Math Clinic (Pre-MBA)

January 2026

Lecture Notes & Guided Practice

Instructor: Saleem Kamaludin(Assoc.)
ALJ-GSB

January 9, 2026

Preface

These notes support the Math Clinic for incoming MBA students. The goal is to refresh essential quantitative skills used in operations, economics, finance, and data-driven decision-making.

Each chapter follows a consistent pattern:

- short theory written for quick recall,
- worked examples with commentary,
- practice questions (with solutions when appropriate).

How to use these notes. Try the practice problems before reading the solutions. If you get stuck, identify exactly which line you cannot justify; then use the solution as a model for writing your own reasoning clearly.

Contents

Chapter 1

Critical Thinking

1-1 How does management accounting differ from financial accounting?

Answer. Management accounting produces information mainly for *internal* users (managers) to support planning, control, and decision-making. It is forward-looking, often detailed (by product, customer, process, or department), and can be tailored to the firm's needs without being constrained by external reporting rules. Financial accounting produces information mainly for *external* users (investors, lenders, regulators). It is largely historical, aggregated for the organization as a whole, and must follow prescribed standards (e.g., IFRS/GAAP) with emphasis on comparability and reliability.

1-2 "Management accounting should not fit the straitjacket of financial accounting." Explain and give an example.

Answer. The point is that internal decisions often require information that is *useful*, not merely information that is *acceptable for external reporting.* Financial accounting rules are designed for consistent reporting to outsiders, but those same rules can hide the economic trade-offs managers must evaluate (such as incremental costs, capacity constraints, and customer profitability). Management accounting should therefore be flexible: it can use estimates, nonfinancial measures, and decision-relevant costs even if they would not appear (or would appear differently) in published financial statements.

Example. For a special one-time order, a manager should compare the *incremental* (avoidable) costs of producing the extra units to the extra revenue. Financial statements include allocations of fixed overhead to products, but if that fixed overhead will not change with the order (and capacity exists), it is not decision-relevant. Rejecting a profitable special order just because the "full cost" (including allocated fixed overhead) exceeds the price would be an example of letting financial-accounting allocations create an unhelpful "straitjacket."

1-3 How can a management accountant help formulate a strategy?

Answer. A management accountant supports strategy by translating goals into measurable plans and by providing decision-focused analysis. For example, they can:

- analyze profitability by product, customer, channel, or region to identify where the firm truly creates value;
- estimate costs and benefits of strategic options (new product launches, pricing changes, outsourcing, automation, capacity expansion);
- build budgets and forecasts that link operational drivers (volume, mix, utilization, defect rates, delivery times) to financial outcomes;
- design performance measures (financial and nonfinancial) that align employee actions with strategic priorities, and monitor variance/trend reports to keep execution on track;
- assess risk and sensitivity ("what if demand drops 10%? what if input prices rise?") so leadership understands trade-offs before committing.

In short, they help ensure strategy is both economically sound and operationally executable.

Exercise 1.0.1

1. Management accounting deals only with costs. Do you agree? Explain.
2. How can management accountants help improve quality and achieve timely product deliveries?
3. Describe the five-step decision-making process.
4. Distinguish planning decisions from control decisions.

Chapter 2

Percentages and inequalities

2.1 Numeracy Toolkit for Accounting: Percentages, Rates, and Ratios

Accounting computations are often simple in structure but easy to mishandle under time pressure. In this clinic section we focus on the small set of numerical moves that appear repeatedly in cost per unit, variance calculations, budgets, and financial statement analysis.

2.1.1 Percentages and percentage change

Definition 2.1.1 **Percent as a multiplier**

A percentage $r\%$ corresponds to the multiplier $r/100$. Thus:

$$r\% \text{ of } X \;=\; \frac{r}{100}X.$$

An increase of $r\%$ means multiply by $1+\frac{r}{100}$, while a decrease of $r\%$ means multiply by $1-\frac{r}{100}$.

Proposition 2.1.1 **Percentage change formula**

If a quantity changes from A (old) to B (new), then the percentage change is

$$\% \text{ change} = \frac{B - A}{A} \times 100\%.$$

A positive result indicates an increase; a negative result indicates a decrease.

Example 2.1.1 **(Markup and markdown are different)**

A product has cost \$80 and is sold for \$100. Compute the markup on cost and the margin (markup) on selling price.

Solution.

The profit is $100 - 80 = \$20$.

Markup on cost compares profit to cost:

$$\frac{20}{80} \times 100\% = 25\%.$$

Margin on selling price compares profit to the selling price:

$$\frac{20}{100} \times 100\% = 20\%.$$

So the same transaction yields 25% markup on cost, but only 20% margin on selling price. This distinction matters in pricing and performance reporting.

