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Decentralized Formation Control With Prescribed Distance Constraints and Shape Uniqueness

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***Abstract*— For Multi-Agent Systems (MAS) formation problem, this paper presents a novel decentralized control protocol that can guarantee the uniqueness of formation shape and restrain the distance between neighboring agents within a prescribed range. The proposed control law is decentralized, in the sense that each agent merely employs local relative information regarding its neighbors to obtain the control input. Using a delicately designed gain matrix, we avoid the problem of non-uniqueness of shape that is existed in the majority of the decentralized formation methods, especially the distance-based methods. Meanwhile, the distance constraints, like connectivity maintenance and collision avoidance, required in many practical scenarios are also addressed. Furthermore, the convergence of formation is rigorously proved under a less restrictive assumption on the communication graph. Finally, a comparative simulation verifies the effectiveness and superiority of proposed approach.**

***Index Terms*— Multi-agent systems, Shape uniqueness, Hysteresis switching control, Collision avoidance, Connectivity maintenance.**

I. Introduction

In recent years, formation control of MAS, which refers to designing appropriate control protocols to make them establish and maintain a desired shape, has attracted more and more attention [1]–[3]. Existing formation control methods (see [3] and reference therein), according to the difference between the perceptual variables and the controlled variables, can be roughly divided into the control schemes based on position, displacement, bearing and distance. In early days, many formation control methods are developed under a centralized framework [4]–[7]. Later, fully decentralized strategies [8]–[9], on the other hand, with naturally parallelized computation ability and in comparison have better scalability and resiliency to global information loss. Hence, it can work in a GPS-denied environment and with limited on-board computation source, such as search and rescue operations, planetary exploration, indoor navigation and so on [10].

As a celebrated decentralized method, distance-based techniques usually assume that the agent can measure its relative position with respect to the adjacent agents. Thus, this type of method is favorable as they do not require global position measurements (such as GPS) or pre-alignment of agent local coordinate system. The distance-based method enables us to design a decentralized control law to actively control the distance among agents. Despite the success of the convergence of the distances to desired values, it may lead to a final formation with a topology property that completely different from the desired formation shape (the example of different formation shape sharing the same distance constraints is illustrated in Fig. 1). This is actually a major drawback of the distance-based technique. This problem has been studied in detail in [11] and [12]. A possible solution to this issue is to pose some restrictive initial conditions as did in [13]-[15], so as to avoid forming an undesirable formation shape. Other methods to deal with the problem is design a new controller by introducing additional controlled variables like the symbolic area of triangles [16] [17], or add additional information to assist control, such as absolute angles of agents relative to the leader [18], angle information between agents [19], and volume information [20].

Inspired by above discussions, this paper proposes a decentralized control protocol that guarantees uniqueness of formation shape without making any unrealistic initial condition hypothesis or adding extra information. Instead, by solving a semidefinite programming (SDP) problem, we use formation control gains that are offline designed for agents with the single-integrator model to ensure the convergence and the uniqueness of the formation shape.

For the sake of practical implementation, we also consider the collision avoidance and the connectivity maintenance issue of MAS, which are two basic requirements to ensure safety and success of formation. These two requirements are essentially to keep the distance among agents staying within a certain range. The most widely employed solutions to this issue are the potential function-based method [21]-[23] and Control Barrier Function(CBF)-based methods [24] [25]. Our idea is also similar to the above two methods. To be specific, we transform the original constrained system into a new equivalent unconstrained system, and design the suitable barrier function to deal with the problem of connectivity maintenance and collision avoidance among neighboring agents. Apart from the aforementioned features, another significance of the presented method worth mentioning is the removal of the the minimal rigidity assumption for communication graph which exists in many other distance-based control methods [13].

II. Preliminaries

Consider an inter-agent communication structure described by an undirected graph with n agents and l edges, denoted by $\mathcal{G} \triangleq (\mathcal{V}, \mathcal{E})$ where $\mathcal{V} = \{1, 2, ..., n\}$ is the set of agents and

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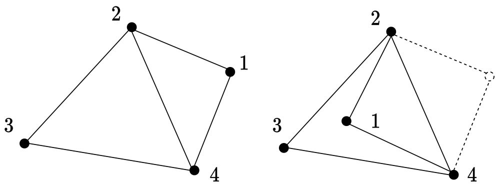

Fig. 1: Graph (a) is a desired in infinitesimally rigid graph. Graph (b) satisfies distance condition, but the shape is not desired.

$\mathcal{E} = \{(i,j)|i,j \in \mathcal{V}, i \neq j\}$ is the set of edges. The set of neighbors of agent i is denoted by $\mathcal{N}_i = \{j \in \mathcal{V}|(i,j) \in \mathcal{E}\}$. Let $q_i \in \mathbb{R}^2$ denote the position that is assigned to agent $i \in \mathcal{V}$. Then, we define the overall vector $q = \mathrm{col}(q_i) \triangleq [q_1^\top, ..., q_n^\top] \in \mathbb{R}^{2n}$ which represents the formation realization of $\mathcal{G}$ in $\mathbb{R}^2$. The pair $\mathcal{F} \triangleq (\mathcal{G}, q)$ denotes the framework of $\mathcal{G}$ in $\mathbb{R}^2$.

Rigidity is an important concept in graph theory. In a rigid framework, preserving the length of the graph edges guarantees that all distances among all agents of the graph remain unaltered (i.e., the shape is preserved). The rigidity of a framework can be easily checked by the following lemmas.

Lemma 1: [27] A graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is said to be rigid in $\mathbb{R}^2$ if and only if there is an induced graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}')$ where $\mathcal{E}' \subseteq \mathcal{E}$ satisfying:

(1). $|\mathcal{E}'| = 2|\mathcal{V}| - 3$.

(2). $|\mathcal{E}''| \leq 2|\mathcal{V}''| - 3$, where $\mathcal{E}'' \subseteq \mathcal{E}'$ and $\mathcal{V}''$ is the set of agents induced by the edge set $\mathcal{E}''$.

