

Secure multi-party computation

Homework 2

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This is the truth table for : $f_{a,4}(x) = \begin{cases} 1 & \text{if } ax \geq 4 \\ 0 & \text{otherwise} \end{cases}$

<i>input</i>	$x = 0$	$x = 1$	$x = 2$	$x = 3$
$a = 0$	0	0	0	0
$a = 1$	0	0	0	0
$a = 2$	0	0	1	1
$a = 3$	0	0	1	1

So our matrix going to be of size 4*4

First to solve that we will do it according to the algorithm:

So first of all we created the dealer class and with out it nothing going to work

In the dealer class we does all of the computation of offline phase(pre-processing)

1. Get two random number $r, c < -_r\{1, \dots, 4\}$ and shift the truth table based on this value
2. Get a random 2*2 matrix (M_b)
3. Compute $M_a = M_b[i, j] \oplus T_{r,c}[i, j]$ // 1 operation

All of that computation going to be In the `__init__` and it's stored in private variable of the dealer class assuming trusted dealer

4 . (r, M_a) to alice when she aske (c, M_b) to bob

In the dealer class we will have two functions `rand_a` which sends message to Alice and `rand_b` send to bob

In the other two class we do the things needed the in the online phase:

Alice calculate it u and send it to bob by `send()` and `receive()` function for receiving back the calculation from bob (v, z_B)

And then just after that Alice able to show the output.

As we proven in the lecture the correctness of OTTT:

$$\begin{aligned}
 z &= M_A[u, v] \oplus z_B && // \text{def of } z \\
 &= M_A[u, v] \oplus M_B[u, v] && // \text{def of } z_B \\
 &= T[u - r, v - c] && // \text{def of } M_A \\
 &= T[x, y] && // \text{def of } u, v \\
 &= f(x, y) && // \text{def of } T
 \end{aligned}$$

And the privacy of it:

The real view is:

$$view_A = (A's \text{ input}, A \text{ randomness}, \text{msgs received})$$

$$= (x, \perp, (r, M_A, v, z_B))$$

$$view_B = (B's \text{ input}, B \text{ randomness}, \text{msgs received})$$

$$= (a, \perp, (c, M_B, u))$$

simulator for Alice

S_A : input $x \in \{0,1,2,3\}$ $z \in \{0,1\}$

sample $z_B \leftarrow \{0,1\}$, $v, r \leftarrow_r \{0,1\}^n$

Construct M_A follows:

$$M_A [x+r, v] = z \oplus z_B$$

$$M_A [i, j] \leftarrow_R \{0,1\} \quad \forall (i, j) \neq (x+r, v)$$

Output: "simulated-view" $:= (x, \perp, (r, M_A, v, z_B))$

In both views

x is identical (the input)

r, v and $M_A [i, j]$ for all $(i, j) \neq (u, v)$ are independent. uniformly random

$(M_A [u, v], z_B)$ is uniformly random subject to $M_A [u, v] \oplus z_B = z$.

So (real) $view_A \equiv$ simulated- $view_A$

simulator for Bob

S_B : input $a \in \{0,1,2,3\}$

sample $c, u \leftarrow_r \{0,1\}^n$

Construct M_B as follows:

$M_B[i, j] \leftarrow_R \{0,1\} \quad \forall (i, j)$

Output: "simulated-view" $:= (a, \perp, (c, M_B, u))$

In both views

a is identical (the input)

c, u and $M_B[i, j]$ for all (i, j) are independent. uniformly random

So $(\text{real}) \text{view}_B \equiv \text{simulated-view}_B$

Time complexity:

Offline phase:

The dealer have truth table from length 4×4

the dealer make XOR between M_a Xor $M_b \rightarrow$ we have 16 XOR

in this example the Time is $O(1)$, Because the inputs between $[0,3]$, but if we assume that the input between $[0,n]$ we will have complexity $= O(2^n)$ for the offline phase

Online phase:

Alice computes $u \rightarrow$ make one +

Bob computes $v \rightarrow$ make one +

Alice make XOR between the Z_b with $M_a \rightarrow$ One XOR

So we make 3 operations $\rightarrow T=3$

Test.py:

this file make check for all the possible inputs of a and x ($a, x \rightarrow \{0..3\}$)

so we have 16 inputs ,and the test file check if every input get the answer that we need to get

We have 16 inputs=16 checks -> we have 16T(T is the number of operations that Alice and Bob make)

T=3

T is CONST in this case -> 16T=48=CONST -> complixety time =O(1)