# LAB 2 - Report

VARUN AGGARWAL

Total Iterations: 5

$$\widehat{\sigma_0^2} = 2.167e - 04$$

 $Threshold = 10^{-8}$ 

### **Summary**

	хo	YO	zo	Omega (deg)	Phi (deg)	Kappa (deg)
Initial Approximation	1.6000e+00	3.2000e+00	3.5000e+00	0.0000e+00	0.0000e+00	0.0000e+00
Final EOP Value	1.8897e+00	3.0359e+00	3.7350e+00	-1.9184e+01	-4.3450e+00	2.0501e+00
SD for EOPs	9.8035e-03	8.8936e-03	3.9152e-03	2.1493e-03	2.2192e-03	9.0243e-04
Delta EOP from BASC	1.9622e-08	1.5206e-08	5.9034e-09	1.8372e-07	1.9285e-07	6.1227e-08

### **Dispersion Matrix**

	хo	YO	ZO	Omega (deg)	Phi (deg)	Kappa (deg)
¥0	0.61000.05	-2.7239e-06	1 63000 06	0.65210.07	2.1529e-05	E 01350 06
XO YO	9.6109e-05 -2.7239e-06	7.9096e-05	-1.6399e-06 -2.2975e-05	9.6521e-07 -1.8865e-05	-5.9816e-07	5.0135e-06 5.4154e-07
ZO	-1.6399e-06	-2.2975e-05	1.5329e-05	5.8205e-06	-5.0113e-07	-3.3482e-07
Omega (deg)	9.6521e-07	-1.8865e-05	5.8205e-06	4.6197e-06	2.0314e-07	-6.5548e-08
Phi (deg)	2.1529e-05	-5.9816e-07	-5.0113e-07	2.0314e-07	4.9247e-06	1.0929e-06
Kappa (deg)	5.0135e-06	5.4154e-07	-3.3482e-07	-6.5548e-08	1.0929e-06	8.1437e-07

	хo	YO	ZO	Omega (deg)	Phi (deg)	Kappa (deg)
Initial Approximation	1.6000e+00	3.2000e+00	3.5000e+00	0.0000e+00	0.0000e+00	0.0000e+00
Iteration 1	1.9589e+00	2.5775e+00	3.4054e+00	-2.2255e-01	-4.0115e-02	3.0624e-02
Iteration 2	1.9190e+00	2.9273e+00	3.7355e+00	-3.1565e-01	-6.9540e-02	3.6687e-02
Iteration 3	1.8905e+00	3.0351e+00	3.7370e+00	-3.3466e-01	-7.5685e-02	3.5860e-02
Iteration 4	1.8897e+00	3.0359e+00	3.7350e+00	-3.3483e-01	-7.5834e-02	3.5782e-02
Iteration 5	1.8897e+00	3.0359e+00	3.7350e+00	-3.3483e-01	-7.5834e-02	3.5782e-02

#### Residuals X Y -9.1259e-03 -7.2668e-03 -5.5392e-03 2.4577e-04 -8.9980e-04 8.7198e-04 7.6428e-03 2.9308e-03 2.1732e-02 6.8278e-03 1.7264e-02 -6.9440e-03 1.5714e-03 -4.2503e-03 -1.0076e-03 -7.3490e-03 -6.8217e-04 -2.2903e-03 5.5982e-03 4.6218e-03 2.2853e-02 5.0299e-03 2.0849e-03 5.6196e-03 -5.1236e-03 -1.0008e-03 -5.5518e-03 3.1556e-03 -1.5575e-02 1.2531e-02 -9.6309e-04 3.4986e-03 1.1395e-03 8.9576e-03 -6.4612e-04 1.7559e-03 -6.3297e-04 7.7563e-04 -1.4522e-02 1.6327e-02 -1.8579e-02 -1.0490e-02 -4.9213e-03 -5.4186e-03 2.6750e-03 -1.9738e-02 2.5594e-03 -1.3976e-02 -1.0906e-02 5.3875e-03

### Measured Image Coordinates (mm)

 $\nu_{\alpha}$ 

 $\chi_{\alpha}$ 

$\lambda a$	$y_a$				
-5.3877e+00	4.9374e+00				
-2.9219e+00	4.8443e+00				
-4.9439e-01	4.6817e+00				
1.7983e+00	4.4987e+00				
3.9337e+00	4.2987e+00				
-5.0479e+00	2.4249e+00				
-2.7741e+00	2.3434e+00				
-5.5643e-01	2.2389e+00				
1.5572e+00	2.1432e+00				
3.5525e+00	2.0444e+00				
-4.7123e+00	3.0738e-01				
-2.6387e+00	2.4322e-01				
-6.1275e-01	1.7794e-01				
1.3400e+00	1.3305e-01				
3.1989e+00	9.2280e-02				
-4.4315e+00	-1.4901e+00				
-2.5313e+00	-1.5383e+00				
-6.6436e-01	-1.5756e+00				
1.1514e+00	-1.5975e+00				
2.8854e+00	-1.5967e+00				
-4.1956e+00	-3.0278e+00				
-2.4407e+00	-3.0661e+00				
-7.0131e-01	-3.0987e+00				
9.9217e-01	-3.0956e+00				
2.6228e+00	-3.0774e+00				

