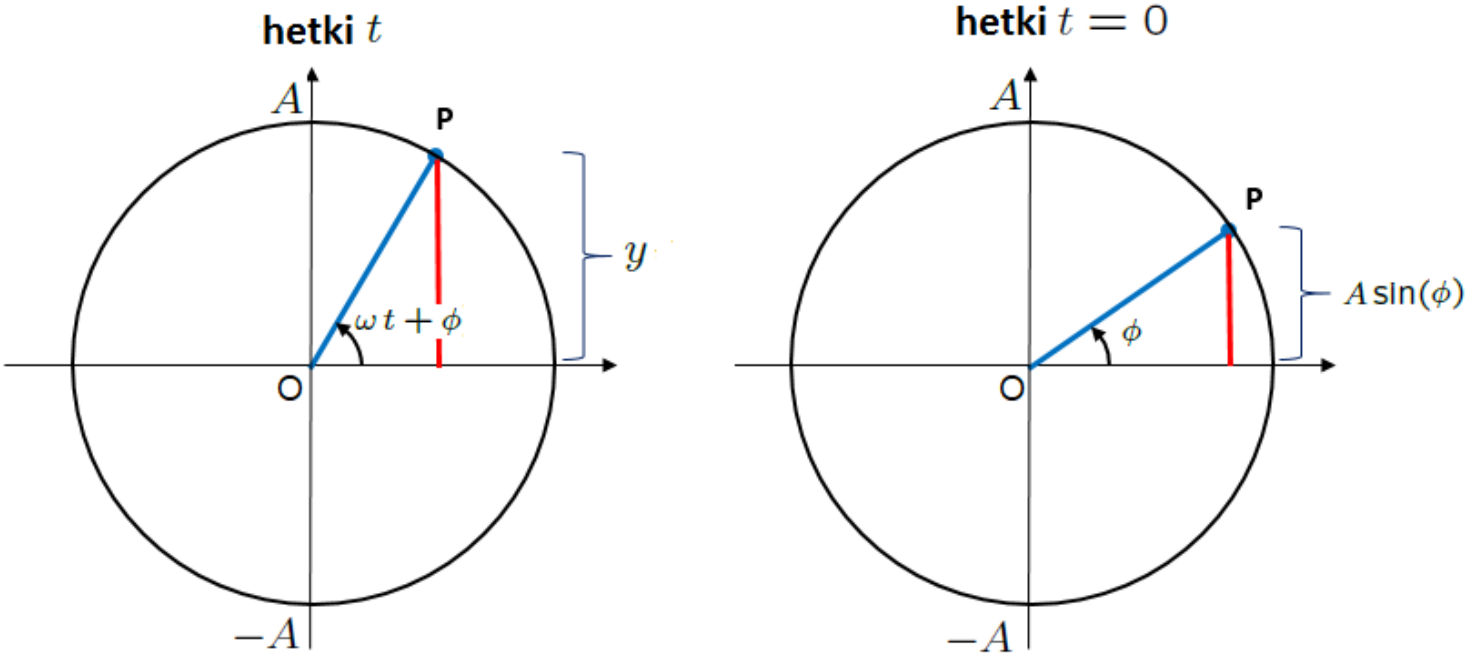
Sinikäyrä $y = A\sin(\omega t + \phi)$



kertoo pisteen P korkeuden hetkellä t, kun se pyörii A-säteistä ympyrää vastapäivään nopeudella ω **rad**iaania sekunnissa ja kiertokulma on ϕ hetkellä $t = 0$.

Kosinikäyrä $x = A\cos(\omega t + \phi)$ kertoo P:n vaakasuoran paikan.

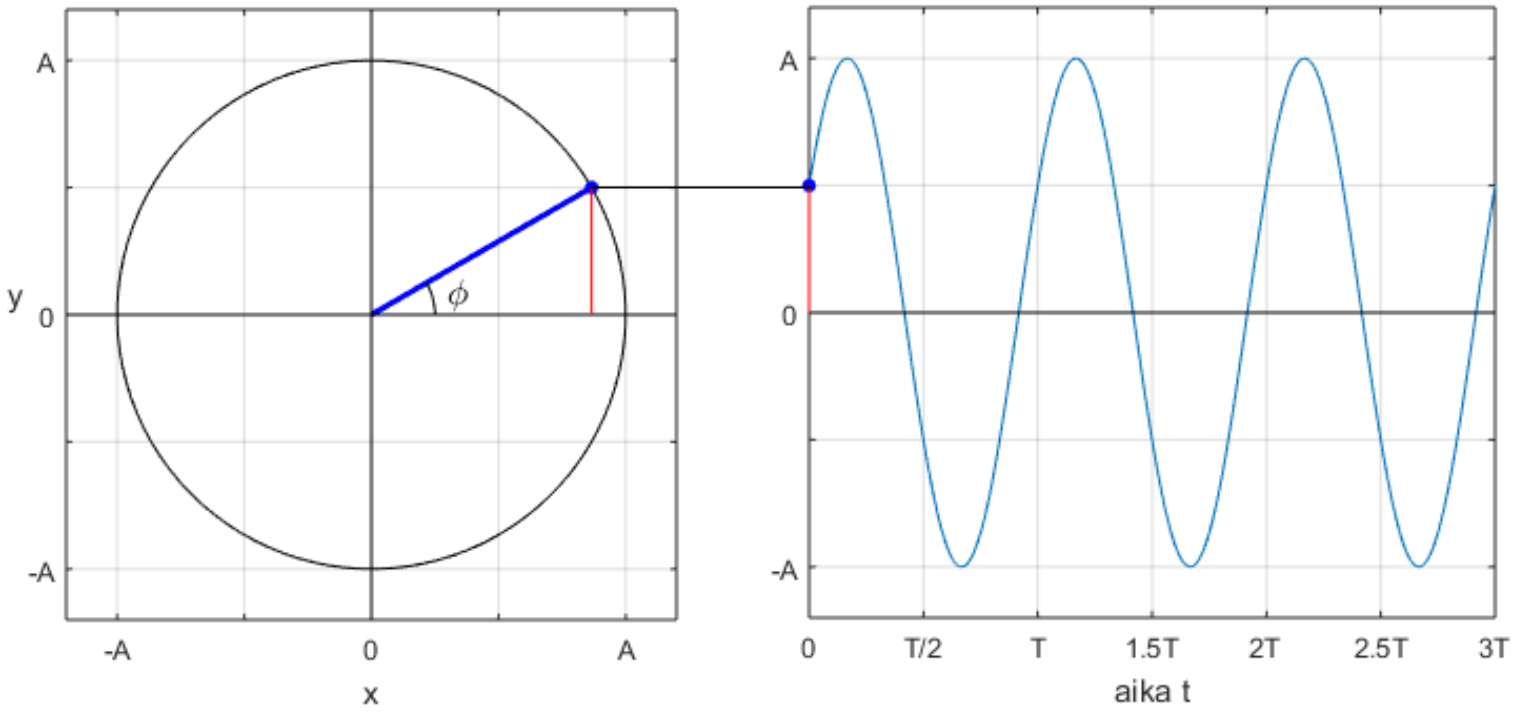
$A =$ amplitudi

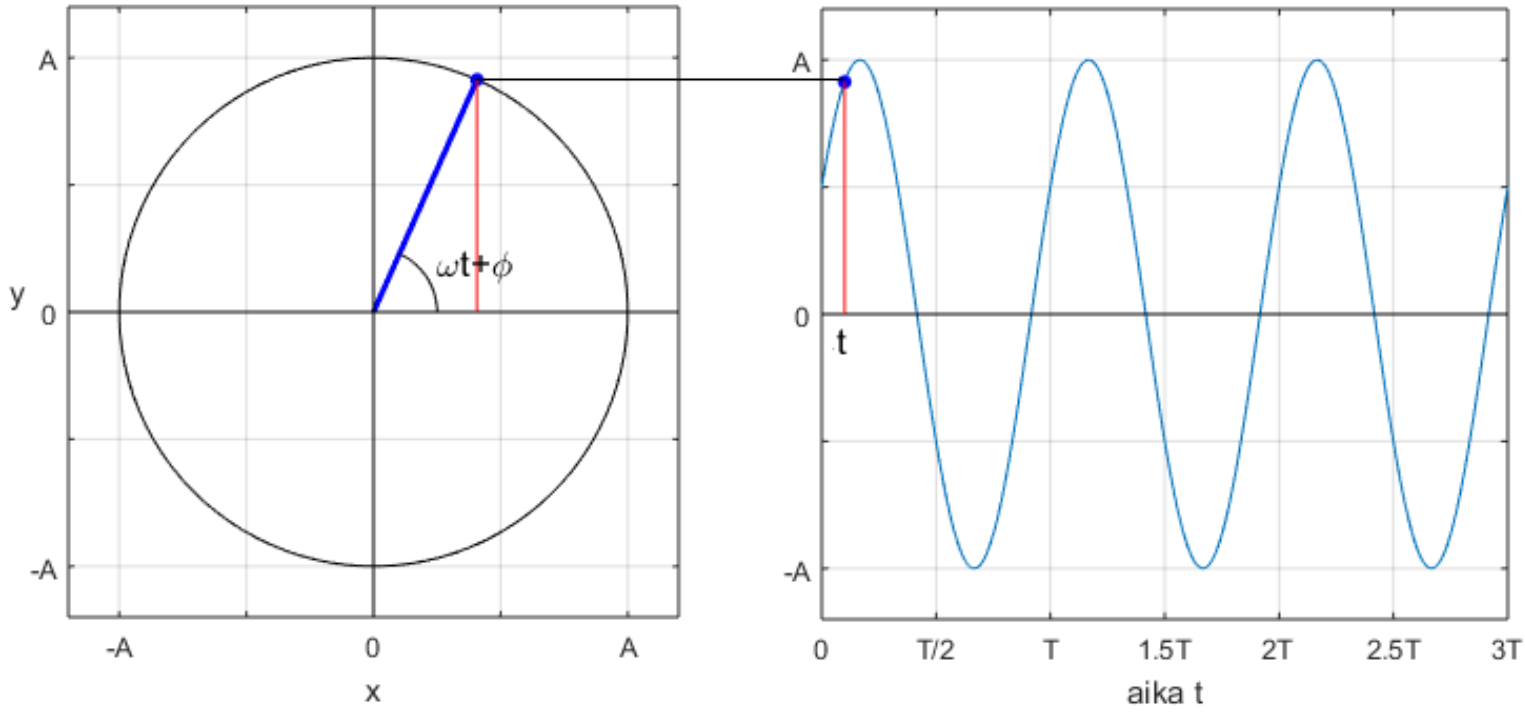
$\omega = 2\pi f =$ kulmataajuus (**rad**/sek)

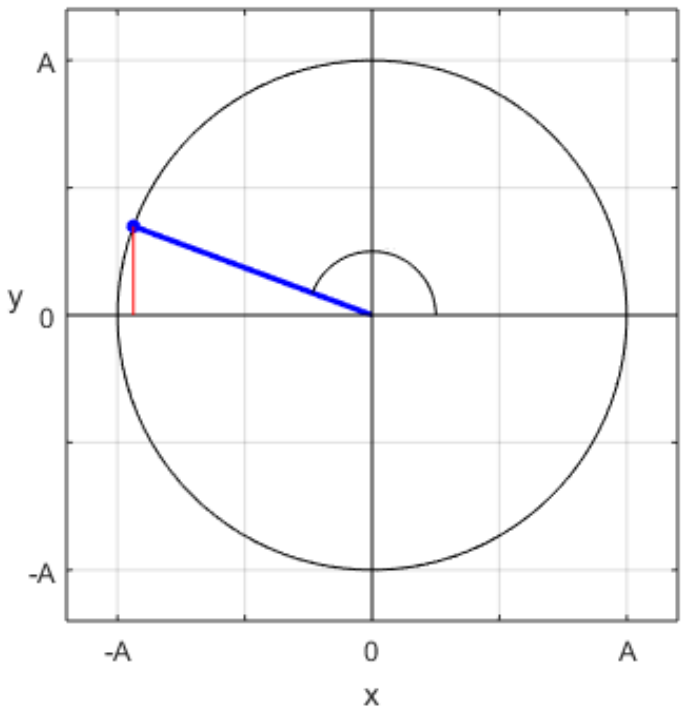
$f =$ taajuus (kierrosta/sek)

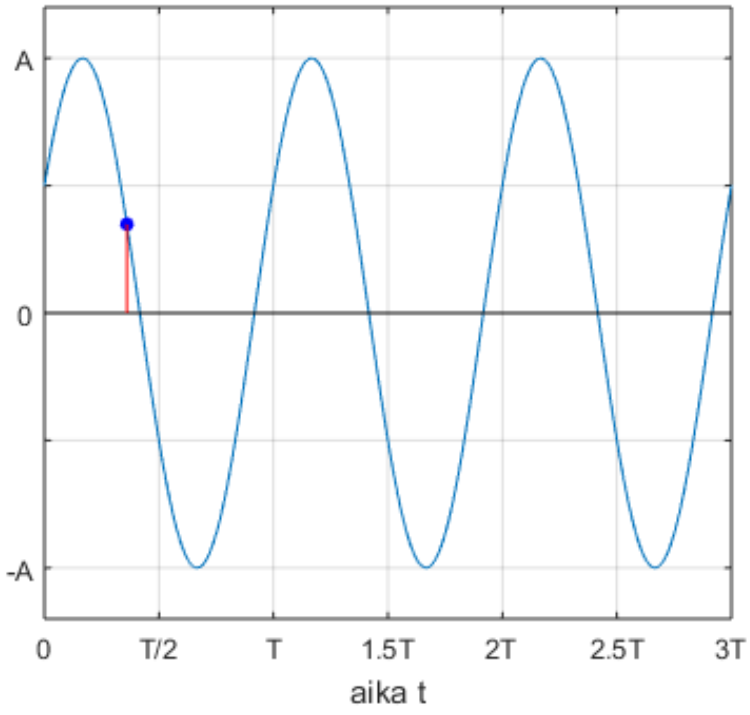
$T = 2\pi/\omega = 1/f =$ jakso =
yhteen kierrokseen kuluva aika