Example 2.1.2 **(Reverse percentages: recovering the original amount)**

A budget line shows "Advertising expense after a 12% increase is \$56,000." Find the original amount.

Solution.

An increase of 12% means the new amount equals the old amount multiplied by 1.12. Let the original amount be x. Then

$$1.12x = 56{,}000.$$

Solving gives

$$x = \frac{56{,}000}{1.12} = 50{,}000.$$

So the original advertising expense was \$50,000.

2.1.2 Rates, "per unit" thinking, and ratios

Definition 2.1.2 **Rate and unit cost**

A *rate* is a quantity per unit of another quantity. In cost accounting, the most common rate is

$$\text{Unit cost} = \frac{\text{Total cost}}{\text{Number of units}}.$$

Conversely, Total cost = (Unit cost)(Number of units).

Proposition 2.1.2 **Weighted average as a cost rate**

If you combine quantities bought at different unit costs, the combined unit cost is a weighted average:

$$\bar{c} = \frac{c_1q_1 + c_2q_2 + \cdots + c_kq_k}{q_1 + q_2 + \cdots + q_k},$$

where q_i are quantities and c_i are unit costs.

Example 2.1.3 **(Weighted average unit cost)**

A firm purchases 200 units at \$18 each and later 300 units at \$21 each. Find the weighted-average unit cost of the 500 units.

Solution.

Compute total cost across both purchases:

$$\text{Total cost} = 200(18) + 300(21) = 3600 + 6300 = 9900.$$

Total units $= 200 + 300 = 500$. Hence the weighted-average unit cost is

$$\bar{c} = \frac{9900}{500} = 19.8.$$

So the weighted-average unit cost is \$19.80 per unit.

Example 2.1.4 **(A ratio used for interpretation)**

A company has current assets of \$420,000 and current liabilities of \$280,000. Compute the current ratio and interpret it.

Solution.

The current ratio is

$$\text{Current ratio} = \frac{\text{Current assets}}{\text{Current liabilities}} = \frac{420{,}000}{280{,}000} = 1.5.$$

Interpreting the number: the firm has \$1.50 in current assets for every \$1.00 of current liabilities. (As always, whether this is "good" depends on the industry and the quality/liquidity of the assets.)

Checkpoint skills (quick self-test)

You should be able to do the following accurately and quickly: (i) convert between percent, decimal, and multiplier; (ii) compute percent change and reverse a percent change; (iii) move between total cost, unit cost, and quantity; (iv) compute and interpret a simple ratio.

2.2 Linear Cost Models and CVP Algebra

In managerial and cost accounting, many planning questions reduce to a linear model: how total cost and profit change as activity (units produced/sold) changes. This section develops the basic algebra needed for break-even and target-profit calculations.

2.2.1 Cost, revenue, and profit as linear functions

Definition 2.2.1 **Linear cost model**

Let Q denote the activity level (typically units produced or units sold). A *linear cost model* assumes total cost has the form

$$C(Q) = F + vQ,$$

where $F \geq 0$ is the *fixed cost* (does not change with Q in the relevant range) and $v \geq 0$ is the *variable cost per unit.*

Definition 2.2.2 **Revenue and profit**

If the selling price is p dollars per unit, then revenue is

$$R(Q) = pQ.$$

Profit (operating income) is defined as

$$\Pi(Q) = R(Q) - C(Q) = pQ - (F + vQ) = (p - v)Q - F.$$

The quantity $p - v$ is called the *contribution margin per unit.*

Proposition 2.2.1 **Break-even units**

Assume $p > v$. The break-even activity level (in units) is the value of Q for which profit is zero:

$$\Pi(Q) = 0 \quad \iff \quad Q = \frac{F}{p - v}.$$

Proposition 2.2.2 **Target-profit units**

Assume $p > v$. To earn a target profit of T dollars (where $T \geq 0$), the required units satisfy

$$\Pi(Q) = T \quad \iff \quad Q = \frac{F+T}{p-v}.$$

2.2.2 Worked applications

Example 2.2.1 **(Break-even analysis)**

A training unit sells a short course for \$250 per participant. Variable cost is \$90 per participant and fixed costs for the month are \$24,000. Find the break-even number of participants.

Solution.

Here $p = 250$, $v = 90$, and $F = 24{,}000$. The contribution margin per participant is

$$p - v = 250 - 90 = 160.$$

By Proposition 2.2.1,

$$Q = \frac{F}{p-v} = \frac{24{,}000}{160} = 150.$$

So the unit breaks even at 150 participants (within the relevant range of the model).

Example 2.2.2 **(Target profit and the “round up” rule)**

A firm sells a product at $75 per unit. Variable cost is $43 per unit and fixed costs are $96,000. How many units must be sold to earn a target profit of $40,000?

Solution.