Total Iterations: 4

$$\widehat{\sigma_0^2} = 4.191e - 05$$

 $Threshold = 10^{-8}$ 

### **Summary**

	AU	10	20	Onlega (deg)	Fill (deg)	Kappa (deg)
Initial Approximation	1.6361e+00	2.1841e+00	3.7271e+00	-1.8415e-02	-9.1752e-02	1.6060e+00
Final EOP Value	1.6805e+00	2.1659e+00	3.5086e+00	-4.1494e-01	-4.9369e+00	9.2477e+01
SD for EOPs	3.6917e-03	3.6434e-03	1.1079e-03	9.0229e-04	9.1565e-04	3.0840e-04
Delta EOP from BASC	2.6532e-05	2.9775e-06	4.9556e-06	8.3212e-05	3.1282e-04	4.9575e-05

### **Dispersion Matrix**

	X0	YO	<b>Z</b> 0	Omega (deg)	Phi (deg)	Kappa (deg)
x0	1.3628e-05	-1.9510e-06	2.2989e-07	5.0942e-07	3.3402e-06	-9.4792e-08
YO	-1.9510e-06	1.3275e-05	-8.9664e-07	-3.2459e-06	-5.1142e-07	6.2440e-08
Z0	2.2989e-07	-8.9664e-07	1.2274e-06	2.4118e-07	4.8197e-08	-5.3369e-09
Omega (deg)	5.0942e-07	-3.2459e-06	2.4118e-07	8.1412e-07	1.3356e-07	-8.6030e-09
Phi (deg)	3.3402e-06	-5.1142e-07	4.8197e-08	1.3356e-07	8.3842e-07	-1.9525e-08
Kappa (deg)	-9.4792e-08	6.2440e-08	-5.3369e-09	-8.6030e-09	-1.9525e-08	9.5113e-08

	X0	YO	ZO	Omega (deg)	Phi (deg)	Kappa (deg)
	18-	3	:: <del></del>	: <del></del> e	-	18
Initial Approximation	1.6361e+00	2.1841e+00	3.7271e+00	-1.8415e-02	-9.1752e-02	1.6060e+00
Iteration 1	1.6849e+00	2.1596e+00	3.4951e+00	-5.8678e-03	-8.5812e-02	1.6146e+00
Iteration 2	1.6805e+00	2.1658e+00	3.5085e+00	-7.2273e-03	-8.6160e-02	1.6140e+00
Iteration 3	1.6805e+00	2.1659e+00	3.5086e+00	-7.2420e-03	-8.6166e-02	1.6140e+00
Iteration 4	1.6805e+00	2.1659e+00	3.5086e+00	-7.2420e-03	-8.6166e-02	1.6140e+00

### Residuals

X	Y
-6.6692e-03	-2.5122e-03
4.1185e-04	1.4938e-03
2.8644e-03	5.6031e-04
5.0626e-03	-2.5870e-03
3.0542e-03	4.3497e-04
-4.5266e-04	-1.6212e-03
-4.7744e-03	-8.5173e-04
1.8819e-03	-1.3400e-03
2.1246e-03	-1.0629e-03
1.9392e-03	-2.2475e-03
-3.0407e-03	-3.9211e-03
-1.5160e-03	-7.0483e-03
2.4444e-03	-5.4390e-03
1.3557e-03	-4.9984e-03
-1.6863e-04	-1.5924e-03
-3.9452e-03	5.6887e-03
-5.3150e-04	3.7276e-03
-1.2248e-03	1.1518e-03
-9.9875e-04	-5.0982e-04
5.8577e-03	1.4762e-02
6.6226e-03	5.0771e-03
-2.2598e-03	5.0348e-04
-7.7244e-03	-6.6451e-05
1.3230e-03	2.8813e-03

### Measured Image Coordinates (mm)

$x_a$	$y_a$
4.2568e+00	4.6788e+00
4.0854e+00	2.3093e+00
3.8851e+00	4.1800e-02
3.6800e+00	-2.0823e+00
3.4790e+00	-4.0521e+00
1.9221e+00	4.7589e+00
1.7977e+00	2.3888e+00
1.6796e+00	1.1750e-01
1.5550e+00	-2.0216e+00
1.4339e+00	-4.0200e+00
-3.9670e-01	4.8045e+00
-4.7240e-01	2.4500e+00
-5.3340e-01	1.9210e-01
-5.9150e-01	-1.9492e+00
-6.4540e-01	-3.9707e+00
-2.6632e+00	4.8258e+00
-2.7100e+00	2.5321e+00
-2.7285e+00	2.9180e-01
-2.7338e+00	-1.8528e+00
-2.7224e+00	-3.8693e+00
-4.8100e+00	4.8076e+00
<b>-4.</b> 8779e+00	2.5851e+00
-4.8782e+00	3.6970e-01
-4.8346e+00	-1.7626e+00