Moreover, we define the *rigidity matrix* $R : \mathbb{R}^{2n} \to \mathbb{R}^{l \times 2n}$ of $\mathcal{F} = (\mathcal{G}, q)$ as:

$$R(q) = \frac{1}{2}\frac{\partial \Phi_{\mathcal{G}}(q)}{\partial q} \tag{1}$$

For an arbitrary sequence of edges in $\mathcal{E}$, an edge function (rigidity function) $\Phi_{\mathcal{G}} : \mathbb{R}^{2n} \to \mathbb{R}^l$ associated with $\mathcal{F}$ is given as $\Phi_{\mathcal{G}}(q) = [..., ||q_i - q_j||^2, ...]^\top, (i,j) \in \mathcal{E}$ such that its kth component, i.e., $||q_i - q_j||^2$, relates to the kth edge of $\mathcal{E}$ connecting the ith and jth agents. Hence, each row of the rigidity matrix $R(q)$ takes the following form:

$$[\mathbf{0}_{1\times 2}^\top ... q_{ij}^\top ... \mathbf{0}_{1\times 2}^\top ... - q_{ij}^\top ... \mathbf{0}_{1\times 2}^\top].$$

Since the rigidity matrix only depends on the relative positions, it can be expressed as $R(\tilde{q})$ where $\tilde{q} = \mathrm{col}(q_{ij}) \in \mathbb{R}^{2l}$ in which $q_{ij} = q_i - q_j, (i,j) \in \mathcal{E}$. It is shown in [28] that $\mathrm{rank}[R(q)] \leq 2n - 3$ in $\mathbb{R}^2$.

Definition 1: [29] A rigid framework is minimally rigid if no single inter-agent distance constraint can be removed without losing its rigidity. In $\mathbb{R}^2$ a rigid framework $(\mathcal{G}, q)$ is minimally rigid if $l = 2n - 3$.

Definition 2: [30] A framework $\mathcal{F} = (\mathcal{G}, q)$ is infinitesimally rigid in 2-dimensional space if: $\mathrm{rank}[R(q)] = 2n - 3$.

Lemma 2: [31] If $\mathcal{F}_p = (\mathcal{G}, p)$ is infinitesimally rigid then there exists a small positive constant v such that all frameworks $\mathcal{F}_q = (\mathcal{G}, q)$ that satisfy $\Psi(\mathcal{F}_p, \mathcal{F}_q) \triangleq \sum_{(i,j)\in\mathcal{E}}(||q_i - q_j|| - ||p_i - p_j||)^2 \leq v$, are also infinitesimally rigid.

Lemma 3: [32] If the framework $\mathcal{F} = (\mathcal{G}, q)$ is minimally and infinitesimally rigid in 2-dimensional space, then the matrix $R(q)R(q)^\top$ is positive definite.

III. Problem Statement

Motion of agents with single-integrator dynamics is expressed as

$$\dot{q}_i = u_i, \quad i = 1, ..., n \tag{2}$$

where $q_i := [x_i, y_i]^\top \in \mathbb{R}^2$ represents the position of agent i in a global coordinate frame, and u_i is the control input. The relative position between neighboring agents is

$$q_{ij} = q_i - q_j, \quad (i,j) \in \mathcal{E},$$

thus each agent can measure it in their local coordinate frames. $||q_{ij}||$ denotes the actual distance between agent i and j. In what follows, collision avoidance and connectivity maintenance among neighboring agents are formulated with respect to $||q_{ij}||$.

A. Collision Avoidance of Neighboring Agents

In order to avoid the collision of agents in the process of movement, it is assumed that there is a circular security distance among the pairs of agents with connection. The controller tends to ensure that the safety security distance will not be transgressed. As shown in Fig. 2, let $r_{si} \in \mathbb{R}^+$, $r_{sj} \in \mathbb{R}^+$ be the safety radius of the neighboring agents i and j, respectively. Then, the safety security range is defined as $r_{sij} = (r_{si} + r_{sj}) > 0, (i,j) \in \mathcal{E}$. Thus, in order to ensure collision avoidance between neighboring agents, it is required that $||q_{ij}(t)|| > r_{sij}$ for all $t \geq 0$.

B. Connectivity Maintenance of Neighboring Agents

In practice, because each agent has limited sensing ability, the controller should limit the distances between neighboring agents to ensure the connectivity maintenance. As shown in Fig. 2, let $r_{ci} \in \mathbb{R}^+$, $r_{cj} \in \mathbb{R}^+$ be the sensing radius of the neighboring agents i and j, respectively. Then, define $r_{cij} = \min\{r_{ci}, r_{cj}\}, (i,j) \in \mathcal{E}$. Therefore, $||q_{ij}(t)|| < r_{cij}$ for all $t \geq 0$ is sufficient to ensure connectivity maintenance. Notice that, $r_{ci} > r_{si}, \forall i \in \mathcal{V}$ always holds. Moreover, collision avoidance and connectivity maintenance conditions together indicate that $r_{cij} > r_{sij}, (i,j) \in \mathcal{E}$.

C. Control Objective

Let the desired formation be defined by a infinitesimally rigid framework $\mathcal{F}^* = (\mathcal{G}^*, q^*)$ where $\mathcal{G}^* = (\mathcal{V}^*, \mathcal{E}^*)$, $\dim(\mathcal{V}^*) = n$, $\dim(\mathcal{E}^*) = l$, and $q^* = \mathrm{col}(q_i^*) \in \mathbb{R}^{2n}$. Moreover, assume that the actual framework (actual formation) of the agents, which shares the same graph with $\mathcal{F}^*$, is represented by $\mathcal{F}(t) = (\mathcal{G}^*, q(t))$, where $q(t) = \mathrm{col}(q_i(t)) \in \mathbb{R}^{2n}$. Let $\mathrm{TRS}(\mathcal{F}^*)$ denote the set of all frameworks (up to a translation, a rotation, and a scaling along the x-y directions of the global coordinate frame) of the desired formation $\mathcal{F}^*$.

Assumption 1: Given the upper bound r_{cij} and lower bound r_{sij} of the distance between connected agents, for

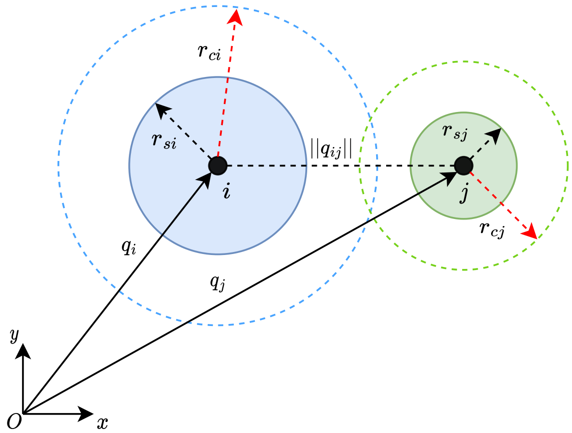

Fig. 2: The safety and sensing ranges of two neighboring agents.

desired formation $\mathcal{F}^* = (\mathcal{G}^*, q^*)$, there exists a positive constant α that satisfies

$$r_{sij} \leq \alpha ||q_{ij}^*|| \leq r_{cij} \quad (i,j) \in \mathcal{E}^*$$

Assumption 2: For a graph with n agents, the required number of connected edges needs to satisfy that $l \geq \max\{\frac{1}{4}[(n-2)^2+2n], 2n-3\}$.