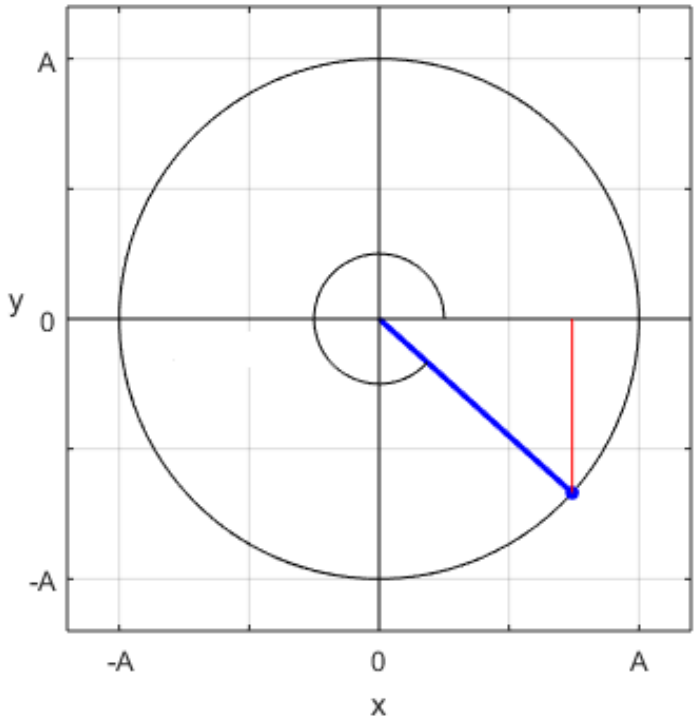
$\phi =$ vaihekulma (**rad**, yleensä välillä $-\pi \ldots \pi$)

