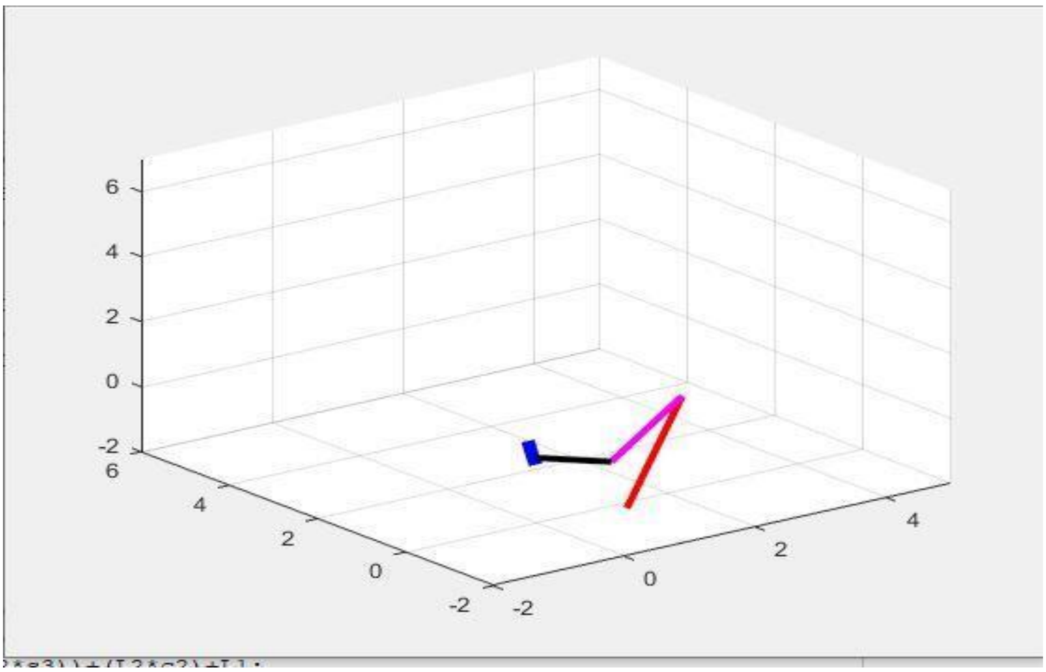


Figure 4 Inverse kinematics with negative angle

Now the trajectory is defined so the simulations were run on the MATLAB using the above derived trajectory, the joint torques have been calculated.

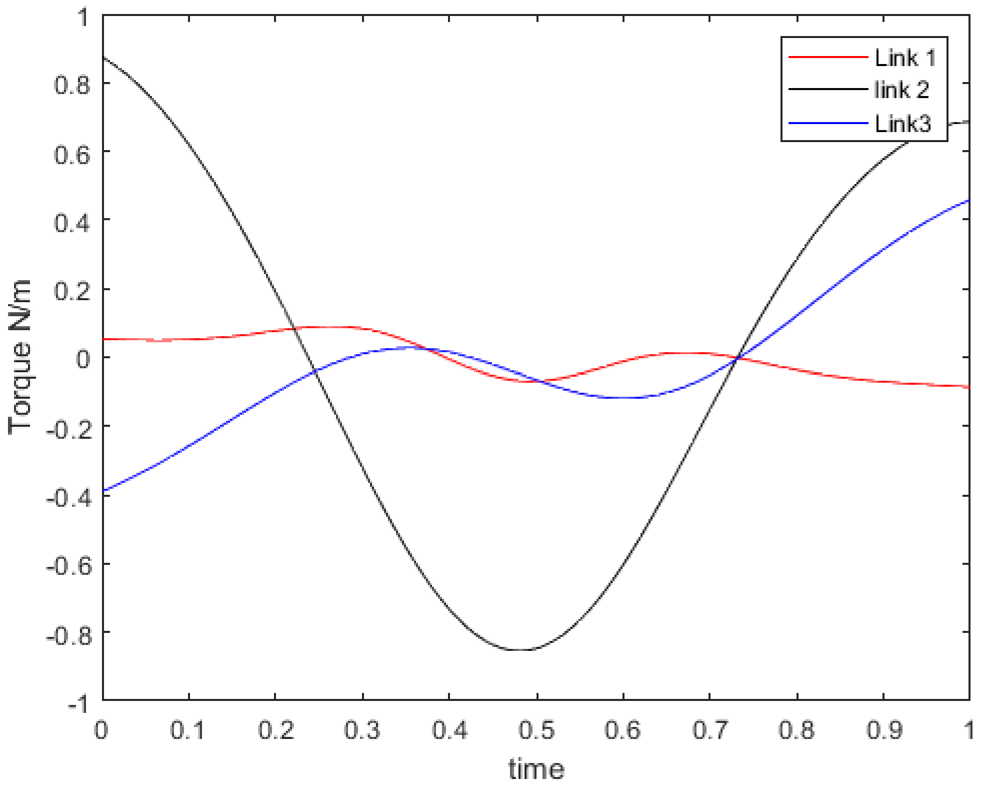


Figure 3 Joint Torques at different joints

The above graph shows the variations in torque at different joints.

