Síntesis de automatismos empleando Mapas de Karnaugh

Ejemplos de Autómatas

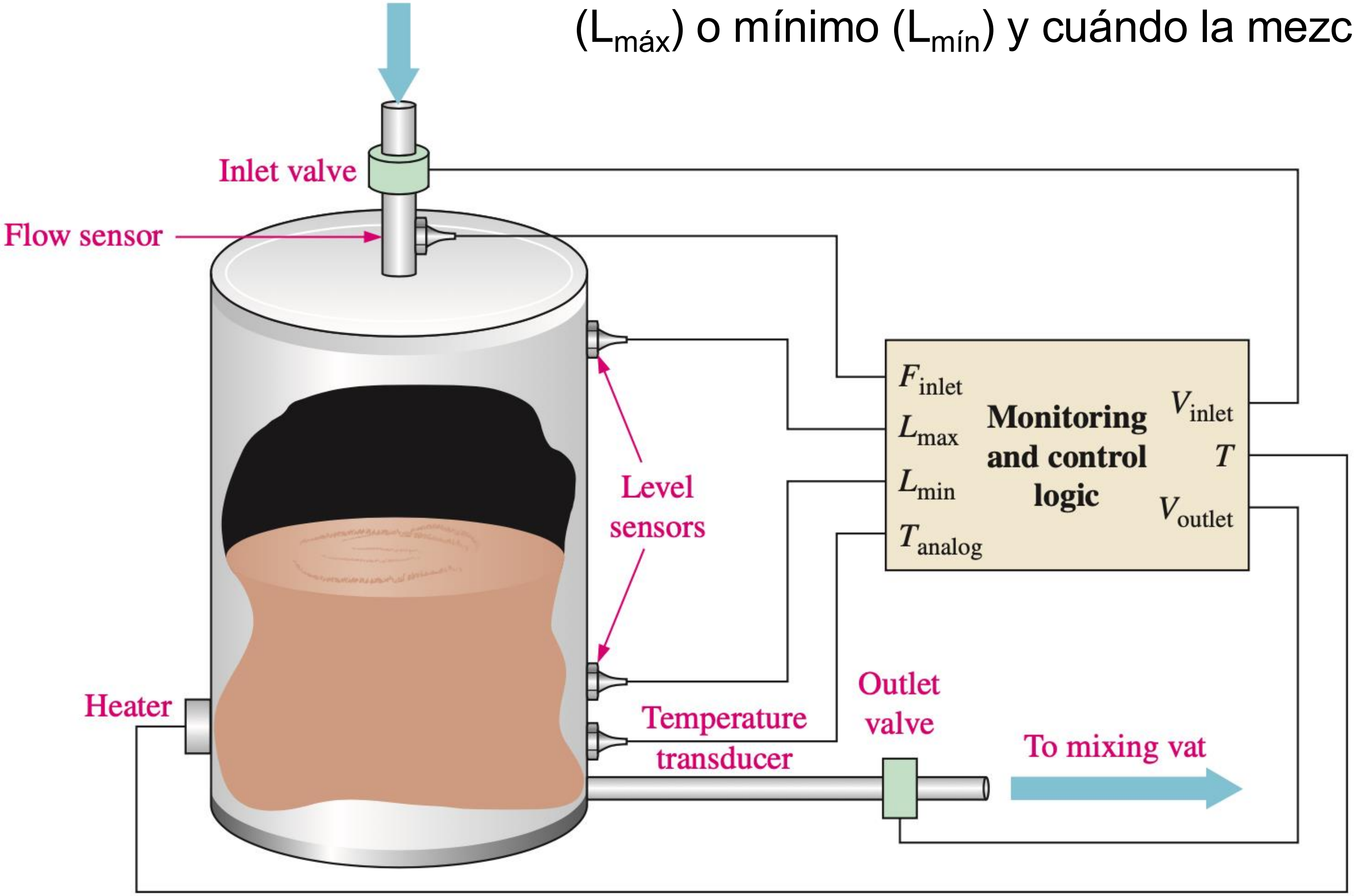
- Mezclador
- Llenado de un tanque
- Sistema hidráulico

Equivalencia de las funciones lógicas y su cálculo en $\mathbb{R}$

Simulación

Ejemplo: Mezcla para arepuelas

La lógica de control de la válvula detecta cuándo se alcanza el nivel máximo ($L_{máx}$) o mínimo ($L_{mín}$) y cuándo la mezcla fluye hacia el tanque (F_{inlet}).



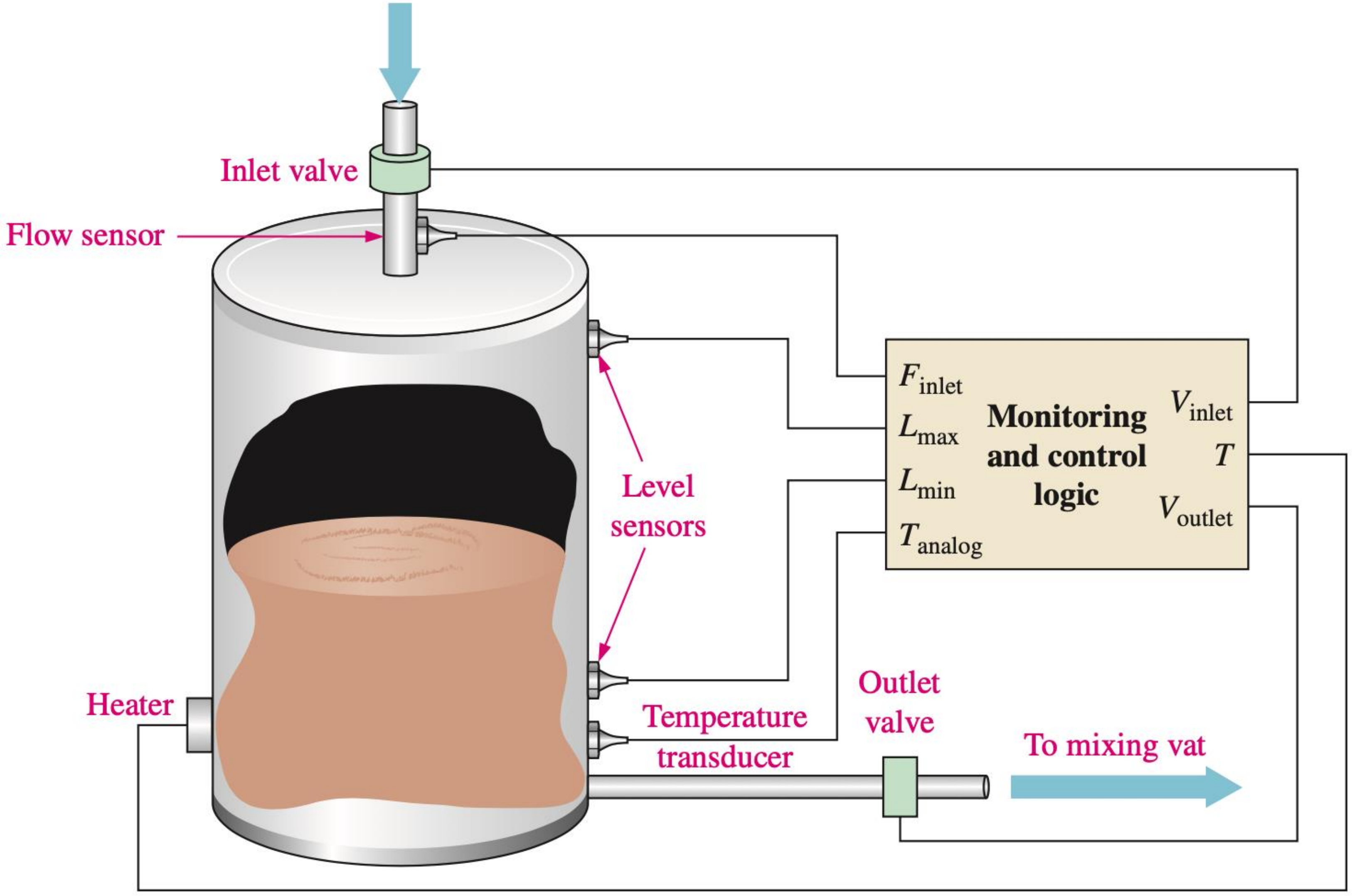
Inputs			Output
L_{max}	L_{min}	F_{inlet}	V_{inlet}
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	X
1	0	1	X
1	1	0	0
1	1	1	0

Inputs			Output
L_{max}	L_{min}	F_{inlet}	V_{inlet}
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	X
1	0	1	X
1	1	0	0
1	1	1	0

$$V_{inlet} = \overline{L}_{max}\overline{L}_{min}\overline{F}_{inlet} + \overline{L}_{max}\overline{L}_{min}F_{inlet} + \overline{L}_{max}L_{min}F_{inlet}$$

$$V_{inlet} = \overline{L}_{min} + \overline{L}_{max}F_{inlet}$$

Usar mapas de Karnough para simplificar la expresión



Inputs				Output
L_{max}	L_{min}	F_{inlet}	T	V_{outlet}
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	X
1	0	0	1	X
1	0	1	0	X
1	0	1	1	X
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

