Determining Mass of the Neutron Through Measurement of Hydrogen-Deuterium Spectral Line Splitting

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Abstract

By measuring the difference in wavelength of the Balmer-alpha line in the emission spectrum of a Hydrogen-Deuterium source we were able to determine the mass of the neutron. The calculation is based on the reduced-mass correction to the Rydberg formula for one-electron atoms and the expression of the Rydberg constant in terms of fundamental constants due to Bohr. We measured a mass of 1.6739×10^{-27} kg $\pm$ 2.99×10^{-33} kg which is not in accordance with the accepted value of 1.6749×10^{-27} kg within the uncertainty of our measurements.

1 Introduction

1.1 Physical Motivation

The Bohr model of the Hydrogen atom formed an early framework for exploration into the microscopic qualities of matter as well as a platform for the discovery of more complicated atoms and molecules. This foundation ultimately led to Harold Urey's discovery of Deuterium which we will leverage to measure one of the most fundamental constants in atomic physics: the mass of the neutron.

1.2 Theory

The Bohr model of the atom presumes that electrons orbit a stationary nucleus in concentric circular "shells". For atoms with one electron, the energies of the shells are given by the Rydberg equation: [1]

$$E = hcR\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \tag{1}$$

Transitions between energy states require an energy source or an energy sink (for transition to higher states and lower states respectively) by conservation of energy. The transition to lower states corresponds to the emission of an electron. The energy of the transition and the wavelength of the emitted photon are related by the following expression

$$E = \frac{hc}{\lambda} \tag{2}$$

Applying a voltage to the gas lamps in the experiment accelerates electrons through the source causing collisions. The voltage applied to the gas lamp sources induces a kinetic energy in the electrons much greater than the energies of bound electrons. Collisions between highly energetic electrons and atoms in the tube are responsible for the falling of bound electrons to lower energy states and thus the emission of photons with wavelengths according to Eq. 2.

The mass term is tucked away into the Rydberg constant R which in full form is [2]

$$R = \frac{m_e e^4}{8\epsilon_0^2 h^3 c} \tag{3}$$

Rydberg's original analysis provided a purely numerical expression for R. It was not until Schrodinger and Bohr's further insights into Quantum Mechanics that an expression in terms of fundamental constant was derived. This expression is only valid in the "infinite-mass" approximation which assumes that the mass of the nucleus is much greater than the mass of the electron and thus remains stationary. In order to relax this assumption and obtain slightly better results we can borrow a result from Classical Mechanics common in the analysis of two-body problems. By replacing the m_e term in the Rydberg constant with the reduced-mass expression we obtain a better approximation of the orbital energies and thus transition wavelengths [1]. Following the procedure in [3], the correction for Hydrogen is

$$R_H = \frac{e^4}{8\epsilon_0^2 h^3 c}\left(\frac{m_e m_p}{m_e + m_p}\right) \tag{4}$$

Where m_e and m_p are the mass of the electron and the mass of the proton respectively. For Hydrogen, the reduced mass is nearly identical to m_e so for the purposes of our analysis we will use the approximation $R_H \approx m_e$. Similarly for Deuterium the correction is

$$R_D = \frac{e^4}{8\epsilon_0^2 h^3 c}\left(\frac{m_e m_p m_n}{m_e + m_p m_n}\right) \tag{5}$$

Here we see the mass of the neutron m_n appear. Rewriting (1) in terms of wavelength yields

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \tag{6}$$

The inverse proportionality between wavelength and Rydberg constant implies that the emission spectrum of Deuterium will be shifted to a slightly lower wavelength due to its slightly larger reduced mass. Using Equations (4), (5), and (6) allows us to write (ratio gets rid of constants and inverse proportionality gives equality)

$$\frac{\Delta\lambda}{\lambda_H} = \frac{1/m_e - 1/m_D}{1/m_e} = \frac{m_e}{m_p} - \frac{m_e}{m_D} \tag{7}$$

Solving for m_D gives

$$m_D = \frac{m_e}{\frac{m_e}{m_p} - \frac{\Delta\lambda}{\lambda_H}} \tag{8}$$

The mass of the neutron should be to great accuracy (due to the small mass of the electron) the difference between the mass of Deuterium and the mass of the proton

$$m_n = m_D - m_p \tag{9}$$

Substituting (8) for the mass of Deuterium then gives us our final expression for the mass of the neutron

$$m_n = \frac{m_e}{\left(\frac{m_e}{m_p} - \frac{\Delta\lambda}{\lambda_H}\right)} - m_p \tag{10}$$

Taking the masses of the electron and proton as given, the mass of the neutron can be deduced by measuring the difference in wavelength corresponding to the same transition in Hydrogen and Deuterium.