Total Iterations: 4

 $\widehat{\sigma_0^2} = 7.2144e - 05$ 

 $Threshold = 10^{-8}$ 

### **Summary**

	хo	¥0	ZO	Omega (deg)	Phi (deg)	Kappa (deg)
Initial Approximation	9.3801e-01	2.4928e+00	2.9883e+00	-1.4337e-01	-3.0529e-01	1.5814e+00
Final EOP Value	1.0095e+00	2.5332e+00	2.8064e+00	-8.9398e+00	-1.7052e+01	9.1181e+01
SD for EOPs	3.8604e-03	3.7749e-03	2.7233e-03	1.1152e-03	1.2504e-03	5.4805e-04
Delta EOP from BASC	7.6935e-06	1.7155e-05	7.1523e-06	3.2238e-04	1.9592e-04	6.1085e-05

### **Dispersion Matrix**

	X0	10	20	Omega (deg)	PHI (deg)	Rappa (deg)
хo	1.4903e-05	6.8329e-07	7.2055e-06	2.3765e-07	4.6886e-06	2.0271e-07
YO	6.8329e-07	1.4250e-05	-3.4952e-06	-4.0880e-06	-2.2047e-08	-1.1044e-06
<b>Z</b> 0	7.2055e-06	-3.4952e-06	7.4166e-06	1.4050e-06	2.6646e-06	4.5797e-07
Omega (deg)	2.3765e-07	-4.0880e-06	1.4050e-06	1.2437e-06	1.6500e-07	3.2188e-07
Phi (deg)	4.6886e-06	-2.2047e-08	2.6646e-06	1.6500e-07	1.5636e-06	1.0213e-07
Kappa (deg)	2.0271e-07	-1.1044e-06	4.5797e-07	3.2188e-07	1.0213e-07	3.0036e-07

	хo	YO	ZO	Omega (deg)	Phi (deg)	Kappa (deg)
Initial Approximation	9.3801e-01	2.4928e+00	2.9883e+00	-1.4337e-01	-3.0529e-01	1.5814e+00
Iteration 1	1.0147e+00	2.5411e+00	2.7957e+00	-1.5778e-01	-2.9739e-01	1.5918e+00
Iteration 2	1.0095e+00	2.5332e+00	2.8063e+00	-1.5604e-01	-2.9761e-01	1.5914e+00
Iteration 3	1.0095e+00	2.5332e+00	2.8064e+00	-1.5603e-01	-2.9761e-01	1.5914e+00
Iteration 4	1.0095e+00	2.5332e+00	2.8064e+00	-1.5603e-01	-2.9761e-01	1.5914e+00

#### Residuals X Y -9.1494e-03 7.1838e-03 -7.1017e-03 1.0709e-05 6.2206e-05 -5.1577e-03 2.1545e-03 -5.2476e-04 3.7755e-03 2.8729e-03 5.4394e-04 -6.8885e-04 2.6088e-03 -1.0071e-02 -3.2874e-03 -6.9874e-03 8.8390e-03 -2.9649e-03 3.5038e-03 -1.4876e-03 7.0983e-04 -4.2513e-03 -1.2045e-02 2.3366e-03 2.6080e-04 5.4908e-03 6.5868e-03 4.6639e-03 2.4318e-04 3.8286e-03 -7.2762e-03 6.7173e-03 1.4691e-03 5.8241e-03

1.2703e-03

3.3365e-04

3.4599e-03

-1.0075e-03

#### ordinates (mm) $x_a$ $y_a$ 5.0408e+00 -3.3540e-01 4.5063e+00 -2.7179e+00 4.0613e+00 -4.6482e+00 2.4057e+00 2.6935e+00 2.1457e+00 -1.7670e-01 1.9236e+00 -2.5023e+00 1.7394e+00 -4.4140e+00 -5.4660e-01 2.7038e+00 -5.0720e-01 -2.9400e-02 -4.8000e-01 -2.2763e+00 -4.5620e-01 -4.1595e+00 -3.1775e+00 2.7274e+00 -2.9114e+00 1.3230e-01 -2.6867e+00 -2.0413e+00 -2.4950e+00 -3.8797e+00 2.7125e+00 -5.4743e+00 2.5380e-01 -5.0707e+00