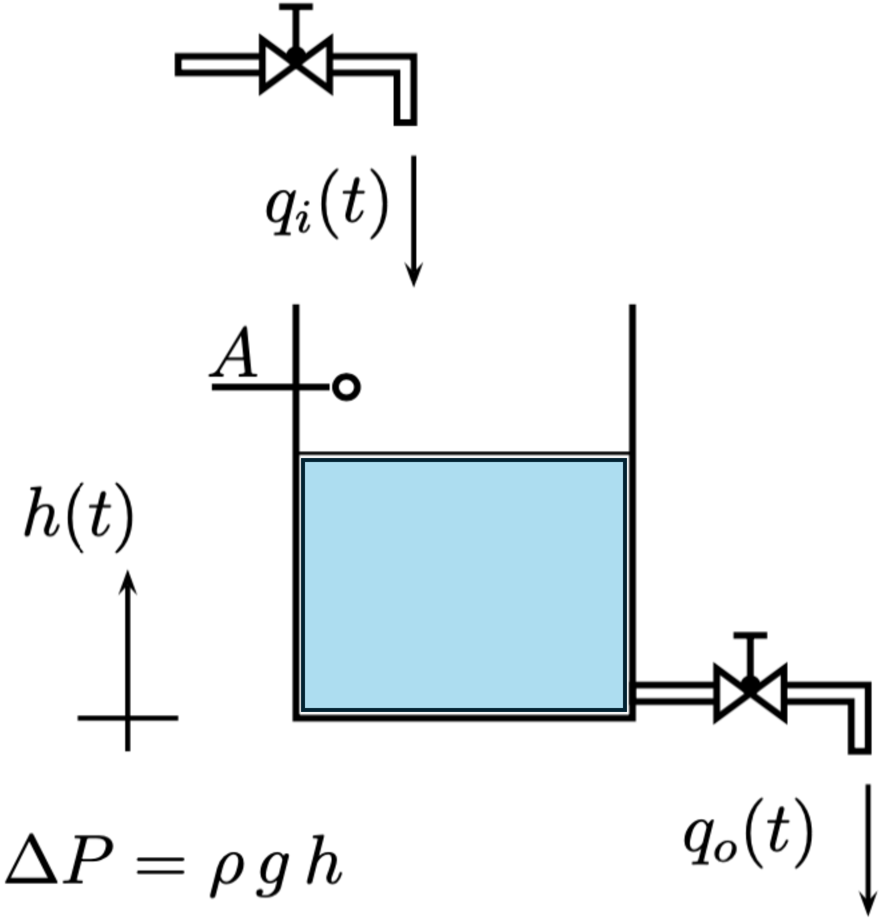
Inputs				Output
L_{max}	L_{min}	F_{inlet}	T	V_{outlet}
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	X
1	0	0	1	X
1	0	1	0	X
1	0	1	1	X
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

$$V_{outlet} = \overline{L}_{max} L_{min} \overline{F}_{inlet} T + L_{max} L_{min} \overline{F}_{inlet} T$$

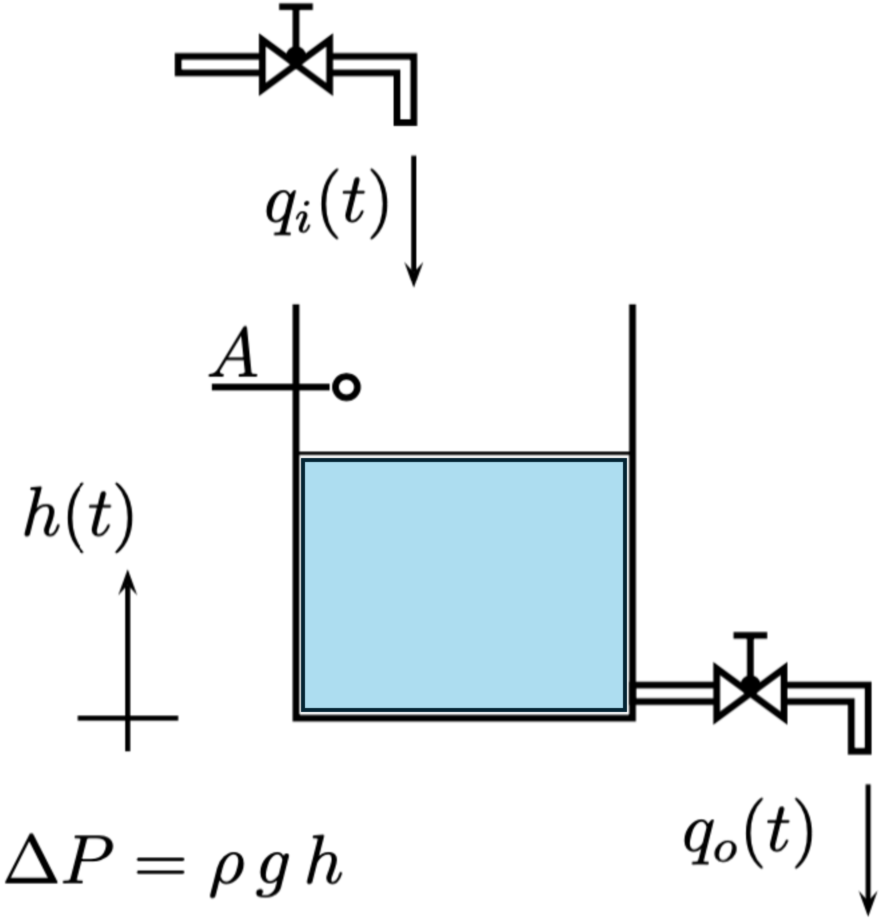
$$V_{outlet} = L_{min} \overline{F}_{inlet} T$$

Usar mapas de Karnough para simplificar la expresión

Ejemplo: Sistema de llenado de un tanque



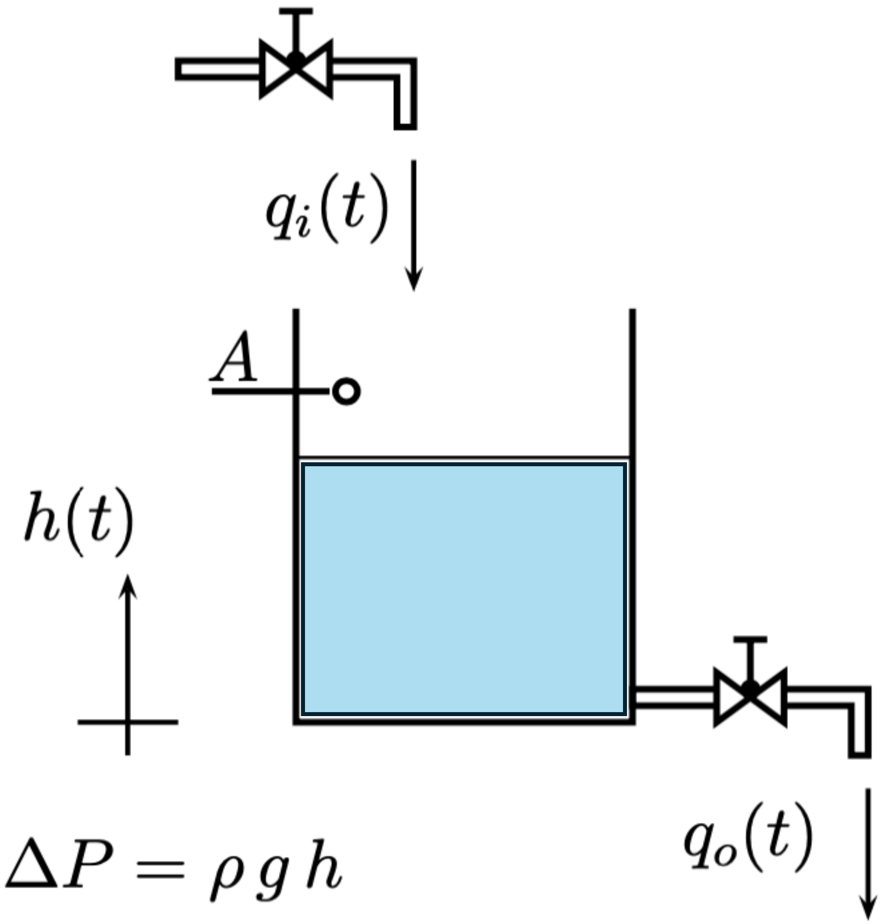
Ejemplo: Sistema de llenado de un tanque



$$\Delta P = \rho\, g\, h$$

$$C_H \frac{d\Delta P}{dt} = q_i - q_o$$

Ejemplo: Sistema de llenado de un tanque



$$\Delta P = \rho\, g\, h$$

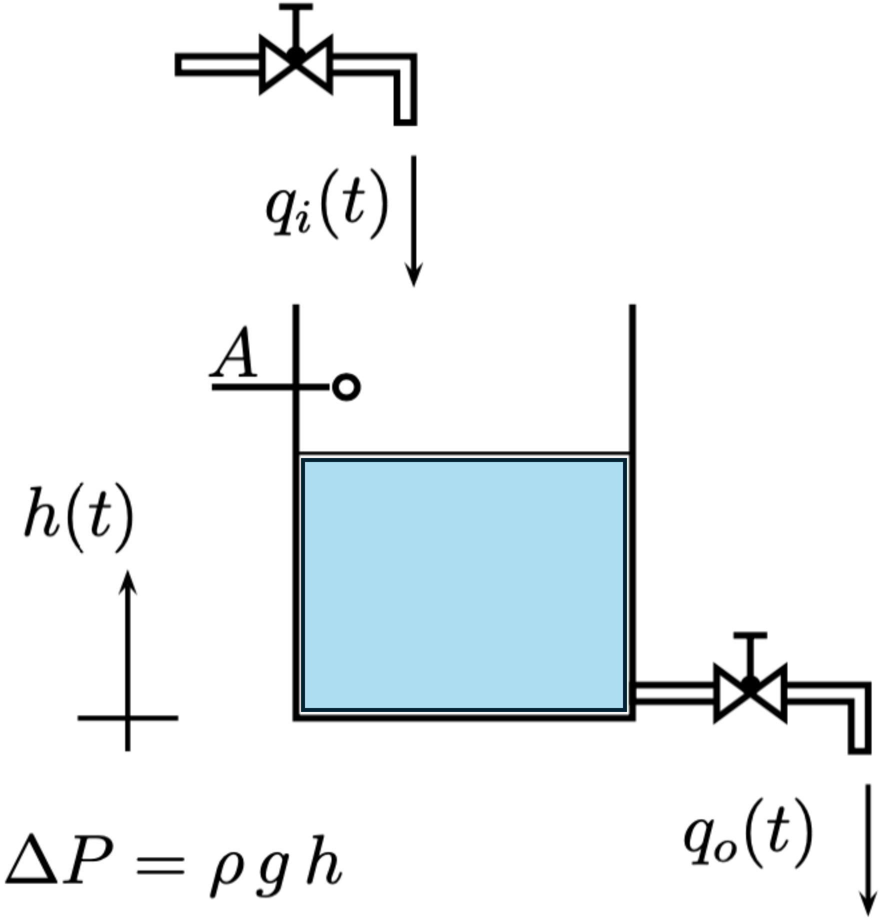
$$C_H \frac{d\Delta P}{dt} = q_i - q_o$$

C_H : Capacitancia hidráulica del tanque.

q_i : Caudal de entrada con retardo $q_i(t - T_d)$.

q_o : Caudal de salida dado por $q_o = \frac{\Delta P}{R_H}$.

