EEE 310 Project

Signal Sampling and Reconstruction at Sub-Nyquist Sampling Frequencies using TPSL-ADC Sampling Method

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Three Parallel Sub-Nyquist Low-rate Analog-to-Digital Converters (TPSL-ADC) Method

- Samples and Reconstructs Time Domain Signal while sampling below Nyquist Frequency.
- Uses three parallel samplers to sample the signal, each below the Nyquist Frequency of the target signal.
- Information from the Three Samplers are combined in a Matrix.
- Compressive Sensing Algorithm is used to recover the Nyquist Frequency equivalent signal.

TPSL-ADC Method can reconstruct a signal below Nyquist Frequency, which is impossible using any conventional methods. For example: Whittaker-Shannon Interpolation

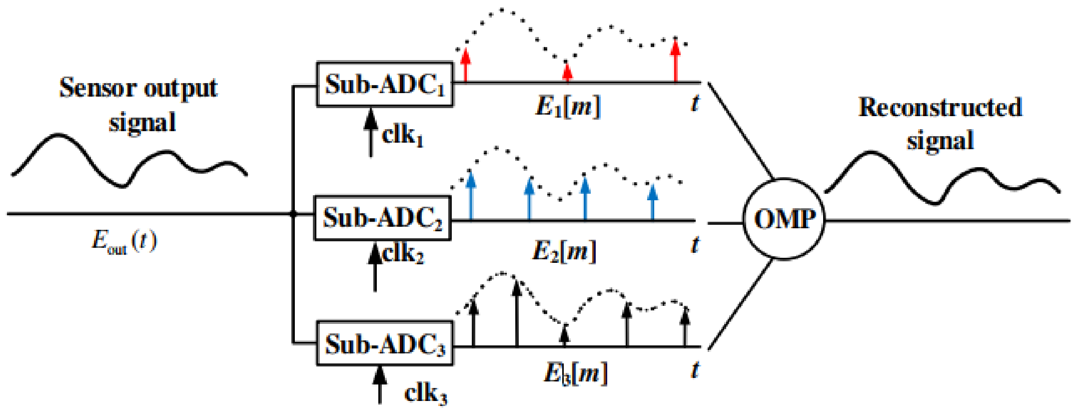
Overview on Compressive Sensing

- Central Idea of our project.
- Most signals in frequency domain has a sparse nature, I.e., apart from some frequencies, the rest are zero. Such signal can be recovered by Compressive Sensing.
- It is used to find sparsest solution of the Fourier spectra of a sub-sampled signal
- The reconstruction requires the pre-modulation of the original signal with a pseudo-random sequence.
- In TPSL-ADC method, the samplers are set in such a way so that parallelly sampled sequence is pseudo-random, and the full potential of the CS method can be achieved.

Input Signal → Sub-Sampler 1,2,3 → Multiplication with Measurement Matrix → Sparse Frequency Domain Representation

→ Compressive Sensing Signal Recovery → Inverse Fourier Transform → Nyquist Sampled Equivalent Signal → Comparison with Input

Workflow



Scheme of the proposed TPSL-ADC sapling method.

