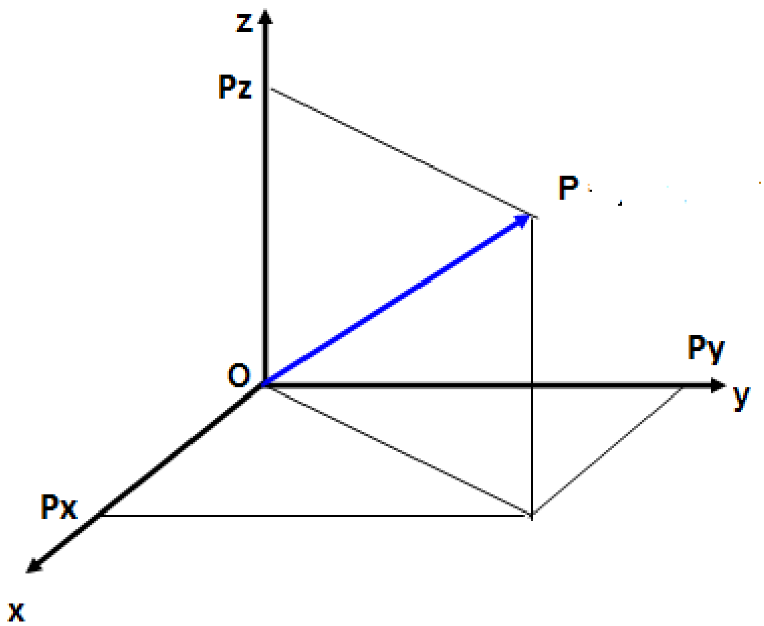
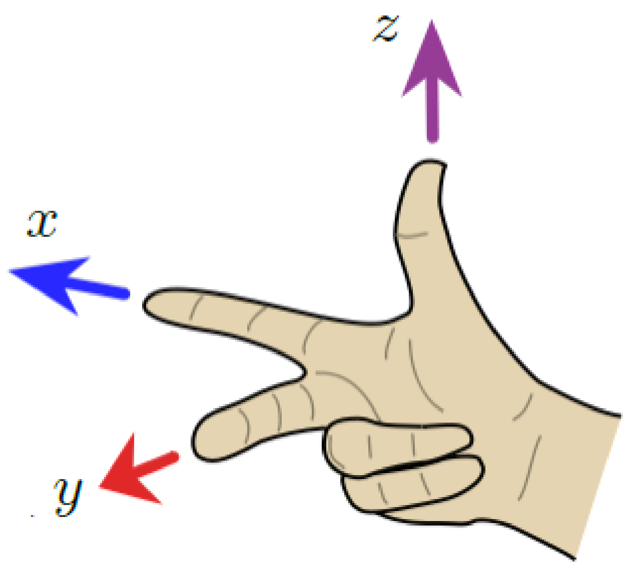
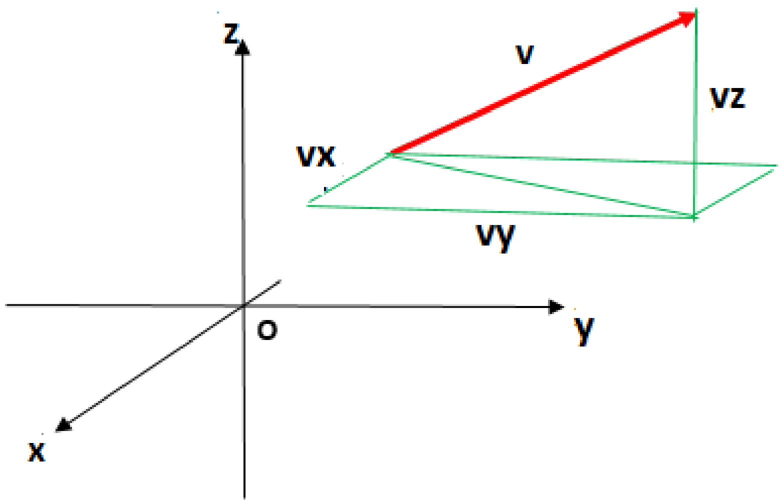
Suorakulmainen koordinaatisto

oikeakätinen järjestys







Origo $O = [0, 0, 0]$

Piste $P = [Px, Py, Pz]$

koordinaatit Px, Py, Pz

Vektori $\mathbf{v} = [vx, vy, vz]$

komponentit vx, vy, vz

Vektorin $\mathbf{v}$ pituus

$\|\mathbf{v}\| = \sqrt{(vx)^2 + (vy)^2 + (vz)^2}$

MATLAB/Octave: norm($\mathbf{v}$)

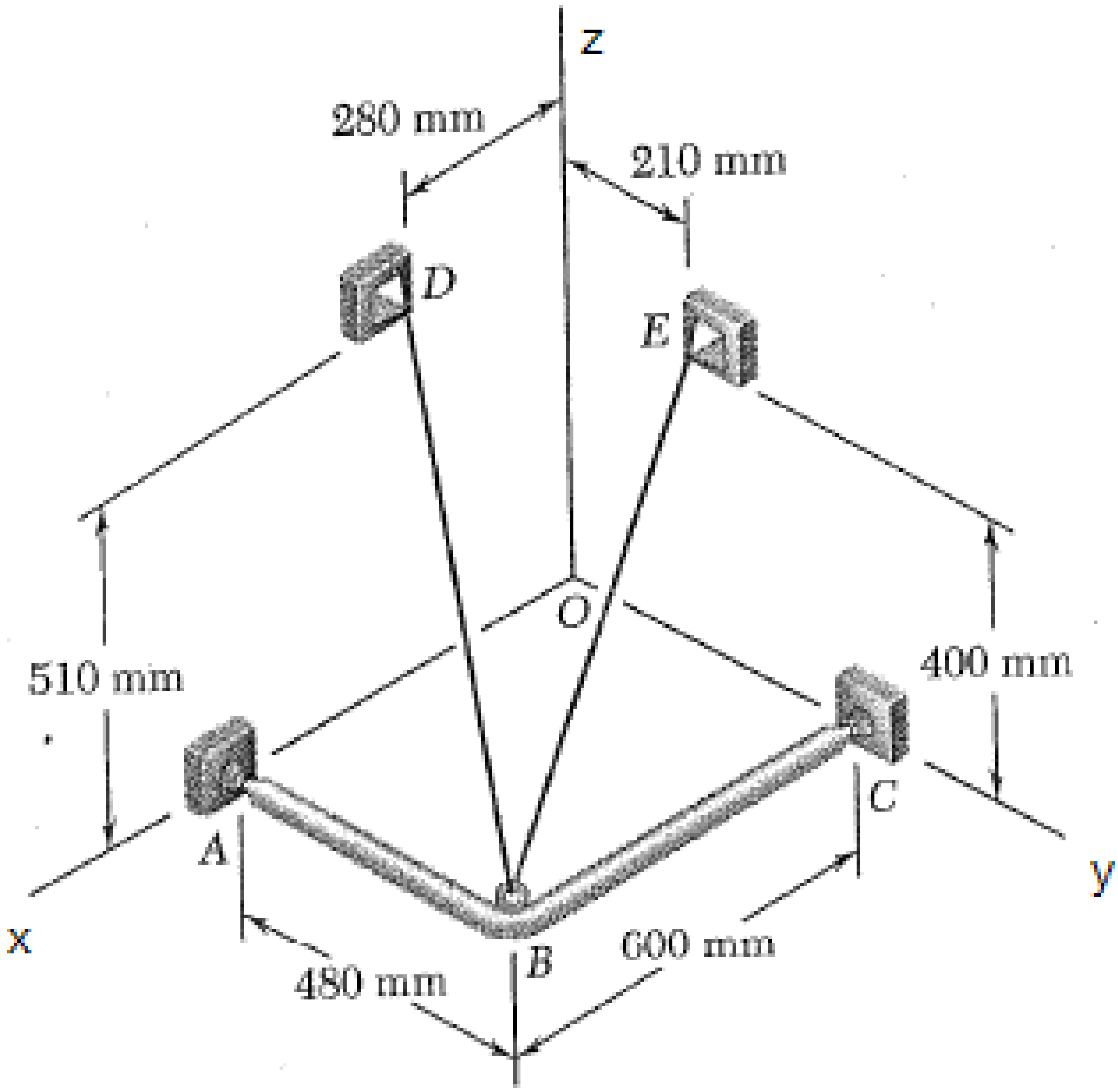
Vektori pisteestä P pisteeseen Q

$\mathbf{PQ} = Q - P = [Qx - Px, Qy - Py, Pz - Qz]$

$PQ = \|\mathbf{PQ}\| = P$:n ja Q:n välinen etäisyys

$= \sqrt{(Qx - Px)^2 + (Qy - Py)^2 + (Qz - Pz)^2}$

Esim.



$B = [600, 480, 0]$, $D = [280, 0, 510]$,
$E = [0, 210, 400]$

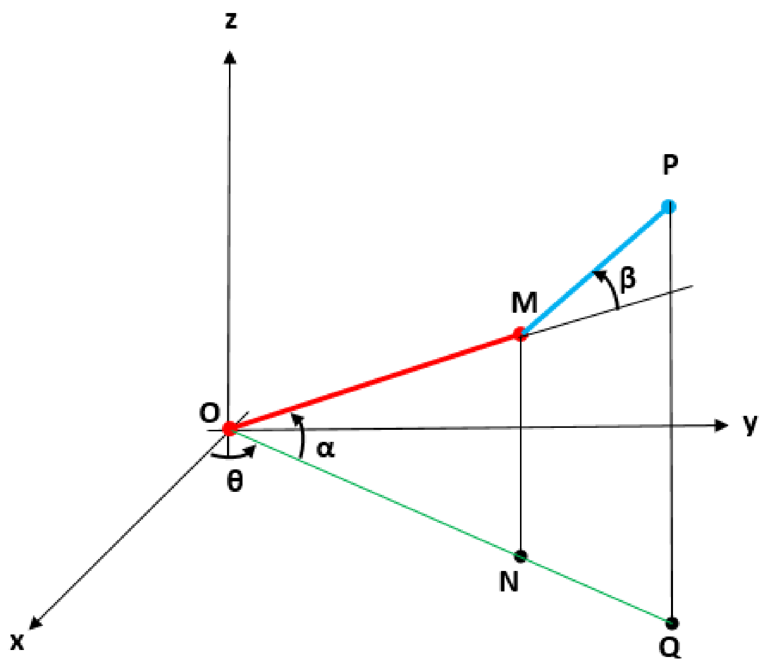
$$\mathbf{BD} = D - B = [-320, -480, 510]$$

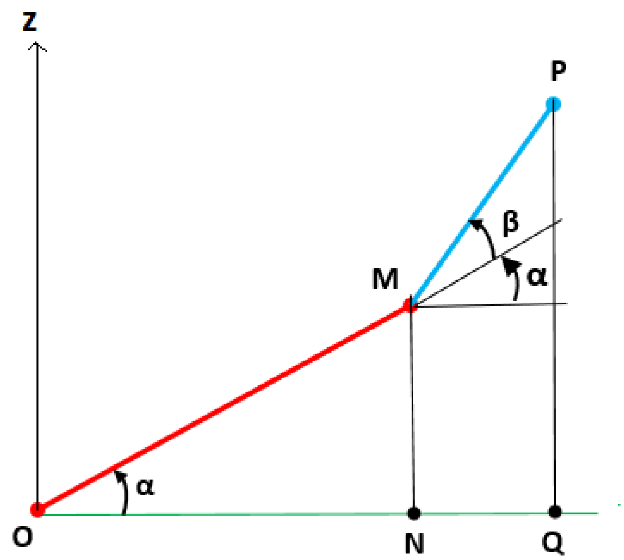
$$\mathbf{BE} = E - B = [-600, -270, 400]$$

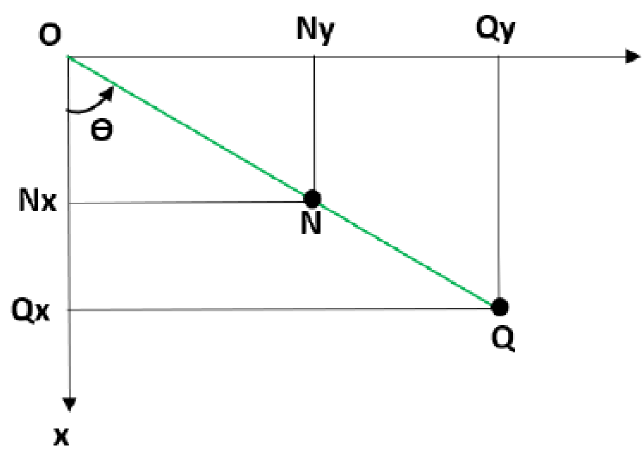
$$\begin{aligned} \|\mathbf{BD}\| &= \sqrt{(-320)^2 + (-480)^2 + 510^2} \\ &= 770 \end{aligned}$$

$$\begin{aligned} \|\mathbf{BE}\| &= \sqrt{(-600)^2 + (-270)^2 + 400^2} \\ &= 770 \end{aligned}$$

Esim: 3D-käsivarsi, varsien pituudet OM ja MP







Suora kinematiikka: $\theta, \alpha, \beta \to Px, Py, Pz$

$$Mz = OM \sin(\alpha)$$

$$ON = OM \cos(\alpha)$$

$$Mx = Nx = ON \cos(\theta)$$

$$My = Ny = ON \sin(\theta)$$

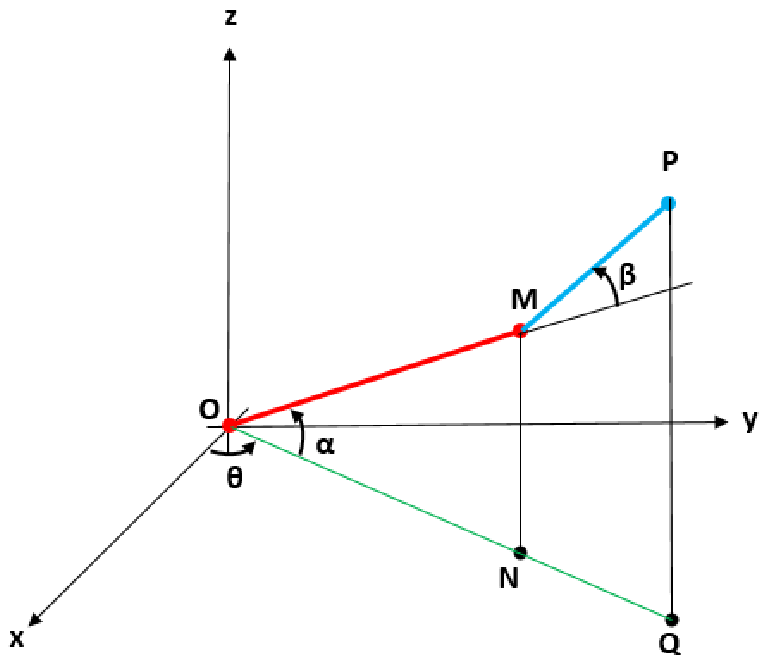
$$Pz = Mz + MP \sin(\alpha + \beta)$$

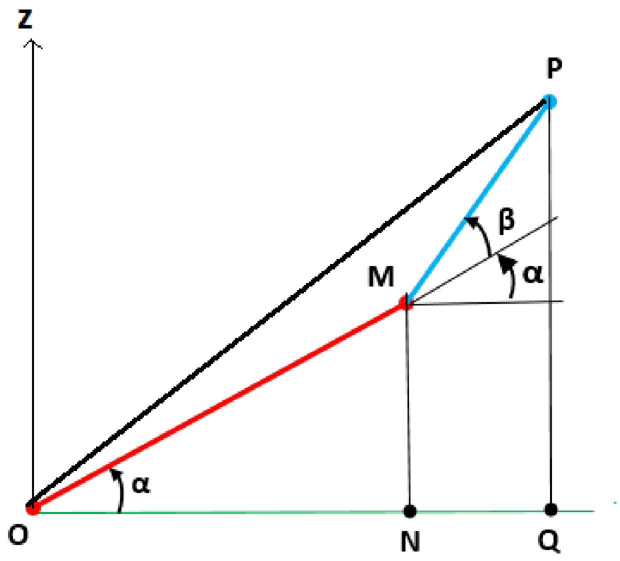
$$NQ = MP \cos(\alpha + \beta)$$

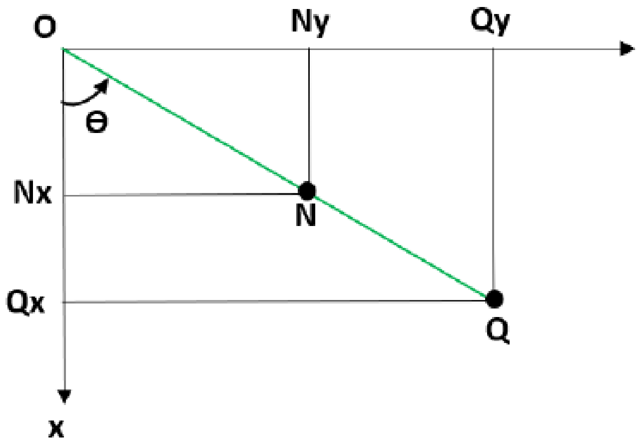
$$OQ = ON + NQ$$

$$Px = Qx = OQ \cos(\theta)$$

$$Py = Qy = OQ \sin(\theta)$$


z
P
M
β
O
y
α
θ
N
x
Q


z
P
β
M
α
α
O
N
Q


O
Ny
Qy
Θ
Nx
N
Qx
Q
x

Käänteinen kinematiikka:

$Px, Py, Pz \rightarrow \theta, \alpha, \beta$

$$Qx = Px, Qy = Py,\ \theta = \mathrm{atan2}(Qy, Qx)$$

$$OQ = \sqrt{Qx^2 + Qy^2}$$

$$\angle POQ = \tan^{-1}(Pz/OQ)$$

$$OP = \sqrt{Px^2 + Py^2 + Pz^2}$$

$$\angle POM = \cos^{-1}\left(\frac{OP^2 + OM^2 - MP^2}{2 \cdot OP \cdot OM}\right)$$

$$\alpha = \angle POQ - \angle POM$$

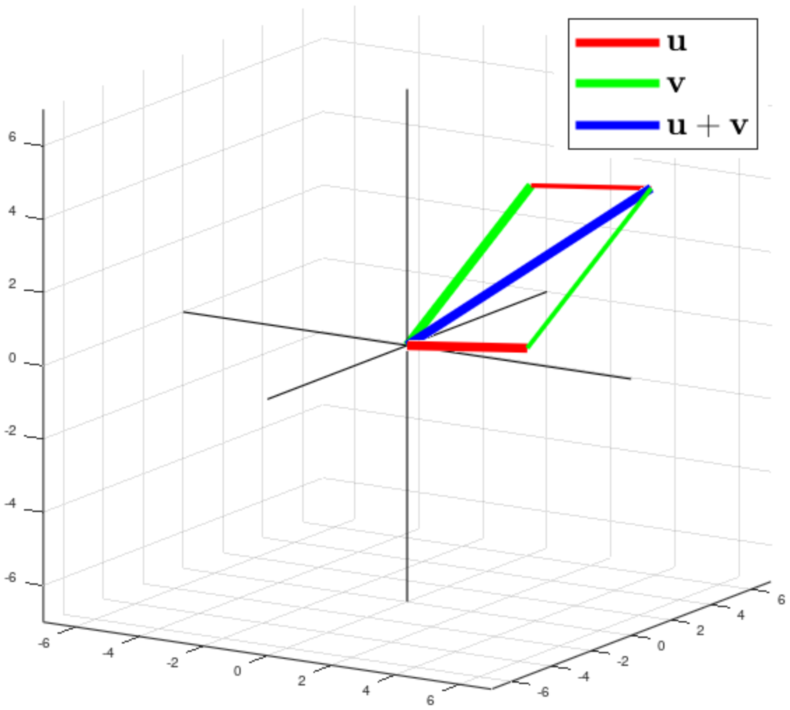
$$\angle OMP = \cos^{-1}\left(\frac{OM^2 + MP^2 - OP^2}{2 \cdot OM \cdot MP}\right)$$

$$\beta = 180^\circ - \angle OMP$$

Laskutoimitukset

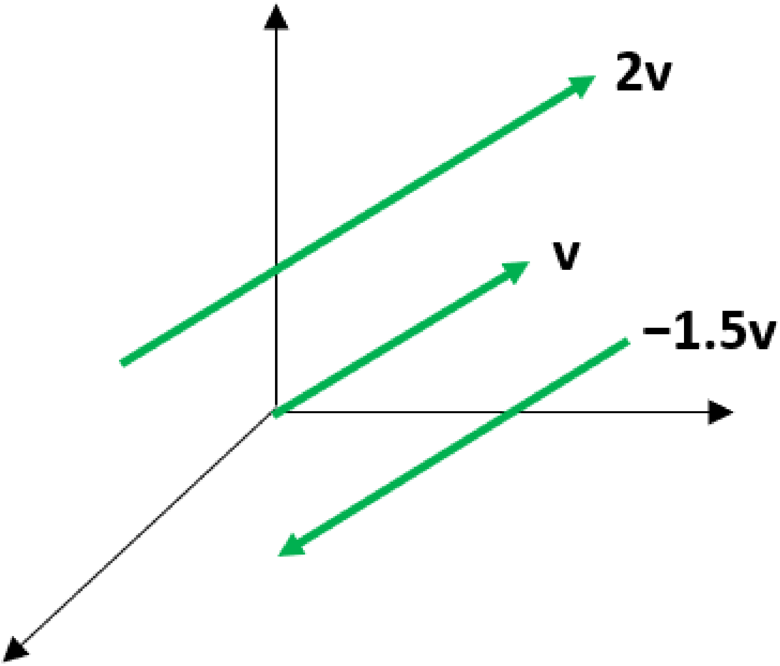
Yhteenlasku:

$$\mathbf{u} + \mathbf{v} = [ux + vx,\, uy + vy, uz + vz]$$



Luvulla kertominen:

$$t\mathbf{v} = t * \mathbf{v} = [t * vx,\ t * vy,\ t * vz]$$



$t\mathbf{v}$ on $\mathbf{v}$:n kanssa

samansuuntainen, jos $t > 0$

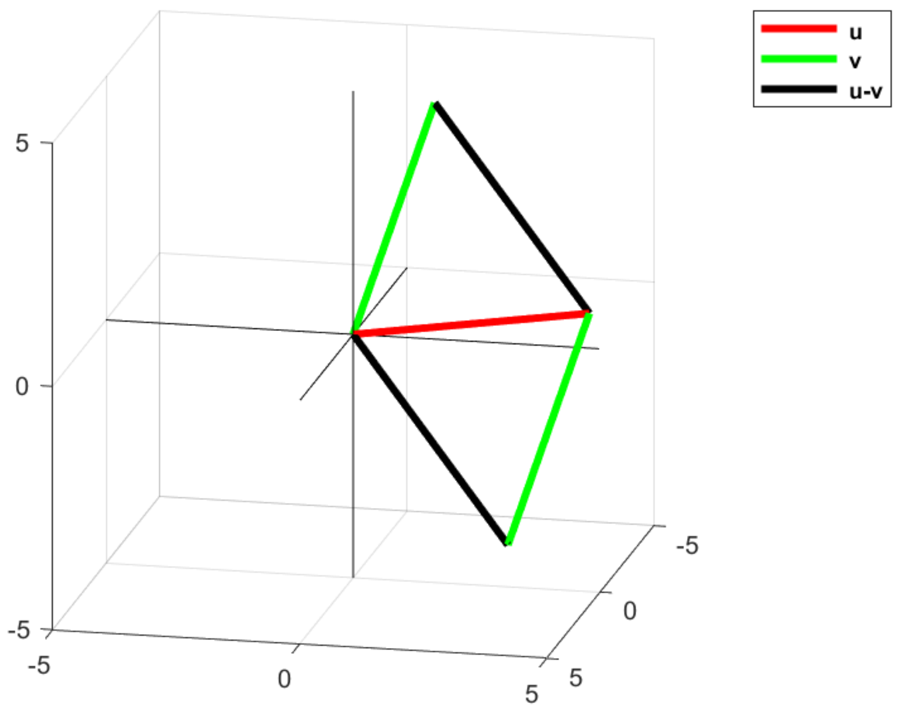
vastakkaisuuntainen, jos $t < 0$

pituus $\|t\mathbf{v}\| = |t| * \|\mathbf{v}\|$

Huom: vähennyslasku

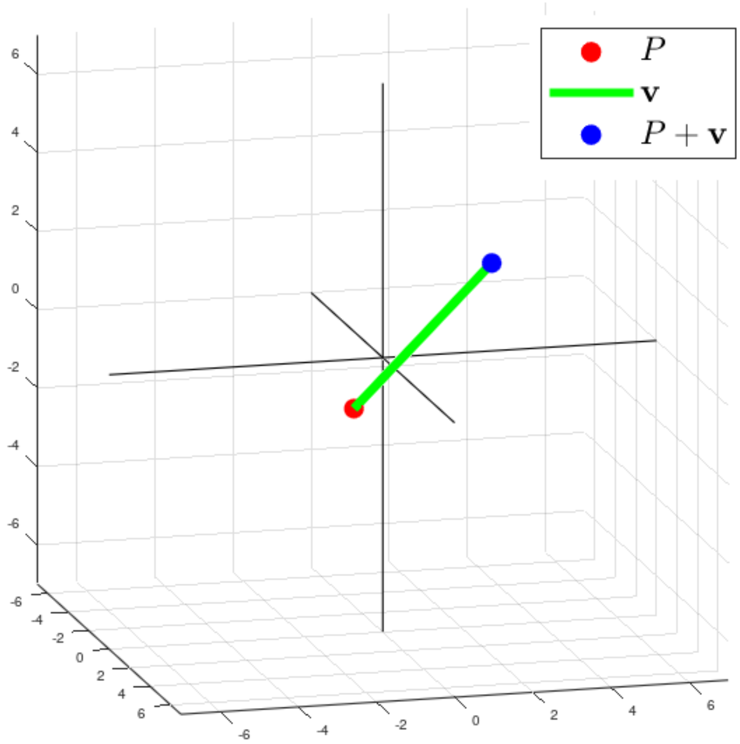
$$\mathbf{u} - \mathbf{v} = [ux - vx, uy - vy, uz - vz]$$

$$= \mathbf{u} + (-\mathbf{v})$$



Huom: piste + vektori = piste

$$P + \mathbf{v} = [Px + vx, Py + vy, Pz + vz]$$

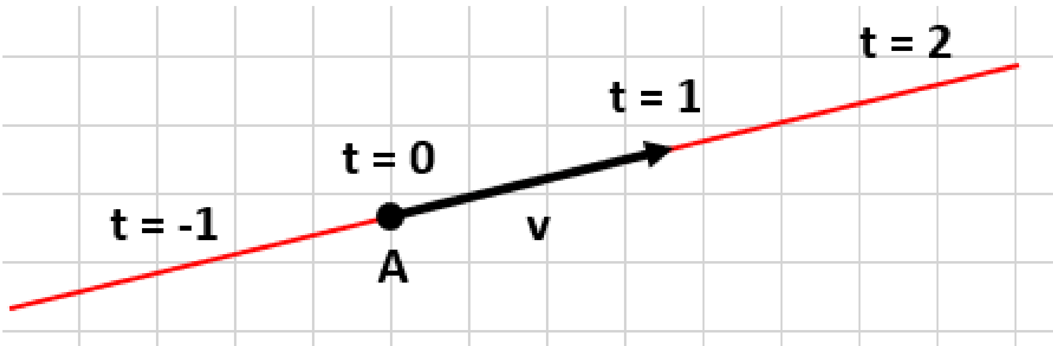


Esim: Suoran parametrimuoto

Pisteen $A = [Ax, Ay, Az]$ kautta kulkevalla, vektorin $\mathbf{v} = [vx, vy, vz]$ suuntaisella suoralla ovat pisteet

$$P = A + t * \mathbf{v}$$

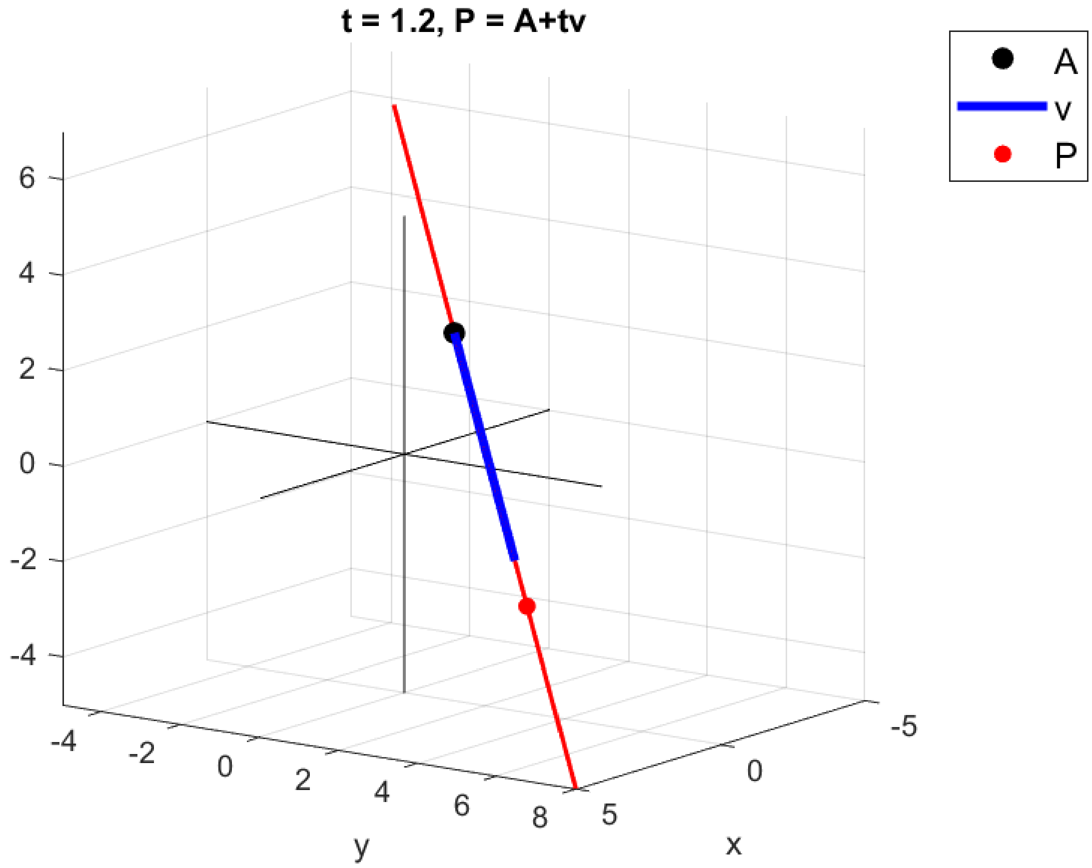
$$= [Ax + t * vx, Ay + t * vy, Az + t * vz]$$



Huom: Pisteiden A ja B kautta kulkeva suora: $\mathbf{v} = \mathbf{AB}$

Esim: $A = [1, 2, 3]$, $\mathbf{v} = [2, 3, -4]$

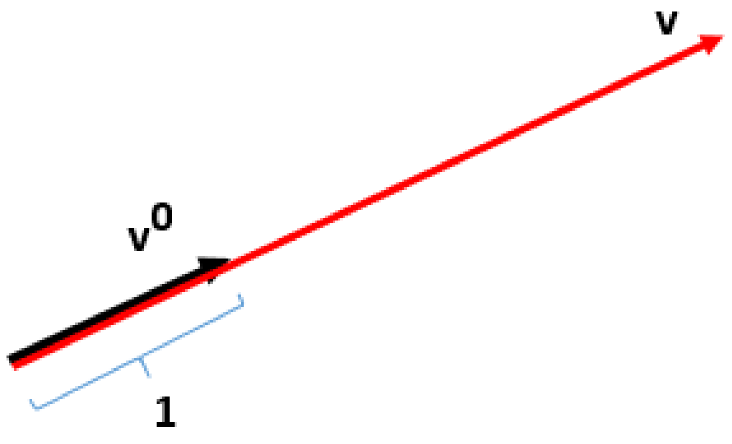
$$\begin{aligned} P = &\ A + t * \mathbf{v} \\ = &\ [1, 2, 3] + t * [2, 3, -4] \\ = &\ [1 + 2t, 2 + 3t, 3 - 4t] \end{aligned}$$



Yksikkövektori

Vektorin $\mathbf{v} = [vx, vy, vz]$ suuntainen yksikkövektori $\mathbf{v}^0$ on $\mathbf{v}$:n suuntainen ja ykkösen pituinen vektori eli $\mathbf{v}$ jaettuna pituudellaan,

$$\mathbf{v}^0 = \frac{\mathbf{v}}{\|\mathbf{v}\|} = \left[\frac{vx}{\|\mathbf{v}\|}, \frac{vy}{\|\mathbf{v}\|}, \frac{vz}{\|\mathbf{v}\|}\right]$$