Here $p = 75$, $v = 43$, $F = 96{,}000$, and $T = 40{,}000$. The contribution margin per unit is

$$p - v = 75 - 43 = 32.$$

Using Proposition 2.2.2,

$$Q = \frac{F+T}{p-v} = \frac{96{,}000 + 40{,}000}{32} = \frac{136{,}000}{32} = 4250.$$

Thus 4,250 units are required.

Interpretation note: If the calculation produces a non-integer, one must round *up* to the next whole unit, because selling a fraction of a unit is not feasible and rounding down would miss the profit target.

Example 2.2.3 **(Profit at least a target: an inequality)**

A service charges $180 per job, has variable cost $60 per job, and fixed costs $18,000 per month. Find the minimum number of jobs needed to earn at least $12,000 profit.

Solution.

Let Q be the number of jobs. Profit is

$$\Pi(Q) = (p - v)Q - F = (180 - 60)Q - 18{,}000 = 120Q - 18{,}000.$$

We want $\Pi(Q) \geq 12{,}000$, so

$$120Q - 18{,}000 \geq 12{,}000 \iff 120Q \geq 30{,}000 \iff Q \geq 250.$$

Therefore the minimum is 250 jobs.

Checkpoint skills (quick self-test)

You should be able to: (i) write $C(Q) = F + vQ$, $R(Q) = pQ$, $\Pi(Q) = (p - v)Q - F$; (ii) solve $(p - v)Q = F$ and $(p - v)Q = F + T$; (iii) interpret $p - v$ as contribution margin and justify rounding up in unit problems.

2.3 Variance Arithmetic and Decomposition

Once students are comfortable with linear profit models, the next mathematical building block is *variance thinking*: measuring deviations from a plan (standard or budget) and separating the deviation into interpretable components (for example, a *price/rate* effect and a *quantity/efficiency* effect). This is the quantitative backbone of budgeting and performance evaluation.

2.3.1 Core definitions

Definition 2.3.1 **Absolute and percentage variance**

Given a benchmark (budget/standard) value B and an actual value A, the *absolute variance* is $A - B$, and the *percentage variance* (when $B \neq 0$) is

$$\frac{A-B}{B} \times 100\%.$$

Definition 2.3.2 **Favourable vs unfavourable: the sign rule**

A variance is called *favourable* if it improves profit relative to the benchmark, and *unfavourable* if it reduces profit relative to the benchmark.

- For *costs*: $A - B < 0$ is favourable (actual cost lower than budget), while $A - B > 0$ is unfavourable.
- For *revenues*: $A - B > 0$ is favourable (actual revenue higher than budget), while $A - B < 0$ is unfavourable.

2.3.2 The key algebra: splitting a product variance

Many accounting quantities are products of a *rate* and a *quantity* (e.g. dollars per unit × units). The crucial algebra is an identity that splits the deviation of one product from another into two effects.

Proposition 2.3.1 **Two-way decomposition of a product**

Let a, s be two rates (actual and standard) and let q, Q be two quantities (actual and standard). Then

$$aq - sQ \;=\; (a-s)q \;+\; s(q-Q).$$

The first term $(a - s)q$ isolates the *rate (price) effect* at actual quantity q, and the second term $s(q - Q)$ isolates the *quantity (usage/efficiency) effect* at the standard rate s.

2.3.3 Worked applications

Example 2.3.1 **(Direct materials variance: price and usage)**

A product has a standard material cost of \$8 per kg and a standard usage of 3 kg per unit. In a month, 1,000 units were produced. Actual material purchased/used was 3,200 kg at \$7.50 per kg.
Compute: (i) total material cost variance, (ii) price variance, (iii) usage variance.

Solution.
Step 1: Identify actual and standard totals.
Standard quantity allowed for output:

$$Q = 3 \times 1{,}000 = 3{,}000 \text{ kg}.$$

Standard cost $= sQ = 8(3{,}000) = \$24{,}000$.
Actual cost $= aq = 7.50(3{,}200) = \$24{,}000$.
Thus the *total* variance is

$$A - B = 24{,}000 - 24{,}000 = 0,$$

so there is no net material cost variance overall.
Step 2: Split the variance into price and usage components.
Using Proposition 2.3.1 with $a = 7.50$, $s = 8$, $q = 3{,}200$, $Q = 3{,}000$:

$$aq - sQ = (a - s)q + s(q - Q).$$

Price variance:

$$(a - s)q = (7.50 - 8.00)(3{,}200) = (-0.50)(3{,}200) = -\$1{,}600.$$

Since this is a *cost* variance and negative, it is *favourable*: materials were cheaper than standard by \$1,600.
Usage variance:

$$s(q - Q) = 8(3{,}200 - 3{,}000) = 8(200) = \$1{,}600.$$

This is positive for a *cost*, hence *unfavourable*: 200 extra kg were used relative to standard, costing \$1,600 at the standard rate.
Check: $-1{,}600 + 1{,}600 = 0$, matching the total variance.