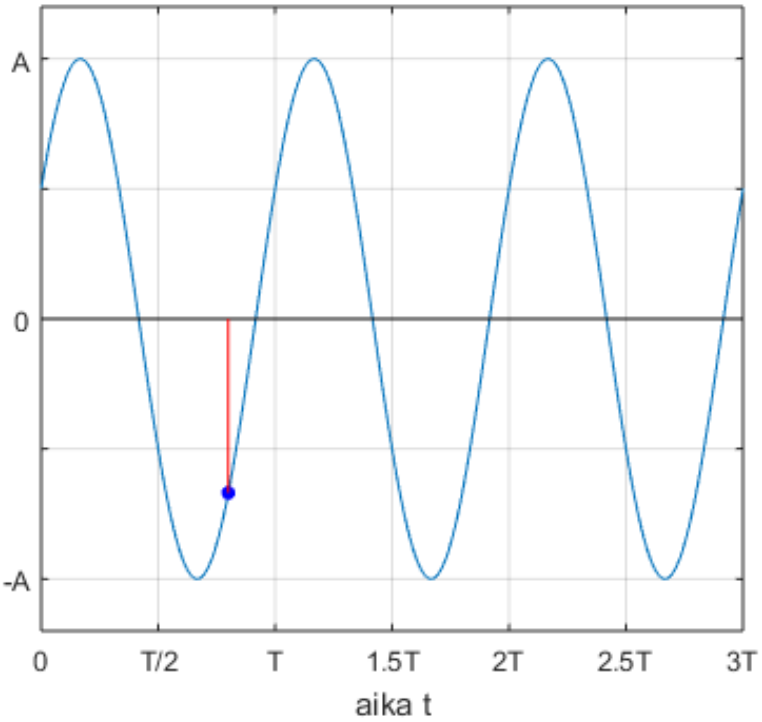

A
y
0
-A
ϕ
-A
0
A
x
A
0
-A
0
T/2
T
1.5T
2T
2.5T
3T
aika t


A
y
0
-A
$\omega t+\phi$
-A
0
A
x
A
0
-A
t
0
T/2
T
1.5T
2T
2.5T
3T
aika t

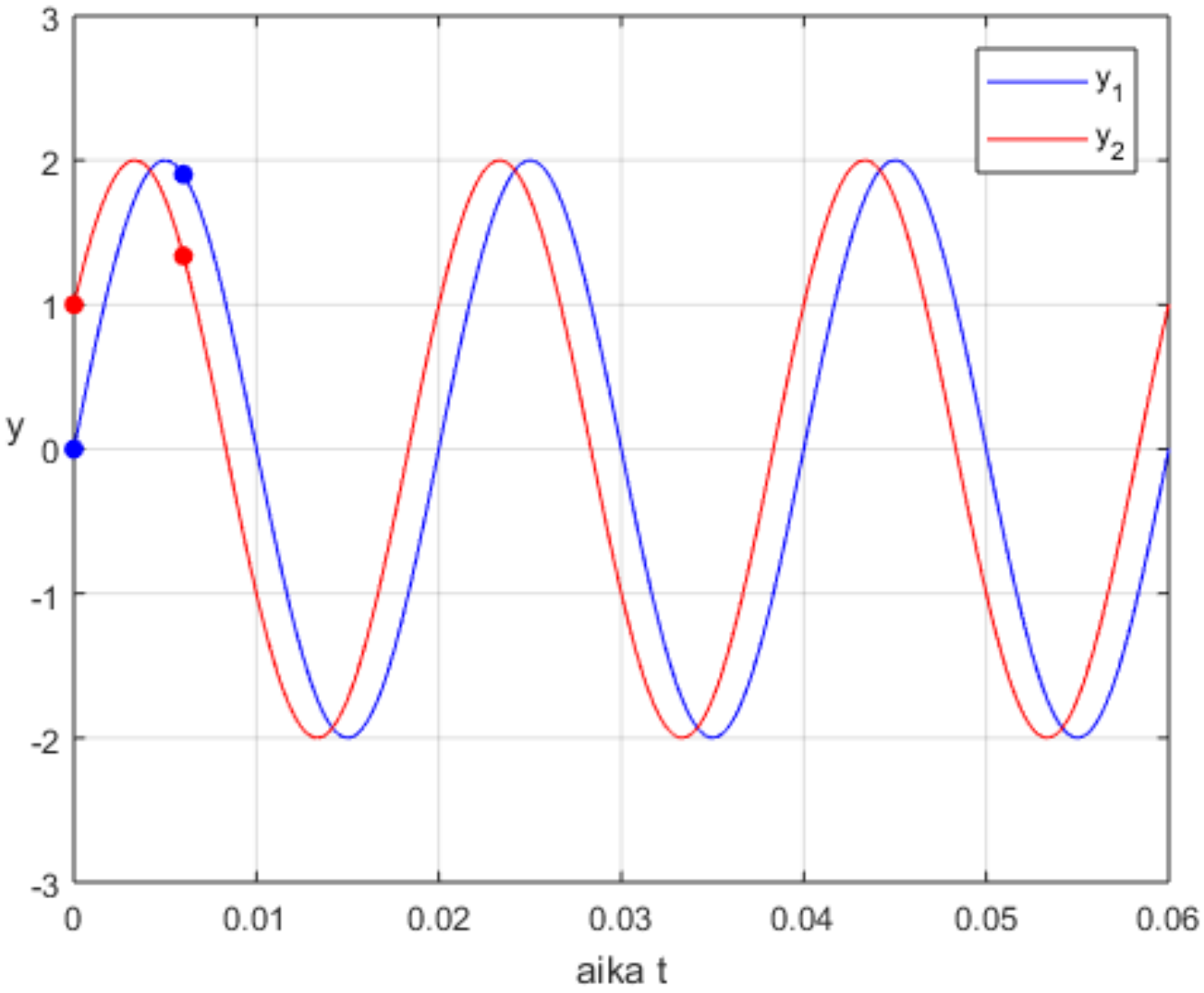

A
y
0
-A
-A
0
A
x

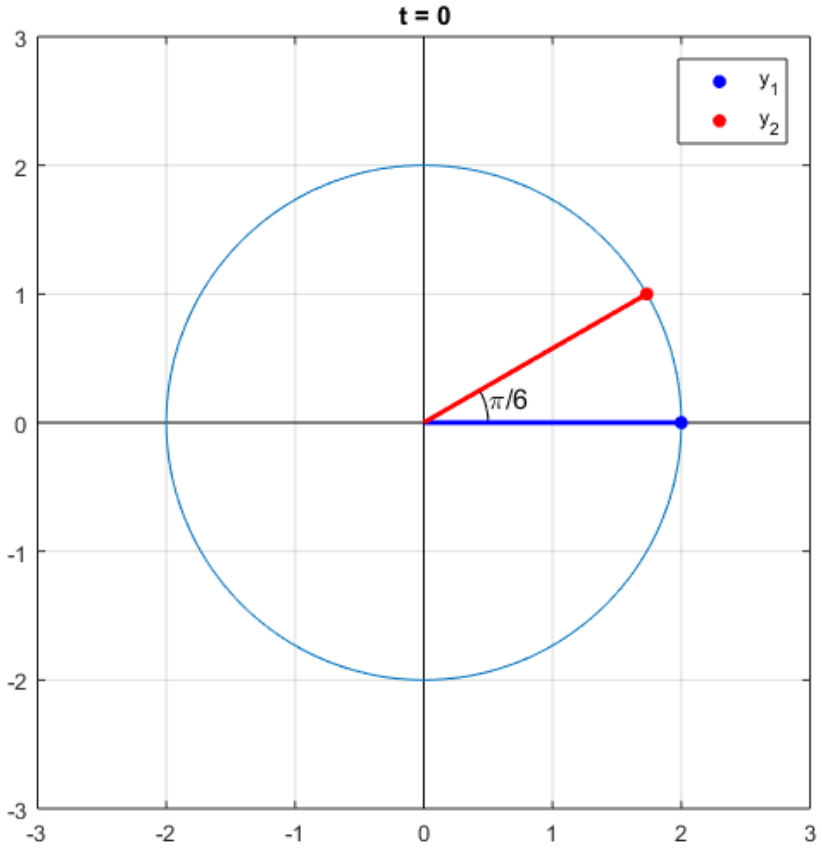

A
0
-A
0
T/2
T
1.5T
2T
2.5T
3T
aika t

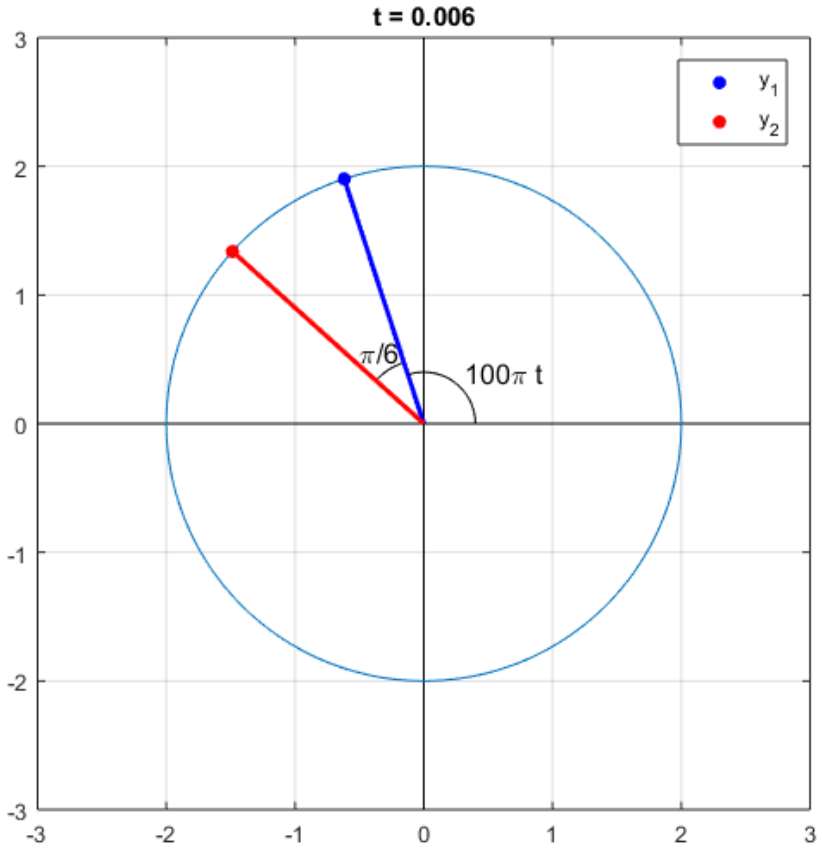

A
y
0
-A
-A
0
A
x


A
0
-A
0
T/2
T
1.5T
2T
2.5T
3T
aika t

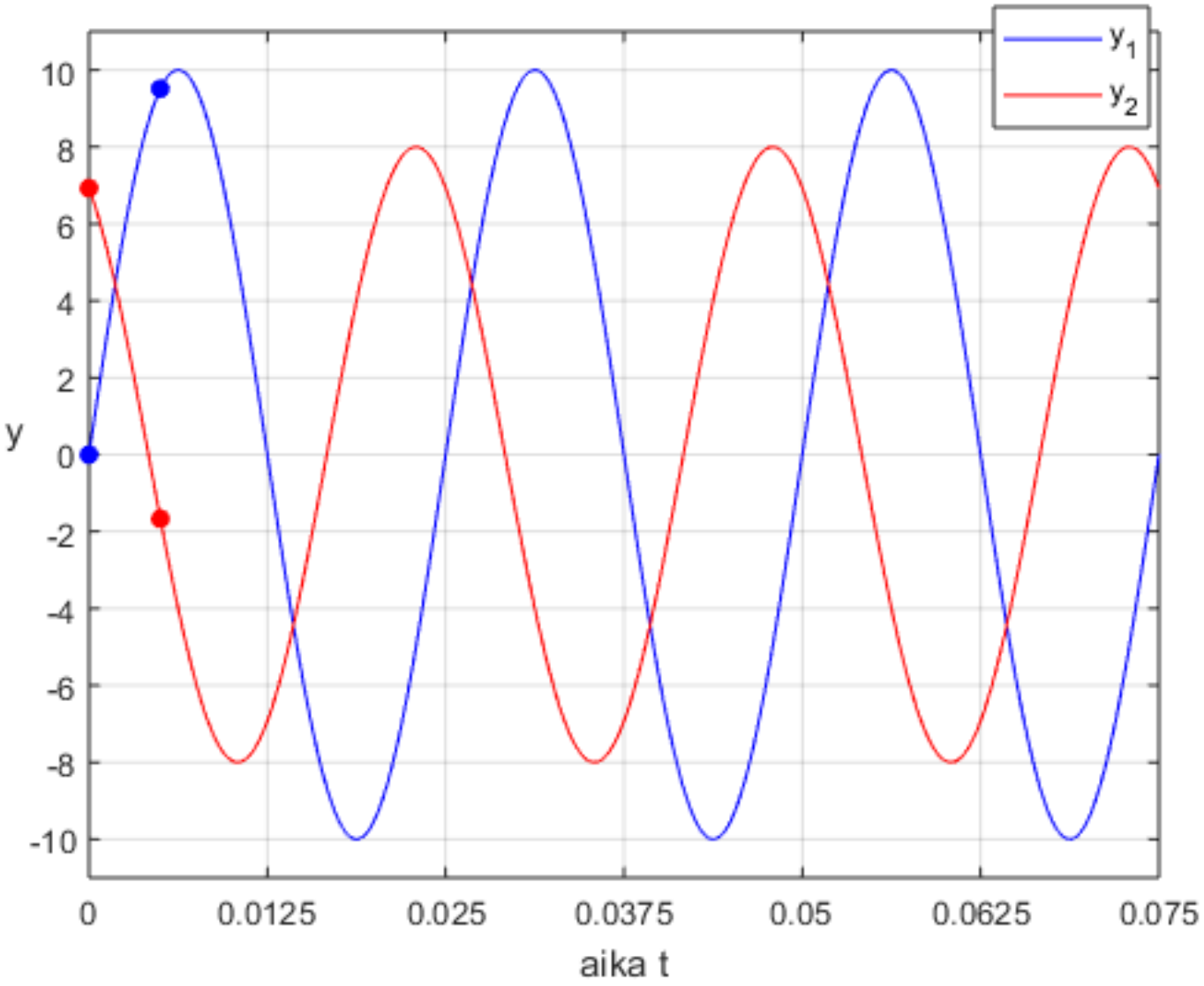
$y_1 = 2\sin(100\pi\, t)$
$y_2 = 2\sin(100\pi\, t + \pi/6)$

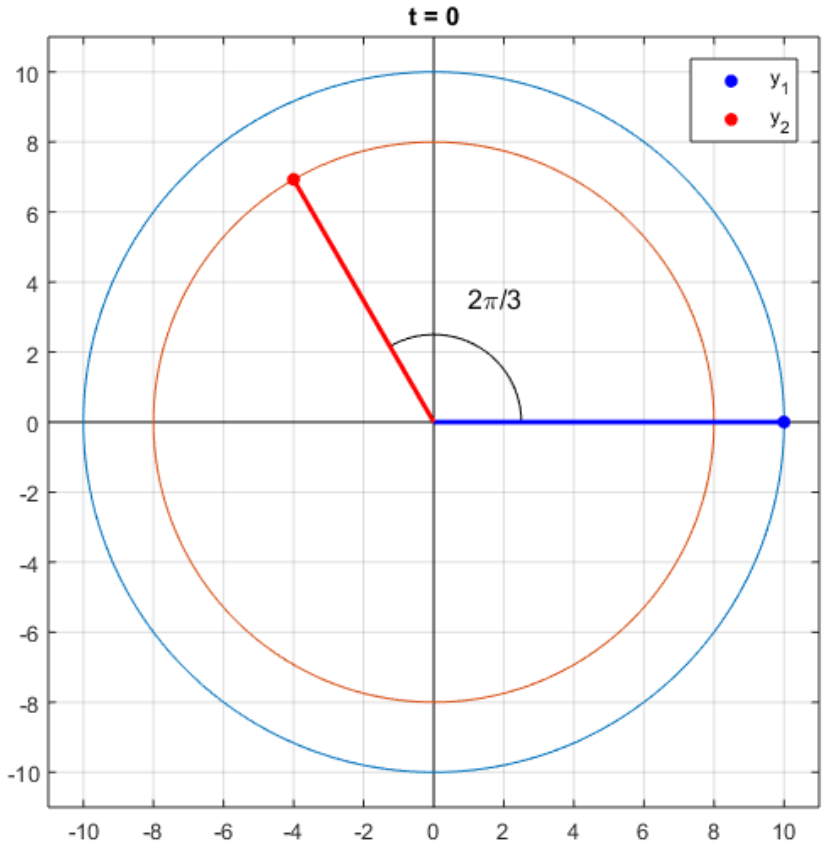


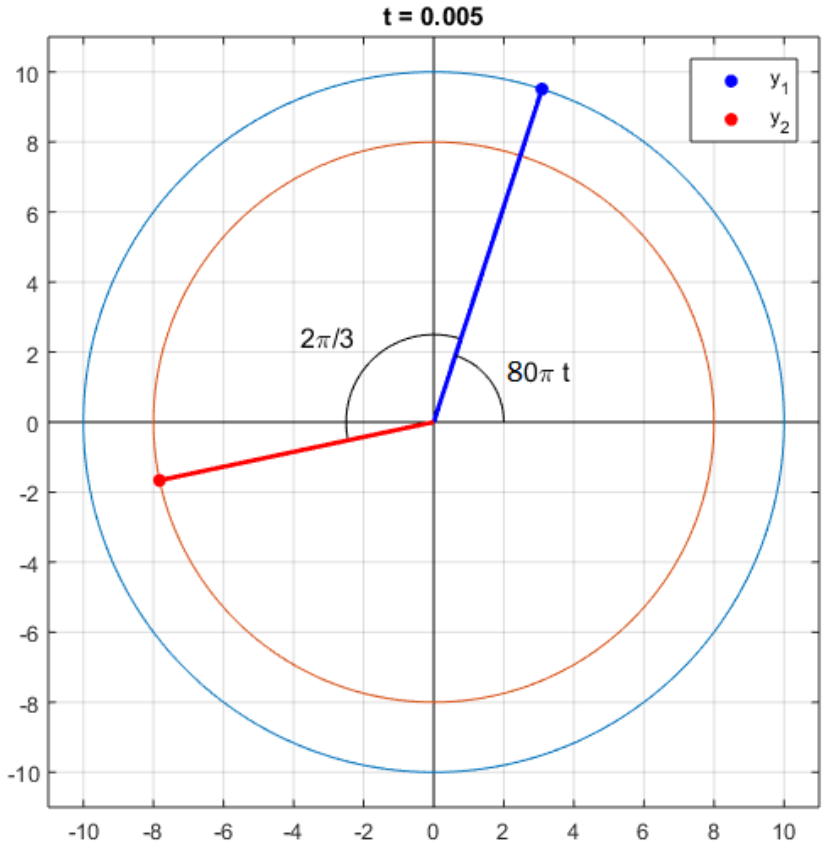

t = 0
y1
y2
π/6
3
2
1
0
-1
-2
-3
-3
-2
-1
0
1
2
3


t = 0.006
y1
y2
π/6
100π t
3
2
1
0
-1
-2
-3
-3
-2
-1
0
1
2
3

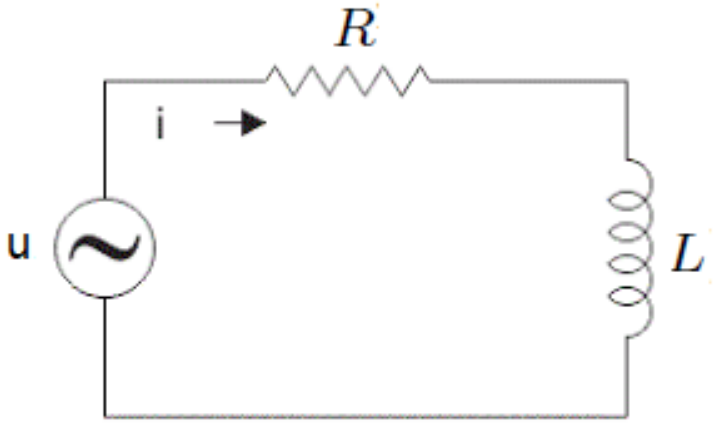
$y_1 = 10\sin(80\pi\, t)$
$y_2 = 8\sin(80\pi\, t + 2\pi/3)$




t = 0
y1
y2
2π/3
10
8
6
4
2
0
-2
-4
-6
-8
-10
-10
-8
-6
-4
-2
0
2
4
6
8
10


t = 0.005
y1
y2
2π/3
80π t
10
8
6
4
2
0
-2
-4
-6
-8
-10
-10
-8
-6
-4
-2
0
2
4
6
8
10

RL-piiri:

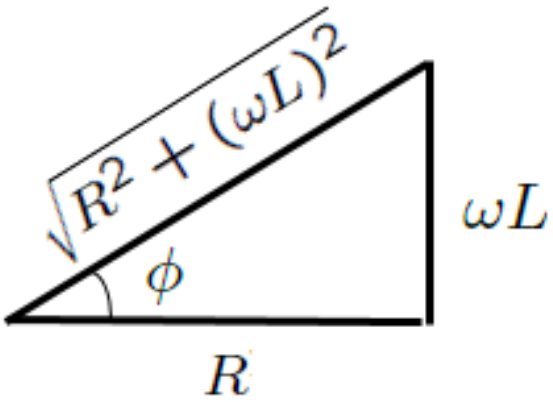


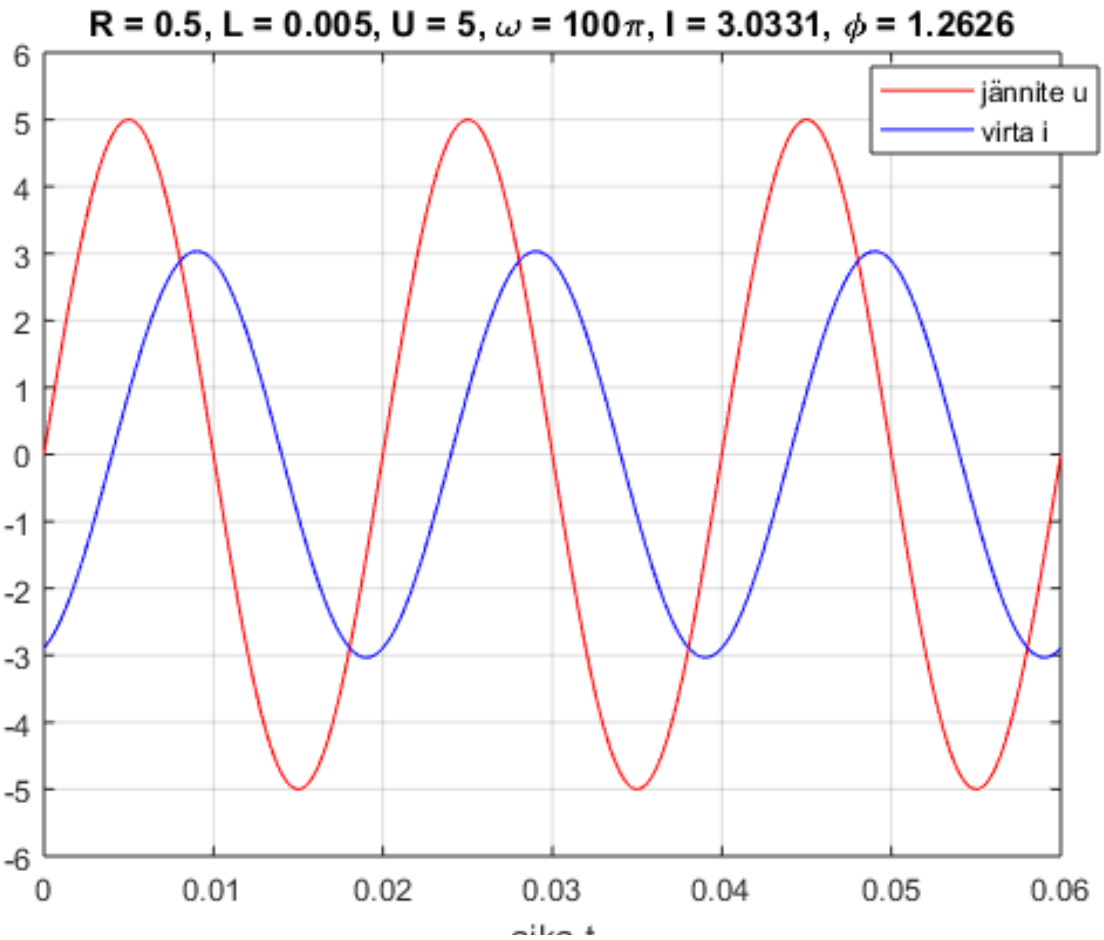
Jos jännite $u = U \sin(\omega t)$, niin virta