MATLAB/Octave: $\mathbf{v0} = \mathbf{v}/\text{norm}(\mathbf{v})$

Esim: jos $\mathbf{v} = [1, 2, 3]$, niin

$$\mathbf{v}^0 = \left[\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right] \approx [0.267, 0.534, 0.802]$$

Yksikkövektorin avulla on helppo muodostaa annetun vektorin suuntainen ja halutun pituinen vektori: kerrotaan yksikkövektori halutulla pituudella

Esimerkiksi, $\mathbf{v} = [1, 2, 3]$:n suuntainen vektori, jonka pituus on 2

$$2 * \mathbf{v}^0 = 2 * \left[\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right] \approx [0.53, 1.07, 1.60]$$

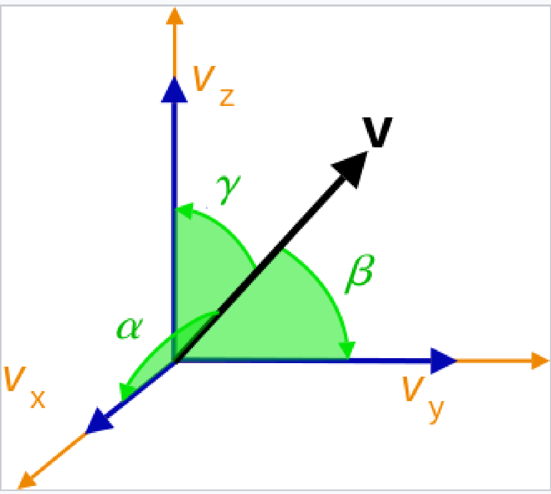
Huom: Yksikkövektorin

$$\mathbf{v}^0 = \frac{\mathbf{v}}{\|\mathbf{v}\|} = \left[\frac{vx}{\|\mathbf{v}\|}, \frac{vy}{\|\mathbf{v}\|}, \frac{vz}{\|\mathbf{v}\|}\right]$$

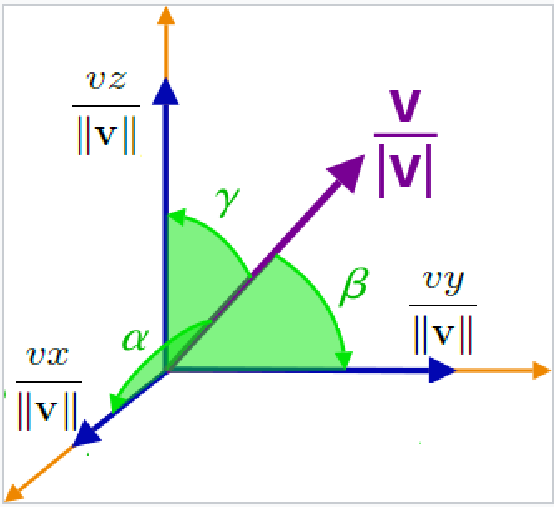
komponentit ovat vektorin $\mathbf{v}$ **suuntako-sinit** (direction cosine)

$$\frac{vx}{\|\mathbf{v}\|} = \cos(\alpha), \frac{vy}{\|\mathbf{v}\|} = \cos(\beta), \frac{vz}{\|\mathbf{v}\|} = \cos(\gamma)$$

missä α, β, γ ovat $\mathbf{v}$:n ja koordinaattiak-seleiden väliset kulmat

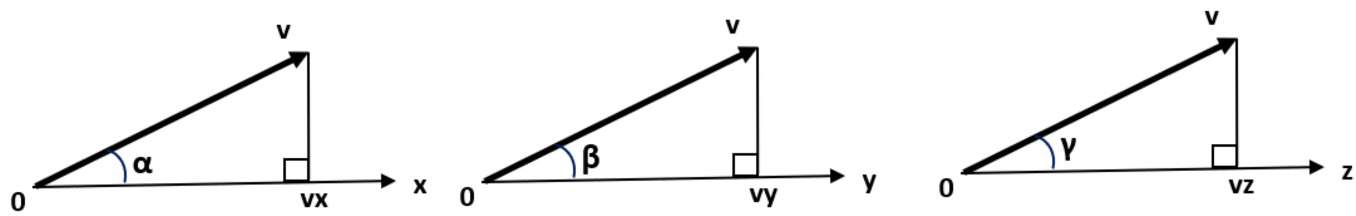


Vector **v** in **R**^{3}



Direction cosines and direction angles for the unit vector **v**/|**v**|

$$\mathbf{v} = \|\mathbf{v}\| * [\cos(\alpha), \cos(\beta), \cos(\gamma)]$$



Esim: jos $\mathbf{v} = [1, 2, 3]$, niin

$$\mathbf{v}^0 = \left[\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right]$$

eli

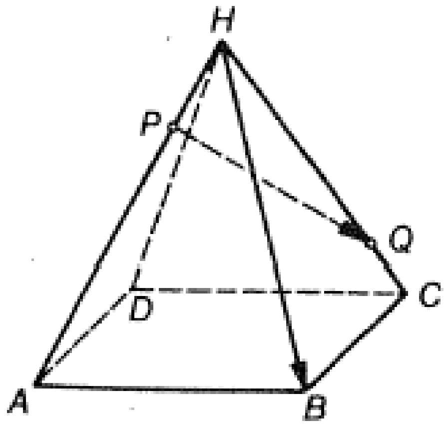
$$\alpha = \cos^{-1}\left(\frac{1}{\sqrt{14}}\right) = 74.5^\circ$$

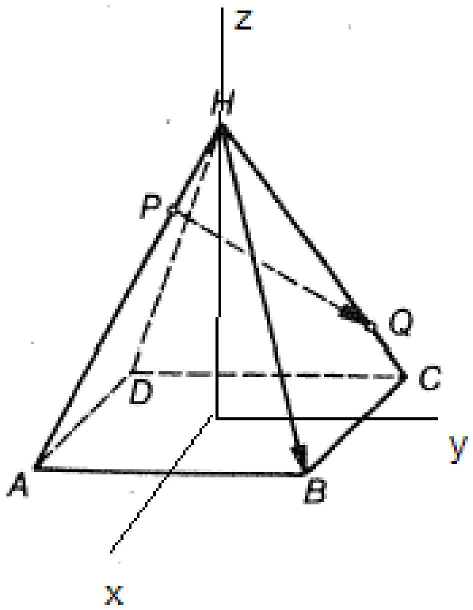
$$\beta = \cos^{-1}\left(\frac{2}{\sqrt{14}}\right) = 57.7^\circ$$

$$\gamma = \cos^{-1}\left(\frac{3}{\sqrt{14}}\right) = 36.7^\circ$$

Esim. Allaolevan pyramidin korkeus on h ja pohjaneliön sivun pituus on s.

Laske etäisyys $\|\mathbf{PQ}\|$, kun $\|\mathbf{HP}\| = hp$ ja $\|\mathbf{HQ}\| = hq$.





Kuvan mukaisessa koordinaatistossa

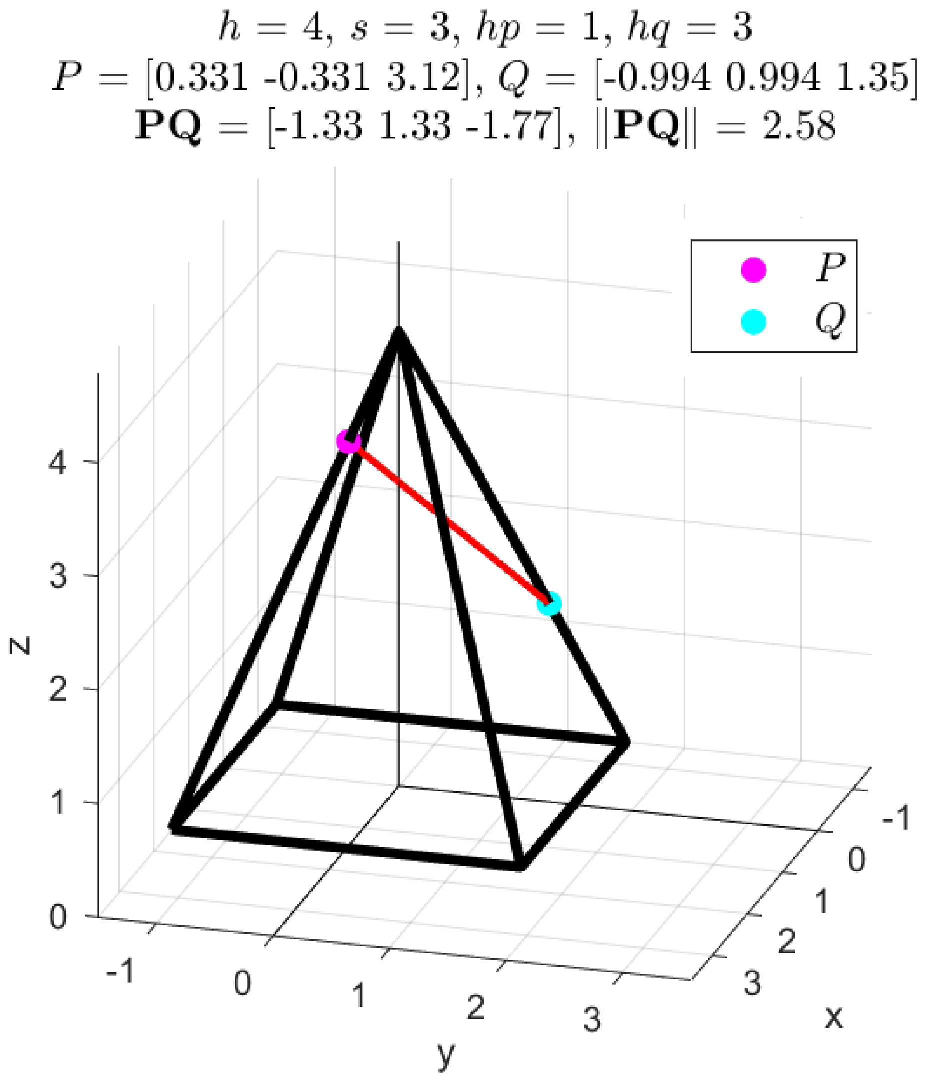
$A = [s/2, -s/2, 0]$, $C = [-s/2, s/2, 0]$,
$H = [0, 0, h]$

$$\mathbf{HA} = A - H,\ \mathbf{HC} = C - H$$

$$\mathbf{HP} = hp * \frac{\mathbf{HA}}{\|\mathbf{HA}\|},\ \mathbf{HQ} = hq * \frac{\mathbf{HC}}{\|\mathbf{HC}\|}$$

$$P = H + \mathbf{HP},\ Q = H + \mathbf{HQ}$$

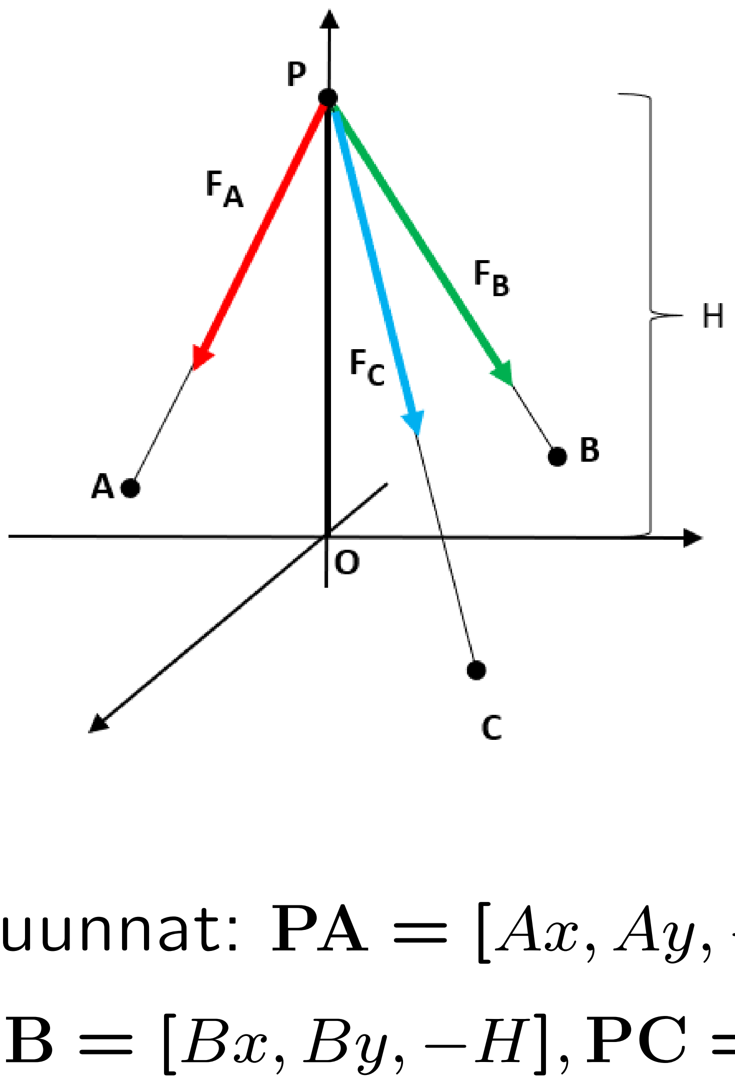
$$\mathbf{PQ} = Q - P \rightarrow \|\mathbf{PQ}\|$$



Esim: $A = [Ax, Ay, 0]$, $B = [Bx, By, 0]$, $C = [Cx, Cy, 0]$, $P = [0, 0, H]$.

Laske $\|\mathbf{F}_A\|$, $\|\mathbf{F}_B\|$ ja $\|\mathbf{F}_C\|$, kun

$\mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C = [0, 0, -F]$



Suunnat: $\mathbf{PA} = [Ax, Ay, -H]$,

$\mathbf{PB} = [Bx, By, -H], \mathbf{PC} = [Cx, Cy, -H]$

Yksikkövektorit

$$\mathbf{u} = \frac{\mathbf{PA}}{\|\mathbf{PA}\|}, \mathbf{v} = \frac{\mathbf{PB}}{\|\mathbf{PB}\|}, \mathbf{w} = \frac{\mathbf{PC}}{\|\mathbf{PC}\|}$$

$$\mathbf{F}_A = \|\mathbf{F}_A\| * \mathbf{u},\ \mathbf{F}_B = \|\mathbf{F}_B\| * \mathbf{v}$$

$$\mathbf{F}_C = \|\mathbf{F}_C\| * \mathbf{w}$$

$$\mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C = [0, 0, -F]$$

$$\rightarrow \begin{cases} \|\mathbf{F}_A\| ux + \|\mathbf{F}_B\| vx + \|\mathbf{F}_C\| wx = 0 \\ \|\mathbf{F}_A\| uy + \|\mathbf{F}_B\| vy + \|\mathbf{F}_C\| wy = 0 \\ \|\mathbf{F}_A\| uz + \|\mathbf{F}_B\| vz + \|\mathbf{F}_C\| wz = -F \end{cases}$$

$$\rightarrow \begin{cases} \|\mathbf{F}_A\| = \dfrac{vy\, wx - vx\, wy}{D} * F \\ \|\mathbf{F}_B\| = \dfrac{ux\, wy - uy\, wx}{D} * F \\ \|\mathbf{F}_C\| = \dfrac{uy\, vx - ux\, vy}{D} * F \end{cases}$$

$$\begin{aligned} D &= ux\, vy\, wz - ux\, vz\, wy - uy\, vx\, wz \\ &+ uy\, vz\, wx + uz\, vx\, wy - uz\, vy\, wx \end{aligned}$$

solve f1*a1+f2*b1+f3*c1=0, f1*a2+f2*b2+f3*c2=0, f1*a3+f2*b3+f3*c3=-f,f1,f2,f3

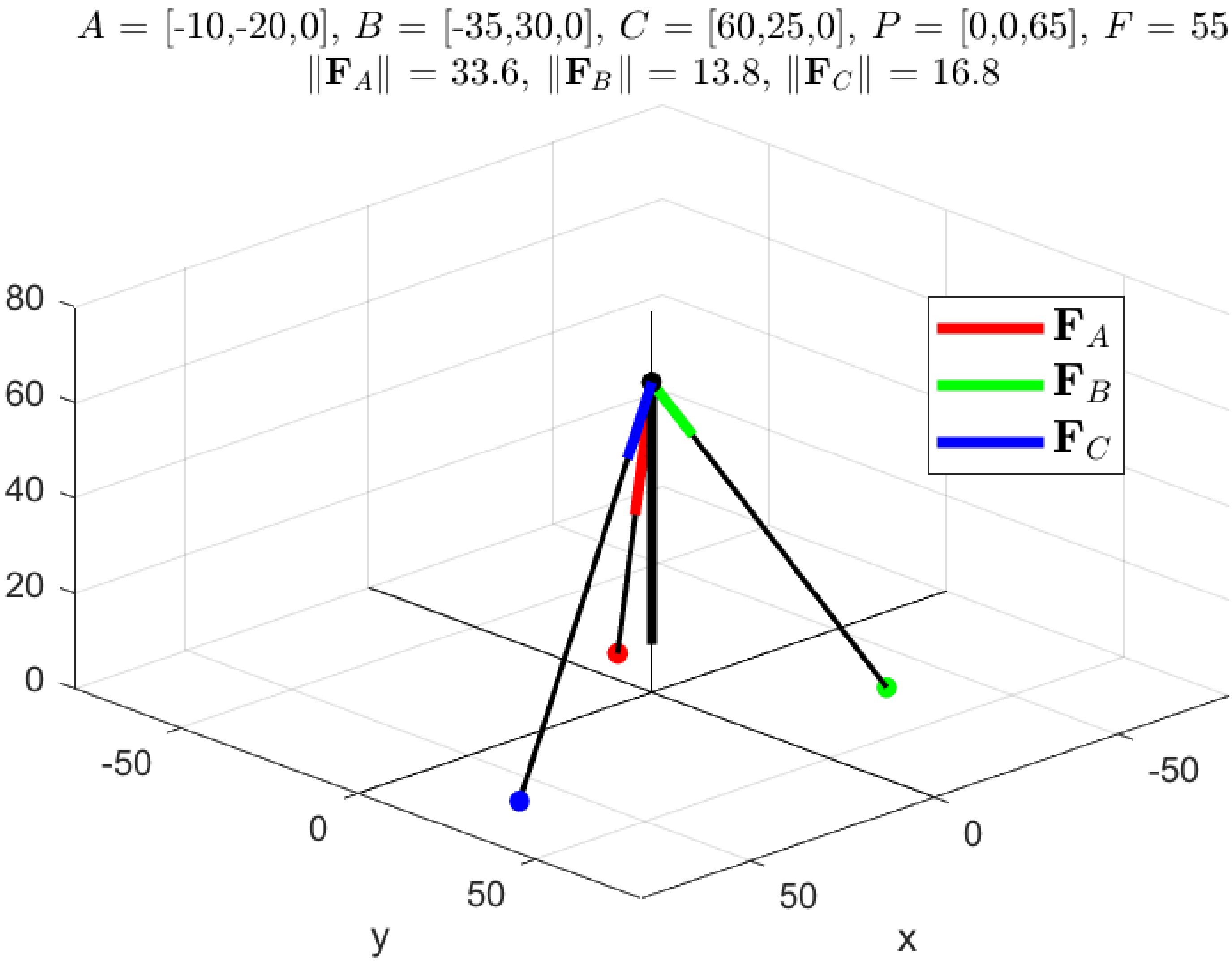
$$\text{solve}\quad \begin{cases} f1\,a1 + f2\,b1 + f3\,c1 = 0 \\ f1\,a2 + f2\,b2 + f3\,c2 = 0 \\ f1\,a3 + f2\,b3 + f3\,c3 = -f \end{cases} \quad \text{for}\quad f1, f2, f3$$

$$f1 = -\frac{f\,(b2\,c1 - b1\,c2)}{-a1\,b2\,c3 + a1\,b3\,c2 + a2\,b1\,c3 - a2\,b3\,c1 - a3\,b1\,c2 + a3\,b2\,c1},$$

$$f2 = -\frac{f\,(a2\,c1 - a1\,c2)}{a1\,b2\,c3 - a1\,b3\,c2 - a2\,b1\,c3 + a2\,b3\,c1 + a3\,b1\,c2 - a3\,b2\,c1},$$

$$f3 = -\frac{f\,(a2\,b1 - a1\,b2)}{-a1\,b2\,c3 + a1\,b3\,c2 + a2\,b1\,c3 - a2\,b3\,c1 - a3\,b1\,c2 + a3\,b2\,c1}$$

Nimet u1,v1,w1,... tuntuvat olevan huonoja wolfram alphalle



A = [-10,-20,0], B = [-35,30,0], C = [60,25,0], P = [0,0,65], F = 55
$\|\mathbf{F}_A\|$ = 33.6, $\|\mathbf{F}_B\|$ = 13.8, $\|\mathbf{F}_C\|$ = 16.8
80
60
40
20
0
-50
0
50
y
-50
0
50
x
$\mathbf{F}_A$
$\mathbf{F}_B$
$\mathbf{F}_C$

PISTETULO

(skalaaritulo, dot product)

Vektoreiden $\mathbf{u} = [ux, uy, uz]$ ja $\mathbf{v} = [vx, vy, vz]$ pistetulo on luku

$$\mathbf{u} \bullet \mathbf{v} = ux * vx + uy * vy + uz * vz$$

Esim: jos $\mathbf{u} = [1, 2, 3]$ ja $\mathbf{v} = [2, -1, 5]$, niin $\mathbf{u} \bullet \mathbf{v} = 1 * 2 + 2 * (-1) + 3 * 5 = 15$

MATLAB/Octave: dot$(\mathbf{u}, \mathbf{v})$

Laskusäännöt:

$$\mathbf{u} \bullet \mathbf{v} = \mathbf{v} \bullet \mathbf{u}$$

$$\mathbf{u} \bullet (\mathbf{v} + \mathbf{w}) = \mathbf{u} \bullet \mathbf{v} + \mathbf{u} \bullet \mathbf{w}$$

$$(t * \mathbf{u}) \bullet \mathbf{v} = \mathbf{u} \bullet (t * \mathbf{v}) = t * (\mathbf{u} \bullet \mathbf{v})$$

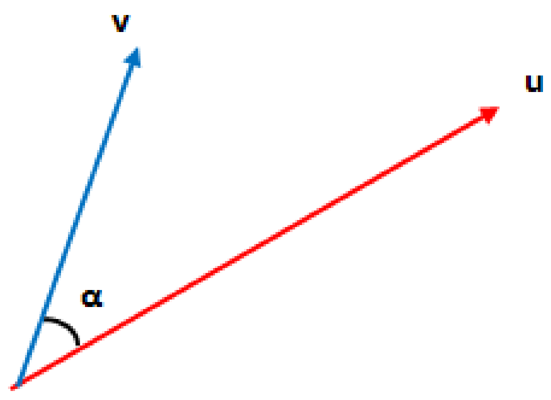
$$\mathbf{u} \bullet \mathbf{u} = \|\mathbf{u}\|^2$$

Pistetulon avulla voidaan laskea kulmia:

$$\mathbf{u} \bullet \mathbf{v} = \|\mathbf{u}\| * \|\mathbf{v}\| * \cos(\alpha)$$

eli $\mathbf{u}$:n ja $\mathbf{v}$:n välinen kulma

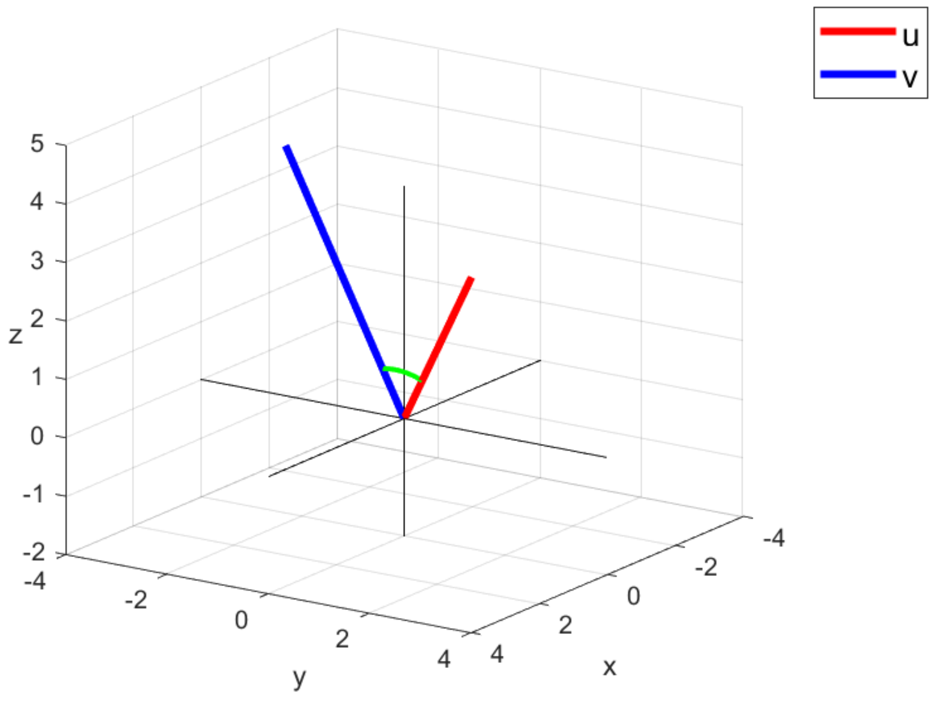
$$\alpha = \cos^{-1}\left(\frac{\mathbf{u} \bullet \mathbf{v}}{\|\mathbf{u}\| * \|\mathbf{v}\|}\right)$$



Esim: $\mathbf{u} = [1, 2, 3], \mathbf{v} = [2, -1, 5]$

$\|\mathbf{u}\| = \sqrt{14}$, $\|\mathbf{v}\| = \sqrt{30}$ ja $\mathbf{u} \bullet \mathbf{v} = 15$

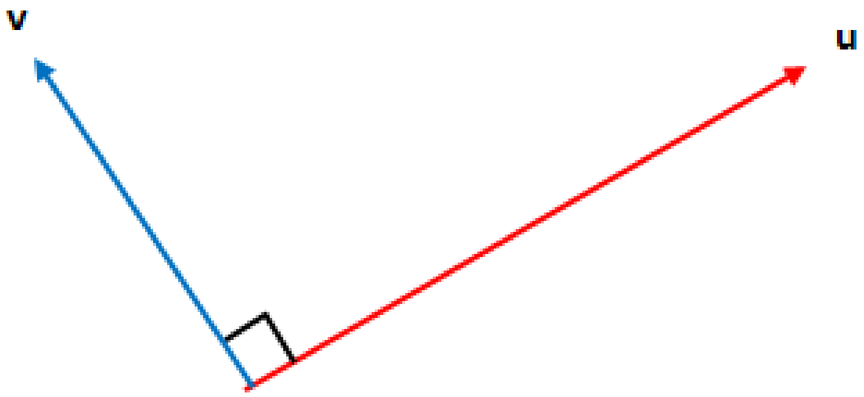
$$\alpha = \cos^{-1}\left(\frac{15}{\sqrt{14} * \sqrt{30}}\right) \approx 43^\circ$$



Pistetulon avulla on helppo testata vektoreiden kohtisuoruutta:

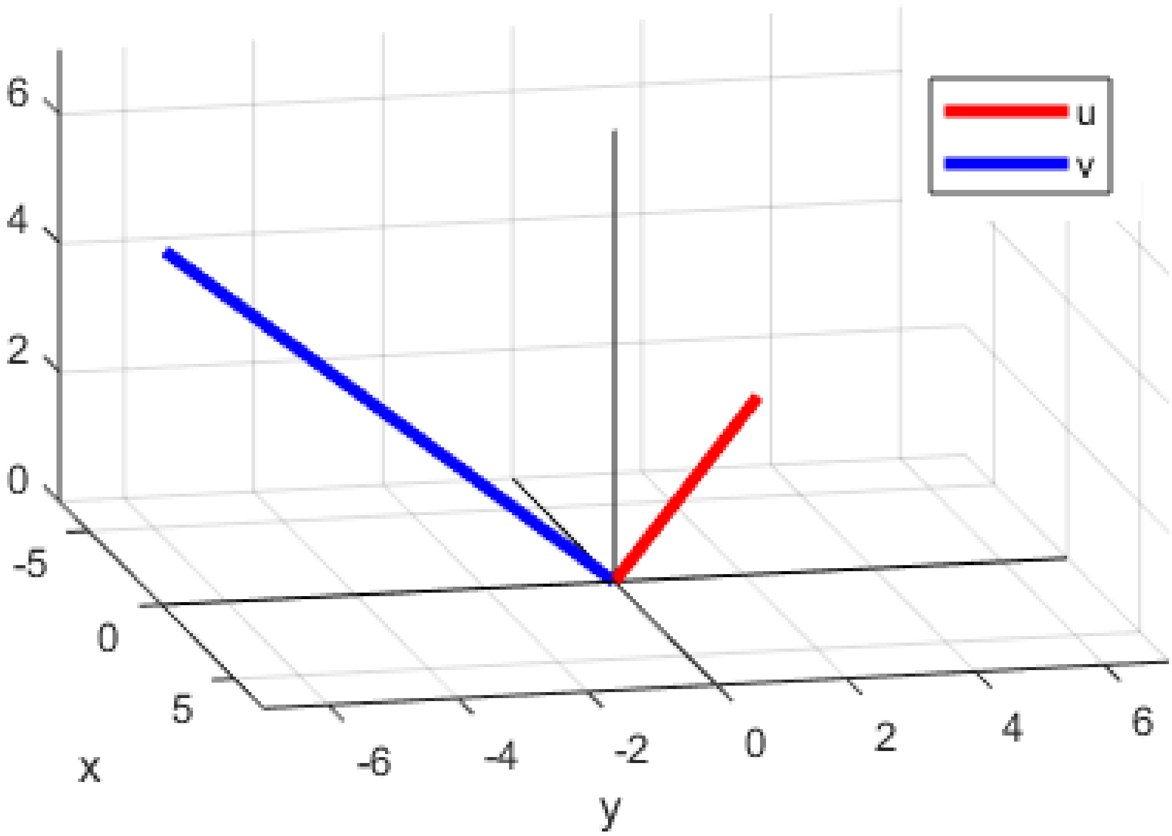
$\mathbf{u}$ ja $\mathbf{v}$ ovat kohtisuoria eli $\mathbf{u} \perp \mathbf{v}$

$\leftrightarrow \alpha = 90^\circ \leftrightarrow \cos\alpha = 0 \leftrightarrow \mathbf{u} \bullet \mathbf{v} = 0$



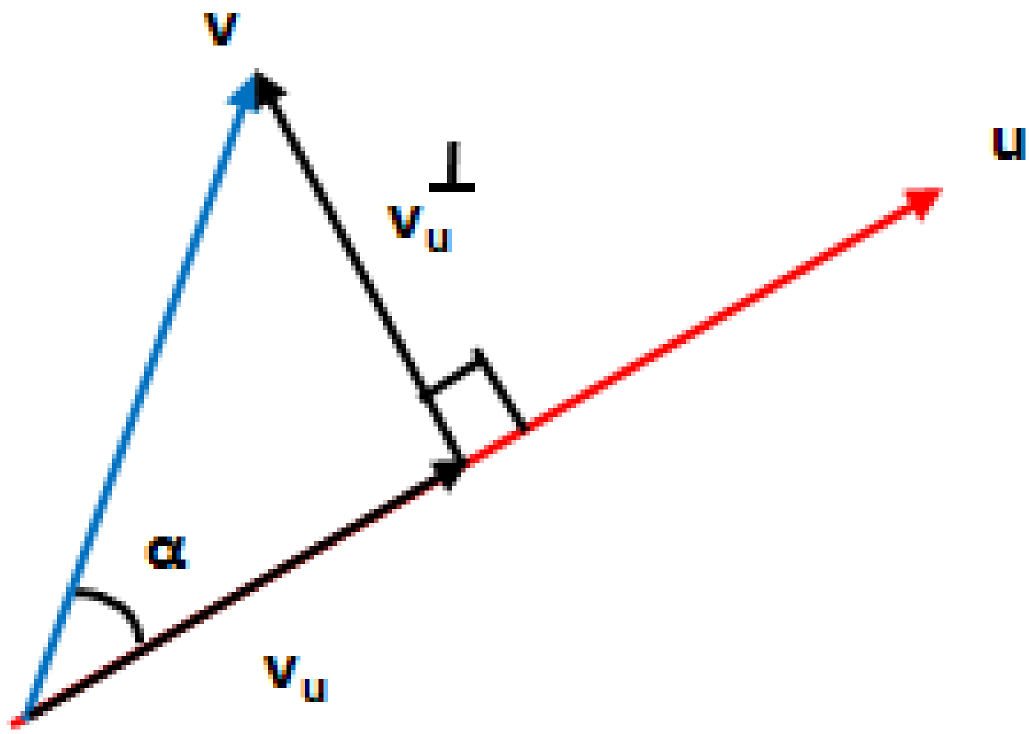
Esim: $\mathbf{u} = [1, 2, 3]$ ja $\mathbf{v} = [-2, y, 5]$ ovat kohtisuoria, kun