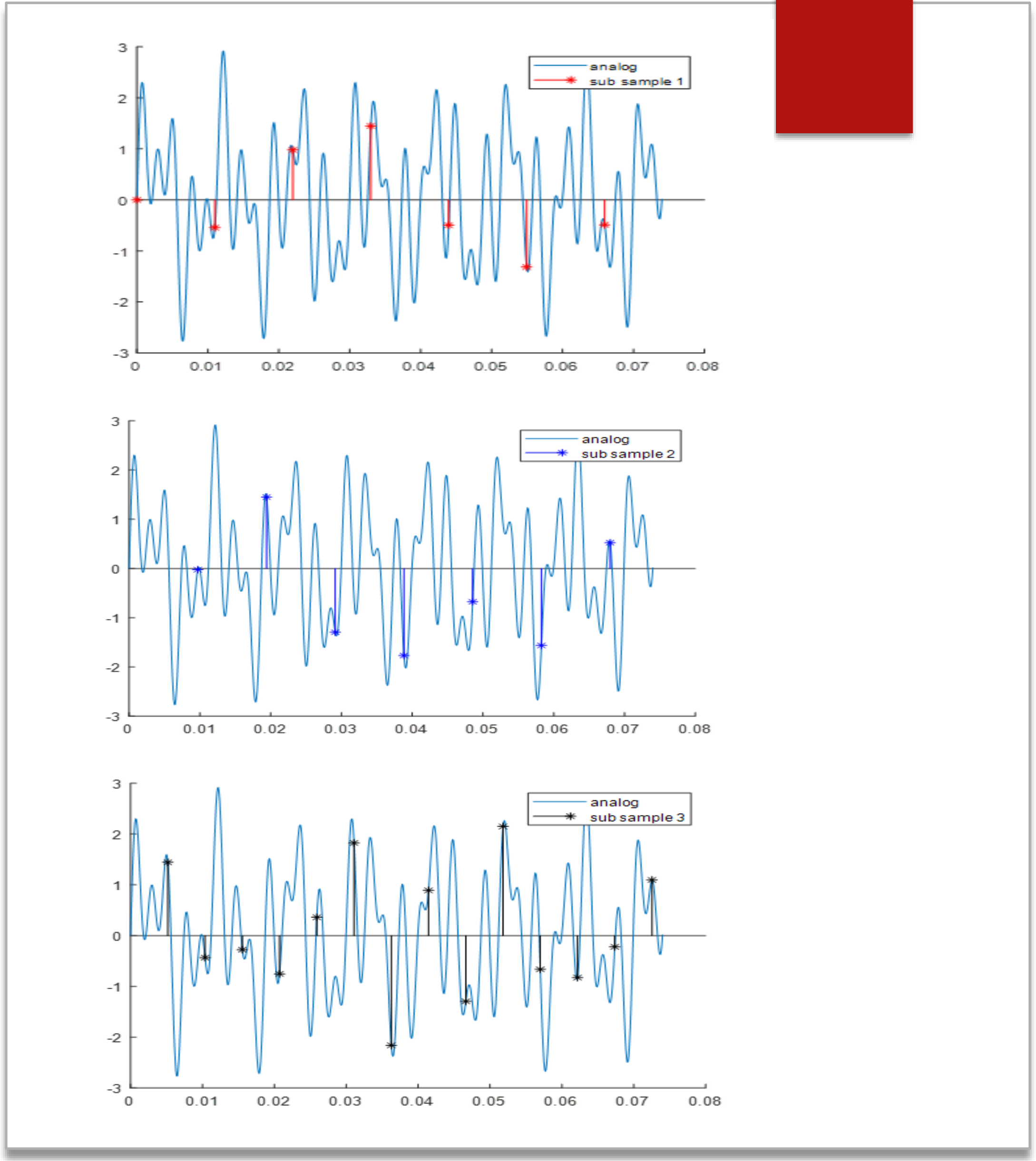
Sampling

- Input Signal:

  sin(2*pi*70*t) + sin(2*pi*275*t) + sin(2*pi*400*t)
- Three Samplers are used Parallel
- Each having a Sampling Frequency Lower than Nyquist Frequency of the Signal
- In this example,

  F1 = 91, F2 = 103, F3 = 193 Hz

  M1 = 91, M2 = 102, M3 = 192 Samples
- Whereas Main sampler frequency = 1000 Hz
- Signal Size = 1000 Samples
- Sampler 2 and 3 is delayed once by their corresponding periods
- When combined, they make a pusedorandom sequence.



Measurement Matrices

In order to reconstruct the original signal from subsampled values, a relationship needs to be established

The measurement matrices links the sub-sampled and Nyquist signals

This matrix uses Whittaker-Shannon interpolation on N-point signal to make continuous signal and then extracts several Sub-sampled values

These matrices satisfy restricted isometry property, which is required in signal reconstruction.

$$\begin{cases} \phi(m,n) = \operatorname{sinc}\left(\frac{mN_1T_e}{T_e} - n\right), \ 1 \le m \le M_1, 1 \le n \le N \\ \varphi(m,n) = \operatorname{sinc}\left(\frac{mN_2T_e+\tau_{21}}{T_e} - n\right), \ 1 \le m \le M_2, 1 \le n \le N. \\ \gamma(m,n) = \operatorname{sinc}\left(\frac{mN_1T_e+\tau_{31}}{T_e} - n\right), \ 1 \le m \le M_3, 1 \le n \le N \end{cases}$$

$$\boldsymbol{\Phi} = [\phi^{\mathrm{T}}, \varphi^{\mathrm{T}}, \gamma^{\mathrm{T}}]^{\mathrm{T}}.$$