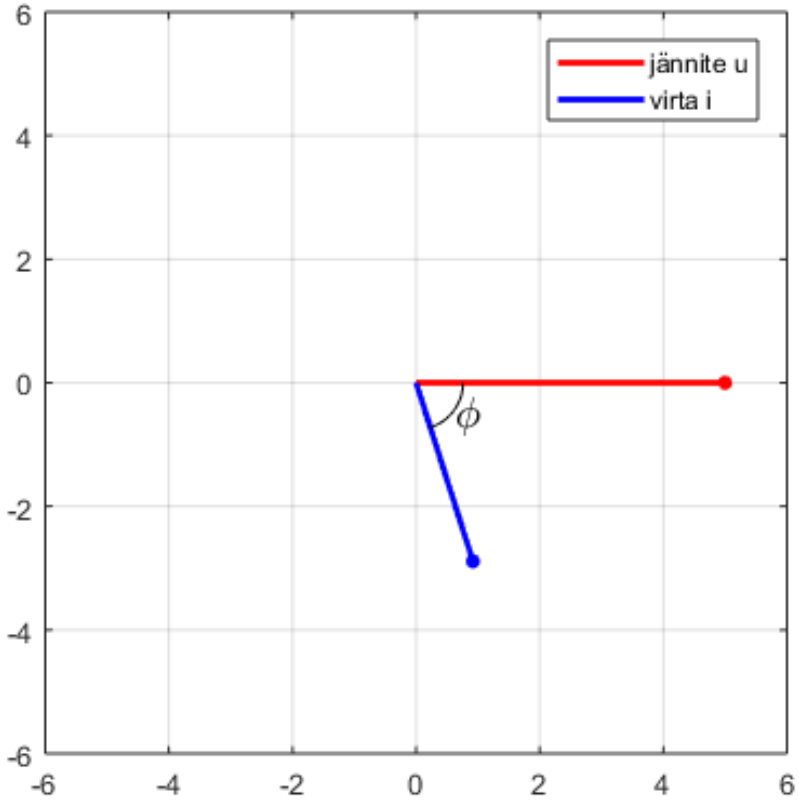
$i = I \sin(\omega t - \phi)$, missä

$$I = \frac{U}{\sqrt{R^2 + (\omega L)^2}}$$

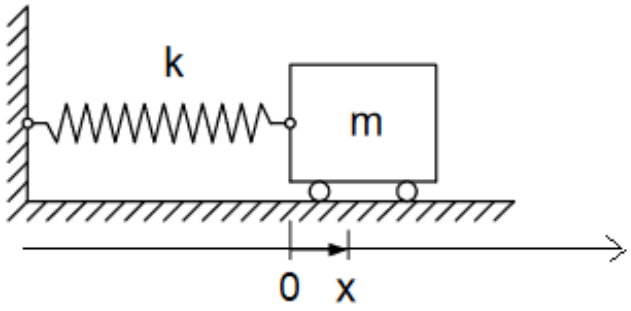
$$\phi = \tan^{-1}(\omega L / R)$$




R = 0.5, L = 0.005, U = 5, ω = 100π, I = 3.0331, ϕ = 1.2626
jännite u
virta i
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
0
0.01
0.02
0.03
0.04
0.05
0.06


jännite u
virta i
ϕ
6
4
2
0
-2
-4
-6
-6
-4
-2
0
2
4
6

Mekaaninen värähtelijä:
massa m, jousivakio k, alkupaikka x_0 ja -nopeus v_0

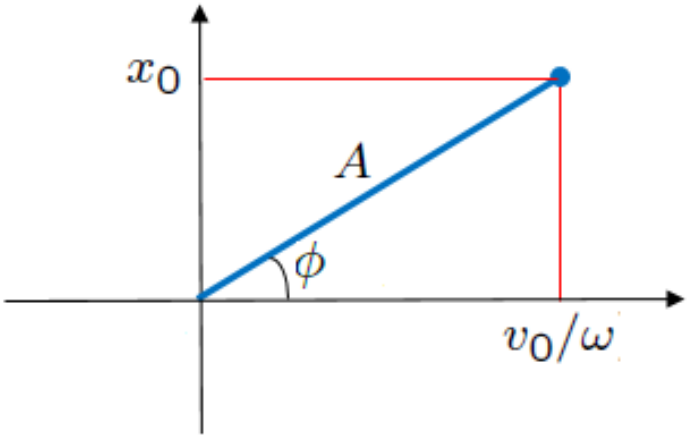


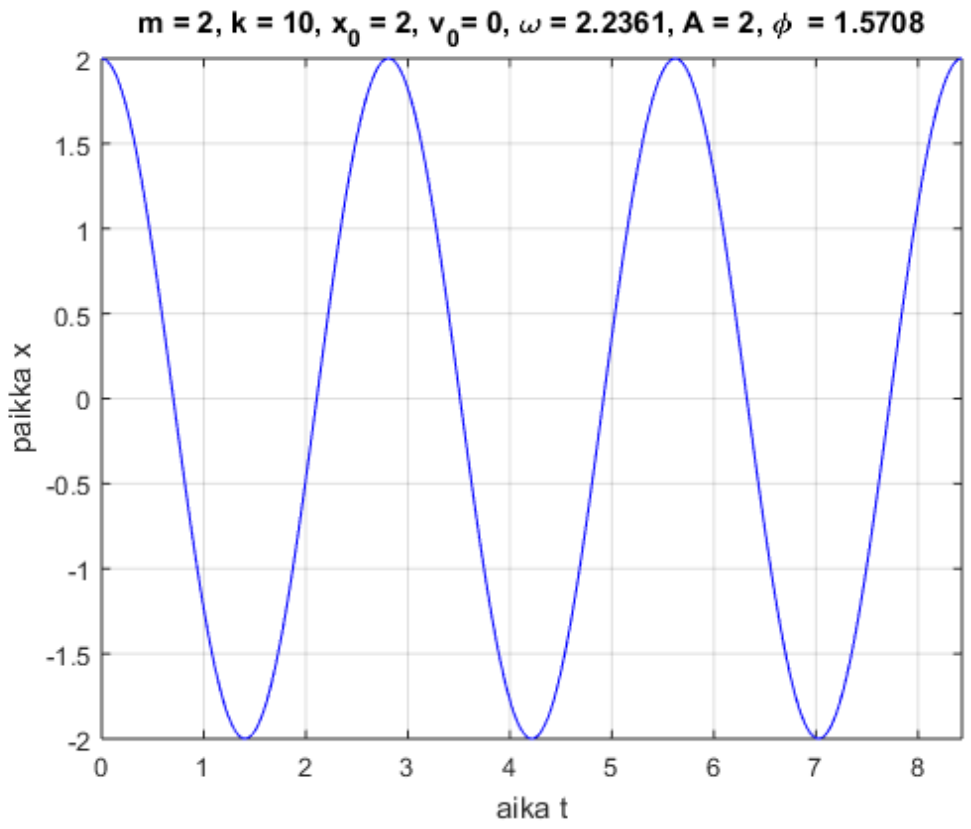
Paikka $x = A\sin(\omega t + \phi)$, missä

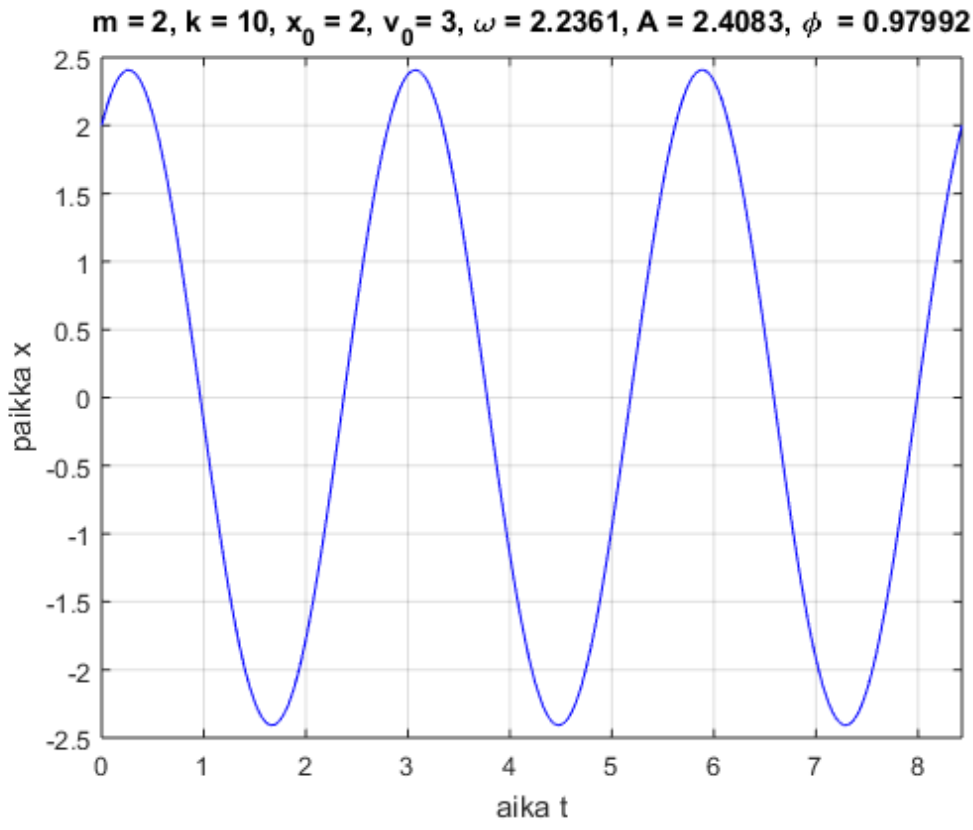
$\omega = \sqrt{k/m}$

$A = \sqrt{x_0^2 + (v_0/\omega)^2}$

$\phi = \text{atan2}(x_0, v_0/\omega)$.



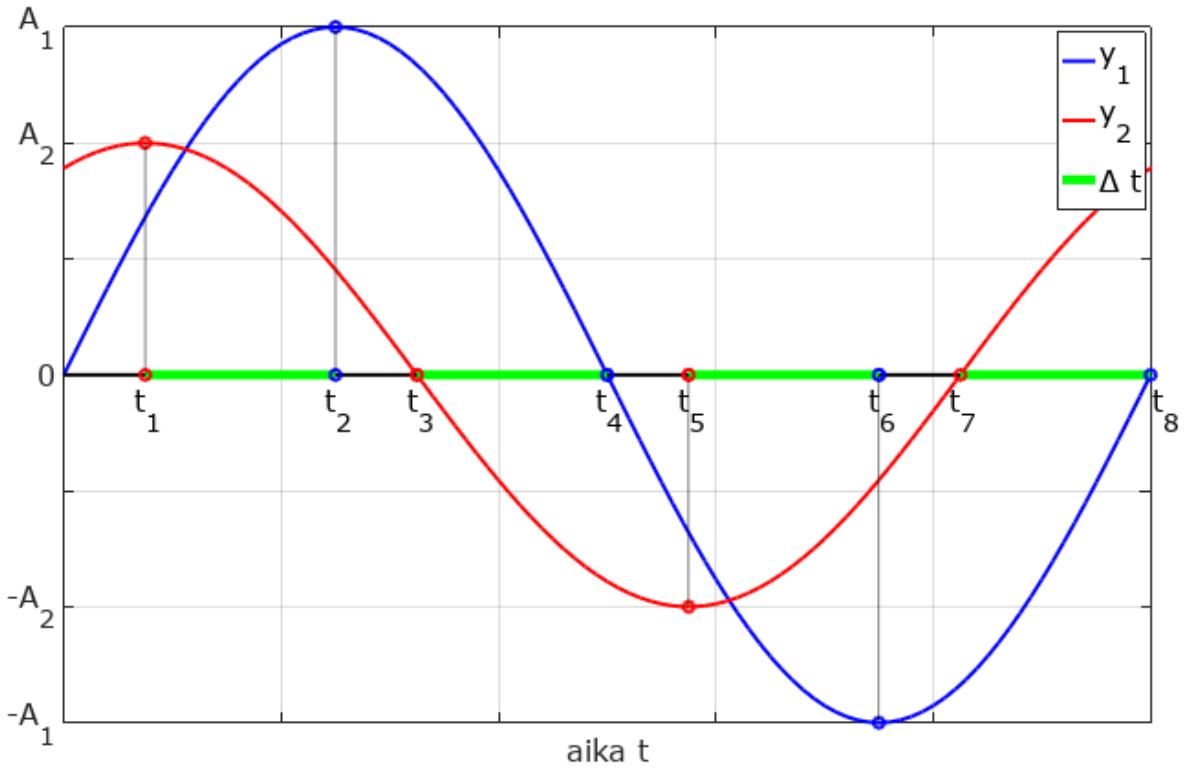

m = 2, k = 10, x0 = 2, v0= 0, ω = 2.2361, A = 2, ϕ = 1.5708
paikka x
aika t
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
1
2
3
4
5
6
7
8


m = 2, k = 10, x0 = 2, v0= 3, ω = 2.2361, A = 2.4083, ϕ = 0.97992
paikka x
aika t
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
0
1
2
3
4
5
6
7
8