Example 2.3.2 **(Labour variance: rate and efficiency)**

A job has a standard labour rate of \$30 per hour and a standard time of 2.5 hours per unit. In a week, 400 units were completed. Actual labour was 1,100 hours at \$32 per hour.
Compute the total labour cost variance and split it into a rate variance and an efficiency variance.

Solution.
Standard hours allowed:

$$Q = 2.5(400) = 1{,}000 \text{ hours.}$$

Standard cost $= sQ = 30(1{,}000) = \$30{,}000$.
Actual cost $= aq = 32(1{,}100) = \$35{,}200$.
Total cost variance:

$$A - B = 35{,}200 - 30{,}000 = \$5{,}200 \quad \text{(unfavourable, since it is a cost).}$$

Decompose using Proposition 2.3.1 with $a = 32$, $s = 30$, $q = 1{,}100$, $Q = 1{,}000$:

$$aq - sQ = (a - s)q + s(q - Q).$$

Rate variance:

$$(a - s)q = (32 - 30)(1{,}100) = 2(1{,}100) = \$2{,}200 \quad \text{(unfavourable).}$$

Efficiency variance:

$$s(q - Q) = 30(1{,}100 - 1{,}000) = 30(100) = \$3{,}000 \quad \text{(unfavourable).}$$

Check: $2{,}200 + 3{,}000 = 5{,}200$, matching the total variance.

Checkpoint skills (quick self-test)

You should be able to: (i) compute absolute and percentage variances; (ii) apply the favourable/unfavourable sign rule correctly for costs vs revenues; (iii) split $aq - sQ$ into a rate (price) effect and a quantity (usage/efficiency) effect and reconcile back to the total variance.

2.4 Financial Statement Mathematics: Common-Size Analysis, Trend Analysis, and Ratio Algebra

After variance work, the next building block accounting and finance is *financial statement mathematics*: expressing items as percentages of a base (common-size), measuring change over time (trend/horizontal analysis), and computing/interpreting ratios. The mathematics is straightforward, but accuracy and interpretation depend on clean definitions.

2.4.1 Common-size (vertical) analysis

Definition 2.4.1 **Common-size percentage**

Given a base amount $B > 0$ and a line item amount x, the *common-size percentage* of x relative to B is

$$\text{Common-size } \% = \frac{x}{B} \times 100\%.$$

Typical choices of B are: (i) net sales (income statement), and (ii) total assets (balance sheet).

Example 2.4.1 **(Common-size income statement)**

A firm reports net sales of \$2,400,000, cost of goods sold of \$1,560,000, and selling & administrative expenses of \$480,000. Compute each as a common-size percentage of sales and determine the gross margin percentage.

Solution.

Use net sales as the base $B = 2{,}400{,}000$.
Cost of goods sold as a percentage of sales:

$$\frac{1{,}560{,}000}{2{,}400{,}000} \times 100\% = 65\%.$$

Selling & administrative expenses as a percentage of sales:

$$\frac{480{,}000}{2{,}400{,}000} \times 100\% = 20\%.$$

Gross profit $= 2{,}400{,}000 - 1{,}560{,}000 = 840{,}000$, so gross margin percentage is

$$\frac{840{,}000}{2{,}400{,}000} \times 100\% = 35\%.$$

Thus, for every \$1.00 of sales, about \$0.65 goes to COGS, \$0.20 to S&A, leaving \$0.35 gross profit before S&A and other items.

2.4.2 Trend (horizontal) analysis

Definition 2.4.2 **Dollar and percentage change**

If a line item changes from A (prior period) to B (current period), then:

$$\text{Dollar change} = B - A, \qquad \text{Percent change} = \frac{B-A}{A} \times 100\% \quad (A \neq 0).$$

Example 2.4.2 **(Trend analysis across two years)**

Net sales increased from \$1,920,000 to \$2,400,000, while COGS increased from \$1,210,000 to \$1,560,000. Compute dollar and percent changes for both items, and comment on whether gross profit improved.

Solution.

For sales:

$$\text{Dollar change} = 2{,}400{,}000 - 1{,}920{,}000 = 480{,}000,$$

$$\text{Percent change} = \frac{480{,}000}{1{,}920{,}000} \times 100\% = 25\%.$$

For COGS:

$$\text{Dollar change} = 1{,}560{,}000 - 1{,}210{,}000 = 350{,}000,$$

$$\text{Percent change} = \frac{350{,}000}{1{,}210{,}000} \times 100\% \approx 28.93\%.$$

Gross profit prior year $= 1{,}920{,}000 - 1{,}210{,}000 = 710{,}000$. Gross profit current year $= 2{,}400{,}000 - 1{,}560{,}000 = 840{,}000$. So gross profit increased by \$130,000.
However, COGS grew ($\approx 28.93\%$) faster than sales (25%), which typically signals margin pressure unless pricing, product mix, or other factors offset it.

