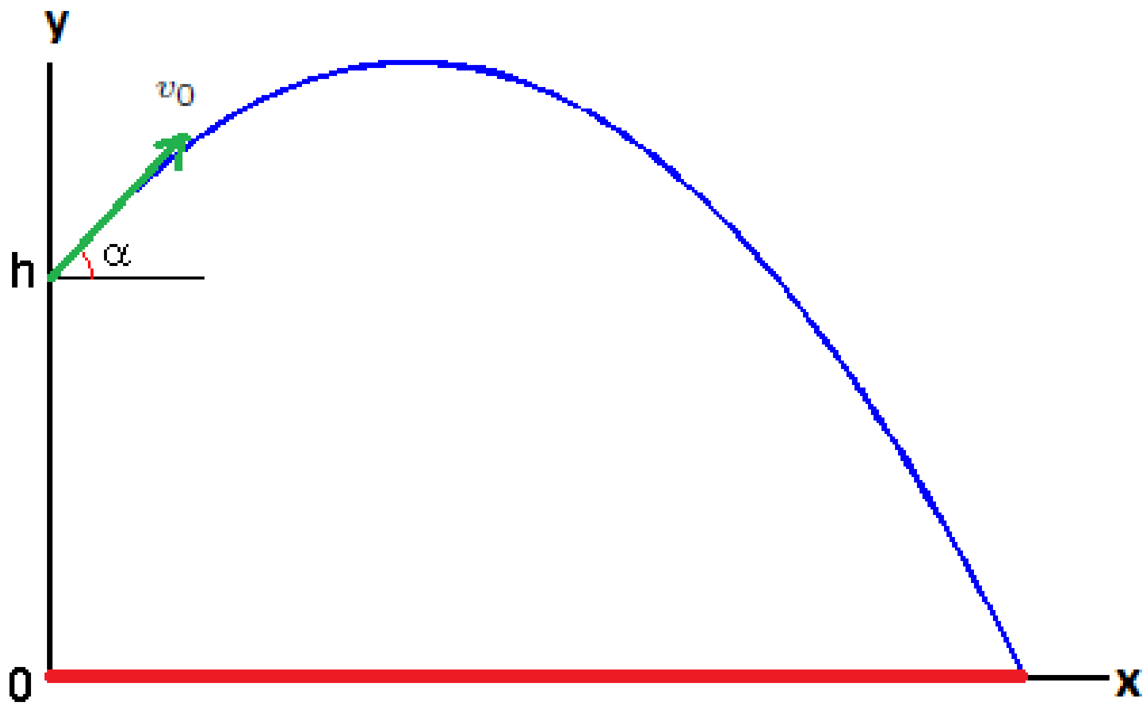
# Heittoliike




$$y = ax^2 + bx + c$$

$$a = -\frac{g}{2v_0^2\cos(\alpha)^2},\ b = \tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)},\ c = h$$

## Vaakasuora lentomatka:

$$y = 0 \leftrightarrow x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-\frac{\sin(\alpha)}{\cos(\alpha)} - \sqrt{\left(\frac{\sin(\alpha)}{\cos(\alpha)}\right)^2 - 4\left(-\frac{g}{2v_0^2\cos(\alpha)^2}\right)h}}{2\left(-\frac{g}{2v_0^2\cos(\alpha)^2}\right)}$$

$$= \frac{\frac{\sin(\alpha)}{\cos(\alpha)} + \sqrt{\frac{\sin(\alpha)^2}{\cos(\alpha)^2} + \frac{2gh}{v_0^2\cos(\alpha)^2}}}{\left(\frac{g}{v_0^2\cos(\alpha)^2}\right)}$$

$$= \frac{\frac{\sin(\alpha)}{\cos(\alpha)} + \frac{1}{\cos(\alpha)}\sqrt{\sin(\alpha)^2 + \frac{2gh}{v_0^2}}}{\left(\frac{g}{v_0^2\cos(\alpha)^2}\right)}$$

$$= \frac{1}{\cos(\alpha)}\left(\sin(\alpha) + \sqrt{\sin(\alpha)^2 + \frac{2gh}{v_0^2}}\right) \cdot \frac{v_0^2\cos(\alpha)^2}{g}$$

$$= \frac{v_0^2}{g}\cos(\alpha)\left(\sin(\alpha) + \sqrt{\sin(\alpha)^2 + \frac{2gh}{v_0^2}}\right)$$