Conclusion:

RRR manipulator gives two configurations for the same end effector position. One is elbow up configuration and one is elbow-down configurations. Singularity also occurs when c3 becomes greater then 0, due to which negative value occurs in the square root. By using the MATLAB code, the torques at different joints are also determined. While using this data, selection of motors can become pretty easy.

References

[1] D. Kostic and S. , "Modeling and Identification for High-Performance Robot Control: An RRR-Robotic Arm Case Study," *Researchgate.*

[2] A. M. van Beek, "RRR-robot : design of an industrial-like test facility for," *Eindhoven University of Technology,* 1998.

Appendix A: The details dynamic analysis is given below:

1

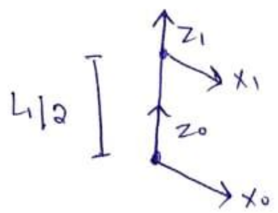
Dynamics Analysis.

Lagrangian Approach.

For link 1

$$K_1 = \frac{1}{2} m_1 V_{c_1}^T V_{c_1} + \frac{1}{2} {}^1\omega_1^T \, {}^{c_1}I_1 \, {}^1\omega_1 \rightarrow \text{eq (A)}$$

First for velocity of link 1 center can be found as follows.



i	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	0	$\frac{L_1}{2}$	θ_i

$x = 0$

$y = 0$

$z = \frac{L_1}{2}$

$$V_{c_1} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

②

$$ {}^{1}\omega_1^{T} = \begin{bmatrix} 0 & 0 & \dot{\theta}_1 \end{bmatrix} $$

So

$$ {}^{1}\omega_1^{T}\, {}^{c_1}I_1\, {}^{1}\omega_1 = \begin{bmatrix} 0 & 0 & \dot{\theta}_1 \end{bmatrix} \begin{bmatrix} \frac{m_1L_1^2}{12} & 0 & 0 \\ 0 & \frac{m_1L_1^2}{12} & 0 \\ 0 & 0 & \frac{m_1L_1^2}{12} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} = \frac{m_1L_1^2}{12}\dot{\theta}_1^2 $$

So eq (A) becomes

$$ \boxed{K.E_1 = \frac{1}{2}\frac{m_1L_1^2}{12}\dot{\theta}_1^2} \longrightarrow (1) $$

Also P.E for link 1 is

$$ U_1 = -m_1\, {}^{0}g^{T}\, {}^{0}P_{c_1} $$

$$ = -m_1 \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix}^{T} \begin{bmatrix} 0 \\ 0 \\ \frac{L_1}{2} \end{bmatrix} $$

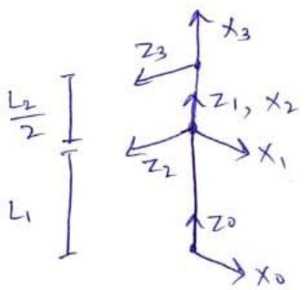
$$ \boxed{U_1 = \frac{m_1 g L_1}{2}} \longrightarrow (2) $$

(3)

Now For link 2

$$K_2 = \frac{1}{2} m_2 V_{c_2}^T V_{c_2} + \frac{1}{2} {}^2\omega_2^T \, {}^{c_2}I_2 \, {}^2\omega_2 \rightarrow (B)$$

To find V_{c_2}



i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	L_1	θ_1
2	$\frac{\pi}{2}$	0	0	$\theta_2 + \frac{\pi}{2}$
3	0	$\frac{L_2}{2}$	0	0

Now from ${}^0_3T$, we obtain ${}^0_3P$ which is

$${}^0_3P = \begin{bmatrix} \dfrac{-L_2 c_1 s_2}{2} \\ \\ \dfrac{-L_2 s_2 s_1}{2} \\ \\ L_1 + \dfrac{L_2 c_2}{2} \end{bmatrix}$$

$$x_{c_2} = \frac{-L_2 c_1 s_2}{2} \; ; \; \dot{x}_{c_2} = -\frac{L_2}{2}\left[s_1 s_2 \dot\theta_1 - c_1 c_2 \dot\theta_2\right]$$

(4)

$$\dot{y}_{c_2} = -\frac{L_2}{2}\left[C_2 \dot{\theta}_2 S_1 + S_2 C_1 \dot{\theta}_1 \right]$$

$$\dot{z}_{c_2} = \frac{-L_2}{2} S_2 \dot{\theta}_2$$

So

$$V_{c_2} = \begin{bmatrix} \frac{L_2}{2} S_1 S_2 \dot{\theta}_1 - \frac{L_2}{2} C_1 C_2 \dot{\theta}_2 \\ -\frac{L_2}{2} C_2 S_1 \dot{\theta}_2 - \frac{L_2}{2} S_2 C_1 \dot{\theta}_1 \\ -\frac{L_2}{2} S_2 \dot{\theta}_2 \end{bmatrix}$$

~~So~~ Also

$$V_{c_2}^T V_{c_2} = \frac{L_2^2 S_2^2 \dot{\theta}_1^2}{4} + \frac{L_2^2 \dot{\theta}_2^2}{4}$$

and

$${}^2\omega_2^T \, {}^{c_2}I_2 \, {}^2\omega_2 = \begin{bmatrix} C_2\dot{\theta}_1 & -S_2\dot{\theta}_1 & \dot{\theta}_2 \end{bmatrix} \begin{bmatrix} \frac{m_2 L_2^2}{12} & 0 & 0 \\ 0 & \frac{m_2 L_2^2}{12} & 0 \\ 0 & 0 & \frac{m_2 L_2^2}{12} \end{bmatrix} \begin{bmatrix} C_2\dot{\theta}_1 \\ -S_2\dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$$= \frac{m_2 L_2^2}{12}\dot{\theta}_1^2 + \frac{m_2 L_2^2}{12}\dot{\theta}_2^2$$

So eq (B) becomes.