2.4.3 Ratio computation and ratio algebra

Definition 2.4.3 **A ratio and its interpretation**

A *ratio* is a quotient of two financial quantities. Ratios summarize a relationship, such as liquidity, profitability, leverage, or efficiency. The meaning of a ratio depends on the definition of its numerator and denominator.

Definition 2.4.4 **Selected ratios**

Let CA be current assets, CL current liabilities, NI net income, S net sales, and TA total assets. Common examples include:

$$\text{Current ratio} = \frac{CA}{CL}, \qquad \text{Net profit margin} = \frac{NI}{S}, \qquad \text{Return on assets (ROA)} = \frac{NI}{TA}.$$

Proposition 2.4.1 **Solving for an unknown from a ratio**

If a ratio is defined by $\rho = \dfrac{X}{Y}$ with $Y \neq 0$, then

$$X = \rho Y \quad \text{and} \quad Y = \frac{X}{\rho} \ (\rho \neq 0).$$

Thus ratio constraints can be converted into algebraic constraints.

Example 2.4.3 **(Compute and interpret key ratios)**

A company reports: $CA = \$510{,}000$, $CL = \$300{,}000$, $NI = \$126{,}000$, $S = \$2{,}400{,}000$, $TA = \$1{,}800{,}000$. Compute the current ratio, net profit margin, and ROA.

Solution.

Current ratio:

$$\frac{CA}{CL} = \frac{510{,}000}{300{,}000} = 1.7.$$

Interpretation: about \$1.70 of current assets for every \$1.00 of current liabilities.

Net profit margin:

$$\frac{NI}{S} = \frac{126{,}000}{2{,}400{,}000} = 0.0525 = 5.25\%.$$

Interpretation: about 5.25 cents of net income per dollar of sales.

ROA:

$$\frac{NI}{TA} = \frac{126{,}000}{1{,}800{,}000} = 0.07 = 7\%.$$

Interpretation: the firm generates about 7% net income relative to total assets.

Example 2.4.4 **(Ratio constraint and solving for a required amount)**

A lender requires a minimum current ratio of 1.8. A firm has current assets $CA = \$450{,}000$. What is the maximum current liabilities CL the firm can have while meeting the requirement?

Solution.

The requirement is

$$\frac{CA}{CL} \geq 1.8.$$

Substitute $CA = 450{,}000$:

$$\frac{450{,}000}{CL} \geq 1.8.$$

Since $CL > 0$, we may multiply both sides by CL without changing the inequality direction:

$$450{,}000 \geq 1.8\,CL \quad \Longleftrightarrow \quad CL \leq \frac{450{,}000}{1.8} = 250{,}000.$$

Thus the firm can have at most \$250,000 in current liabilities and still satisfy the lender's minimum current ratio.

Checkpoint skills (quick self-test)

You should be able to: (i) compute common-size percentages and interpret them; (ii) compute dollar and percentage changes across periods; (iii) compute key ratios and solve simple ratio constraints for an unknown.

2.5 Time Value of Money: Compounding, Discounting, and Annuities

A central mathematical idea in financial management is that cash flows at different times are not directly comparable. To compare them, we translate everything to a *common time* using compounding (moving forward in time) or discounting (moving backward).

2.5.1 Compound growth and present value

Definition 2.5.1 **Effective rate per period**

Let i be the interest rate *per period* (for example, per year or per month). Compounding means that a balance B grows over one period to $B(1+i)$.

Proposition 2.5.1 **Compound interest formula**

Let P be a principal invested at rate i per period, compounded once each period. After n periods, the future value is

$$FV = P(1+i)^n.$$

Definition 2.5.2 **Present value**

If an amount F is due n periods in the future, its present value at rate i is the amount PV such that

$$PV(1+i)^n = F.$$

Equivalently,

$$PV = \frac{F}{(1+i)^n}.$$

Example 2.5.1 **(Future value and present value)**

An investment of \$12,000 earns 9% per year compounded annually for 5 years. (i) Find the future value. (ii) What is the present value of \$18,000 due in 5 years at the same rate?

Solution.

(i) Here $P = 12{,}000$, $i = 0.09$, $n = 5$. By Proposition 2.5.1,

$$FV = 12{,}000(1.09)^5 \approx 12{,}000(1.53862) \approx \$18{,}463.44.$$

(ii) By Definition 2.5.2,

$$PV = \frac{18{,}000}{(1.09)^5} \approx \frac{18{,}000}{1.53862} \approx \$11{,}699.53.$$

2.5.2 Annuities and geometric-series algebra

Many business problems involve a *level payment* repeated over time (loan payments, lease payments, savings contributions). The mathematics is a geometric series.

