EE160: Introduction of Control

2022-2023 Final Exam

Teacher: Dr. Yang Wang

Time and Location

2023-01-03, Tuesday, 10:30-12:30 , 120 mins, Online

Regulation

This is an "closed-book" exam, but you can bring one A4 cheat sheet with you. Note that, you are NOT allowed to use mobile phones or other electronic devices except calculator.

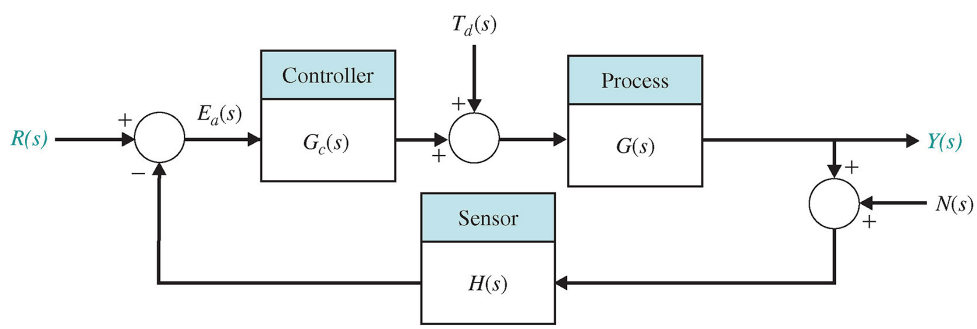
Scores

Question 1:10 points
Question 2:10 points
Question 3:10 points
Question 4:10 points
Question 5:10 points

Max 50 points in total.

Question 1: True of False (10 points)

1. The dimension of a state space model must equal to the order of the system it describes.
2. Any Single-Input-Single-Output Linear system can be described by a transfer function with constant coefficients.
3. The matrix exponential equality $e^A \cdot e^A = e^{2A}$ holds for all non-zero matrix A.
4. Compared to open loop systems, an important advantage of feedback control systems is the ability to reduce the effect of the variation of parameters.
5. Consider the closed-loop system described by the block diagram below. For a fixed $G(s)$, as the loop gain increases over the frequency of interest, the effect of $T\mathrm{d}(s)$ on the tracking error increases.



6. If system 1 is said to be more stable than system 2, it indicates that the transient response of system 1 will decay faster than the transient response of system 2.
7. A BIBO minimum phase system has no zero located at the right-half complex plane.
8. For a standard second order closed-loop system, the percentage overshoot is irrelevant with the natural frequency of its loop-gain function.
9. The transient response (in terms of overshoot and settling time) of a third-order system is always worse than the approximated second-order system governed by its dominant roots, that is, the third pole always degrades the transient performance of approximated system.
10. A linear system satisfies the properties of superposition and homogeneity.

Question 2 System Modelling (10 points)

Consider the block diagram in Figure 1.

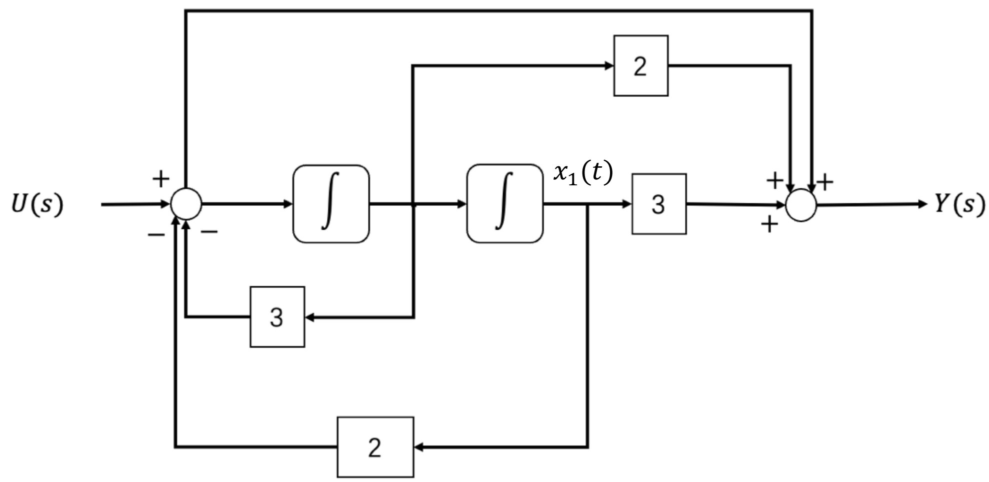


Figure 1 Signal flow graph model

1）Using the block diagram as a guide, obtain a second-order state variable model of the system in the form of