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$$\boxed{K_2 = \frac{m_2 L_2^2 s_2^2 \dot{\theta}_1^2}{8} + \frac{m_2 L_2^2 \dot{\theta}_2^2}{6} + \frac{m_2 L_2^2 \dot{\theta}_1^2}{24}} \rightarrow 3$$

Also

$$U_2 = -m_2 \, {}^0g^T \, {}^0P_{c_2}$$

$$= -m_2 \begin{bmatrix} 0 & 0 & -g \end{bmatrix} \begin{bmatrix} \frac{-L_2 c_1 s_2}{2} \\ \frac{-L_2 s_2 s_1}{2} \\ L_1 + \frac{L_2 c_2}{2} \end{bmatrix}$$

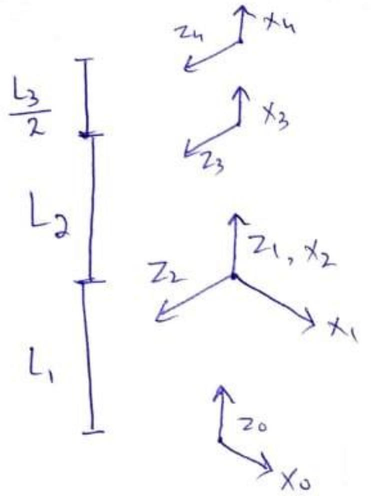
$$\boxed{U_2 = m_2 g L_1 + \frac{m_2 L_2 c_2 g}{2}} \rightarrow 4$$

Now for link "3"

$$K_3 = \frac{1}{2} m_3 V_{c_3}^T V_{c_3} + \frac{1}{2} {}^3\omega_3^T \, {}^{c_2}I_2 \, {}^3\omega_3 \longrightarrow C$$

To find V_{c_3}

(6)



i	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	0	L_1	θ_1
2	$\frac{\pi}{2}$	0	0	$\theta_2+\frac{\pi}{2}$
3	L_2	0	0	θ_3
4	$\frac{L_3}{2}$	0	0	0

So from ${}^0_4T$ we can get x_{c_3}, y_{c_3} & z_{c_3}

$$x_{c_3} = -L_2 c_1 s_2 - \frac{L_3}{2}\left(c_1 s_3 c_2 + c_1 c_3 s_2\right)$$

$$y_{c_3} = -L_2 s_2 s_1 - \frac{L_3}{2}\left(s_1 s_3 c_2 + c_3 s_2 s_1\right)$$

$$z_{c_3} = L_1 + \frac{L_3}{2}\left(c_3 c_2 - s_2 s_3\right) + L_2 c_2$$

$$\dot{x}_{c_3} = -L_2\left(-s_1 s_2 \dot{\theta}_1 + c_1 c_2 \dot{\theta}_2\right) - \frac{L_3}{2}\left(-s_1 s_{23} \dot{\theta}_1 + c_1 c_{23} \dot{\theta}_{23}\right)$$

$$\dot{y}_{c_3} = -L_2\left(c_2 s_1 \dot{\theta}_2 + s_2 c_1 \dot{\theta}_1\right) - \frac{L_3}{2}\left(c_1 s_{32} \dot{\theta}_1 + s_1 c_{32} \dot{\theta}_{32}\right)$$

$$\dot{z}_{c_3} = -\frac{L_3}{2} s_{23} \dot{\theta}_{23} - L_2 s_2 \dot{\theta}_2$$

(7)

$$V_{c_3} = \begin{bmatrix} L_2 S_1 S_2 \dot{\theta}_1 - L_2 C_1 C_2 \dot{\theta}_2 + \frac{L_3}{2} S_1 S_{23} \dot{\theta}_1 - \frac{L_3}{2} C_1 C_{23} \dot{\theta}_{23} \\ -L_2 C_2 S_1 \dot{\theta}_2 - L_2 S_2 C_1 \dot{\theta}_1 - \frac{L_3}{2} C_1 S_{32} \dot{\theta}_1 - \frac{L_3}{2} S_1 C_{32} \dot{\theta}_{32} \\ -\frac{L_3}{2} S_{32} \dot{\theta}_{23} - L_2 S_2 \dot{\theta}_2 \end{bmatrix}$$