Remark 1: Note that our requirements for the graph only need to satisfy Assumption 2. Many distance-based formation control methods require that the number of edges must be $l = 2n-3$, so that Lemma 3 can be used [13] [14]. However, our method releases this condition, which makes more options for connecting edges.

Assumption 3: The initial and desired formation frameworks are infinitesimally rigid.

Under Assumptions 1-3, the objective is to design a decentralized control law u_i such that

$$\mathcal{F}(t) \to \mathrm{TRS}(\mathcal{F}^*) \quad as \quad t \to \infty \tag{3}$$

while ensuring the connectivity maintenance and collision avoidance among each pair of neighboring agents, that is

$$r_{sij} < ||q_{ij}(t)|| < r_{cij}, \quad (i,j) \in \mathcal{E}^*, \forall t \geq 0. \tag{4}$$

Remark 2: So far, the proposed method only ensures collision avoidance and connection maintenance among agents with communication connections. Extension to collision avoidance between arbitrary two agents is currently under investigation, possibly via a switching communication topology.

Remark 3: In many distance-based formation control scheme, the formation shape is not unique, that is, although the formation results satisfy the distance conditions, the shape is completely different from the desired one. As shown in Fig. 1, graph (a) is the desired formation shape, while in graph (b), agent 1 is located inside the triangle formed by the other three agents where the formation also satisfies the distance requirement, but the final shape is completely inconsistent. The proposed control law can perfectly guarantee the uniqueness of the shape.

IV. Controller Design

In this section, we propose a novel control law consisting of a convergence term and a robust term. The former ensures the convergence of the formation and the uniqueness of the shape, while the latter ensures the collision avoidance and connectivity maintenance of the connected agents through a switching term based on hysteresis switching.

A. Design of Gain Matrices

Let us consider the control gain matrices A_{ij} that commute with rotation matrices R_{θ_i} are of the form:

$$A_{ij} = \begin{bmatrix} a_{ij} & -b_{ij} \\ b_{ij} & a_{ij} \end{bmatrix}, \quad a_{ij}, b_{ij} \in \mathbb{R}$$

From the commutativity property of A_{ij} matrices (which holds due to their special structure) it follows that the closed-loop dynamics with coordinates q_i and q_j expressed in agents' local coordinate frames are identical to the case that coordinates are expressed in a common global frame.

Without considering collision avoidance and communication maintenance, the control law is as follows:

$$u_i = -\sum_{j \in \mathcal{N}_i} A_{ij} q_{ij} \tag{5}$$

Then the closed-loop dynamics of the agents can be collectively expressed as:

$$\dot{q} = Aq \tag{6}$$

where $q := [q_1^\top, q_2^\top, \ldots, q_n^\top]^\top \in \mathbb{R}^{2n}$ denotes the aggregate position vector, and $A \in \mathbb{R}^{2n \times 2n}$ is the aggregate gain matrix given by:

$$A = \begin{bmatrix} -\sum_{j\neq 1} A_{1j} & A_{12} & \cdots & A_{1n} \\ A_{21} & -\sum_{j\neq 2} A_{2j} & \cdots & A_{2n} \\ \vdots & & \ddots & \vdots \\ A_{n1} & A_{n2} & \cdots & -\sum_{j\neq n} A_{nj} \end{bmatrix}$$

in which for $j \notin \mathcal{N}_i$(i.e., when agents are not neighbors) the A_{ij} blocks are defined as zeros. Note that the 2×2 diagonal blocks of A are the negative sum of the rest of the blocks on the same row. Thus, A has a block Laplacian structure.

Property 1: Define vectors

$$\mathbf{1} := [1, ..., 1]^\top \in \mathbb{R}^{2n}, \bar{\mathbf{1}} := [-1, 1, ..., -1, 1]^\top \in \mathbb{R}^{2n}.$$

From the block Laplacian structure of A together with control gain structure, it follows that $A\mathbf{1} = \mathbf{0}, A\bar{\mathbf{1}} = \mathbf{0}$.

Consider an arbitrary embedding of agents at their desired formation in the global coordinate frame. In the previous section, q^* is defined as the desired position. Then define the $90°$ rotated coordinates $\bar{q}_i^* := R_{\frac{\pi}{2}} q_i^*$ and the aggregate coordinate vectors $\bar{q}^* := [\bar{q}_1^{*\top}, \bar{q}_2^{*\top}, ..., \bar{q}_n^{*\top}]^\top \in \mathbb{R}^{2n}$. q^* is an equilibrium of (6), the block Laplacian structure of A implies $Aq^* = \mathbf{0}, A\bar{q}^* = \mathbf{0}$.

Property 1 implies that A has four zero eigenvalues with corresponding eigenvectors $\mathbf{1}$, $\bar{\mathbf{1}}$, q^*, $\bar{q}^*$.

Property 2: Assume that A_{ij} in the design satisfies the following conditions:

(i). vectors $\mathbf{1}$, $\bar{\mathbf{1}}$, q^*, $\bar{q}^*$ form a basis for $\ker(A)$.

(ii). all nonzero eigenvalues of A have negative real parts.

Next, we will show how to design the gain matrices to meet the conditions in Property 2. The proposed approach is based on a semidefinite programming (SDP) formulation. Let $N := [\mathbf{1}, \bar{\mathbf{1}}, q^*, \bar{q}^*] \in \mathbb{R}^{2n\times 4}$ denote the desired set of bases for $\ker(A)$. Let $USV^\top = N$ be the (full) singular value decomposition (SVD) of N, and further let $Q \in \mathbb{R}^{2n\times(2n-4)}$ be the last $2n-4$ columns of U, then matrix A and

$$\bar{A} := Q^\top A Q \in \mathbb{R}^{(2n-4)\times(2n-4)}$$

have the same set of nonzero eigenvalues. Consequently, matrix A can be computed by solving the convex optimization problem

$$\begin{aligned} A = &\underset{a_{ij}, b_{ij}}{\operatorname{argmin}} \quad \lambda_{\max}(\bar{A}) \\ &\text{subject} \quad \text{to} \quad AN = \mathbf{0} \end{aligned} \tag{7}$$

where $\lambda_{\max}(\cdot)$ denotes the largest eigenvalue of a matrix. By minimizing the largest eigenvalues of $\bar{A}$, we aim to make $\bar{A}$ as negative definite as possible.

