LAB 3 - Report

VARUN
AGGARWAL

IMAGE 09

$RMSE\ Residual$: 1.62E−02

$Lab\ 2$: $\widehat{\sigma_0^{1/2}} = 1.47e-02$

Summary

	c (mm)	X0	Y0	Z0	omega(deg)	phi(deg)	kappa(deg)
Lab 1	8.167E+00	1.890E+00	3.036E+00	3.735E+00	-1.918E+01	-4.345E+00	2.050E+00
Lab 2	8.167E+00	1.890E+00	3.036E+00	3.735E+00	-1.918E+01	-4.345E+00	2.050E+00
Lab 3	7.177E+00	1.923E+00	2.712E+00	3.351E+00	-1.627E+01	-4.304E+00	2.084E+00
Delta from Lab 1	9.906E-01	3.371E-02	3.239E-01	3.836E-01	2.914E+00	4.088E-02	3.375E-02
Delta from Lab 2	9.906E-01	3.371E-02	3.239E-01	3.836E-01	2.914E+00	4.088E-02	3.375E-02

IMAGE 09

Image No	Residual X (mm)	Residual Y (mm)
Image 1	-6.00E-02	4.91E-03
Image 2	-7.96E-03	8.17E-03
Image 3	2.34E-02	-7.01E-03
Image 4	9.42E-03	-1.59E-02
Image 5	-1.07E-02	-8.20E-03
Image 6	-8.74E-03	-2.47E-02
Image 7	6.24E-03	-1.45E-02
Image 8	1.29E-02	-1.05E-02
Image 9	8.02E-03	3.14E-03
Image 10	-3.07E-04	1.77E-02
Image 11	3.45E-02	-3.67E-04
Image 12	7.49E-03	2.66E-03
Image 13	-1.36E-05	-3.77E-03
Image 14	1.67E-03	8.93E-03
Image 15	-5.98E-03	2.32E-02
Image 16	2.54E-02	3.03E-02
Image 17	1.83E-03	1.64E-02
Image 18	-2.78E-03	3.62E-03
Image 19	1.60E-03	2.14E-03
Image 20	-4.20E-03	1.78E-02
Image 21	-1.03E-02	1.64E-02
Image 22	-1.68E-02	-9.85E-03
Image 23	-6.42E-03	-3.21E-02
Image 24	1.00E-03	-2.42E-02
Image 25	-1.89E-03	-6.29E-03

IMAGE 10

$RMSE\ Residual$: 1.56E−02

$$Lab\ 2\text{:}\ \widehat{\sigma_0^{1/2}} = 6.50e - 03$$

Summary

	c (mm)	X0	Y0	Z0	omega(deg)	phi(deg)	kappa(deg)
Lab 1	8.167E+00	1.680E+00	2.166E+00	3.509E+00	-4.149E-01	-4.937E+00	9.248E+01
Lab 2	8.167E+00	1.680E+00	2.166E+00	3.509E+00	-4.149E-01	-4.937E+00	9.248E+01
Lab 3	8.167E+00	1.603E+00	2.155E+00	3.514E+00	-2.673E-01	-6.055E+00	9.250E+01
Delta from Lab 1	0.000E+00	7.767E-02	1.039E-02	5.673E-03	1.475E-01	1.118E+00	2.109E-02
Delta from Lab 2	0.000E+00	7.769E-02	1.039E-02	5.668E-03	1.476E-01	1.118E+00	2.104E-02