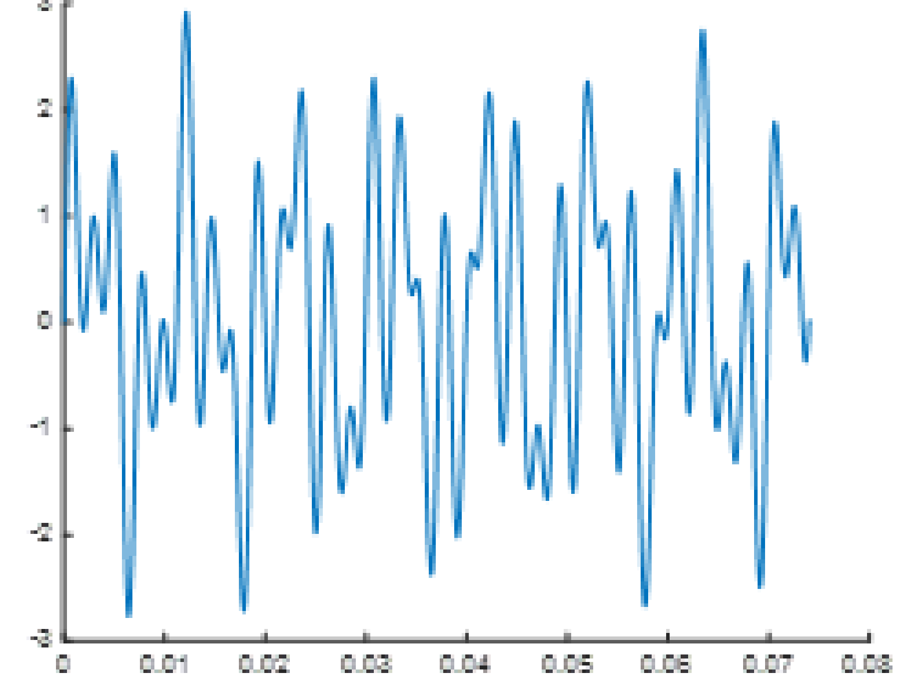
Relation between Sub-Nyquist and Nyquist Sampled Signals

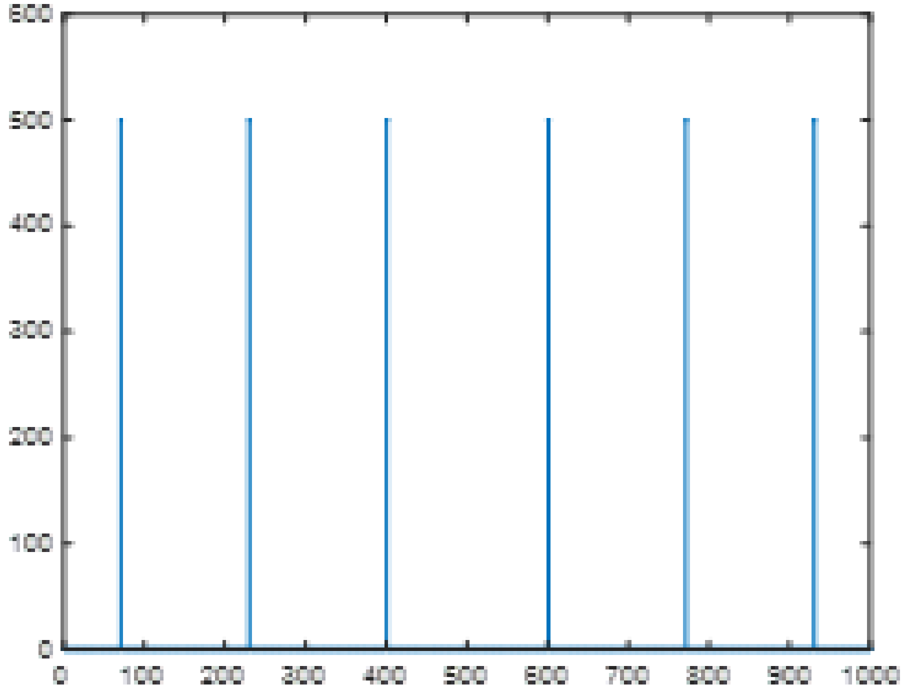
Sub-Nyquist Sampled Sequence = (Measurement Matrix) x (Sampled Sequence in equivalent sampling/Nyquist frequency)

- $E = \Phi.\ E_{nyquist} = \Phi \cdot \Psi \cdot X = AX$, where $A = \Phi\Psi$ is sensing matrix, and X is sparse Fourier spectra of the signal sampled above Nyquist rate – yet to be solved.
- Size of the Sub-Nyquist Sampled Sequence ($M = 385$) is always less than Equivalent/Nyquist Sampled Sequence ($N = 1000$)
- So, this equation cannot be solved using Cramer's rule or Matrix Inversion.
- Because it is an underdetermined system and thus has an infinite number of solutions .