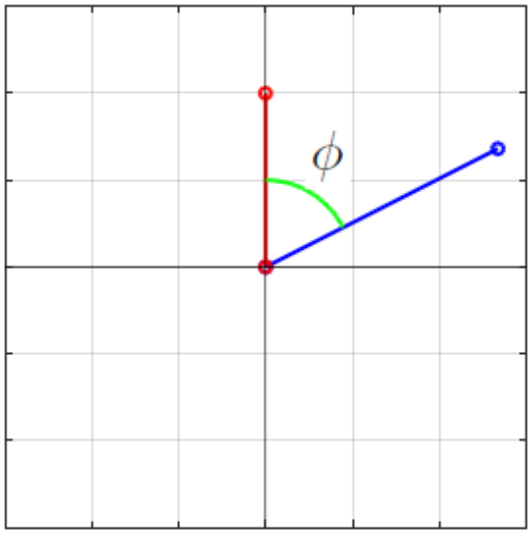
Samantaajuisten sinikäyrien

$y_1 = A_1 \sin(\omega t)$ ja $y_2 = A_2 \sin(\omega t + \phi)$

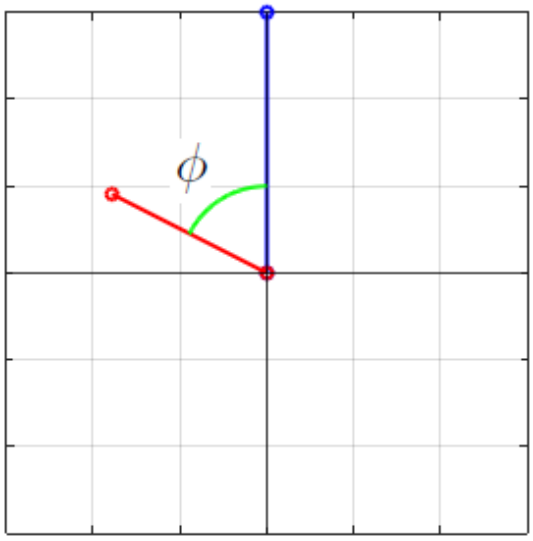
vaihe-ero $\phi = \omega \cdot \Delta t$, missä Δt on sinikäyrien huippu- tai nollakohtien aika-ero.



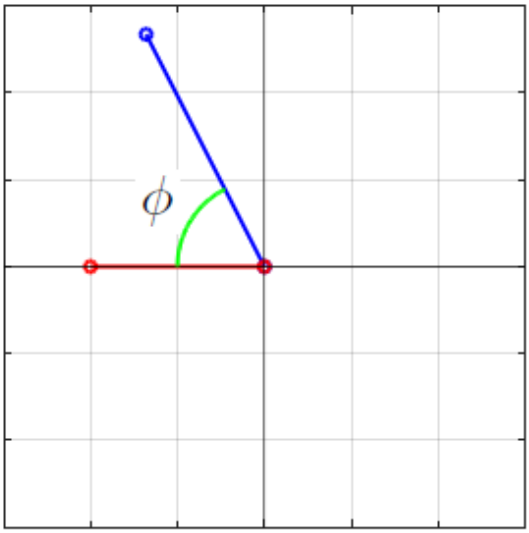
hetki t_1



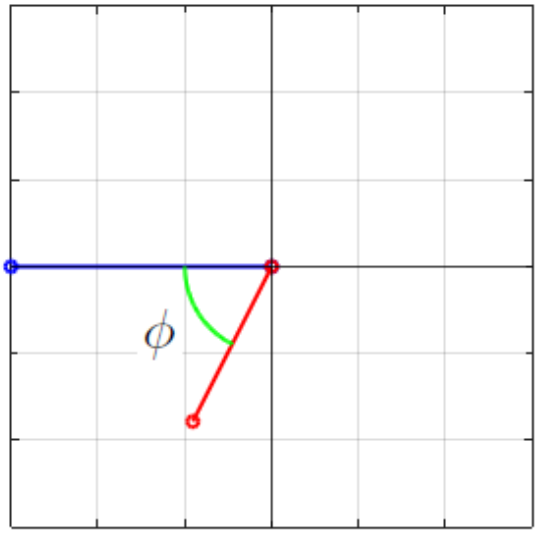
hetki t_2

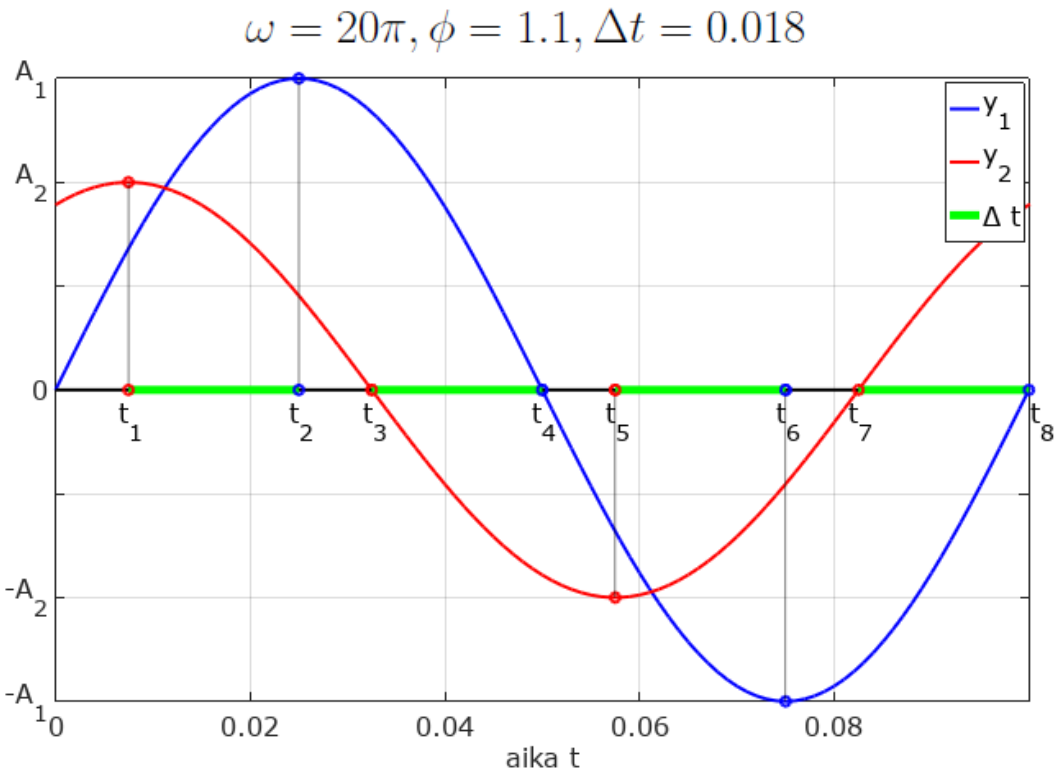


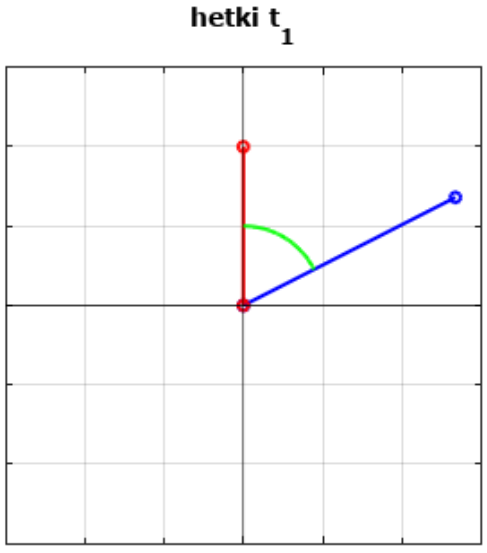
hetki t_3

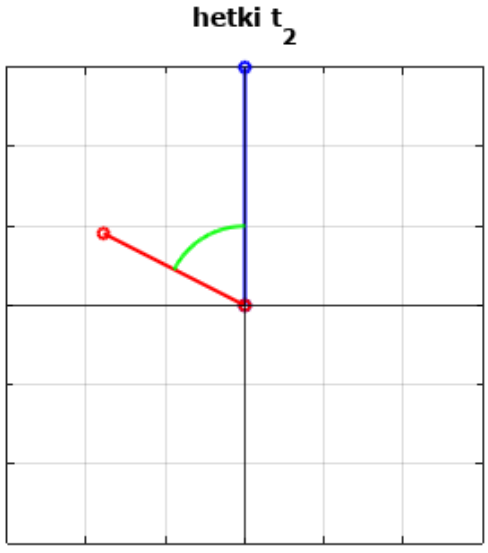


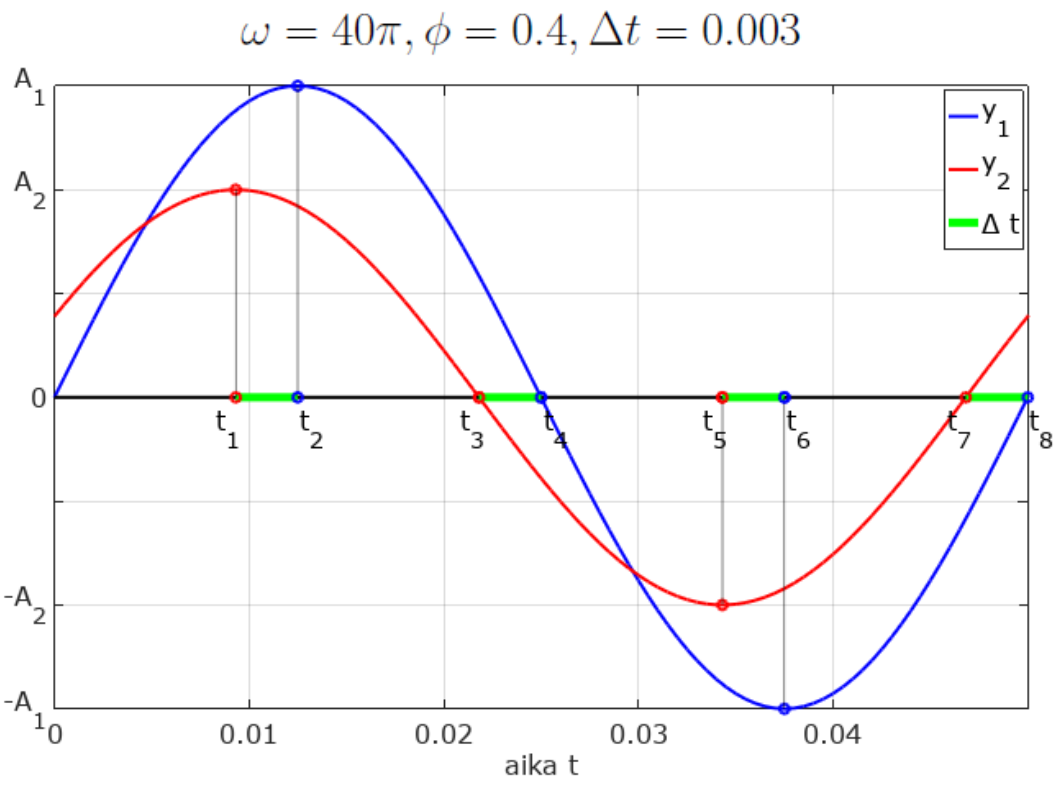
hetki t_4

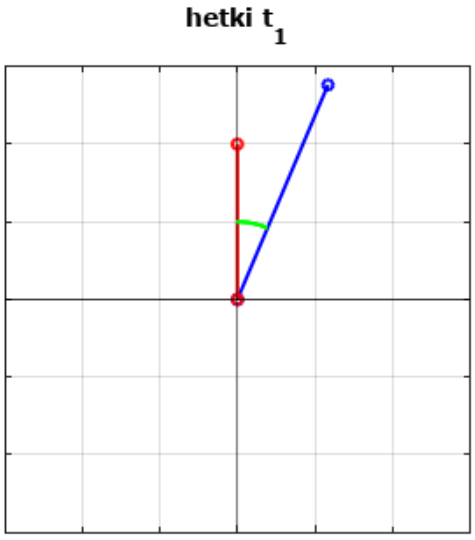


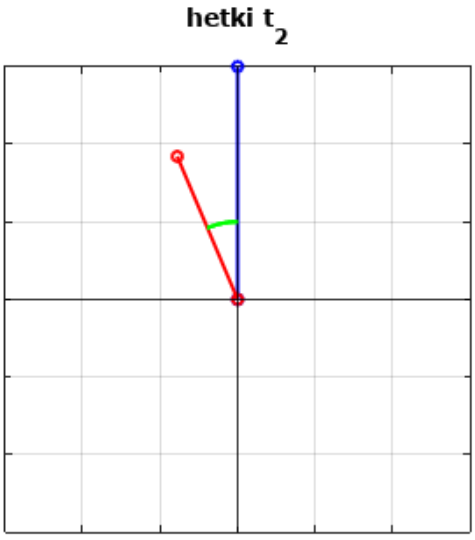

$\omega = 20\pi, \phi = 1.1, \Delta t = 0.018$
A1
A2
0
-A2
-A1
y1
y2
Δ t
t1
t2
t3
t4
t5
t6
t7
t8
0
0.02
0.04
0.06
0.08
aika t


