Math Clinic Solutions

Session 2: Quadratics

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Instructions.
Solve each quadratic equation below. (When necessary, round numerical answers to *two* decimal places.)

(a) $x^2 - 100 = 0$

(b) $2x^2 - 8 = 0$

(c) $x^2 - 3 = 0$

(d) $x^2 - 5.72 = 0$

(e) $x^2 + 1 = 0$

(f) $3x^2 + 6.21 = 0$

(g) $x^2 = 0$

(a) $x^2 - 100 = 0 \Longrightarrow x^2 = 100 \Longrightarrow x = \pm\sqrt{100} = \pm 10.$

(b) $2x^2 - 8 = 0 \Longrightarrow x^2 = 4 \Longrightarrow x = \pm\sqrt{4} = \pm 2.$

(c) $x^2 - 3 = 0 \Longrightarrow x^2 = 3 \Longrightarrow x = \pm\sqrt{3} \approx \boxed{\pm 1.73}.$

(d) $x^2 - 5.72 = 0 \Longrightarrow x^2 = 5.72 \Longrightarrow x = \pm\sqrt{5.72} \approx \boxed{\pm 2.39}.$

(e) $x^2 + 1 = 0 \Longrightarrow x^2 = -1 \Longrightarrow x = \pm i.$

(f) $3x^2 + 6.21 = 0 \Longrightarrow x^2 = -\dfrac{6.21}{3} = -2.07 \Longrightarrow x = \pm i\sqrt{2.07} \approx \boxed{\pm 1.44\, i}.$

(g) $x^2 = 0 \Longrightarrow x = 0$ (a double root).

Solving Quadratic Equations

Instructions. Solve each quadratic equation shown below. Where appropriate, give exact answers; if no real roots exist, state this explicitly.

(a) $2x^2 - 19x - 10 = 0$

(b) $4x^2 + 12x + 9 = 0$

(c) $x^2 + x + 1 = 0$

(d) $x^2 - 3x + 10 = 2x + 4$

(a)

The quadratic
$$2x^2 - 19x - 10 = 0$$
can be factored by splitting the middle term. We seek two numbers whose *product* is $2(-10) = -20$ and whose *sum* is -19. These numbers are -20 and 1, so

$$\begin{aligned} 2x^2 - 19x - 10 &= 2x^2 - 20x + x - 10 \\ &= 2x(x-10) + 1(x-10) \\ &= (2x+1)(x-10). \end{aligned}$$

Setting each factor to zero gives

$$2x + 1 = 0 \implies x = -\frac{1}{2}, \qquad x - 10 = 0 \implies x = 10.$$

Hence
$$\boxed{\{\,10,\ -\tfrac{1}{2}\,\}}.$$

(b)

Observe that
$$4x^2 + 12x + 9 = (2x+3)^2.$$

Setting the perfect square to zero,

$$(2x+3)^2 = 0 \implies 2x + 3 = 0 \implies x = -\frac{3}{2}.$$

The root occurs twice (a repeated or “double” root):

$$\boxed{\{-\tfrac{3}{2}\}}.$$

(c)

For
$$x^2 + x + 1 = 0$$
the discriminant is
$$\Delta = b^2 - 4ac = 1^2 - 4(1)(1) = -3 < 0,$$
so **no real roots** exist. Over the complex numbers,

$$x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-1 \pm i\sqrt{3}}{2}.$$

Hence

$$\boxed{\left\{ \frac{-1+i\sqrt{3}}{2},\ \frac{-1-i\sqrt{3}}{2} \right\}}.$$

(d)

First bring all terms to one side:

$$\begin{aligned} x^2 - 3x + 10 &= 2x + 4 \\ x^2 - 3x + 10 - 2x - 4 &= 0 \\ x^2 - 5x + 6 &= 0. \end{aligned}$$

Factor the resulting quadratic:

$$x^2 - 5x + 6 = (x-2)(x-3).$$

Setting each factor to zero yields

$$x - 2 = 0 \implies x = 2, \qquad x - 3 = 0 \implies x = 3.$$

Therefore

$$\boxed{\{\,2,\ 3\,\}}.$$

Quadratic Equations (Factored Form)

Instructions. Write down the solution set for each quadratic equation below. *(No need to expand the brackets.)*

(a) $(x-4)(x+3)=0$

(b) $x(10-2x)=0$

(c) $(2x-6)^2=0$

(a)

$$(x-4)(x+3)=0 \quad\Longrightarrow\quad x-4=0 \text{ or } x+3=0 \quad\Longrightarrow\quad x=4 \text{ or } x=-3.$$

$$\boxed{\{4,\,-3\}}$$

(b)

$$x(10-2x)=0 \quad\Longrightarrow\quad x=0 \text{ or } 10-2x=0 \quad\Longrightarrow\quad x=0 \text{ or } x=5.$$

$$\boxed{\{0,\,5\}}$$

(c)

$$(2x-6)^2=0 \quad\Longrightarrow\quad 2x-6=0 \quad\Longrightarrow\quad x=3.$$

The root $x=3$ is a *double* (repeated) root.

$$\boxed{\{3\}}$$

Function Tables and Quadratic Sketches

Task. *For each quadratic function below,*

1. calculate the function values in the specified table;
2. sketch the graph using the completed points, indicating the vertex and axis of symmetry.