$\mathbf{u} \bullet \mathbf{v} = 2y + 13 = 0$ eli $y = -6.5$



Komponentteihin jako:

Jaetaan vektori $\mathbf{v}$ kahteen osaan, vektorin $\mathbf{u}$ suuntaiseen ja $\mathbf{u}$:ta vastaan kohtisuoraan: $\mathbf{v} = \mathbf{v_u} + \mathbf{v_u^\perp}$



$$\mathbf{v_u} = \frac{\mathbf{v} \bullet \mathbf{u}}{\|\mathbf{u}\|^2} * \mathbf{u} = \frac{\mathbf{v} \bullet \mathbf{u}}{\|\mathbf{u}\|} * \frac{\mathbf{u}}{\|\mathbf{u}\|}$$

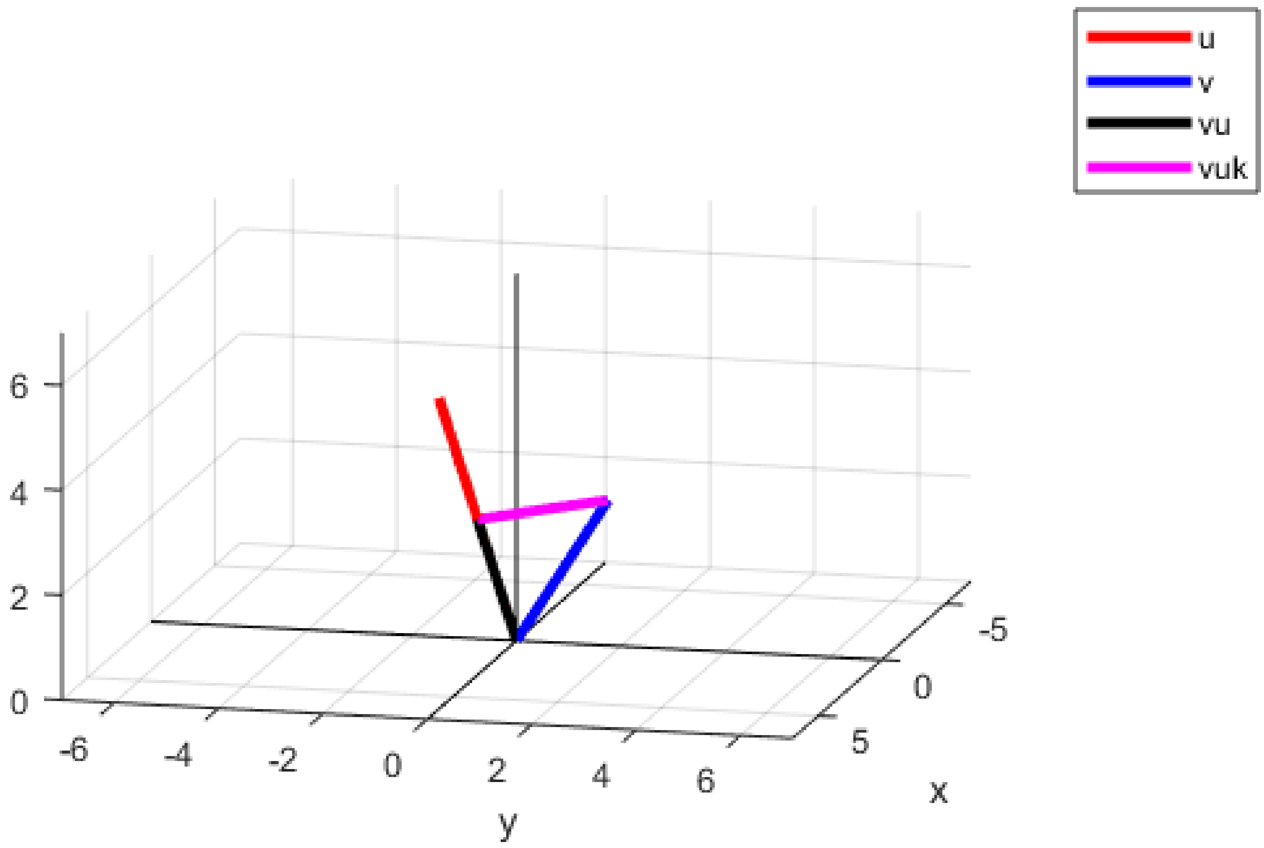
$$\mathbf{v_u^\perp} = \mathbf{v} - \mathbf{v_u}$$

Esim. $\mathbf{v} = [1, 2, 3]$ ja $\mathbf{u} = [2, -1, 5]$

$\mathbf{v} \bullet \mathbf{u} = 15$ ja $\|\mathbf{u}\| = \sqrt{30}$

$$\mathbf{v_u} = \frac{15}{(\sqrt{30})^2} * \mathbf{u} = 0.5 * \mathbf{u} = [1, -0.5, 2.5]$$

$$\mathbf{v}_{\mathbf{u}}^{\perp} = [1, 2, 3] - [1, -0.5, 2.5] = [0, 2.5, 0.5]$$



RISTITULO

(vektoritulo, cross product)

Vektoreiden $\mathbf{u} = [ux, uy, uz]$ ja $\mathbf{v} = [vx, vy, vz]$ ristitulo on vektori

$\mathbf{u} \times \mathbf{v} =$

$[uy\,vz - uz\,vy\,,\; uz\,vx - ux\,vz\,,\; ux\,vy - uy\,vx]$

$= -\mathbf{v} \times \mathbf{u}$

Esim. jos $\mathbf{u} = [1, 2, 3]$ ja $\mathbf{v} = [2, -1, 5]$, niin $\mathbf{u} \times \mathbf{v} = [13, 1, -5]$

MATLAB/Octave: cross($\mathbf{u}, \mathbf{v}$)

1. <u>$\mathbf{u} \times \mathbf{v}$ on kohtisuorassa $\mathbf{u}$:tä ja $\mathbf{v}$:tä vastaan</u>, ja sen suunta määräytyy oikean käden säännöllä

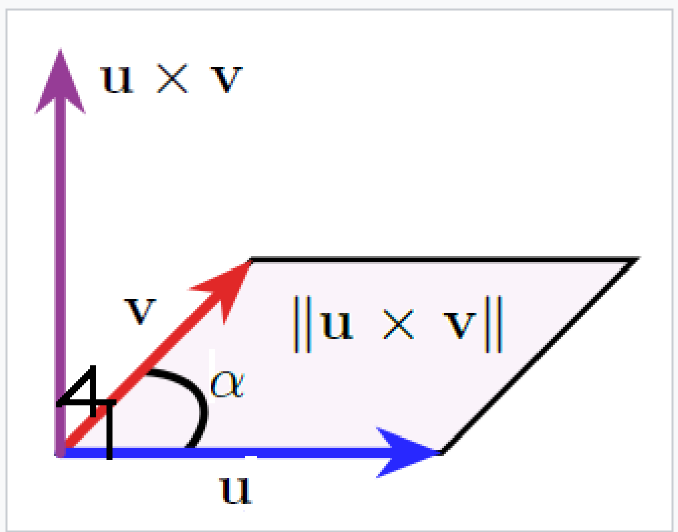
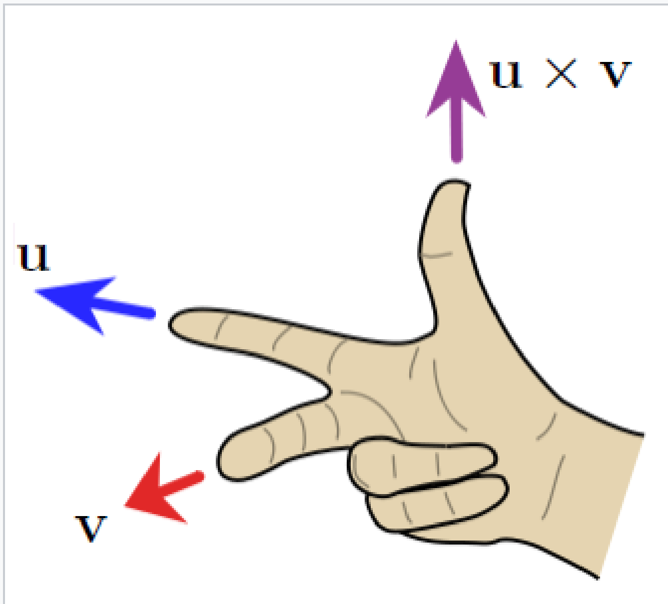


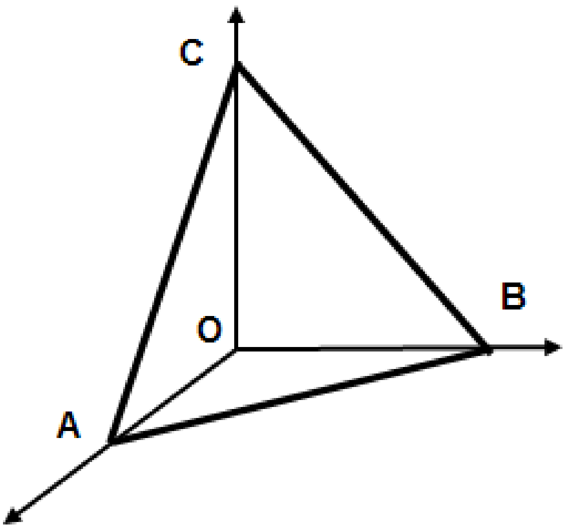
Figure 1. The area of a parallelogram as the magnitude of a cross product



Finding the direction of the cross product by the right-hand rule

2. pituus $\|\mathbf{u} \times \mathbf{v}\| = \|\mathbf{u}\| * \|\mathbf{v}\| * \sin(\alpha)$, missä α on $\mathbf{u}$:n ja $\mathbf{v}$:n välinen kulma, on $\mathbf{u}$:n ja $\mathbf{v}$:n määräämän suunnikkaan pinta-ala.

Esim. Pisteiden A, B ja C muodostaman kolmion pinta-ala on $\frac{1}{2} * \|\mathbf{AB} \times \mathbf{AC}\|$



$A = [2, 0, 0], B = [0, 3, 0],\ C = [0, 0, 4]$

$\mathbf{AB} = [-2, 3, 0],\ \mathbf{AC} = [-2, 0, 4]$

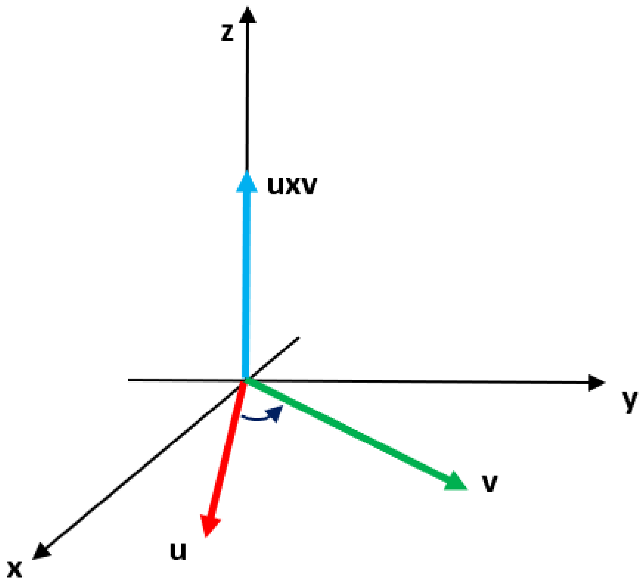
$\mathbf{AB} \times \mathbf{AC} = [12, 8, 6]$

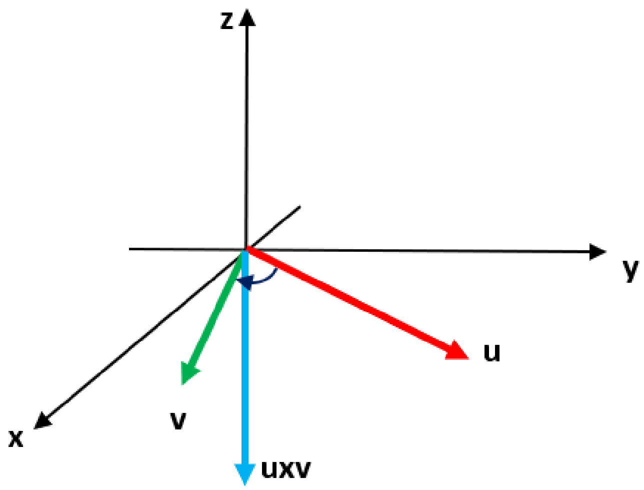
$\frac{1}{2} * \|\mathbf{AB} \times \mathbf{AC}\| = 7.8$

Esim: $\mathbf{u} = [ux, uy, 0]$, $\mathbf{v} = [vx, vy, 0]$

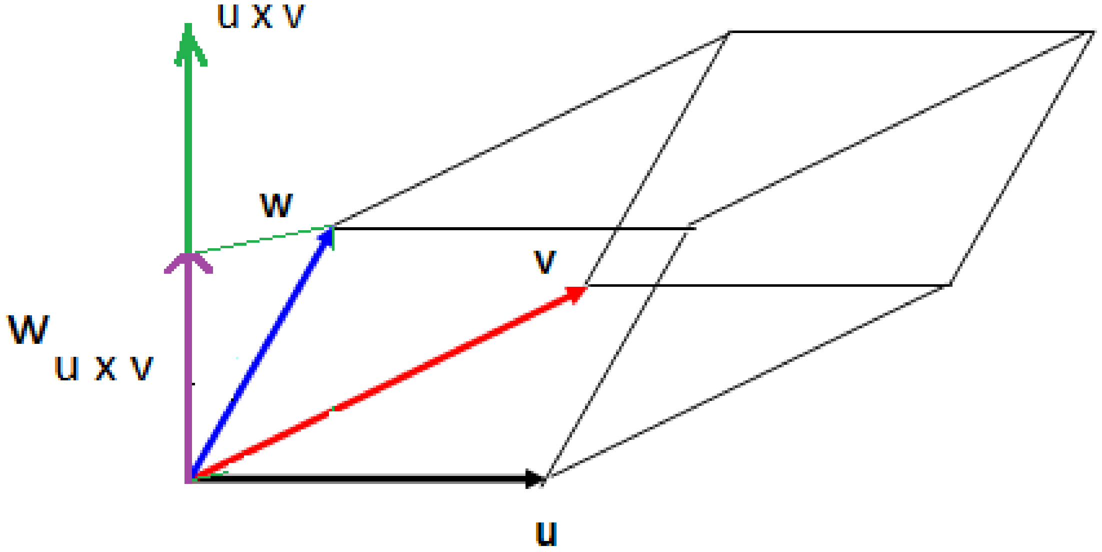
$$\mathbf{u} \times \mathbf{v} = [0, 0, ux\,vy - uy\,vx]$$

z-komponentti on 2D-ristitulo





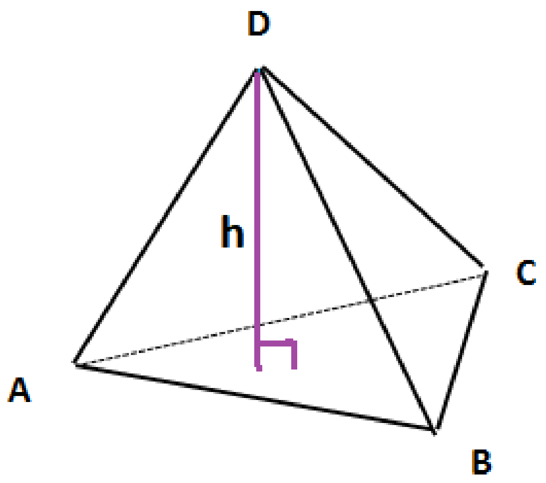
Esim. Vektoreiden $\mathbf{u}$, $\mathbf{v}$ ja $\mathbf{w}$ määräämän särmiön tilavuus on skalaarikolmitulon $(\mathbf{u} \times \mathbf{v}) \bullet \mathbf{w}$ itseisarvo



$$|(\mathbf{u} \times \mathbf{v}) \bullet \mathbf{w}| = \|\mathbf{u} \times \mathbf{v}\| * \underbrace{\frac{|(\mathbf{u} \times \mathbf{v}) \bullet \mathbf{w}|}{\|\mathbf{u} \times \mathbf{v}\|}}_{=\|\mathbf{w}_{\mathbf{u}\times\mathbf{v}}\|}$$

$=$ pohjan ala $*$ korkeus

Esim: Pisteiden A, B, C ja D muodostaman tetraedrin tilavuus

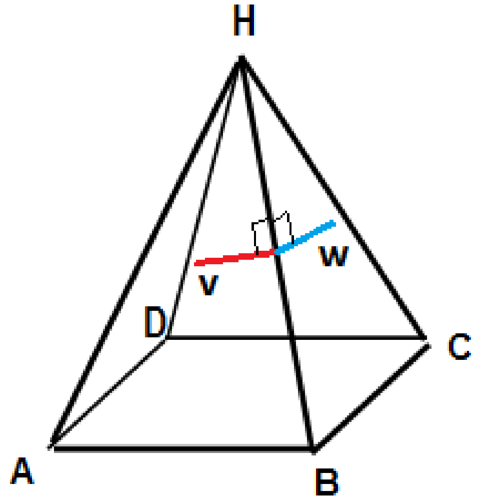


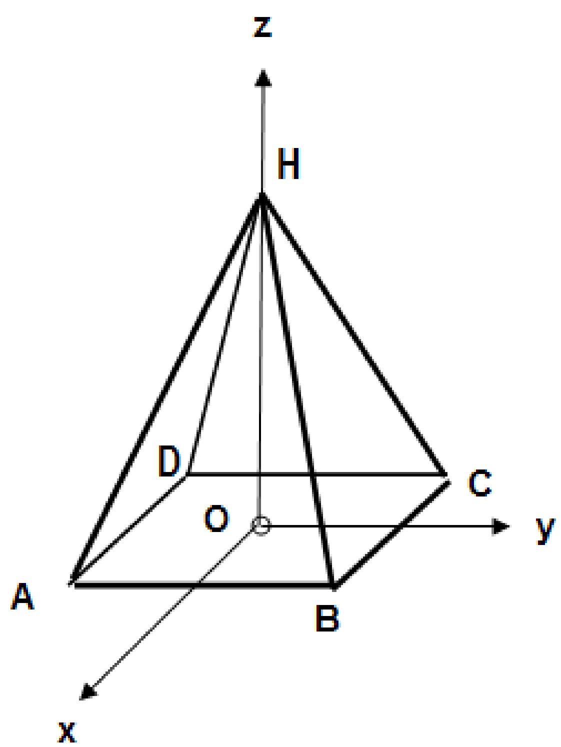
$= \frac{1}{3} *$ kolmion ABC ala $*$ korkeus h

$= \frac{1}{6} *$ vektoreiden $\mathbf{AB}, \mathbf{AC}$ ja $\mathbf{AD}$ määräämän särmiön tilavuudesta

$= \frac{1}{6} * |(\mathbf{AB} \times \mathbf{AC}) \bullet \mathbf{AD}|$

Esim. Pyramidin korkeus $h = 4$ ja pohjaneliön sivun pituus $s = 2$. Muodosta etuseinän ABH suuntainen, särmää BH vastaan kohtisuora vektori $\mathbf{v}$ ja sivuseinän BCH suuntainen, särmää BH vastaan kohtisuora vektori $\mathbf{w}$ ja laske niiden välinen kulma (= seinien ABH ja BCH välinen kulma).





Kuvan koordinaatistossa

$A = [s/2, -s/2, 0], B = [s/2, s/2, 0],$

$C = [-s/2, s/2, 0], H = [0, 0, h]$

Etuseinän ABH (ulospäin sojottava)

normaali $\mathbf{n}_1 = \mathbf{AB} \times \mathbf{AH} = [8, 0, 2]$

Sivuseinän BCH (oikealle sojottava)

normaali $\mathbf{n}_2 = \mathbf{BC} \times \mathbf{BH} = [0, 8, 2]$

Esimerkiksi

$\mathbf{v} = \mathbf{n}_1 \times \mathbf{BH} = [2, -34, -8]$ ja

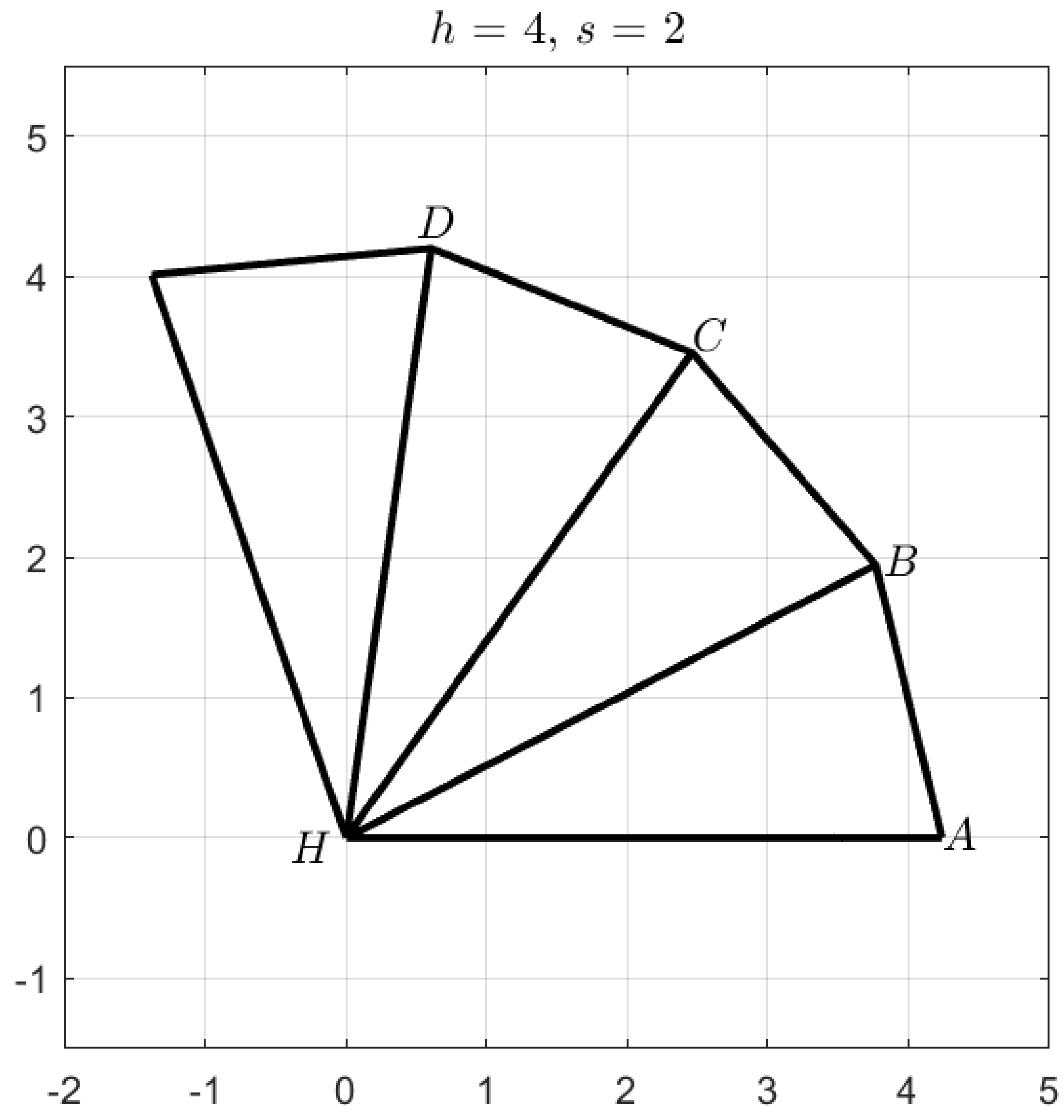
$\mathbf{w} = \mathbf{BH} \times \mathbf{n}_2 = [-34, 2, -8]$

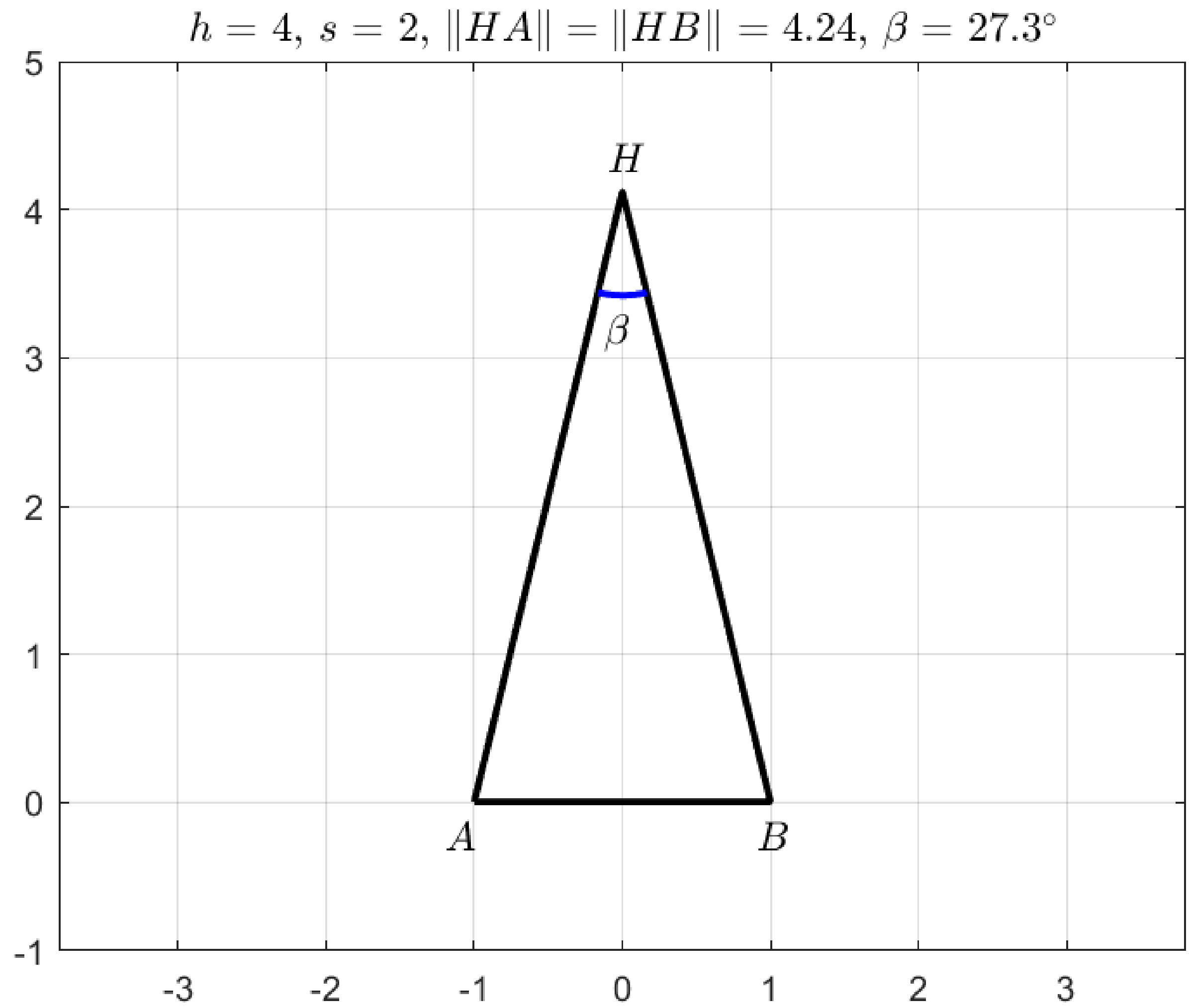
ovat halutun suuntaisia ja niiden välinen

kulma

$$\alpha = \cos^{-1}\left(\frac{\mathbf{v} \bullet \mathbf{w}}{\|\mathbf{v}\| * \|\mathbf{w}\|}\right) = 93.4^\circ$$

Eli, pyramidi syntyy taivuttamalla alla-olevaa levyä viivoja HB, HC, HD pitkin kulman $180° - \alpha = 86.6°$ verran



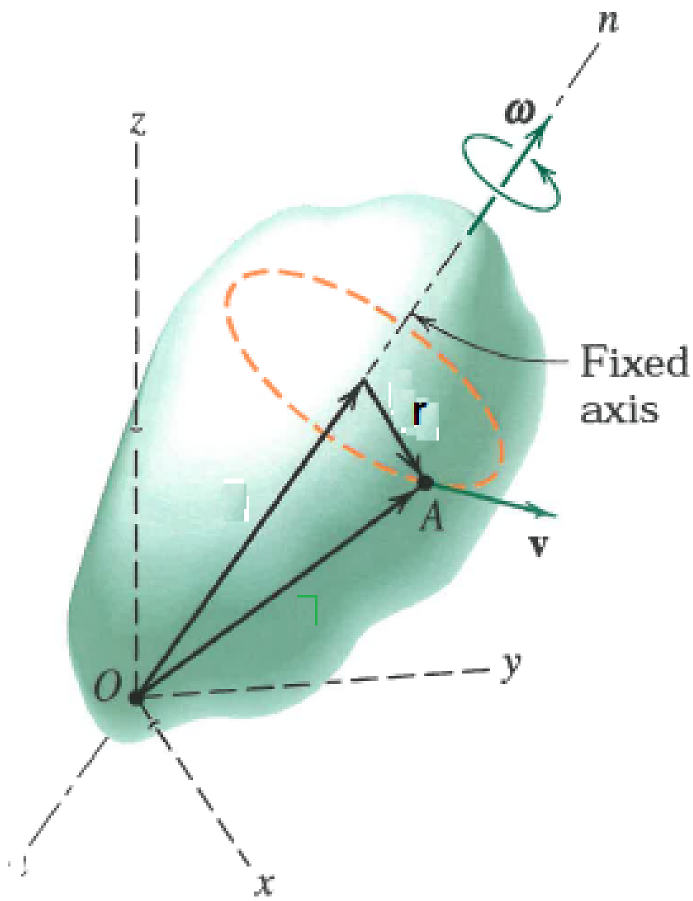


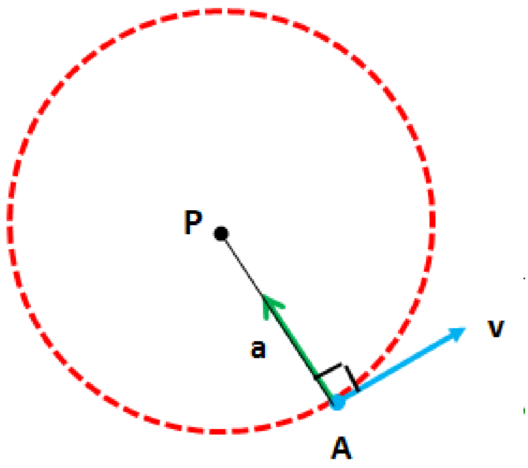
$$\beta = \cos^{-1}\left(\frac{\mathbf{HA} \bullet \mathbf{HB}}{\|\mathbf{HA}\| * \|\mathbf{HB}\|}\right) = 27.3^\circ$$

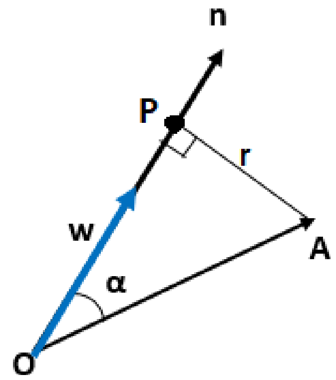
Esim. Kappale pyörii O:n kautta kulkevan, vektorin $\mathbf{n}$ suuntaisen akselin ympäri kulmanopeudella ω (rad/sek)

Kulmanopeusvektori $\mathbf{w} = \omega * \dfrac{\mathbf{n}}{\|\mathbf{n}\|}$

Pisteen A nopeus $\mathbf{v} = \mathbf{w} \times \mathbf{OA}$ ja kiihtyvyys $\mathbf{a} = \mathbf{w} \times \mathbf{v}$