hetki t1


hetki t2


$\omega = 40\pi, \phi = 0.4, \Delta t = 0.003$
A1
A2
0
-A2
-A1
y1
y2
Δ t
t1
t2
t3
t4
t5
t6
t7
t8
0
0.01
0.02
0.03
0.04
aika t


hetki t1


hetki t2

Samantaajuisten sinikäyrien summa on myös sinikäyrä, eli jos

$y_1 = A_1 \sin(\omega t)$ ja $y_2 = A_2 \sin(\omega t + \phi)$, niin

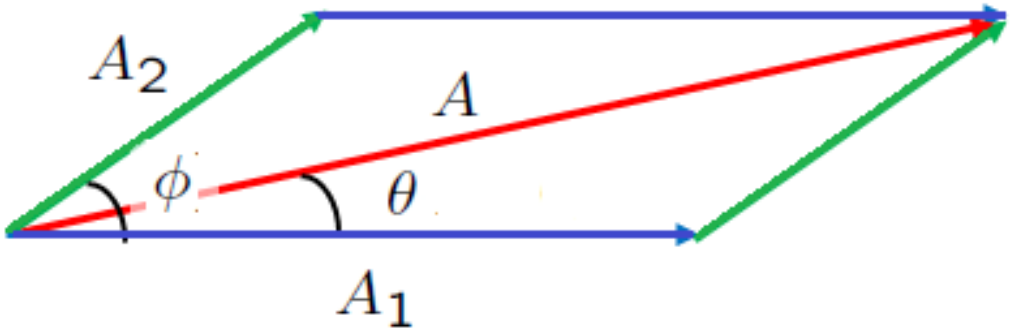
$y_1 + y_2 = A \sin(\omega t + \theta)$

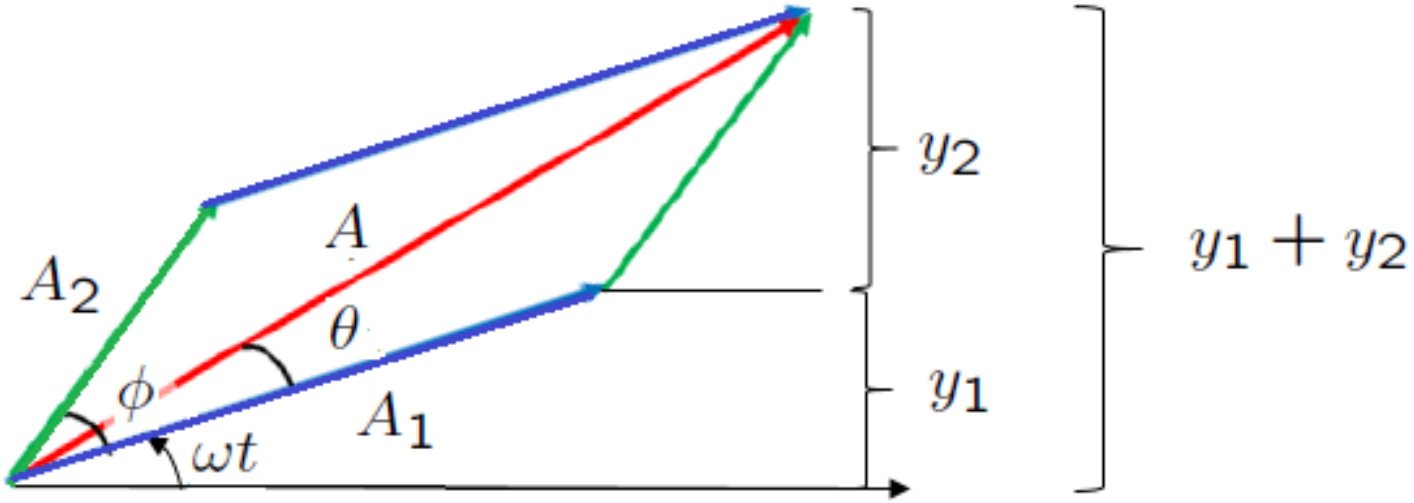
missä amplitudi

$$A = \sqrt{(A_1 + A_2 \cos(\phi))^2 + (A_2 \sin(\phi))^2}$$

ja vaihekulma

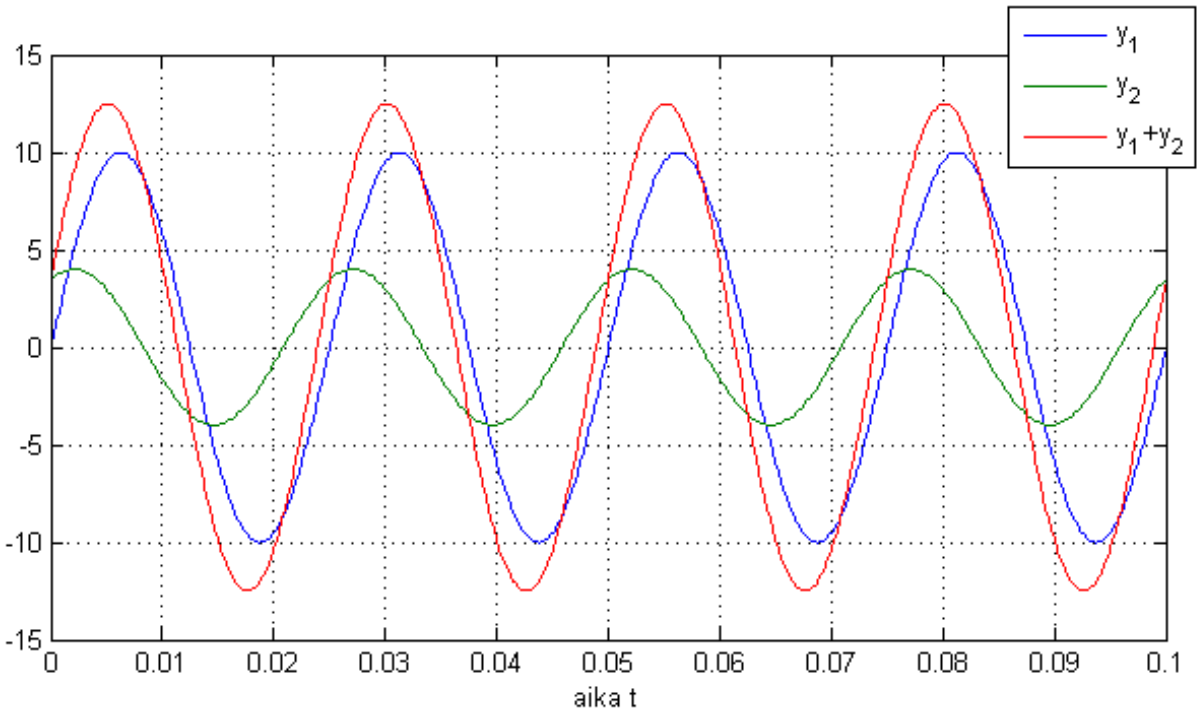
$$\theta = \operatorname{atan2}(A_2 \sin(\phi), A_1 + A_2 \cos(\phi))$$

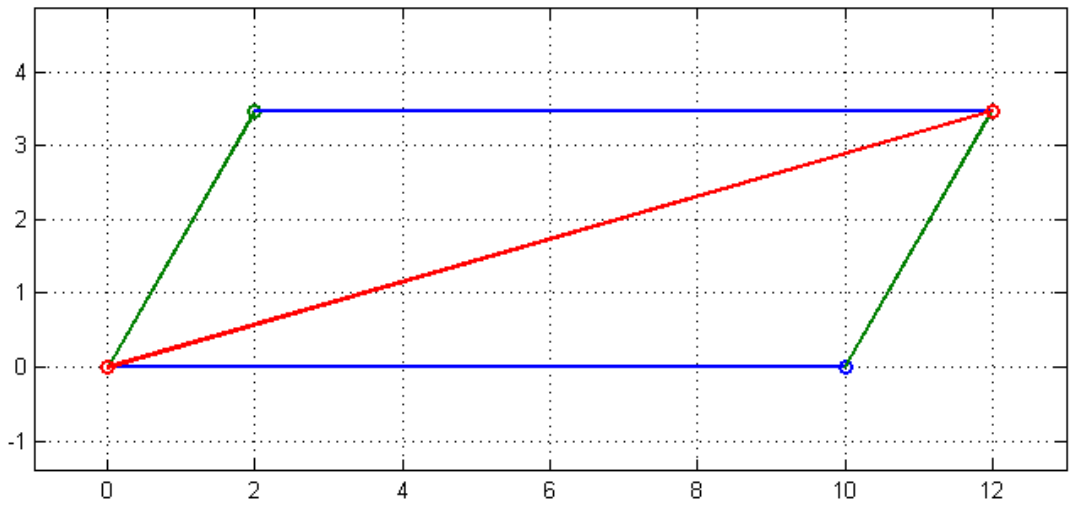

A_2
A
ϕ
θ
A_1


y_2
$y_1 + y_2$
A
A_2
θ
ϕ
y_1
A_1
ωt

$$y_1 = 10\sin(\omega t),\ y_2 = 4\sin(\omega t + \pi/3)$$

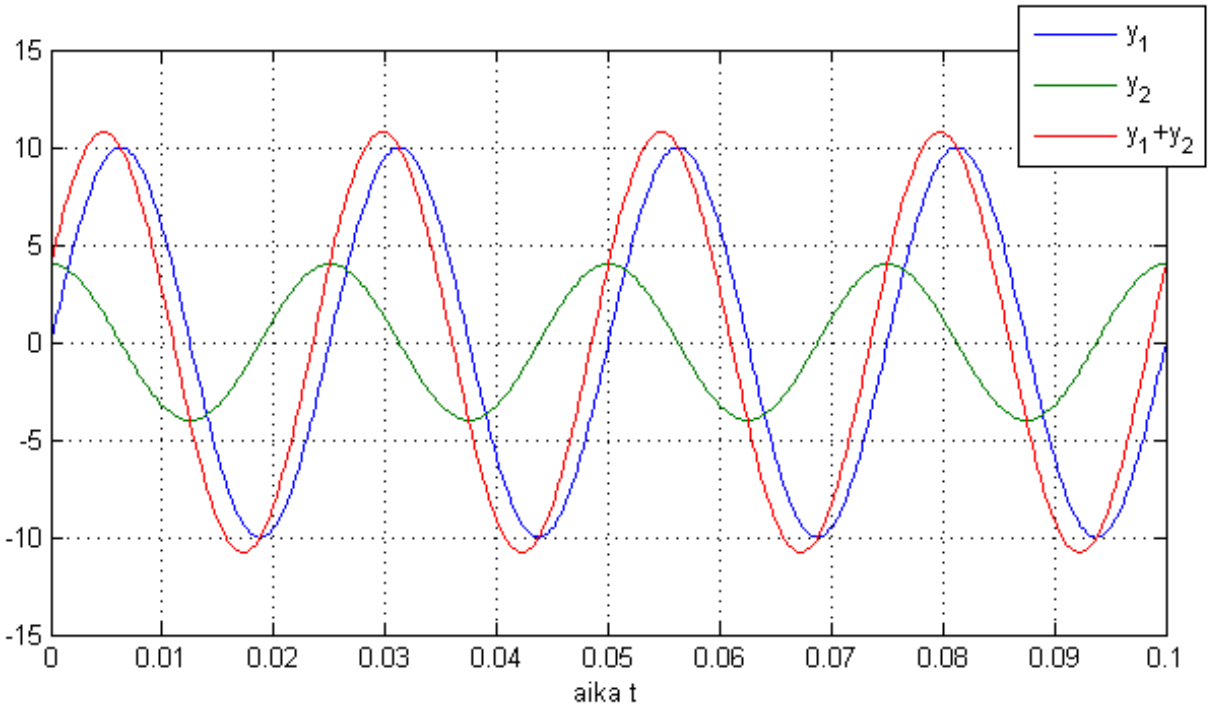
$$y_1 + y_2 = 12.5\sin(\omega t + 0.28)$$