IMAGE 10

Image No	Residual X (mm)	Residual Y (mm)
Image 1	1.65E-02	2.97E-02
Image 2	2.10E-02	-1.99E-03
Image 3	-3.04E-03	-1.99E-02
Image 4	-2.15E-02	2.09E-04
Image 5	-1.80E-02	2.90E-02
Image 6	-1.28E-02	1.94E-02
Image 7	-1.29E-02	-4.44E-03
Image 8	2.31E-03	-1.23E-02
Image 9	7.45E-03	-6.82E-03
Image 10	1.54E-02	1.99E-03
Image 11	-2.21E-02	-1.92E-02
Image 12	-1.06E-02	-8.07E-03
Image 13	1.78E-03	-6.08E-03
Image 14	1.11E-02	-1.08E-02
Image 15	1.82E-02	-1.77E-02
Image 16	1.12E-02	-3.06E-02
Image 17	1.41E-03	9.95E-03
Image 18	-9.56E-04	1.13E-02
Image 19	1.81E-03	-9.65E-04
Image 20	1.16E-02	-5.38E-03
Image 21	3.58E-02	-7.95E-03
Image 22	-1.15E-02	2.51E-02
Image 23	-2.67E-02	1.94E-02
Image 24	-1.53E-02	5.99E-03

IMAGE 14

$RMSE\ Residual$: 9.87E−03

$Lab\ 2\!: \widehat{\sigma_0^{1/2}} = 8.50e-03$

Summary

	c (mm)	X0	Y0	Z0	omega(deg)	phi(deg)	kappa(deg)
Lab 1	8.167E+00	1.010E+00	2.533E+00	2.806E+00	-8.940E+00	-1.705E+01	9.118E+01
Lab 2	8.167E+00	1.010E+00	2.533E+00	2.806E+00	-8.940E+00	-1.705E+01	9.118E+01
Lab 3	7.976E+00	1.010E+00	2.475E+00	2.753E+00	-8.072E+00	-1.729E+01	9.138E+01
Delta from Lab 1	1.915E-01	7.311E-05	5.828E-02	5.291E-02	8.678E-01	2.353E-01	2.032E-01
Delta from Lab 2	1.915E-01	8.080E-05	5.826E-02	5.290E-02	8.675E-01	2.351E-01	2.031E-01

IMAGE 14

Image No	Residual X (mm)	Residual Y (mm)
Image 3	2.44E-02	-1.56E-02
Image 4	-8.49E-03	1.13E-02
Image 5	-2.62E-03	2.86E-02
Image 7	-3.84E-03	8.49E-03
Image 8	1.29E-03	-1.22E-02
Image 9	1.05E-03	-6.84E-03
Image 10	8.02E-03	-3.36E-03
Image 12	-2.29E-02	-3.49E-03
Image 13	-7.96E-03	-8.25E-03
Image 14	-3.13E-03	-8.00E-03
Image 15	-3.15E-04	-1.63E-02
Image 17	-3.51E-03	1.66E-03
Image 18	2.67E-03	6.43E-03
Image 19	1.99E-03	6.37E-03
Image 20	-4.48E-03	-4.06E-03
Image 22	1.21E-02	4.70E-03
Image 23	6.18E-03	6.20E-03
Image 24	3.66E-03	6.21E-03
Image 25	-4.61E-03	-2.30E-03

IMAGE 18

$RMSE\ Residual$: 1.76E−02

$Lab\ 2\!: \widehat{\sigma_0^{1/2}} = 1.17e-02$

Summary

	c (mm)	X0	Y0	Z0	omega(deg)	phi(deg)	kappa(deg)
Lab 1	8.167E+00	2.102E+00	1.195E+00	3.700E+00	1.337E+01	1.466E+00	9.023E+01
Lab 2	8.167E+00	2.102E+00	1.195E+00	3.700E+00	1.337E+01	1.465E+00	9.023E+01
Lab 3	8.167E+00	2.012E+00	1.181E+00	3.711E+00	1.352E+01	2.426E-01	9.042E+01
Delta from Lab 1	8.167E+00	8.993E-02	1.319E-02	1.132E-02	1.542E-01	1.223E+00	1.921E-01
Delta from Lab 2	8.167E+00	8.992E-02	1.318E-02	1.133E-02	1.542E-01	1.223E+00	1.921E-01