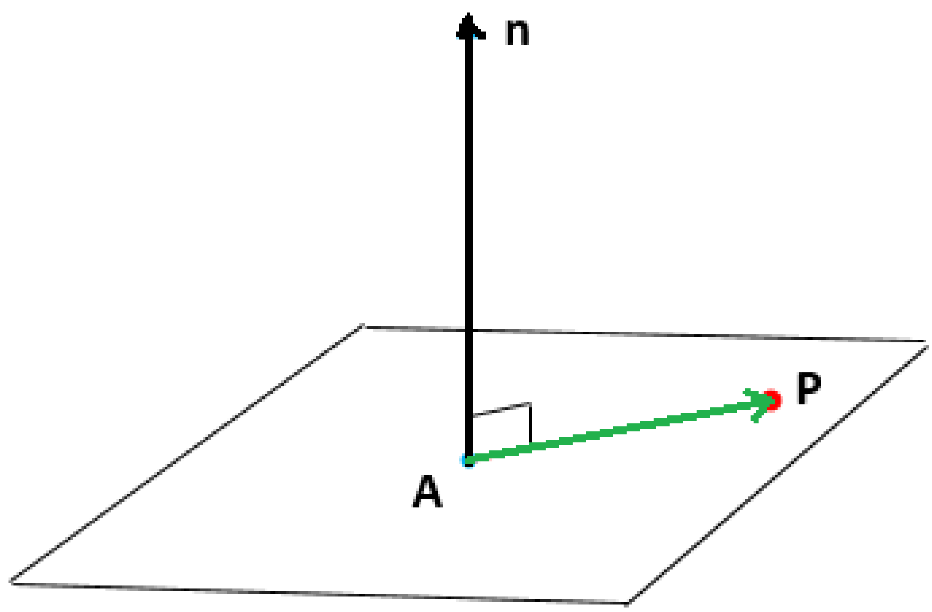


Selitys: Jos α on $\mathbf{w}$:n ja $\mathbf{OA}$:n välinen kulma, niin A:n rataympyrän säde $r = \|\mathbf{OA}\| * \sin(\alpha)$, joten A:n vauhti $\|\mathbf{v}\| = \|\mathbf{w}\| * \|\mathbf{OA}\| * \sin(\alpha) = \omega r$ ja $\mathbf{v}$:n suunta on ympyrän tangentti

Koska $\mathbf{w}$ ja $\mathbf{v}$ ovat kohtisuoria, niin $\|\mathbf{a}\| = \|\mathbf{w}\| * \|\mathbf{v}\| * \sin(90°) = \omega^2 r$ ja $\mathbf{a}$:n suunta on kohti A:n rataympyrän keskipistettä P

Esim. Tason normaalimuoto

Pisteen $A = [Ax, Ay, Az]$ kautta kulkevalla, vektoria $\mathbf{n} = [a, b, c]$ (tason **normaali**) vastaan kohtisuoralla tasolla



ovat ne pisteet $P = [x, y, z]$, joille vektorit $\mathbf{AP}$ ja $\mathbf{n}$ ovat kohtisuoria eli

$\mathbf{n} \bullet \mathbf{AP} = 0$

$a(x - Ax) + b(y - Ay) + c(z - Az) = 0$

$ax + by + cz = d$, missä

$d = aAx + bAy + cAz = \mathbf{n} \bullet \mathbf{OA}$

(vrt. suora 2D:ssä: $ax + by = c$)

Jos $c \neq 0$, niin tason yhtälö voidaan kirjoittaa muotoon

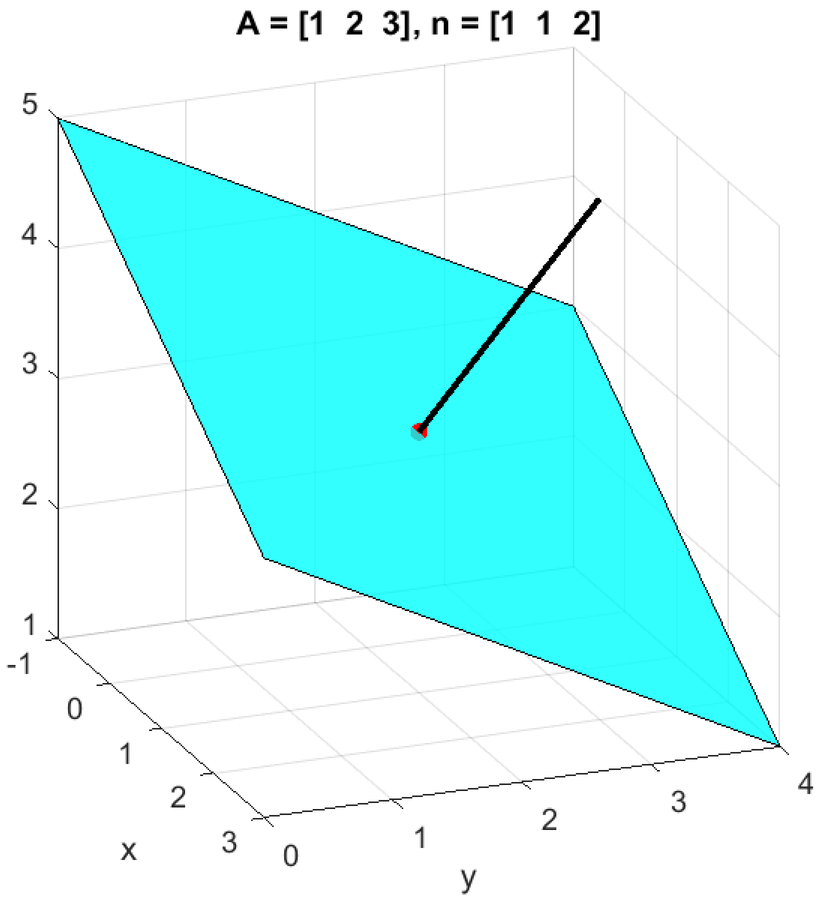
$$z = \frac{1}{c}(-ax - by + d) = -\frac{a}{c}x - \frac{b}{c}y + \frac{d}{c}$$

(vrt. suora 2D:ssä: $y = kx + b$)

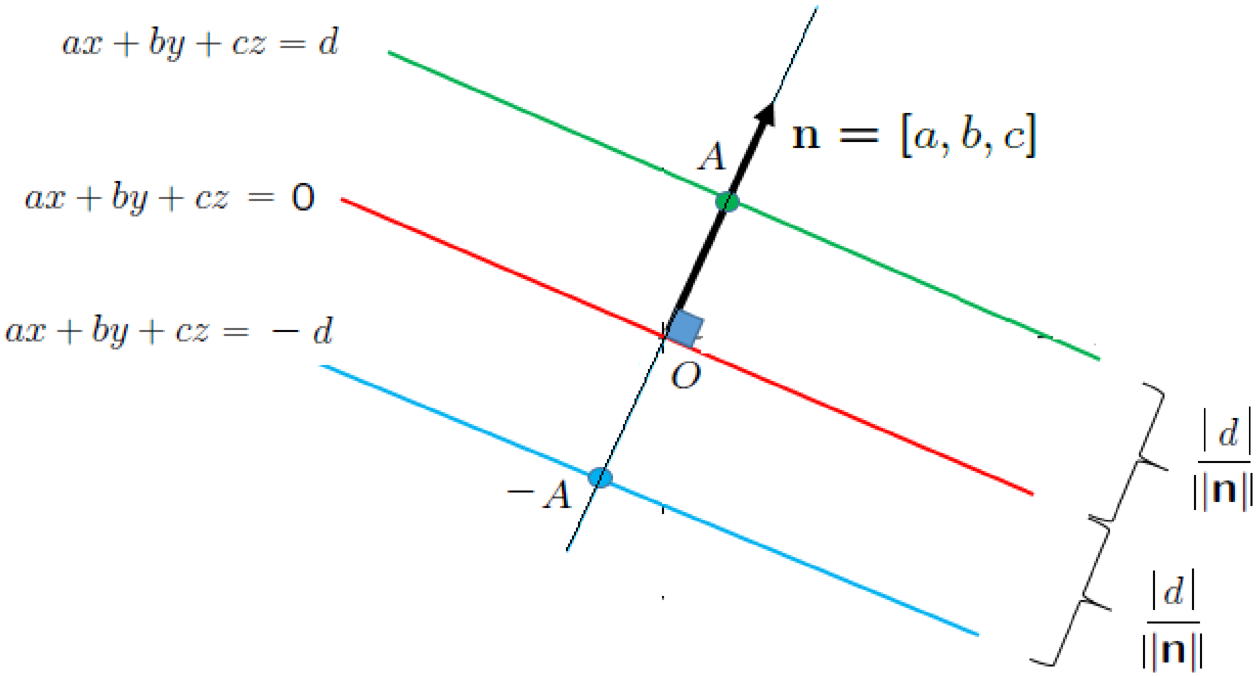
Esim. $A = [1, 2, 3]$, $\mathbf{n} = [1, 1, 2]$

$1(x - 1) + 1(y - 2) + 2(z - 3) = 0$ eli

$x + y + 2z = 9$ tai $z = \frac{1}{2}(-x - y + 9)$



Huom: taso $ax + by + cz = d$ kulkee pisteen $A = \frac{d}{\|\mathbf{n}\|} * \frac{\mathbf{n}}{\|\mathbf{n}\|}$ kautta ja sen etäisyys O:sta on $\|\mathbf{OA}\| = \frac{|d|}{\|\mathbf{n}\|}$



Syy: $a\,Ax + b\,Ay + c\,Ay = \mathbf{n} \bullet \mathbf{OA}$

$$= \mathbf{n} \bullet \left(\frac{d}{\|\mathbf{n}\|^2} * \mathbf{n} \right) = \frac{d}{\|\mathbf{n}\|^2} * (\mathbf{n} \bullet \mathbf{n})$$

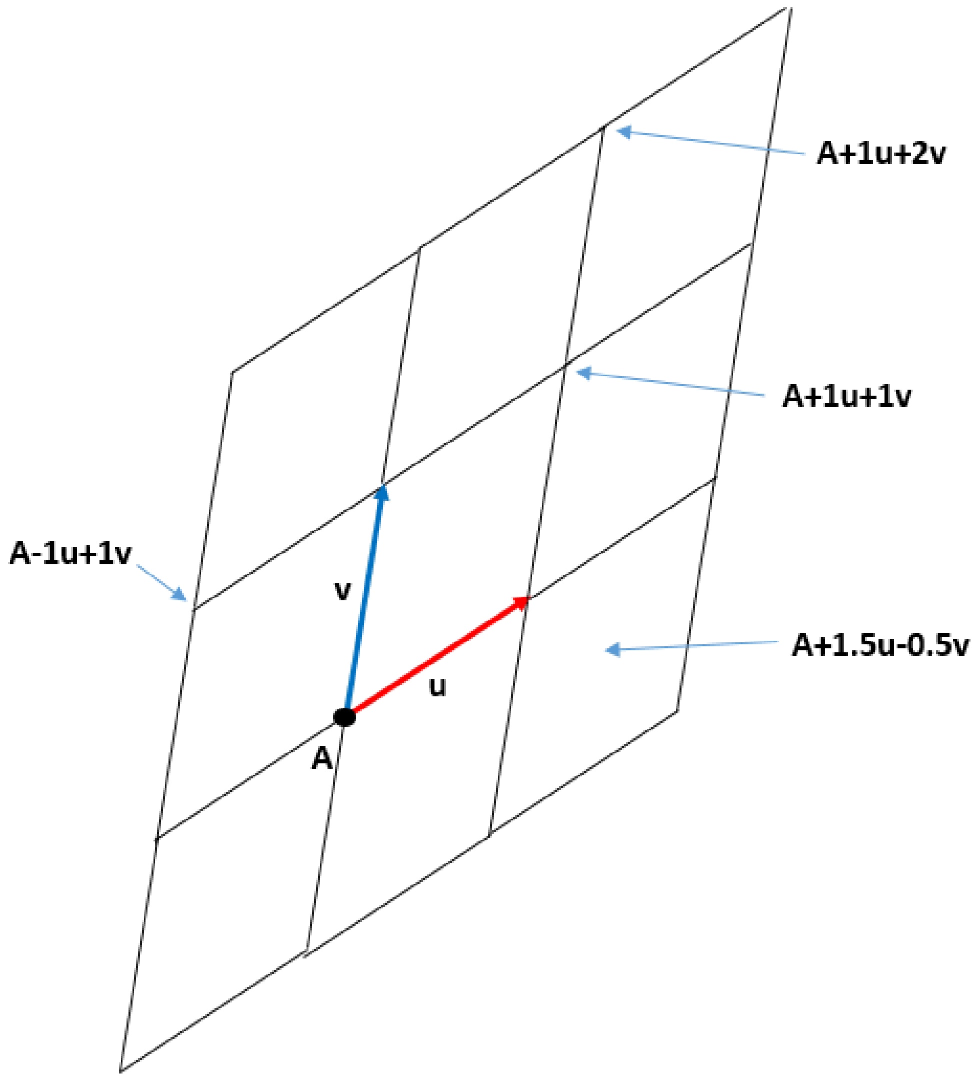
$$= \frac{d}{\|\mathbf{n}\|^2} * \|\mathbf{n}\|^2 = d$$

Esim. Tason parametrimuoto

Pisteen A kautta kulkevalla, vektoreiden $\mathbf{u}$ ja $\mathbf{v}$ suuntaisella tasolla ovat pisteet

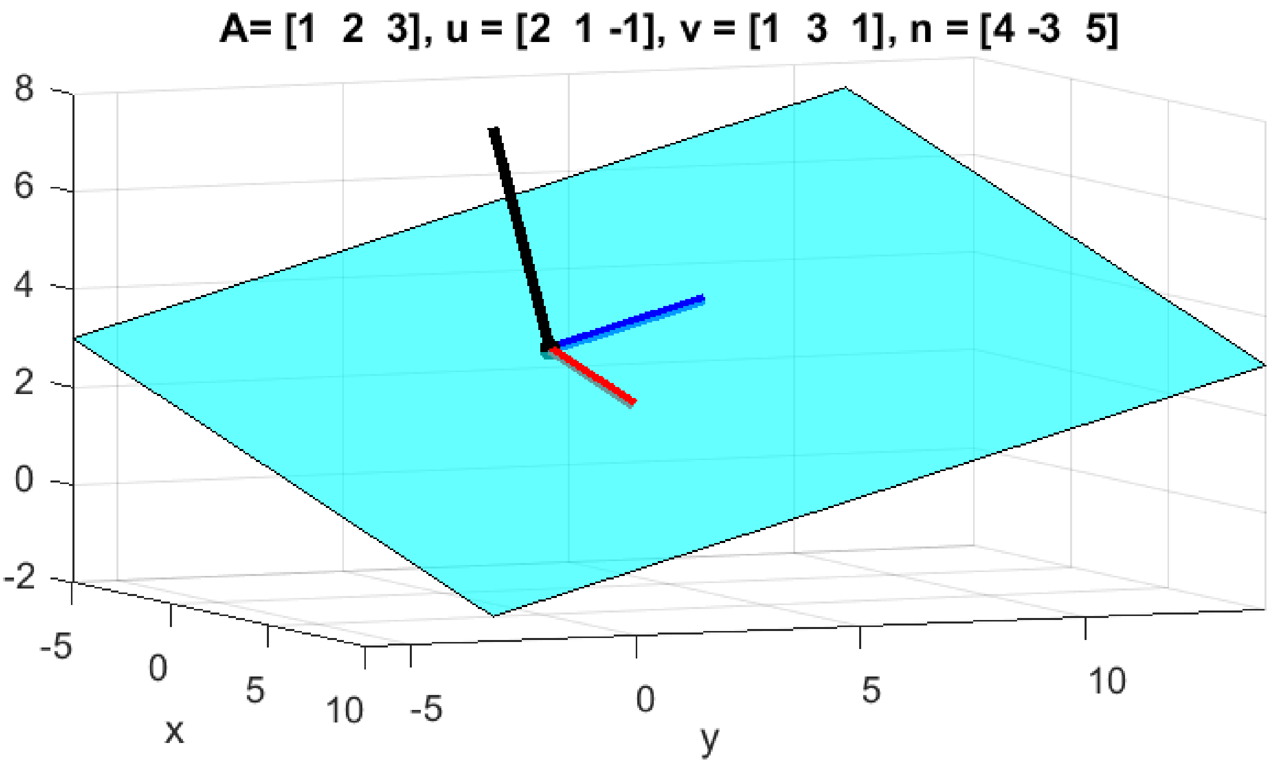
$$P = A + s * \mathbf{u} + t * \mathbf{v}$$

Tason normaali on esimerkiksi $\mathbf{n} = \mathbf{u} \times \mathbf{v}$


A+1u+2v
A+1u+1v
A-1u+1v
v
u
A+1.5u-0.5v
A

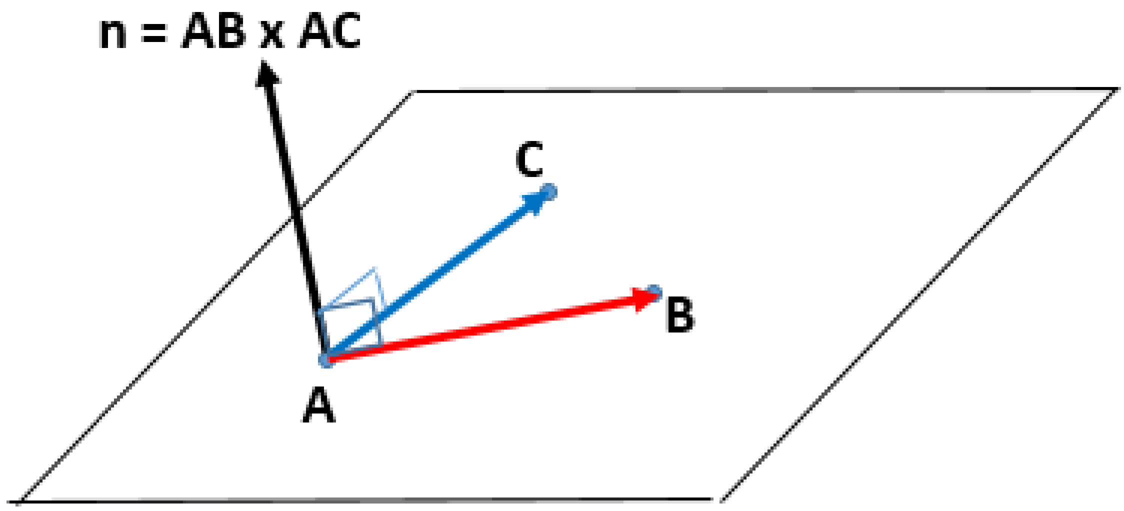
Esim: Jos $A = [1,2,3]$, $\mathbf{u} = [2,1,-1]$ ja $\mathbf{v} = [1,3,1]$, niin tasolla ovat pisteet

$$\begin{aligned} P &= A + s * \mathbf{u} + t * \mathbf{v} \\ &= [1 + 2s + 1t, 2 + 1s + 3t, 3 - 1s + 1t] \end{aligned}$$

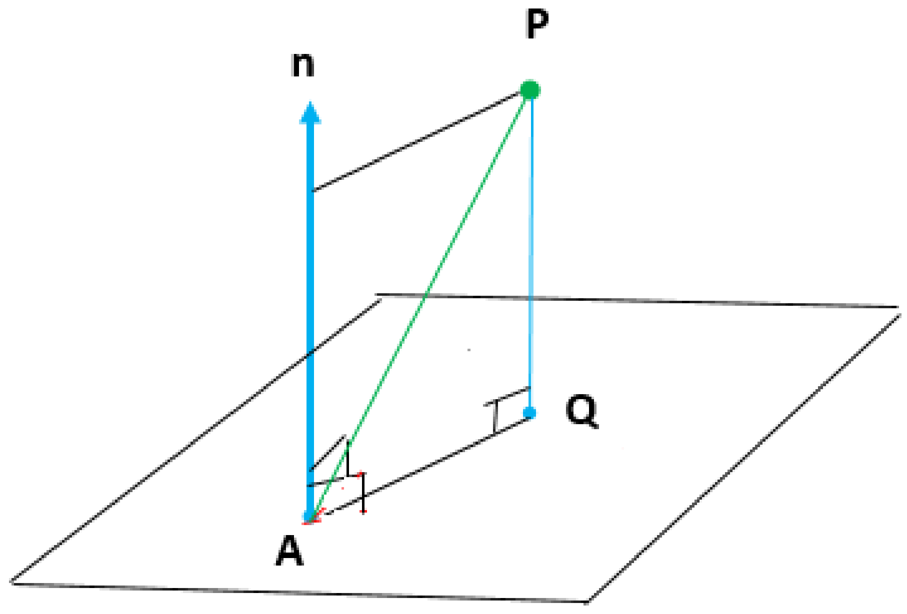


Tason normaali on $\mathbf{n} = \mathbf{u} \times \mathbf{v} = [4, -3, 5]$

Esim: Pisteiden A, B ja C määräämä taso on vektoreiden $\mathbf{u} = \mathbf{AB}$ ja $\mathbf{v} = \mathbf{AC}$ suuntainen, ja sen normaali on esimerkiksi $\mathbf{n} = \mathbf{AB} \times \mathbf{AC}$



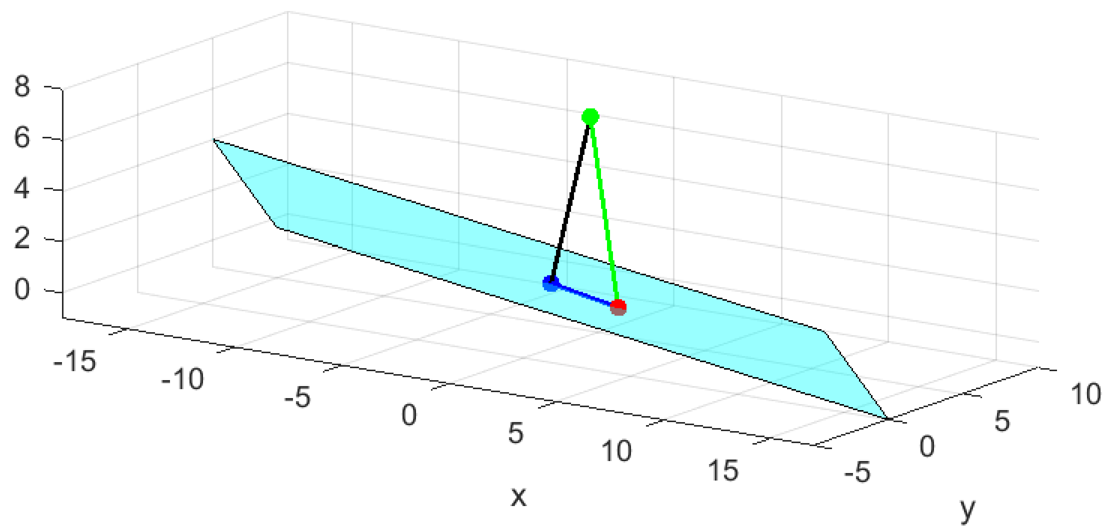
Esim. Pisteen P kohtisuora projektio tasolle $A, \mathbf{n}$ on se tason piste Q, joka on lähimpänä P:tä eli $\mathbf{PQ}$ on kohtisuorassa tasoa vastaan eli normaalin $\mathbf{n}$ suuntainen



$$\mathbf{QP} = \mathbf{AP_n} = \frac{\mathbf{AP} \bullet \mathbf{n}}{\|\mathbf{n}\|^2} * \mathbf{n}$$

$$\mathbf{AQ} = \mathbf{AP}_{\mathbf{n}}^{\perp} = \mathbf{AP} - \mathbf{AP_n}$$

$$\begin{aligned} Q &= A + \mathbf{AQ} \\ &= P + \mathbf{PQ} = P - \mathbf{QP} \end{aligned}$$

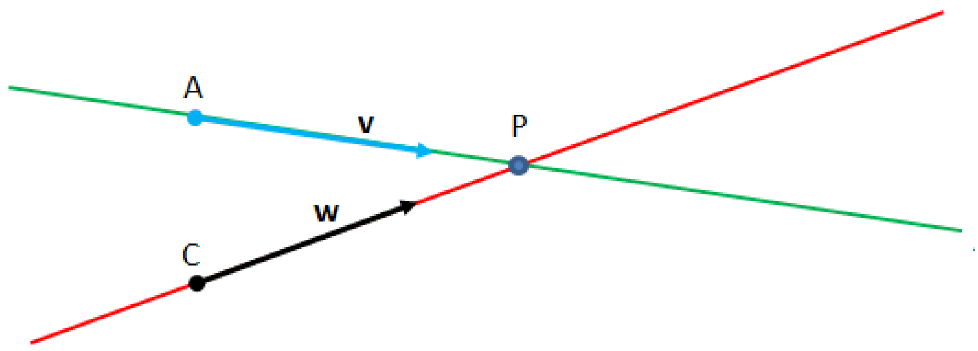


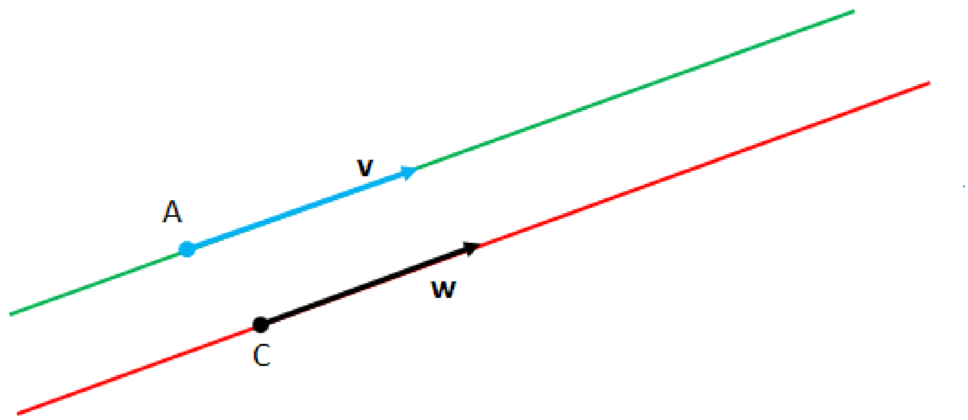
Esim: Kaksi 3D-suoraa voivat

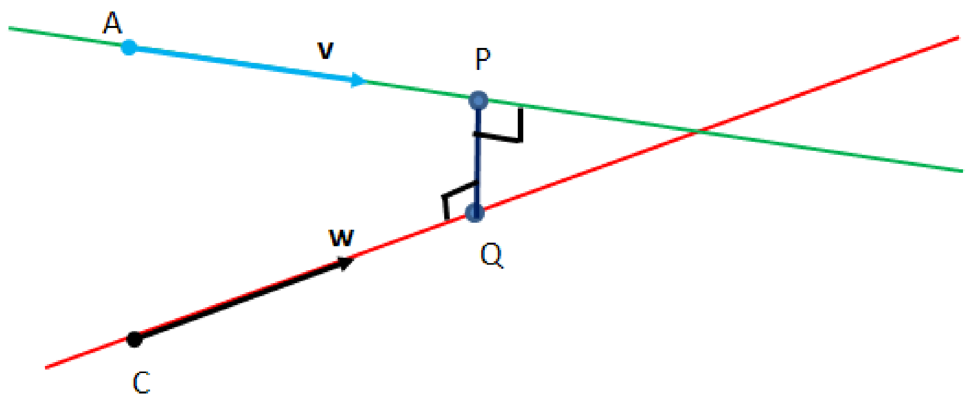
i) leikata toisensa pisteessä P

ii) olla yhdensuuntaisia (eivät leikkaa ja ovat samassa tasossa)

iii) olla ristikkäisiä (eivät leikkaa eivätkä ole samassa tasossa)


A
v
P
w
C


v
A
w
C


A
v
P
w
Q
C

Jos

$$\mathbf{n} = \mathbf{v} \times \mathbf{w}, \quad \mathbf{v}^{\perp} = \mathbf{n} \times \mathbf{v}, \quad \mathbf{w}^{\perp} = \mathbf{n} \times \mathbf{w}$$

$P = A + t * \mathbf{v}, \quad Q = C + s * \mathbf{w}$, missä

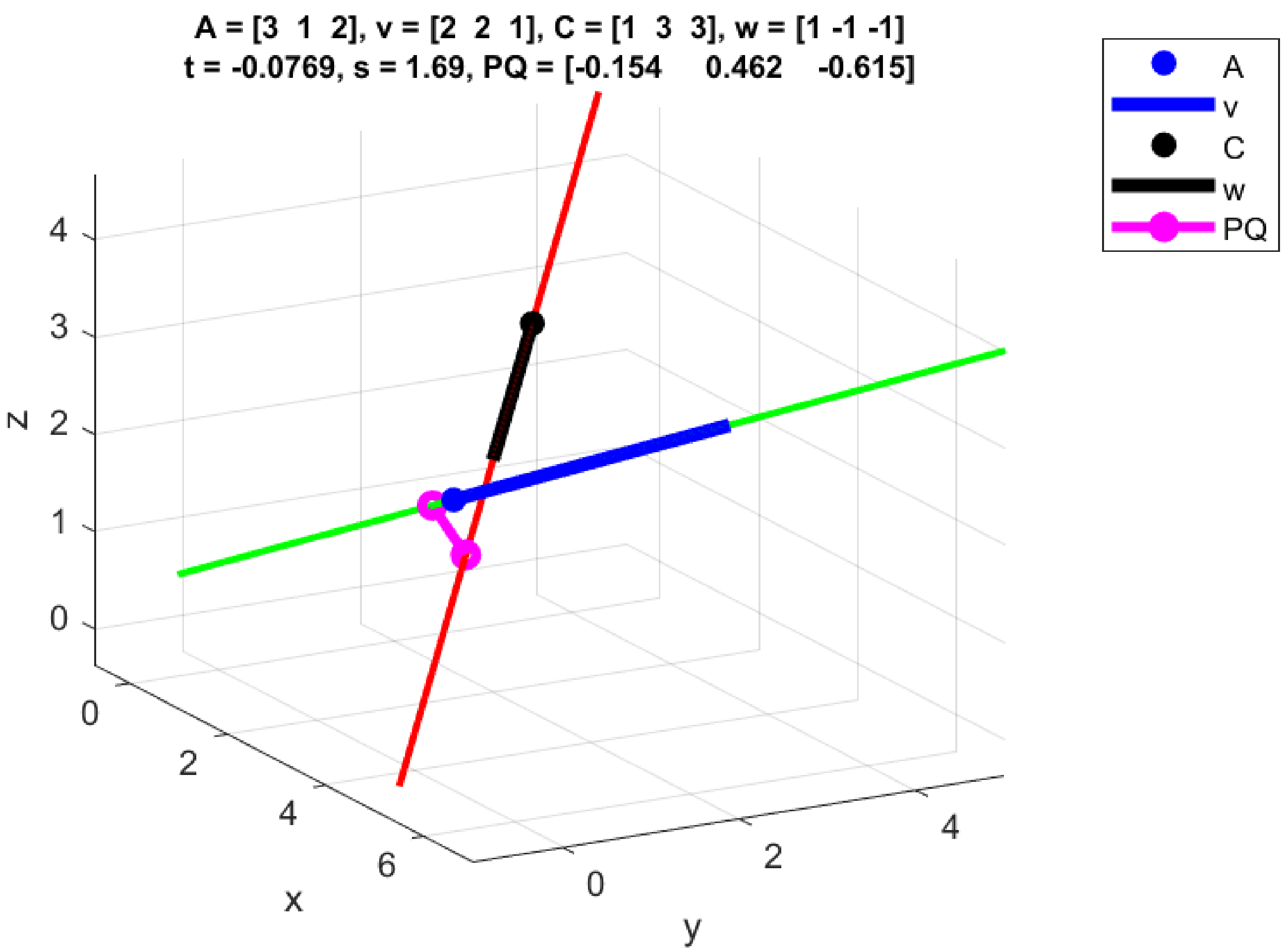
$$t = \frac{\mathbf{AC} \bullet \mathbf{w}^{\perp}}{\mathbf{v} \bullet \mathbf{w}^{\perp}}, \quad s = -\frac{\mathbf{AC} \bullet \mathbf{v}^{\perp}}{\mathbf{w} \bullet \mathbf{v}^{\perp}}$$

niin suorat

i) ovat yhdensuuntaisia, jos $\mathbf{n} = [0, 0, 0]$

ii) leikkaavat pisteessä $P = Q$

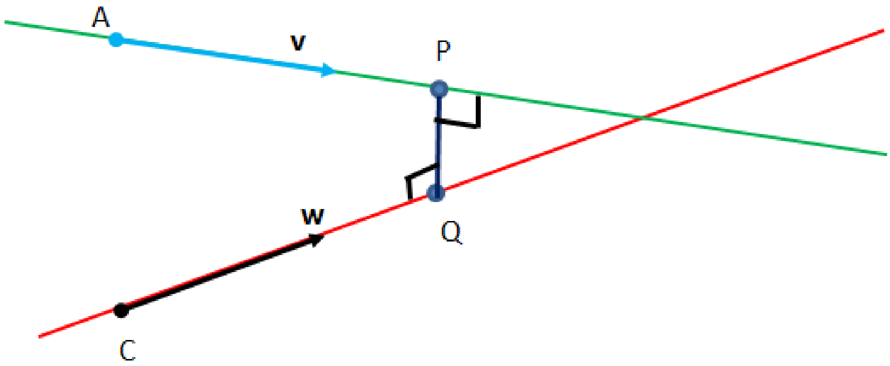
iii) ovat ristikkäisiä ja niiden välinen lyhin etäisyys on $\|\mathbf{PQ}\|$, jos $P \neq Q$
(eli $\mathbf{PQ}$ on kohtisuorassa molempia suoria vastaan).