Lemma 2.5.1 (Finite geometric series)

For any real number $r \neq 1$,

$$1 + r + r^2 + \cdots + r^{n-1} = \frac{1 - r^n}{1 - r}.$$

Proposition 2.5.2 Present value of an ordinary annuity

Let payments of amount A occur at the *end* of each period for n periods, discounted at rate $i > 0$ per period. Then the present value is

$$PV = A\left(\frac{1 - (1+i)^{-n}}{i}\right).$$

Proof. The present value is the sum of each payment discounted back to time 0:

$$PV = \frac{A}{1+i} + \frac{A}{(1+i)^2} + \cdots + \frac{A}{(1+i)^n} = A\sum_{k=1}^{n}(1+i)^{-k}.$$

Factor out one discount factor:

$$PV = A(1+i)^{-1}\sum_{k=0}^{n-1}(1+i)^{-k}.$$

This is a geometric series with ratio $r = (1+i)^{-1} \neq 1$. By Lemma 2.5.1,

$$\sum_{k=0}^{n-1} r^k = \frac{1 - r^n}{1 - r}.$$

Substitute $r = (1+i)^{-1}$:

$$\sum_{k=0}^{n-1}(1+i)^{-k} = \frac{1 - (1+i)^{-n}}{1 - (1+i)^{-1}}.$$

Since $1 - (1+i)^{-1} = \frac{i}{1+i}$, we obtain

$$PV = A(1+i)^{-1} \cdot \frac{1 - (1+i)^{-n}}{i/(1+i)} = A\left(\frac{1 - (1+i)^{-n}}{i}\right).$$

$\square$

Example 2.5.2 **(Loan payment (ordinary annuity))**

A loan of \$50,000 is to be repaid with equal monthly payments over 4 years at 12% per year compounded monthly. Find the required monthly payment.

Solution.

Monthly rate $i = \frac{0.12}{12} = 0.01$. Number of payments $n = 4 \times 12 = 48$. The loan amount is the present value of the payment stream:

$$50{,}000 = A\left(\frac{1-(1.01)^{-48}}{0.01}\right).$$

Solve for A:

$$A = 50{,}000 \cdot \frac{0.01}{1-(1.01)^{-48}}.$$

Compute $(1.01)^{48} \approx 1.611$, so $(1.01)^{-48} \approx 0.621$. Hence

$$A \approx 50{,}000 \cdot \frac{0.01}{1-0.621} = 50{,}000 \cdot \frac{0.01}{0.379} \approx 50{,}000 \cdot 0.02638 \approx \$1{,}319.00.$$

Therefore the monthly payment is approximately \$1,319 (to the nearest dollar).

Checkpoint skills (quick self-test)

You should be able to: (i) move between PV and FV using $(1+i)^n$; (ii) prove and use the finite geometric-series identity; (iii) set up and solve $PV = A\left(\frac{1-(1+i)^{-n}}{i}\right)$ for A in loan/payment problems.

2.6 Percentage-Based Reasoning and Ratio Algebra for Business Decisions

In accounting and financial management, many conclusions are driven not by raw dollar values but by *relative* quantities: percentages, ratios, and growth rates. This section strengthens algebraic fluency with ratio constraints, reverse percentages, and multi-step percentage effects.

2.6.1 Percentages as multipliers and chained changes

Lemma 2.6.1 (Percent change as a multiplier)

If a quantity X increases by $r\%$, the new value is $X(1 + r/100)$. If it decreases by $r\%$, the new value is $X(1 - r/100)$.

Proof. An increase by $r\%$ means add $r\%$ of X to X:

$$X + \frac{r}{100}X = X\left(1 + \frac{r}{100}\right).$$

A decrease by $r\%$ means subtract $r\%$ of X from X:

$$X - \frac{r}{100}X = X\left(1 - \frac{r}{100}\right).$$

□

Proposition 2.6.1 Chained percentage changes multiply

Suppose X undergoes successive percentage changes of $r_1\%, r_2\%, \ldots, r_k\%$. Then the final value is

$$X\prod_{j=1}^{k}\left(1 + \frac{r_j}{100}\right),$$

where decreases are represented by negative r_j.

Proof. After the first change, the value becomes $X\left(1 + \frac{r_1}{100}\right)$. Applying the second change multiplies the current value by $\left(1 + \frac{r_2}{100}\right)$, giving

$$X\left(1 + \frac{r_1}{100}\right)\left(1 + \frac{r_2}{100}\right).$$

Continuing similarly, after k changes we obtain the stated product. □

Example 2.6.1 **(A discount followed by a markup is not "net zero")**

A retailer discounts a product by 20% during a sale and later increases the sale price by 20%. If the original price was \$250, what is the final price? What is the net percentage change?

Solution.