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t)$$
$$y(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}u(t)$$

and write down the general solution of the state $x(t)$. (3 points)

2）Calculate the transfer function T(s) from U(s) to Y(s) utilizing the state space model obtained in question 1). (2 points)

3) Determine the high frequency gain and the steady state response of $y(t)$ with respect to a unit step input. (2 points)

4) Sketch the amplitude figure of the bode plot of T(s). (3 points)

Question 3 Root locus and the Performance of the feedback control system (10 points)

A three-dimensional cam for generating a function of two variables is shown in Figure 2. Both x and y may be controlled using a position control system.

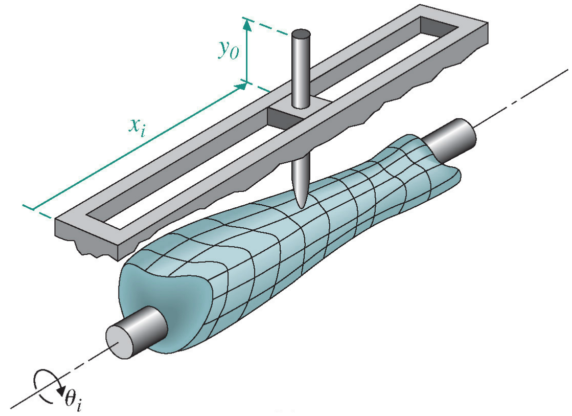


Figure 2 Three-dimensional cam

The control of x may be achieved with a DC motor whose transfer function represented by

$$G(s) = \frac{K}{s^2 + ps + q}$$

1) Figure 3 describes the time response of G(s) with respect to a unit step input. Determine the value of p, q and K. (3 points)

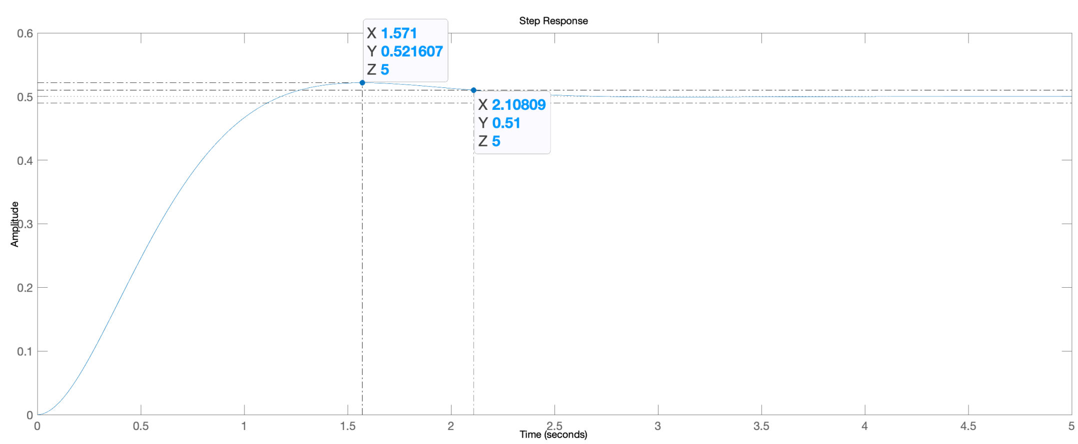


Figure 3 Time response of G(s)

2) A good position control of x features a position feedback loop of the form shown in Figure 4,

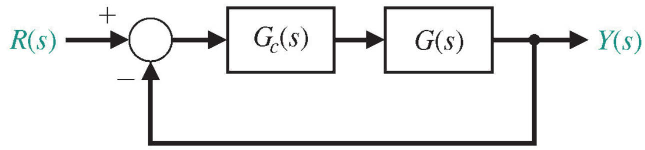


Figure 4 x-axis control system

where the plant is connected with a PI-controller

$$G_c(s) = Kp + \frac{Ki}{s}$$

If $p = 2, q = 3.5, K = 0.5$ and $Kp = 1$, using Routh-Hurwitz criteria to determine the value of Ki for which the closed-loop system is marginally stable and identify the frequency(Hz) of the oscillating mode it contains. (3 points)

3) If $p = 4, q = 3, K = 0.5$ and $Ki = 3.01K_p$, sketch the root locus plot of the system as Kp varies. Furthermore, determine the roughly value of Kp for which the damping ratio of the dominate roots is 0.707.(4 points)