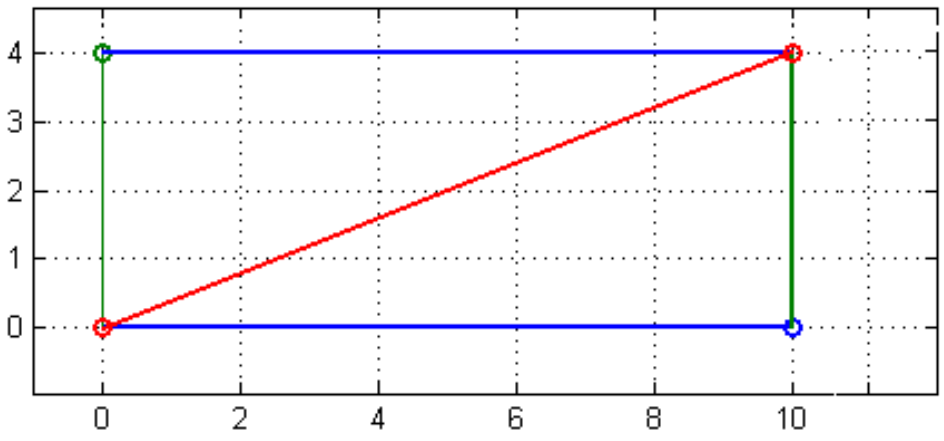




$$y_1 = 10\sin(\omega t),\ y_2 = 4\sin(\omega t + \pi/2)$$

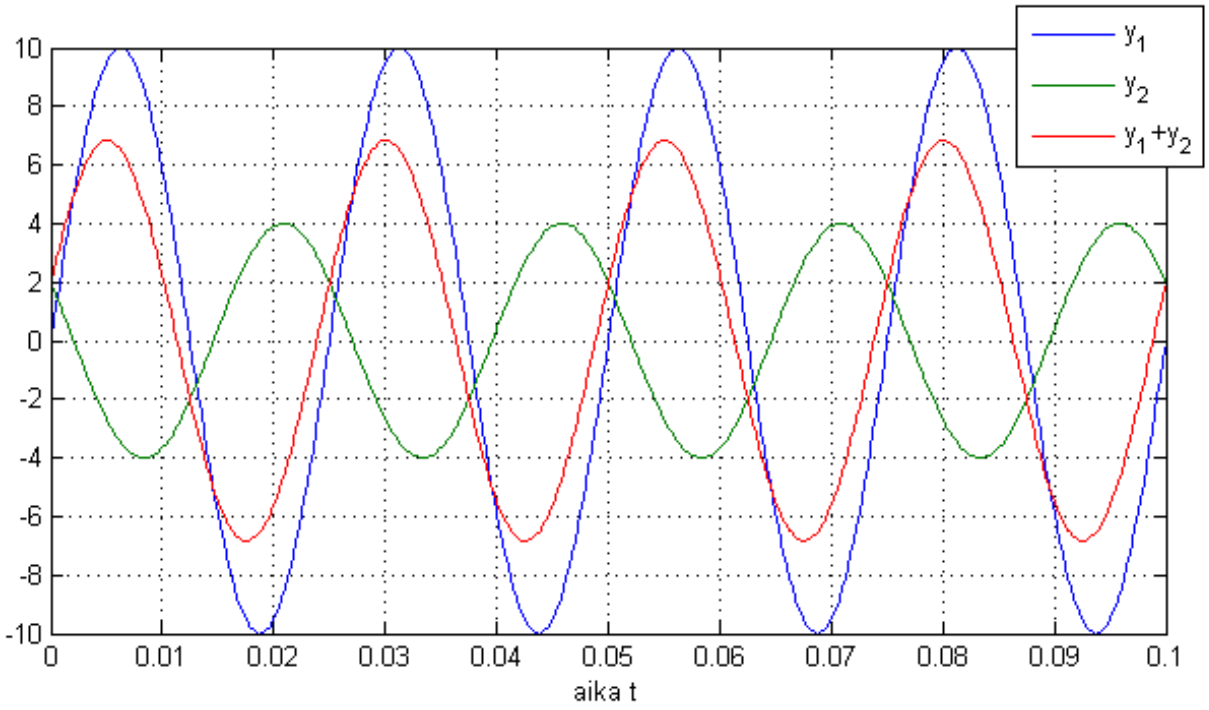
$$y_1 + y_2 = 10.8\sin(\omega t + 0.38)$$

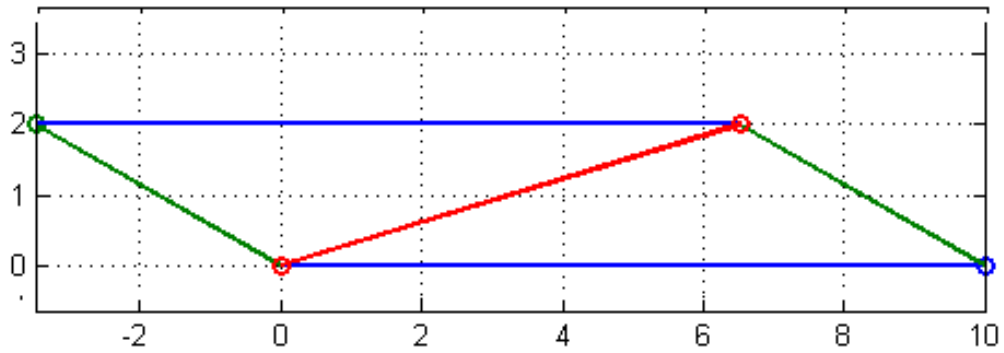




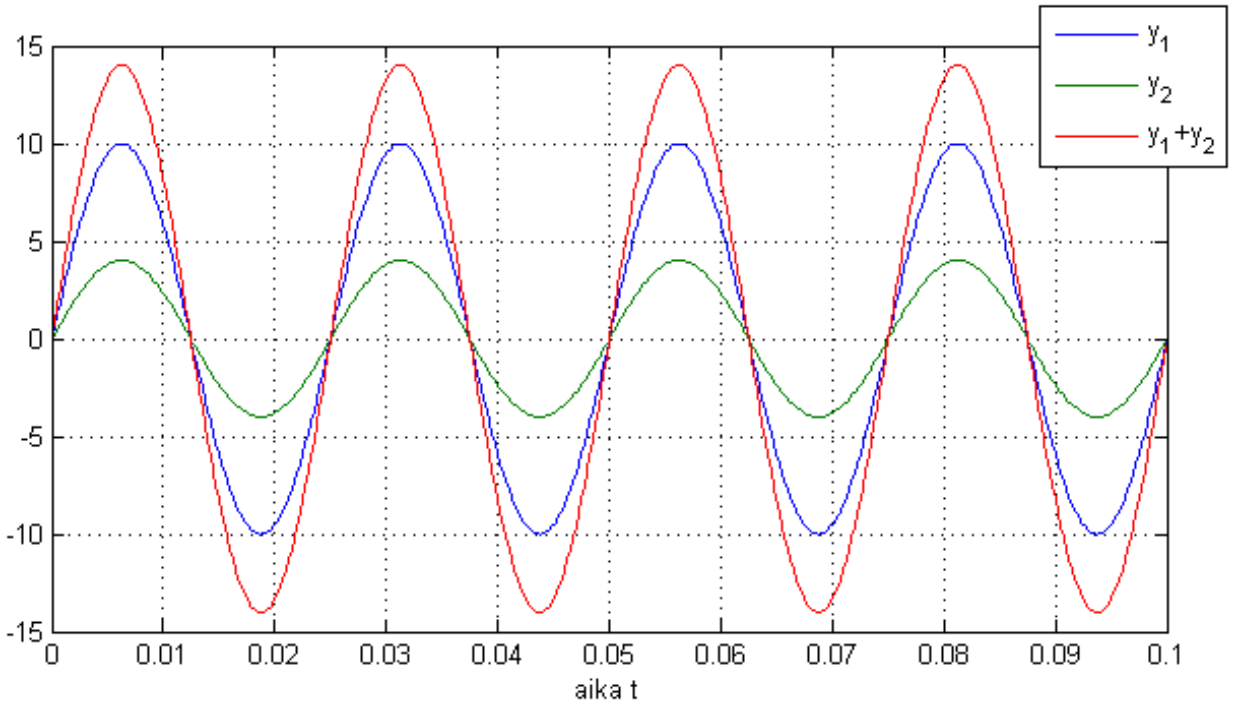
$$10\sin(\omega t) + 4\sin(\omega t + 5\pi/6)$$

$$= 6.8\sin(\omega t + 0.30)$$

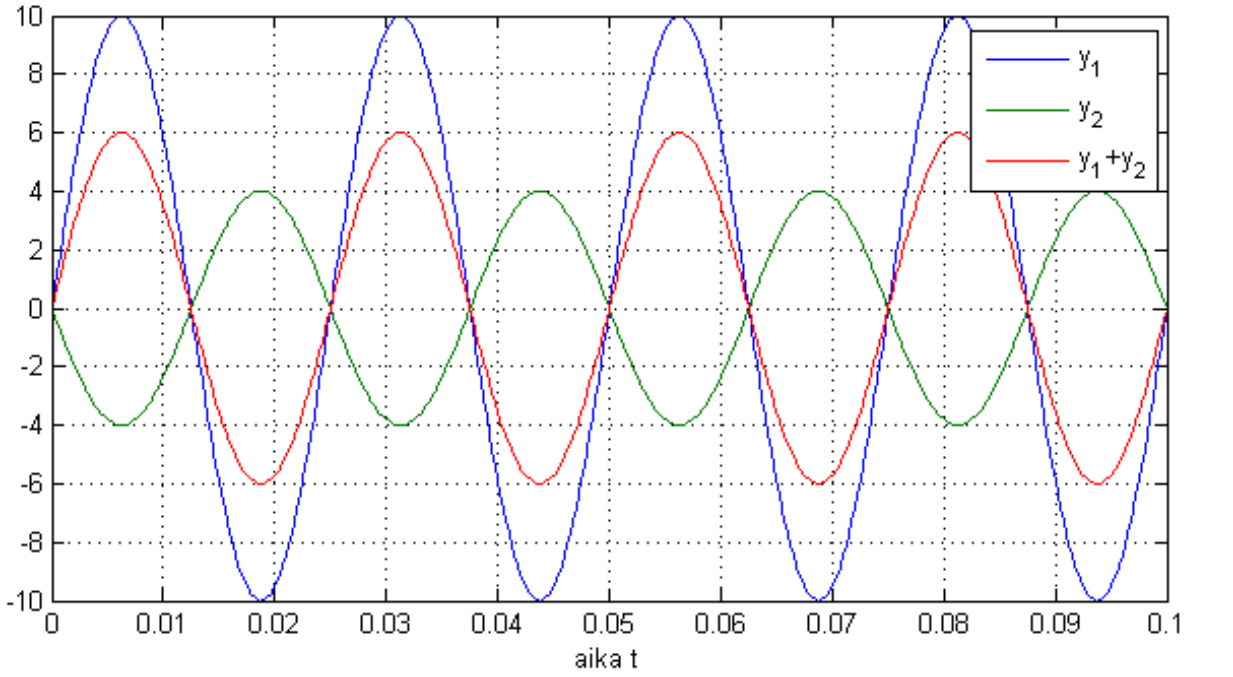




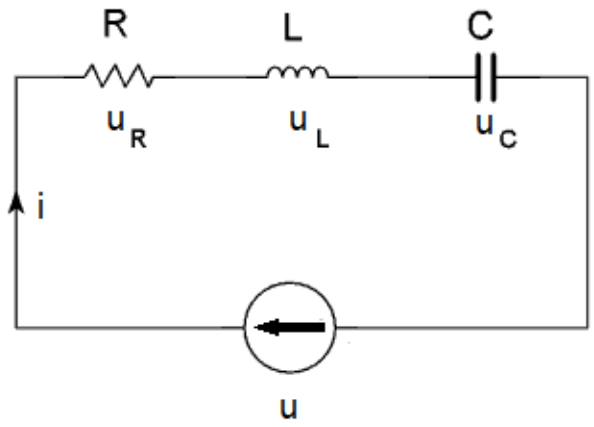
$$10\sin(\omega t) + 4\sin(\omega t) = 14\sin(\omega t)$$



$$10\sin(\omega t) + 4\sin(\omega t + \pi) = 6\sin(\omega t)$$



RLC-piiri:



jos virta $i = I \sin(\omega t)$, niin jännitteet

$$u_R = RI \sin(\omega t)$$

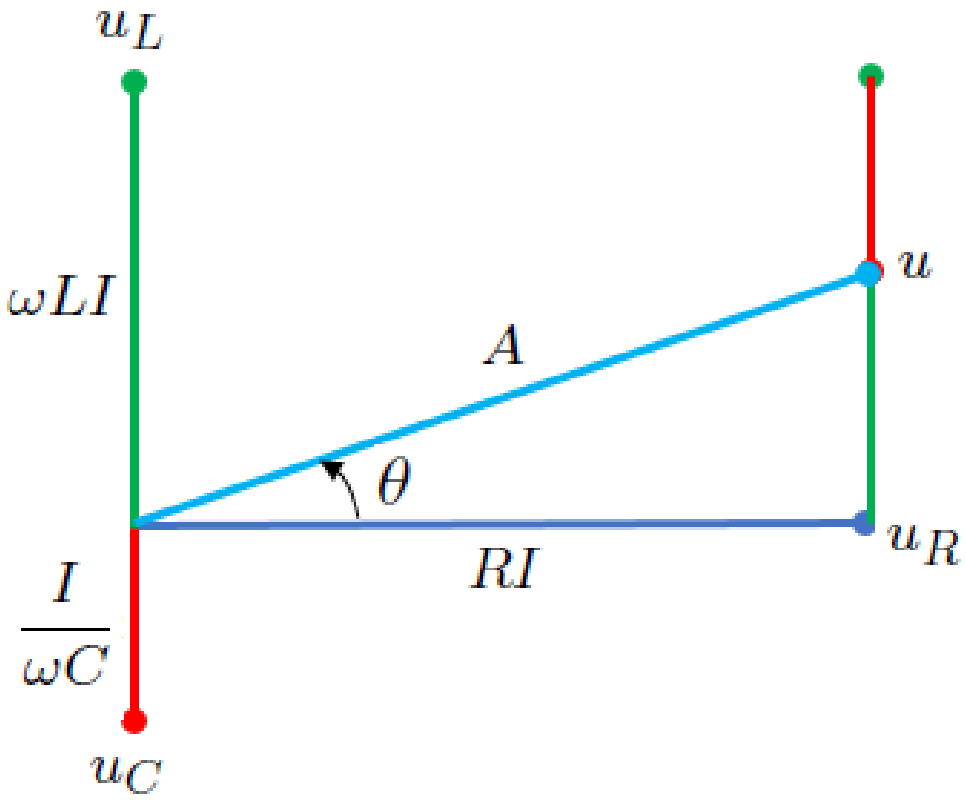
$$u_L = \omega LI \sin(\omega t + \pi/2)$$

