Laplacian and its use in Blur Detection

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> *According to Wikipedia, the Laplacian of a function f at a point p is (up to a factor) the rate at which the average value of f over spheres centered at p deviates from f(p) as the radius of the sphere shrinks towards 0. It is usually denoted by the symbols ∇·∇, ∇^2 (where ∇ is the nabla operator) or Δ.*

Simply speaking laplician operator is defined as divergance of gradient of a function f.

△f(x,y) = div(grad(f))

And Gradient is slope of steepest accent. Fundamentally, it gives information about the direction, the point(in the curve) should walk towards to reach the highest accent ie local maxima. Likewise negative gradient gives the direction of local minima.

For example if we consider the graph generated by the function *z=x*exp(-x^2 +y^2))* and the gradient at each point on the curve(represented by line with arrow). We can obtain figure like the following.

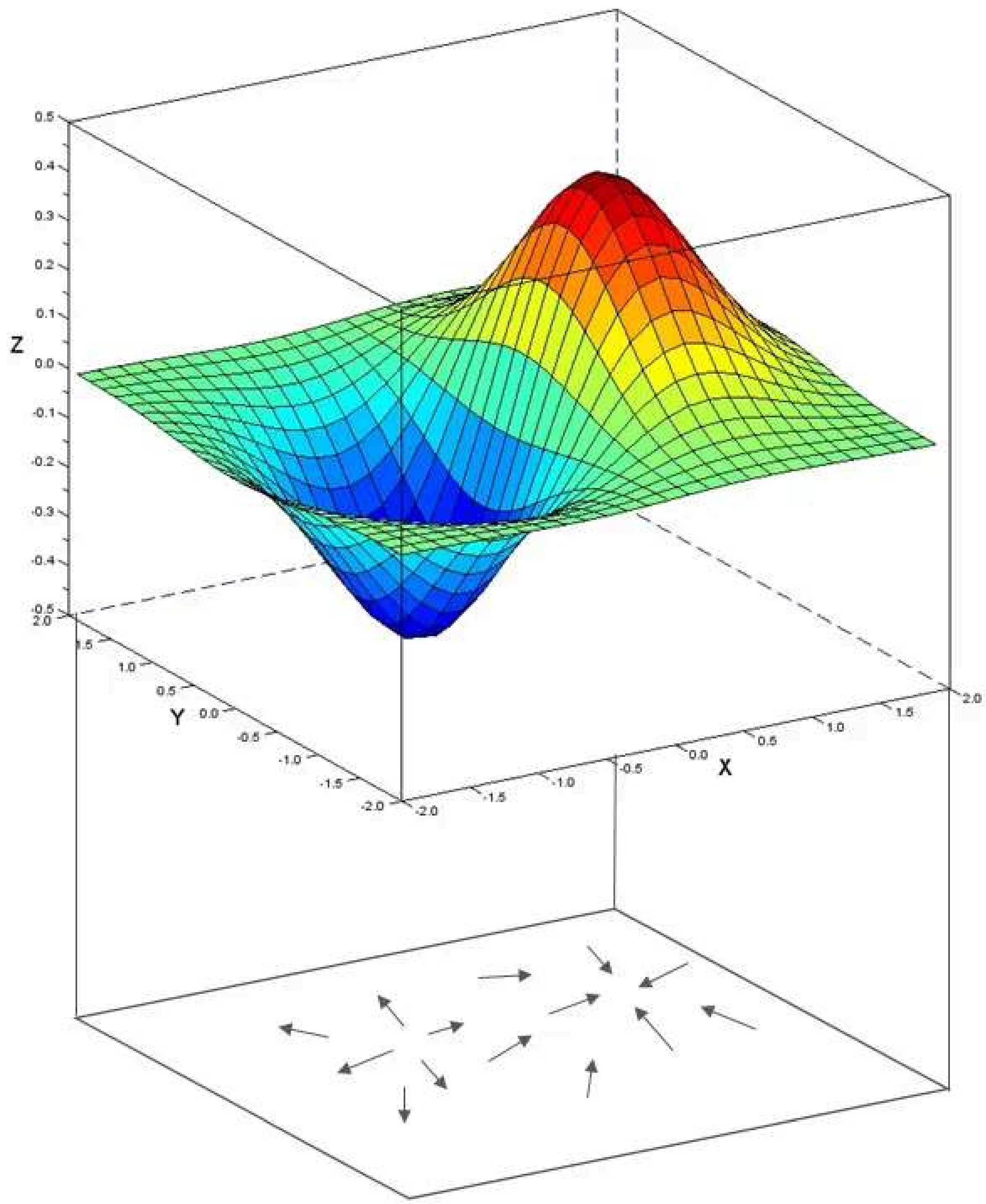


figure 1: graph of function z=x*exp(-x^2 +y^2)) and illustration of its gradient at various points of curve

> *Note: As gradient gives direction of steepest accent we can observe all arrows(vectors) are pointing towards hill and are pointing away from the valley.*

> *Although, it may not be clear from the figure, the magnitude of vector, ie length of arrow gives us idea about how steep the curve is at that particular point.*