Question 4 Frequency Response method (10 points)

A unity feedback system is shown below in Figure 5

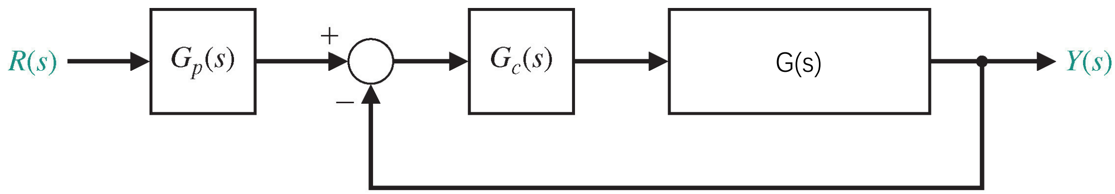
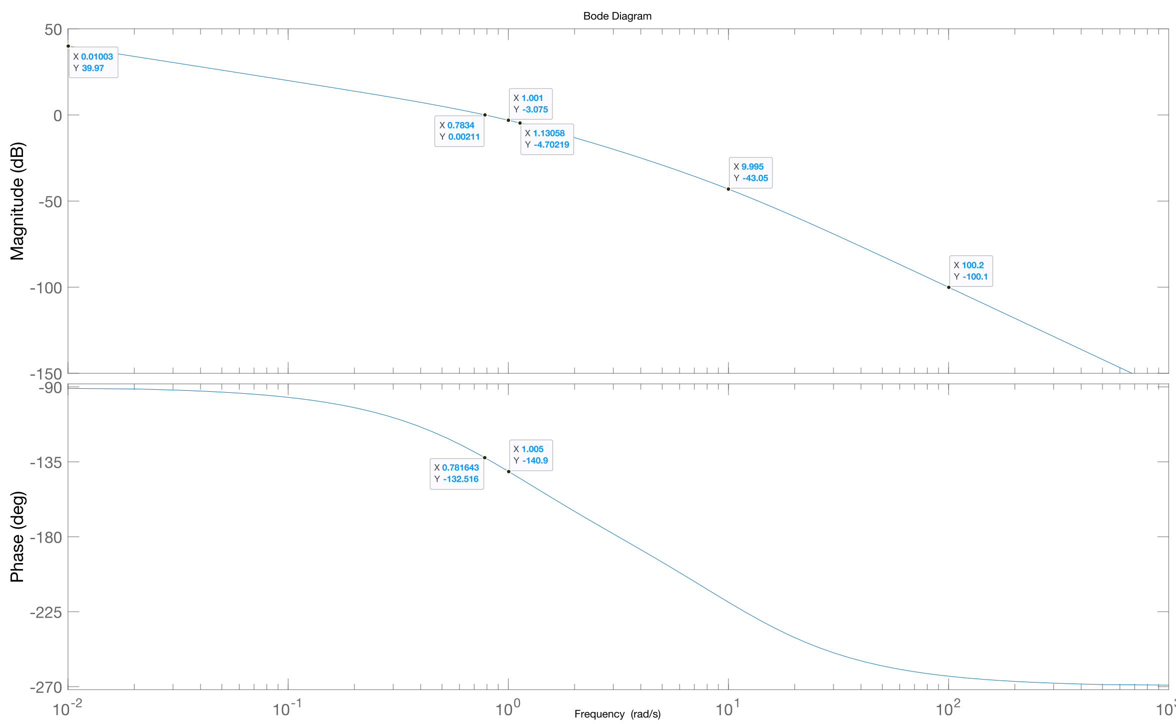


Figure 5 block diagram of closed-loop system

with

$$G(s) = \frac{10}{s(s+p)(s+10)}$$

verifying the following bode diagram



1) Determine the value of an integer parameter p (1points)

2) When $G_p(s) = 1$, design a phase-lead compensator $G_c(s)$ such that the closed-loop system has a pair of dominate roots with damping ratio $\xi = 0.7$. (5 points)

3) When $G_p(s) = 1$, calculate the velocity error constant K_v of the system equipped with the compensator you designed in question 2).(2 points)

4) Design a prefilter $G_p(s)$ that can cancel the effect of the zero introduced by the compensator while maintain the steady state error of the closed-loop system with respect to a unit step input equal to 0. (2 points)

Question 5: Controller Design based on the state space model (10 points)

The motion control of a lightweight hospital transport vehicle can be represented by a system of two masses, as shown in Figure 6, where $m_1 = m_2 = 1, k_1 = k_2 = 1$.