Sparse Representation of Input Signal

- CS uses algorithms that can reconstruct matrices that are sparse in nature
- However, our time domain signal is ill-suited for such a task
- But if any signal is taken into frequency domain, only the frequencies present in the signal has a non-zero value
- Our example sinusoid has only 6 nonzero values in frequency domain (Some cases the signal is subject to Spectral Leakage)



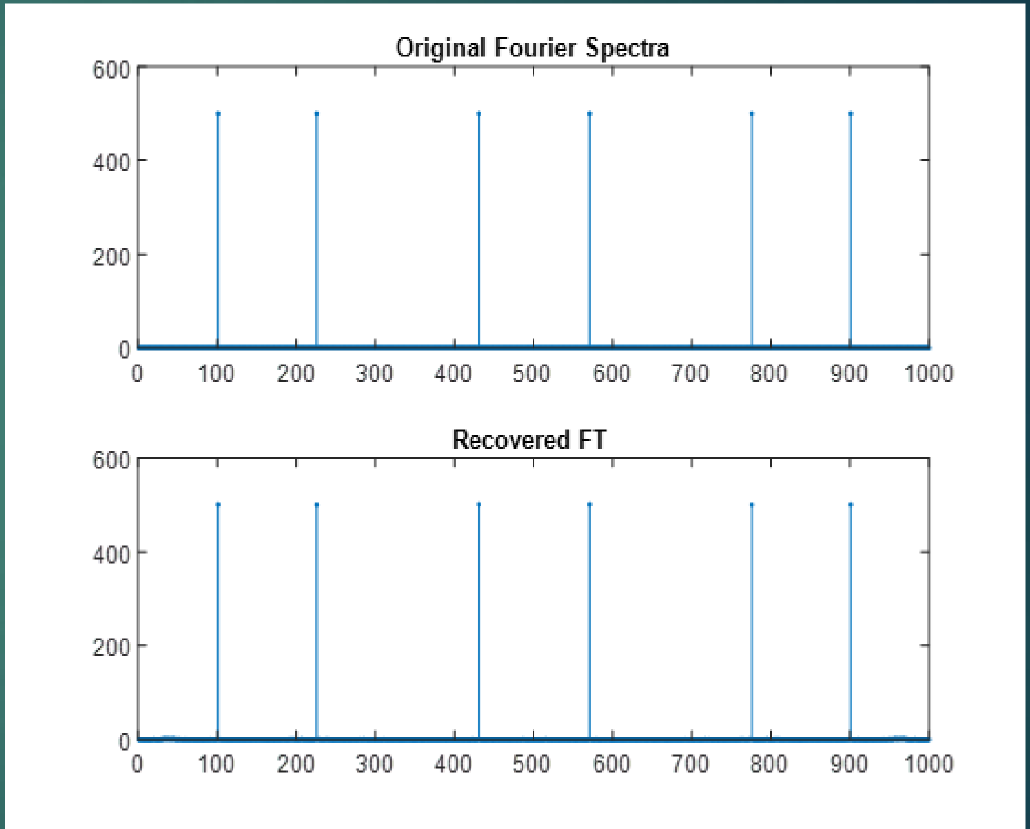


Compressive Sensing and Signal Reconstruction