-1.8410e+00

-3.6312e+00

-4.6992e+00

-4.3740e+00

Measured Image Co-

Total Iterations: 4

 $\widehat{\sigma_0^2} = 1.3664e - 04$ 

 $Threshold = 10^{-8}$ 

### **Summary**

	ХO	YO	ZO	Omega (deg)	Phi (deg)	Kappa (deg)
				S	<del>2</del> 5	27
Initial Approximation	1.9813e+00	1.1559e+00	3.9338e+00	2.2549e-01	2.7793e-03	1.5747e+00
Final EOP Value	2.1016e+00	1.1947e+00	3.6996e+00	1.3367e+01	1.4655e+00	9.0231e+01
SD for EOPs	7.2693e-03	7.0928e-03	2.5152e-03	1.6975e-03	1.6901e-03	6.4748e-04
Dolta FOR from PACC	1 22510 05	2 60170 06	1 06350 06	1 00520 05	1 32060 04	1 00020 05

### **Dispersion Matrix**

	X0	Y0	ZO	Omega (deg)	Phi (deg)	Kappa (deg)
	<u> </u>	1	-	<u> </u>		7
жo	5.2843e-05	-1.1007e-06	-4.0257e-06	3.5920e-07	1.2145e-05	-2.2374e-06
YO	-1.1007e-06	5.0308e-05	8.4483e-06	-1.1905e-05	-2.2545e-07	5.2030e-07
20	-4.0257e-06	8.4483e-06	6.3260e-06	-2.0936e-06	-9.9653e-07	2.6862e-07
Omega (deg)	3.5920e-07	-1.1905e-05	-2.0936e-06	2.8815e-06	7.8427e-08	-1.1986e-07
Phi (deg)	1.2145e-05	-2.2545e-07	-9.9653e-07	7.8427e-08	2.8563e-06	-4.9485e-07
Kappa (deg)	-2.2374e-06	5.2030e-07	2.6862e-07	-1.1986e-07	-4.9485e-07	4.1923e-07

	хo	YO	<b>Z</b> 0	Omega (deg)	Phi (deg)	Kappa (deg)
Initial Approximation	9.3801e-01	2.4928e+00	2.9883e+00	-1.4337e-01	-3.0529e-01	1.5814e+00
Iteration 1	1.0147e+00	2.5411e+00	2.7957e+00	-1.5778e-01	-2.9739e-01	1.5918e+00
Iteration 2	1.0095e+00	2.5332e+00	2.8063e+00	-1.5604e-01	-2.9761e-01	1.5914e+00
Iteration 3	1.0095e+00	2.5332e+00	2.8064e+00	-1.5603e-01	-2.9761e-01	1.5914e+00
Iteration 4	1.0095e+00	2.5332e+00	2.8064e+00	-1.5603e-01	-2.9761e-01	1.5914e+00

#### Residuals

#### X Y -4.5648e-03 -1.8875e-03 -2.4678e-03 -2.3467e-03 -1.1436e-03 -1.7240e-05 -4.1860e-04 3.4880e-03 -8.8269e-03 5.6334e-03 2.0109e-04 -1.1697e-03 1.1524e-03 -5.4694e-04 5.6302e-03 2.1803e-03 5.2687e-03 1.2950e-03 2.0063e-03 -3.0703e-03 6.5752e-03 3.3518e-03 6.0511e-03 3.7739e-03 8.8111e-03 2.0679e-03 8.4652e-03 -4.4021e-03 -1.2135e-02 1.1348e-03 5.8902e-03 7.1434e-04 2.4951e-03 4.4472e-03 4.3101e-04 5.0749e-03 -1.4891e-03 -3.8245e-03 -1.5643e-03 -6.0432e-03 3.0723e-03 -1.6205e-02 -1.4727e-02 -5.0023e-03 -2.0604e-02 1.1902e-03 -1.2807e-02 1.2664e-03 1.4222e-02 2.1839e-02

### Measured Image Coordinates (mm)

$x_a$	$y_a$
3.2893e+00	3.8357e+00
3.2950e+00	2.0059e+00
3.2772e+00	1.6290e-01
3.2471e+00	-1.6697e+00
3.2149e+00	-3.4760e+00
1.5691e+00	4.0453e+00
1.5740e+00	2.1277e+00
1.5795e+00	1.8440e-01
1.5697e+00	-1.7649e+00
1.5570e+00	-3.6961e+00
-3.3780e-01	4.2673e+00
-3.4410e-01	2.2563e+00
-3.3750e-01	2.0580e-01
-3.3810e-01	-1.8635e+00
-3.4120e-01	-3.9356e+00
-2.4459e+00	4.4805e+00
-2.4810e+00	2.4023e+00
-2.4964e+00	2.4660e-01
-2.5036e+00	-1.9536e+00
-2.4987e+00	-4.1554e+00
-4.7066e+00	4.6783e+00
-4.8269e+00	2.5441e+00
-4.8928e+00	2.7650e-01
-4.9179e+00	-2.0524e+00
-4.8917e+00	-4.3690e+00