IMAGE 18

Image No	Residual X (mm)	Residual Y (mm)
Image 1	1.94E-02	2.10E-02
Image 2	2.16E-02	5.96E-06
Image 3	-5.28E-03	-1.51E-02
Image 4	-3.17E-02	-3.27E-03
Image 5	-3.84E-02	1.46E-02
Image 6	-3.38E-03	1.50E-02
Image 7	-2.62E-03	-6.08E-04
Image 8	8.73E-03	-3.19E-03
Image 9	7.55E-03	-4.01E-03
Image 10	9.98E-03	-3.92E-03
Image 11	-1.91E-02	-1.29E-02
Image 12	-5.80E-03	4.01E-03
Image 13	1.20E-02	7.04E-03
Image 14	2.13E-02	-4.94E-03
Image 15	2.59E-02	-2.45E-02
Image 16	-3.61E-03	-4.02E-02
Image 17	-2.22E-03	1.08E-02
Image 18	2.63E-03	2.09E-02
Image 19	8.50E-03	3.46E-03
Image 20	1.63E-02	-2.09E-02
Image 21	1.75E-02	-3.74E-02
Image 22	-1.97E-02	2.41E-02
Image 23	-2.96E-02	3.19E-02
Image 24	-1.90E-02	1.52E-02
Image 25	8.04E-03	3.58E-03

Issues and Explanation

- The EOPs for all the images, when compared to output of BASC and Lab 2, the difference is of the order of $10^0\ to\ 10^{-2}$
- The principal distance, c, when compared to output of BASC and Lab 2, the difference is of the order of 10^{-1}
- The difference between the values can be attributed to the assumption that object plane is flat
- The residuals are of the order of 10^{-3}
- The RMSE when compared to Lab 2's square root of variance are very close
- For image 10 and 18, the c was imaginary. It would occur in instances when images are very vertical.
- To address this issue, c was manually set to 8.16
- The major issue for this lab was with debugging the code
- While coding the mathematical equations, a misplaced plus or minus within the code let to wrong output
- Debugging required a lot of focus because of the number of equations and variables in the model

Computer Pseudo Code

main.m

Declare common parameters -> IOPs, Ground co-ordinates of targets

For each image:
Get image co-ordinates for each image

Call **LSA.m function**

LSA.m

Remove distortions from image co-ordinates **remove_dist.m**
Calculate A matrix by calling **calc_A.m**
Calculate y matrix by calling **calc_y.m**
Calculate x_hat by calling **calc_x_hat.m**
Calculate c, X0, Y0, Z0

Calculate S matrix by calling **calc_S.m**
Calculate omega phi kappa by calling **calc_opk.m**
Calculate residuals by calling **residuals.m**

Code on Github:
https://github.com/Salazar-Prime/photogrammetry/tree/master/Project%203

Computer Pseudo Code

remove_dist.m

Calculate radial distortion
Calculate de-centering lens distortion
Subtract from image co-ordiantes

Calc_A.m

Calculate A for both X and Y

Calc_y.m

Calculate y by alternating xi and yi without distortions

Calc_x_hat.m

```
x_hat = inv(A'*A)*A'*y;
```

Calc_S.m

Loop over all GCPs
get xi quaternion and normalize it
get Xi quaternion and normalize it
multiply C for both quaternions to get Si
sum Si to get the S matrix

Code on Github:
https://github.com/Salazar-Prime/photogrammetry/tree/master/Project%203

Computer Pseudo Code

Calc_opk.m

Get eigenvalue and eigenvector for S
Get the eigen vector against maximum eigenvalue
Get rotation matrix
Calculate Omega, Phi, kappa from rotation matrix