2 Experimental Setup

2.1 Apparatus

To observe the wavelengths emitted by the various lamps we used an OceanView HR4000 High Resolution Spectrometer paired with OceanView software. The computer was connected to the spectrometer via USB and the spectrometer collected light data through a fiber optic cable positioned in a clamp in front of the gas lamp.

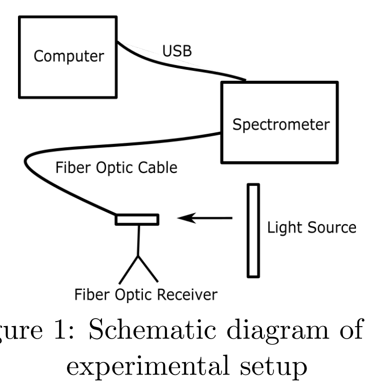

Figure 1: Schematic diagram of the experimental setup

[1] For a more detailed discussion of the reduced mass expression see the derivation in the Appendix

The bandwidth of the spectrometer is 638nm-680nm. The observable emission lines in this range are the Lyman-alpha lines of Hydrogen and Deuterium. This bandwidth also allowed us to observe eight peaks in the Neon spectrum that we used for calibration.

2.2 Data Collection

Using the OceanView software we collected three sets of tabular data corresponding to the intensity as a function of wavelength of each of the three lamps: Hydrogen, Neon, and Hydrogen-Deuterium. The software provided tabulated data which was saved as a text file for analysis.

3 Analysis and Results

In order to obtain an accurate value for the wavelength difference in the Hydrogen-Deuterium spectrum we first calculated a calibration curve for the spectrometer using the known emission spectrum of Neon.

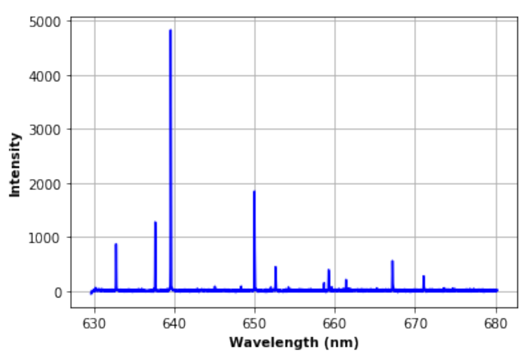

Figure 2: Observed emission spectrum of Neon

We recorded the tabular data produced by the OceanView software and extracted the data points surrounding the eight significant intensity peaks (those with relative intensity greater than 100). By fitting a Gaussian curve to each of the isolated peaks and extracting the mean I was able to come up with a set of observed emission wavelengths to compare to the accepted values provided by the NIST [4]. Upon closer inspection of the fits on the last two peaks I found that the data was too noisy to provide a reliable calibration so I omitted them from the calibration calculation. To obtain the calibration curve I fit a linear model to the set of accepted and observed peaks.

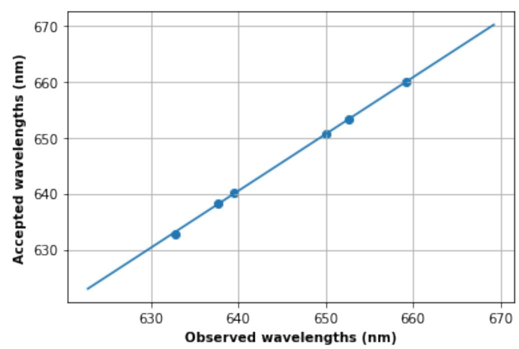

Figure 3: Calibration line for Neon emission spectrum

I used this curve to calibrate the Hydrogen-Deuterium data and then fit a sum of two Gaussian curves to the Hydrogen-Deuterium spectrum.

