Percentages

Objectives

At the end of this section you should be able to:

- Understand what a percentage is.
- Solve problems involving a percentage increase or decrease.
- Write down scale factors associated with percentage changes.
- Work out overall percentage changes.
- Calculate and interpret index numbers.(For later)
- Adjust value data for inflation.(For later)

Advice

This this section provides a leisurely revision of the idea of a percentage as well as reminding you about how to use scale factors to cope with percentage changes. These ideas are crucial to any understanding of financial mathematics.

In order to be able to handle financial calculations, it is necessary to use percentages proficiently. The word ‘percentage’ literally means ‘per cent’, i.e. per hundredth, so that whenever we speak of r% of something, we simply mean the fraction $(r/100)$ths of it.

For example,

25% is the same as $\frac{25}{100} = \frac{1}{4}$

30% is the same as $\frac{30}{100} = \frac{3}{10}$

50% is the same as $\frac{50}{100} = \frac{1}{2}$

Example

Calculate

(a) 15% of 12 **(b)** 98% of 17 **(c)** 150% of 290

Solution

(a) 15% of 12 is the same as

$$\frac{15}{100} \times 12 = 0.15 \times 12 = 1.8$$

(b) 98% of 17 is the same as

$$\frac{98}{100} \times 17 = 0.98 \times 17 = 16.66$$

(c) 150% of 290 is the same as

$$\frac{150}{100} \times 290 = 1.5 \times 290 = 435$$

Practice Problem

1 Calculate

(a) 10% of \$2.90 **(b)** 75% of \$1250 **(c)** 24% of \$580

Whenever any numerical quantity increases or decreases, it is customary to refer to this change in percentage terms. The following example serves to remind you how to perform calculations involving percentage changes.

Example

(a) An investment rises from \$2500 to \$3375. Express the increase as a percentage of the original.

(b) At the beginning of a year, the population of a small village is 8400. If the annual rise in population is 12%, find the population at the end of the year.

(c) In a sale, all prices are reduced by 20%. Find the sale price of a good originally costing \$580.

Solution

(a) The rise in the value of the investment is

$$3375 - 2500 = 875$$

As a fraction of the original this is

$$\frac{875}{2500} = 0.35$$

This is the same as 35 hundredths, so the percentage rise is 35%.

(b) As a fraction

$$12\% \text{ is the same as } \frac{12}{100} = 0.12$$

so the rise in population is

$$0.12 \times 8400 = 1008$$

Hence the final population is

$$8400 + 1008 = 9408$$

(c) As a fraction

$$20\% \text{ is the same as } \frac{20}{100} = 0.2$$

so the fall in price is

$$0.2 \times 580 = 116$$

Hence the final price is

$$580 - 116 = \$464$$

Practice Problem

2 (a) A firm's annual sales rise from 50 000 to 55 000 from one year to the next. Express the rise as a percentage of the original.

(b) The government imposes a 15% tax on the price of a good. How much does the consumer pay for a good priced by a firm at $1360?

(c) Investments fall during the course of a year by 7%. Find the value of an investment at the end of the year if it was worth $9500 at the beginning of the year.

In the previous example and in Practice Problem 2, the calculations were performed in two separate stages. The actual rise or fall was first worked out, and these changes were then applied to the original value to obtain the final answer. It is possible to obtain this answer in a single calculation, and we now describe how this can be done. Not only is this new approach quicker, but it also enables us to tackle more difficult problems. To be specific, let us suppose that the price of good is set to rise by 9%, and that its current price is $78. The new price consists of the original (which can be thought of as 100% of the \$78) plus the increase (which is 9% of \$78). The final price is therefore

$$100\% + 9\% = 109\% \text{ (of the \$78)}$$

which is the same as

$$\frac{109}{100} = 1.09$$

In other words, in order to calculate the final price all we have to do is to multiply by the ***scale factor***, 1.09. Hence the new price is

$$1.09 \times 78 = \$85.02$$

One advantage of this approach is that it is then just as easy to go backwards and work out the original price from the new price. To go backwards in time we simply *divide* by the scale factor. For example, if the final price of a good is \$1068.20 then before a 9% increase the price would have been

$$1068.20 \div 1.09 = \$980$$

In general, if the percentage rise is r% then the final value consists of the original (100%) together with the increase (r%), giving a total of

$$\frac{100}{100} + \frac{r}{100} = 1 + \frac{r}{100}$$

To go forwards in time we multiply by this scale factor, whereas to go backwards we divide.

Example

(a) If the annual rate of inflation is 4%, find the price of a good at the end of a year if its price at the beginning of the year is \$25.

(b) The cost of a good is \$799 including 17.5% VAT (value added tax). What is the cost excluding VAT?