We can understand divergence as the vector field corresponding to some kind of motion generated by fluid flow. For more information refer here. In the hill of the graph as each vector field points towards the region causing divergence to be negative and in the valley where each vector points away the divergence is positive.

So in a way, Laplician operator is a kind of measure how much a minimum point is x,y. Thus, acting like second derivative for multivariable function with scalar value.

Second Derivative can be used to determine local extrema for a function. If a function has a critical point $f'(x) = 0$ on which second derivative of the point $f''(x) < 0$, ie negative, then f has the local maximum at the point. This can be furthur illustrated with the figure below:

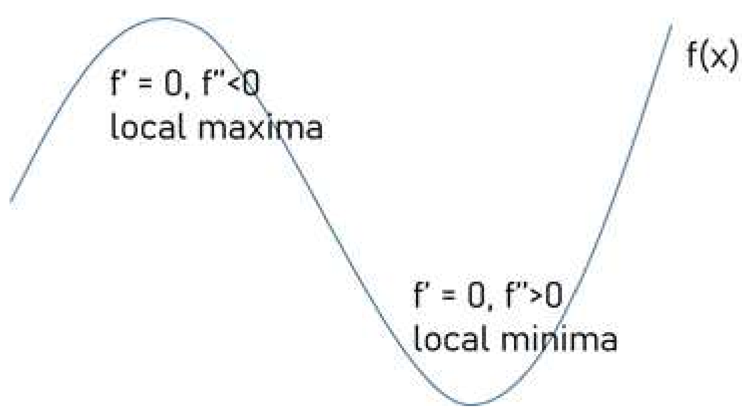


figure: local minima and maxima for function f(x)

The laplacian operator can be given as:

$$\nabla^2 f = \frac{\partial^2 f}{\partial^2 x} + \frac{\partial^2 f}{\partial^2 y}$$

And partial derivative in x direction can be given as:

$$\frac{\partial^2 f}{\partial^2 x} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

Similarly, partial derivative in y direction can be given as:

$$\frac{\partial^2 f}{\partial^2 y} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

And combining these equations for partial derivatives, we can obtain:

$$\nabla^2 f = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)] - 4f(x, y)$$

Now Lets consider what a filter looks like. 3*3 filter for a image can be represented for coordinate (x,y) and can be represented as:

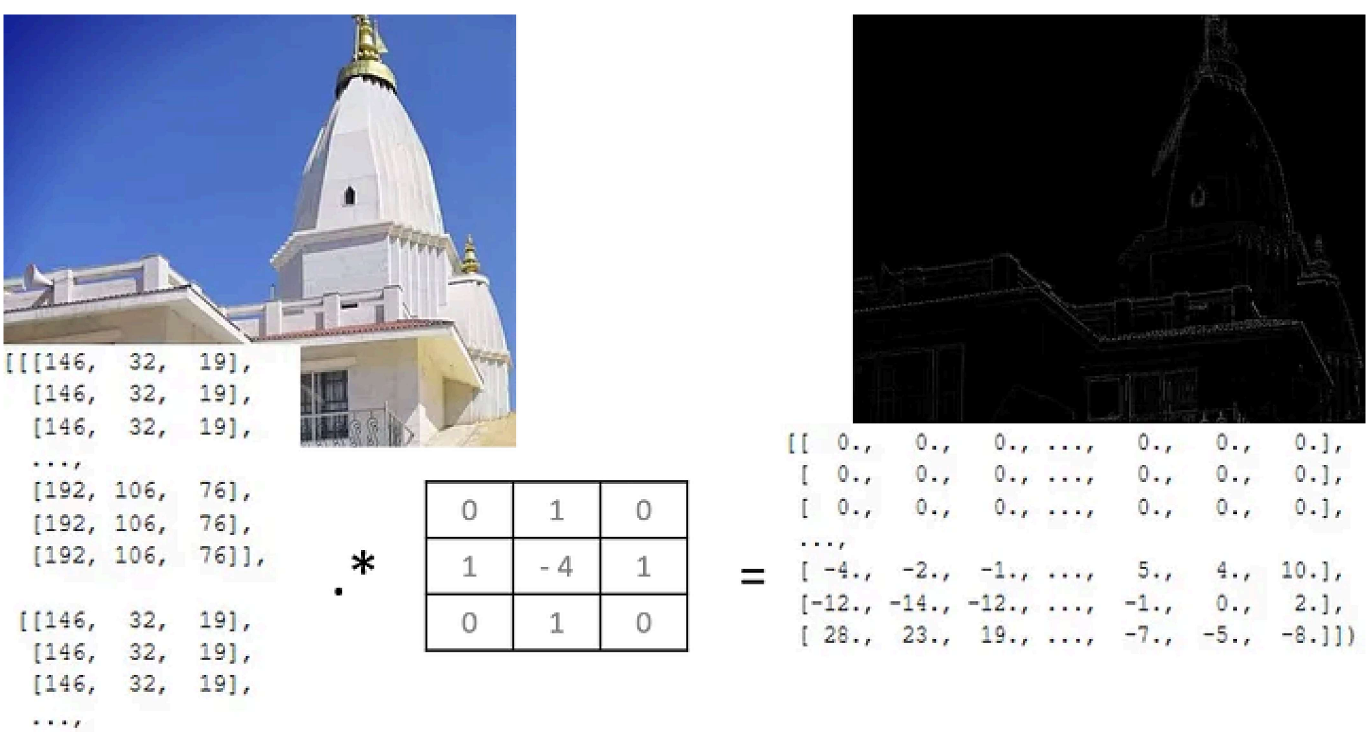
x-1,y-1	x,y-1	x+1,y-1
x-1,y	x,y	x,y+1
x-1,y+1	x,y+1	x+1,y+1

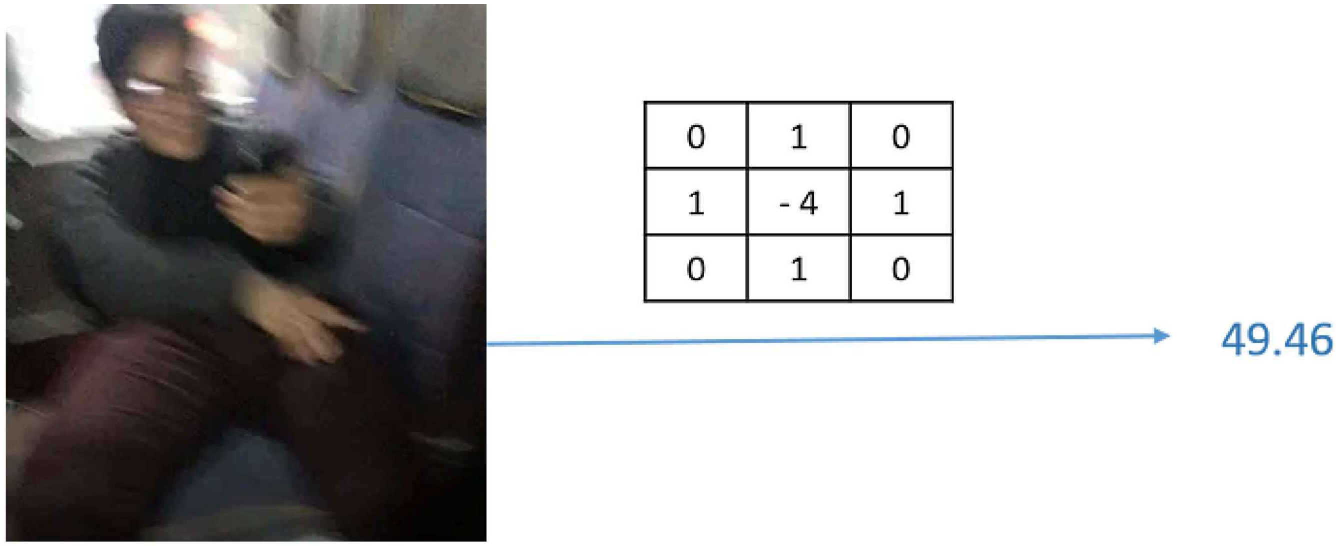
If we put the coefficients of the equation in the image we can obtain following matrix.