Code on Github:

https://github.com/Salazar-Prime/photogrammetry/tree/master/Project%203

Compute Code – main.m

```
%% Common Parameters

% IOPs
 IOP = [6.7451660984e-2,-1.1709829919e-1,8.1671200690];
dist = [-2.9350008918e-4, 9.2190322166e-6,-2.2562559450e-7,6.1878890685e-
5,-7.2907688047e-5,0];

% Ground co-ordinates of targets
XA = [-
0.044038221723;0.89829592555;1.8428648416;2.7928032424;3.731135328;-
0.036927916146;0.90108606018;1.8511589932;2.8010339963;3.7422959221;-
0.037704361429;0.90892725171;1.8588605461;2.8062554595;3.7520655984;-
0.024722332105;0.91454137562;1.8596674595;2.8078118825;3.7544680979;0;0.9
263914178;1.8642980884;2.8111736654;3.7628755375];
YA =
[3.751130948;3.7594793917;3.7736433303;3.7823238561;3.7801397656;2.805767
5589;2.8180218203;2.8257925207;2.8332144474;2.8378518827;1.8634346772;1.8
678901457;1.8758730363;1.8838128654;1.8922797004;0.91840024749;0.92066047
041;0.92760553219;0.93553829291;0.94557055946;0;-0.00026178267339;-
0.0021390199929;-0.0019752230297;0];
ZA = [0.072640804133;0.048121990404;0;-0.047649472361;-
0.070838477818;0.054102199693;0.026008316122;0.0059282538012;-
0.023295585379;-
0.044574905144;0.016369939657;0.010208246675;0.0071618170061;-
0.0096698096332;-0.020910207448;-
0.0091692255168;0.003877584251;0.003049069041;-0.0030509411619;-
0.0098127723745;0;0.017342094476;0.016047411487;0.006582607018;0];

%% Image 1 - image_09

[EOP_basc,xa,ya] = data(9); % get data for image 9
[c,EOP,res_x,res_y] = LSA(xa,ya,XA,YA,ZA,IOP,dist)
%% Image 1 - image_10

[EOP_basc,xa,ya] = data(10); % get data for image 10
[c,EOP,res_x,res_y] = LSA(xa,ya,XA(1:24),YA(1:24),ZA(1:24),IOP,dist)
%% Image 1 - image_14

[EOP_basc,xa,ya] = data(14); % get data for image 14
[c,EOP,res_x,res_y] =
LSA(xa,ya,XA([3:5,7:10,12:15,17:20,22:25]),YA([3:5,7:10,12:15,17:20,22:25
]),ZA([3:5,7:10,12:15,17:20,22:25]),IOP,dist);

%% Image 1 - image_18

[EOP_basc,xa,ya] = data(18); % get data for image 18
[c,EOP,res_x,res_y] = LSA(xa,ya,XA,YA,ZA,IOP,dist)
```

Code on Github:

https://github.com/Salazar-Prime/photogrammetry/tree/master/Project%203

Compute Code – LSA.m

```
function [c,EOP,res_x,res_y] = LSA(xa,ya,XA,YA,ZA,IOP,dist)
%% parameters
num_GCP = length(xa);

% Unpack IOPs
[xp,yp,~] = assign_IOP(IOP);

%% A and y matrix
[xa,ya] = remove_dist(xa,ya,dist,xp,yp);
[A] = calc_A(num_GCP,xa,ya,XA,YA);
[y] = calc_y(xa,ya);

%% calculate x_hat

[C1,C2,C3,C4,C5,C6,C7,C8] = calc_x_hat(A,y);
%% calculate c
c = sqrt(-1 * (C1*C2+C4*C5)/(C7*C8));
if not(isreal(c))
    c = 8.1671200690;
end
%% calculate A'*A
A =
[C1,C4,C7;C2,C5,C8;C3,C6,1]*[1,0,0;0,1,0;0,0,c*c]*[C1,C2,C3;C4
,C5,C6;C7,C8,1];

%% calculate EOP (X0,Y0,Z0)
X0 = - A(1,3)/A(1,1);
Y0 = - A(2,3)/A(1,1);
Z0 = sqrt(A(3,3)/A(1,1) - X0^2 - Y0^2);

%% calculate EOP (omega,phi,kappa)
S = calc_S(num_GCP,xa,ya,xp,yp,c,XA,YA,ZA,X0,Y0,Z0);
[omega,phi,kappa] = calc_opk(S);
EOP = [X0,Y0,Z0,rad2deg(omega),rad2deg(phi),rad2deg(kappa)];

%% residuals
[res_x,res_y] =
residuals(C1,C2,C3,C4,C5,C6,C7,C8,xa,ya,XA,YA);
end
```