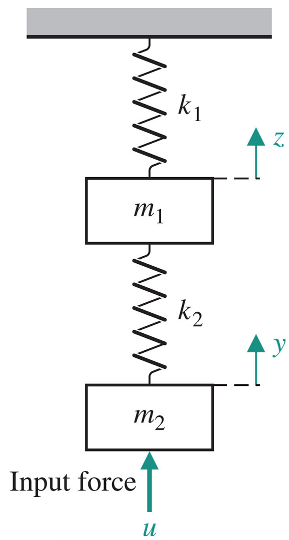


Figure 6 sketch of a lightweight hospital transport vehicle

1) Neglect the gravity force and establish the state space model of the system with $x = [z, \dot{z}, y, \dot{y}]$.(2points)

2) Write down the algebraic condition under which the system is completely controllable. (2 points)

3) Given the state x is fully measurable, describe the procedure of designing an optimal controller that minimizes the velocity of two masses and limits the control effort. (2 points)

4) Consider the subsystem for mass 1. Treat the force brought by mass 2 as an external force to be designed, i.e. $f_{ext} = k_2 y$ and note that only the position z is accessible. Now, design an observer-based full-state feedback law for f_{ext} such that
   a) the observation error achieves a deadbeat behavior with settling time Ts=4.82s.
   b) the position of the mass 1 asymptotically converge to its origin with a speed of $e^{-0.1t}$ after the observation error have reached zero. (4 points)

Appendix A Laplace Transform

Table D.1

$F(s)$	$f(t), t \supseteq 0$
1. 1	$\delta(t_0)$, unit impulse at $t = t_0$
2. $1/s$	1, unit step
3. $\dfrac{n!}{s^{n+1}}$	t^n
4. $\dfrac{1}{(s+a)}$	e^{-at}
5. $\dfrac{1}{(s+a)^n}$	$\dfrac{1}{(n-1)!}t^{n-1}e^{-at}$
6. $\dfrac{a}{s(s+a)}$	$1 - e^{-at}$
7. $\dfrac{1}{(s+a)(s+b)}$	$\dfrac{1}{(b-a)}(e^{-at} - e^{-bt})$
8. $\dfrac{s+\alpha}{(s+a)(s+b)}$	$\dfrac{1}{(b-a)}[(\alpha - a)e^{-at} - (\alpha - b)e^{-bt}]$
9. $\dfrac{ab}{s(s+a)(s+b)}$	$1 - \dfrac{b}{(b-a)}e^{-at} + \dfrac{a}{(b-a)}e^{-bt}$
10. $\dfrac{1}{(s+a)(s+b)(s+c)}$	$\dfrac{e^{-at}}{(b-a)(c-a)} + \dfrac{e^{-bt}}{(c-a)(a-b)} + \dfrac{e^{-ct}}{(a-c)(b-c)}$
11. $\dfrac{s+\alpha}{(s+a)(s+b)(s+c)}$	$\dfrac{(\alpha-a)e^{-at}}{(b-a)(c-a)} + \dfrac{(\alpha-b)e^{-bt}}{(c-b)(a-b)} + \dfrac{(\alpha-c)e^{-ct}}{(a-c)(b-c)}$
12. $\dfrac{ab(s+\alpha)}{s(s+a)(s+b)}$	$\alpha - \dfrac{b(\alpha-a)}{(b-a)}e^{-at} + \dfrac{a(\alpha-b)}{(b-a)}e^{-bt}$
13. $\dfrac{\omega}{s^2+\omega^2}$	$\sin \omega t$
14. $\dfrac{s}{s^2+\omega^2}$	$\cos \omega t0$
15. $\dfrac{s+\alpha}{s^2+\omega^2}$	$\dfrac{\sqrt{\alpha^2+\omega^2}}{\omega}\sin(\omega t + \phi), \phi = \tan^{-1}\omega/\alpha$
16. $\dfrac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\sin \omega t$
17. $\dfrac{(s+\alpha)}{(s+a)^2+\omega^2}$	$e^{-at}\cos \omega t$
18. $\dfrac{s+\alpha}{(s+a)^2+\omega^2}$	$\dfrac{1}{\omega}[(\alpha-a)^2+\omega^2]^{1/2}e^{-at}\sin(\omega t + \phi)$, $\phi = \tan^{-1}\dfrac{\omega}{\alpha - a}$
19. $\dfrac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\dfrac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin \omega_n\sqrt{1-\zeta^2}t, \zeta < 1$