Ejemplo: Sistema de llenado de un tanque



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$$C_H \rho\, g \frac{dh}{dt} = q_i(t - T_0) - \frac{\rho\, g}{R_H} h(t)$$

$$C_H \rho\, g \frac{dh}{dt} = q_i(t - T_0) - \frac{\rho\, g}{R_H} h(t)$$

$$\mathcal{L}\left\{C_H \rho\, g \frac{dh}{dt}\right\} = \mathcal{L}\left\{q_i(t - T_0)\right\} - \mathcal{L}\left\{\frac{\rho\, g}{R_H} h(t)\right\}$$

$$C_H \rho\, g H(s) s = Q_i(s) e^{-T_0 s} - \frac{\rho\, g}{R_H} H(s)$$

$$H(s)\left(C_H \rho\, g s + \frac{\rho\, g}{R_H}\right) = Q_i(s) e^{-T_0 s}$$

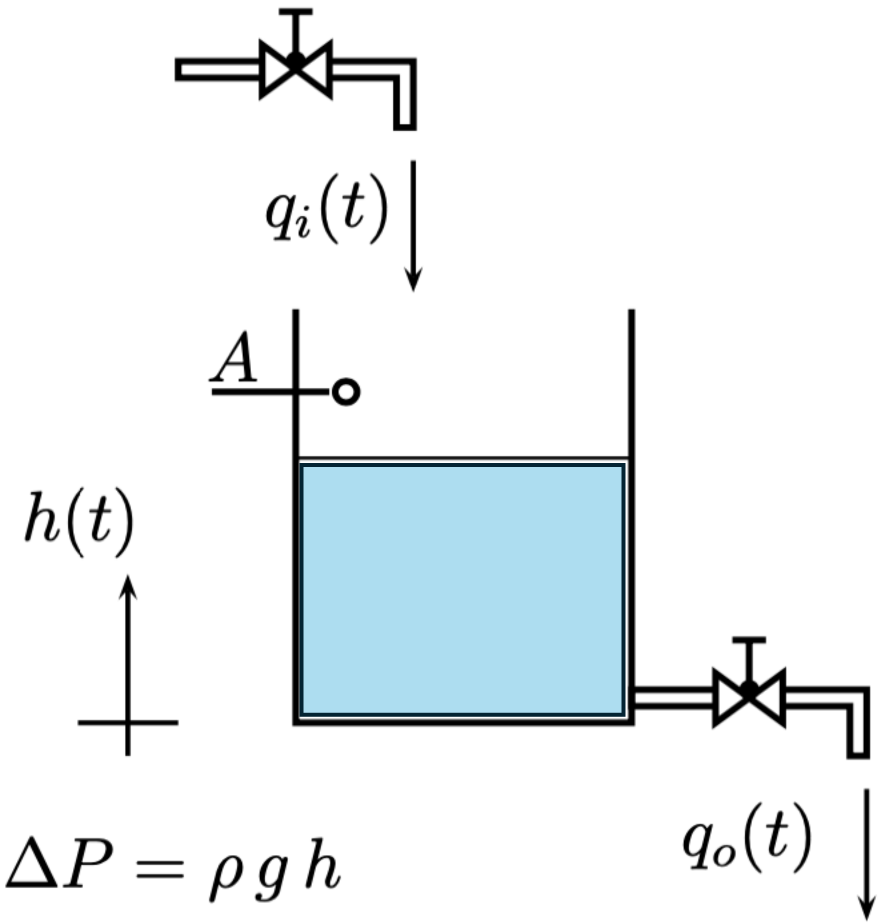
$$G(s) = \frac{H(s)}{Q_i(s)} = \frac{e^{-T_0 s}}{C_H \rho\, g s + \frac{\rho\, g}{R_H}}$$

$$G(s) = \frac{\frac{R_H}{\rho\, g} e^{-T_0 s}}{C_H R_H s + 1}$$

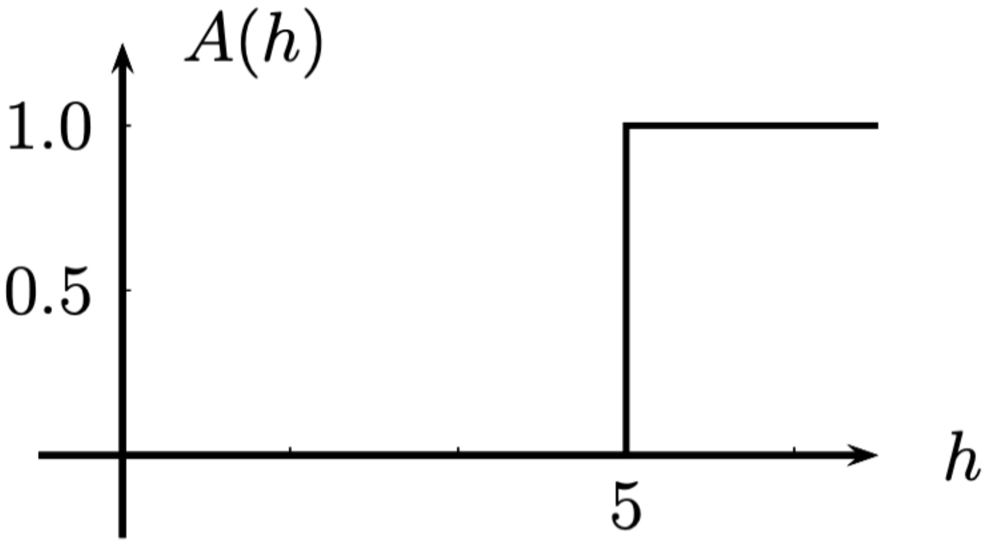
$$\boxed{G(s) = \frac{K e^{-T_0 s}}{\tau s + 1}}$$

$K = 10$, $\tau = 1$ y $T_0 = 0{,}5$

$$G(s) = \frac{10e^{-0{,}5s}}{s+1}$$



$\Delta P = \rho\, g\, h$



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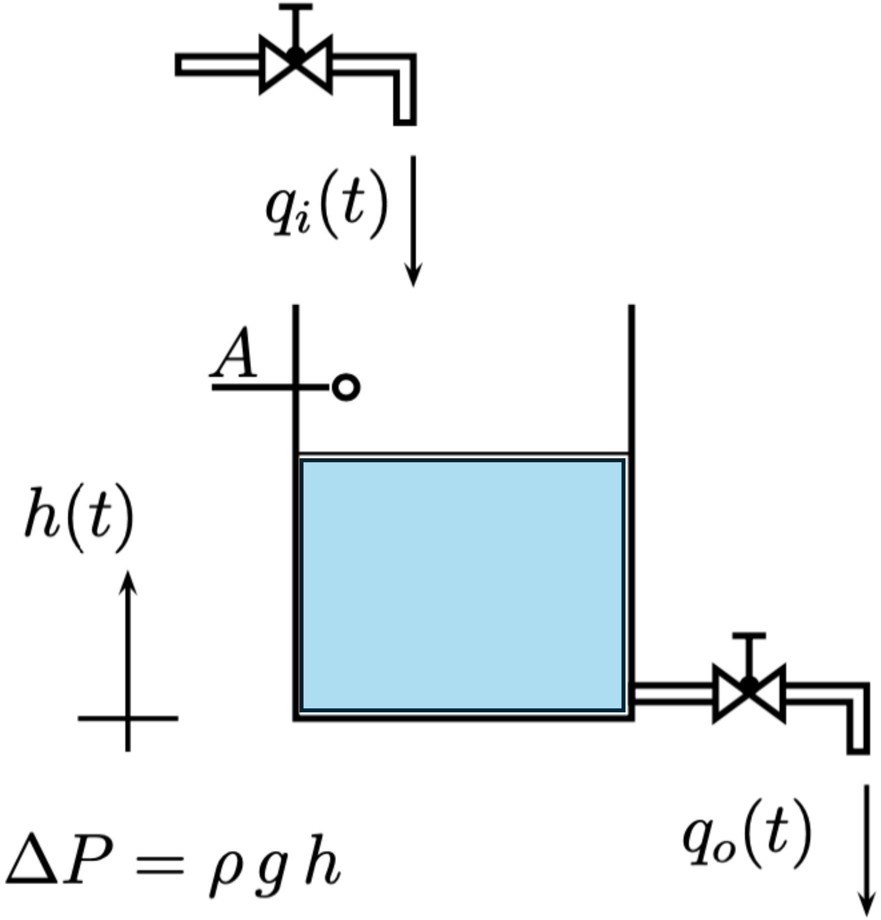
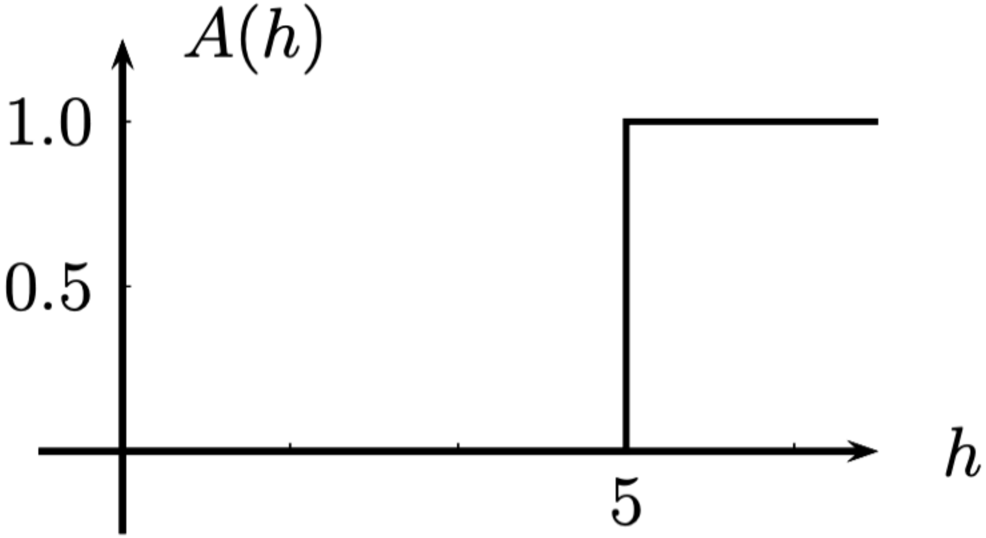