# Issues and Explanation

- The EOPs for all the images, when compared to output of BASC, the difference is of the order of  $10^{-4}\ to\ 10^{-7}$
- The residuals from this lab exercise are also very close to residuals from BASC
- It took around 4 to 5 iterations for the solution to converge with a threshold for change in  $\Delta \widehat{\sigma_0^2} = 10^{-8}$
- $\widehat{\sigma_0^2}$  is of the order of  $10^{-4}$  and  $10^{-5}$
- IOPs were used from Lab 1
- One issue encountered during this project was with the linearization of Co-linearity equations
- There were multiple terms with very similar variable name hence to completely understand the meaning of each variable, derivation had to be done from scratch
- While, coding in the mathematical equations, a misplaced plus or minus within the code let to diverging output
- Debugging required a lot of focus because of the number of equations and variables in the model

# Computer Pseudo Code

#### main.m

Declare common parameters -> IOPs, Ground coordinates of targets

For each image:

Get image co-ordinates for each image

Call LSA.m function

Generate various results by calling **results.m** function

#### LSA.m

Define threshold for convergence

Start loop (max 100 iterations):

Calculate A matrix by calling calc\_A.m

Calculate y matrix by calling calc y.m

Calculate x hat

Update EOPs -> old\_EOPs + x\_hat

Calculate e vector and  $\widehat{\sigma_0^2}$ 

If 
$$\left|\widehat{\sigma_{0ld}^2} - \widehat{\sigma_{new}^2}\right| < threshold$$

Then Calculate D and break out of loop

# Computer Pseudo Code

### Calc\_A.m

```
Calculate rotation matrix by calling rotation.m
Calculate N_x, N_y and D
For i=1 to num_of_targets
Calculate a_i, b_i \rightarrow i = 1,2,3,4,5,6
add to A matrix
End loop
```

### Calc\_y.m

Calculate  $\bar{x}$ ,  $\bar{y}$ , r

Calculate  $\Delta x_r$ ,  $\Delta y_r$  (radial distortions)

Calculate  $\Delta x_d$ ,  $\Delta y_d$  (de-centering distortion)

Remove distortions from image co-ordinate  $(x_{corr}, y_{corr})$ 

Calculate y vector  $(x_{corr} - x_p + c \frac{N_x}{D}, y_{corr} - y_p + c \frac{N_y}{D})$ 

#### rotation.m

Calculate rotation  $\omega$ 

Calculate rotation  $\phi$ 

Calculate rotation  $\kappa$ 

Multiply the rotation matrices to get final rotation matrix

#### **Code on Github:**

https://github.com/Salazar-Prime/photogrammetry/tree/master/Project%202

# Compute Code – main.py

```
%% Common Parameters
% IOPs
 IOP = [6.7451660984e-2, -1.1709829919e-1, 8.1671200690];
\mathtt{dist} = [-2.9350008918e - 4, \ 9.2190322166e - 6, -2.2562559450e - 7, 6.1878890685e - 5, -7.2907688047e - 1.000864e - 1.000864e - 1.00086e 
% Ground co-ordinates of targets
0.037704361429; 0.90892725171; 1.8588605461; 2.8062554595; 3.7520655984; -
884;2.8111736654;3.7628755375];
[3.751130948;3.7594793917;3.7736433303;3.7823238561;3.7801397656;2.8057675589;2.8180218203;2
.8257925207;2.8332144474;2.8378518827;1.8634346772;1.8678901457;1.8758730363;1.8838128654;1.
8922797004; 0.91840024749; 0.92066047041; 0.92760553219; 0.93553829291; 0.94557055946; 0; -
0.00026178267339;-0.0021390199929;-0.0019752230297;0];
ZA = [0.072640804133; 0.048121990404; 0; -0.047649472361; -
0.070838477818;0.054102199693;0.026008316122;0.0059282538012;-0.023295585379;-
0.020910207448;-0.0091692255168;0.003877584251;0.003049069041;-0.0030509411619;-
0.0098127723745;0;0.017342094476;0.016047411487;0.006582607018;0];
%% Image 1 - image 09
[EOP i, EOP basc, xa, ya] = data(9); % get data for image 9
[EOP 9,e 9,sigma hat 9,iter 9,D 9,EOP history 9] = LSA(xa,ya,XA,YA,ZA,IOP,dist,EOP i); % run
[table 9 summary,table 9 D,table 9 hist] =
results(EOP i, EOP 9,D 9,EOP basc, EOP history 9,iter 9); % complie results
%% Image 1 - image 10
[EOP i, EOP basc, xa, ya] = data(10); % get data for image 10
[EOP_10,e_10,sigma_hat 10,iter 10,D 10,EOP history 10] =
LSA(xa,ya,XA(1:24),YA(1:24),ZA(1:24),IOP,dist,EOP i); % run LSA
[table 10 summary, table 10 D, table 10 hist] =
results(EOP i,EOP 10,D 10,EOP_basc,EOP_history_10,iter_10); % complie results
%% Image 1 - image 14
[EOP i, EOP basc, xa, ya] = data(14); % get data for image 14
[EOP 14,e 14,sigma hat_14,iter_14,D_14,EOP_history_14] =
LSA(xa,ya,XA([3:5,7:10,12:15,17:20,22:25]),YA([3:5,7:10,12:15,17:20,22:25]),ZA([3:5,7:10,12:
15,17:20,22:25]), IOP, dist, EOP i); % run LSA
[table_14_summary,table_14_D,table_14_hist] =
results(EOP i,EOP 14,D 14,EOP base,EOP history 14,iter 14); % complie results
%% Image 1 - image 18
[EOP i, EOP basc, xa, ya] = data(18); % get data for image 14
[EOP 18,e 18, sigma hat 18, iter 18, D 18, EOP history 18] = LSA(xa, ya, XA, YA, ZA, IOP, dist, EOP i);
% run LSA
[table 18 summary, table 18 D, table 18 hist] =
results (EOP i, EOP 18, D 18, EOP basc, EOP history 14, iter 14); % complie results
```