- The equivalent/Nyquist sampled signal must be sparsely represented

- The Sub-Nyquist Sampled Sequence and Sensing Matrix is known

- Solving for X in E=AX, using least L1 norm algorithm



Results

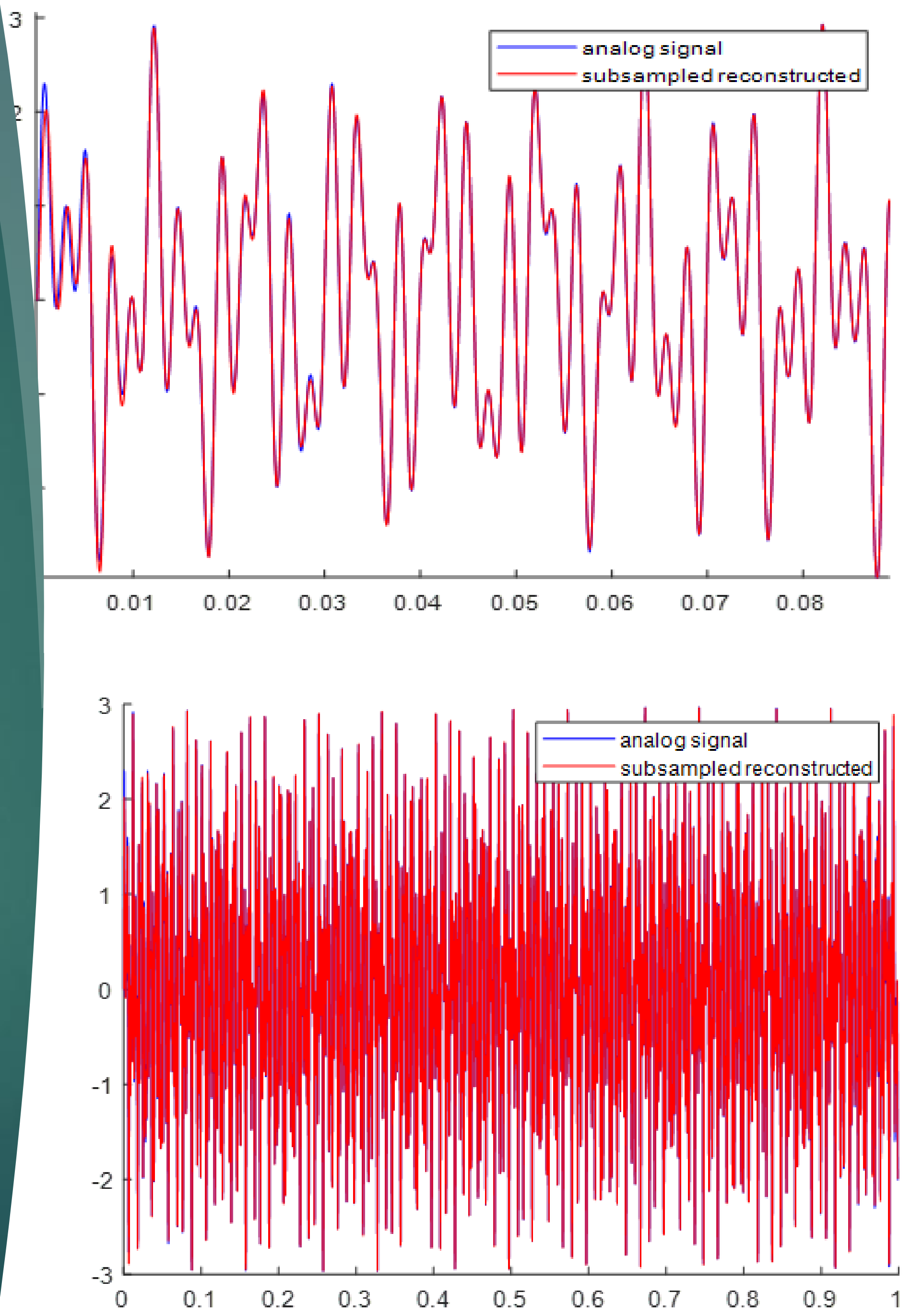
- Applying Inverse Fourier Transform on the Reconstructed Signal, we can get the time domain signal and compare it with the input.

- The signal has been recovered satisfactorily

- Although there are some distortion at the two ends of the signal which is due to Whittaker-Shannon interpolation of finite length signals