$$\Delta P = \rho\, g\, h$$

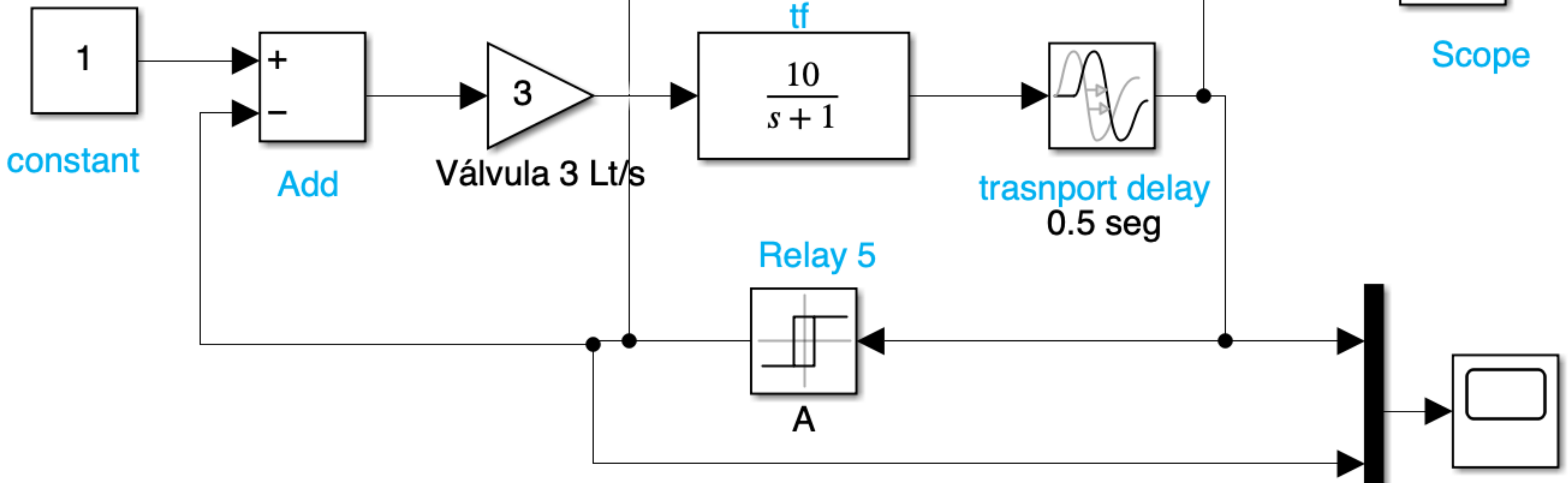


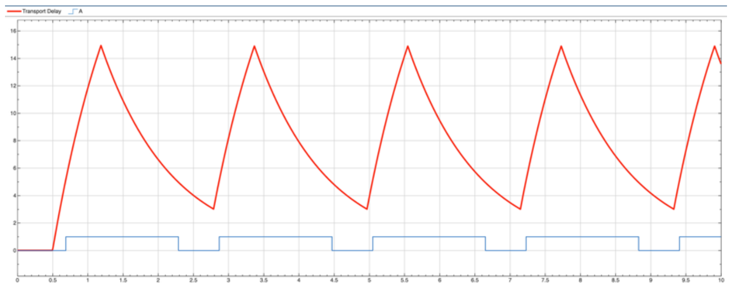
A	Y_i
0	1
1	0

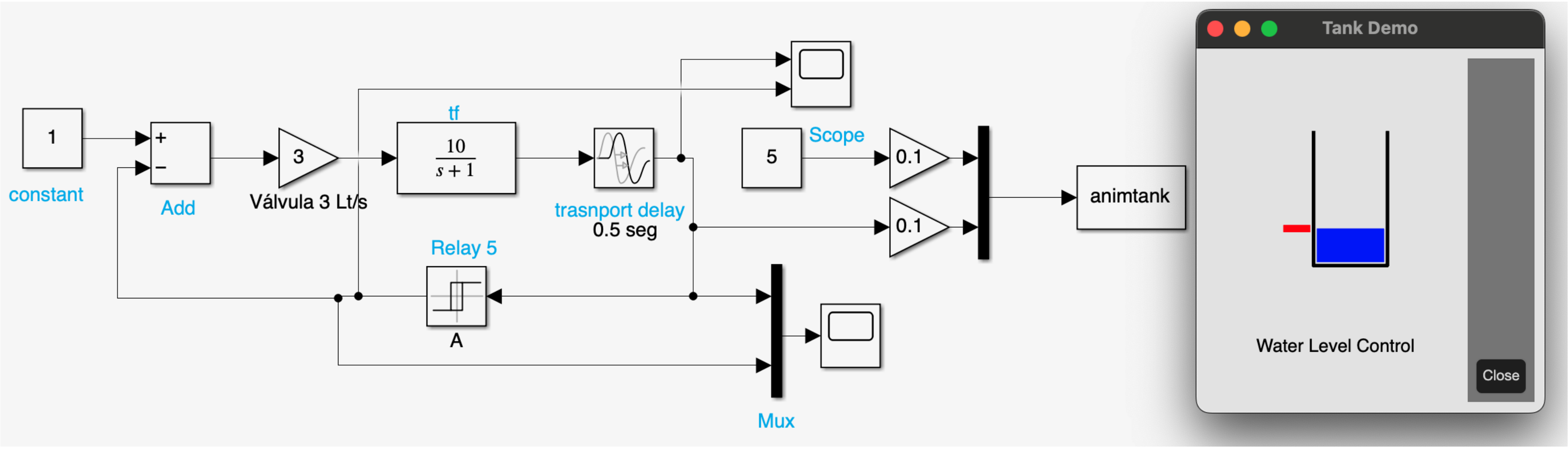
$$\boxed{Y_i = \overline{A}}$$

$K = 10$, $\tau = 1$ y $T_0 = 0{,}5$

$$G(s) = \frac{10e^{-0{,}5s}}{s+1}$$



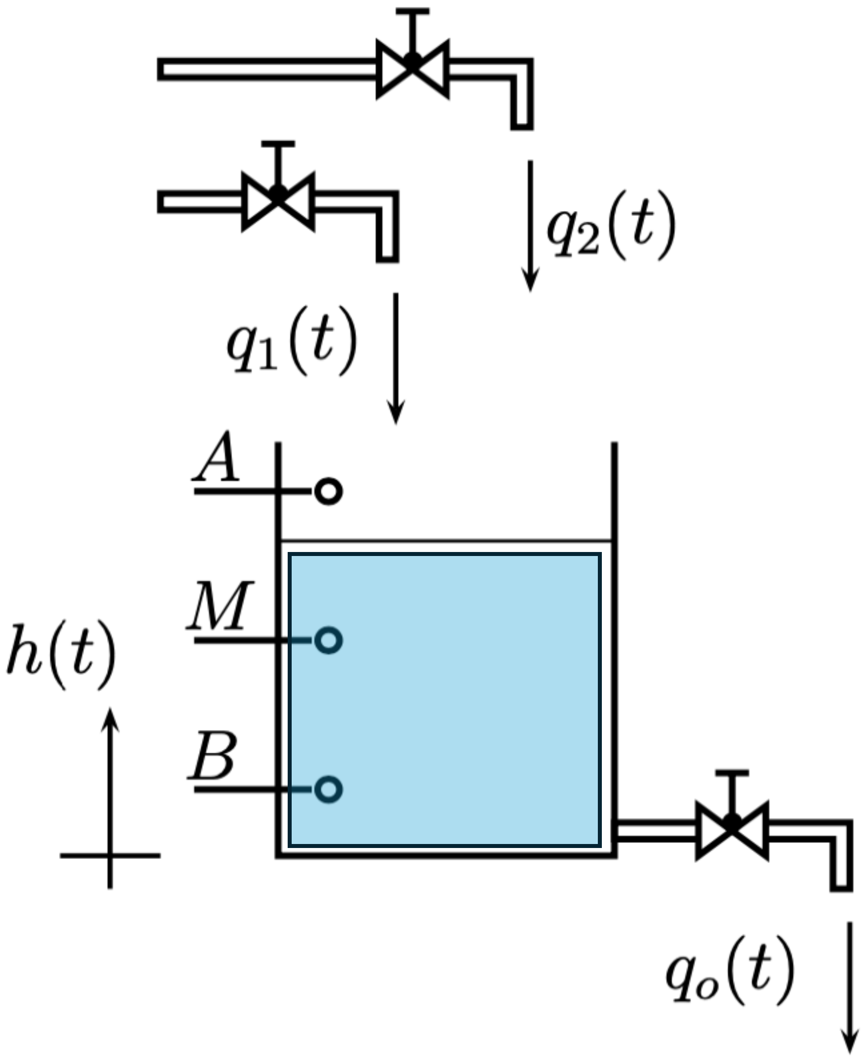




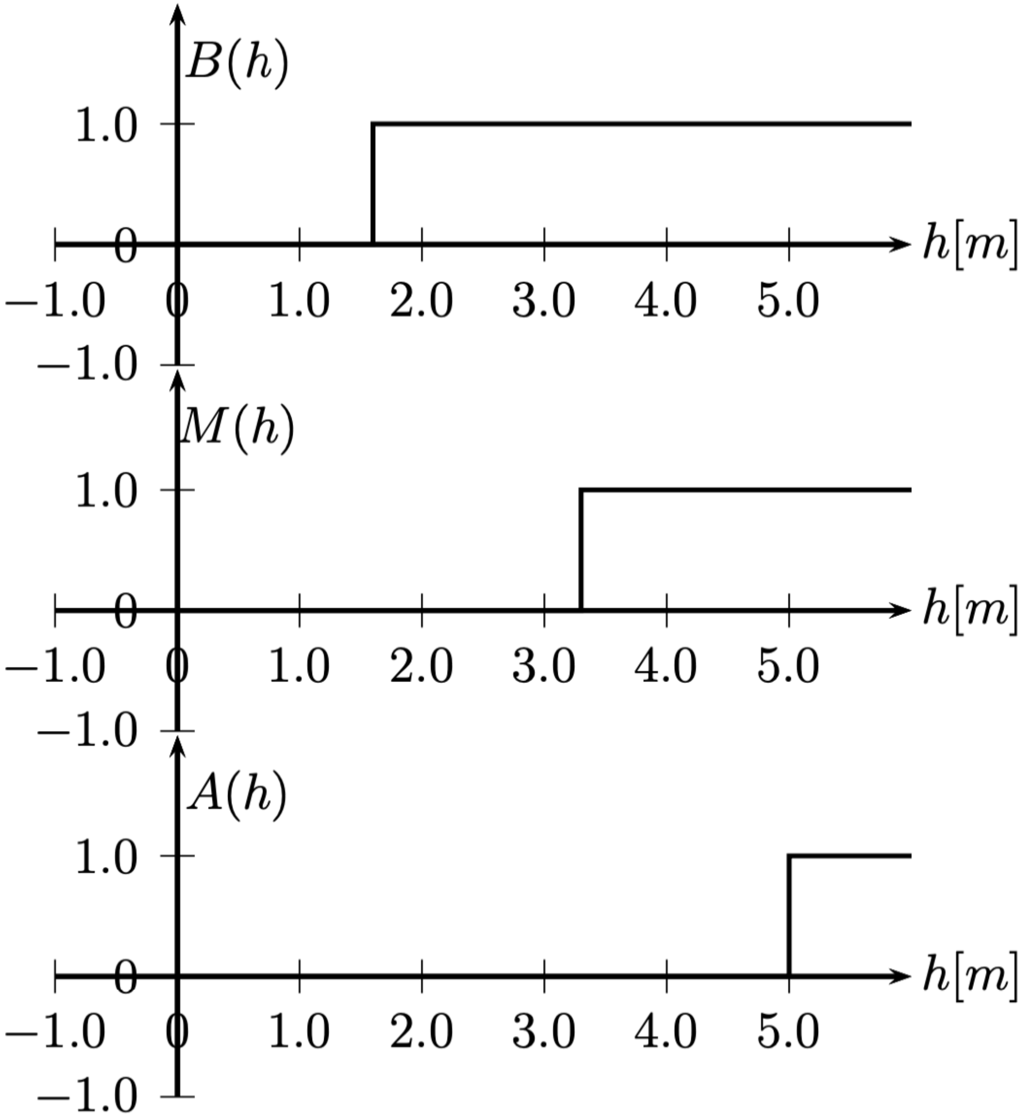
open_system("sltank")

