

Poista sulut ja sievennä (eli yhdistä termejä), jos mahdollista:

1.

$$\begin{aligned}& -(3a^2b + b^2a) - (-a^2b - ab^2) \\&= -3a^2b - b^2a + a^2b + ab^2 \\&= -2a^2b\end{aligned}$$

2.

$$\begin{aligned}& a^2b - (5ab^2 - (ab^2 - (a^2b - 2ab^2))) \\&= a^2b - (5ab^2 - (ab^2 - a^2b + 2ab^2)) \\&= a^2b - (5ab^2 - ab^2 + a^2b - 2ab^2) \\&= a^2b - 5ab^2 + ab^2 - a^2b + 2ab^2 \\&= -2ab^2\end{aligned}$$

3.

$$\begin{aligned}& 3n(7m - 6) - 2m(3n + 1) \\&= 3n \cdot 7m - 3n \cdot 6 - 2m \cdot 3n - 2m \cdot 1 \\&= 21mn - 18n - 6mn - 2m \\&= 15mn - 2m - 18n\end{aligned}$$

4.

$$\begin{aligned}& (2t - 1)(3t - 2) \\&= 2t \cdot 3t - 2t \cdot 2 - 1 \cdot 3t + 1 \cdot 2 \\&= 6t^2 - 4t - 3t + 2 \\&= 6t^2 - 7t + 2\end{aligned}$$

5.

$$\begin{aligned}& (-2pq^2)^3 + (3p^2q)^3 - (-5pq^2p^2q^4 + 20p^3p^3q^3) \\&= (-2)^3p^3(q^2)^3 + 3^3(p^2)^3q^3 + 5pq^2p^2q^4 - 20p^3p^3q^3 \\&= -8p^3q^6 + 27p^6q^3 + 5p^3q^6 - 20p^6q^3 \\&= -3p^3q^6 + 7p^6q^3\end{aligned}$$

6.

$$\begin{aligned}& (a+b)^3 \\&= (a+b)^2(a+b) \\&= (a^2 + 2ab + b^2)(a+b) \\&= a^3 + a^2b + 2a^2b + 2ab^2 + b^2a + b^3 \\&= a^3 + 3a^2b + 3ab^2 + b^3\end{aligned}$$

Supista, jos mahdollista

7.

$$\begin{aligned}& \frac{12r-4}{3r-1} \\&= \frac{4(3r-1)}{3r-1} \\&= 4\end{aligned}$$

8.

$$\begin{aligned}& \frac{R^2 + RH}{R + H} \\&= \frac{R(R + H)}{R + H} \\&= R\end{aligned}$$

9.

$$\begin{aligned}\frac{a+b}{a^2-b^2} \\&= \frac{a+b}{(a+b)(a-b)} \\&= \frac{1}{a-b}\end{aligned}$$

10.

$$\frac{a+b}{a^2+b^2} \quad \text{ei supistu}$$

11.

$$\begin{aligned}\frac{r^2-2r}{r^2-4} \\&= \frac{r(r-2)}{r^2-2^2} \\&= \frac{r(r-2)}{(r+2)(r-2)} \\&= \frac{r}{r+2}\end{aligned}$$

12.

$$\begin{aligned}\frac{ab-3b}{a^2-6a+9} \\&= \frac{(a-3)b}{a^2-2 \cdot a \cdot 3+3^2} \\&= \frac{(a-3)b}{(a-3)^2} \\&= \frac{b}{a-3}\end{aligned}$$

Kirjoita yhdellä jakoviivalla ja supista, jos mahdollista

13.

$$\begin{aligned} & 1 - \frac{R}{R+H} \\ &= \frac{R+H}{R+H} - \frac{R}{R+H} \\ &= \frac{R+H-R}{R+H} \\ &= \frac{H}{R+H} \end{aligned}$$

14.

$$\begin{aligned} & \left(\frac{H+h}{2}\right)^2 - \left(\frac{H-h}{2}\right)^2 \\ &= \frac{(H+h)^2}{2^2} - \frac{(H-h)^2}{2^2} \\ &= \frac{(H+h)^2 - (H-h)^2}{4} \\ &= \frac{(H^2 + 2Hh + h^2) - (H^2 - 2Hh + h^2)}{4} \\ &= \frac{4Hh}{4} = Hh \end{aligned}$$

15.