So

$$V_{c_3}^T \; V_{c_3} = \begin{bmatrix} L_2 \dot{\theta}_1^2 + \frac{L_3^2}{4} \dot{\theta}_{32} + L_2^2 S_2^2 \dot{\theta}_1^2 + \frac{L_3^2}{4} S_{32}^2 \dot{\theta}_1^2 + 2 L_2 \frac{L_3}{2} S_1 S_2 S_{23} \dot{\theta}_1^2 \\ + 2 L_2 \frac{L_3}{2} \dot{\theta}_2 \dot{\theta}_{23} C(\theta_2 - \theta_{23}) \end{bmatrix}$$

Also

$${}^3\omega_3^T \; {}^{C_3}I_3 \; {}^3\omega_3 =$$

$$\begin{bmatrix} \dot{\theta}_1 C_{23} & -\dot{\theta}_1 S_{23} & \dot{\theta}_2 + \dot{\theta}_3 \end{bmatrix} \begin{bmatrix} \frac{m_3 l_3^2}{12} & 0 & 0 \\ 0 & \frac{m_3 l_3^2}{12} & 0 \\ 0 & 0 & \frac{m_3 l_3^2}{12} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 C_{23} \\ -\dot{\theta}_1 S_{23} \\ \dot{\theta}_{23} \end{bmatrix}$$

$$= \frac{m_3 l_3^2}{12} \dot{\theta}_1^2 + \frac{m_3 l_3^2}{12} \dot{\theta}_{23}^2$$

So eq (c) becomes

(8)

$$K_3 = \left[\frac{m_3 L_2^2 \dot\theta_1^2}{2} + \frac{m_3 L_3^2 \dot\theta_{32}}{8} + \frac{m_3 L_2^2 S_2^2 \dot\theta_1^2}{2} + \frac{m_3 L_3^2 S_{23}^2 \dot\theta_1^2}{8} \right.$$

$$\frac{m_3 L_2 L_3 S_2 S_{23} \dot\theta_1^2}{2} + \frac{m_3 L_2 L_3 \dot\theta_2 \dot\theta_{23} C_3}{2} + \frac{m_3 L_3^2 \dot\theta_1^2}{24}$$

$$\left. + \frac{m_3 L_3^2 \dot\theta_{23}^2}{24} \right]$$

And $$U_3 = -m_3 \begin{bmatrix} 0 & 0 & -g \end{bmatrix} \begin{bmatrix} -L_2 C_1 S_2 - \frac{L_3}{2} C_1 S_{23} \\ -L_2 S_2 S_1 - \frac{L_3 S_1 S_{32}}{2} \\ L_1 + \frac{L_3}{2} C_{32} + L_2 C_2 \end{bmatrix}$$

$$\boxed{U_3 = m_3 L_1 g + \frac{m_3 L_3 C_{32} g}{2} + m_3 L_2 C_2 g} \rightarrow (6)$$

(9)

Now adding all the K.E i-e

$$K_1+K_2+K_3 = \frac{m_1 L_1^2 \dot{\theta}_1^2}{24} + \frac{m_2 L_2^2 S_2^2 \dot{\theta}_1^2}{8} + \frac{3m_2 L_2^2 \dot{\theta}_2^2}{24}$$

$$+ \frac{m_2 L_2^2 \dot{\theta}_1^2}{24} + \frac{m_3 L_2^2 \dot{\theta}_1^2}{2} + \frac{m_3 L_3^2 \dot{\theta}_{32}}{8} + \frac{m_3 L_2^2 S_2^2 \dot{\theta}_1^2}{2}$$

$$+ \frac{m_3 L_3^2 S_{23}^2 \dot{\theta}_1^2}{8} + \frac{m_3 L_3 L_2 S_2 S_{32} \dot{\theta}_1^2}{2} + \frac{m_3 L_2 L_3 \dot{\theta}_2 \dot{\theta}_{23} C_3}{2}$$

$$+ \frac{m_3 L_3^2 \dot{\theta}_1^2}{24} + \frac{m_3 L_3^2 \dot{\theta}_{23}^2}{24}.$$

Also sum of all P.E is

$$U = \frac{m_1 g L_1}{2} + m_2 g L_1 + \frac{m_2 L_2 C_2 g}{2} + m_3 L_1 g$$

$$+ \frac{m_3 L_3 C_{32} g}{2} + m_3 L_2 C_2 g$$

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Now $\tau_1 = \frac{d}{dt}\frac{\partial k}{\partial \dot{\theta}_1} - \frac{\partial k}{\partial \theta_1} + \frac{\partial u}{\partial \theta_1}$

So

$$\tau_1 = \left[\frac{m_1 L_1^2 \ddot{\theta}_1}{12} + \frac{m_2 L_2^2}{4}\left(2 S_2 C_2 \dot{\theta}_1 \dot{\theta}_2 + S_2^2 \ddot{\theta}_1\right) + \frac{m_2 L_2^2 \ddot{\theta}_1}{12} + m_3 L_2^2 \ddot{\theta}_1 + m_3 L_2 \left(2 S_2 C_2 \dot{\theta}_2 \dot{\theta}_1 + S_2^2 \ddot{\theta}_1\right) + \frac{m_3 L_3^2}{4}\left(2 S_{23} C_{23} \dot{\theta}_{23} \dot{\theta}_1 + S_{23}^2 \ddot{\theta}_1\right) + \frac{m_3 L_3^2 \ddot{\theta}_1}{12}\right]$$