(c) Express the rise from 950 to 1007 as a percentage.

Solution

(a) The scale factor is

$$1 + \frac{4}{100} = 1.04$$

We are trying to find the price *after* the increase, so we *multiply* to get

$$25 \times 1.04 = \$26$$

(b) The scale factor is

$$1 + \frac{17.5}{100} = 1.175$$

This time we are trying to find the price *before* the increase, so we *divide* by the scale factor to get

$$799 \div 1.175 = \$680$$

(c) The scale factor is

$$\frac{\text{new value}}{\text{old value}} = \frac{1007}{950} = 1.06$$

which can be thought of as

$$1 + \frac{6}{100}$$

so the rise is 6%.

Practice Problem

3 **(a)** The value of a good rises by 13% in a year. If it was worth \$6.5 million at the beginning of the year, find its value at the end of the year.

(b) The GNP of a country has increased by 63% over the past 5 years and is now \$124 billion. What was the GNP 5 years ago?

(c) Sales rise from 115 000 to 123 050 in a year. Find the annual percentage rise.

It is possible to use scale factors to solve problems involving percentage decreases. To be specific, suppose that an investment of \$76 falls by 20%. The new value is the original (100%) less the decrease (20%), so is 80% of the original. The scale factor is therefore 0.8, giving a new value of

$$0.8 \times 76 = \$60.80$$

In general, the scale factor for an r% decrease is

$$\frac{100}{100} - \frac{r}{100} = 1 - \frac{r}{100}$$

Once again, you multiply by this scale factor when going forwards in time and divide when going backwards.

Example

(a) The value of a car depreciates by 25% in a year. What will a car, currently priced at \$43 000, be worth in a year's time?

(b) After a 15% reduction in a sale, the price of a good is \$39.95. What was the price before the sale began?

(c) The number of passengers using a rail link fell from 190 205 to 174 989. Find the percentage decrease.

Solution

(a) The scale factor is

$$1 - \frac{25}{100} = 0.75$$

so the new price is

$$43\ 000 \times 0.75 = \$32\ 250$$

forwards in time so multiply

(b) The scale factor is

$$1 - \frac{15}{100} = 0.85$$

so the original price was

$$39.95 \div 0.85 = \$47$$

backwards in time so divide

(c) The scale factor is

$$\frac{\text{new value}}{\text{old value}} = \frac{174\ 989}{190\ 205} = 0.92$$

which can be thought of as

$$1 - \frac{8}{100}$$

so the fall is 8%.

not 92%!

Practice Problem

4 **(a)** Current monthly output from a factory is 25 000. In a recession, this is expected to fall by 65%. Estimate the new level of output.

(b) As a result of a modernization programme, a firm is able to reduce the size of its workforce by 24%. If it now employs 570 workers, how many people did it employ before restructuring?

(c) Shares originally worth \$10.50 fall in a stock market crash to \$2.10. Find the percentage decrease.

The final application of scale factors that we consider is to the calculation of overall percentage changes. It is often the case that over various periods of time the price of a good is subject to several individual percentage changes. It is useful to be able to replace these by an equivalent single percentage change spanning the entire period. This can be done by simply multiplying together successive scale factors.

Example

(a) Share prices rise by 32% during the first half of the year and rise by a further 10% during the second half. What is the overall percentage change?

(b) Find the overall percentage change in the price of a good if it rises by 5% in a year but is then reduced by 30% in a sale.

Solution

(a) To find the value of shares at the end of the first 6 months we would multiply by

$$1 + \frac{32}{100} = 1.32$$

and at the end of the year we would multiply again by the scale factor

$$1 + \frac{10}{100} = 1.1$$

The net effect is to multiply by their product

$$1.32 \times 1.1 = 1.452$$

which can be thought of as

$$1 + \frac{45.2}{100}$$

so the overall change is 45.2%.

Notice that this is not the same as

$$32\% + 10\% = 42\%$$

This is because during the second half of the year we not only get a 10% rise in the original value, but we also get a 10% rise on the gain accrued during the first 6 months.

(b) The individual scale factors are 1.05 and 0.7, so the overall scale factor is

$$1.05 \times 0.7 = 0.735$$

The fact that this is less than 1 indicates that the overall change is a decrease. Writing

$$0.735 = 1 - 0.265 = 1 - \frac{26.5}{100}$$

we see that this scale factor represents a 26.5% decrease.

Practice Problem

5 Find the single percentage increase or decrease equivalent to

(a) an increase of 30% followed by an increase of 40%

(b) a decrease of 30% followed by a decrease of 40%

(c) an increase of 10% followed by a decrease of 50%.

We conclude this section by describing two applications of percentages in macroeconomics:

- index numbers
- inflation.

We consider each of these in turn.