A = [3 1 2], v = [2 2 1], C = [1 3 3], w = [1 -1 -1]
t = -0.0769, s = 1.69, PQ = [-0.154   0.462   -0.615]
A
v
C
w
PQ
z
x
y
0
1
2
3
4
0
2
4
6
0
2
4

Syy: Etsitään suorilta pisteet

$P = A + t * \mathbf{v}$ ja $Q = C + s * \mathbf{w}$ niin, että

$\mathbf{PQ} \perp \mathbf{v}$ ja $\mathbf{PQ} \perp \mathbf{w}$



$$\begin{aligned}
\mathbf{PQ} &= Q - P \\
&= (C + s * \mathbf{w}) - (A + t * \mathbf{v}) \\
&= \underbrace{C - A}_{=\mathbf{AC}} + s * \mathbf{w} - t * \mathbf{v}
\end{aligned}$$

Ratkaistaan t: koska $\mathbf{PQ} \perp \mathbf{v}$ ja $\mathbf{PQ} \perp \mathbf{w}$, niin $\mathbf{PQ}$ on vektorin $\mathbf{n} = \mathbf{v} \times \mathbf{w}$ suuntainen eli kohtisuorassa vektoreiden

$\mathbf{v}^{\perp} = \mathbf{n} \times \mathbf{v}$ ja $\mathbf{w}^{\perp} = \mathbf{n} \times \mathbf{w}$ kanssa eli

$$\mathbf{PQ} \bullet \mathbf{w}^{\perp} = 0$$

$$(\mathbf{AC} + s * \mathbf{w} - t * \mathbf{v}) \bullet \mathbf{w}^{\perp} = 0$$

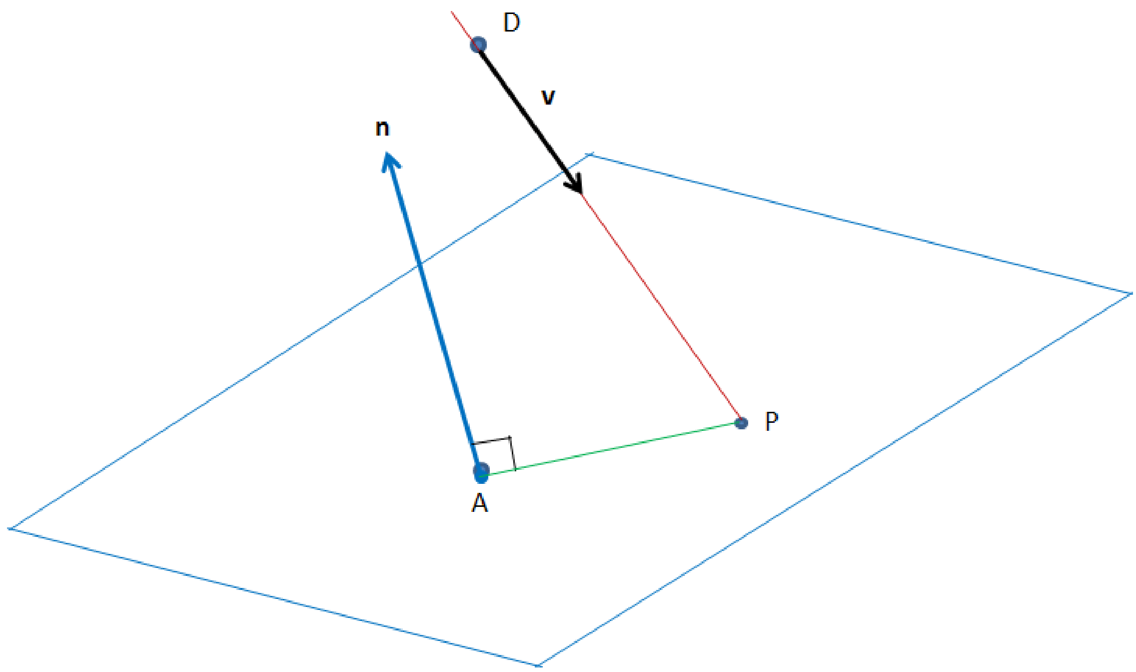
$$\mathbf{AC} \bullet \mathbf{w}^{\perp} + s * \underbrace{\mathbf{w} \bullet \mathbf{w}^{\perp}}_{=0} - t * \mathbf{v} \bullet \mathbf{w}^{\perp} = 0$$

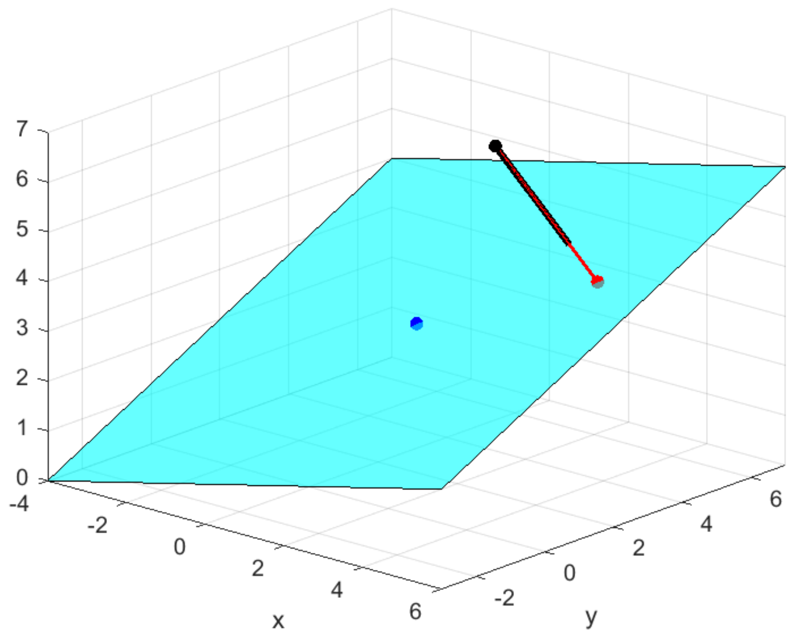
$$t = \frac{\mathbf{AC} \bullet \mathbf{w}^{\perp}}{\mathbf{v} \bullet \mathbf{w}^{\perp}}$$

Vastaavasti

$$\mathbf{PQ} \bullet \mathbf{v}^{\perp} = 0 \rightarrow s = -\frac{\mathbf{AC} \bullet \mathbf{v}^{\perp}}{\mathbf{w} \bullet \mathbf{v}^{\perp}}$$

Esim. Suoran $D, \mathbf{v}$ ja tason $A, \mathbf{n}$ leikkauspiste P





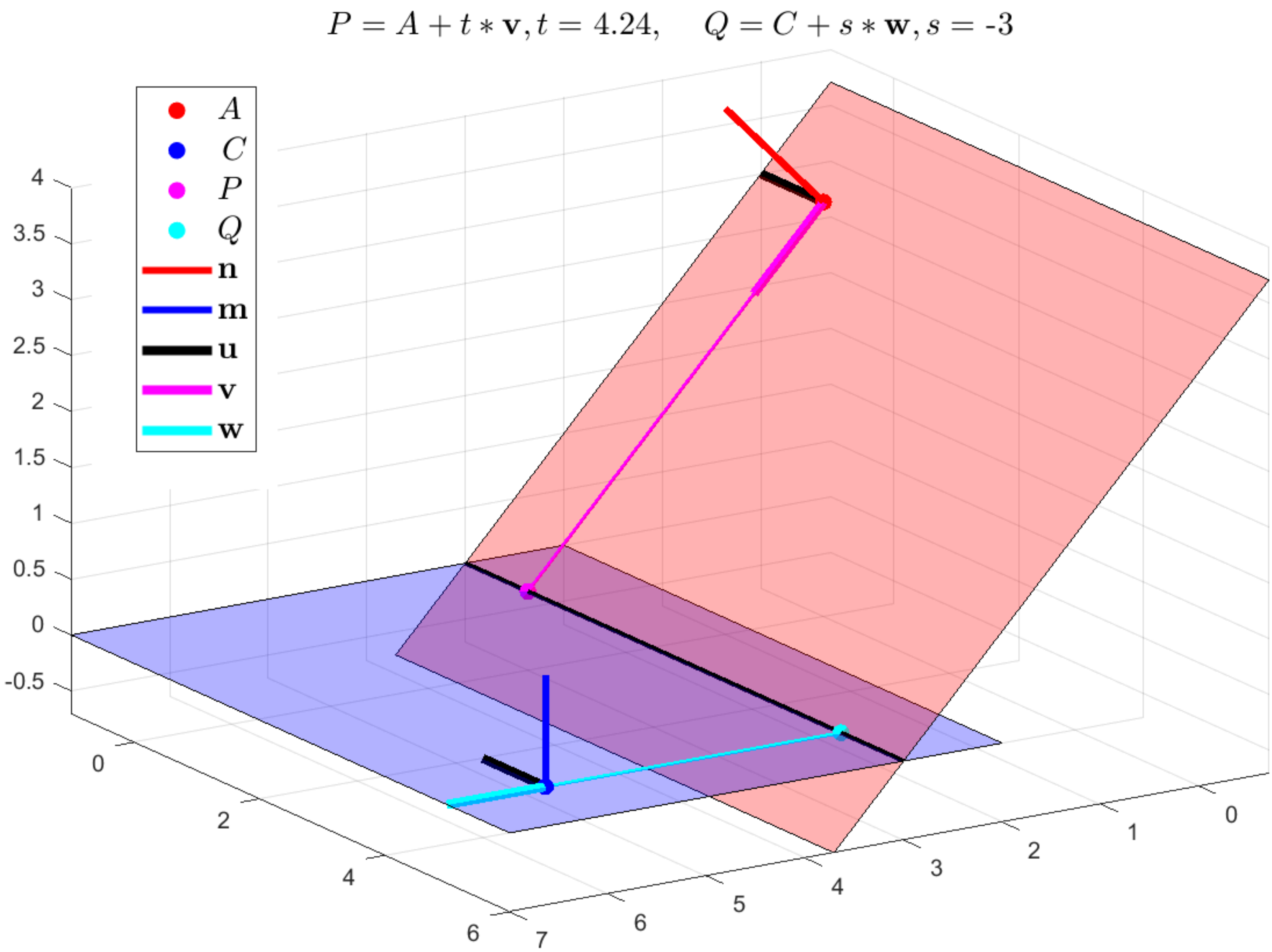
Suoran piste $P = D + t * \mathbf{v}$ on tasolla $A, \mathbf{n}$
jos $\mathbf{AP} \perp \mathbf{n}$ eli $\mathbf{AP} \bullet \mathbf{n} = 0$

$$\mathbf{AP} = \mathbf{AD} + t * \mathbf{v}$$

$$\begin{aligned} \mathbf{AP} \bullet \mathbf{n} &= (\mathbf{AD} + t * \mathbf{v}) \bullet \mathbf{n} \\ &= \mathbf{AD} \bullet \mathbf{n} + t * (\mathbf{v} \bullet \mathbf{n}) = 0 \end{aligned}$$

$$\rightarrow t = -\frac{\mathbf{AD} \bullet \mathbf{n}}{\mathbf{v} \bullet \mathbf{n}}$$

Esim: Tasojen $A, \mathbf{n}$ ja $C, \mathbf{m}$ leikkaussuora



Yksikkövektorit:

$$\mathbf{u} = \frac{\mathbf{n} \times \mathbf{m}}{\|\mathbf{n} \times \mathbf{m}\|}$$ on leikkaussuoran

suuntainen

$\mathbf{v} = \dfrac{\mathbf{n} \times \mathbf{u}}{\|\mathbf{n} \times \mathbf{u}\|}$ on tason $A, \mathbf{n}$ suuntainen,

kohtisuorassa leikkaussuoraa vastaan

$\mathbf{w} = \dfrac{\mathbf{m} \times \mathbf{u}}{\|\mathbf{m} \times \mathbf{u}\|}$ on tason $C, \mathbf{m}$ suuntainen,

kohtisuorassa leikkaussuoraa vastaan

Suoran $A, \mathbf{v}$ ja tason $C, \mathbf{m}$ leikkauspiste

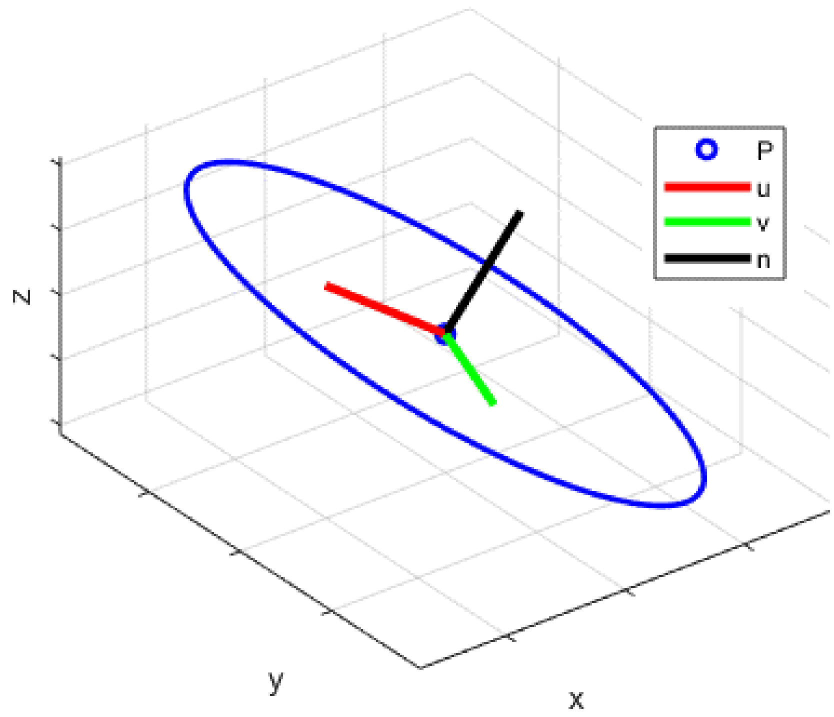
$$P = A + t * \mathbf{v}, \quad t = -\frac{\mathbf{CA} \bullet \mathbf{m}}{\mathbf{v} \bullet \mathbf{m}}$$

Suoran $C, \mathbf{w}$ ja tason $A, \mathbf{n}$ leikkauspiste

$$Q = C + s * \mathbf{w}, \quad s = -\frac{\mathbf{AC} \bullet \mathbf{n}}{\mathbf{w} \bullet \mathbf{n}}$$

Leikkaussuora kulkee pisteiden P ja Q kautta ja on vektorin $\mathbf{u}$ suuntainen

Esim. Ympyrä: keskipiste P, säde r, ympyrän tason normaali $\mathbf{n}$.



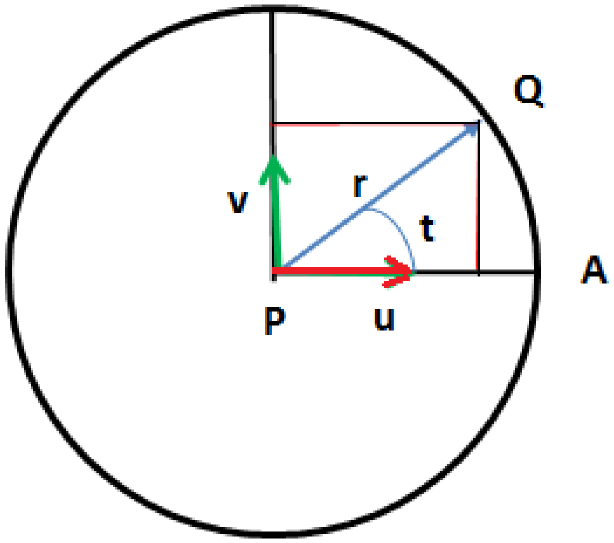
Kohtisuorat yksikkövektorit $\mathbf{u}$ ja $\mathbf{v}$ ympyrän tasossa:

Tapa 1: Jos $\mathbf{w}$ on $\mathbf{n}$:n kanssa erisuuntainen vektori, niin

$$\mathbf{u} = \frac{\mathbf{n} \times \mathbf{w}}{\|\mathbf{n} \times \mathbf{w}\|}, \qquad \mathbf{v} = \frac{\mathbf{n} \times \mathbf{u}}{\|\mathbf{n} \times \mathbf{u}\|}$$

Tapa 2: Jos A on ympyrän piste, niin

$$\mathbf{u} = \frac{\mathbf{PA}}{\|\mathbf{PA}\|}, \quad \mathbf{v} = \frac{\mathbf{n} \times \mathbf{u}}{\|\mathbf{n} \times \mathbf{u}\|}$$



Ympyrän pisteiden koordinaatit:

$$Q = P + r\cos(t) * \mathbf{u} + r\sin(t) * \mathbf{v} \text{ eli}$$

$$\begin{cases} Qx = Px + r\cos(t) * ux + r\sin(t) * vx \\ Qy = Py + r\cos(t) * uy + r\sin(t) * vy \\ Qz = Pz + r\cos(t) * uz + r\sin(t) * vz \end{cases}$$

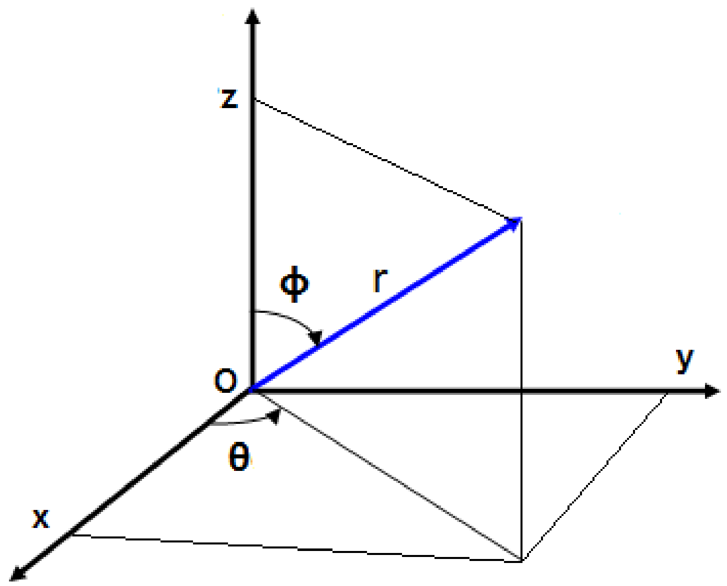
Pallokoordinaatit

(spherical coordinates)

$r =$ etäisyys O:sta

$\theta = 0 \ldots 360^\circ$ (pituuspiiri)

$\phi = 0 \ldots 180^\circ$ (leveyspiiri)



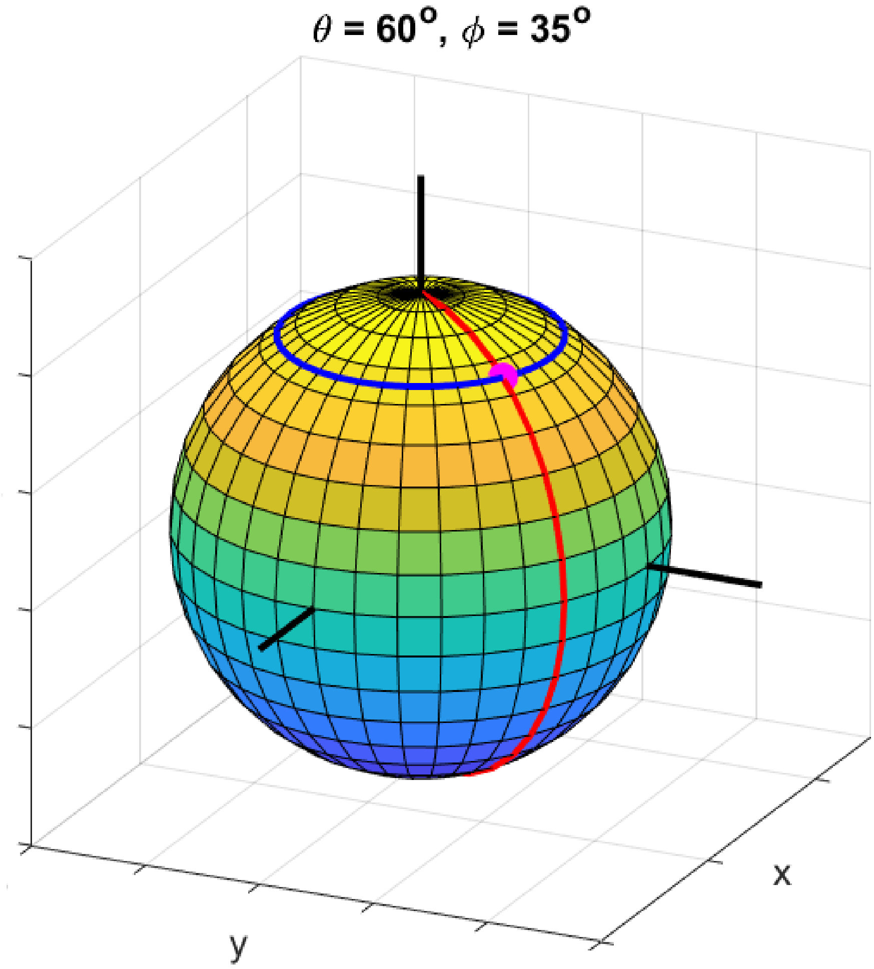
Seinäjoki: 22.8° E, 62.8° N eli

$\theta = 22.8^\circ$, $\phi = 90^\circ - 62.8^\circ = 27.2^\circ$

$$\begin{cases} x = r\sin(\phi)\cos(\theta) \\ y = r\sin(\phi)\sin(\theta) \\ z = r\cos(\phi) \end{cases}$$

$$\begin{cases} r = \sqrt{x^2 + y^2 + z^2} \\ \theta = \mathrm{atan2}(y, x) \quad (\text{-}180^\circ \ldots 180^\circ) \\ \phi = \cos^{-1}(z/r) \end{cases}$$

Esim. Pallo, keskipiste $[x_0, y_0, z_0]$, säde r

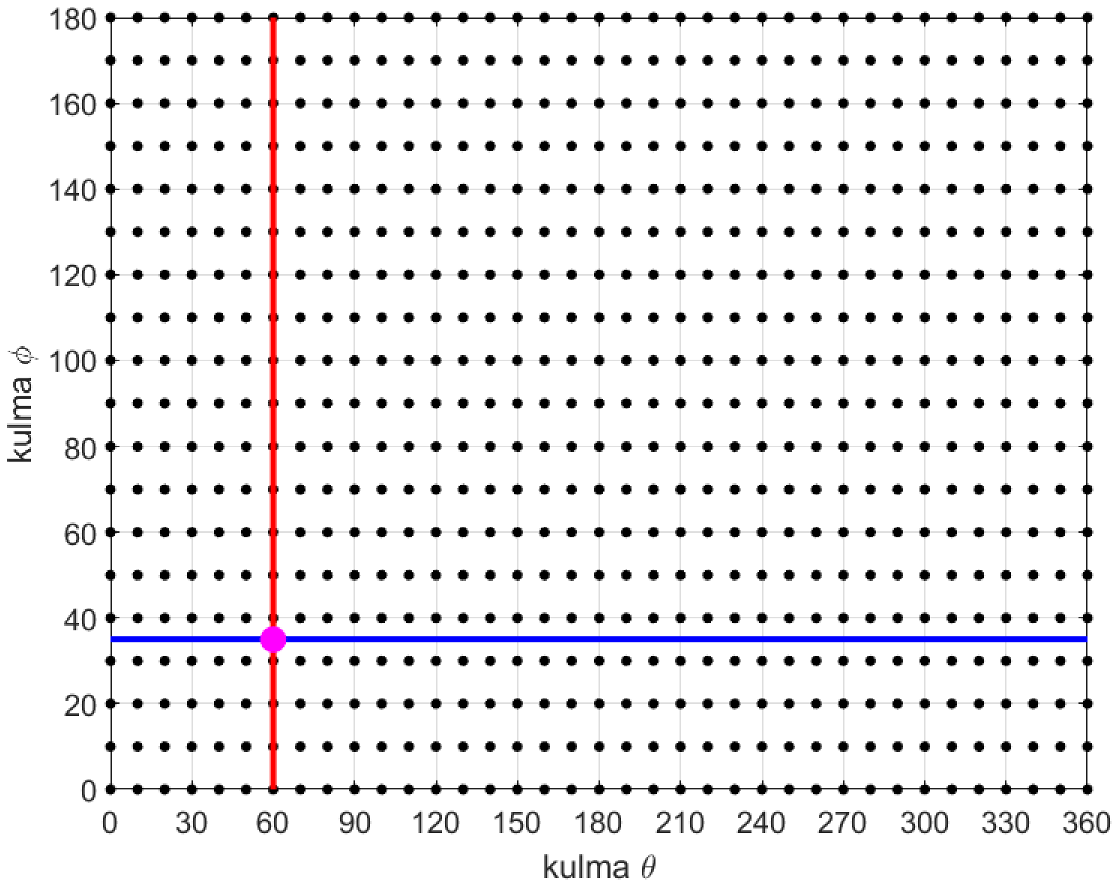


Yhtälö:

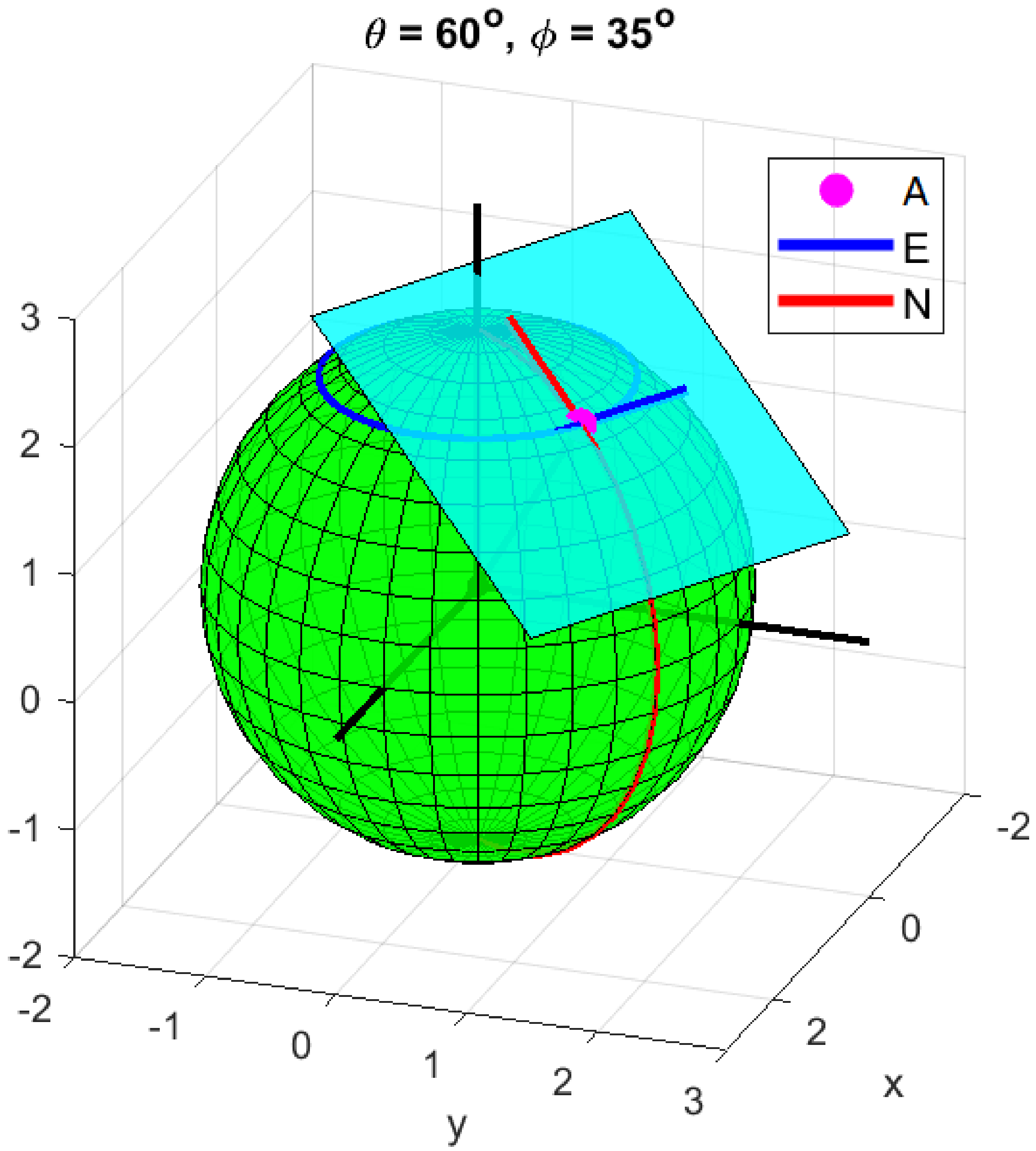
$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$$

Pisteiden koordinaatit

$$\begin{cases} x = x_0 + r\sin(\phi)\cos(\theta) \\ y = y_0 + r\sin(\phi)\sin(\theta) \\ z = z_0 + r\cos(\phi) \end{cases}$$



Esim. Pallon (keskipiste P) tangenttitaso eli pallon pinnan suuntainen taso pisteessä A



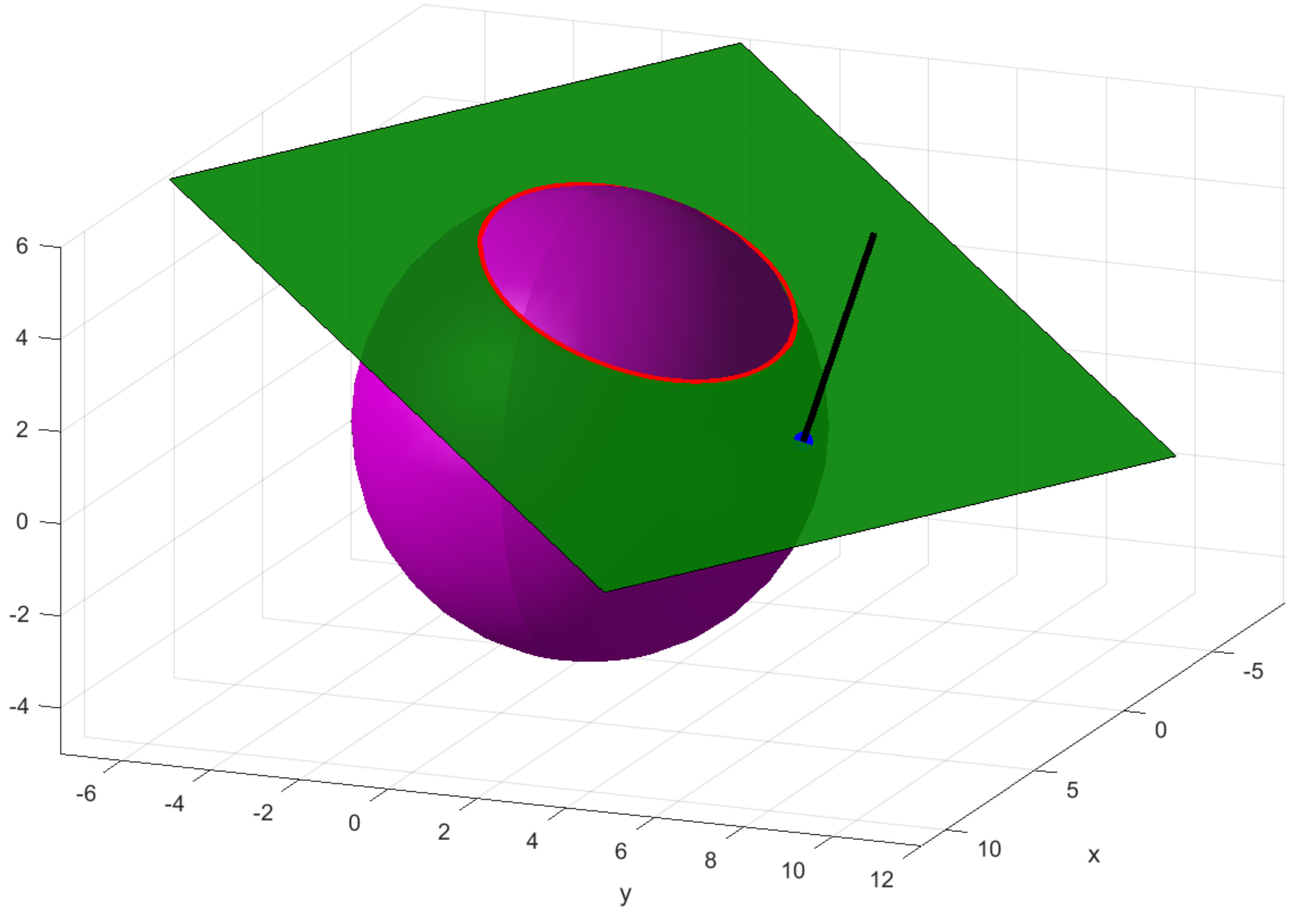
tangenttitason normaali $\mathbf{n} = \mathbf{PA}$

