

Pallojen leikkauspiste  $x, y, z$ :

$$\begin{cases} (x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 = r_1^2 \\ (x - x_2)^2 + (y - y_2)^2 + (z - z_2)^2 = r_2^2 \\ (x - x_3)^2 + (y - y_3)^2 + (z - z_3)^2 = r_3^2 \end{cases}$$

poistetaan sulut:

$$\begin{cases} x^2 - 2x_1x + x_1^2 + y^2 - 2y_1y + y_1^2 + z^2 - 2z_1z + z_1^2 = r_1^2 & (1) \\ x^2 - 2x_2x + x_2^2 + y^2 - 2y_2y + y_2^2 + z^2 - 2z_2z + z_2^2 = r_2^2 & (2) \\ x^2 - 2x_3x + x_3^2 + y^2 - 2y_3y + y_3^2 + z^2 - 2z_3z + z_3^2 = r_3^2 & (3) \end{cases}$$

Vähennetään (1) - (2) ja (1) - (3) :

$$\begin{cases} 2(x_2 - x_1)x + (x_1^2 - x_2^2) + 2(y_2 - y_1)y + (y_1^2 - y_2^2) + 2(z_2 - z_1)z + (z_1^2 - z_2^2) = r_1^2 - r_2^2 & (4) \\ 2(x_3 - x_1)x + (x_1^2 - x_3^2) + 2(y_3 - y_1)y + (y_1^2 - y_3^2) + 2(z_3 - z_1)z + (z_1^2 - z_3^2) = r_1^2 - r_3^2 & (5) \end{cases}$$

ratkaistaan  $x$  ja  $y$ :

$$\left\{ \begin{array}{l} \overbrace{2(x_2 - x_1)}^A x + \overbrace{2(y_2 - y_1)}^B y = \overbrace{-2(z_2 - z_1)}^E z + \overbrace{r_1^2 - r_2^2 - (x_1^2 - x_2^2) - (y_1^2 - y_2^2) - (z_1^2 - z_2^2)}^F \\ \overbrace{2(x_3 - x_1)}^C x + \overbrace{2(y_3 - y_1)}^D y = \overbrace{-2(z_3 - z_1)}^G z + \overbrace{r_1^2 - r_3^2 - (x_1^2 - x_3^2) - (y_1^2 - y_3^2) - (z_1^2 - z_3^2)}^H \end{array} \right.$$

$$\left\{ \begin{array}{l} x = \frac{\overbrace{BG - DE}^I}{BC - AD} z + \frac{\overbrace{HB - FD}^J}{BC - AD} \\ y = \frac{\overbrace{CE - AG}^K}{BC - AD} z + \frac{\overbrace{FC - AH}^L}{BC - AD} \end{array} \right.$$

sijoitetaan (1):een:

$$(Iz + J)^2 - 2x_1(Iz + J) + x_1^2 + (Kz + L)^2 - 2y_1(Kz + L) + y_1^2 + z^2 - 2z_1z + z_1^2 = r_1^2$$

$$I^2z^2 + 2IJz + J^2 - 2x_1Iz - 2x_1J + x_1^2 + K^2z^2 + 2KLz + L^2 - 2y_1Kz - 2y_1L + y_1^2 + z^2 - 2z_1z + z_1^2 = r_1^2$$

$$\underbrace{(I^2 + K^2 + 1)}_a z^2 + \underbrace{(2IJ - 2x_1I + 2KL - 2y_1K - 2z_1)}_b z + \underbrace{(J^2 - 2x_1J + x_1^2 + L^2 - 2y_1L + y_1^2 + z_1^2 - r_1^2)}_c = 0$$

ja

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$