$$\begin{aligned} & \frac{a}{a+b} + \frac{b}{a-b} \\ &= \frac{a(a-b)}{(a+b)(a-b)} + \frac{b(a+b)}{(a-b)(a+b)} \\ &= \frac{a(a-b) + b(a+b)}{(a+b)(a-b)} \\ &= \frac{a^2 + b^2}{(a+b)(a-b)} \end{aligned}$$

16.

$$\begin{aligned} & m \cdot \frac{n-m}{m+n} + m \\ = & \frac{m(n-m)}{m+n} + \frac{m(m+n)}{m+n} \\ = & \frac{m(n-m) + m(m+n)}{m+n} \\ = & \frac{2mn}{m+n} \end{aligned}$$

17.

$$\begin{aligned} & \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \\ = & \frac{1}{\frac{R_2}{R_1 R_2} + \frac{R_1}{R_1 R_2}} \\ = & \frac{1}{\left(\frac{R_2 + R_1}{R_1 R_2} \right)} \\ = & \frac{R_1 R_2}{R_1 + R_2} \end{aligned}$$

18.

$$\begin{aligned}& \frac{1 + \frac{1}{s+1}}{1 + \frac{s}{s+1}} \\&= \frac{\frac{s+1}{s+1} + \frac{1}{s+1}}{\frac{s+1}{s+1} + \frac{s}{s+1}} \\&= \frac{\left(\frac{s+2}{s+1}\right)}{\left(\frac{2s+1}{s+1}\right)} \\&= \frac{(s+2)(s+1)}{(2s+1)(s+1)} \\&= \frac{s+2}{2s+1}\end{aligned}$$

19.

$$\begin{aligned}& \frac{1-x}{1+\sqrt{x}} \\&= \frac{1^2 - (\sqrt{x})^2}{1+\sqrt{x}} \\&= \frac{(1+\sqrt{x})(1-\sqrt{x})}{1+\sqrt{x}} \\&= 1-\sqrt{x}\end{aligned}$$

20.

$$\begin{aligned}& \frac{\sqrt{Rr}}{\sqrt{Rr}+R} - \frac{\sqrt{Rr}}{\sqrt{Rr}+r} \\&= \frac{\sqrt{R}\sqrt{r}}{\sqrt{R}\sqrt{r}+\sqrt{R}\sqrt{R}} - \frac{\sqrt{R}\sqrt{r}}{\sqrt{R}\sqrt{r}+\sqrt{r}\sqrt{r}} \\&= \frac{\sqrt{R}\sqrt{r}}{\sqrt{R}(\sqrt{r}+\sqrt{R})} - \frac{\sqrt{R}\sqrt{r}}{\sqrt{r}(\sqrt{R}+\sqrt{r})} \\&= \frac{\sqrt{r}}{\sqrt{r}+\sqrt{R}} - \frac{\sqrt{R}}{\sqrt{R}+\sqrt{r}} \\&= \frac{\sqrt{r}-\sqrt{R}}{\sqrt{r}+\sqrt{R}}\end{aligned}$$

21.

$$\begin{aligned}\frac{a+b}{2} - \sqrt{ab} &= \frac{a+b-2\sqrt{ab}}{2} \\ &= \frac{(\sqrt{a})^2 - 2\sqrt{a}\sqrt{b} + (\sqrt{b})^2}{2} \\ &= \frac{(\sqrt{a} - \sqrt{b})^2}{2} \geq 0\end{aligned}$$

Ratkaise yhtälöt:

1.

$$2(x + 5) - 3(x - 1) = 7 - 2(3x - 2)$$

$$2x + 10 - 3x + 3 = 7 - 6x + 4$$

$$2x - 3x + 6x = 7 + 4 - 10 - 3$$

$$5x = -2$$

$$x = -\frac{2}{5}$$

2.

$$\frac{2u + 1}{3 - 4u} = 5$$

$$2u + 1 = 5(3 - 4u)$$

$$2u + 1 = 15 - 20u$$

$$2u + 20u = 15 - 1$$

$$22u = 14$$

$$u = \frac{14}{22} = \frac{7}{11}$$

3.

$$E = U + R \cdot I, \quad I = ?$$

$$RI = E - U$$

$$I = \frac{E - U}{R}$$

4.

$$Ia = Rm(g + a), \quad a = ?$$

$$Ia = Rmg + Rma$$

$$Ia - Rma = Rmg$$

$$(I - Rm)a = Rmg$$

$$a = \frac{Rmg}{I - Rm}$$

5.