# Compute Code – LSA.py

```
function [EOP,e,sigma hat,iter,D,EOP_history] = LSA(xa,ya,XA,YA,ZA,IOP,dist,EOP)
%% parameters
max iter = 10000;
thres = 1e-8;
sigma hat = 0;
num GCP = length(xa);
% Unpack EOPs and IOPs
[X0,Y0,Z0,omega,phi,kappa] = update EOP(EOP, zeros(6,1));
[xp,yp,c] = assign IOP(IOP);
EOP_history = EOP;
%% LSA iterations
for iter=1:max iter
    % update sigma hat prev
    sigma_hat_prev = sigma_hat;
    % OAX, OAY, OAZ
    OAX = XA - X0;
    OAY = YA - Y0;
    OAZ = ZA - Z0;
    % calculate A 50x6 (num GCP,c,OAX,OAY,OAZ,omega,phi,kappa)
    [A,Nx,Ny,D] = calc_A(num_GCP, c, OAX, OAY, OAZ, omega, phi, kappa);
    % calculate y 50x1 (dist,xa,ya,xp,yp,c,Nx mat,Ny mat,D mat)
    y = calc y(dist, xa, ya, xp, yp, c, Nx, Ny, D);
    % calculate x hat
    P = eye(num_GCP*2);
    x hat = inv(A'*P*A)*A'*P*y;
    % update EOPs
    [X0,Y0,Z0,omega,phi,kappa,EOP] = update EOP(EOP, x hat);
    EOP history = [EOP history; EOP]
    % calculate e and sigma hat
    e = y - A*x hat;
    sigma hat = (e'*P*e)/(num_GCP - length(x_hat));
    % break out of loop is threshold is reached
    if abs(sigma hat - sigma hat prev) < thres</pre>
         D = sigma_hat*inv(A'*P*A);
          e = [e(1:2:end), e(2:2:end)];
        EOP = [X0,Y0,Z0,rad2deg(omega),rad2deg(phi),rad2deg(kappa)];
        break
    end
end
```

# Compute Code – calc A.py

```
function [A,Nx mat,Ny mat,D mat] = calc A(num GCP,c,OAX,OAY,OAZ,omega,phi,kappa)
%% calculate rotation
r = rotation(omega, phi, kappa);
%% calculate N and D (OAX,OAY,OAZ,rot)
Nx \text{ mat} = r(1,1)*OAX + r(2,1)*OAY + r(3,1)*OAZ;
Ny_mat = r(1,2)*OAX + r(2,2)*OAY + r(3,2)*OAZ;
D mat = r(1,3)*OAX + r(2,3)*OAY + r(3,3)*OAZ;
%% calculate aij, bij
% create an empty A matrix
A = zeros(num GCP*2, 6);
% loop over all ground control points
for i=1:num GCP
    % Nx, Ny, D for ith target
    Nx = Nx mat(i);
    Ny = Ny mat(i);
    D = D mat(i);
    % aij for ith GCP
    a1 = r(1,1)*D - r(1,3)*Nx;
    a2 = r(2,1)*D - r(2,3)*Nx;
    a3 = r(3,1)*D - r(3,3)*Nx;
    a4 = D*(r(3,1)*OAY(i) - r(2,1)*OAZ(i)) + Nx*(r(2,3)*OAZ(i) - r(3,3)*OAY(i));
    a5 = D*D*cos(kappa) - Nx*(-Nx*cos(kappa) + Ny*sin(kappa));
    a6 = -1 * Ny*D;
    % bij for ith GCP
    b1 = r(1,2)*D - r(1,3)*Ny;
    b2 = r(2,2) *D - r(2,3) *Ny;
    b3 = r(3,2)*D - r(3,3)*Ny;
    b4 = D*(r(3,2)*OAY(i) - r(2,2)*OAZ(i)) + Ny*(r(2,3)*OAZ(i) - r(3,3)*OAY(i));
    b5 = -1*D*D*sin(kappa) - Ny*(-Nx*cos(kappa) + Ny*sin(kappa));
    b6 = Nx*D;
    % multiplying by c/D^2 before appending into A
    A(2*i-1,:) = [a1,a2,a3,a4,a5,a6]*c/D^2;
      A(2*i,:) = [b1,b2,b3,b4,b5,b6]*c/D^2;
end
```