Lemma 4: Consider agents with dynamics (2) and the controller (5) under Assumption 2. Then agents globally converge to the desired formation up to a translation, a rotation, and a scaling along the x-y directions of the global coordinate frame, i.e. all trajectories of (6) converge to the invariant set $q \in \ker(A)$.

The proof of Lemma 4 can be found in APPENDIX.

Remark 4: As indicated in [33], the convex optimization problem (7) has a solution, i.e. matrix A exists, if Assumption 2 holds. According to the property of matrix A in Property 2 and the Lemma 4 we know that the positions of the agents will converge to the invariant set $q \in \ker(A)$. Thus, the objective (3) can be achieved while not forming any inconsistent shapes.

B. Transformed System

Define the squared distance $\eta_{ij} = ||q_{ij}||^2$, $(i,j) \in \mathcal{E}^*$, squared minimum and maximum distances $\underline{r}_{ij} = r_{sij}^2$, $\overline{r}_{ij} = r_{cij}^2$. Then the original objective (3) (4) with bounded distances becomes:

$$\mathcal{F}(t) \to \text{TRS}(\mathcal{F}^*) \quad as \quad t \to \infty \tag{8}$$

$$\underline{r}_{ij} < \eta_{ij}(t) < \overline{r}_{ij}, \quad (i,j) \in \mathcal{E}^*, \forall t \geq 0. \tag{9}$$

The time derivative of η_{ij} is given by:

$$\dot{\eta}_{ij} = 2q_{ij}^\top (u_i - u_j)$$

Utilizing the rigidity matrix defined in (1), it can be written in a compact form as:

$$\dot{\eta} = 2R(q)u \tag{10}$$

where $u = \text{col}(u_i) \in \mathbb{R}^{2n}$, $\eta = \text{col}(\eta_{ij}) \in \mathbb{R}^l$, $(i,j) \in \mathcal{E}^*$.

To transform the constrained system into an equivalent unconstrained one, we introduce the following transformation corresponding to each edge:

$$\sigma_{ij} = T_{ij}(\eta_{ij}) = \frac{1}{2}\ln\left(\frac{\overline{r}_{ij}\eta_{ij} - \overline{r}_{ij}\underline{r}_{ij}}{\overline{r}_{ij}\underline{r}_{ij} - \underline{r}_{ij}\eta_{ij}}\right), \quad (i,j) \in \mathcal{E}^* \tag{11}$$

where σ_{ij} is the transformed edge variable corresponding to η_{ij} and $T_{ij}(\cdot)$: $(\underline{r}_{ij}, \overline{r}_{ij}) \to (-\infty, +\infty)$ is a smooth, strictly increasing bijective mapping. Now, taking the time derivative of σ_{ij}, yields $\dot{\sigma}_{ij} = (\mathrm{d}T_{ij}/\mathrm{d}\eta_{ij})\dot{\eta}_{ij} = 0.5\xi_{ij}\dot{\eta}_{ij}$, where

$$\xi_{ij} = \frac{1}{\eta_{ij} - \underline{r}_{ij}} - \frac{1}{\eta_{ij} - \overline{r}_{ij}}, \quad (i,j) \in \mathcal{E}^* \tag{12}$$

Then $\dot{\sigma}_{ij}$ is given in compact form as:

$$\dot{\sigma} = \frac{1}{2}\xi\dot{\eta} \tag{13}$$

where $\sigma = \text{col}(\sigma_{ij}) \in \mathbb{R}^l$, $\xi = \text{diag}(\xi_{ij}) \in \mathbb{R}^{l\times l}$.

Because we adopt hysteresis switching control, as shown in the Fig. 3, we divide the control process into the following 6 cases by judging the different states of the variable η_{ij} and its derivative $\dot{\eta}_{ij}$:

$$\begin{array}{ll}
(C1) & \underline{r}_{ij} + c + \delta \leq \eta_{ij} \leq \overline{r}_{ij} - c - \delta \\
(C2) & \underline{r}_{ij} + c \leq \eta_{ij} \leq \underline{r}_{ij} + c + \delta \quad \& \quad \dot{\eta}_{ij} < 0 \\
(C3) & \overline{r}_{ij} - c - \delta \leq \eta_{ij} \leq \overline{r}_{ij} - c \quad \& \quad \dot{\eta}_{ij} \geq 0 \\
(C4) & \underline{r}_{ij} < \eta_{ij} \leq \underline{r}_{ij} + c \quad or \quad \overline{r}_{ij} - c \leq \eta_{ij} < \overline{r}_{ij} \\
(C5) & \underline{r}_{ij} + c \leq \eta_{ij} \leq \underline{r}_{ij} + c + \delta \quad \& \quad \dot{\eta}_{ij} \geq 0 \\
(C6) & \overline{r}_{ij} - c - \delta \leq \eta_{ij} \leq \overline{r}_{ij} - c \quad \& \quad \dot{\eta}_{ij} < 0
\end{array}$$

where c and δ are positive constants. Then, our proposed hysteresis switching control law is:

$$\begin{aligned} u_i &= -\sum_{j\in\mathcal{N}_i}[A_{ij}q_{ij} + f_{ij}(\eta_{ij})k_{ij}\xi_{ij}\sigma_{ij}I_2 q_{ij}] \\ f_{ij}(\eta_{ij}) &= \begin{cases} 0 & C1, C2, C3 \\ 1 & C4, C5, C6 \end{cases} \end{aligned} \tag{14}$$

where k_{ij} is positive constant, I_2 is a identity matrix.

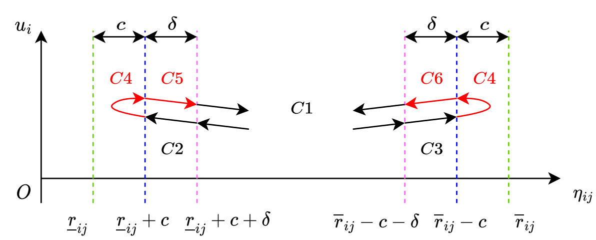

Fig. 3: Diagram of cases $C1$-$C6$ during the switching control process.