$$\frac{x}{H} = \frac{L - x}{h}, \quad x = ?$$

$$hx = H(L - x)$$

$$hx = HL - Hx$$

$$hx + Hx = HL$$

$$(h + H)x = HL$$

$$x = \frac{HL}{h + H}$$

6.

$$\varepsilon = \frac{T_1}{T_1 - T_2}, \quad T_1 = ?$$

$$\varepsilon(T_1 - T_2) = T_1$$

$$\varepsilon T_1 - \varepsilon T_2 = T_1$$

$$\varepsilon T_1 - T_1 = \varepsilon T_2$$

$$(\varepsilon - 1)T_1 = \varepsilon T_2$$

$$T_1 = \frac{\varepsilon T_2}{\varepsilon - 1}$$

7.

$$v_k = \frac{2v_1v_2}{v_1 + v_2}, \quad v_2 = ?$$

$$v_k(v_1 + v_2) = 2v_1v_2$$

$$v_kv_1 + v_kv_2 = 2v_1v_2$$

$$v_kv_2 - 2v_1v_2 = -v_kv_1$$

$$(v_k - 2v_1)v_2 = -v_kv_1$$

$$v_2 = \frac{-v_kv_1}{v_k - 2v_1} = \frac{v_kv_1}{2v_1 - v_k}$$

8.

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}, \quad R = ?$$

$$\frac{1}{R} = \frac{R_2 + R_1}{R_1 R_2}$$

$$R_1 R_2 = R(R_2 + R_1)$$

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

9.

$$\frac{H}{L + \frac{H}{R}} = S$$

$$\frac{HR}{LR + H} = S$$

$$HR = (LR + H)S$$

$$HR = LRS + HS$$

$$HR - HS = LRS$$

$$(R - S)H = LRS$$

$$H = \frac{LRS}{R - S}$$

10.

$$x(x+2) = 3$$

$$x^2 + 2x = 3$$

$$x^2 + 2x - 3 = 0$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot (-3)}}{2 \cdot 1}$$

$$x = \frac{-2 \pm 4}{2}$$

$$x = 1 \quad \text{tai} \quad x = -3$$

11.

$$\frac{4-r}{2+r} = 2r+5$$

$$4-r = (2+r)(2r+5)$$

$$4-r = 4r+10+2r^2+5r$$

$$2r^2 + 10r + 6 = 0$$

$$r = \frac{-10 \pm \sqrt{10^2 - 4 \cdot 2 \cdot 6}}{2 \cdot 2}$$

$$r = \frac{-10 \pm \sqrt{52}}{4}$$

$$r = \frac{-10 \pm 2\sqrt{13}}{4}$$

$$r = \frac{-5 \pm \sqrt{13}}{2}$$

12.

$$h = H - \frac{1}{2}gt^2, \quad t = ?$$

$$2h = 2H - gt^2$$

$$t^2 = \frac{2H - 2h}{g}$$

$$t = \pm \sqrt{\frac{2(H - h)}{g}}$$

13.

$$g = \frac{GM}{(R + h)^2}, \quad h = ?$$

$$g(R + h)^2 = GM$$

$$(R + h)^2 = \frac{GM}{g}$$

$$R + h = \pm \sqrt{\frac{GM}{g}}$$

$$h = -R \pm \sqrt{\frac{GM}{g}}$$

14.

$$(R + h)^2 = R^2 + L^2$$

$$R^2 + 2Rh + h^2 = R^2 + L^2$$

$$h^2 + 2Rh - L^2 = 0$$

$$h = \frac{-2R \pm \sqrt{(2R)^2 + 4L^2}}{2}$$

$$h = -R \pm \sqrt{R^2 + L^2}$$

15.

$$A = \pi r^2 + 2\pi r h$$

$$\pi r^2 + 2\pi h r - A = 0$$

$$r = \frac{-2\pi h \pm \sqrt{(2\pi h)^2 + 4\pi A}}{2\pi}$$

$$r = \frac{-2\pi h}{2\pi} \pm \sqrt{\frac{4\pi^2 h^2 + 4\pi A}{(2\pi)^2}}$$

$$r = -h \pm \sqrt{\frac{4\pi^2 h^2 + 4\pi A}{4\pi^2}}$$

$$r = -h \pm \sqrt{h^2 + A/\pi}$$

16.

