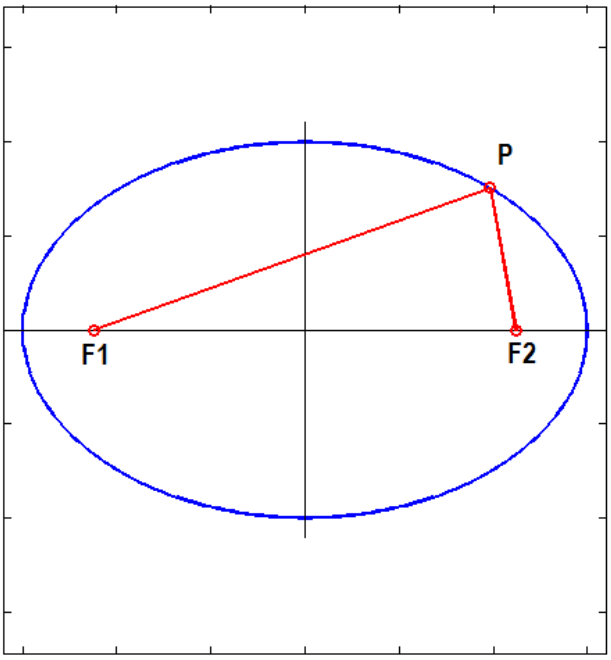
ELLIPSIN YHTÄLÖ



$$F_1 = -\sqrt{a^2 - b^2},\ F_2 = \sqrt{a^2 - b^2}$$

P:n koordinaatit x ja y

$$PF_1 + PF_2 = 2a \quad \leftrightarrow \quad \left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

$$\text{merkintä: } F = \sqrt{a^2 - b^2}$$

$$PF_1 + PF_2 = 2a$$

$$\sqrt{(x+F)^2 + y^2} + \sqrt{(x-F)^2 + y^2} = 2a$$

$$\sqrt{(x+F)^2 + y^2} = 2a - \sqrt{(x-F)^2 + y^2} \qquad |(\,)^2$$

$$(x+F)^2 + y^2 = 4a^2 - 4a\sqrt{(x-F)^2 + y^2} + (x-F)^2 + y^2$$

$$x^2 + 2xF^2 + F^2 + y^2 = 4a^2 - 4a\sqrt{(x-F)^2 + y^2} + x^2 - 2xF + F^2 + y^2$$