$$\tau_2 = \left[m_2 L_2^2 \ddot{\theta}_2 + m_3 L_2 L_3 \left(-S_3 \dot{\theta}_3 \dot{\theta}_2 + C_3 \ddot{\theta}_2\right) + \frac{m_3 L_2 L_3}{2}\left(-S_3 \dot{\theta}_3^2 + C_3 \ddot{\theta}_3\right) + \frac{m_3 L_3^2}{12}\left(\ddot{\theta}_2 + \ddot{\theta}_3\right) - m_3 L_2^2 \dot{\theta}_1^2 C_2 S_2 - \frac{m_3 L_3^2 S_{23} C_{23} \dot{\theta}_1^2}{4} - m_3 L_2 L_3 \dot{\theta}_1^2 \left(C_2 S_{23} + S_2 C_{23}\right) - \frac{m_2 L_2 S_2 g}{2} - \frac{m_3 L_3 S_{32} g}{2} - m_3 L_2 S_2 g\right]$$

(11)

$$\tau_3 = \left[\frac{m_3 L_3^2 (\ddot{\theta}_3 + \ddot{\theta}_2)}{3} - \frac{m_3 L_3^2 S_{23} C_{23} \dot{\theta}_1^2}{4} - \frac{m_3 L_2 L_3 S_2 C_{23} \dot{\theta}_1^2}{2} + \frac{m_3 L_2 L_3 S_3 \dot{\theta}_2 \dot{\theta}_{23}}{2} - \frac{m_3 L_3 S_{32} g}{2} \right]$$

Now In state space model.

$$\tau = M(\theta)\ddot{\theta} + V(\theta, \dot{\theta}) + G(\theta).$$

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$$M = \begin{bmatrix} \frac{m_1 L_1^2}{12} + \frac{m_2 L_2^2 S_2^2}{4} + \frac{m_2 L_2^2}{12} + m_3 L_2^2 + m_3 L_2 S_2^2 + \frac{m_3 L_3^2 S_{23}^2}{4} + \frac{m_3 L_3^2}{12} & 0 & 0 \\ 0 & \frac{m_2 L_2^2}{4} + m_3 L_2 L_3 C_3 + \frac{m_3 L_3^2}{12} & \frac{m_3 L_2 L_3 C_3}{2} \\ 0 & \frac{m_3 L_3^2}{3} & \frac{m_3 L_3^2}{3} \end{bmatrix}$$

Appendix B:

MATLAB Code for Joint Trajectory

```
clc;clear all;
format short
%   syms pi;
L1=1;L2=3;L3=3;
```

```
% q1=0;q2=0;q3=0;
syms t;
%initial position of robot
      x=1.3;
      y=1.3;
      z=1.3;
      c1=x;
      s1=y;
q1o=atan2(s1,c1);
c1=cos(q1o);
s1=sin(q1o);
x11=x/c1;
y11=y/s1;
z11=z-L1;
c3=(x11^2+z11^2-L3^2-L2^2)/(2*L3*L2);
s3=sqrt((1-c3^2));
q3o=atan2(s3,c3);
NMS=[-L3*c1*s3 x; -L3*c1*s3 y];
DMS=[-L3*c1*s3 (-L3*c1*c3)-(L2*c1); -L3*c1*s3 (-L3*s1*c3)-(L2*s1)];
NMC=[x (-L3*c1*c3)-(L2*c1); y (-L3*s1*c3)-(L2*s1)];
DMC=[-L3*c1*s3 (-L3*c1*c3)-(L2*c1); -L3*c1*s3 (-L3*s1*c3)-(L2*s1)];
DS1=det(NMS);
DS2=det(DMS);
DC1=det(NMC);
DC2=det(DMC);
s2=DS1/DS2;
c2=DC1/DC2;
q2o=atan2(s2,c2);

%final position of robot
     x=2;
     y=2.5;
     z=2.5;

c1=x;
s1=y;
q1f=atan2(s1,c1);
c1=cos(q1f);
s1=sin(q1f);
x11=x/c1;
y11=y/s1;
z11=z-L1;
c3=(x11^2+z11^2-L3^2-L2^2)/(2*L3*L2);
s3=sqrt((1-c3^2));
q3f=atan2(s3,c3);
NMS=[-L3*c1*s3 x; -L3*c1*s3 y];
DMS=[-L3*c1*s3 (-L3*c1*c3)-(L2*c1); -L3*c1*s3 (-L3*s1*c3)-(L2*s1)];
NMC=[x (-L3*c1*c3)-(L2*c1); y (-L3*s1*c3)-(L2*s1)];
DMC=[-L3*c1*s3 (-L3*c1*c3)-(L2*c1); -L3*c1*s3 (-L3*s1*c3)-(L2*s1)];
DS1=det(NMS);
DS2=det(DMS);
DC1=det(NMC);
DC2=det(DMC);
s2=DS1/DS2;
c2=DC1/DC2;
q2f=atan2(s2,c2);

%joint velocities at t=0 and t=tf
dq1o=0;dq2o=0;dq3o=0;dq1f=0;dq2f=0;dq3f=0;
a10=q1o;
a20=q2o;
```

```
a30=q3o;
a11=dq1o;
a21=dq2o;
a31=dq3o;
%for a13 and a12
a13=dq1f-2*q1f+2*a10-a11;
a12=q1f-a10-a11-a13;
%for a23 and a22
a23=dq2f-2*q2f+2*a20-a21;
a22=q2f-a20-a21-a23;
%for a33 and a32
a33=dq3f-2*q3f+2*a30-a31;
a32=q3f-a30-a31-a33;
t=(0:0.01:1);

q1 = a10 + a11.*t + a12.*t.^2 + a13.*t.^3
q2 = a20 + a21.*t + a22.*t.^2 + a23.*t.^3
q3 = a30 + a31.*t + a32.*t.^2 + a33.*t.^3
dq1 = a11 + 2*a12.*t + 3*a13.*t.^2
dq2 = a21 + 2*a22.*t + 3*a23.*t.^2
dq3 = a31 + 2*a32.*t + 3*a33.*t.^2

subplot(2,1,1);
plot(t,q1);
hold on
plot(t,q2);
hold on
plot(t,q3);
xlim([0 1])
ylim([0 4])
hold off

subplot(2,1,2);
plot(t,dq1);
hold on
plot(t,q2);
hold on
plot(t,dq3);
xlim([0 1])
ylim([-1 4])
hold off
```