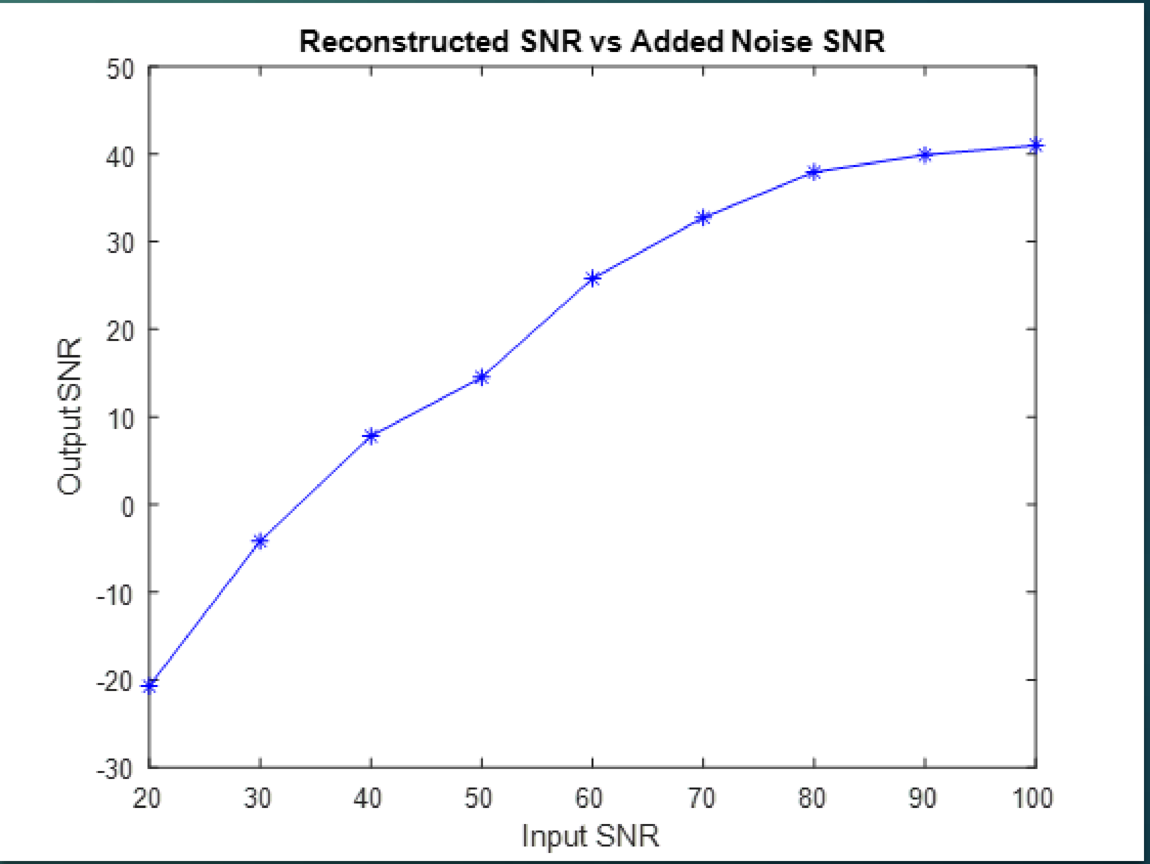
- Signal to Noise Ratio of the reconstructed signal is:

40.167 dB

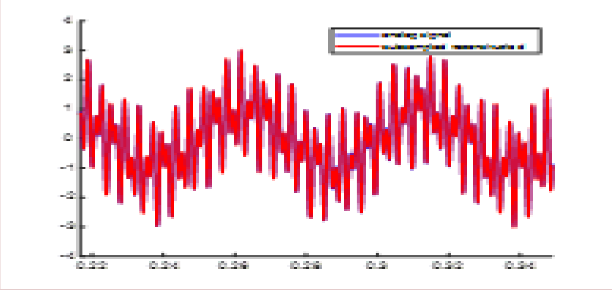
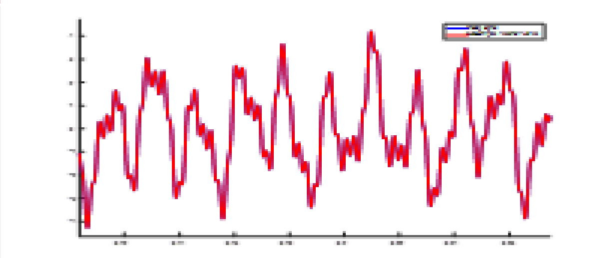
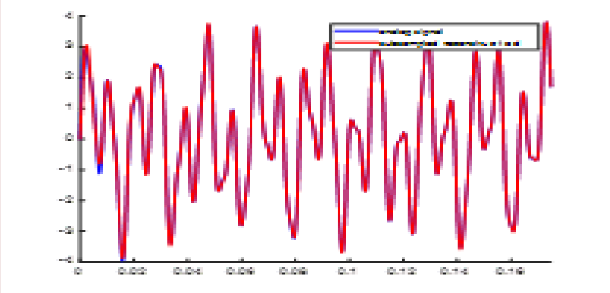