Use multipliers. A 20% discount multiplies by 0.80, and a 20% increase multiplies by 1.20. Thus

$$250(0.80)(1.20) = 250(0.96) = \$240.$$

The final price is \$240, which is \$10 less than the original.

Net percentage change:

$$\frac{240 - 250}{250} \times 100\% = -4\%.$$

So the overall effect is a 4% decrease, not zero.

Example 2.6.2 **(Reverse percentages: before and after VAT)**

An invoice total is \$1,920 after VAT of 12.5% is added. Find the pre-VAT amount.

Solution.

If the pre-VAT amount is x, then adding 12.5% gives

$$x\left(1 + \frac{12.5}{100}\right) = 1.125x = 1{,}920.$$

Hence

$$x = \frac{1{,}920}{1.125} = \$1{,}706.666\ldots \approx \$1{,}706.67.$$

So the pre-VAT amount is approximately \$1,706.67.

2.6.2 Ratios: computation, constraints, and solving for unknowns

Definition 2.6.1 **Ratio as a constraint**

A ratio $\rho = \dfrac{X}{Y}$ (with $Y \neq 0$) can be rewritten algebraically as $X = \rho Y$. Thus a ratio requirement (e.g. $\frac{X}{Y} \geq \rho_0$) becomes an inequality in the underlying variables.

Lemma 2.6.2 **(Solving a ratio inequality)**

Let $X > 0$, $Y > 0$, and $\rho_0 > 0$. Then

$$\frac{X}{Y} \geq \rho_0 \quad \Longleftrightarrow \quad X \geq \rho_0 Y,$$

and

$$\frac{X}{Y} \leq \rho_0 \quad \Longleftrightarrow \quad X \leq \rho_0 Y.$$

Proof. Since $Y > 0$, multiplying an inequality by Y does not change its direction:

$$\frac{X}{Y} \geq \rho_0 \Longleftrightarrow X \geq \rho_0 Y, \qquad \frac{X}{Y} \leq \rho_0 \Longleftrightarrow X \leq \rho_0 Y.$$

□

Example 2.6.3 **(Current ratio constraint: solve for the maximum liabilities)**

A lender requires a current ratio of at least 2.0. A firm has current assets $CA = \$620{,}000$. What is the maximum current liabilities CL permitted?

Solution.

The requirement is

$$\frac{CA}{CL} \geq 2.0.$$

Substitute $CA = 620{,}000$:

$$\frac{620{,}000}{CL} \geq 2.0.$$

Since $CL > 0$, multiply both sides by CL:

$$620{,}000 \geq 2.0\,CL \quad \Longleftrightarrow \quad CL \leq 310{,}000.$$

Thus the firm can have at most \$310,000 in current liabilities.

Example 2.6.4 **(Working capital requirement)**

A company must maintain working capital of at least \$120,000, where working capital is $WC = CA - CL$. If current liabilities are \$455,000, what is the minimum current assets required?

Solution.

The constraint is

$$CA - CL \geq 120{,}000.$$

Substitute $CL = 455{,}000$:

$$CA - 455{,}000 \geq 120{,}000 \quad \Longleftrightarrow \quad CA \geq 575{,}000.$$

So current assets must be at least \$575,000.

Example 2.6.5 **(Debt-to-assets constraint: solve for maximum debt)**

A bank requires the debt-to-assets ratio to be at most 0.60. If total assets are \$3,500,000, what is the maximum allowable total debt?

Solution.

Debt-to-assets is $\frac{D}{TA}$. The requirement is

$$\frac{D}{3{,}500{,}000} \leq 0.60.$$

Since $3{,}500{,}000 > 0$, multiply both sides:

$$D \leq 0.60(3{,}500{,}000) = 2{,}100{,}000.$$

Thus total debt must not exceed \$2,100,000.

2.6.3 Ratios involving profit margins and sales targets

Definition 2.6.2 **Profit margin**

If net income is NI and sales are S with $S > 0$, then the net profit margin is

$$m = \frac{NI}{S}.$$

Equivalently, $NI = mS$.

Example 2.6.6 **(Sales required for a target net income)**

A business targets net income of \$180,000 and expects a net profit margin of 6%. What sales level is required?

Solution.

Here $NI = 180{,}000$ and $m = 0.06$. Using $NI = mS$,

$$180{,}000 = 0.06\,S \quad \Longleftrightarrow \quad S = \frac{180{,}000}{0.06} = 3{,}000{,}000.$$

So required sales are \$3,000,000.

Example 2.6.7 **(Maintaining margin after a cost increase)**

A product sells for \$120 and has unit cost \$84. (i) Compute the gross margin percentage on sales. (ii) If unit cost increases by 10%, what new selling price maintains the same gross margin percentage?

Solution.