Matlab code for Joint torques:

```
clc;
clear all;
format short
M1=0.05;M2=0.05;M3=0.05;
L1=1;L2=1;L3=1;

 q1o=0;q2o=0;q3o=0;
q=q1o;d=0;a=0;alpha=0; %for i=1;
T01=[cos(q) -sin(q) 0 a;sin(q)*cos(alpha) cos(q)*cos(alpha) -sin(alpha) -
sin(alpha)*d;sin(q)*sin(alpha) cos(q)*sin(alpha) cos(alpha) cos(alpha)*d;0
0 0 1];
q=q2o+(pi/2);d=L1;a=0;alpha=pi/2;  %fori=2;
T12=[cos(q) -sin(q) 0 a;sin(q)*cos(alpha) cos(q)*cos(alpha) -sin(alpha) -
sin(alpha)*d;sin(q)*sin(alpha) cos(q)*sin(alpha) cos(alpha) cos(alpha)*d;0
0 0 1];
```

```
q=q3o;d=0;a=L2;alpha=0; % for i=3;
T23=[cos(q) -sin(q) 0 a;sin(q)*cos(alpha) cos(q)*cos(alpha) -sin(alpha) -
sin(alpha)*d;sin(q)*sin(alpha) cos(q)*sin(alpha) cos(alpha) cos(alpha)*d;0
0 0 1];
q=0;d=0;a=L3;alpha=0;
T34=[cos(q) -sin(q) 0 a;sin(q)*cos(alpha) cos(q)*cos(alpha) -sin(alpha) -
sin(alpha)*d;sin(q)*sin(alpha) cos(q)*sin(alpha) cos(alpha) cos(alpha)*d;0
0 0 1];
T02=T01*T12;
T03=T01*T12*T23;
T04=T01*T12*T23*T34;

axis([-2 3 -2 3 -2 3]);

Ax1 = [T01(1,4),T02(1,4)];
Ay1 = [T01(2,4),T02(2,4)];
Az1 = [T01(3,4),T02(3,4)];

Ax2 = [T02(1,4),T03(1,4)];
Ay2 = [T02(2,4),T03(2,4)];
Az2 = [T02(3,4),T03(3,4)];

Ax3 = [T03(1,4),T04(1,4)];
Ay3 = [T03(2,4),T04(2,4)];
Az3 = [T03(3,4),T04(3,4)];
Ax4 = [-.1,.1];
Ay4=  [0,0];
Az4= [0,0];

  p1  = line(Ax1,Ay1,Az1,'LineWidth',[3],'Color','k');
  p2  = line(Ax2,Ay2,Az2,'LineWidth',[3],'Color','M');
  p3  = line(Ax3,Ay3,Az3,'LineWidth',[3],'Color','R');
  p4  = line(Ax4,Ay4,Az4,'LineWidth',[12],'Color','B');
  drawnow

 pause()
n=1;
g=9.80;
  for t=0:.01:1

q1=0.7854+(0.332*t^2)-(0.2213*t^3);
q2=3.1416-(14.1372*(t^2))+(9.4248*t^3);
q3=2.5102-(1.8867*t^2)+(1.2578*(t^3));
dq1=(0.664*t)-(0.6639*(t^2));
ddq1=-1.3278*t+0.664;
dq2=(-28.2744*t)+(28.2744*(t^2));
ddq2=-28.2744+(56.5488*t);
dq3=-3.7734*t+(3.7734*(t^2));
ddq3=-3.7734+(7.5468*t);

q=q1;d=0;a=0;alpha=0; %for i=1;
T01=[cos(q) -sin(q) 0 a;sin(q)*cos(alpha) cos(q)*cos(alpha) -sin(alpha) -
sin(alpha)*d;sin(q)*sin(alpha) cos(q)*sin(alpha) cos(alpha) cos(alpha)*d;0
0 0 1];
q=q2+(pi/2);d=L1;a=0;alpha=pi/2;  %fori=2;
T12=[cos(q) -sin(q) 0 a;sin(q)*cos(alpha) cos(q)*cos(alpha) -sin(alpha) -