$$4xF - 4a^2 = -4a\sqrt{(x-F)^2 + y^2} \qquad | : 4,\ (\,)^2$$

$$x^2F^2 - 2xFa^2 + a^4 = a^2(x^2 - 2xF + F^2 + y^2)$$

$$x^2(a^2 - b^2) - 2xFa^2 + a^4 = a^2x^2 - 2xFa^2 + a^2(a^2 - b^2) + a^2y^2$$

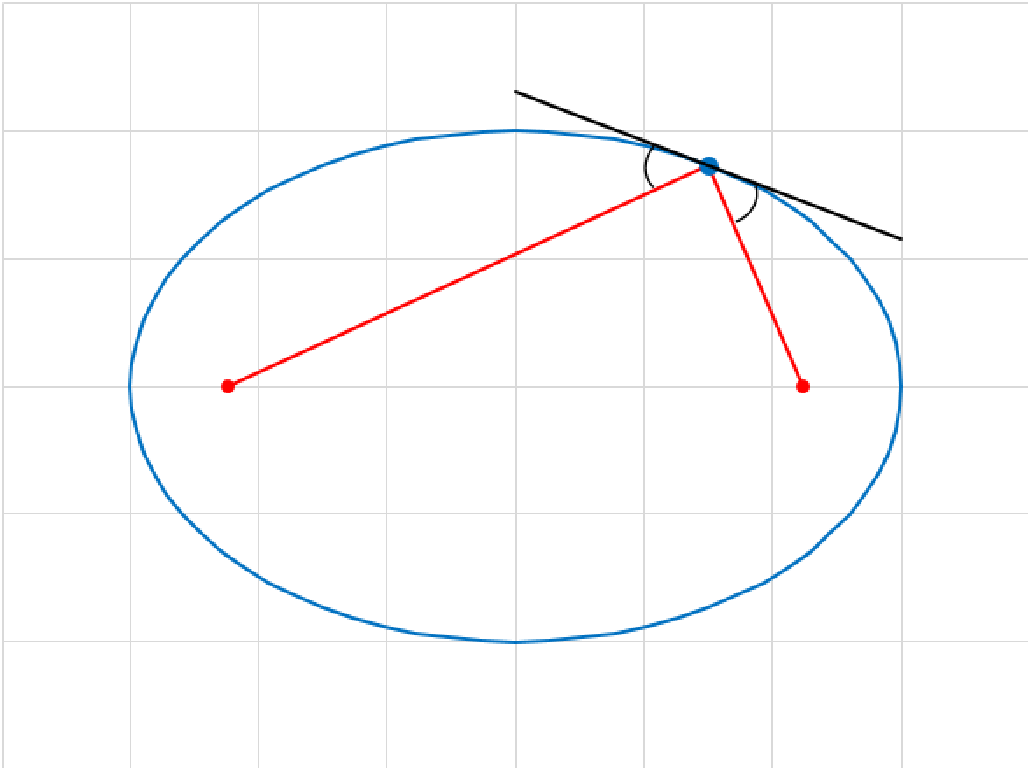
$$b^2x^2 + a^2y^2 = a^2b^2$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

ELLIPSIN TANGENTTI

Ellipsin pisteen x_0, y_0 kautta kulkevan tangentin kulmakerroin

$$k = -\frac{x_0 y_0}{a^2 - x_0^2}$$



Syy: etsitään k niin, että pisteen x_0, y_0 kautta kulkevalla suoralla $y = kx + y_0 - kx_0$ ja ellipsillä

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

on vain yksi leikkauspiste.

$$\frac{x^2}{a^2} + \frac{(kx + y_0 - kx_0)^2}{b^2} = 1 | \cdot a^2 b^2$$

$$b^2 x^2 + a^2 (kx + y_0 - kx_0)^2 = a^2 b^2$$

$$b^2 x^2 + a^2 (k^2 x^2 + 2k(y_0 - kx_0)x + (y_0 - kx_0)^2) = a^2 b^2$$

$$(b^2 + a^2 k^2) x^2 + 2a^2 k (y_0 - kx_0) x + a^2 (y_0 - kx_0)^2 - a^2 b^2 = 0$$

Yksi ratkaisu, kun

$$(2a^2k(y_0 - kx_0))^2 - 4(b^2 + a^2k^2)(a^2(y_0 - kx_0)^2 - a^2b^2) = 0$$

$$4a^4k^2(y_0-kx_0)^2-4b^2a^2(y_0-kx_0)^2+4a^2b^4-4a^4k^2(y_0-kx_0)^2+4a^4b^2k^2 = 0 \quad | : 4a^2b^2$$

$$-(y_0 - kx_0)^2 + b^2 + a^2k^2 = 0$$

$$(a^2 - x_0^2)k^2 + 2x_0y_0k + b^2 - y_0^2 = 0$$

$$k = \frac{-2x_0y_0 \pm \sqrt{4x_0^2y_0^2 - 4(a^2 - x_0^2)(b^2 - y_0^2)}}{2(a^2 - x_0^2)}$$

$$k = \frac{-2x_0y_0 \pm \sqrt{-4a^2b^2 + 4a^2y_0^2 + 4b^2x_0^2}}{2(a^2 - x_0^2)}$$