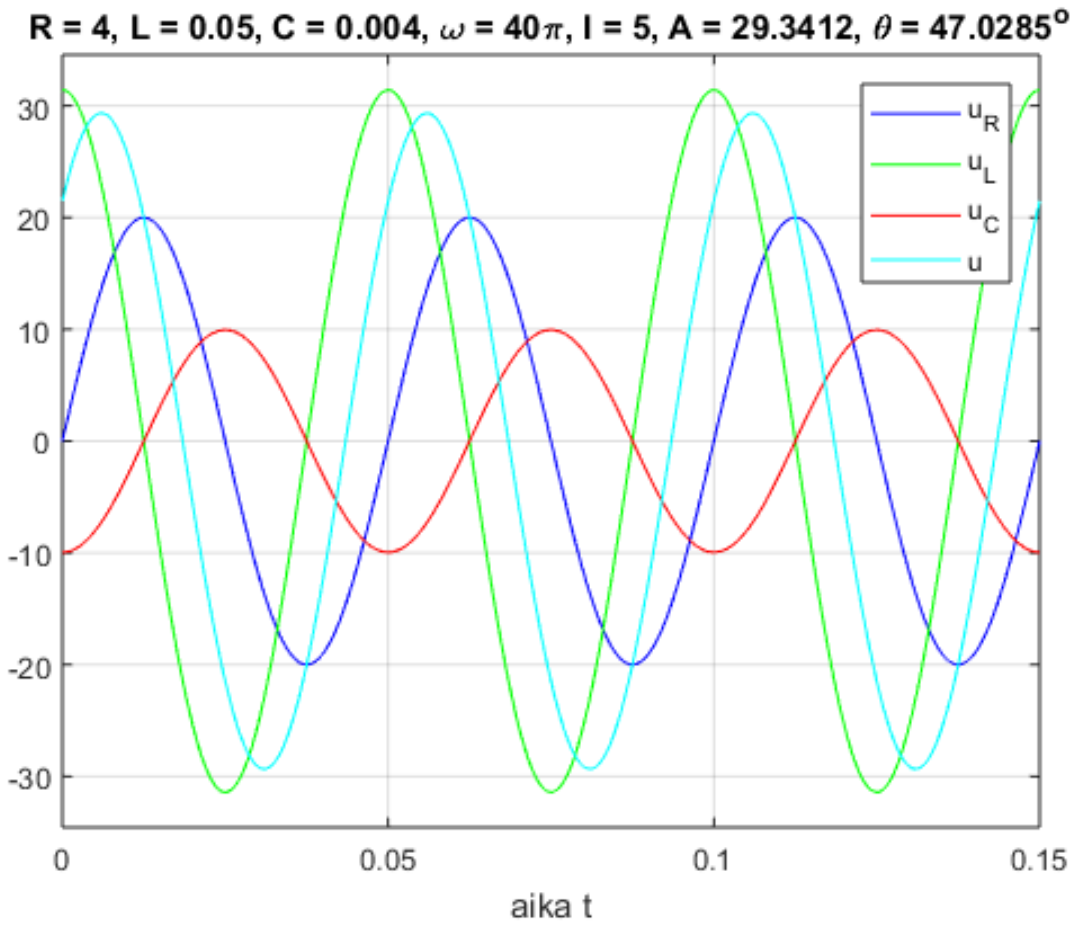
$$u_C = \frac{I}{\omega C} \sin(\omega t - \pi/2)$$

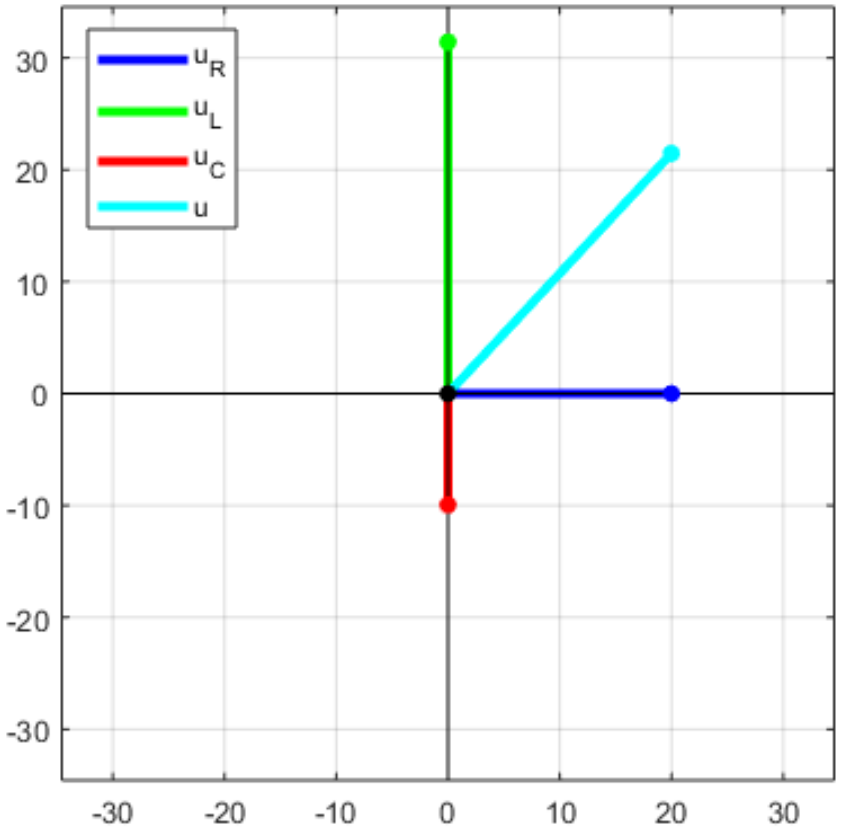
$$u = u_R + u_L + u_C = A\sin(\omega t + \theta)$$

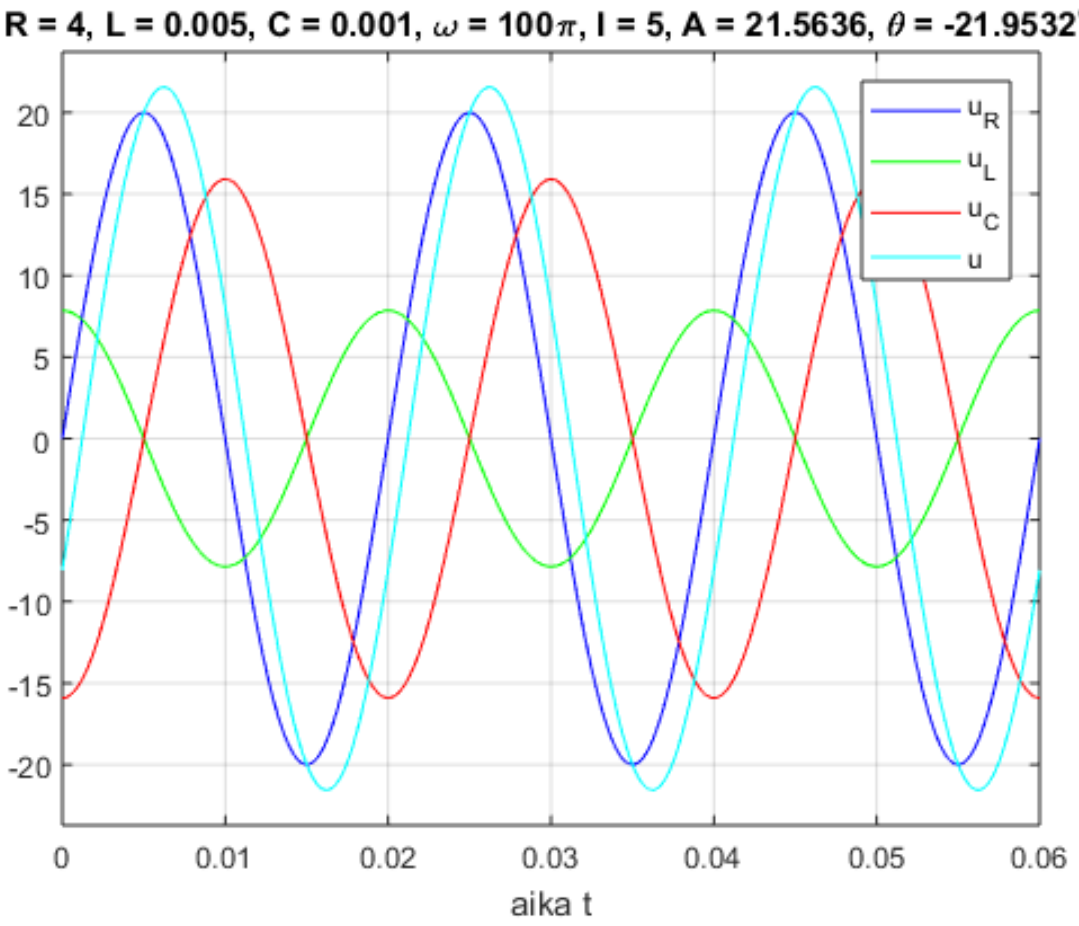
$$A = \sqrt{(RI)^2 + \left(\omega LI - \frac{I}{\omega C}\right)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}\, I$$

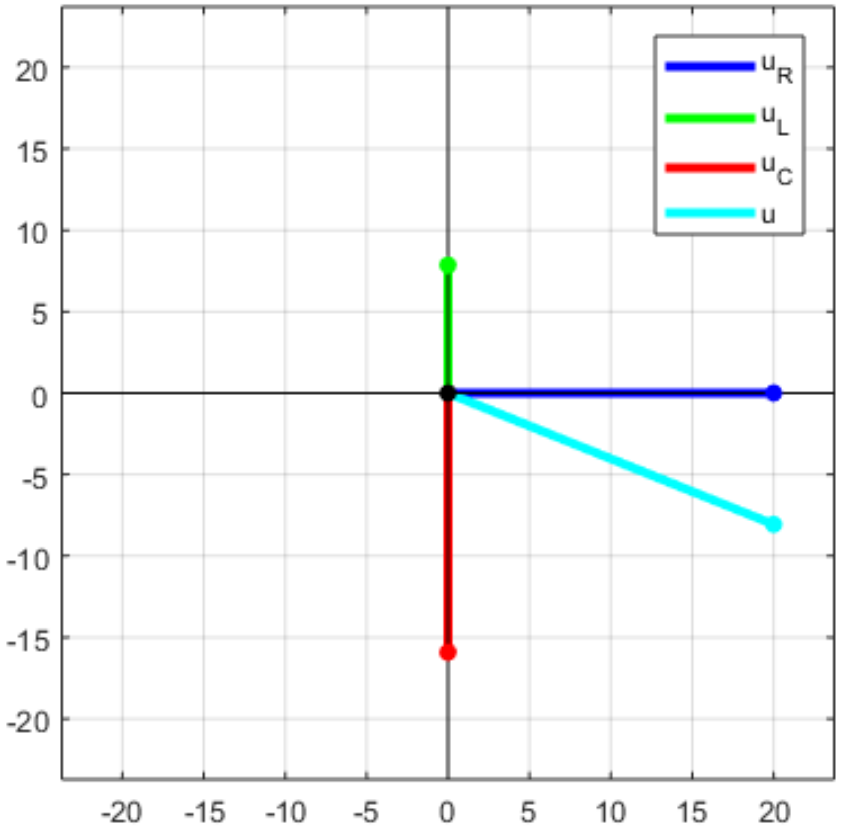
$$\theta = \operatorname{atan2}\left(\omega LI - \frac{I}{\omega C}, RI\right) = \operatorname{atan2}\left(\omega L - \frac{1}{\omega C}, R\right)$$




R = 4, L = 0.05, C = 0.004, ω = 40π, I = 5, A = 29.3412, θ = 47.0285°
30
20
10
0
-10
-20
-30
0
0.05
0.1
0.15
aika t
u_R
u_L
u_C
u


30
20
10
0
-10
-20
-30
-30
-20
-10
0
10
20
30
u_R
u_L
u_C
u


R = 4, L = 0.005, C = 0.001, ω = 100π, I = 5, A = 21.5636, θ = -21.9532°
u_R
u_L
u_C
u
aika t


u_R
u_L
u_C
u