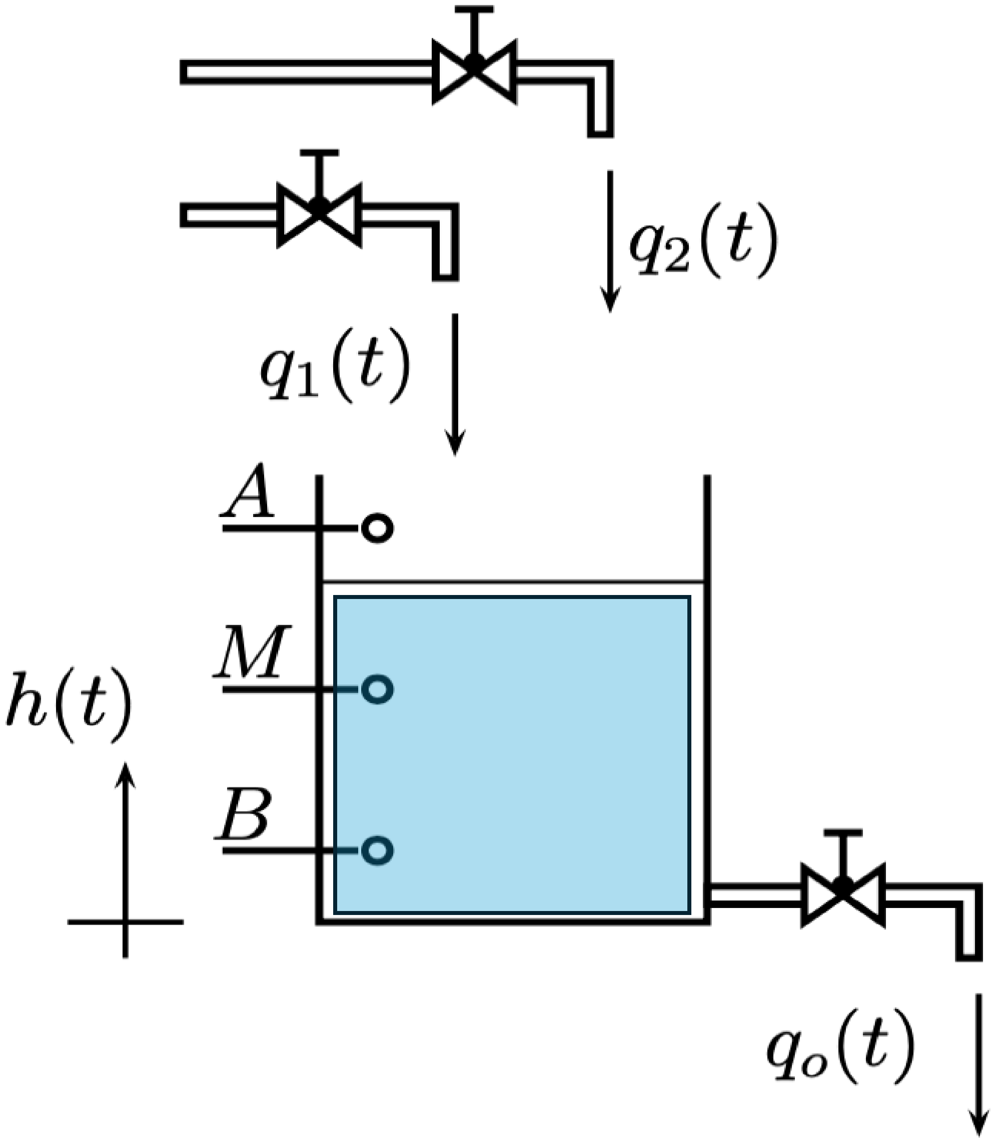
Ejemplo: Sistema hidráulico

$$q_i = Y_1 q_1 + Y_2 q_2$$

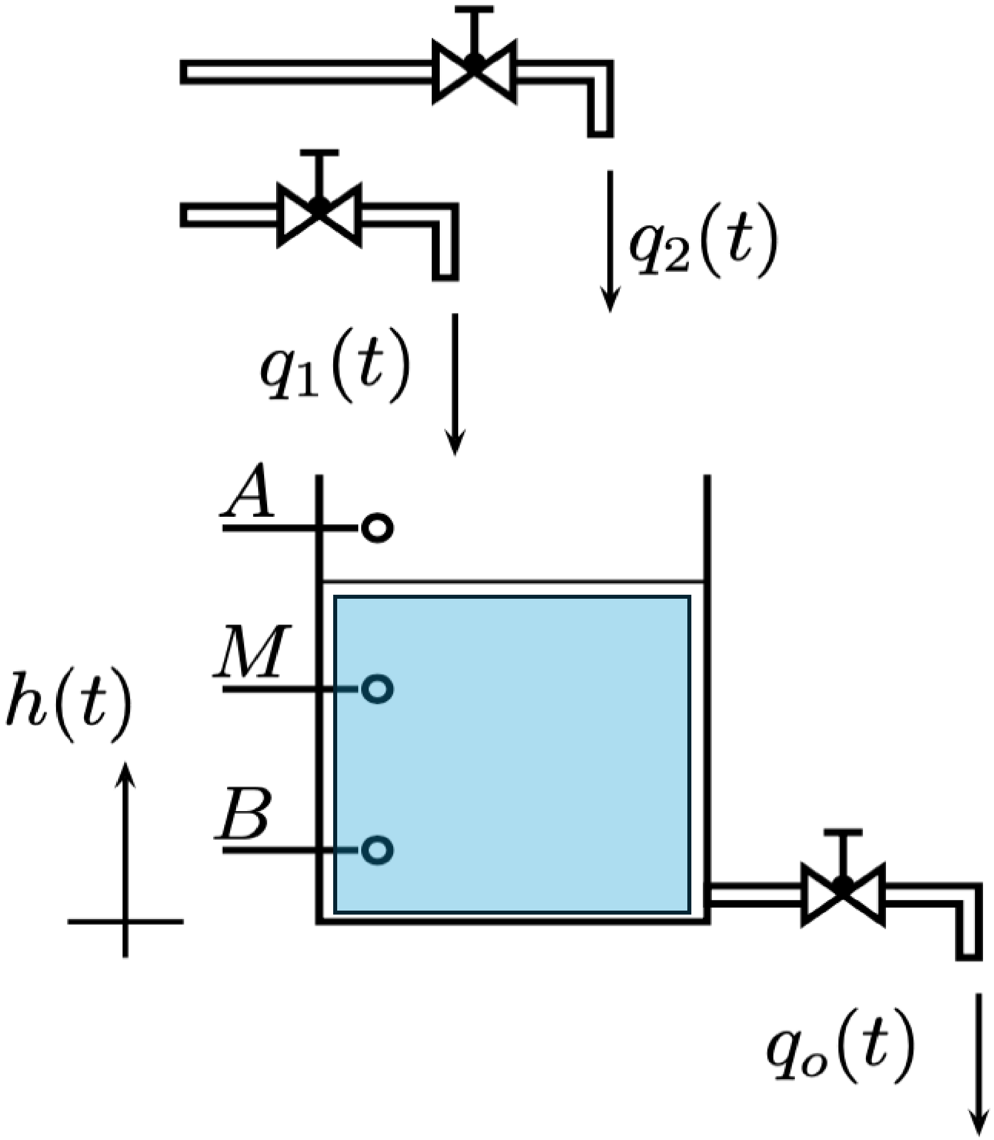


$$q_1 = 2, q_2 = 1$$





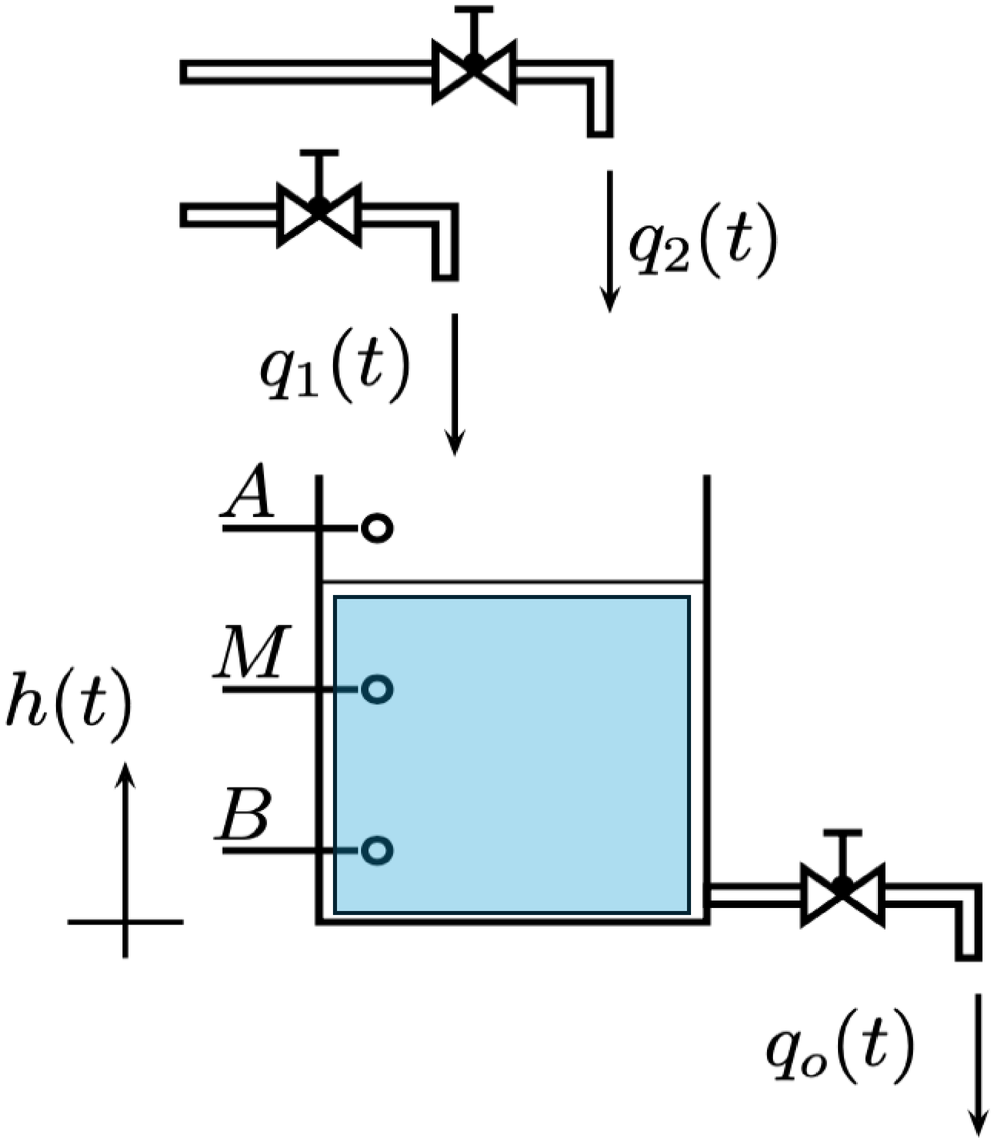
A	M	B	Y_1	Y_2
0	0	0	1	1
0	0	1	0	1
0	1	1	1	0
1	1	1	0	0



A	M	B	Y_1	Y_2
0	0	0	1	1
0	0	1	0	1
0	1	1	1	0
1	1	1	0	0

M \ AB	00	01	11	10
0	1	0	X	X
1	X	1	0	X

$$Y_1 = \overline{B} + \overline{A}M$$



A	M	B	Y_1	Y_2
0	0	0	1	1
0	0	1	0	1
0	1	1	1	0
1	1	1	0	0

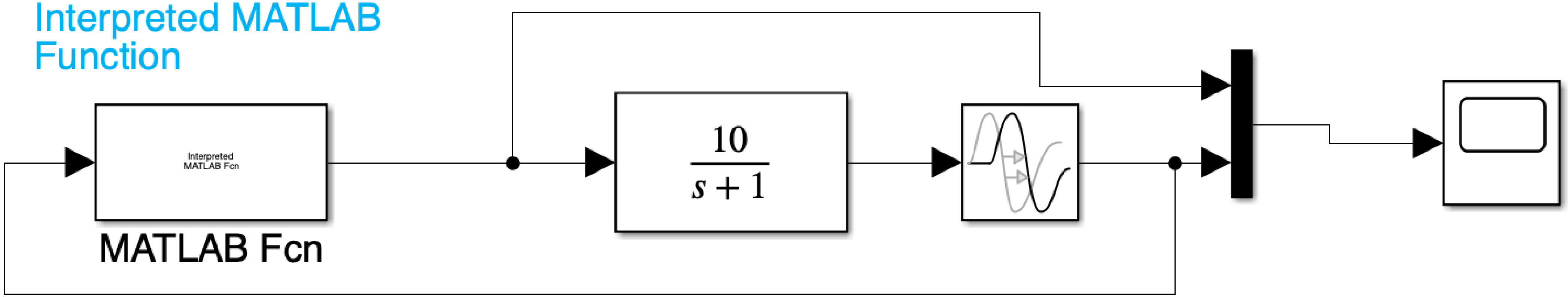
M \ AB	00	01	11	10
0	1	0	X	X
1	X	1	0	X

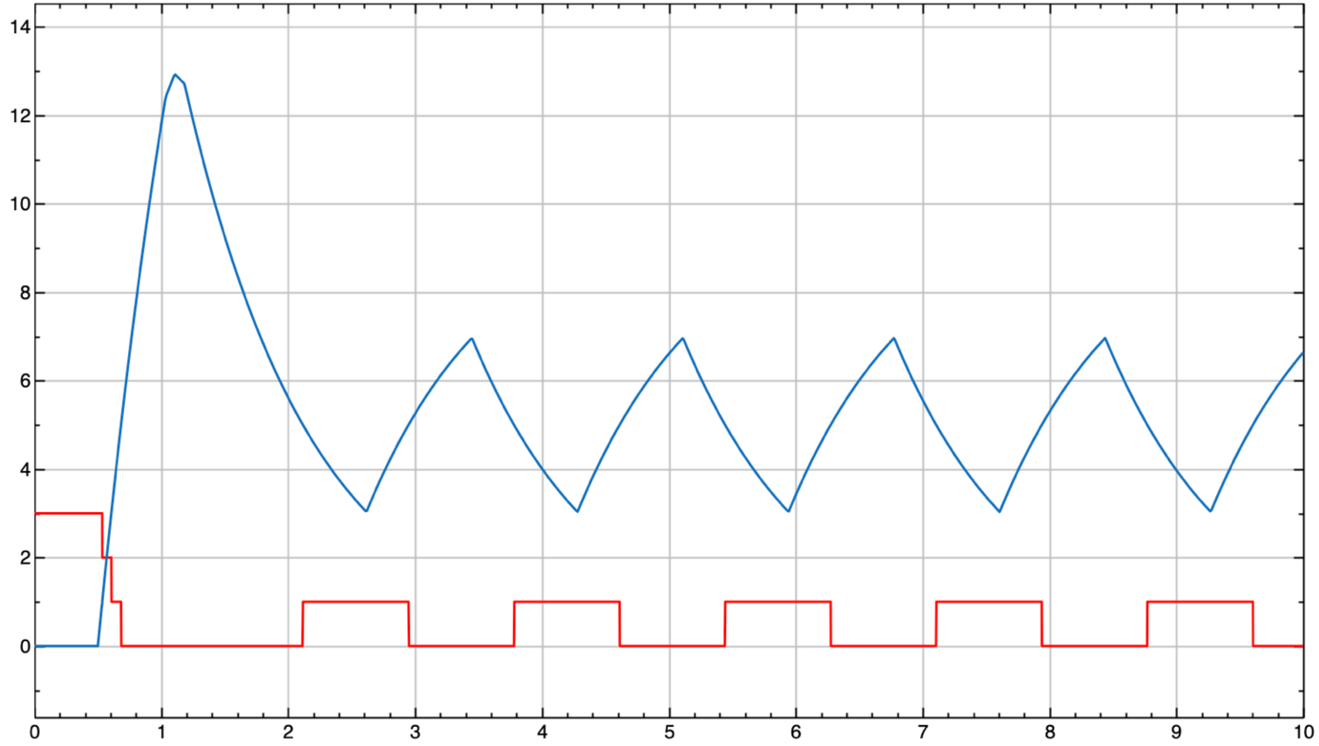
$$Y_1 = \overline{B} + \overline{A}M$$

1	1	X	X
X	0	0	X

$$Y_2 = \overline{M}$$