Koska x_0, y_0 on ellipsin piste, niin

$$\frac{x_0^2}{a^2} + \frac{y_0^2}{b^2} = 1$$

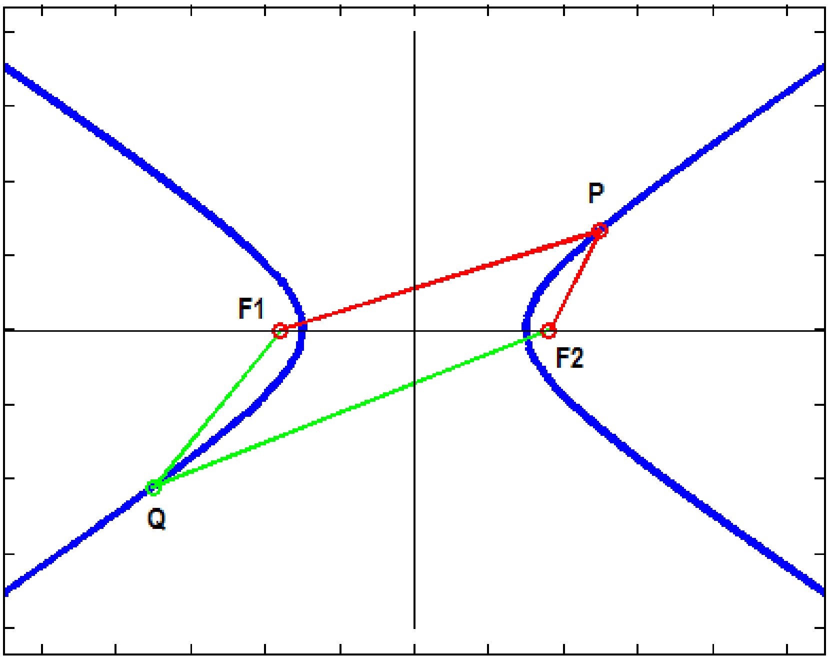
eli

$$b^2x_0^2 + a^2y_0^2 = a^2b^2$$

joten neliöjuuren sisus $= 0$ ja

$$k = -\frac{x_0y_0}{a^2 - x_0^2}$$

HYPERBELIN YHTÄLÖ



$$F1 = -\sqrt{a^2 + b^2},\ F2 = \sqrt{a^2 + b^2}$$

$$F = \sqrt{a^2 + b^2}$$

$$PF1 - PF2 = 2a$$

$$\sqrt{(x+F)^2 + y^2} - \sqrt{(x-F)^2 + y^2} = 2a$$

$$\sqrt{(x+F)^2 + y^2} = 2a + \sqrt{(x-F)^2 + y^2} \qquad |(\,)^2$$

$$(x+F)^2 + y^2 = 4a^2 + 4a\sqrt{(x-F)^2 + y^2} + (x-F)^2 + y^2$$

$$x^2 + 2xF^2 + F^2 + y^2 = 4a^2 + 4a\sqrt{(x-F)^2 + y^2} + x^2 - 2xF + F^2 + y^2$$

$$4xF - 4a^2 = 4a\sqrt{(x-F)^2 + y^2} \qquad | : 4,\ (\,)^2$$

$$x^2F^2 - 2xFa^2 + a^4 = a^2(x^2 - 2xF + F^2 + y^2)$$

$$x^2(a^2 + b^2) - 2xFa^2 + a^4 = a^2x^2 - 2xFa^2 + a^2(a^2 + b^2) + a^2y^2$$

$$b^2x^2 - a^2y^2 = a^2b^2$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Vastaavasti,