(a) $f(x) = 4x^2 - 12x + 5$

Computations.

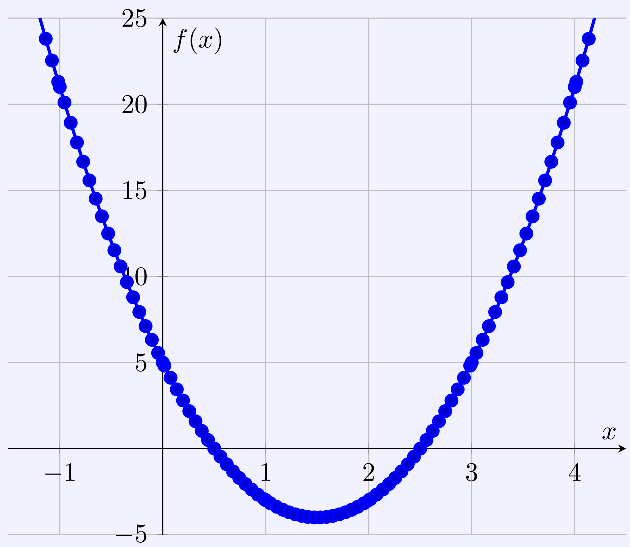
$$f(x) = 4x^2 - 12x + 5 \quad \Longrightarrow \quad \begin{array}{c|cccccc} x & -1 & 0 & 1 & 2 & 3 & 4 \\ \hline f(x) & 21 & 5 & -3 & -3 & 5 & 21 \end{array}$$

Explanation. Each entry is obtained by substituting the given x into $f(x)$. For instance, at $x = 1$,

$$f(1) = 4(1)^2 - 12(1) + 5 = 4 - 12 + 5 = -3.$$

Graph Features.

- The parabola opens **upwards** because the leading coefficient $4 > 0$.
- Vertex at $x = -\frac{b}{2a} = \frac{12}{8} = 1.5$, halfway between the equal y-values at $x = 1$ and $x = 2$.
- Axis of symmetry: $x = 1.5$.



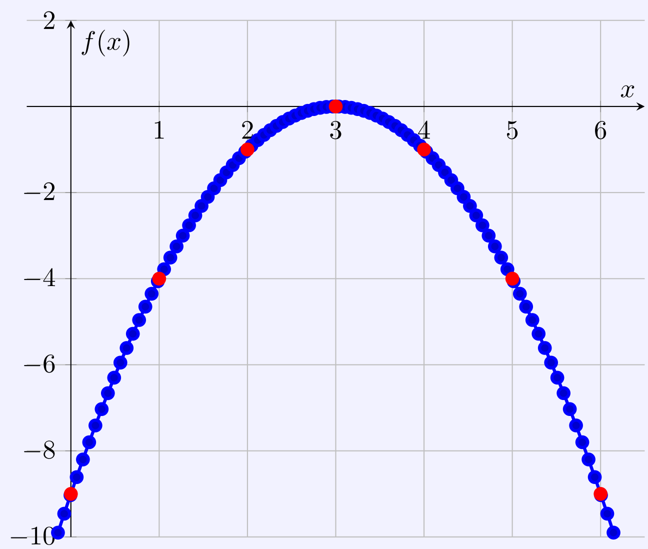
(b) $f(x) = -x^2 + 6x - 9$

Computations.

$$f(x) = -x^2 + 6x - 9 \quad \Longrightarrow \quad \begin{array}{c|ccccccc} x & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline f(x) & -9 & -4 & -1 & 0 & -1 & -4 & -9 \end{array}$$

Graph Features.

- Opens **downwards** because the leading coefficient $-1 < 0$.
- Vertex at $x = -\frac{b}{2a} = \frac{-6}{-2} = 3$ with maximum value $f(3) = 0$.
- Axis of symmetry: $x = 3$.