sin(alpha)*d;sin(q)*sin(alpha) cos(q)*sin(alpha) cos(alpha) cos(alpha)*d;0
0 0 1];
q=q3;d=0;a=L2;alpha=0; % for i=3;
```

```
T23=[cos(q) -sin(q) 0 a;sin(q)*cos(alpha) cos(q)*cos(alpha) -sin(alpha) -
sin(alpha)*d;sin(q)*sin(alpha) cos(q)*sin(alpha) cos(alpha) cos(alpha)*d;0
0 0 1];
q=0;d=0;a=L3;alpha=0; % for i=3;
T34=[cos(q) -sin(q) 0 a;sin(q)*cos(alpha) cos(q)*cos(alpha) -sin(alpha) -
sin(alpha)*d;sin(q)*sin(alpha) cos(q)*sin(alpha) cos(alpha) cos(alpha)*d;0
0 0 1];
T02=T01*T12;
T03=T01*T12*T23;
T04=T01*T12*T23*T34;
Ax1 = [T01(1,4),T02(1,4)];
Ay1 = [T01(2,4),T02(2,4)];
Az1 = [T01(3,4),T02(3,4)];

Ax2 = [T02(1,4),T03(1,4)];
Ay2 = [T02(2,4),T03(2,4)];
Az2 = [T02(3,4),T03(3,4)];

Ax3 = [T03(1,4),T04(1,4)];
Ay3 = [T03(2,4),T04(2,4)];
Az3 = [T03(3,4),T04(3,4)];
Ax4 = [-.1,.1];
Ay4=  [0,0];
Az4= [0,0];

  set(p1,'X', Ax1, 'Y',Ay1,'Z',Az1)
  set(p2,'X', Ax2, 'Y',Ay2,'Z',Az2)
  set(p3,'X', Ax3, 'Y',Ay3,'Z',Az3)
  set(p4,'X', Ax4, 'Y',Ay4,'Z',Az4)

))/2];
M=[(M1*L1^2)/12+(M2*L2^2*sin(q2)*sin(q2))/4+(M2*L2^2)/12+(M3*L2^2)+(M3*L2*s
in(q2)*sin(q2))+(M3*L3^2*(sin(q2+q3))^2)+(M3*L3^2)/12 0 0;0
(M2*L2^2)/4+(M3*L2*L3*cos(q3))+(M3*L3^2)/12
(M3*L2*L3*cos(q3))/2+(M3*L3^2)/12;0 (M3*L3^2)/3 (M3*L3^2)/3];
V=[(M2*L2^2*sin(q2)*cos(q2)*dq1*dq2)/2+(2*M3*L2*sin(q2)*cos(q2)*dq1*dq2)+(M
3*L3^2*sin(q2+q3)*cos(q2+q3)*dq1*(dq1+dq2))/2;(-M3*L2*L3*sin(q3)*dq3*dq2)-
(M3*L2*L3*sin(q3)*(dq3)^2)/2-(M3*L2^2*sin(q2)*cos(q2)*(dq1^2))-
(M3*L3^2*sin(q2+q3)*cos(q2+q3)*(dq1^2))/4-
(M3*L2*L3*dq1^2*((cos(q2)*sin(q2+q3)+(sin(q2)*cos(q2+q3)))));(-
M3*L3^2*sin(q2+q3)*cos(q2+q3)*(dq1^2))/4-
(M3*L2*L3*sin(q2)*cos(q2+q3)*(dq1^2))/2+(M3*L2*L3*sin(q3)*dq1*(dq2+dq3))/2]
;
G=[0;(-M2*L2*sin(q2)*g)/2-(M3*L3*sin(q2+q3)*g)/2-(M3*L2*sin(q2)*g);(-
M3*L3*sin(q2+q3)*g)/2];
T=M*[ddq1;ddq2;ddq3]+V+G;

  A1(n,1)=t;%Time of traj.
  A2(n,1)=T(1,1);%T at joint 1
  A3(n,1)=T(2,1); %T at joint 2
  A4(n,1)=T(3,1); %T at joint 3

  drawnow
  pause(.03)
  n=n+1;
  end
hold on
figure
plot(A1(:,1),A2(:,1),'r')
hold on
```

```
plot(A1(:,1),A3(:,1),'k')
hold on
plot(A1(:,1),A4(:,1),'b')
xlabel('time')
ylabel('Torque N/m')
legend('Link 1','link 2','Link3');
```