$$QF2 - QF1 = 2a$$

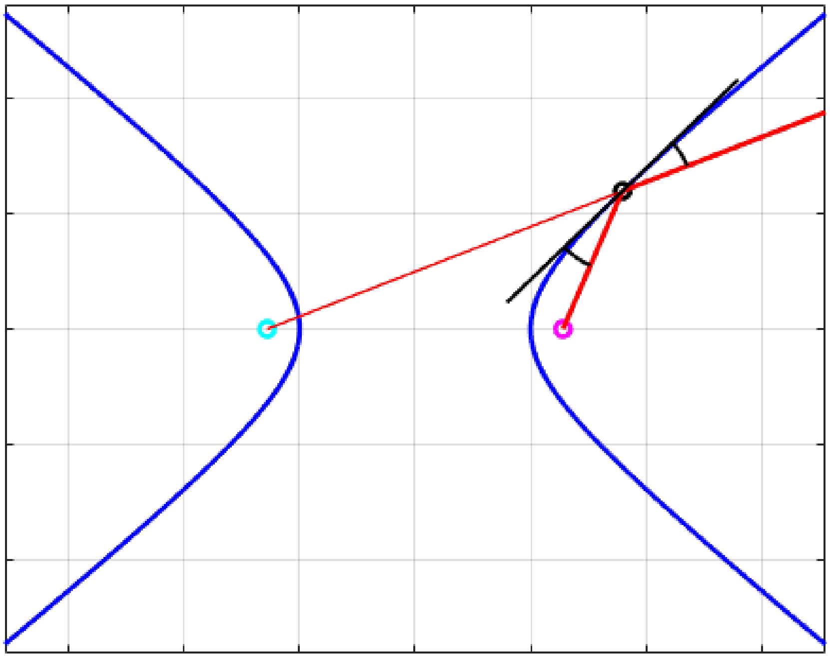
$$\sqrt{(x-F)^2 + y^2} - \sqrt{(x+F)^2 + y^2} = 2a$$

$$\rightarrow \qquad \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

HYPERBELIN TANGENTTI

Hyperbelin pisteen x_0, y_0 kautta kulkevan tangentin kulmakerroin

$$k = -\frac{x_0 y_0}{a^2 - x_0^2}$$



Syy: etsitään k niin, että pisteen x_0, y_0 kautta kulkevalla suoralla $y = kx + y_0 - kx_0$ ja hyperbelillä

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

on vain yksi leikkauspiste.

$$\frac{x^2}{a^2} - \frac{(kx + y_0 - kx_0)^2}{b^2} = 1 | \cdot a^2 b^2$$

$$b^2 x^2 - a^2 (kx + y_0 - kx_0)^2 = a^2 b^2$$

$$b^2 x^2 - a^2 (k^2 x^2 + 2k(y_0 - kx_0)x + (y_0 - kx_0)^2) = a^2 b^2$$

$$(b^2 - a^2 k^2) x^2 - 2a^2 k(y_0 - kx_0)x - a^2 (y_0 - kx_0)^2 - a^2 b^2 = 0$$

Yksi ratkaisu, kun

$$(2a^2k(y_0 - kx_0))^2 + 4(b^2 - a^2k^2)(a^2(y_0 - kx_0)^2 + a^2b^2) = 0$$

$$4a^4k^2(y_0-kx_0)^2+4b^2a^2(y_0-kx_0)^2+4a^2b^4-4a^4k^2(y_0-kx_0)^2-4a^4b^2k^2 = 0 \quad | : 4a^2b^2$$

$$(y_0 - kx_0)^2 + b^2 - a^2k^2 = 0$$

$$(-a^2 + x_0^2)k^2 - 2x_0y_0k + b^2 + y_0^2 = 0$$

$$k = \frac{2x_0y_0 \pm \sqrt{4x_0^2y_0^2 - 4(-a^2 + x_0^2)(b^2 + y_0^2)}}{2(-a^2 + x_0^2)}$$

$$k = \frac{2x_0y_0 \pm \sqrt{4a^2b^2 + 4a^2y_0^2 - 4b^2x_0^2}}{2(-a^2 + x_0^2)}$$

Koska x_0, y_0 on hyperbelin piste, niin

$$\frac{x_0^2}{a^2} - \frac{y_0^2}{b^2} = 1$$

eli

$$b^2x_0^2 - a^2y_0^2 = a^2b^2$$

joten neliöjuuren sisus $= 0$ ja

$$k = -\frac{x_0y_0}{a^2 - x_0^2}$$