Remark 5: Hysteresis switching can avoid the infinite fast switch, that is, there is a certain buffer to avoid the instability caused by frequent switching.

Remark 6: It worth pointing out that the proposed method is decentralized instead of distributed in the sense that: the users need the initial conditions of all agents to verify the assumptions and calculate the A matrix at the beginning of the formation. However, during the formation process, each agent merely requires the information of adjacent agents to calculate its own control input.

V. Stability Analysis

Rearranging the dynamics (2) and switching control law (14), the closed-loop system is given as:

$$\dot{q} = u = -R^\top F \xi K \sigma + Aq \tag{15}$$

where R is the rigidity matrix, $F = \mathrm{diag}(f_{ij}(\eta_{ij})) \in \mathbb{R}^{l \times l}$, $K = \mathrm{diag}(k_{ij}) > 0$, $\sigma = \mathrm{col}(\sigma_{ij})$, and $\xi = \mathrm{diag}(\xi_{ij})$ with σ_{ij}, ξ_{ij} defined in (11) and (12), respectively.

For the subsequent stability analysis, we first split the whole graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ into m subgraphs $\mathcal{G}_i = (\mathcal{V}, \mathcal{E}_i)$ such that: 1) $|\mathcal{E}_i| = l' = 2n-3$; 2) $\cup \mathcal{E}_i = \mathcal{E}$; 3) $\mathcal{E}_i \neq \mathcal{E}_j$. In this way, (15) can be written as:

$$\dot{q} = u = -\sum_{i=1}^{m} R^\top N_i^\top F_i \xi_i K_i N_i \sigma + Aq \tag{16}$$

where $F_i, \xi_i, K_i \in \mathbb{R}^{l' \times l'}$, $N_i = \mathrm{col}[N_i^j] \in \mathbb{R}^{l' \times l}$,

$$N_i^j = \begin{cases} \mathbf{0}_{l' \times 1} & \text{others} \\ \bar{N}_i^j & j\text{-th edge} \in \mathcal{E}_i \end{cases}, \bar{N}_i^j = [\cdots, e_k, \cdots]^\top \in \mathbb{R}^{l'}$$

$$e_k = \begin{cases} 0 & k \neq j \\ \sqrt{1/n_j} & k = j \end{cases},$$

n_j represents the number of times that the j-th edge is included in all subgraphs.

Lemma 5: For the connected egde $(i,j) \in \mathcal{E}$, if it has an upper bound, such as $||q_i - q_j|| \leq \bar{r}$, then there exists a positive constant μ satisfies: $||q_j|| \leq \mu ||q_i||$.

Lemma 6: By transforming $\sigma_{ij} = T_{ij}(\eta_{ij})$, we can get $\eta_{ij} = F_{ij}(\sigma_{ij})$. Then, in case $C4, C5, C6$, there exists a positive constant L_{ij} satisfies: $\eta_{ij} = F_{ij}(\sigma_{ij}) \leq L_{ij}\sigma_{ij}^2$.

For a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, define the following sets:

$$\mathcal{I}_e = \{(i,j) \in \mathcal{E} | f_{ij}(\eta_{ij} = 1)\} \tag{17}$$

$$\mathcal{I}_v = \{i \in \mathcal{V} | (i,j) \in \mathcal{I}_e, \forall j \in \mathcal{N}_i\} \tag{18}$$

$$\mathcal{J}_e = \{(i,j) \in \mathcal{E} | i \in \mathcal{I}_v, \forall j \in \mathcal{N}_i\} \tag{19}$$

$$\mathcal{J}_v = \{i \in \mathcal{V} | (i,j) \in \mathcal{J}_e, \forall j \in \mathcal{N}_i\} \tag{20}$$

where $\mathcal{I}_e$ represents the set of constrained edges, $\mathcal{I}_v$ represents the set of points corresponding to the constrained edge, $\mathcal{J}_e$ represents the set of constrained edges and their adjacent edges, and $\mathcal{J}_v$ represents the set of points corresponding to the constrained edges and their adjacent points.

Lemma 7: There exists a block diagonal matrix $G = \mathrm{blockdiag}(G_i) \in \mathbb{R}^{2n \times 2n}$ such that $FRAq = FRAGq$, where

$$G_i = \begin{cases} G_A & i \in \mathcal{J}_v \\ G_B & i \in \mathcal{V} - \mathcal{J}_v \end{cases}, G_A = \begin{bmatrix} 1 & \\ & 1 \end{bmatrix}, G_B = \begin{bmatrix} 0 & \\ & 0 \end{bmatrix}$$

The proofs of Lemma 5-7 can be found in APPENDIX.

Theorem 1: Consider n agents with dynamics (2) and the actual formation $\mathcal{F}(t) = (\mathcal{G}^*, q(t))$, suppose that Assumptions 1-3 hold. Then, under the hysteresis switching control law (14), agents converge to the desired formation $\mathrm{TRS}(\mathcal{F}^*)$ while ensuring connectivity maintenance and collision avoidance among neighboring agents.

Proof: Consider the Lyapunov function:

$$\begin{aligned} V(\sigma, q) &= \frac{1}{2}\sigma^\top K F \sigma + \rho q^\top P_1 q \\ &= \frac{1}{2}\sum_{i=1}^{m} \sigma^\top N_i^\top K_i F_i N_i \sigma + \rho q^\top P_1 q \end{aligned}$$

where ρ is a positive constant, $P_1 \succ 0$ and it satisfies that $Q_1 = -(P_1 A + A^\top P_1) \succeq 0$ which is shown in the proof of Lemma 4. Then, by taking the derivative of V along the solution of q and employing (10) and (13), we arrive at:

$$\dot{V} = \sum_{i=1}^{m} \sigma^\top N_i^\top K_i F_i \xi_i N_i R u + \rho \dot{q}^\top P_1 q + \rho q^\top P_1 \dot{q} \tag{21}$$

For the sake of notational simplicity, we define $\phi_i := \sigma^\top N_i^\top K_i F_i \xi_i N_i R$ and rewrite (21) as:

$$\dot{V} = \sum_{i=1}^{m} \phi_i u + \rho \dot{q}^\top P_1 q + \rho q^\top P_1 \dot{q}$$