$$f = \frac{1}{2\pi\sqrt{LC}}, \quad C = ?$$

$$2\pi f\sqrt{LC} = 1$$

$$4\pi^2 f^2 LC = 1$$

$$C = \frac{1}{4\pi^2 f^2 L}$$

17.

$$\frac{H}{v_1} = \frac{\sqrt{H^2 + L^2}}{v_2}, \quad H = ?$$

$$v_2 H = v_1 \sqrt{L^2 + H^2}$$

$$v_2^2 H^2 = v_1^2 (L^2 + H^2)$$

$$(v_2^2 - v_1^2) H^2 = v_1^2 L^2$$

$$H^2 = \frac{v_1^2 L^2}{v_2^2 - v_1^2}$$

$$H = \sqrt{\frac{v_1^2 L^2}{v_2^2 - v_1^2}} = \frac{v_1 L}{\sqrt{v_2^2 - v_1^2}}$$

18.

$$\begin{cases} 5(x-1) = 3(1-2y) \\ 4x-2 = 2-5y \end{cases}$$

$$\begin{cases} 5x+6y = 8 & | \cdot (-4) \\ 4x+5y = 4 & | \cdot 5 \end{cases}$$

$$\rightarrow \begin{cases} -20x-24y = -32 \\ 20x+25y = 20 \end{cases}$$

Lasketaan yhtälöt yhteen $\rightarrow y = -12$

$$\begin{cases} 5x+6y = 8 & | \cdot 5 \\ 4x+5y = 4 & | \cdot (-6) \end{cases}$$

$$\rightarrow \begin{cases} 25x+30y = 40 \\ -24x-30y = -24 \end{cases}$$

Lasketaan yhtälöt yhteen $\rightarrow x = 16$

19.

$$\begin{cases} m_1 a = T - m_1 g \\ -m_2 a = T - m_2 g \end{cases}, a = ?, T = ?$$

$$\rightarrow \begin{cases} T - m_1 a = m_1 g \\ T + m_2 a = m_2 g \end{cases}$$

vähennetään

$$\rightarrow (-m_1 - m_2)a = (m_1 - m_2)g \rightarrow a = \frac{(m_1 - m_2)g}{-m_1 - m_2} = \frac{(m_2 - m_1)g}{m_1 + m_2}$$

$$\begin{cases} T - m_1 a = m_1 g & | \cdot m_2 \\ T + m_2 a = m_2 g & | \cdot m_1 \end{cases}$$

$$\rightarrow \begin{cases} m_2 T - m_2 m_1 a = m_2 m_1 g \\ m_1 T + m_1 m_2 a = m_1 m_2 g \end{cases}$$

lasketaan yhteen

$$\rightarrow (m_1 + m_2)T = 2m_1 m_2 g \rightarrow T = \frac{2m_1 m_2 g}{m_1 + m_2}$$

20.

$$\begin{cases} \frac{h}{x} = t_1 \\ \frac{h}{L+x} = t_2 \end{cases} \rightarrow \begin{cases} h = t_1 x \\ h = t_2 L + t_2 x \end{cases}$$

$$\rightarrow \begin{cases} h - t_1 x = 0 \\ h - t_2 x = t_2 L \end{cases}$$

vähennetään

$$\rightarrow (-t_1 + t_2)x = -t_2 L \rightarrow x = \frac{-t_2 L}{-t_1 + t_2} = \frac{t_2 L}{t_1 - t_2}$$

$$\rightarrow h = t_1 x = \frac{t_1 t_2 L}{t_1 - t_2}$$

21.

$$\begin{aligned} R &= R_1 (1 + \alpha_1 \Delta T) + R_2 (1 + \alpha_2 \Delta T) \\ &= (R_1 + R_2) + (R_1 \alpha_1 + R_2 \alpha_2) \Delta T \end{aligned}$$

$$\rightarrow \begin{cases} R_1 + R_2 = R_0 & | \cdot \alpha_1 \\ R_1 \alpha_1 + R_2 \alpha_2 = 0 \end{cases}$$

$$\rightarrow \begin{cases} R_1 \alpha_1 + R_2 \alpha_1 = R_0 \alpha_1 \\ R_1 \alpha_1 + R_2 \alpha_2 = 0 \end{cases}$$

