Hyperbeli-paikannus

$F_1 = [0,0],\ F_2 = [x_2, y_2],\ F_3 = [x_3, y_3],\ P = [x, y]$

$$\begin{cases} PF_1 - PF_2 = d_{12} \\ \\ PF_1 - PF_3 = d_{13} \end{cases}$$

$$\begin{cases} \sqrt{x^2+y^2} - \sqrt{(x-x_2)^2+(y-y_2)^2} = d_{12} \\ \\ \sqrt{x^2+y^2} - \sqrt{(x-x_3)^2+(y-y_3)^2} = d_{13} \end{cases}$$

Ratkaistaan yhtälöpari ns. Bancroftin menetelmällä:

Merkitään $PF_1 = \sqrt{x^2+y^2} = R$

$\rightarrow\ PF_2 = R - d_{12},\ PF_3 = R - d_{13}$

$$\xrightarrow{(\,)^2} \begin{cases} x^2+y^2 = R^2 \quad (1) \\ (x-x_2)^2+(y-y_2)^2 = (R-d_{12})^2 \quad (2) \\ (x-x_3)^2+(y-y_3)^2 = (R-d_{13})^2 \quad (3) \end{cases}$$

$$\rightarrow \begin{cases} 2x_2\,x + 2y_2\,y = 2d_{12}R + x_2^2 + y_2^2 - d_{12}^2 \quad (1)-(2) \\ 2x_3\,x + 2y_3\,y = 2d_{13}R + x_3^2 + y_3^2 - d_{13}^2 \quad (1)-(3) \end{cases}$$

$$\rightarrow \begin{cases} ax + by = e_1R + e_2 \\ \\ cx + dy = f_1R + f_2 \end{cases}$$

$$\rightarrow \begin{cases} x = \dfrac{d(e_1R+e_2)-b(f_1R+f_2)}{ad-bc} = \dfrac{de_1-bf_1}{ad-bc}\,R + \dfrac{de_2-bf_2}{ad-bc} \\ y = \dfrac{a(f_1R+f_2)-c(e_1R+e_2)}{ad-bc} = \dfrac{af_1-ce_1}{ad-bc}\,R + \dfrac{af_2-ce_2}{ad-bc} \end{cases}$$

$$\rightarrow \begin{cases} x = \alpha R + \beta \\ y = \gamma R + \delta \end{cases}$$

$$\xrightarrow{(\,)^2} \begin{cases} x^2 = \alpha^2 R^2 + 2\alpha\beta R + \beta^2 \quad (4) \\ y^2 = \gamma^2 R^2 + 2\gamma\delta R + \delta^2 \quad (5) \end{cases}$$

$$\rightarrow R^2 \overset{(1)}{=} x^2 + y^2 \overset{(4)+(5)}{=} (\alpha^2+\gamma^2)R^2 + 2(\alpha\beta+\gamma\delta)R + \beta^2 + \delta^2$$

$$\rightarrow (\alpha^2+\gamma^2-1)R^2 + 2(\alpha\beta+\gamma\delta)R + \beta^2 + \delta^2 = 0$$

$$\rightarrow AR^2 + BR + C = 0 \leftrightarrow R = \frac{-B \pm \sqrt{B^2-4AC}}{2A}$$

$$\rightarrow x = \alpha R + \beta,\; y = \gamma R + \delta$$