In order to avoid confusion, we consider the previous part first and substitue (16) into $\dot{V}$, then we get:

$$\dot{V} = -\sum_{i=1}^{m} \phi_i \phi_i^\top - 2\sum_{i \neq j, i<j}^{i,j \leq m} \phi_i \phi_j^\top + \sum_{i=1}^{m} \phi_i A q + \rho \dot{q}^\top P_1 q + \rho q^\top P_1 \dot{q} \tag{22}$$

For the cross product $2\sum_{i \neq j, i<j}^{i,j \leq m} \phi_i \phi_j^\top = 2\sum_{i \neq j, i<j}^{i,j \leq m} \sigma^\top K F \xi N_i^\top N_i R R^\top N_j^\top N_j \xi F K \sigma$, where $N_i^\top N_i \succeq 0, RR^\top \succeq 0, N_j^\top N_j \succeq 0$, so it is semi-positive definite, that is, it can be discarded. And according to the Lemma 7, (22) is equivalent to the following form:

$$\dot{V} \leq -\sum_{i=1}^{m} \phi_i \phi_i^\top + \sum_{i=1}^{m} \phi_i A G_i q + \rho \dot{q}^\top P_1 q + \rho q^\top P_1 \dot{q}$$

Applying Young's inequality to $\sum_{i=1}^{m} \phi_i A G_i q$ we get: $\sum_{i=1}^{m} \phi_i A G_i q \leq 0.5\sum_{i=1}^{m} \phi_i \phi_i^\top + 0.5\sum_{i=1}^{m} q^\top G_i A^\top A G_i q$, then we have:

$$\dot{V} \leq -\frac{1}{2}\sum_{i=1}^{m} \phi_i \phi_i^\top + \frac{1}{2}\lambda_{\max}(A^\top A)\sum_{i=1}^{m} q^\top G_i q + \rho \dot{q}^\top P_1 q + \rho q^\top P_1 \dot{q}$$

Now, we expand the part $\rho \dot{q}^\top P_1 q + \rho q^\top P_1 \dot{q}$ and employ Lemma 4, then we get:

$$\begin{aligned} \dot{V} \leq & -\frac{1}{2}\sum_{i=1}^{m} \phi_i \phi_i^\top + \frac{1}{2}\lambda_{\max}(A^\top A)\sum_{i=1}^{m} q^\top G_i q \\ & - \rho q^\top Q_1 q - 2\rho \sum_{i=1}^{m} \phi_i P_1 q \end{aligned}$$

Let α be a positive constant and invoke Young's inequality on $-2\rho\sum_{i=1}^{m} \phi_i P_1 q$, we further obtain:

$$\begin{aligned} \dot{V} \leq & -[\frac{1}{2} - 2\alpha^2 \rho \lambda_{\max}(P_1^2)] \\ & \sum_{i=1}^{m} \lambda_{\min}(N_i R R^\top N_i^\top) \lambda_{\min}(\xi_i^2) \lambda_{\min}(K_i^2) \sigma^\top N_i^\top F_i N_i \sigma \\ & + \frac{1}{2}\lambda_{\max}(A^\top A)\sum_{i=1}^{m} q^\top G_i q - \rho q^\top Q_1 q + \frac{m}{2\alpha^2} q^\top q \end{aligned} \tag{23}$$

Let's define $\lambda_i = \lambda_{\min}(N_i R R^\top N_i^\top)\lambda_{\min}(\xi_i^2)\lambda_{\min}(K_i^2)$, $\lambda_A = \frac{1}{2}\lambda_{\max}(A^\top A)$, $\lambda_0 = \frac{1}{2} - 2\alpha^2\rho\lambda_{\max}(P_1^2)$ where the selection of α and ρ satisfies $\lambda_0 > 0$. Then (23) can be written as:

$$\dot{V} \leq -\lambda_0 \sum_{i=1}^{m} \lambda_i \sigma^\top N_i^\top F_i N_i \sigma + \lambda_A \sum_{i=1}^{m} q^\top G_i q - \rho q^\top Q_1 q + \frac{m}{2\alpha^2} q^\top q$$

According to the the properties of subgraph $\mathcal{G}_i$ mentioned above, if λ_i does not exist, then $\sum_{i=1}^{m} \sigma^\top N_i^\top F_i N_i \sigma$ is equal to $||\sigma_s||^2$, where $\sigma_s = [..., \sigma_{ij}, ...]^\top, (i,j) \in \mathcal{I}_e$. Then define $\lambda_{\min}(\cdot) := \min\{\lambda_i\}$, we have:

$$\dot{V} \leq -\lambda_0\lambda_{\min}(\cdot)||\sigma_s||^2 + \lambda_A \sum_{i=1}^{m} q^\top G_i q - \rho q^\top Q_1 q + \frac{m}{2\alpha^2} q^\top q$$

For $\sum_{i=1}^{m} q^\top G_i q$, from the definition of G_i, there exists a constant $\beta < m$, such that:

$$\dot{V} \leq -\lambda_0\lambda_{\min}(\cdot)||\sigma_s||^2 + \lambda_A\beta(||q_s||^2 + ||q_{s'}||^2) - \rho q^\top Q_1 q + \frac{m}{2\alpha^2} q^\top q$$

where $q_s = [..., q_i, ...]^\top, i \in \mathcal{I}_e$, $q_{s'} = [..., q_j, ...]^\top, j \in \mathcal{J}_e - \mathcal{I}_e$. According to the Lemma 5, we have $||q_{s'}||^2 \leq \mu||q_s||^2$. By introducing the matrix $\bar{H} = H \otimes I_2 \in \mathbb{R}^{2l\times 2n}$, where H is the incidence matrix, the relative position vectors corresponding to the edges can be constructed as $d_s = \bar{H}q_s$, where $d_s = \mathrm{col}(\tilde{p}_{ij})$ with $\tilde{p}_{ij} = p_i - p_j, i,j \in \mathcal{I}_e$. So there exists a pseudo inverse matrix S_s satisfying $q_s = S_s d_s$. Further, we can get $||q_s||^2 \leq ||S_s||^2||d_s||^2$. Then replace $||d_s||^2$ with $\sum_{k=1}\eta_k = \sum_{k=1}F_k(\sigma_k)$ and apply Lemma 6 to obtain: $||q_s||^2 \leq ||S_s||^2\sum_{k=1}F_k(\sigma_k) \leq ||S_s||^2\sum_{k=1}L_k\sigma_k^2 \leq ||S_s||^2\sum_{k=1}L_k||\sigma_s||^2$. By sorting out the above results, we get:

$$\dot{V} \leq -\lambda_0\lambda_{\min}(\cdot)||\sigma_s||^2 + \lambda_A\beta(1+\mu)||S_s||^2(\sum_{k=1} L_k)||\sigma_s||^2 - \rho q^\top Q_1 q + \frac{m}{2\alpha^2} q^\top q$$