vähennetään

$$\rightarrow (\alpha_1 - \alpha_2) R_2 = R_0 \alpha_1 \rightarrow R_2 = \frac{R_0 \alpha_1}{\alpha_1 - \alpha_2} = \frac{\alpha_1}{\alpha_1 - \alpha_2} \cdot R_0$$

$$\begin{cases} R_1 + R_2 = R_0 & | \cdot \alpha_2 \\ R_1 \alpha_1 + R_2 \alpha_2 = 0 \end{cases}$$

$$\rightarrow \begin{cases} R_1 \alpha_2 + R_2 \alpha_2 = R_0 \alpha_2 \\ R_1 \alpha_1 + R_2 \alpha_2 = 0 \end{cases}$$

vähennetään

$$\rightarrow (\alpha_2 - \alpha_1) R_1 = R_0 \alpha_2 \rightarrow R_1 = \frac{R_0 \alpha_2}{\alpha_2 - \alpha_1} = \frac{-\alpha_2}{\alpha_1 - \alpha_2} \cdot R_0$$

22.

$$\begin{cases} mv + MV = mu + MU & (1) \\ \frac{1}{2}mv^2 + \frac{1}{2}MV^2 = \frac{1}{2}mu^2 + \frac{1}{2}MU^2 & (2) \end{cases}, u, U = ?$$

$$\begin{cases} m(v - u) = M(U - V) & (3) \\ m(v^2 - u^2) = M(U^2 - V^2) & (4) \end{cases}$$

$$\begin{cases} m(v - u) = M(U - V) & (5) \\ m(v - u)(v + u) = M(U - V)(U + V) & (6) \end{cases}$$

$$(5) \text{ ja } (6) \rightarrow v + u = U + V \rightarrow U = v + u - V \quad (7)$$

$$(1) \text{ ja } (7) \rightarrow mv + MV = mu + M(v + u - V)$$

$$\rightarrow u = \frac{mv + MV - Mv + MV}{m + M} = \frac{m - M}{m + M} \cdot v + \frac{2M}{m + M} \cdot V \quad (8)$$

$$(7) \text{ ja } (8) \rightarrow U = v + u - V = v + \frac{m - M}{m + M} \cdot v + \frac{2M}{m + M} \cdot V - V$$

$$= \frac{2m}{m + M} \cdot v + \frac{M - m}{m + M} \cdot V$$

23.

$$\begin{cases} I = I_1 + I_2 \\ E = RI + R_1 I_1 \\ E = RI + R_2 I_2 \end{cases} \rightarrow \begin{cases} I - I_1 - I_2 = 0 \\ RI + R_1 I_1 = E \\ RI + R_2 I_2 = E \end{cases}$$

E=12;

R=6;

R1=4;

R2=3;

ans =

```
rref([1,-1,-1,0      1.0000      0      0      1.5556
      R,R1,0,E        0      1.0000      0      0.6667
      R,0,R2,E])      0      0      1.0000      0.8889
```

24.

$$\begin{cases} I = I_1 + I_2 \\ I_2 = I_3 + I_4 \\ E = RI + R_1 I_1 \\ E = RI + R_2 I_2 + R_3 I_3 \\ E = RI + R_2 I_2 + R_4 I_4 \end{cases} \rightarrow \begin{cases} I - I_1 + I_2 = 0 \\ I_2 - I_3 - I_4 = 0 \\ RI + R_1 I_1 = E \\ RI + R_2 I_2 + R_3 I_3 = E \\ RI + R_2 I_2 + R_4 I_4 = E \end{cases}$$

E=24.0;

R=5.2;

R1=10.3;

R2=17.2;

R3=11.9;

R4=22.4;

ans =

```
rref([1,-1,-1,0,0,0      1.0000      0      0      0      0      1.9212
      0,0,1,-1,-1,0      0      1.0000      0      0      0      1.3602
      R,R1,0,0,0,E        0      0      1.0000      0      0      0.5610
      R,0,R2,R3,0,E       0      0      0      1.0000      0      0.3664
      R,0,R2,0,R4,E])     0      0      0      0      1.0000      0.1946
```

25.