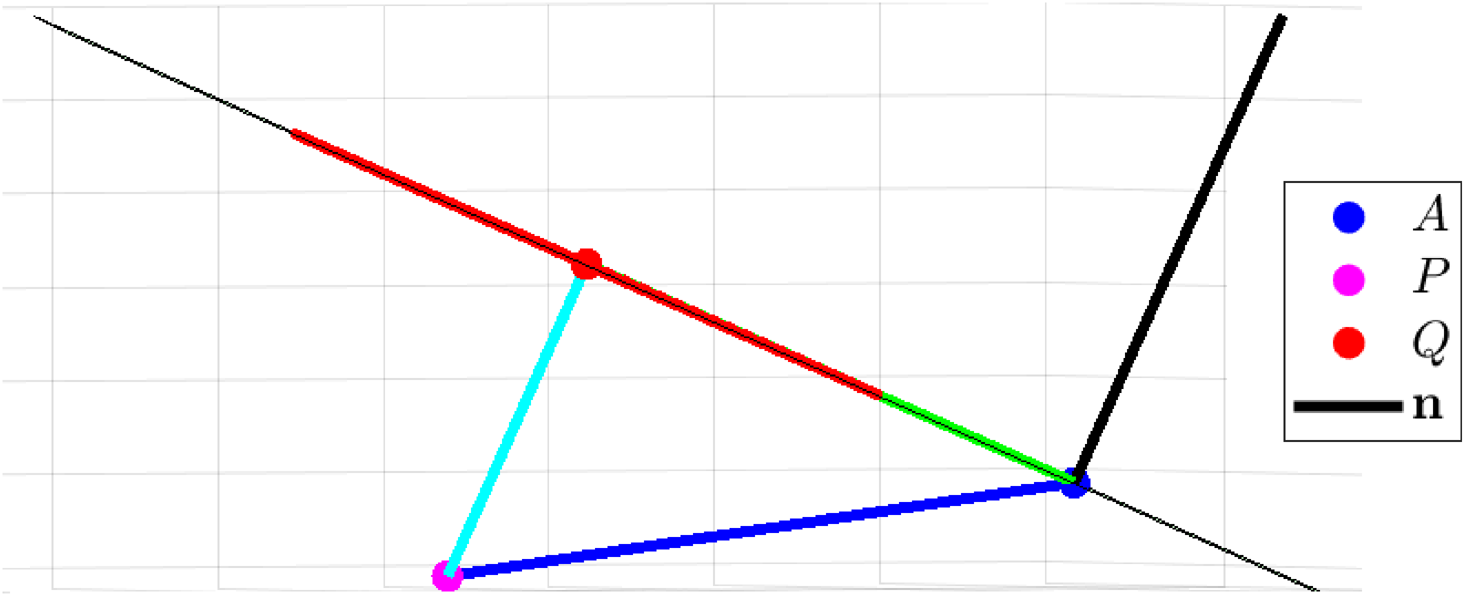
tangenttitason suuntaiset vektorit

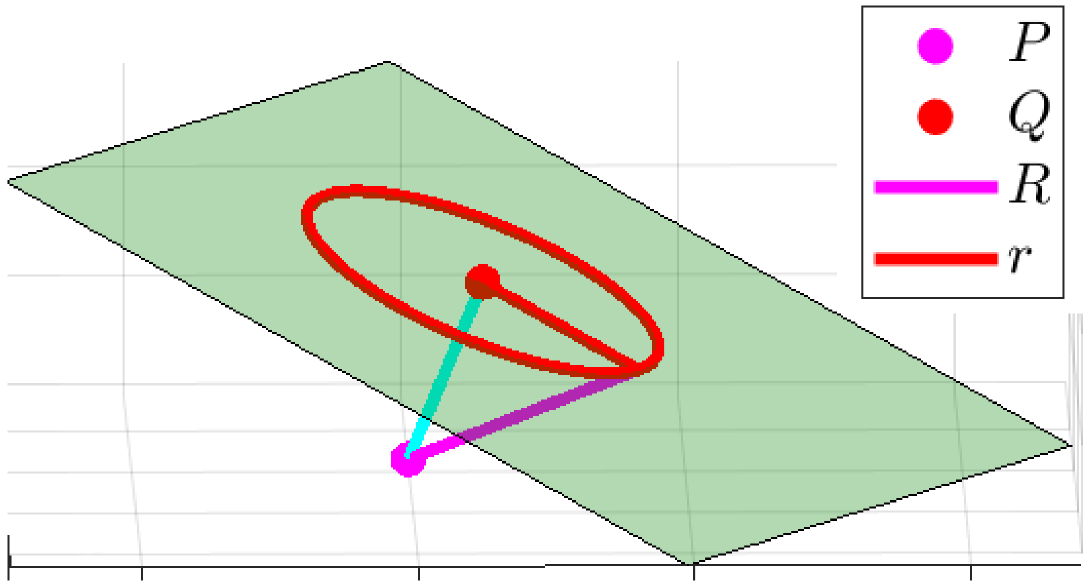
$\mathbf{E} = [0, 0, 1] \times \mathbf{n}$ (A:sta itään)

$\mathbf{N} = \mathbf{n} \times \mathbf{E}$ (A:sta pohjoiseen)

Esim. Pallon P, R ja tason $A, \mathbf{n}$ leikkausympyrä





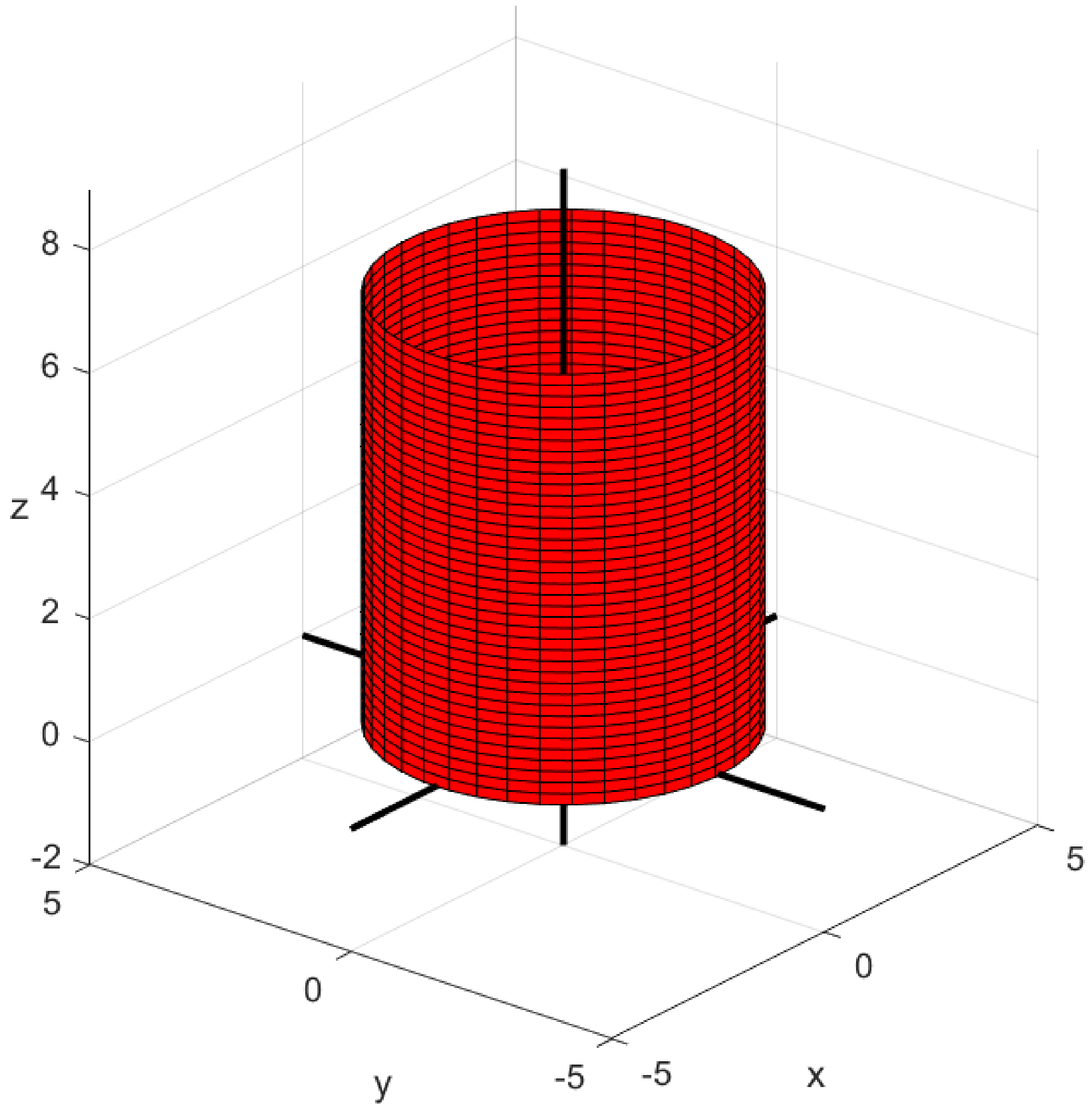


ympyrän keskipiste $Q = P + \mathbf{PA_n}$

säde $r = \sqrt{R^2 - \|\mathbf{PQ}\|^2}$

leikkaavat, jos $\|\mathbf{PQ}\| \le R$

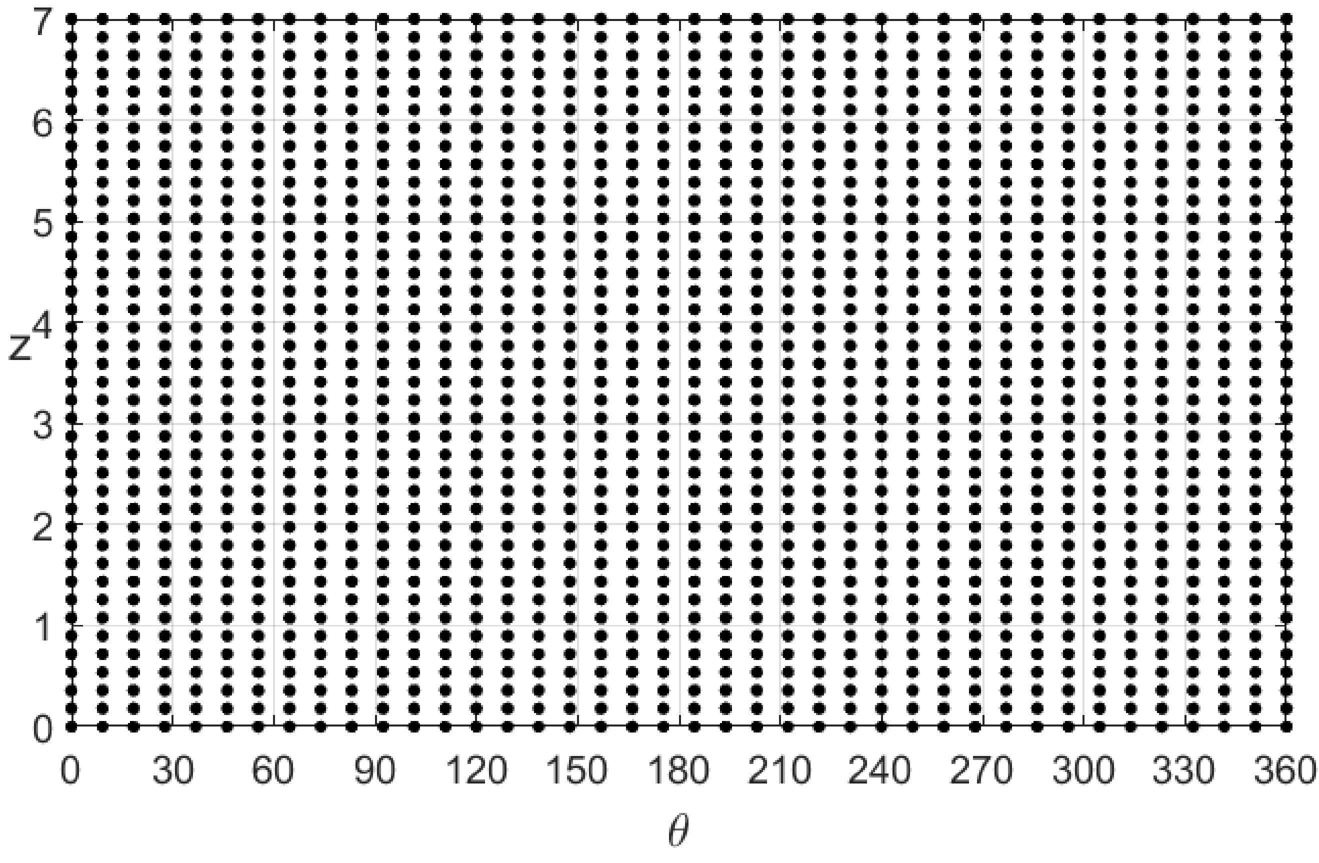
Esim: Lieriö, säde r, korkeus h



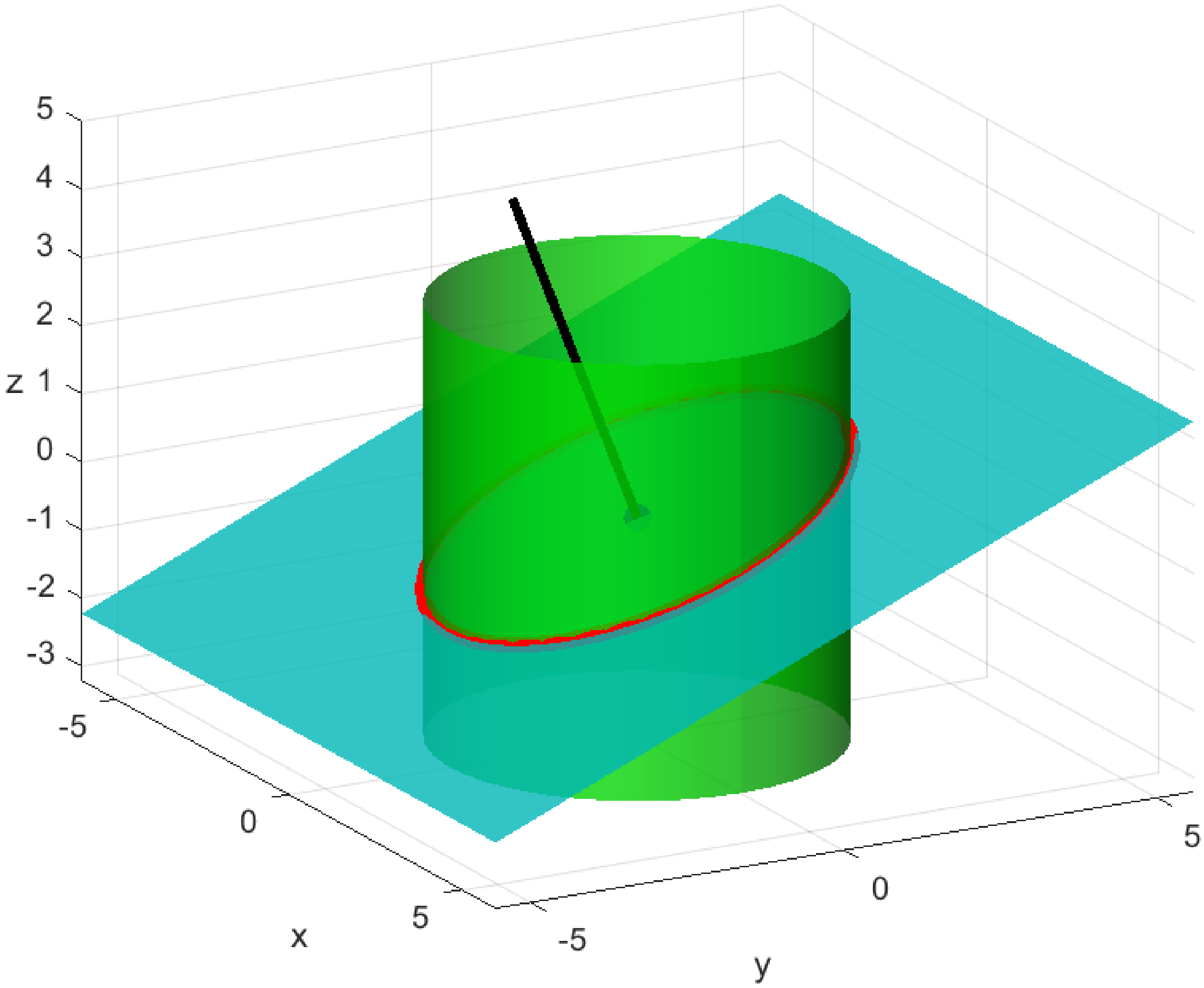
Jos pohjaympyrän keskipiste on [0,0,0] ja akselina z-akseli, niin lieriön pisteiden koordinaatit ovat

$$x = r\cos(\theta),\ y = r\sin(\theta),\ \theta = 0\ldots 360^\circ$$

$$z = 0\ldots h$$

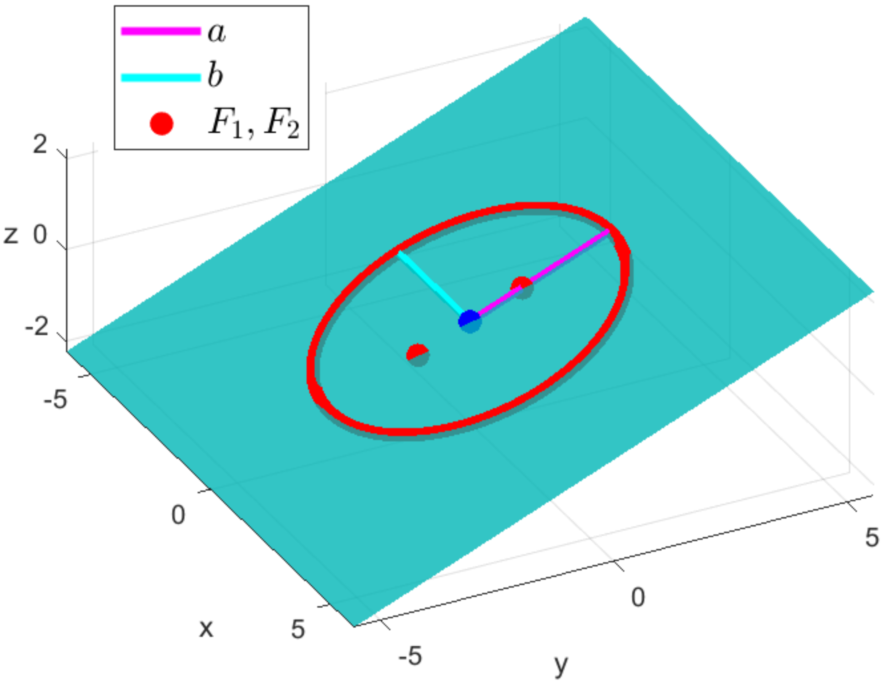


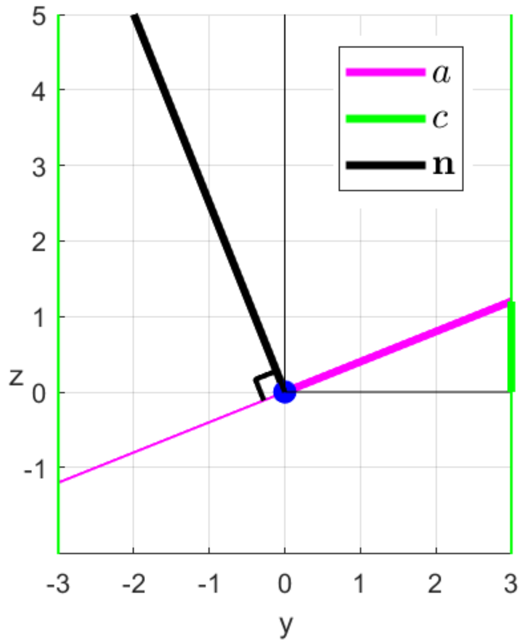
Esim. Lieriön ja tason leikkauskäyrä on ellipsi



Lieriön säde r, akselina z-akseli

Taso $O = [0, 0, 0]$, $\mathbf{n} = [0, ny, nz]$, $ny < 0$





$$\frac{c}{r} = \frac{-ny}{nz} \rightarrow c = \frac{-ny}{nz} \cdot r$$

Puoliakseleiden pituudet

$a = \sqrt{r^2 + c^2}$, $b = r$

Niiden suuntaiset yksikkövektorit

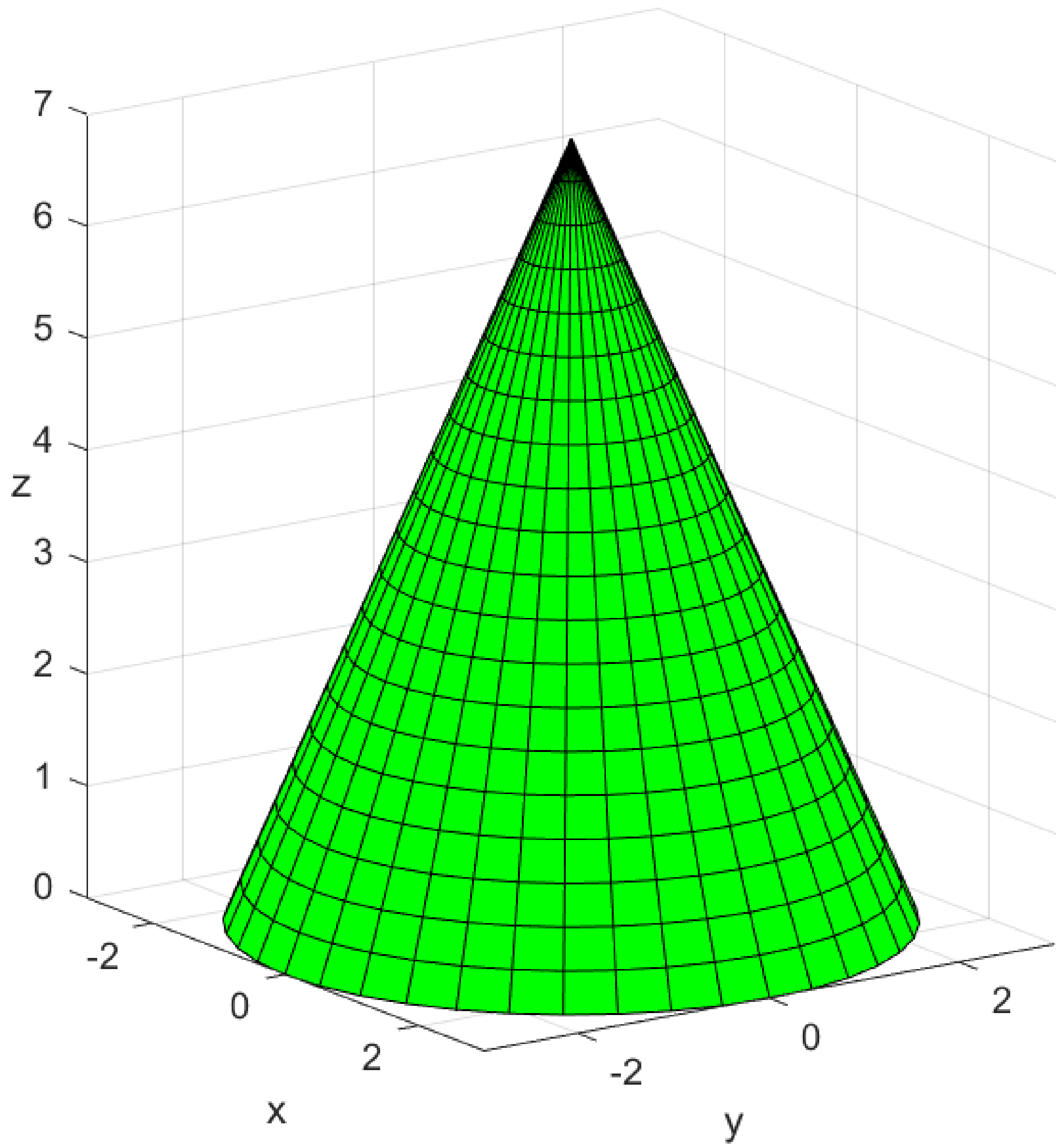
$\mathbf{u} = [0, r/a, c/a]$, $\mathbf{v} = [-1, 0, 0]$

Ellipsin pisteet

$P = a\cos(\theta) * \mathbf{u} + b\sin(\theta) * \mathbf{v}$

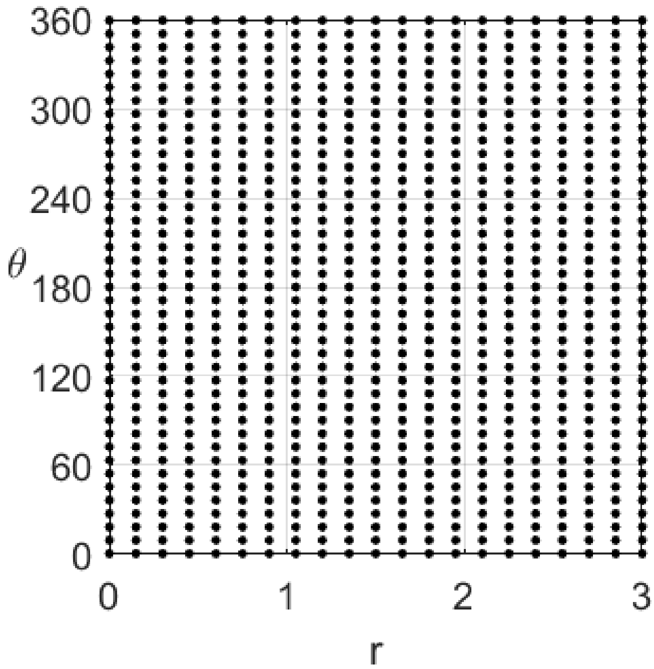
$\theta = 0 \ldots 360^\circ$

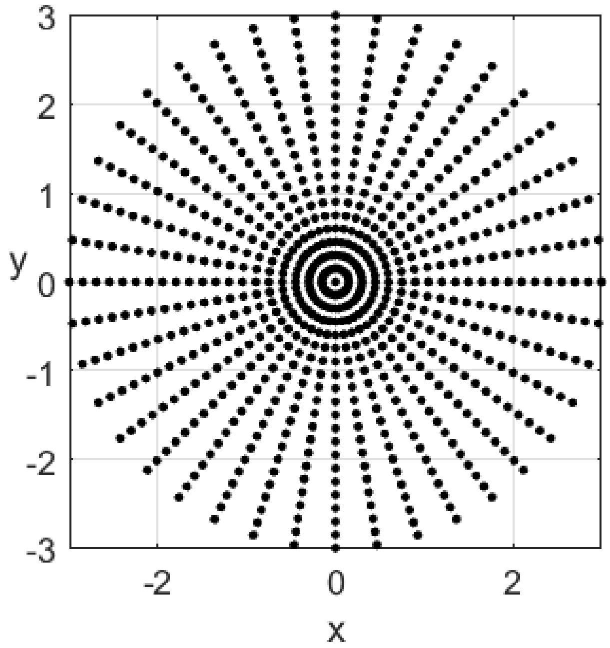
Esim. Kartio, pohjan säde R, korkeus h, akselina z-akseli



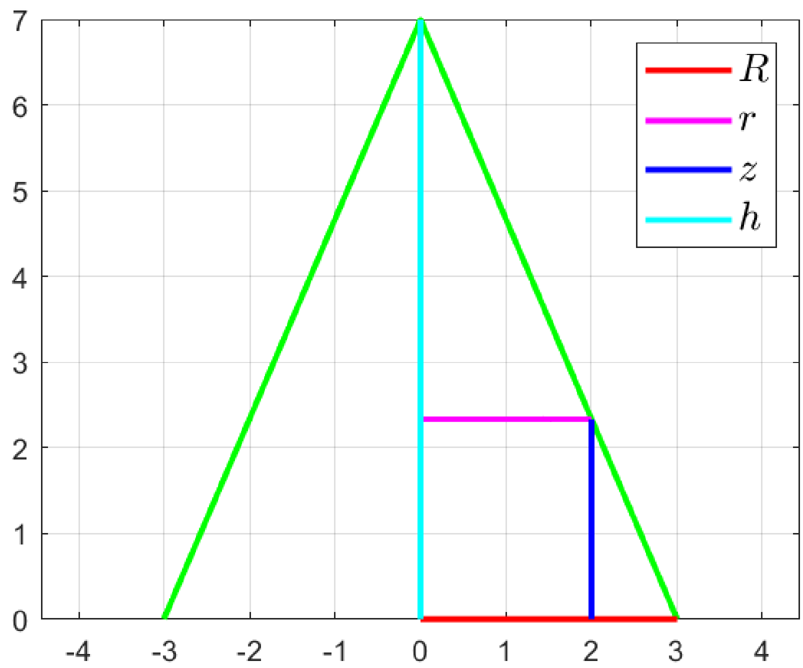
$$x = r\cos(\theta),\ y = r\sin(\theta)$$

$$r = 0 \ldots R,\ \theta = 0 \ldots 360^\circ$$

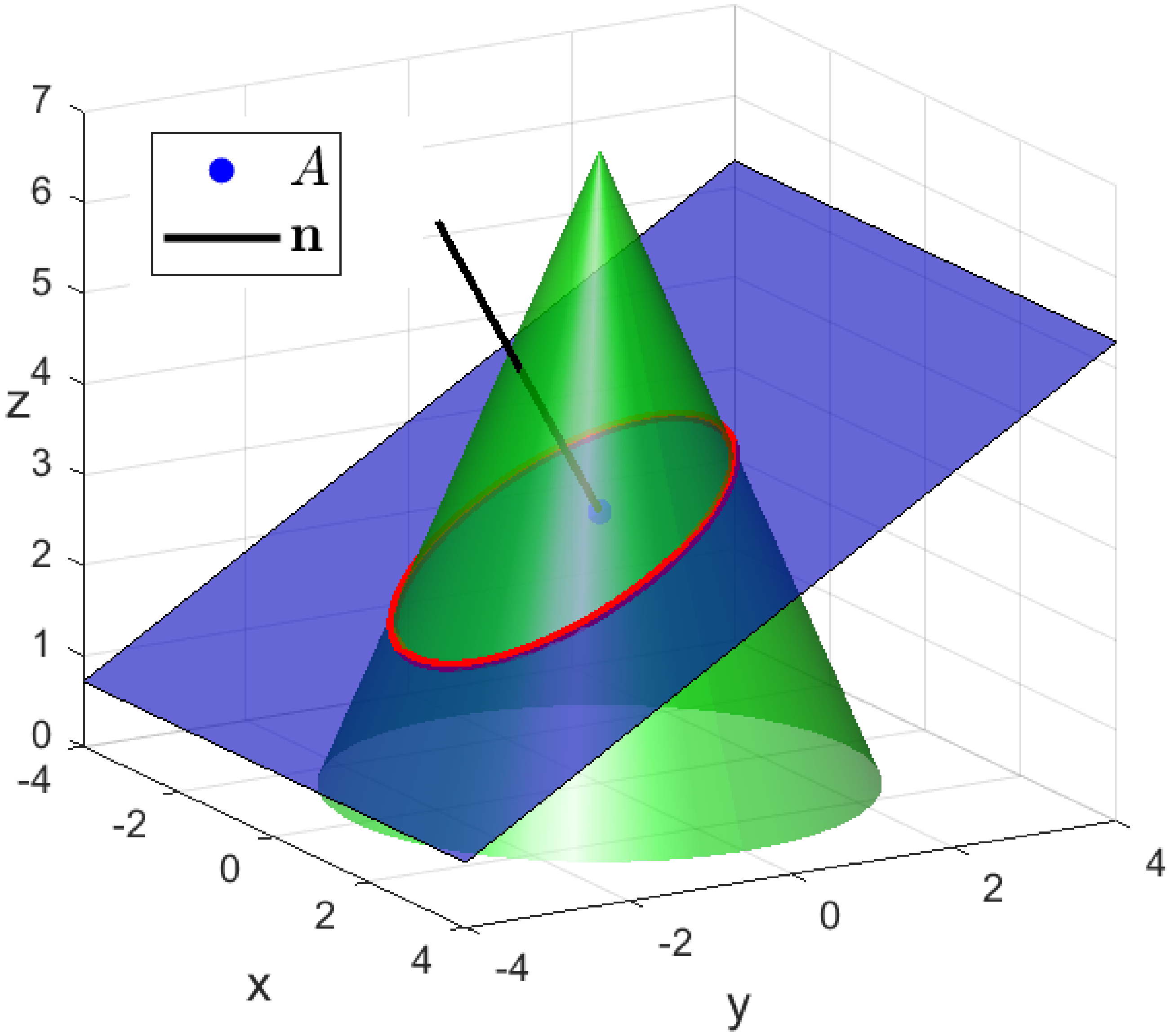




$$\frac{h-z}{r} = \frac{h}{R} \rightarrow z = \left(1 - \frac{r}{R}\right) \cdot h$$