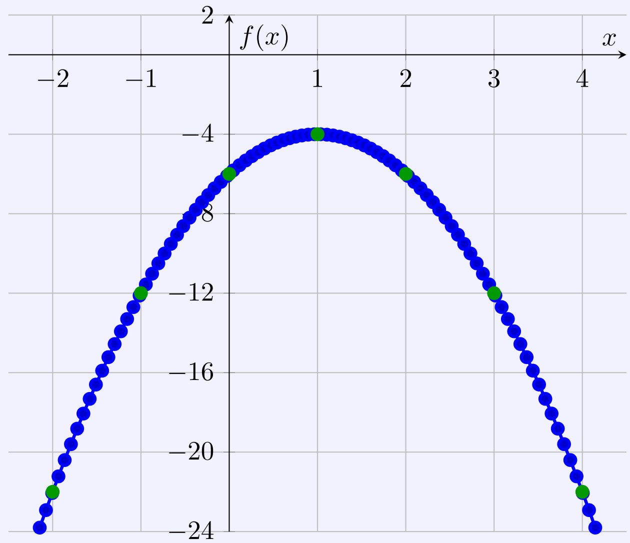
(c) $f(x) = -2x^2 + 4x - 6$

Computations.

$$f(x) = -2x^2 + 4x - 6 \quad \Longrightarrow \quad \begin{array}{c|ccccccc} x & -2 & -1 & 0 & 1 & 2 & 3 & 4 \\ \hline f(x) & -22 & -12 & -6 & -4 & -6 & -12 & -22 \end{array}$$

Graph Features.

- Opens **downwards** since $-2 < 0$.
- Vertex at $x = -\frac{b}{2a} = \frac{-4}{-4} = 1$, giving $f(1) = -4$ (the highest point).
- Axis of symmetry: $x = 1$.



Supply–Demand Equilibrium

Problem 6. Given the supply and demand functions

$$P = 2Q_S^2 + 10Q_S + 10,$$
$$P = -Q_D^2 - 5Q_D + 52,$$

calculate the equilibrium price and quantity.

Set quantity supplied equal to quantity demanded. At equilibrium the quantities are equal, so let $Q_S = Q_D = Q$.

Equate the two price expressions.

$$2Q^2 + 10Q + 10 \;=\; -Q^2 - 5Q + 52.$$

Gather all terms on one side and simplify.

$$2Q^2 + 10Q + 10 + Q^2 + 5Q - 52 = 0$$
$$3Q^2 + 15Q - 42 = 0$$
$$\text{Divide by } 3: \quad Q^2 + 5Q - 14 = 0.$$

Solve the quadratic for Q.

$$Q^2 + 5Q - 14 = 0 \implies (Q+7)(Q-2) = 0 \implies Q = -7 \quad \text{or} \quad Q = 2.$$

Since a negative quantity is not economically relevant, we take

$$Q^* = 2.$$

Substitute Q^* to find the equilibrium price. Using the supply function (the demand function yields the same result):

$$P^* = 2(2)^2 + 10(2) + 10 = 2 \cdot 4 + 20 + 10 = 8 + 20 + 10 = 38.$$

$$\boxed{Q^* = 2 \quad \text{units}}, \qquad \boxed{P^* = 38}.$$

Thus, the market clears at a quantity of 2 units and a price of $38.

Solution
At equilibrium: $Q_S = Q_D = Q$, and supply equals demand, so set the two equations for P equal:

$$2Q^2 + 10Q + 10 = -Q^2 - 5Q + 52$$

Bring all terms to one side:

$$2Q^2 + 10Q + 10 + Q^2 + 5Q - 52 = 0$$
$$(2Q^2 + Q^2) + (10Q + 5Q) + (10 - 52) = 0$$
$$3Q^2 + 15Q - 42 = 0$$

Solve the quadratic equation:

$$Q^2 + 5Q - 14 = 0 \quad \text{(divide both sides by 3 for simplicity, but can also solve as is.)}$$

But let's stick to original quadratic:

$$3Q^2 + 15Q - 42 = 0$$

Use the quadratic formula:

$$Q = \frac{-15 \pm \sqrt{15^2 - 4 \cdot 3 \cdot (-42)}}{2 \cdot 3}$$
$$Q = \frac{-15 \pm \sqrt{225 + 504}}{6}$$
$$Q = \frac{-15 \pm \sqrt{729}}{6}$$
$$Q = \frac{-15 \pm 27}{6}$$

So,

$$Q_1 = \frac{-15 + 27}{6} = \frac{12}{6} = 2$$
$$Q_2 = \frac{-15 - 27}{6} = \frac{-42}{6} = -7$$

Quantity cannot be negative for this context, so $Q = 2$.
Substitute $Q = 2$ into either original equation (use supply for example):

$$P = 2(2)^2 + 10 \times 2 + 10 = 2 \times 4 + 20 + 10 = 8 + 20 + 10 = 38$$

Equilibrium quantity: $\boxed{2}$
Equilibrium price: $\boxed{38}$