Verifying Relativistic Energy-Momentum Relationship of Electrons Scattered By Gamma Rays

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Abstract

By measuring the energy distribution of electrons scattered via gamma radiation from various radioactive isotopes we were able to extract both the energy of the incident gamma rays and the Compton edge energies of maximally scattered electrons. The maximally scattered electrons possess energies on the scale of keV. At this energy, the electrons are safely in the relativistic regime and the predictions of relativity may be probed. In particular, we compare the measured energy against both the classically predicted energy and the relativistically predicted energy in order to make a conclusion on which model is more accurate.

1 Introduction

1.1 Physical Motivation

In 1905 Albert Einstein published "*Zur Elektrodynamik bewegter Korper*" ("On The Electrodynamics of Moving Bodies")[3]. In it, he introduced what is colloquially known as his Special Theory of Relativity. It proposed modifications to Newton's laws of motion for objects travelling close to the speed of light. His theory was based on two postulates: The laws of physics are the same in all inertial (non-accelerating) reference frames (this is the postulate of relativity) and that the speed of light is the same in all inertial frames (which he surmised from the coordinate invariance of Maxwell's equations).

In the same year[1] he published *"Ist die Trgheit eines Krpers von seinem Energieinhalt abhngig?"* ("Does the Inertia of a Body Depend Upon Its Energy Content?")[4]. Here he proposed the so-called Mass-Energy Equivalence Principle which states that massive objects at rest contain energy distinct from their potential or kinetic energies. This is called rest-mass energy and is given by the famous equation $E = mc^2$.

The object of our experiment is to verify the predictions implied by the modifications of dynamics provided by the Special Theory of Relativity and the Mass-Energy Equivalence.

1.2 Theory

1.2.1 Scattering Energy Statistics

We would first like to develop a physical model governing the relationship between gamma radiation of a certain energy and expected spectroscopic observations. This model will allow us to extract the Compton edge as a fit parameter and use it to make our calculations of the rest mass. The mechanisms governing the emission of Gamma radiation is beyond the scope of this discussion. For a detailed discussion of such topics see [5]. We take the energies

[1] This year, 1905, later came to be know as the *Annus Mirabilis* from the latin for *Extraordinary Year* to commemorate Einstein's publishing of four of the most influential papers in modern physics

of gamma radiation from each isotope as given and focus on the interaction between the radiation and matter in the apparatus.

The expected energy spectrum is characterised by several physical processes. The two we are interested in are the photopeak and Compton scattering. The photopeak corresponds to the photoelectric effect completely absorbing the gamma radiation and emitting a photon of the same energy. This corresponds to a sharp peak at the gamma ray energy on the spectrum. The Compton scattering feature corresponds to the gamma radiation being scattered off the electron. The scattering energies are given by [1]

$$E'_\gamma = \frac{E_\gamma}{1 + (E_\gamma/m_e c^2)(1 - \cos\theta)} \tag{1}$$

Ideally, all scattering energies (corresponding to variation of θ) are equally likely and this corresponds to a Heaviside function at lower energies than the photopeak. The idealized expected spectrum is given by

$$P_\gamma(E) = aH(E; E_c) + b\delta(E - E_\gamma) \tag{2}$$

Where $H(E; E_c)$ is a Heaviside step-function centered at the Compton edge denoted by E_c and E_γ is the photopeak energy. This looks something like the following (ideally the delta function would have no width, that however, is not very easy to see on an illustrative plot)

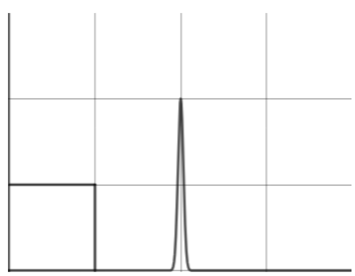

Figure 1: Plot of $P_\gamma(E)$, idealized energy distribution

The initial collision, scintillation, and photocathode reaction all carry an inherent randomness. Therefore, we propose that the expected number of counts C in an energy range $E + dE$ is given by

$$C = \int_E N(n; \mu(E), \sigma(E)) P_\gamma(E) dE \tag{3}$$

Where $N(n; \mu(E), \sigma(E))$ is a normal distribution with mean $\mu(E)$ and standard deviation (width-characteristic) $\sigma(E)$ accounting for the composition of random errors via the Central Limit Theorem.