a)

$$\frac{1}{24} \cdot t + \frac{1}{40} \cdot t = 1 \rightarrow t = 15$$

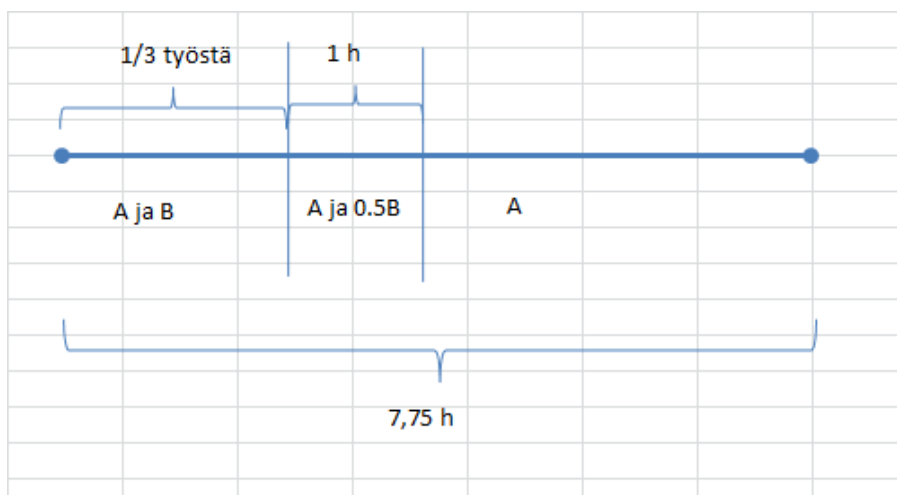
b)

$$\frac{1}{24} \cdot t = \frac{1}{40} \cdot t + 1 \rightarrow t = 60$$

26. A tekee osan x työstä päivässä, B osan y

$$\begin{cases} 5x + 9y = 0.5 \\ 7.5x + 7.5y = 0.5 \end{cases} \rightarrow \begin{cases} x = 1/40 \\ y = 1/24 \end{cases}$$

27. A tekee osan x työstä tunnissa, B osan $0.5x$



A ja B tekevät $\frac{1}{3}$ työstä: aikaa kuluu

$$\frac{1}{3} \cdot \frac{1}{1.5x} \quad \text{tuntia}$$

A ja 0.5B tekevät 1 tunnissa osan $1.25x$ työstä

A tekee osan $1 - \frac{1}{3} - 1.25x$ työstä, aikaa kuluu

$$\left(1 - \frac{1}{3} - 1.25x\right) \cdot \frac{1}{x} \quad \text{tuntia}$$

$$\rightarrow \frac{1}{3} \cdot \frac{1}{1.5x} + 1 + \left(1 - \frac{1}{3} - 1.25x\right) \cdot \frac{1}{x} = 7.75 \rightarrow x = \frac{1}{9}$$

Eli kunnossa ollessaan A ja B tekisivät yhdessä osan $1.5x = \frac{1}{6}$ työstä eli aikaa kuluisi 6 tuntia

28. Glyserolin tiheys 1.26, veden 1.00, liuoksen 1.10 kg/dm³

1 dm³ liuosta, massa 1.10 kg, x kg glyserolia, $1.10 - x$ kg vettä

$$\frac{x}{1.26} + \frac{1.10 - x}{1.00} = 1 \rightarrow x = 0.485$$

Painopitoisuus

$$\frac{x}{1.10} = 0.441$$

29. x kg 12.5 % ja $30 - x$ kg 18 %

$$0.125x + 0.18(30 - x) = 0.14 \cdot 30 \rightarrow x = 21.8$$

30. x tonnia kuparikiisua, $18 - x$ tonnia rikkikiisua

$$0.34x + 0.47(18 - x) = 6.9 \rightarrow x = 12$$

Kuparia $0.33 \cdot 12 = 3.96$

31. A \rightarrow B: x km myötävirtaa, y km virratonta, $70 - x - y$ km vastavirtaa

B \rightarrow A: $70 - x - y$ km myötävirtaa, y km virratonta, x km vastavirtaa

$$\begin{cases} x/20 + y/16 + (70 - x - y)/12 = 4 + \frac{1}{3} \\ (70 - x - y)/20 + y/16 + x/12 = 4 + \frac{48}{60} \end{cases} \rightarrow \begin{cases} x = 30 \\ y = 24 \\ 70 - x - y = 16 \end{cases}$$

32. I täyttää osan x altaasta tunnissa, II osan y , III osan z

$$\begin{cases} x + y = 1/6 \\ y + z = 1/10 \\ x + z = 1/7.5 \end{cases} \rightarrow \begin{cases} x = 1/10 \\ y = 1/15 \\ z = 1/30 \end{cases}$$

Kaikki yhdessä $x + y + z = 1/5$