# Compute Code – calc\_y.py

```
function [y] = calc y(dist,xa,ya,xp,yp,c,Nx,Ny,D)
%% calculate x corr and y corr
% unpack distortion parameters
k1 = dist(1);
k2 = dist(2);
k3 = dist(3);
p1 = dist(4);
p2 = dist(5);
p3 = dist(6);
% calculate x bar, y bar, r
x bar = xa - xp;
y bar = ya - yp;
r = sqrt(x bar.^2 + y bar.^2);
% calculate delta xr, delta yr - radial distortion
delta xr = x bar .* (k1*r.^2 + k2*r.^4 + k3*r.^6);
delta_yr = y_bar.* (k1*r.^2 + k2*r.^4 + k3*r.^6);
% calculate delta xd, delta yd - de-centering lens distortion
delta xd = (1+p3*r.^2).*(p1*(r.^2 + 2*x bar.^2) +
2*p2*x bar.*y bar);
delta yd = (1+p3*r.^2).*(2*p1*x bar.*y bar + p2*(r.^2 +
2*y bar.^2);
% cremove distortions
x corr = xa - delta xr - delta xd;
y corr = ya - delta yr - delta yd;
%% calcualte y
x_{corr} = x_{corr} - (xp - c*Nx./D);
y_{corr} = y_{corr} - (yp - c*Ny./D);
% alternate merge in order : x corr and y corr
y = [x corr y corr]';
y = y(:);
end
```

# Compute Code – rotation.py

```
function [rot]=rotation(omega, phi, kappa)

rot_o = [1 0 0; 0 cos(omega) -sin(omega); 0
  sin(omega) cos(omega)];

rot_p = [cos(phi) 0 sin(phi); 0 1 0; -sin(phi) 0
  cos(phi)];

rot_k = [cos(kappa) -sin(kappa) 0; sin(kappa)
  cos(kappa) 0; 0 0 1];

rot = rot_o * rot_p * rot_k;
end
```

# Compute Code – update\_EOP.py

```
function
[X0,Y0,Z0,omega,phi,kappa,EOP]=update_EOP(EOP, x_hat)

X0 = EOP(1) + x_hat(1);
Y0 = EOP(2) + x_hat(2);
Z0 = EOP(3) + x_hat(3);
omega = EOP(4) + x_hat(4);
phi = EOP(5) + x_hat(5);
kappa = EOP(6) + x_hat(6);
EOP = [X0,Y0,Z0,omega,phi,kappa];
end
```

# Compute Code – results.py

```
function [summary,D,hist,e]=results(EOP_i, EOP_f, D, EOP_basc, hist,
iter)
    out = [EOP_i;EOP_f;transpose(diag(D).^0.5);abs(EOP_basc - EOP_f)];

    row=["Initial Approximation"];
    for i=1:iter
        row=[row,sprintf("Iteration %d",i)];
    end

    summary = array2table(out,'VariableNames',["X0","Y0","Z0","Omega
(deg)","Phi (deg)","Kappa (deg)"],'RowNames',["Initial
Approximation","Final EOP Value","SD for EOPs","Delta EOP from BASC"]);

    D = array2table(D,'VariableNames',["X0","Y0","Z0","Omega (deg)","Phi
(deg)","Kappa (deg)"],'RowNames',["X0","Y0","Z0","Omega (deg)","Phi
(deg)","Kappa (deg)"]);

    hist = array2table(hist,'VariableNames',["X0","Y0","Z0","Omega
(deg)","Phi (deg)","Kappa (deg)"],'RowNames',row);
end
```

# Compute Code – assign\_IOP.py

```
function [xp,yp,c]=assign_IOP(IOP)
    xp = IOP(1);
    yp = IOP(2);
    c = IOP(3);
end
```