Interpreted MATLAB Function





Equivalencia de las funciones lógicas y su cálculo en $\mathbb{R}$

Suma lógica (OR): $A + B = \max\{A, B\}$

Equivalencia de las funciones lógicas y su cálculo en $\mathbb{R}$

Suma lógica (OR): $A + B = \max\{A, B\}$

Producto lógico (AND): $A \cdot B = \min\{A, B\}$

Equivalencia de las funciones lógicas y su cálculo en $\mathbb{R}$

$$\text{Suma lógica (OR): } A + B = \text{máx}\{A, B\}$$

$$\text{Producto lógico (AND): } A \cdot B = \text{mín}\{A, B\}$$

$$\text{Negación (NOT): } \overline{A} = 1 - A$$

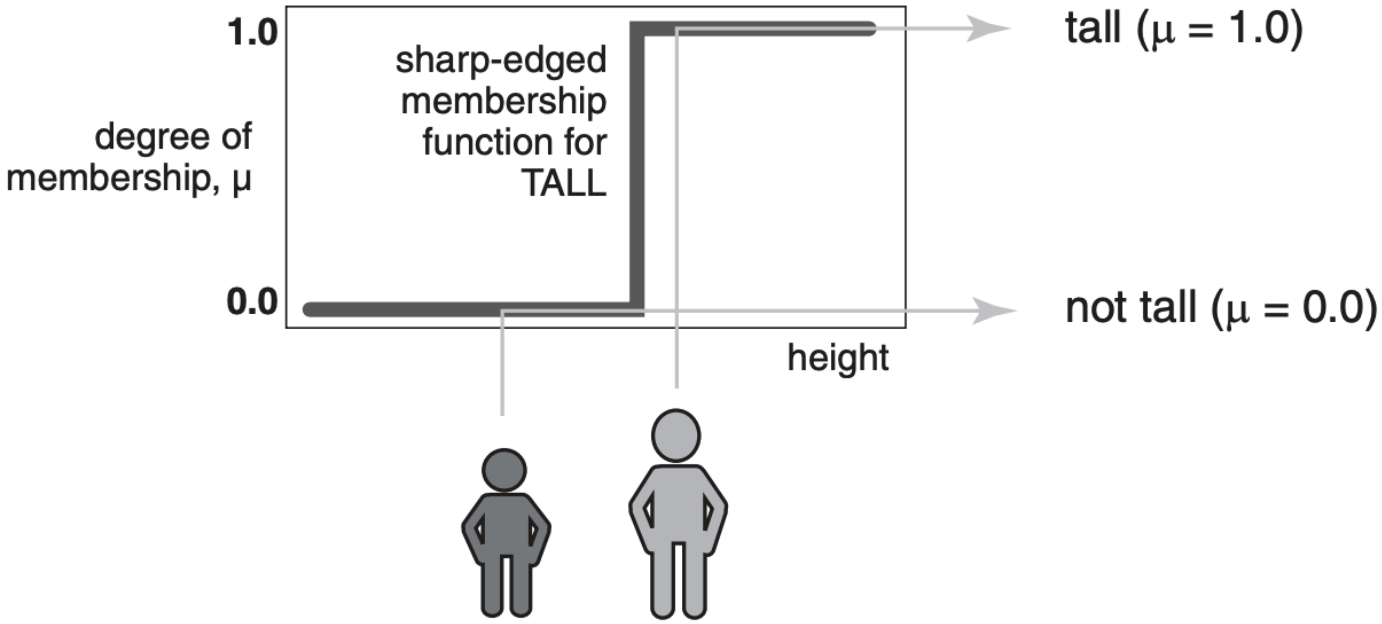
Equivalencia de las funciones lógicas y su cálculo en $\mathbb{R}$