1.2.2 Relativity

The relativistic energy-momentum relation is

$$E^2 = p^2c^2 + m^2c^3 \tag{4}$$

To incorporate the notion of kinetic energy this is rewritten as

$$E = K + mc^2 \tag{5}$$

This means that the relativistic form of kinetic energy is

$$K = \sqrt{p^2c^2 + m^2c^4} - mc^2 \tag{6}$$

The classical form of kinetic energy is given by

$$K = \frac{p^2}{2m} \tag{7}$$

Comparing our measurements to each of the two will reveal which is more valid.

1.2.3 Scattering

In order to extract physical parameters from our measured spectra we need to analyze the scattering mechanics inside the scintillation apparatus. The Compton edge corresponds to a head-on collision between the radiation and the electron (and the radiation being reflected in the opposite direction of incidence). Conservation of momentum implies

$$P_e = P_\gamma + P'_\gamma \tag{8}$$

Conservation of energy implies

$$E_\gamma = E_e + E'_\gamma \tag{9}$$

Combining these two and substituting the classical relationship between photon energy and momentum $P_\gamma = E_\gamma/c$ gives

$$cP_e = 2E_\gamma - E_e \tag{10}$$

Where E_γ is the photopeak energy and E_e is the Compton edge energy. This momentum value can be used to calculate the kinetic energy for the classical prediction and compare to that of the relativistic prediction.

2 Experimental Setup

2.1 Apparatus

The radioactive source is placed inside of a lead slug which is inserted at one end of a sealed metal vessel. On the other end of the vessel there is a scintillator attached to a photomultiplier tube. Gamma rays emitted by spontaneous radioactive decay (as described in Section 1.2.1) stimulate scintillation which is then converted to a voltage by the photomultiplier. The output of the photomultiplier tube is fed into a pre-amplifier. The pre-amplifier then feeds a signal to a spectroscopic amplifier set to shape the incoming pulses into outgoing gaussians. The output of the spectroscopic amplifier is fed in parallel to an oscilloscope for monitoring the signal as well as a pulse counter for recording statistical data.

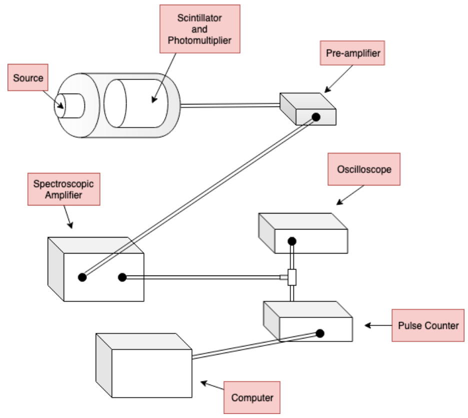

Figure 2: Schematic of experimental apparatus setup

The gaussian waveform produced by the spectroscopic amplifier effectively allows the pulse counter to assign an energy (bin number) to a detected electron from the height of the pulse. It can then perform a running count of measured events for each energy in the detection range.

2.1.1 Scintillator

The role of the scintillator is to produce visible photons to feed to the photomultiplier. When radiation strikes the scintillator crystal it ejects electrons from their bound states in constituent molecules in the crystal. These electrons then settle into other bound states and emit photons in the visible spectrum. The emitted photons then travel through the lattice into the photomultiplier tube. They travel freely because the energy gaps between bound states in the molecules are much larger than the photon energies and thus they are not absorbed. This makes the crystal lattice see-through.

A natural question arises: If the emitted photons do not correspond to energies like those of the band gaps present in the molecules of the crystal how do they get emitted in the first place? The emitted photons correspond to electrons settling into bound states of a different element than the crystal is made out of, one which whose bound states fill those band gaps of the crystal. In our case, we are using a Sodium-Iodide scintillator doped with Thallium. This particular combination works well for the detection of gamma radiation which we are using in our experiment due to its high energy.