Code on Github:

https://github.com/Salazar-Prime/photogrammetry/tree/master/Project%203

Compute Code – calc_A.m

```
function [A] = calc_A(num_GCP,xa,ya,XA,YA)

% create an empty A matrix
A = zeros(num_GCP*2,8);

% loop over all ground control points
for i=1:num_GCP

    A(2*i-1,:) = [XA(i),YA(i),1, 0,0,0, -
1*xa(i)*XA(i), -1*xa(i)*YA(i)];
    A(2*i,  :) = [0,0,0, XA(i),YA(i),1, -
1*ya(i)*XA(i), -1*ya(i)*YA(i)];

end
```

Compute Code – calc_y.m

```
function [y] = calc_y(xa,ya)

% alternate merge in order : xa, ya
y = [xa ya]';
y = y(:);

end
```

Code on Github:

https://github.com/Salazar-Prime/photogrammetry/tree/master/Project%203

Compute Code – calc_S.m

```
function [S] =
calc_S(num_GCP,xa,ya,xp,yp,c,XA,YA,ZA,X0,Y0,Z0)

S = zeros(4,4);
for i=1:num_GCP
    xi = [0,xa(i),ya(i),-c];
    xi  = xi/norm(xi);
    Xi = [0,XA(i)-X0,YA(i)-Y0,ZA(i)-Z0];
    Xi = Xi/norm(Xi);

    mat1 = [xi(1),-xi(2),-xi(3),-xi(4);
            xi(2), xi(1), xi(4),-xi(3);
            xi(3),-xi(4), xi(1), xi(2);
            xi(4), xi(3),-xi(2), xi(1)];
    mat2 = [Xi(1),-Xi(2),-Xi(3),-Xi(4);
            Xi(2), Xi(1),-Xi(4), Xi(3);
            Xi(3), Xi(4), Xi(1),-Xi(2);
            Xi(4),-Xi(3), Xi(2), Xi(1)];
    S = S + mat1'*mat2;
    trace(mat1*mat2)
end
end
```

Compute Code – calc_x_hat.m

```
function [C1,C2,C3,C4,C5,C6,C7,C8] = calc_x_hat(A,y)

x_hat = inv(A'*A)*A'*y;
C1 = x_hat(1);
C2 = x_hat(2);
C3 = x_hat(3);
C4 = x_hat(4);
C5 = x_hat(5);
C6 = x_hat(6);
C7 = x_hat(7);
C8 = x_hat(8);

end
```

Compute Code – calc_opk.m

```
function [omega,phi,kappa] = calc_opk(S)

[V,D] = eig(S);
[~,id] = sort(diag(D),'descend');
ev = V(:,id(1));
% quat2rotm(ev')
r13 = 2*ev(2)*ev(4) + 2*ev(1)*ev(3);
r12 = 2*ev(2)*ev(3) - 2*ev(1)*ev(4);
r11 = ev(2)^2 + ev(1)^2 - ev(3)^2 - ev(4)^2;
r23 = 2*ev(3)*ev(4) - 2*ev(1)*ev(2);
r33 = ev(4)^2 + ev(1)^2 - ev(3)^2 - ev(2)^2;

phi = asin(r13); % r13
kappa = atan2(-r12/cos(phi),r11/cos(phi))
omega = atan2(-r23/cos(phi),r33/cos(phi))
end
```

Compute Code – residuals.m

```
function [res_x,res_y] =
residuals(C1,C2,C3,C4,C5,C6,C7,C8,xa,ya,XA,YA)
    res_x = xa - (C1*XA + C2*YA +
C3)./(C7*XA+C8*YA+1);
    res_y = ya - (C4*XA + C5*YA +
C6)./(C7*XA+C8*YA+1);
end
```