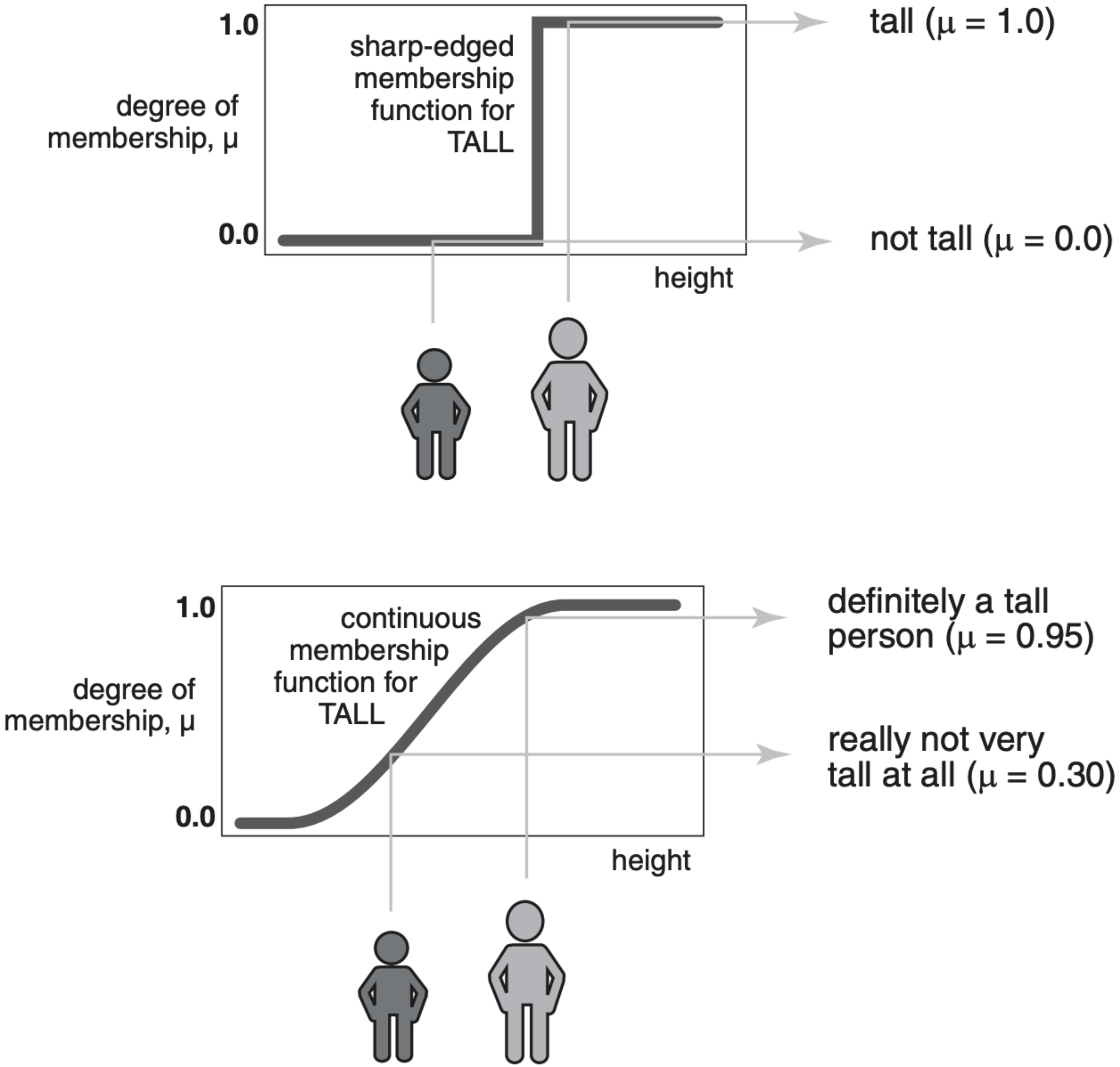
Suma lógica (OR): $A + B = \max\{A, B\}$

Producto lógico (AND): $A \cdot B = \min\{A, B\}$

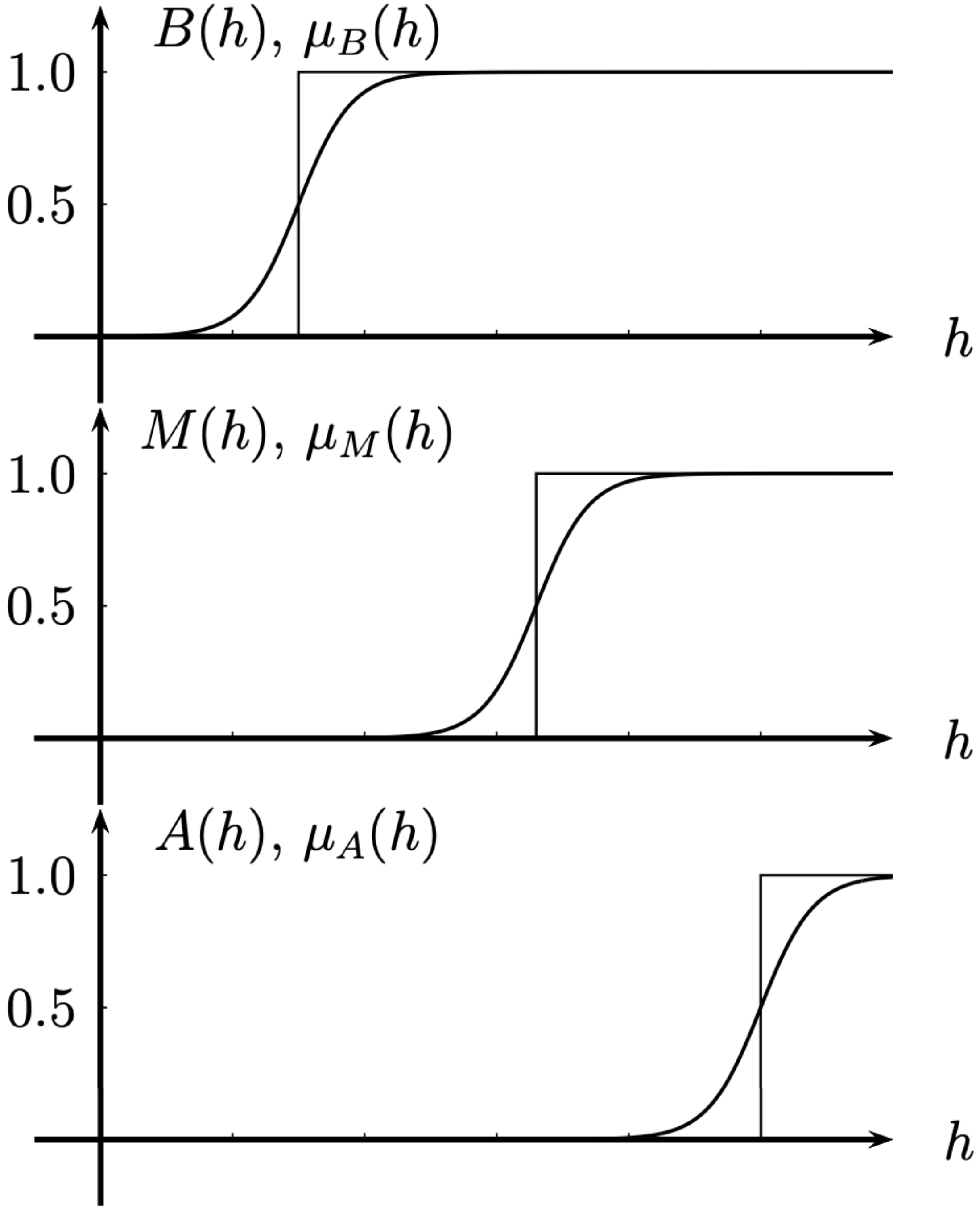
Negación (NOT): $\overline{A} = 1 - A$

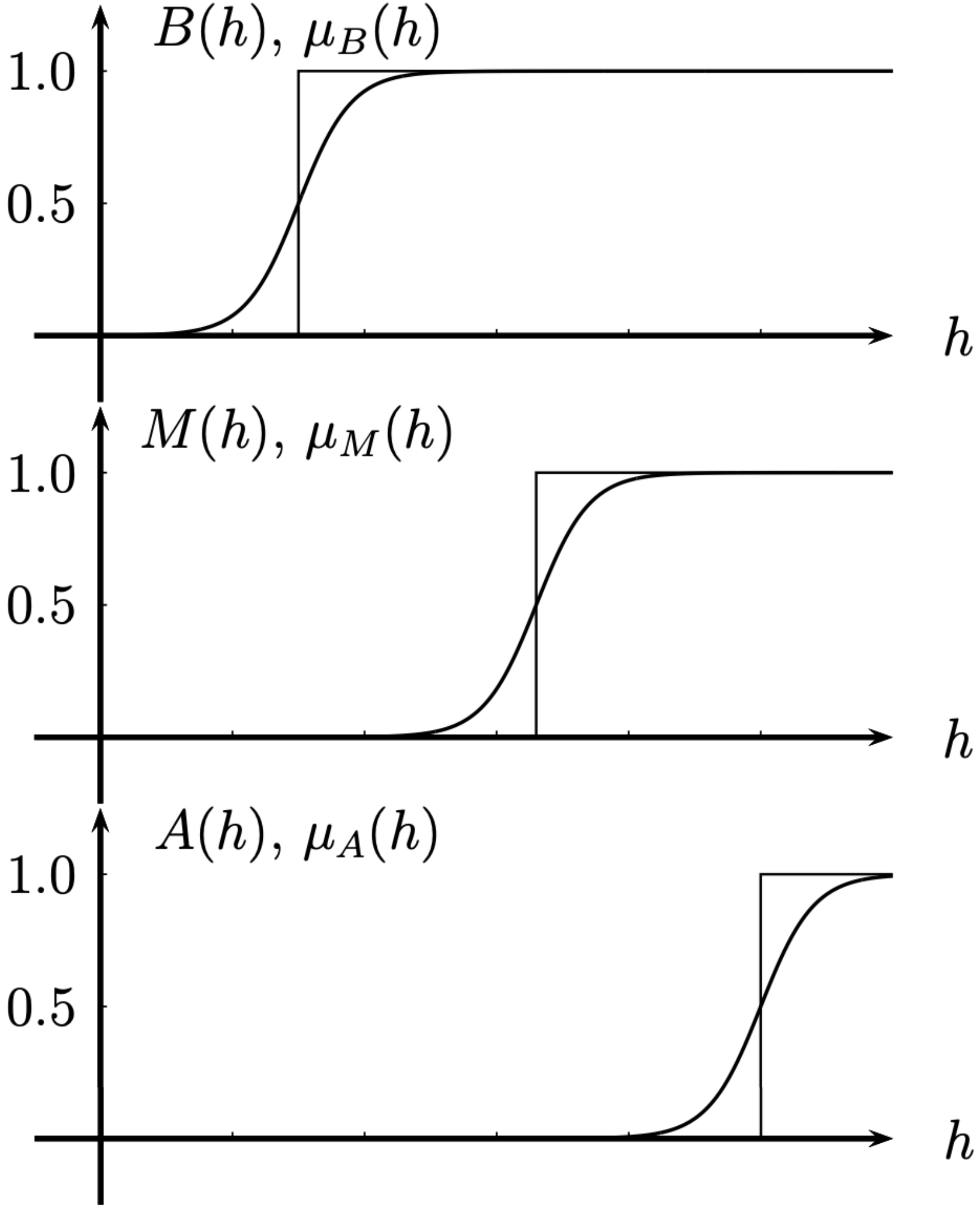
AB	$A+B$	$A \cdot B$	$\overline{A}$	$\max\{A, B\}$	$\min\{A, B\}$	$1 - A$
00	0	0	1	0	0	1
01	1	0	1	1	0	1
10	1	0	0	1	0	0
11	1	1	0	1	1	0