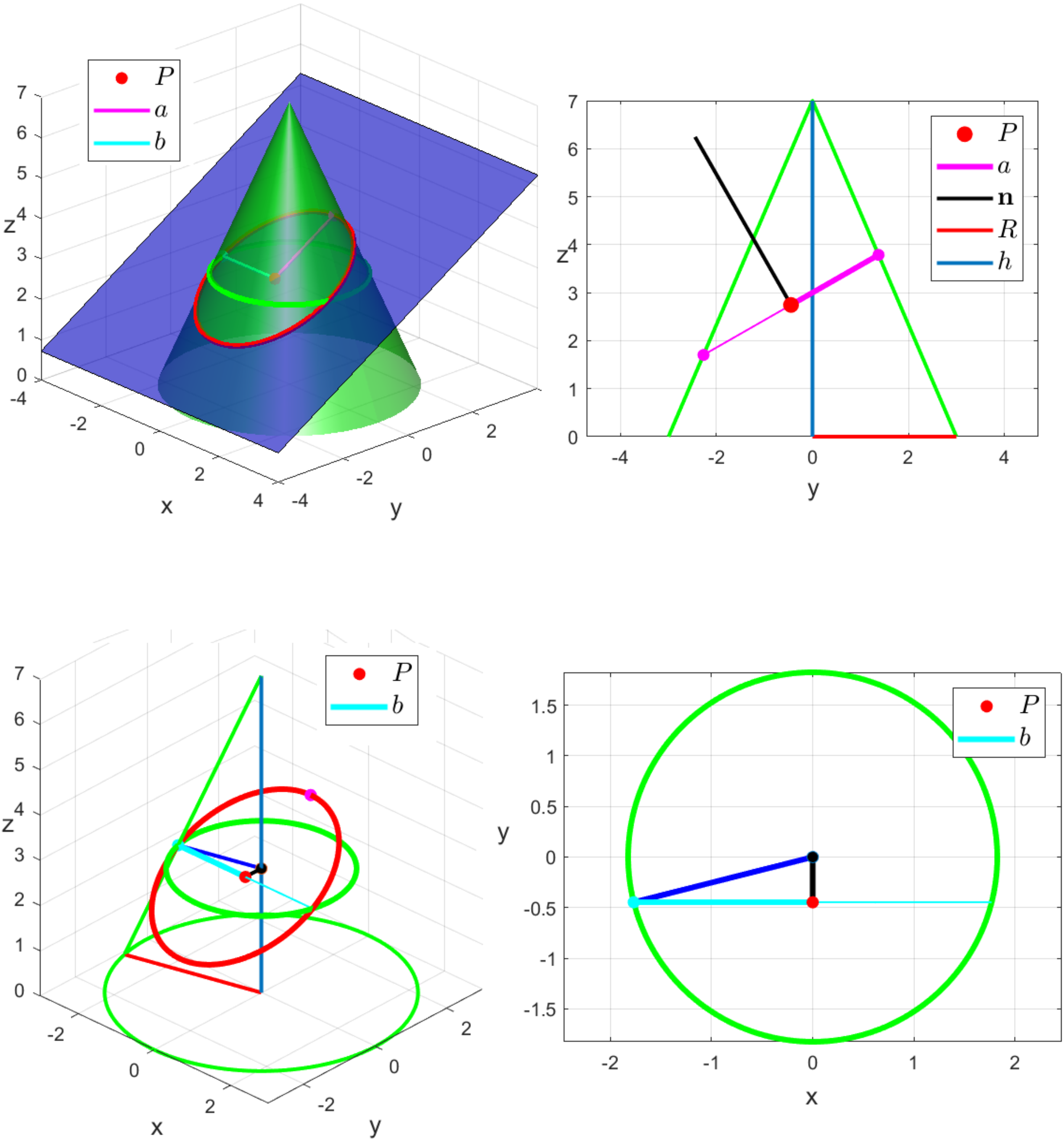
Esim. Kartion ja tason leikkauskäyrä on ellipsi



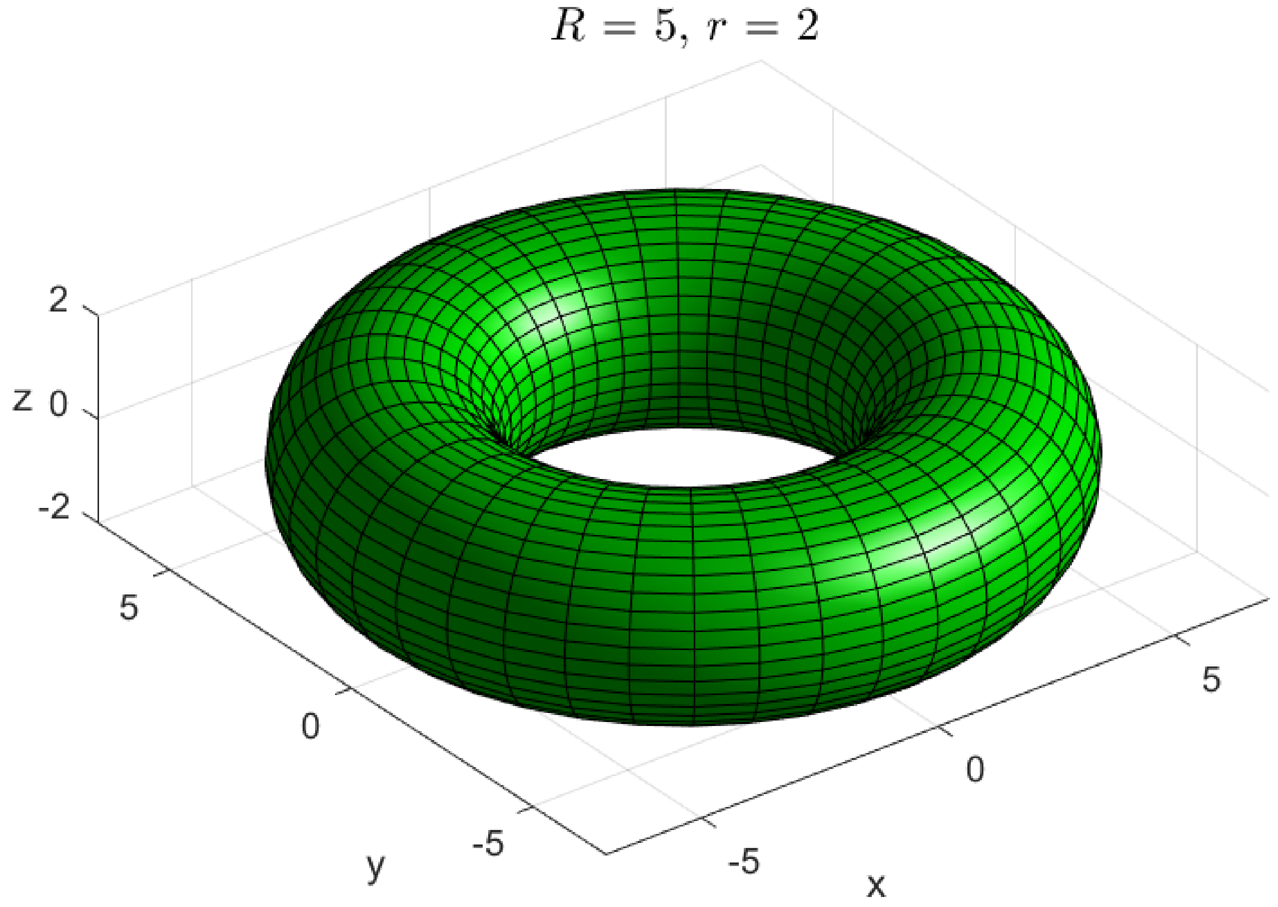
Kartion pohjan säde R, korkeus h, akselina z-akseli

Tason piste $A = [0, 0, Az]$, normaali $\mathbf{n} = [0, ny, nz]$, $ny < 0$

Ellipsin keskipiste P, puoliakselit a ja b



Esim. Torus, säteet R ja r



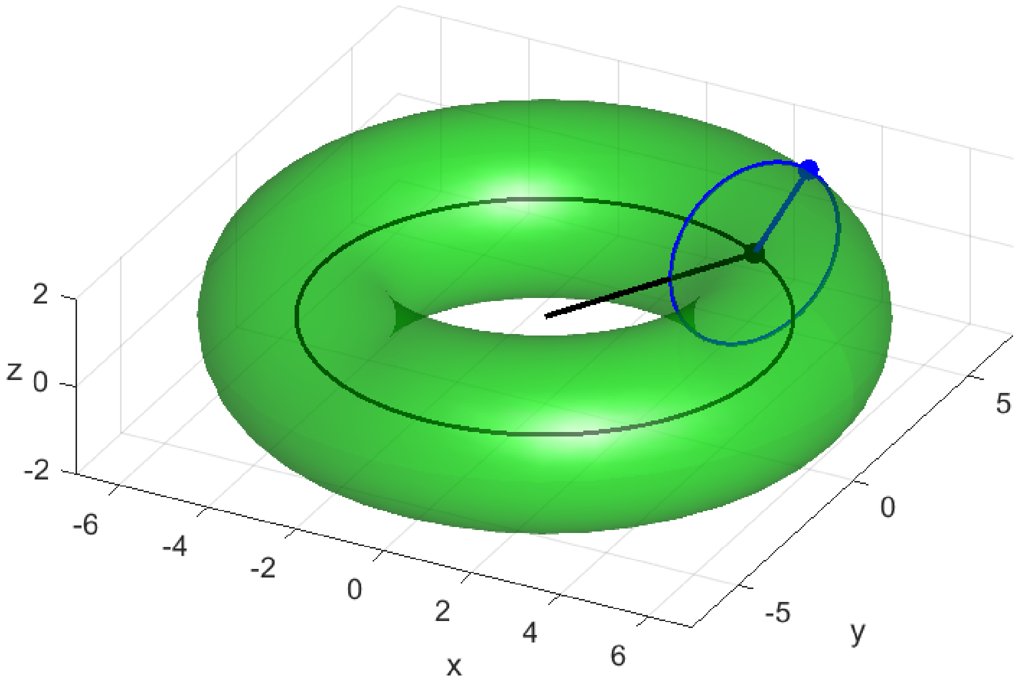
$$x = (R + r\cos(\phi))\cos(\theta)$$

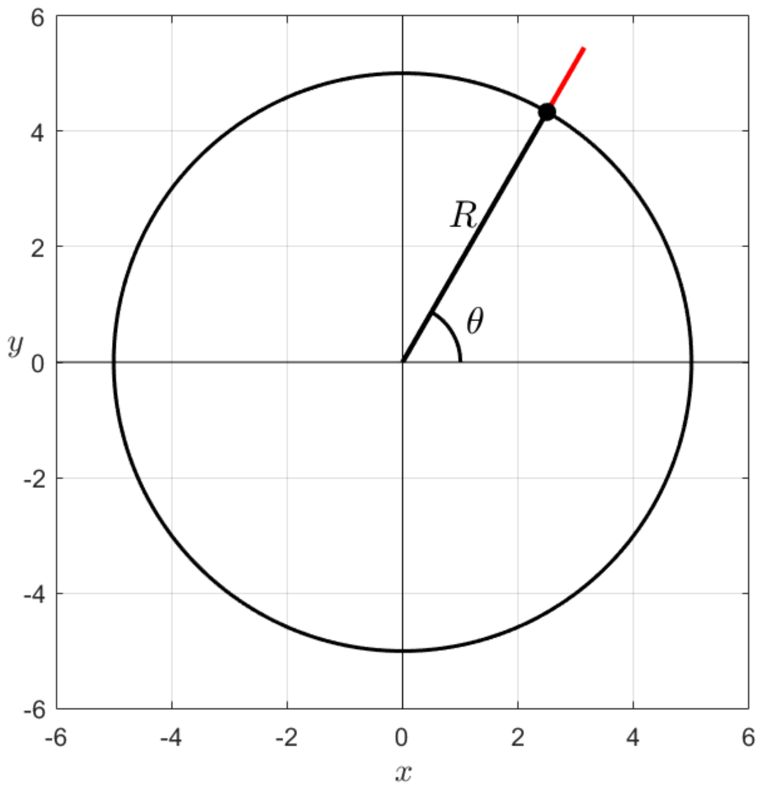
$$y = (R + r\cos(\phi))\sin(\theta)$$

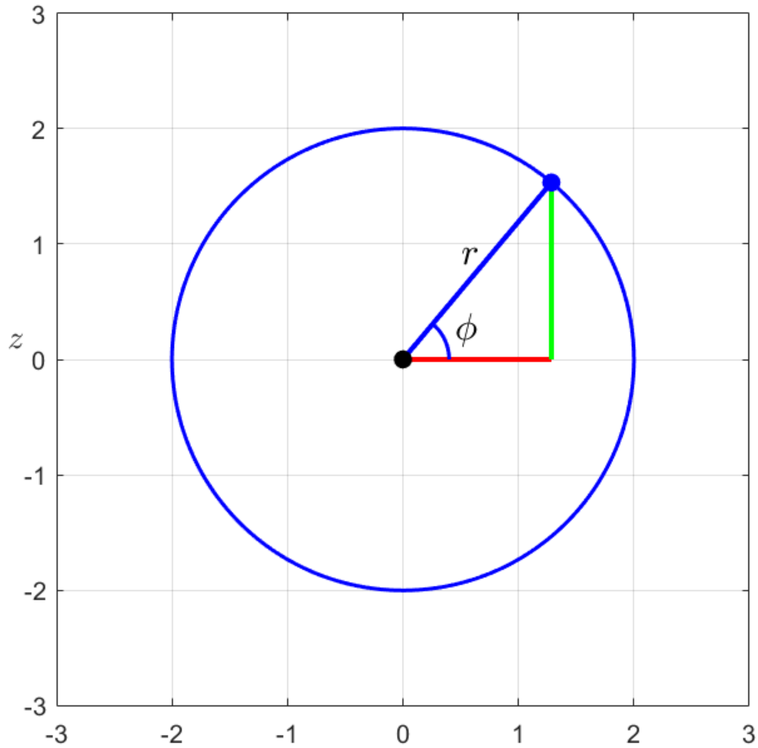
$$z = r\sin(\phi)$$

$$\theta = 0 \ldots 360^\circ$$

$$\phi = 0 \ldots 360^\circ$$

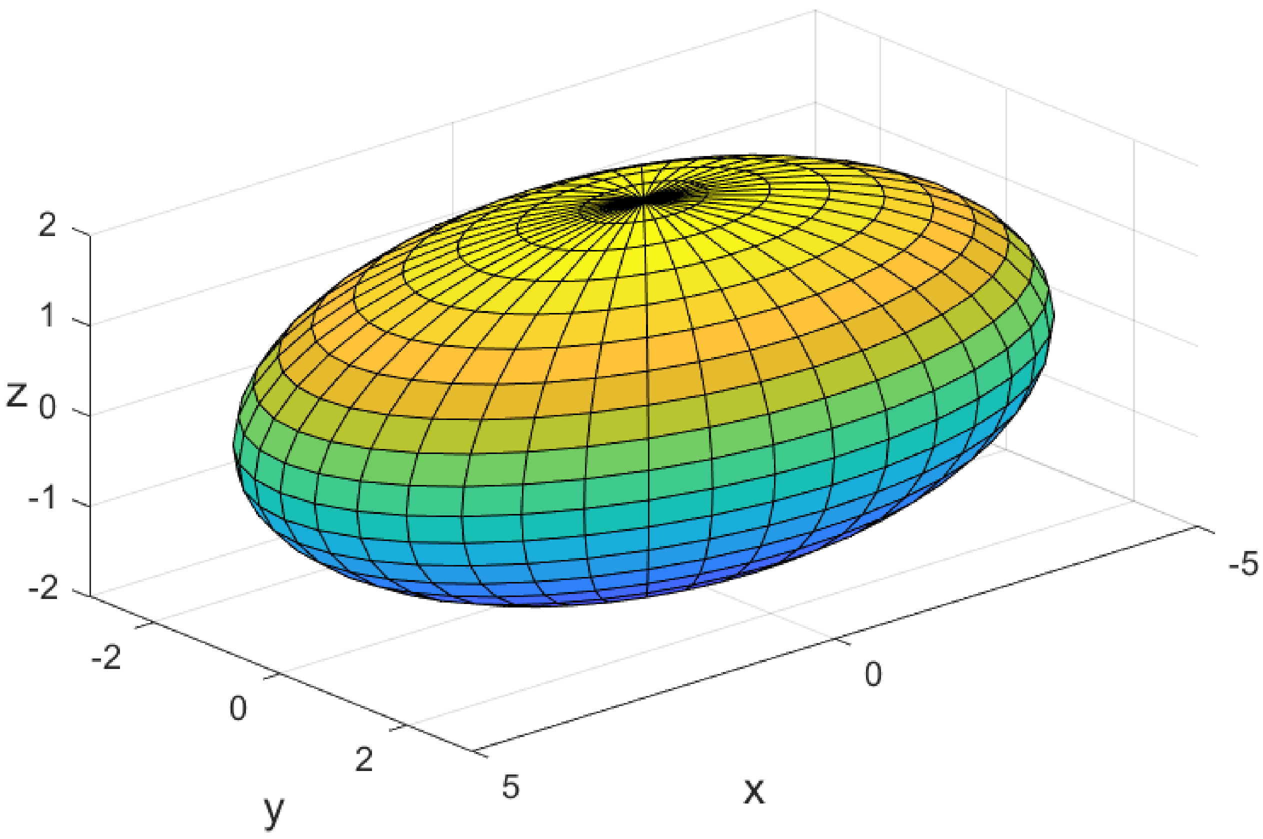

2
z 0
-2
-6
-4
-2
0
2
4
6
x
5
0
-5
y

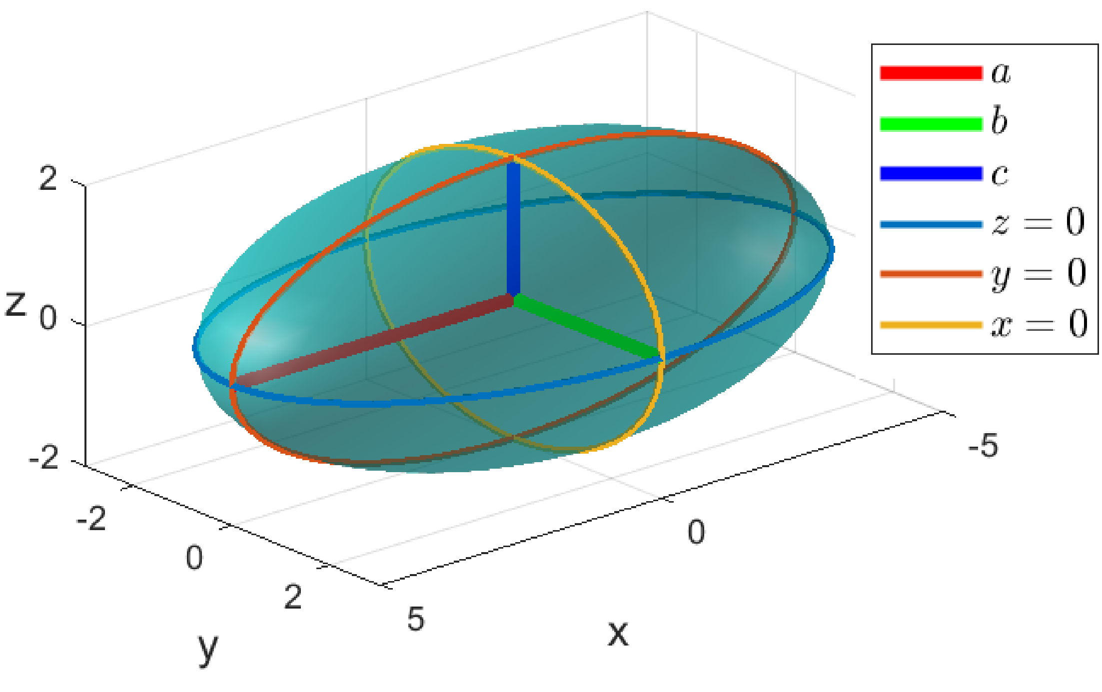

6
4
2
y 0
-2
-4
-6
-6
-4
-2
0
2
4
6
x
R
θ


3
2
1
z 0
-1
-2
-3
-3
-2
-1
0
1
2
3
r
ϕ

Toisen asteen pinnat

Ellipsoidi





Yhtälö: $$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

$$z=\pm h, h<c \rightarrow \frac{x^2}{a^2}+\frac{y^2}{b^2}=1-\frac{h^2}{c^2} \text{ (ellipsi)}$$

$$y=\pm h, h<b \rightarrow \frac{x^2}{a^2}+\frac{z^2}{c^2}=1-\frac{h^2}{b^2} \text{ (ellipsi)}$$

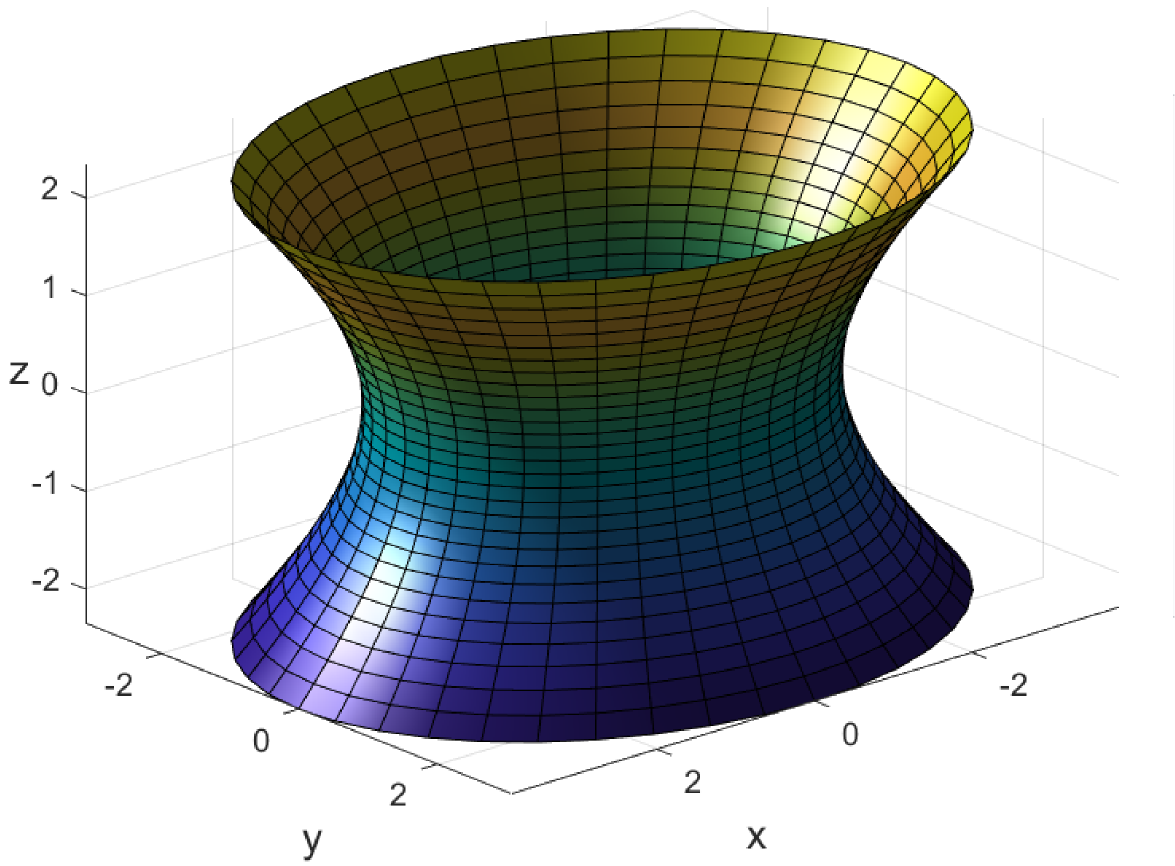
$$x=\pm h, h<a \rightarrow \frac{y^2}{b^2}+\frac{z^2}{c^2}=1-\frac{h^2}{a^2} \text{ (ellipsi)}$$

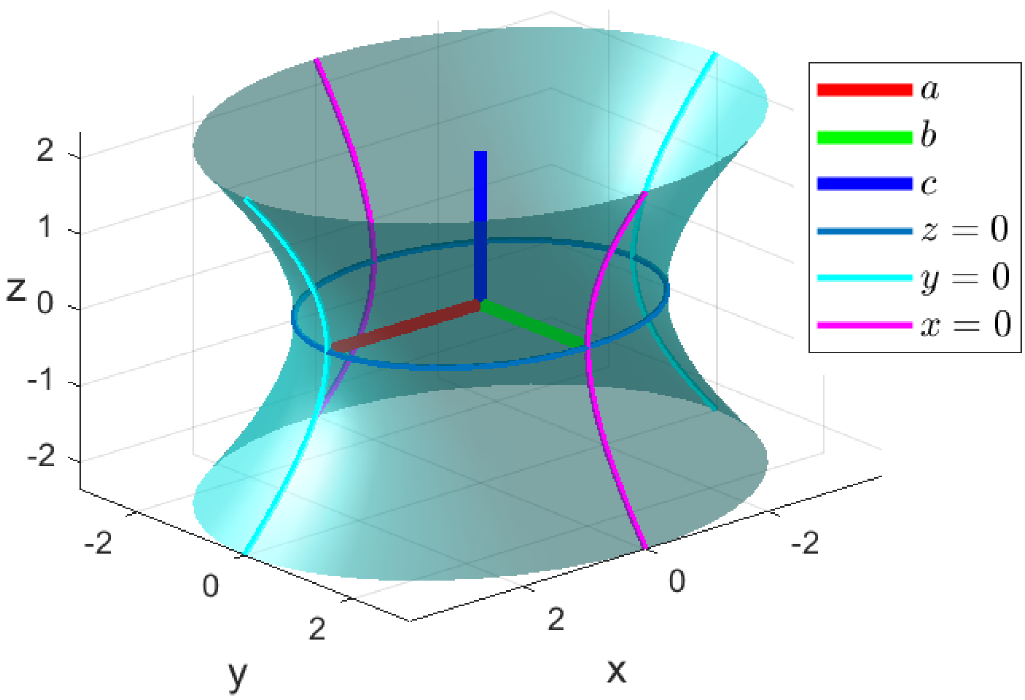
Parametrimuoto: pallokoordinaateissa

$$\begin{cases} x=a\sin(\phi)\cos(\theta) \\ y=b\sin(\phi)\sin(\theta) \\ z=c\cos(\phi) \end{cases} \qquad \begin{array}{l} \theta=0\ldots 360^\circ \\ \phi=0\ldots 180^\circ \end{array}$$

Hyperbolinen hyperboloidi

One-sheeted hyperboloid





Yhtälö: $$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$$

$$z=h\to\frac{x^2}{a^2}+\frac{y^2}{b^2}=1+\frac{h^2}{c^2}\ \text{(ellipsi)}$$

$$y=h\to\frac{x^2}{a^2}-\frac{z^2}{c^2}=1-\frac{h^2}{b^2}\ \text{(hyperbeli)}$$

$$x=h\to\frac{y^2}{b^2}-\frac{z^2}{c^2}=1-\frac{h^2}{a^2}\ \text{(hyperbeli)}$$

Parametrimuoto:

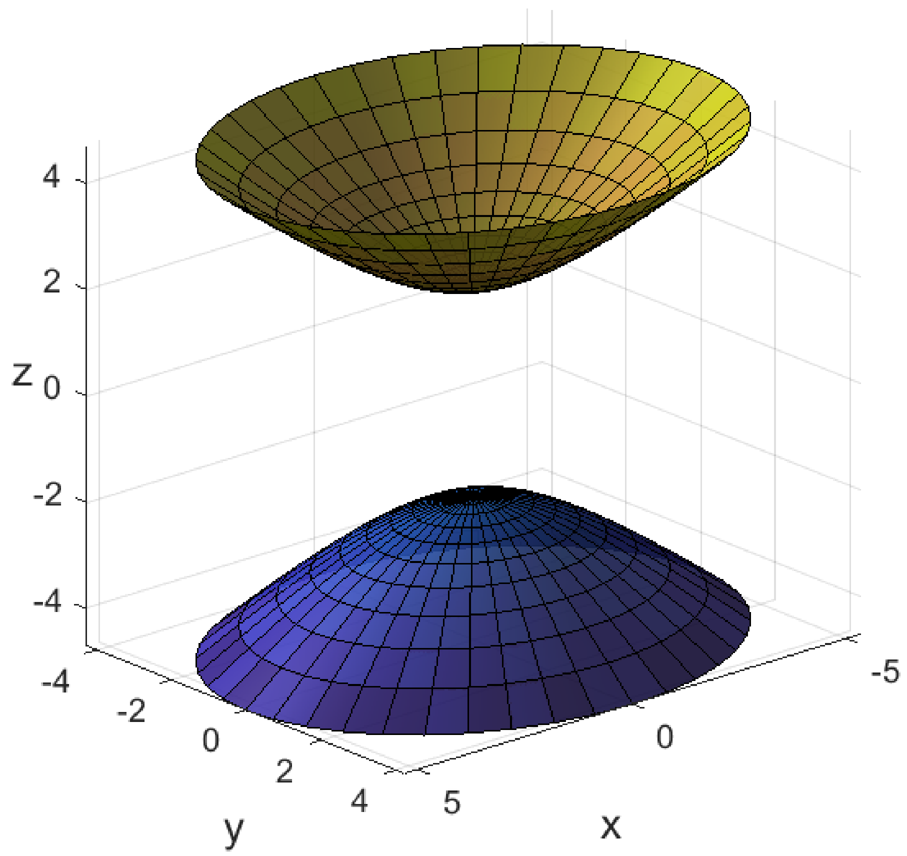
$$\begin{cases} x=a\cosh(t)\cos(\theta) \\ y=b\cosh(t)\sin(\theta) \\ z=c\sinh(t) \end{cases}\qquad \begin{array}{l}\theta=0\ldots360^\circ \\ t=t_{min}\ldots t_{max}\end{array}$$