Suurin arvo:

$$a = -\frac{g}{2v_0^2\cos(\alpha)^2} = -\frac{g}{2v_0^2}(1+\tan(\alpha)^2),\ b = \tan(\alpha),\ c = h$$

$$\left(\frac{1}{\cos(\alpha)^2} = \frac{\cos(\alpha)^2+\sin(\alpha)^2}{\cos(\alpha)^2} = \frac{\cos(\alpha)^2}{\cos(\alpha)^2} + \frac{\sin(\alpha)^2}{\cos(\alpha)^2} = 1 + \tan(\alpha)^2\right)$$

$$ax^2 + bx + c = 0$$

$$-\frac{g}{2v_0^2}(1+\tan(\alpha)^2)x^2 + \tan(\alpha)\,x + h = 0$$

$$\frac{g}{2v_0^2}x^2 = -\frac{g}{2v_0^2}\tan(\alpha)^2x^2 + \tan(\alpha)\,x + h$$

$$x^2 = -(\tan(\alpha)x)^2 + 2\frac{v_0^2}{g}\tan(\alpha)x + \frac{2v_0^2h}{g}$$

$$= -\left((\tan(\alpha)x)^2 - 2\frac{v_0^2}{g}\tan(\alpha)x + \left(\frac{v_0^2}{g}\right)^2\right) + \left(\frac{v_0^2}{g}\right)^2 + \frac{2v_0^2h}{g}$$

$$= \underbrace{-\left(\tan(\alpha)x - \frac{v_0^2}{g}\right)^2}_{\leq 0} + \frac{v_0^2(v_0^2+2gh)}{g^2}$$

$$\leq \frac{v_0^2(v_0^2+2gh)}{g^2}$$

Eli,

$$x \leq \sqrt{\frac{v_0^2(v_0^2+2gh)}{g^2}} = \frac{v_0}{g}\sqrt{v_0^2+2gh}$$

ja

$$x = \frac{v_0}{g}\sqrt{v_0^2 + 2gh}$$

silloin, kun

$$\tan(\alpha)x - \frac{v_0^2}{g} = 0$$

eli

$$\tan(\alpha) = \frac{v_0^2}{gx} = \frac{v_0^2}{g\frac{v_0}{g}\sqrt{v_0^2 + 2gh}} = \frac{v_0}{\sqrt{v_0^2 + 2gh}}$$

**Lentoradan huippu eli ylin piste:**

$$x_0 = -\frac{b}{2a} = -\frac{\tan(\alpha)}{2 \cdot \left(-\frac{g}{2v_0^2\cos(\alpha)^2}\right)} = \frac{\frac{\sin(\alpha)}{\cos(\alpha)} \cdot v_0^2\cos(\alpha)^2}{g}$$

$$= \frac{v_0^2\sin(\alpha)\cos(\alpha)}{g} = \frac{v_0^2}{2g}\sin(2\alpha) \quad | \sin(\alpha)\cos(\alpha) = \frac{1}{2}\sin(2\alpha)$$

$$y_0 = -\frac{b^2}{4a} + c = -\frac{\tan(\alpha)^2}{4 \cdot \left(-\frac{g}{2v_0^2\cos(\alpha)^2}\right)} + h$$

$$= \frac{\frac{\sin(\alpha)^2}{\cos(\alpha)^2} \cdot v_0^2\cos(\alpha)^2}{2g} + h = \frac{v_0^2}{2g}\sin(\alpha)^2 + h$$