2.1.2 Photomultiplier

The photomultiplier converts incoming photons into a voltage which can be measured and used to deduce properties about the incoming radiation. Incoming photons strike the surface of the photomultiplier, called a photocathode, which causes an electron to be ejected. The electron is then accelerated through a series of dynos (metallic surfaces with lose electrons) by a potential. This causes a snowball effect in which a single electron ejects more and more electrons from each dyno which in turn eject their own multiples of electrons. The mass of electrons finally reaches the anode of the potential difference and is converted into a voltage.

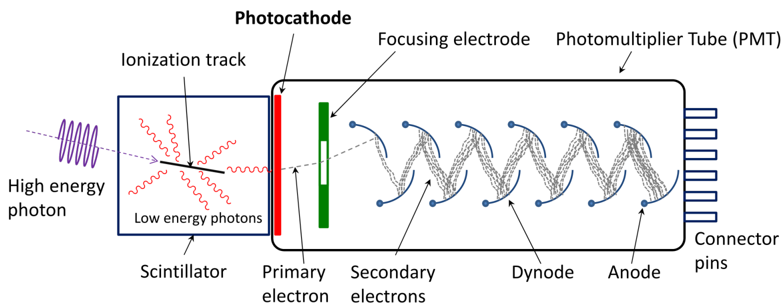

Figure 3: A schematic of the scintillator-photomultiplier apparatus

2.2 Data Collection

We collected spectroscopic data for seven radioactive isotopes in order to cover a broad range of energies from which comparisons could be made with relativistic predictions. The radioactive isotopes used were: Cs-137, Na-22, Co-60, Ba-133, Co-57, Ca-109, and Mn-54.

We collected data using GammaVision software which interfaced with the pulse counter. The data consists of "bins" or channels corresponding to energies along with a corresponding count of how many times that energy was detected. Pairing this with the known gamma radiation energies of our isotopes we were able to identify features in the data corresponding to physical processes occurring inside the detector.

3 Analysis and Results

Below is a sample spectra collected via our experiment

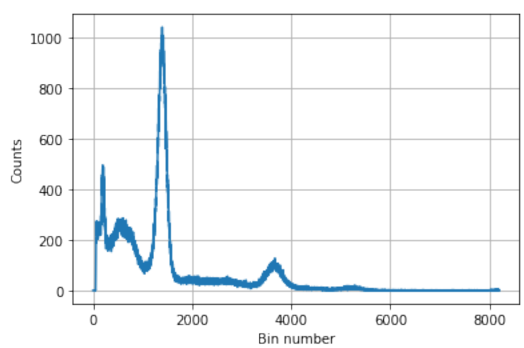

Figure 4: Na-22 Gamma Ray Spectrum

The measured data is as follows

Isotope	$E_\gamma(keV)$	E$_c(keV)$	P (kgm/s)
Ba-133	357	205	1.70×10^{-6}
Cs-137	662	475	2.83×10^{-6}
Co-57	120	39	6.70×10^{-7}
Co-60	1180	960	4.67×10^{-6}
Na-22	510	340	2.26×10^{-6}
Zn-65	1120	910	4.45×10^{-6}

Table 1: Summarized Experiment Results

4 Conclusion

I was unable to produce an adequately accurate fit of the spectra data and settled on eye-balling the quantities from high-resolution plots. I was also unable to produce a convincing argument of the validity of the relativistic equation from the data I did acquire (Table 1).

References

[1] Physics LibreTexts. *University Physics III - Optics and Modern Physics*. Ch 6.3.

[2] P.L. jolivette. N.Rouze. *Compton Scattering, The Electron Mass, and Relativity: A Laboratory Experiment*. 1993.

[3] Einstein, Albert (1905). *Zur Elektrodynamik bewegter Korper*. Annalen der Physik. 17 (10): 891921.

[4] Einstein, Albert (1905). *Ist die Trgheit eines Krpers von seinem Energieinhalt abhngig?*. Annalen der Physik. 18 (13): 639641.

[5] Krane, K.S. *Introductory Nuclear Physics*. Ch.10. Wiley, 1988.