Appendix B Decibel Conversion

Table F.1

M	0	1	2	3	4	5	6	7	8	9
0.0	$m =$	−40.00	−33.98	−30.46	−27.96	−26.02	−24.44	−23.10	−21.94	−20.92
0.1	−20.00	−19.17	−18.42	−17.72	−17.08	−16.48	−15.92	−15.39	−14.89	−14.42
0.2	−13.98	−13.56	−13.15	−12.77	−12.40	−12.04	−11.70	−11.37	−11.06	−10.75
0.3	−10.46	−10.17	−9.90	−9.63	−9.37	−9.12	−8.87	−8.64	−8.40	−8.18
0.4	−7.96	−7.74	−7.54	−7.33	−7.13	−6.94	−6.74	−6.56	−6.38	−6.20
0.5	−6.02	−5.85	−5.68	−5.51	−5.35	−5.19	−5.04	−4.88	−4.73	−4.58
0.6	−4.44	−4.29	−4.15	−4.01	−3.88	−3.74	−3.61	−3.48	−3.35	−3.22
0.7	−3.10	−2.97	−2.85	−2.73	−2.62	−2.50	−2.38	−2.27	−2.16	−2.05
0.8	−1.94	−1.83	−1.72	−1.62	−1.51	−1.41	−1.31	−1.21	−1.11	−1.01
0.9	−0.92	−0.82	−0.72	−0.63	−0.54	−0.45	−0.35	−0.26	−0.18	−0.09
1.0	0.00	0.09	0.17	0.26	0.34	0.42	0.51	0.59	0.67	0.75
1.1	0.83	0.91	0.98	1.06	1.14	1.21	1.29	1.36	1.44	1.51
1.2	1.58	1.66	1.73	1.80	1.87	1.94	2.01	2.08	2.14	2.21
1.3	2.28	2.35	2.41	2.48	2.54	2.61	2.67	2.73	2.80	2.86
1.4	2.92	2.98	3.05	3.11	3.17	3.23	3.29	3.35	3.41	3.46
1.5	3.52	3.58	3.64	3.69	3.75	3.81	3.86	3.92	3.97	4.03
1.6	4.08	4.14	4.19	4.24	4.30	4.35	4.40	4.45	4.51	4.56
1.7	4.61	4.66	4.71	4.76	4.81	4.86	4.91	4.96	5.01	5.06
1.8	5.11	5.15	5.20	5.25	5.30	5.34	5.39	5.44	5.48	5.53
1.9	5.58	5.62	5.67	5.71	5.76	5.80	5.85	5.89	5.93	5.98
2.	6.02	6.44	6.85	7.23	7.60	7.96	8.30	8.63	8.94	9.25
3.	9.54	9.83	10.10	10.37	10.63	10.88	11.13	11.36	11.60	11.82
4.	12.04	12.26	12.46	12.67	12.87	13.06	13.26	13.44	13.62	13.80
5.	13.98	14.15	14.32	14.49	14.65	14.81	14.96	15.12	15.27	15.42
6.	15.56	15.71	15.85	15.99	16.12	16.26	16.39	16.52	16.65	16.78
7.	16.90	17.03	17.15	17.27	17.38	17.50	17.62	17.73	17.84	17.95
8.	18.06	18.17	18.28	18.38	18.49	18.59	18.69	18.79	18.89	18.99
9.	19.08	19.18	19.28	19.37	19.46	19.55	19.65	19.74	19.82	19.91
	0.	**1.**	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**	**9.**

Decibels $= 20 \log_{10} M$.

Appendix C

Table 10.2 Coefficients and Response Measures of a Deadbeat System

System Order	Coefficients α	β	γ	δ	ϵ	Percent Overshoot $P.O.$	Percent Overshoot $P.U.$	90% Rise Time T_r	Settling Time T_s
2nd	1.82					0.10%	0.00%	3.47	4.82
3rd	1.90	2.20				1.65%	1.36%	3.48	4.04
4th	2.20	3.50	2.80			0.89%	0.95%	4.16	4.81
5th	2.70	4.90	5.40	3.40		1.29%	0.37%	4.84	5.43
6th	3.15	6.50	8.70	7.55	4.05	1.63%	0.94%	5.49	6.04

Note: All times are normalized.