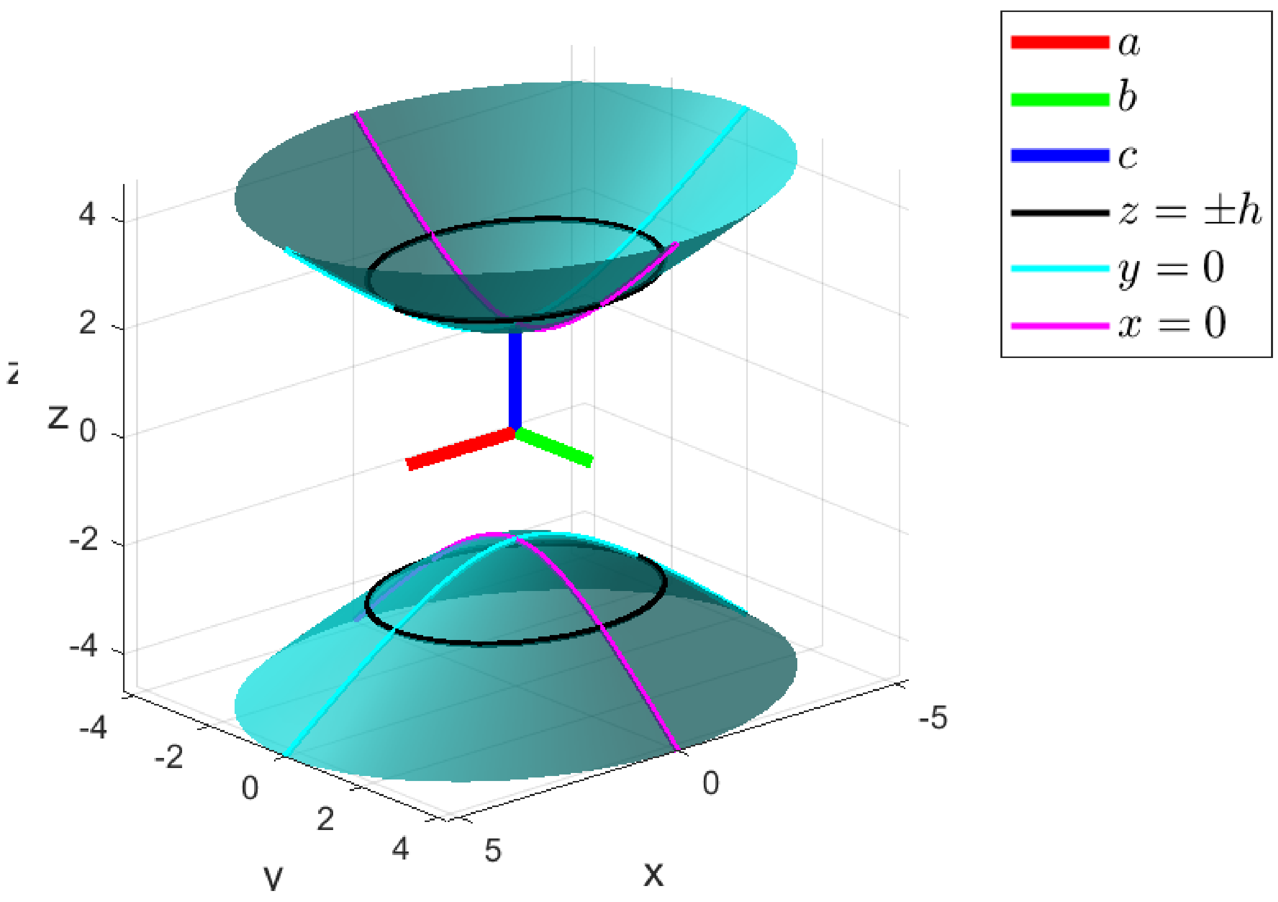
Jos $t_{max}=\text{asinh}(1)$ eli $\sinh(t_{max})=1$ ja

$t_{min}=-t_{max}$, niin $z_{max}=c$, $z_{min}=-c$

Elliptinen hyperboloidi

Two-sheeted hyperboloid





Yhtälö: $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} - \dfrac{z^2}{c^2} = -1$

$$z = \pm h,\ h > c \rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{h^2}{c^2} - 1 \text{ (ellipsi)}$$

$$y = h \rightarrow -\frac{x^2}{a^2} + \frac{z^2}{c^2} = \frac{h^2}{b^2} + 1 \text{ (hyperbeli)}$$

$$x = h \rightarrow -\frac{y^2}{b^2} + \frac{z^2}{c^2} = \frac{h^2}{a^2} + 1 \text{ (hyperbeli)}$$

Parametrimuoto:

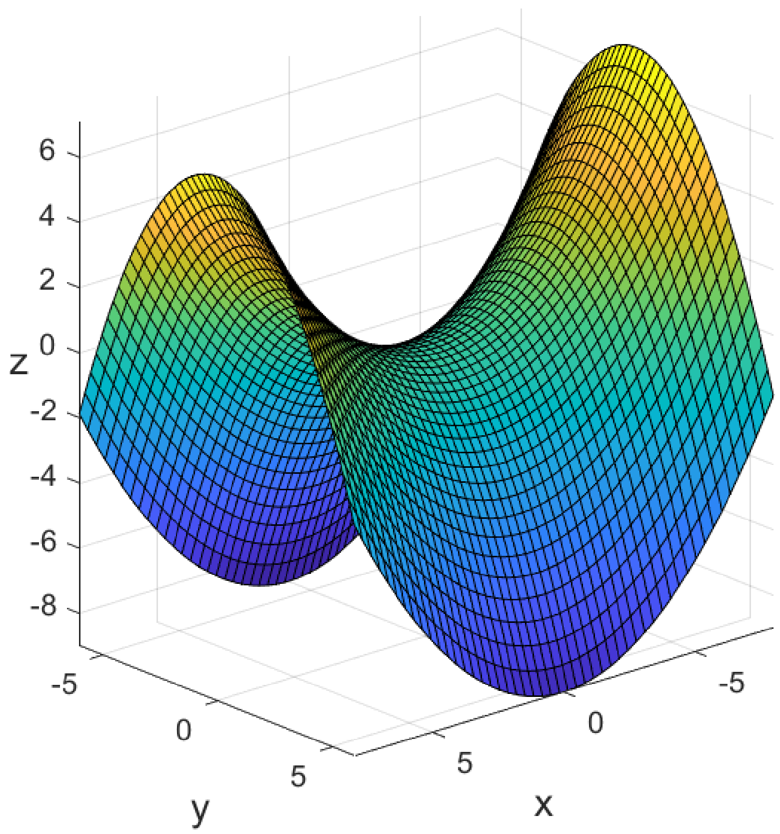
$$\begin{cases} x = a \sinh(t) \cos(\theta) \\ y = b \sinh(t) \sin(\theta) \\ z = \pm c \cosh(t) \end{cases} \qquad \begin{array}{l} \theta = 0 \ldots 360^\circ \\ t = 0 \ldots t_{max} \end{array}$$

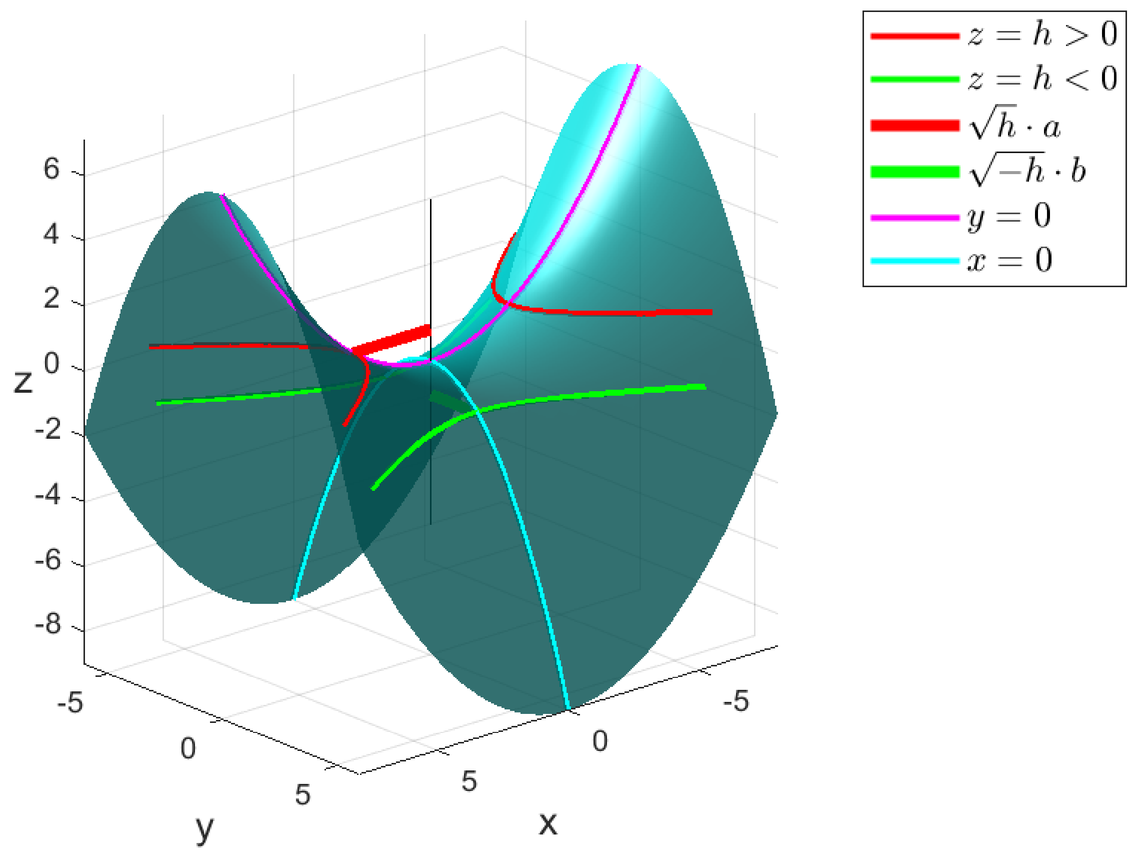
$$t = 0 \rightarrow x = 0, y = 0, z = \pm c$$

$$z_{max} = c \cosh(t_{max}) = H$$

$$\leftrightarrow t_{max} = \operatorname{acosh}(H/c) \rightarrow z_{min} = -H$$

Hyperbolinen paraboloidi





Yhtälö: $z = \dfrac{x^2}{a^2} - \dfrac{y^2}{b^2}$

$z = h \neq 0 \to \dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = h$ (hyperbeli)

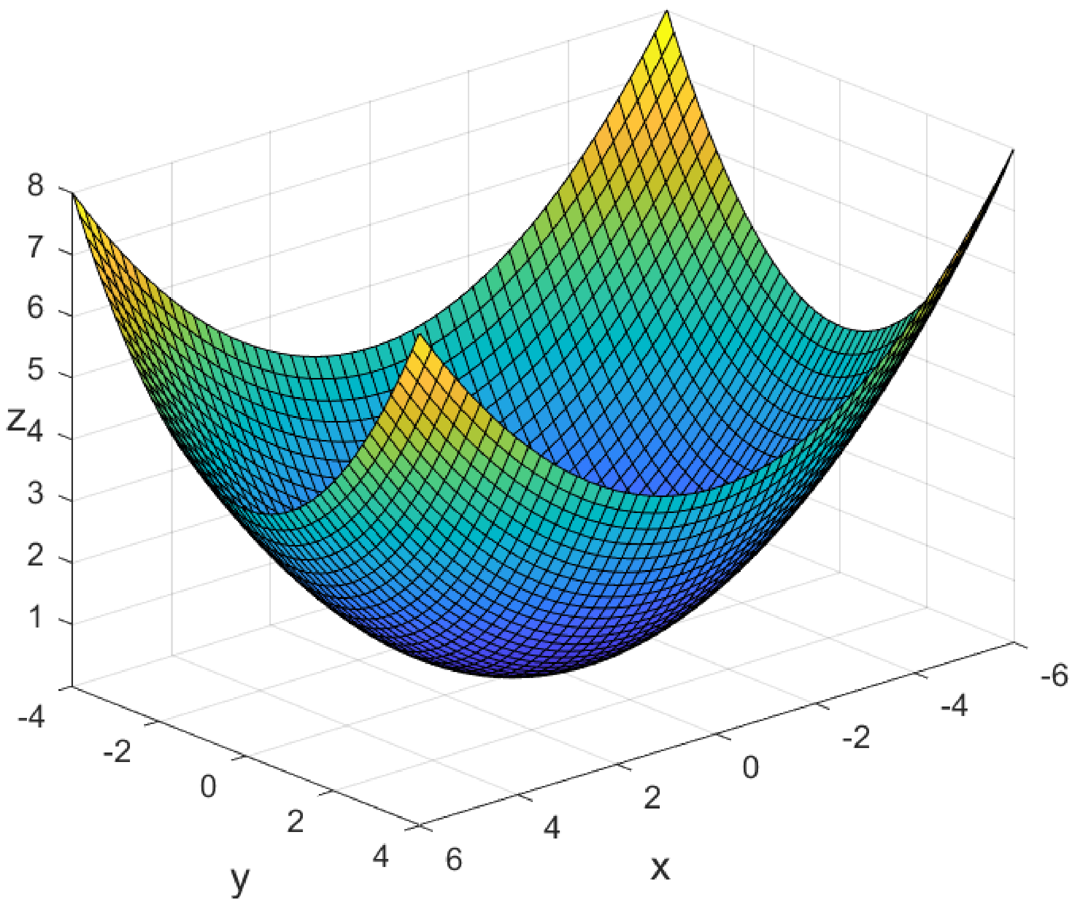
$z = 0 \to \dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = \left(\dfrac{x}{a} + \dfrac{y}{b}\right)\left(\dfrac{x}{a} - \dfrac{y}{b}\right) = 0$

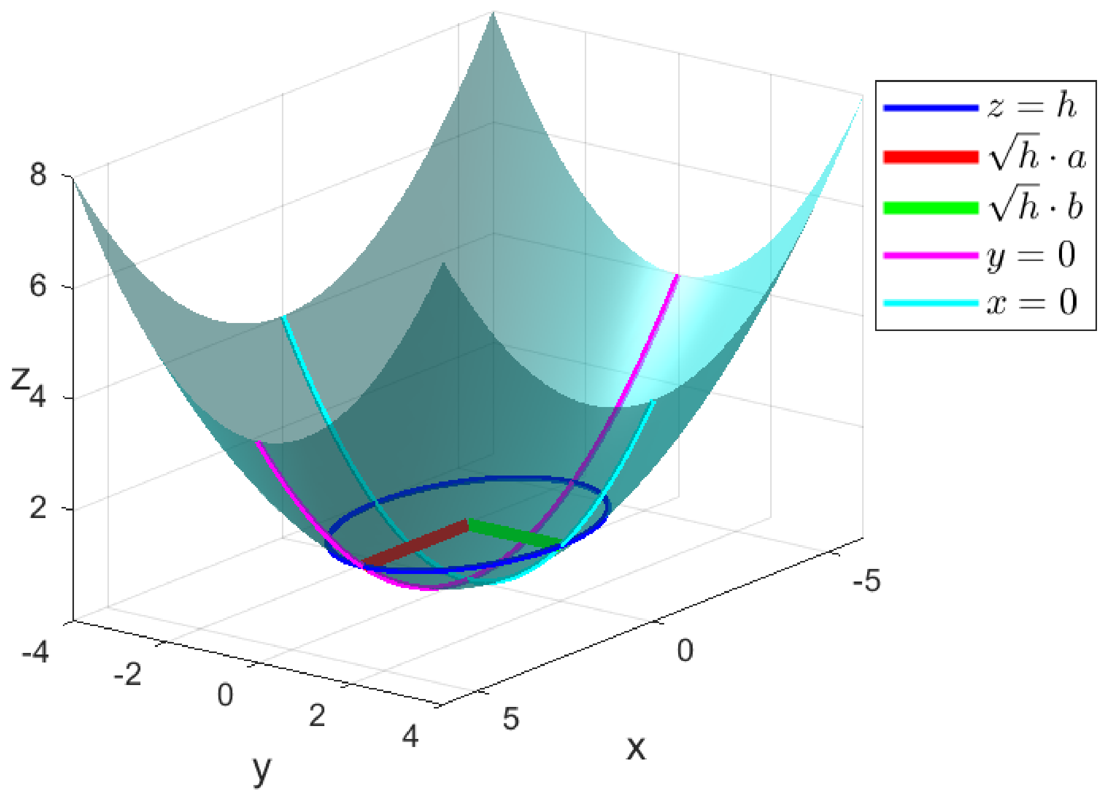
(kaksi suoraa)

$y = h \to z = \dfrac{x^2}{a^2} - \dfrac{h^2}{b^2}$ (paraabeli)

$x = h \to z = -\dfrac{y^2}{b^2} + \dfrac{h^2}{a^2}$ (paraabeli)

Elliptinen paraboloidi





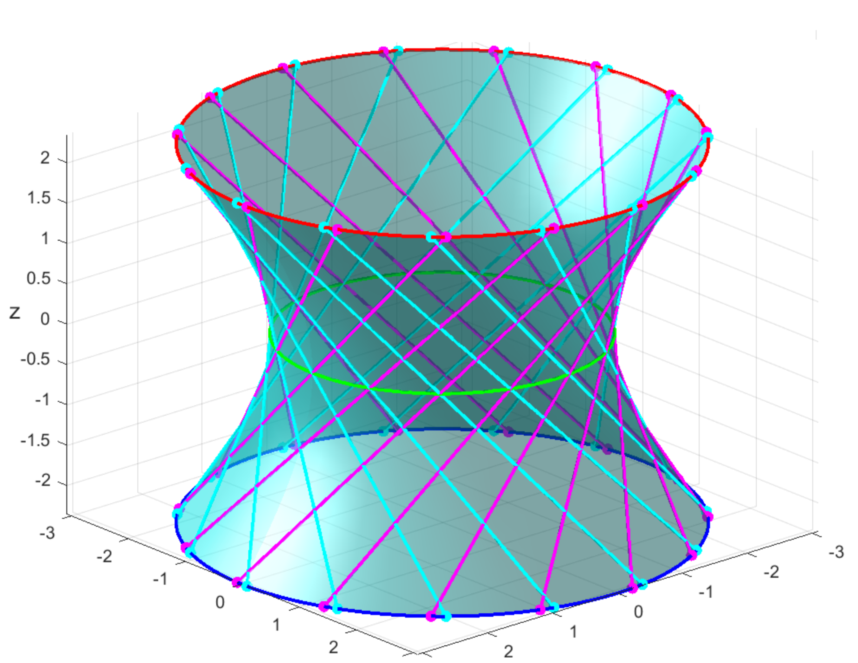
Yhtälö: $z = \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2}$

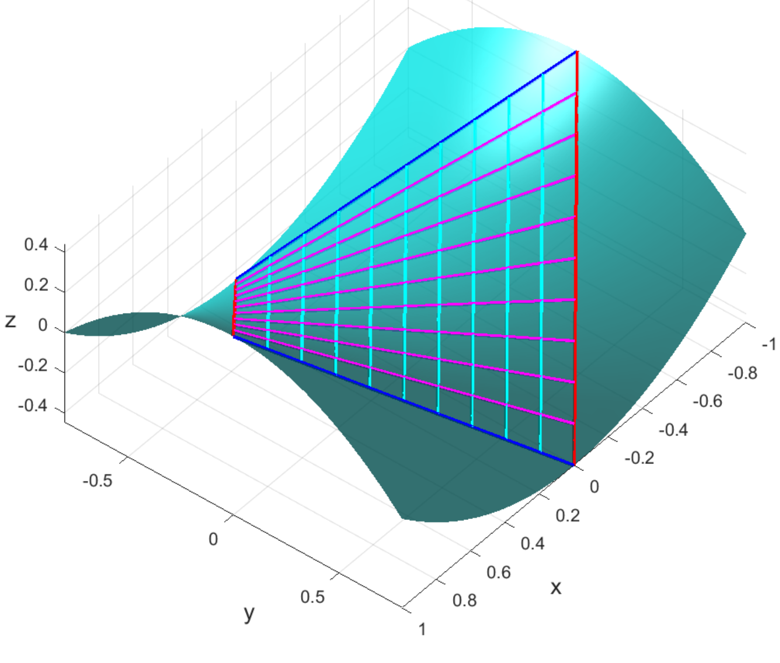
$z = h > 0 \rightarrow \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = h$ (ellipsi)

$y = h \rightarrow z = \dfrac{x^2}{a^2} + \dfrac{h^2}{b^2}$ (paraabeli)

$x = h \rightarrow z = \dfrac{y^2}{b^2} + \dfrac{h^2}{a^2}$ (paraabeli)

Huom: Hyperbolinen hyperboloidi ja hyperbolinen paraboloidi ovat ns. doubly ruled surfaces: niiden jokaisen pisteen kautta kulkee kaksi erisuuntaista, pintaa pitkin kulkevaa suoraa









WKD


Syy: Hyperbolinen hyperboloidi

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

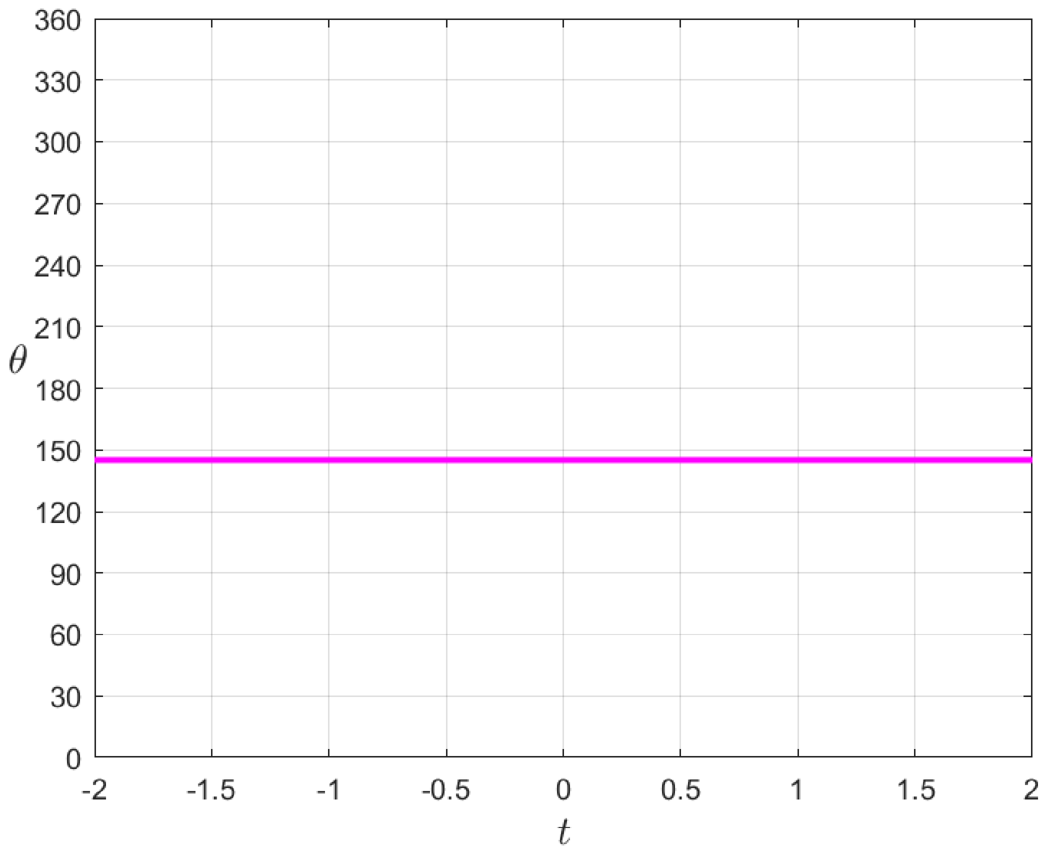
toinen parametrisointi

$$\begin{cases} x = a\cos(\theta) - a\,t\sin(\theta) \\ y = b\sin(\theta) + b\,t\cos(\theta) \\ z = \pm c\,t \end{cases}, \qquad \begin{array}{l} \theta = 0\ldots 360^\circ \\ t = t_{min}\ldots t_{max} \end{array}$$

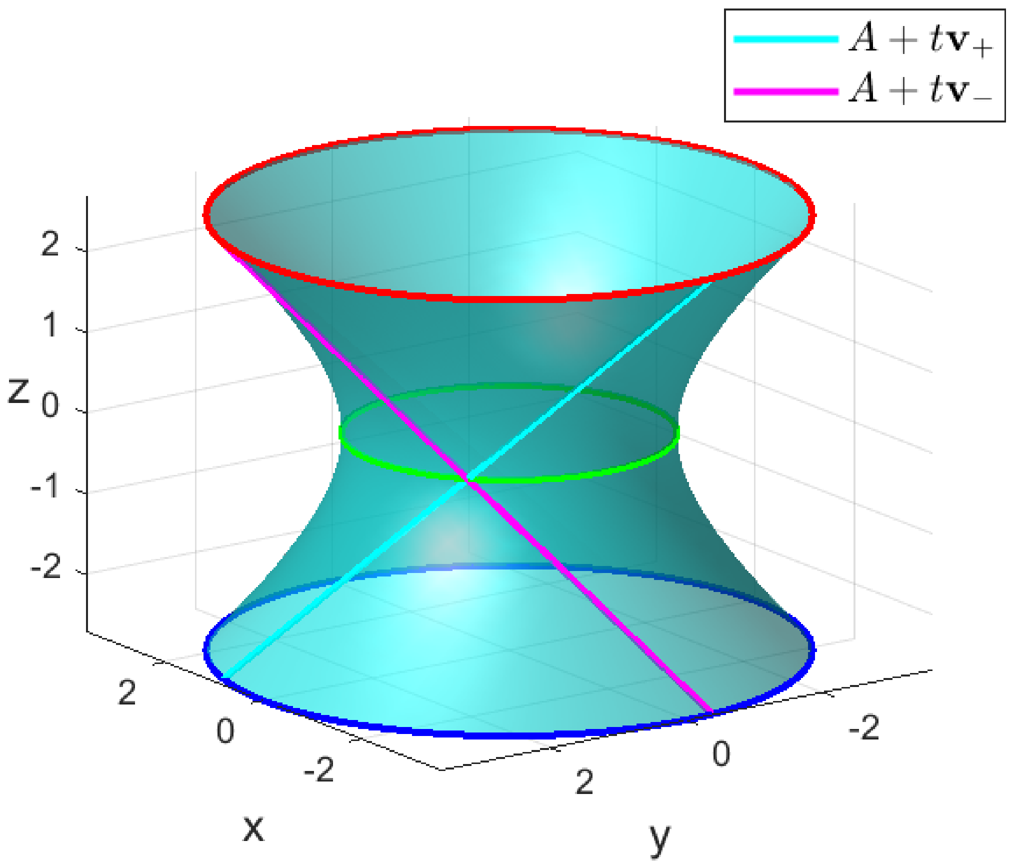
eli kun $\theta = \theta_0$ on vakio, niin

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \underbrace{\begin{bmatrix} a\cos(\theta_0) \\ b\sin(\theta_0) \\ 0 \end{bmatrix}}_{A} + t \underbrace{\begin{bmatrix} -a\sin(\theta_0) \\ b\cos(\theta_0) \\ \pm c \end{bmatrix}}_{\mathbf{v}_\pm}$$

$$= A + t\mathbf{v}_\pm, \quad t = t_{min}\ldots t_{max}$$



360
330
300
270
240
210
180
150
120
90
60
30
0
θ
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
t



$A + t\mathbf{v}_+$
$A + t\mathbf{v}_-$
2
1
z 0
-1
-2
2
0
-2
x
2
0
-2
y

Hyperbolinen paraboloidi $z = \dfrac{x^2}{a^2} - \dfrac{y^2}{b^2}$ parametrisointi

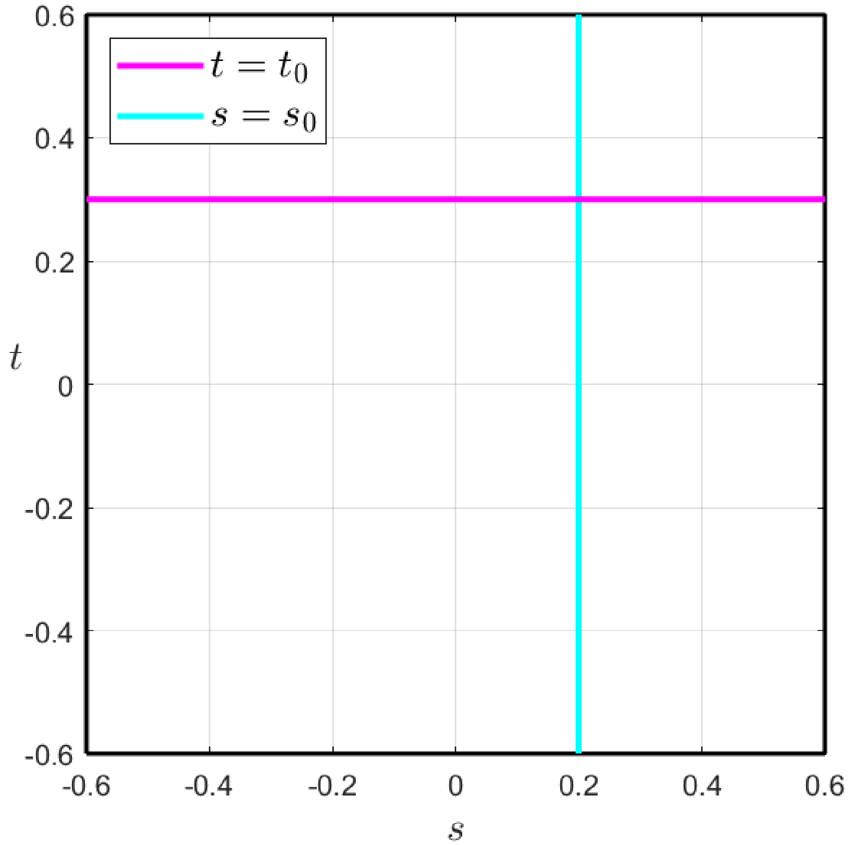
$$\begin{cases} x = a(s+t) \\ y = b(s-t) \\ z = 4st \end{cases}, \qquad \begin{matrix} s = s_{min} \ldots s_{max} \\ t = t_{min} \ldots t_{max} \end{matrix}$$

kun $t = t_0$ on vakio, niin

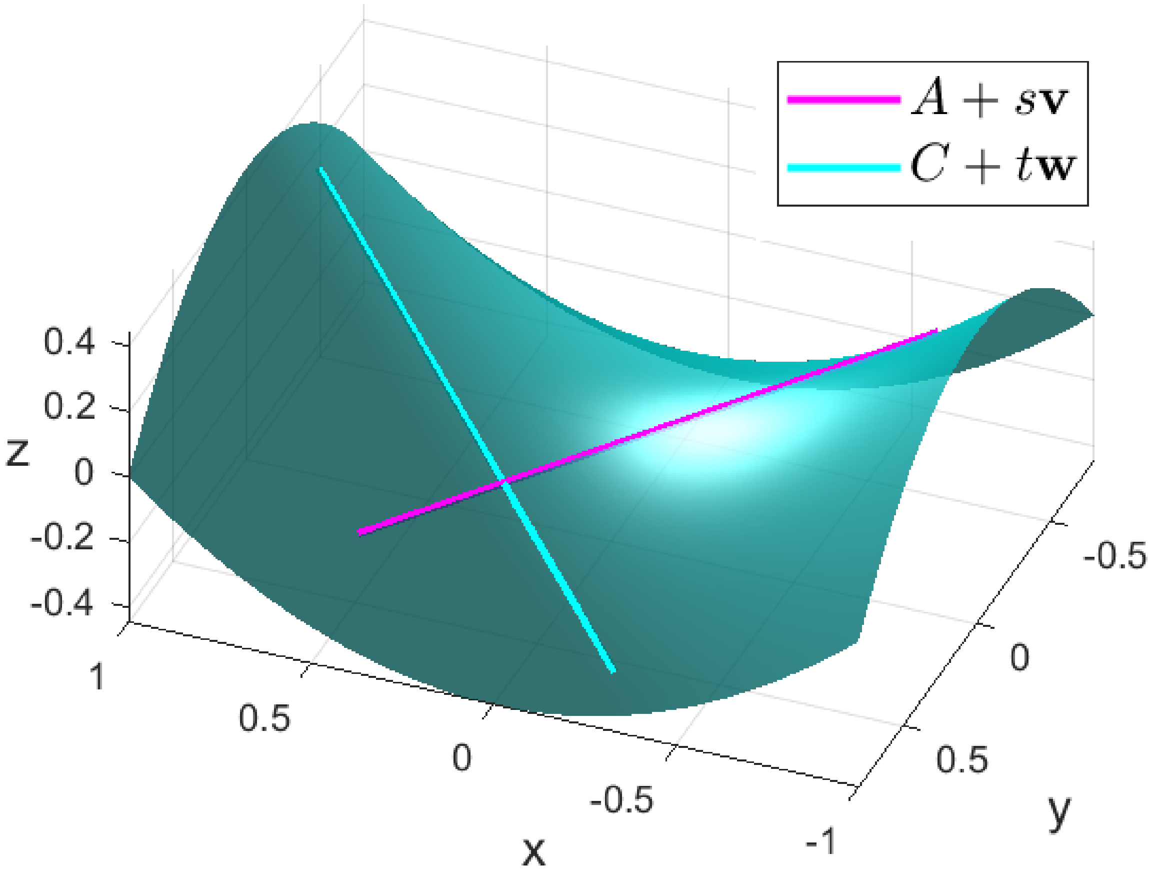
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \underbrace{\begin{bmatrix} at_0 \\ -bt_0 \\ 0 \end{bmatrix}}_{\mathbf{A}} + s \underbrace{\begin{bmatrix} a \\ b \\ 4t_0 \end{bmatrix}}_{\mathbf{v}} = A + s\mathbf{v}$$

kun $s = s_0$ on vakio, niin

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \underbrace{\begin{bmatrix} as_0 \\ bs_0 \\ 0 \end{bmatrix}}_{\mathbf{C}} + t \underbrace{\begin{bmatrix} a \\ -b \\ 4s_0 \end{bmatrix}}_{\mathbf{w}} = C + t\mathbf{w}$$



0.6
0.4
0.2
0
-0.2
-0.4
-0.6
t
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
s
$t = t_0$
$s = s_0$



$A + s\mathbf{v}$
$C + t\mathbf{w}$
0.4
0.2
z
0
-0.2
-0.4
1
0.5
0
-0.5
-1
x
-0.5
0
0.5
y