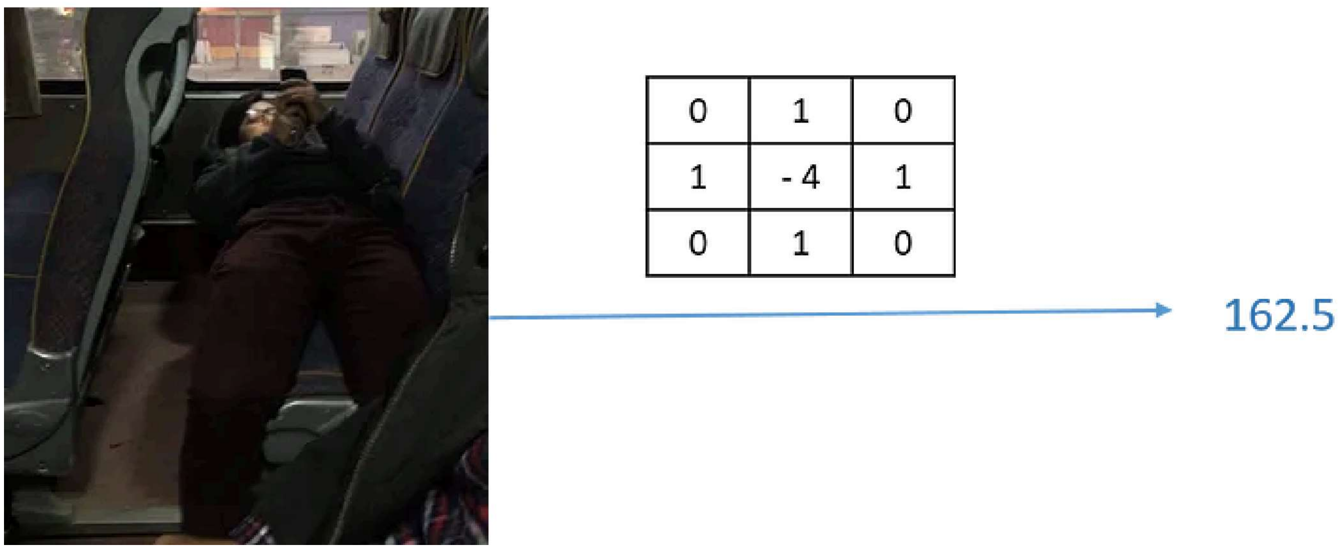
0	1	0
1	- 4	1
0	1	0

This matrix when convoluted with image gives transformed image based on laplician. This matrix is used to find area of rapid changes in images given there is no noise. Fundamentally the operator takes second derivative of the image on some higher dimensional plane. If the image is basically uniform, the result will be zero. Wherever a change occurs, the resultant matrix will have positive elements on the darker side and a negative element on the brighter side.

The convolution operation for the image gives the following result:







Final Words:

In this way we can use laplician operator to determine if the image is blurry or not. The threshold can be set based on performance on the data we have. This theoritically will work best in images obtained from same source, for example images from live feed rather than collection of images obtained from different camera source at different time of day, but please feel free to experiment.

References:

Diatom autofocusign in brightfield microscopy: a comparative study (link)