(i) Gross margin (in dollars) is $120 - 84 = \$36$. Gross margin percentage on sales is

$$\frac{36}{120} \times 100\% = 30\%.$$

(ii) A 10% increase in unit cost gives new cost

$$84(1.10) = \$92.40.$$

To maintain a 30% gross margin on sales, the cost must be 70% of the selling price. Let the required selling price be p. Then

$$0.70p = 92.40 \quad \Longleftrightarrow \quad p = \frac{92.40}{0.70} = 132.$$

So the new selling price must be \$132 to keep the same 30% gross margin.

Checkpoint skills (quick self-test)

You should be able to: (i) convert percentage changes into multipliers and combine multiple changes correctly; (ii) reverse a percentage change by dividing by the appropriate multiplier; (iii) translate ratio requirements into linear inequalities and solve for unknowns; (iv) solve sales/price targets given a margin constraint.

Example 2.6.8

Suppose that x months from now, the price of a certain calculator model will be P dollars per unit, where

$$P(x) = 40 + \frac{30}{x+1}.$$

(a) What will be the price 5 months from now?

Solution.

Substitute $x = 5$ into the given price function:

$$P(5) = 40 + \frac{30}{5+1}.$$

Compute the fraction carefully:

$$\frac{30}{5+1} = \frac{30}{6} = 5.$$

Therefore,

$$P(5) = 40 + 5 = 45.$$

So the price 5 months from now will be \$45.

(b) By how much will the price drop during the fifth month?

Solution.

The *fifth month* refers to the time interval from the end of month 4 to the end of month 5. So the drop during the fifth month is

$$P(4) - P(5).$$

First find $P(4)$:

$$P(4) = 40 + \frac{30}{4+1} = 40 + \frac{30}{5} = 40 + 6 = 46.$$

Next find $P(5)$:

$$P(5) = 40 + \frac{30}{5+1} = 40 + \frac{30}{6} = 40 + 5 = 45.$$

Hence the price drop during the fifth month is

$$P(4) - P(5) = 46 - 45 = 1.$$

So the price drops by \$1 during the fifth month.

(c) When will the price be $\$43$?

Solution.

Set $P(x) = 43$ and solve for x:

$$40 + \frac{30}{x+1} = 43.$$

Subtract 40 from both sides:

$$\frac{30}{x+1} = 3.$$

Now multiply both sides by $(x+1)$:

$$30 = 3(x+1).$$

Expand the right-hand side:

$$30 = 3x + 3.$$

Subtract 3 from both sides:

$$27 = 3x.$$

Divide by 3:

$$x = 9.$$

Therefore, the price will be \$43 in 9 months.

(d) What happens to the price in the long run (as $x \to \infty$)?

Solution.

Consider the term $\dfrac{30}{x+1}$. As $x \to \infty$, the denominator $x+1 \to \infty$, so

$$\frac{30}{x+1} \to 0.$$

Therefore,

$$P(x) = 40 + \frac{30}{x+1} \to 40 + 0 = 40 \quad \text{as } x \to \infty.$$

So in the long run, the price approaches \$40 (and since $\frac{30}{x+1} > 0$, it approaches 40 from above).

Exercise 2.6.1

1. The demand for a certain commodity is $D(x) = -50x + 800$; that is, x units of the commodity will be demanded by consumers when the price is $p = D(x)$ dollars per unit. Total consumer expenditure $E(x)$ is the amount of money consumers pay to buy x units of the commodity.

   (a) **Express consumer expenditure as a function of x, and sketch the graph of $E(x)$.**

   (b) **Use the graph in part (a) to determine the level of production x at which consumer expenditure is largest. What price p corresponds to maximum consumer expenditure?**

2. A retailer can obtain digital cameras from the manufacturer at a cost of $150 apiece. The retailer has been selling the cameras at a price of $340 apiece, and at this price consumers buy 40 cameras per month. The retailer plans to lower the price to stimulate sales and estimates that for each $5 reduction in price, 10 more cameras will be sold each month.

   Express the retailer's monthly profit from the sale of the cameras as a function of the selling price. Draw the graph, and estimate the optimal selling price.

3. Khalil is trying to decide between two competing property tax propositions. With Proposition A, he will pay $100 plus 8% of the assessed value of his home, while Proposition B requires a payment of $1,900 plus 2% of the assessed value. Assuming Khalil's only consideration is to minimize his tax payment, determine which proposition he should choose.

Case Study

Harlan borrows $2000 for 2 years. The lender quotes a "good deal," 4% simple interest. Since 4% simple interest for 2 years is $160, Harlan repays a total of $2160 in 24 equal monthly payments, i.e. $90 per month.

Harlan pays 4% on the $2000 for the entire 2 years, though he repays some of the principal each month. Thus, he really pays more than 4% on what he owes.

What interest rate does Harlan pay if it is computed as interest on the unpaid balance?