Code on Github:

https://github.com/Salazar-Prime/photogrammetry/tree/master/Project%203

Compute Code – data.py

```
function [EOP_basc,xa,ya]=data(image_no)

    switch image_no
        case 9
            EOP_basc = [1.88968674,3.03586621,3.73500036,-19.1841784,-
4.34497759,2.05014903];
            xa = [-5.3877;-2.92187;-0.49439;1.79834;3.93369;-5.04787;-
2.77407;-0.55643;1.5572;3.55246;-4.71235;-2.63868;-
0.61275;1.34004;3.19895;-4.43154;-2.5313;-0.66436;1.1514;2.88537;-
4.19557;-2.4407;-0.70131;0.99217;2.62277];
            ya =
[4.93742;4.84433;4.68172;4.49867;4.29866;2.42494;2.3434;2.23887;2.1432;2
.04441;0.30738;0.24322;0.17794;0.13305;0.09228;-1.49013;-1.53833;-
1.5756;-1.59753;-1.59671;-3.02777;-3.06606;-3.09871;-3.09563;-3.07743];
        case 10
            EOP_basc = [1.68045678e+000   2.16587520e+000
3.50855335e+000 -4.14855263e-001  -4.93725230e+000   9.24769326e+001];
            xa =
[4.2568;4.0854;3.8851;3.68;3.479;1.9221;1.7977;1.6796;1.555;1.4339;-
0.3967;-0.4724;-0.5334;-0.5915;-0.6454;-2.6632;-2.71;-2.7285;-2.7338;-
2.7224;-4.81;-4.8779;-4.8782;-4.8346];
            ya= [4.6788;2.3093;0.0418;-2.0823;-
4.0521;4.7589;2.3888;0.1175;-2.0216;-4.02;4.8045;2.45;0.1921;-1.9492;-
3.9707;4.8258;2.5321;0.2918;-1.8528;-3.8693;4.8076;2.5851;0.3697;-
1.7626];
        case 14
            EOP_basc = [1.00952899e+000   2.53318361e+000
2.80635782e+000 -8.94008150e+000  -1.70515538e+001   9.11810993e+001];
            xa = [5.0408;4.5063;4.0613;2.4057;2.1457;1.9236;1.7394;-
0.5466;-0.5072;-0.48;-0.4562;-3.1775;-2.9114;-2.6867;-2.495;-5.4743;-
5.0707;-4.6992;-4.374];
            ya = [-0.3354;-2.7179;-4.6482;2.6935;-0.1767;-2.5023;-
4.414;2.7038;-0.0294;-2.2763;-4.1595;2.7274;0.1323;-2.0413;-
3.8797;2.7125;0.2538;-1.841;-3.6312];
        case 18
            EOP_basc = [2.10162316e+000   1.19465520e+000
3.69958966e+000 1.33667830e+001   1.46559054e+000   9.02308070e+001];
            xa =
[3.2893;3.295;3.2772;3.2471;3.2149;1.5691;1.574;1.5795;1.5697;1.557;-
0.3378;-0.3441;-0.3375;-0.3381;-0.3412;-2.4459;-2.481;-2.4964;-2.5036;-
2.4987;-4.7066;-4.8269;-4.8928;-4.9179;-4.8917];
            ya = [3.8357;2.0059;0.1629;-1.6697;-
3.476;4.0453;2.1277;0.1844;-1.7649;-3.6961;4.2673;2.2563;0.2058;-
1.8635;-3.9356;4.4805;2.4023;0.2466;-1.9536;-
4.1554;4.6783;2.5441;0.2765;-2.0524;-4.369];

    end
end
```

Compute Code – assign_IOP.py

```
function [xp,yp,c]=assign_IOP(IOP)
    xp = IOP(1);
    yp = IOP(2);
     c = IOP(3);
end
```

Code on Github:

https://github.com/Salazar-Prime/photogrammetry/tree/master/Project%203