We know that $\lambda_A\beta(1+\mu)||S_s||^2(\sum_{k=1}L_k)||$ is bounded and $\lambda_0 > 0$, so there exist large enough k_{ij} to ensure $\lambda \triangleq \lambda_0\lambda_{\min}(\cdot) - \lambda_A\beta(1+\mu)||S_s||^2(\sum_{k=1}L_k)|| > 0$, i.e.

$$\dot{V} \leq -\lambda||\sigma_s||^2 - \rho q^\top Q_1 q + \frac{m}{2\alpha^2} q^\top q \tag{24}$$

According to the Lemma 4, suppose $q = [q_0^\top, q_d^\top]^\top$, $q_0 \in \mathbb{R}^4$, $q_d \in \mathbb{R}^{2n-4}$, $z = [z_0^\top, z_d^\top]^\top$, $z_0 \in \mathbb{R}^4$, $z_d \in \mathbb{R}^{2n-4}$ and the relationship between q and z is:

$$\begin{bmatrix} q_0 \\ q_d \end{bmatrix} = \begin{bmatrix} T_1 & T_2 \\ T_3 & T_4 \end{bmatrix}\begin{bmatrix} z_0 \\ z_d \end{bmatrix}, \quad \begin{bmatrix} z_0 \\ z_d \end{bmatrix} = \begin{bmatrix} M_1 & M_2 \\ M_3 & M_4 \end{bmatrix}\begin{bmatrix} q_0 \\ q_d \end{bmatrix}$$

Then it's easy to get $z_0 = (I - M_1T_1 - M_2T_3)^{-1}(M_1T_2 + M_2T_4)z_d$, and the inequality relationship between $||q||^2$ and $||z_d||^2$ is:

$$\begin{aligned} ||q||^2 &= ||q_0||^2 + ||q_d||^2 \\ &= ||T_1z_0 + T_2z_d||^2 + ||T_3z_0 + T_4z_d||^2 \leq \gamma(\cdot)||z_d||^2 \end{aligned}$$

where $\gamma(\cdot) \triangleq \big[(||T_1||^2 + ||T_3||^2)||(I - M_1T_1 - M_2T_3)^{-1}||^2||M_1T_2 + M_2T_4||^2 + (||T_2||^2 + ||T_4||^2)\big]$. Apply the above results and Lemma 4, (24) can be written as:

$$\dot{V} \leq -\lambda||\sigma_s||^2 - \rho||z_d||^2 + \frac{m}{2\alpha^2}\gamma(\cdot)||z_d||^2$$

By selecting ρ and α, it can be guaranteed that $\theta \triangleq \rho - \frac{m}{2\alpha^2}\gamma(\cdot) > 0$, finally we obtain:

$$\dot{V} \leq -\lambda||\sigma_s||^2 - \theta||z_d||^2 < 0$$

■

All constrained edges will converge to a certain boundary range over time, i.e $||\sigma_s||^2 \to 0$. At this time, the formation shape convergence is achieved by the control law related to the gain matrix A, that is all trajectories converge to the invariant set $q \in \ker(A)$. Therefore, agents converge to the desired formation $\mathrm{TRS}(\mathcal{F}^*)$ while ensuring connectivity maintenance and collision avoidance among neighboring agents.

Remark 7: Note that, in this work, we do not assume that the communication graph is minimally rigid, hence Lemma 3 cannot be directly applied. To solve this issue, we treat the whole graph as a union of several subgraphs where each subgraph satisfies the condition of minimal rigidity. In this way, without any extra operation, it can be proved that the positiveness property of the subgraphs ($R(p)R(p)^\top \succ 0$) is preserved, which enables us to extract $\lambda_{\min}(N_iRR^\top N_i^\top)$ from equation (23) and continue the subsequent analysis. Therefore, our approach relaxes the assumptions of communication graph presented in a state-of-art distance-based method [13].

VI. Simulation Results

In this section, the effectiveness of the proposed algorithm is verified in two numerical examples.

For comparison, we use the classic distance-based gradient control law:

$$u_i = -\sum_{j\in\mathcal{N}_i} k_{ij}(||q_{ij}||^2 - \bar{d}_{ij}^*)q_{ij} = -\sum_{j\in\mathcal{N}_i} k_{ij}e_{ij}q_{ij} \tag{25}$$

where $k_{ij} > 0$ is a feedback gain, $||q_{ij}||^2$ denotes the square of the actual distance, $\bar{d}_{ij}^* := ||q_i^* - q_j^*||^2$ represents the square of the desired distance.

Case (I) Consider a five-agent formation, each agent is modeled by (2) in a two dimensional space. The desired formation is set to be a pentagon defined by a infinitesimally rigid graph with edge set $\mathcal{E}^* = \{(1,2),(1,3),(1,4),(1,5),(2,3),(3,4),(4,5)\}$. Then we design the desired position of agents satisfying the pentagon shape as follows $q_1^* = (0, 0.5\tan(2\pi/5))$, $q_2^* = (-0.5 - \cos(2\pi/5), \sin(2\pi/5))$, $q_3^* = (-0.5, 0)$, $q_4^* = (0.5, 0)$, $q_5^* = (0.5 + \cos(2\pi/5), \sin(2\pi/5))$. Therefore, the desired distances between neighboring agents are $d_{12}^* = d_{23}^* = d_{34}^* = d_{45}^* = d_{15}^* = 1$, $d_{13}^* = d_{14}^* = \sqrt{2(1-\cos(3\pi/5))}$. The initial positions of the agents are given by $q_1(0) = (0,0)$, $q_2(0) = (-3,-1)$, $q_3(0) = (0,-3)$, $q_4(0) = (3,0)$, $q_5(0) = (2,3)$ indicating that $\mathcal{F}(0)$ is infinitesimally rigid.

For simplicity, in this case all agents are assumed to have the same safety and sensing radius, i.e., $r_{si} = 0.2$ and $r_{ci} = 5$ for $i = 1, ..., 5$.