Lähtönopeus $v_0$ ja -korkeus $h = 0$. Määrää lähtökulma $\alpha$ niin, että $aL^2 + bL = H$ eli

$$-\frac{gL^2}{2v_0^2\cos(\alpha)^2} + \tan(\alpha)L = H \quad | \cdot \cos(\alpha)^2$$

$$-\frac{gL^2}{2v_0^2} + \tan(\alpha)\cos(\alpha)^2 L = H\cos(\alpha)^2 \quad | \tan = \sin / \cos$$

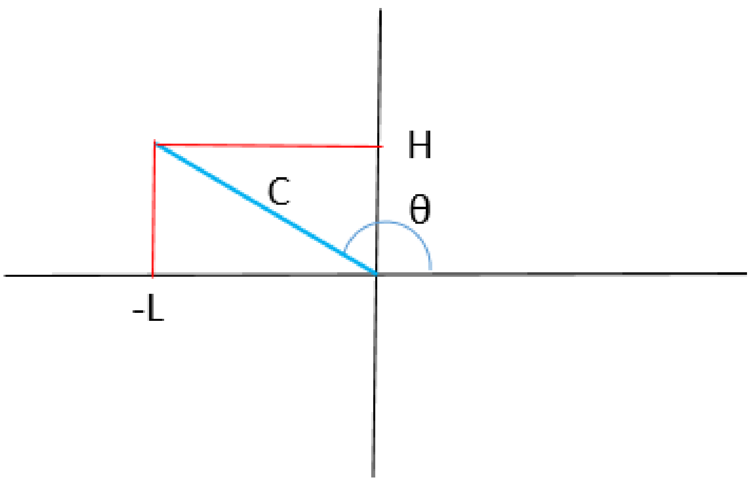
$$-\frac{gL^2}{2v_0^2} + \sin(\alpha)\cos(\alpha)L = H\cos(\alpha)^2 \quad \left| \begin{array}{l} \sin(\alpha)\cos(\alpha) = \frac{1}{2}\sin(2\alpha) \\ \cos(\alpha)^2 = \frac{1}{2}(\cos(2\alpha)+1) \end{array} \right.$$

$$-\frac{gL^2}{2v_0^2} + \frac{1}{2}\sin(2\alpha)L = H\frac{1}{2}(\cos(2\alpha)+1) \quad | \cdot 2$$

$$H\cos(2\alpha) - L\sin(2\alpha) = -\underbrace{\left(\frac{gL^2}{v_0^2} + H\right)}_{=k} \left| \begin{array}{l} A\cos(\delta) + B\sin(\delta) = C\sin(\delta+\theta) \\ C = \sqrt{A^2+B^2},\ \theta = \mathrm{atan2}(A, B) \end{array} \right.$$

$$C\sin(2\alpha+\theta) = -k, \quad C = \sqrt{H^2+L^2},\ \theta = \mathrm{atan2d}(H, -L)$$

$$\sin(2\alpha+\theta) = -\frac{k}{C}$$



Ehto:

$$\frac{k}{C} \leq 1 \leftrightarrow k \leq C \leftrightarrow \frac{gL^2}{v_0^2} + H \leq \sqrt{H^2 + L^2}$$

$$v_0^2 \geq \frac{gL^2}{\sqrt{H^2+L^2} - H} \leftrightarrow v_0 \geq \sqrt{\frac{gL^2}{\sqrt{H^2+L^2} - H}} = \min\ v_0$$

Jos $k/C \leq 1$ ja

$$\beta = \sin^{-1}(k/C)$$

niin

$$\sin(2\alpha + \theta) = -\frac{k}{C}$$

jos

$$2\alpha + \theta = 180^\circ + \beta \quad \text{tai} \quad 2\alpha + \theta = 360^\circ - \beta$$

eli

$$\alpha = \frac{1}{2}(180^\circ + \beta - \theta) \quad \text{tai} \quad \alpha = \frac{1}{2}(360^\circ - \beta - \theta)$$