1.0
0.0
degree of
membership, μ
sharp-edged
membership
function for
TALL
height
tall (μ = 1.0)
not tall (μ = 0.0)



1.0
0.0
degree of membership, μ
sharp-edged membership function for TALL
height
tall (μ = 1.0)
not tall (μ = 0.0)
1.0
0.0
degree of membership, μ
continuous membership function for TALL
height
definitely a tall person (μ = 0.95)
really not very tall at all (μ = 0.30)


$B(h)$, $\mu_B(h)$
1.0
0.5
h
$M(h)$, $\mu_M(h)$
1.0
0.5
h
$A(h)$, $\mu_A(h)$
1.0
0.5
h

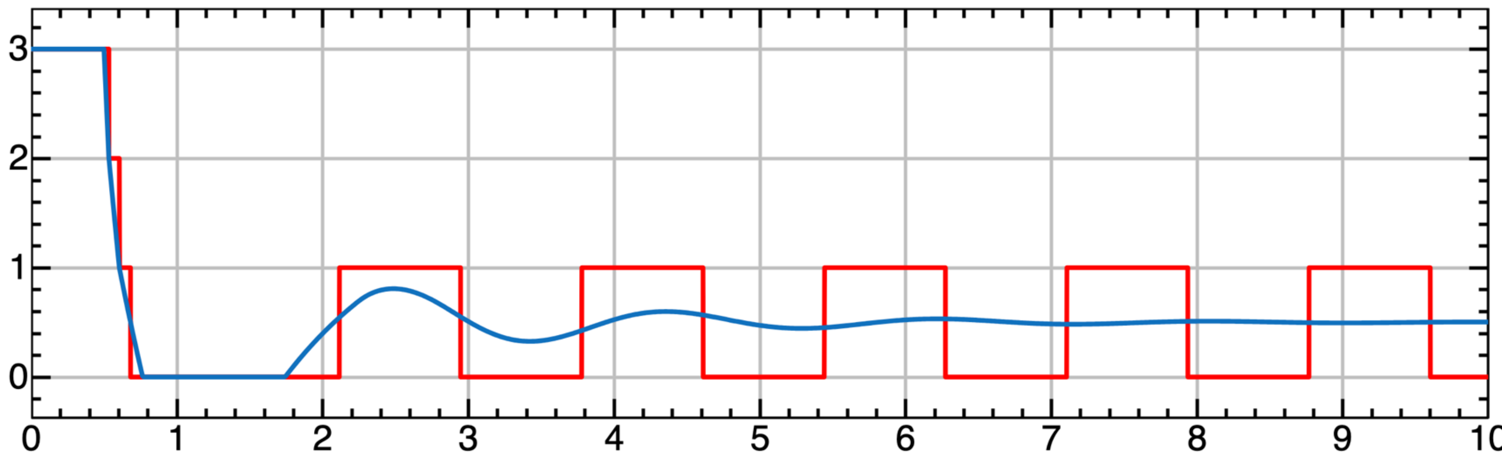


$$Y_2 = \overline{M}$$

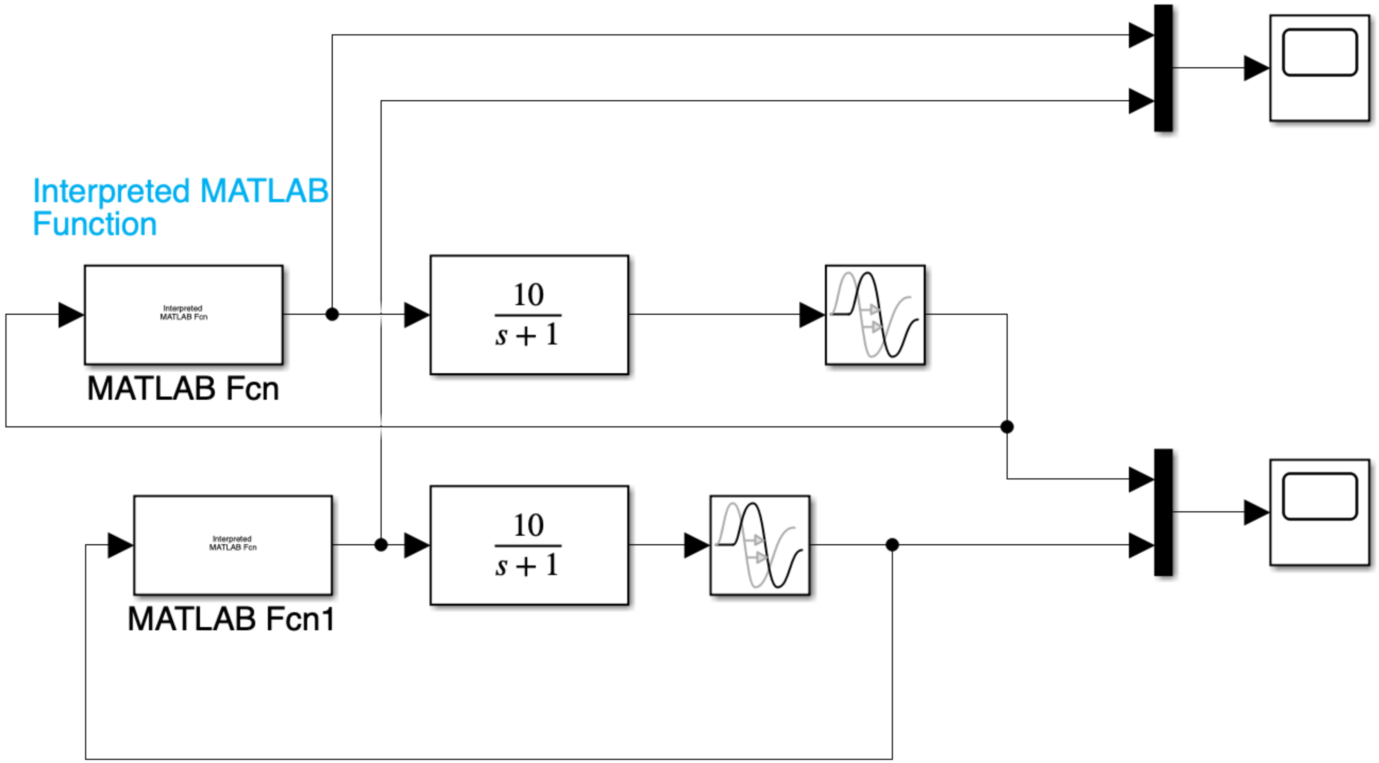
$$Y_2 = 1 - \mu_M$$

$$Y_1 = \overline{B} + \overline{A}M$$

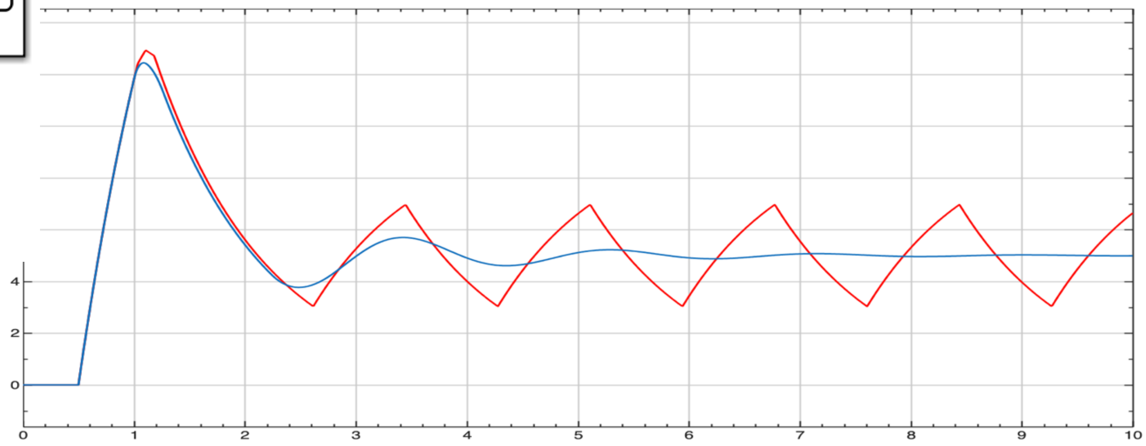
$$Y_1 = \text{máx}\{(1-\mu_B), \text{mín}\{(1-\mu_A), \mu_M\}\}$$



3
2
1
0
0 1 2 3 4 5 6 7 8 9 10



Interpreted MATLAB Function
Interpreted MATLAB Fcn
MATLAB Fcn
10/(s+1)
Interpreted MATLAB Fcn
MATLAB Fcn1
10/(s+1)



4
2
0
0 1 2 3 4 5 6 7 8 9 10