# Compute Code – data.py

```
function [EOP i, EOP basc, xa, ya] = data(image no)
              switch image no
                             case 9
                                           EOP i = [1.60, 3.20, 3.5, 0, 0, 0];
                                           EOP basc = [1.88968674, 3.03586621, 3.73500036, -19.1841784, -
4.34497759,2.050149031;
                                           xa = [-5.3877; -2.92187; -0.49439; 1.79834; 3.93369; -5.04787; -
2.77407; -0.55643; 1.5572; 3.55246; -4.71235; -2.63868; -
0.61275;1.34004;3.19895;-4.43154;-2.5313;-0.66436;1.1514;2.88537;-4.19557;-
2.4407; -0.70131; 0.99217; 2.62277];
                                           ya =
[4.93742;4.84433;4.68172;4.49867;4.29866;2.42494;2.3434;2.23887;2.1432;2.04
441;0.30738;0.24322;0.17794;0.13305;0.09228;-1.49013;-1.53833;-1.5756;-
1.59753; -1.59671; -3.02777; -3.06606; -3.09871; -3.09563; -3.07743];
                             case 10
                                           EOP i = [1.636114 \ 2.184056 \ 3.727135 \ deg2rad(-1.055093)]
deg2rad(-5.256979) deg2rad(92.014892)];
                                                                                                                                                                                                                         3.50855335e+000
                                           EOP basc = [1.68045678e+000]
                                                                                                                                                   2.16587520e+000
-4.14855263e-001 -4.93725230e+000 9.24769326e+001];
[4.2568;4.0854;3.8851;3.68;3.479;1.9221;1.7977;1.6796;1.555;1.4339;-
0.3967; -0.4724; -0.5334; -0.5915; -0.6454; -2.6632; -2.71; -2.7285; -2.7338; -
2.7224; -4.81; -4.8779; -4.8782; -4.8346];
                                           ya= [4.6788;2.3093;0.0418;-2.0823;-
4.0521; 4.7589; 2.3888; 0.1175; -2.0216; -4.02; 4.8045; 2.45; 0.1921; -1.9492; -
3.9707; 4.8258; 2.5321; 0.2918; -1.8528; -3.8693; 4.8076; 2.5851; 0.3697; -1.7626];
                                           EOP i = [0.938012 \ 2.492823 \ 2.988345 \ deg2rad(-8.214459)]
deg2rad(-17.492066) deg2rad(90.607889)];
                                           EOP basc = [1.00952899e+000]
                                                                                                                                                      2.53318361e+000 2.80635782e+000
-8.94008150e+000 -1.70515538e+001 9.11810993e+0011;
                                           xa = [5.0408; 4.5063; 4.0613; 2.4057; 2.1457; 1.9236; 1.7394; -
0.5466; -0.5072; -0.48; -0.4562; -3.1775; -2.9114; -2.6867; -2.495; -5.4743; -
5.0707; -4.6992; -4.3741;
                                           ya = [-0.3354; -2.7179; -4.6482; 2.6935; -0.1767; -2.5023; -
4.414;2.7038;-0.0294;-2.2763;-4.1595;2.7274;0.1323;-2.0413;-
3.8797; 2.7125; 0.2538; -1.841; -3.63121;
                            case 18
                                           EOP i = [1.981303 \ 1.155948 \ 3.933779 \ deg2rad(12.919527)
deg2rad(0.159243) deg2rad(90.221535)];
                                           EOP basc = [2.10162316e+000]
                                                                                                                                                   1.19465520e+000
                                                                                                                                                                                                                         3.69958966e+000
1.33667830e+001 1.46559054e+000 9.02308070e+001];
[3.2893;3.295;3.2772;3.2471;3.2149;1.5691;1.574;1.5795;1.5697;1.557;-
0.3378; -0.3441; -0.3375; -0.3381; -0.3412; -2.4459; -2.481; -2.4964; -2.5036; -2.481; -2.4964; -2.5036; -2.481; -2.4964; -2.5036; -2.481; -2.4964; -2.5036; -2.481; -2.4964; -2.5036; -2.481; -2.4964; -2.5036; -2.481; -2.4964; -2.5036; -2.481; -2.4964; -2.4964; -2.5036; -2.481; -2.4964; -2.5036; -2.481; -2.4964; -2.5036; -2.481; -2.4964; -2.5036; -2.481; -2.4964; -2.5036; -2.481; -2.4964; -2.5036; -2.481; -2.4964; -2.5036; -2.481; -2.4964; -2.5036; -2.481; -2.4964; -2.5036; -2.481; -2.4964; -2.5036; -2.481; -2.4964; -2.4964; -2.5036; -2.481; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4964; -2.4966; -2.4966; -2.4966; -2.4966; -2.4966; -2.4966; -2.4966; -2.4966; -2.4966; -2.4966; -2.4966; -2.4966; -2.4966; -2.4966; -2.4966; -2.4966; -2.
2.4987;-4.7066;-4.8269;-4.8928;-4.9179;-4.8917];
                                           ya = [3.8357; 2.0059; 0.1629; -1.6697; -
3.476; 4.0453; 2.1277; 0.1844; -1.7649; -3.6961; 4.2673; 2.2563; 0.2058; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.8635; -1.86355; -1.8655; -1.8655; -1.8655; -1.8655; -1.8655; -1.8655; -1.8655; -1.8655; -1.8655; -1.8655;
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