For the tuning parameters of our control law, we select $k_{ij} = 1$, $c = 0.1$, $\delta = 0.05$. As for the gradient-based control law, we set the control gain $k_{ij} = 1$. Fig. 4 shows the trajectory diagram and formation shape of agents and we see that both two control laws can finally form a pentagon shape and the results obtained by our method are not fixed in shape scale which is different from the size of the given desired formation.

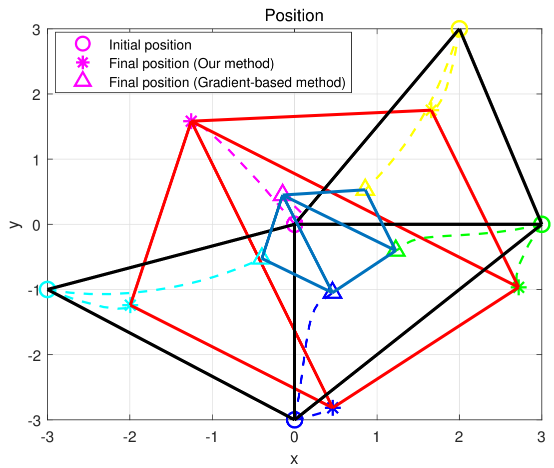

Fig. 4: Case (I): Trajectory diagram and formation shape of agents, the black one is the initial formation shape, the blue one uses the gradient-baesd control law (25) and the red one uses our control law (14).

Case (II) In this case, we keep all settings are the same as ***Case (I)*** except the initial positions of the agents to be $q_1(0) = (0, 0)$, $q_2(0) = (3, -3)$, $q_3(0) = (0, -3)$, $q_4(0) = (3, 0)$, $q_5(0) = (-1, -1)$. The simulation results are shown in the Fig. 6, which is completely different from ***Case (I)***.

Obviously, our control law can still form the desired pentagon formation shape and keep the distances between connected agents within the upper and lower bounds. For gradient-based control law, agents eventually tend to be stable which is shown in the Fig. 5, but they do not form the desired pentagon shape. What's more, according to Fig. 7, in the time of 0.5s-1s, the distance between agent 1 and agent 5 is lower than the safe distance r_{sij}, which may lead to mutual collision between these two agents. The above results show that our control method has greater effectiveness for achieving shape uniqueness, collision avoidance and connectivity maintenance than the gradient-based control method.

Remark 8: The above two numerical examples illustrate that, the classic distance-based method may form an undesired formation shape and/or bring collision with merely a change of the initial position of the agents, while these will not happen with our method. However, one limitation we have to admit is that the proposed method can only guarantee the MAS to form a shape up to a translation, a rotation, and a scaling along the x-y directions of the desired formation. In other words, the exact size of the final formation can not be determined by the designer. Nevertheless, since the CBF term in the control law can force the distance among agents satisfies any lower and upper boundaries, one straightforward

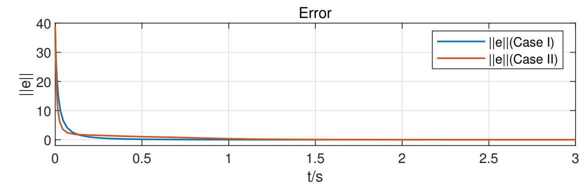

Fig. 5: Convergence diagram of distance error in two cases using gradient-based control law (25).

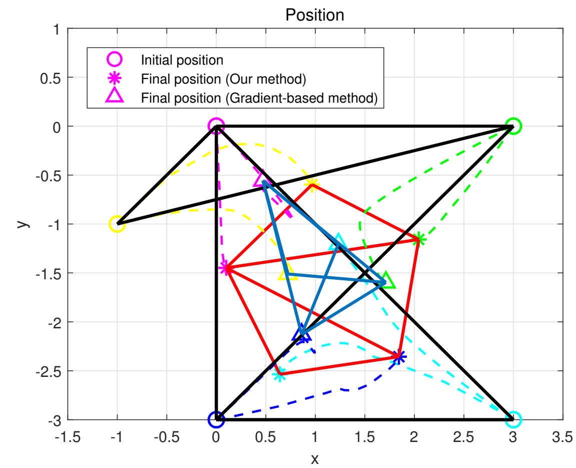

Fig. 6: Case (II): Trajectory diagram and formation shape of agents, the black one is the initial formation shape, the blue one uses the gradient-baesd control law (25) and the red one uses our control law (14).

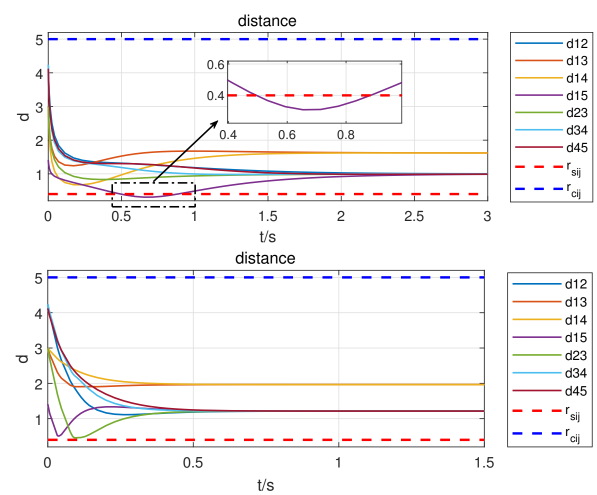

Fig. 7: Case (II): Distances change graph between connected agents, the first one uses the gradient-baesd control law (25) and the second one uses our control law (14).

idea to solve this size issue is to modify the boundary constraints to be time-varying functions to drive the formation shrink or grow to a satisfactory size without altering the shape. This is currently under investigation.

VII. Conclusion

In this paper, a novel formation control algorithm is proposed for a group of agents described by single integrator dynamic equations. The features that distinguish our method from the existing ones include: 1) By employing a pre-calculated gain matrix obtained from the solution of a SDP problem, the uniqueness of the shape of formation can be ensured via a decentralized control law. 2) The collision and connectivity lose among the neighboring agents will never occur due to the use of a hysteresis switching-based barrier function. 3) Some critical but realistic assumptions regarding to the communication graph (minimal rigidity) are relaxed without affecting the convergence analysis. The simulation results show the effectiveness and superiority of our approach compared with the classic distance-based formation control method. Future research efforts will focus on the control of the formation size and extension to 3D case.

Appendix

Due to space limitation, the proofs of the lemmas in this paper are not presented here. The interested reader can refer to the supplementary materials via the link https://www.overleaf.com/read/rvhtchcxgyxw.

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