Matlab code for forward Forward Kinematics:

```
clc;clear all;
format short

% L1=1;L2=.8;L3=.8;
L1=1;L2=1.5;L3=1.5;
q1=0;q2=0;q3=0;

q=q1;d=0;a=0;alpha=0; %for i=1;
T01=[cos(q) -sin(q) 0 a;sin(q)*cos(alpha) cos(q)*cos(alpha) -sin(alpha) -
sin(alpha)*d;sin(q)*sin(alpha) cos(q)*sin(alpha) cos(alpha) cos(alpha)*d;0
0 0 1];
q=(q2+pi/2);d=L1;a=0;alpha=pi/2;  %fori=2;
T12=[cos(q) -sin(q) 0 a;sin(q)*cos(alpha) cos(q)*cos(alpha) -sin(alpha) -
sin(alpha)*d;sin(q)*sin(alpha) cos(q)*sin(alpha) cos(alpha) cos(alpha)*d;0
0 0 1];
q=q3;d=0;a=L2;alpha=0; % for i=3;
T23=[cos(q) -sin(q) 0 a;sin(q)*cos(alpha) cos(q)*cos(alpha) -sin(alpha) -
sin(alpha)*d;sin(q)*sin(alpha) cos(q)*sin(alpha) cos(alpha) cos(alpha)*d;0
0 0 1];
q=0;d=0;a=L3;alpha=0; %for i=4;
T34=[cos(q) -sin(q) 0 a;sin(q)*cos(alpha) cos(q)*cos(alpha) -sin(alpha) -
sin(alpha)*d;sin(q)*sin(alpha) cos(q)*sin(alpha) cos(alpha) cos(alpha)*d;0
0 0 1];
T02=T01*T12;
T03=T01*T12*T23;
T04=T01*T12*T23*T34
  axis([-2 5 -2 6 -2 7]);

  Ax1 = [T01(1,4),T02(1,4)];
  Ay1 = [T01(2,4),T02(2,4)];
  Az1 = [T01(3,4),T02(3,4)];

  Ax2 = [T02(1,4),T03(1,4)];
  Ay2 = [T02(2,4),T03(2,4)];
  Az2 = [T02(3,4),T03(3,4)];

  Ax3 = [T03(1,4),T04(1,4)];
  Ay3 = [T03(2,4),T04(2,4)];
  Az3 = [T03(3,4),T04(3,4)];

  Ax4 = [-.1,.1];
  Ay4=  [0.1,0.1];
  Az4=  [0.1,0.1];
  p1  = line(Ax1,Ay1,Az1,'LineWidth',[3],'Color','k');
  p2  = line(Ax2,Ay2,Az2,'LineWidth',[3],'Color','M');
  p3  = line(Ax3,Ay3,Az3,'LineWidth',[3],'Color','R');
  p4  = line(Ax4,Ay4,Az4,'LineWidth',[12],'Color','B');

   drawnow
   grid
```

```
q1o=q1;q2o=q2;q3o=q3;
pause()
  for t=0:.01:1
      q1=pi/3*t;
      q2=pi/3*t;
      q3=pi/3*t;

q=q1;d=0;a=0;alpha=0; %for i=1;
T01=[cos(q) -sin(q) 0 a;sin(q)*cos(alpha) cos(q)*cos(alpha) -sin(alpha) -
sin(alpha)*d;sin(q)*sin(alpha) cos(q)*sin(alpha) cos(alpha) cos(alpha)*d;0
0 0 1];
q=(q2+pi/2);d=L1;a=0;alpha=pi/2;  %fori=2;
T12=[cos(q) -sin(q) 0 a;sin(q)*cos(alpha) cos(q)*cos(alpha) -sin(alpha) -
sin(alpha)*d;sin(q)*sin(alpha) cos(q)*sin(alpha) cos(alpha) cos(alpha)*d;0
0 0 1];
q=q3;d=0;a=L2;alpha=0; % for i=3;
T23=[cos(q) -sin(q) 0 a;sin(q)*cos(alpha) cos(q)*cos(alpha) -sin(alpha) -
sin(alpha)*d;sin(q)*sin(alpha) cos(q)*sin(alpha) cos(alpha) cos(alpha)*d;0
0 0 1];
q=0;d=0;a=L3;alpha=0; %for i=4;
T34=[cos(q) -sin(q) 0 a;sin(q)*cos(alpha) cos(q)*cos(alpha) -sin(alpha) -
sin(alpha)*d;sin(q)*sin(alpha) cos(q)*sin(alpha) cos(alpha) cos(alpha)*d;0
0 0 1];
T02=T01*T12;
T03=T01*T12*T23;
T04=T01*T12*T23*T34;

  Ax1 = [T01(1,4),T02(1,4)];
  Ay1 = [T01(2,4),T02(2,4)];
  Az1 = [T01(3,4),T02(3,4)];

  Ax2 = [T02(1,4),T03(1,4)];
  Ay2 = [T02(2,4),T03(2,4)];
  Az2 = [T02(3,4),T03(3,4)];

  Ax3 = [T03(1,4),T04(1,4)];
  Ay3 = [T03(2,4),T04(2,4)];
  Az3 = [T03(3,4),T04(3,4)];

  Ax4 = [-.1,.1];
  Ay4=  [0.1,0.1];
  Az4=  [0.1,0.1];

  set(p1,'X', Ax1, 'Y',Ay1,'Z',Az1)
  set(p2,'X', Ax2, 'Y',Ay2,'Z',Az2)
  set(p3,'X', Ax3, 'Y',Ay3,'Z',Az3)
  set(p4,'X', Ax4, 'Y',Ay4,'Z',Az4)

  drawnow
  pause(.01)
  end
```