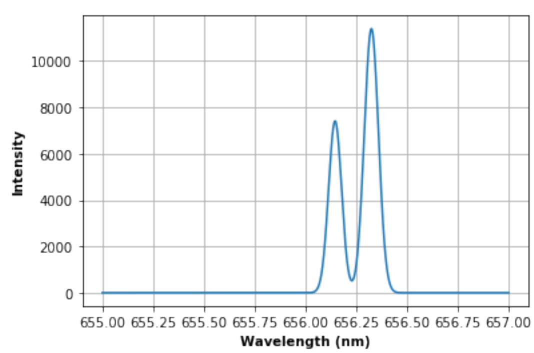

Figure 4: Best fit curve for Hydrogen-Deuterium emission spectrum

The mean values of each curve are $656.3244 \pm .0002$ nm and $656.1456 \pm .0003$ nm. The difference is then $.1788 \pm .0004$. Using this measured value and (10) I calculated a mass for the neutron of 1.6739×10^{-27} kg $\pm 2.99 \times 10^{-33}$ kg. The error propogation for the calculation is

$$\sigma = \sqrt{\left(\frac{m_e}{\lambda_H \left(\frac{m_e}{m_p} - \frac{\Delta\lambda}{\lambda_H}\right)^2}\right)^2 \sigma_{\Delta\lambda}^2}$$

This does not agree with the accepted value of 1.6749×10^{-27} kg within the bounds of our uncertainties. Although the spectrometer measurements were calibrated, the presence of additional noise may have contributed to an inaccurate result. Ideally, the source would be placed in a dark room so as to minimize noise in the spectrum. The exact contribution of the ambient light in the laboratory may not be known without a detailed analysis of light sources and their distances to the detector.

The source code used for this analysis can be found at **https://github.com/Salazar-99/Modern-Laboratory** in the form of an iPython Notebook.

4 Conclusion

In order to measure the mass of the neutron we gathered emission spectra data of various sources using a fiber-optic spectrometer and analyzed it to extract the difference in wavelength for the Balmer-alpha emission line in Hydrogen and Deuterium. Using this value and the framework developed in the theory section we calculated a value of 1.6739×10^{-27} kg $\pm 2.99 \times 10^{-33}$ kg for the mass of the neutron. The accepted value for the mass of the neutron is 7.6749×10^{-27} kg . Thus, we were unable to verify the predictions of the underlying theory within the bounds of our uncertainty.

Appendix

Consider an isolated system (no external forces) of two masses m_1 and m_2 orbiting around each other with position vectors $\mathbf{r_1}$ and $\mathbf{r_2}$ (a good mental picture is the Moon orbiting around the Earth). The first mass exerts a force F_{12} on the second and by Newton's Third Law the second mass exerts a force F_{21} related to the first by

$$F_{12} = -F_{21}$$

The equations of motion of the masses are then

$$m_1 \frac{d^2\mathbf{r_1}}{dt^2} = -F$$
$$m_2 \frac{d^2\mathbf{r_2}}{dt^2} = F$$

Where $F \equiv F_{21}$. The center of mass of the system is given by

$$\mathbf{r_{cm}} = \frac{m_1\mathbf{r_1} + m_2\mathbf{r_2}}{m_1 + m_2}$$

Defining $\mathbf{r} \equiv \mathbf{r_2} - \mathbf{r_1}$ we can rewrite the position vectors of the two masses as

$$\mathbf{r_1} = \mathbf{r_{cm}} - \frac{m_2}{m_1 + m_2}\mathbf{r}$$
$$\mathbf{r_2} = \mathbf{r_{cm}} - \frac{m_1}{m_1 + m_2}\mathbf{r}$$

By substituting either of these two into their equations of motion above we find that they reduce to

$$\mu \frac{d^2\mathbf{r}}{dt^2} = F$$

by taking advantage of the fact that the center of mass of an isolated system does not vary with time. Here μ is the so-called reduced mass defined as

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

References

[1] Davis, Gailey, Whitten. *Principles of Chemistry*. CBS College Publishing, 1984. p.142.

[2] Bohr, Niels. *On the Constitution of Atoms and Molecules*. Philosophical Magazine. Series 6 Vol 26 (1913). p. 1-25.

[3] Fitzpatrick, Richard. *Newtonian Dynamics*. Lulu, 2011.

[4] Physics.nist.gov. NIST: Atomic Spectra Database Lines Form. https://physics.nist.gov/PhysRefData/ASD/linesform.html. Retrieved June 12, 2019.