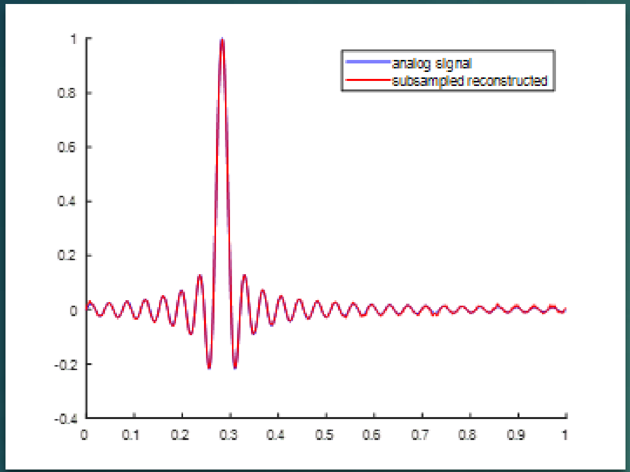
Investigating the effect of Added Noise in Reconstruction

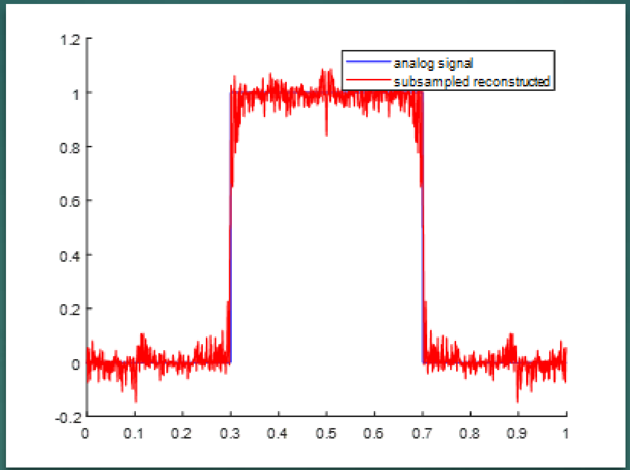
- In practice, when the sub-sampled signal is transmitted, there will be noise introduced.
- This can be introduced while sampling as well. It is more susceptible to noise as it has much lower number of sampling points.
- We investigate a case where we get the reconstructed signal SNR = 40.8 dB without any added gaussian noise.
- However, with more added noise, the reconstruction noise increases, as presented in the graph below.
- Signals are often quantized during transmission which also introduces noise, this must be considered while applying our model in practical applications

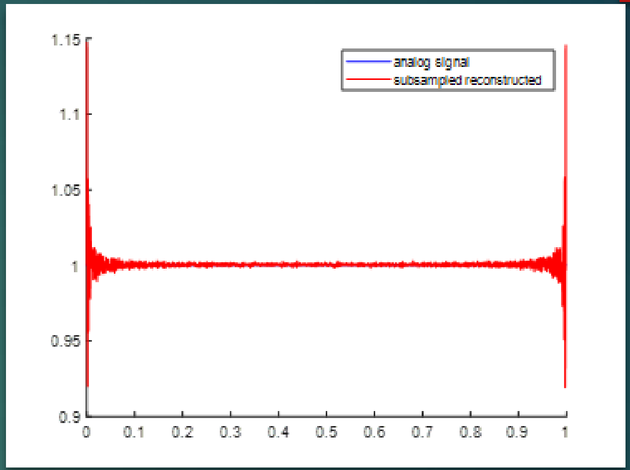


Signal Reconstruction of Some other cases: Sinusoids

Input Signal	The Frequencies (F1, F2, F3, FC)	M/N	SNR (dB)	Input Vs. Output Graph
sin(2*pi*20*t) + sin(2*pi*280*t) + sin(2*pi*490*t)	91, 103, 193, 1000 Hz	385, 1000	29	
sin(2*pi*25*t) + 0.5*sin(2*pi*345*t) + sin(2*pi*120*t) + 2*sin(2*pi*63*t)	83, 97, 117, 1000 Hz	295, 1000	53	
1.5*sin(2*pi*48*t) + 0.5*sin(2*pi*233*t) + 2*sin(2*pi*111*t);	83, 97, 117, 500 Hz	295, 500	38	







Signal Reconstruction of Some other cases:

1. Sinc Function 2. Rectangular Pulse 3. Simple DC

Signal Reconstruction of a practical signal (Voice Signal)

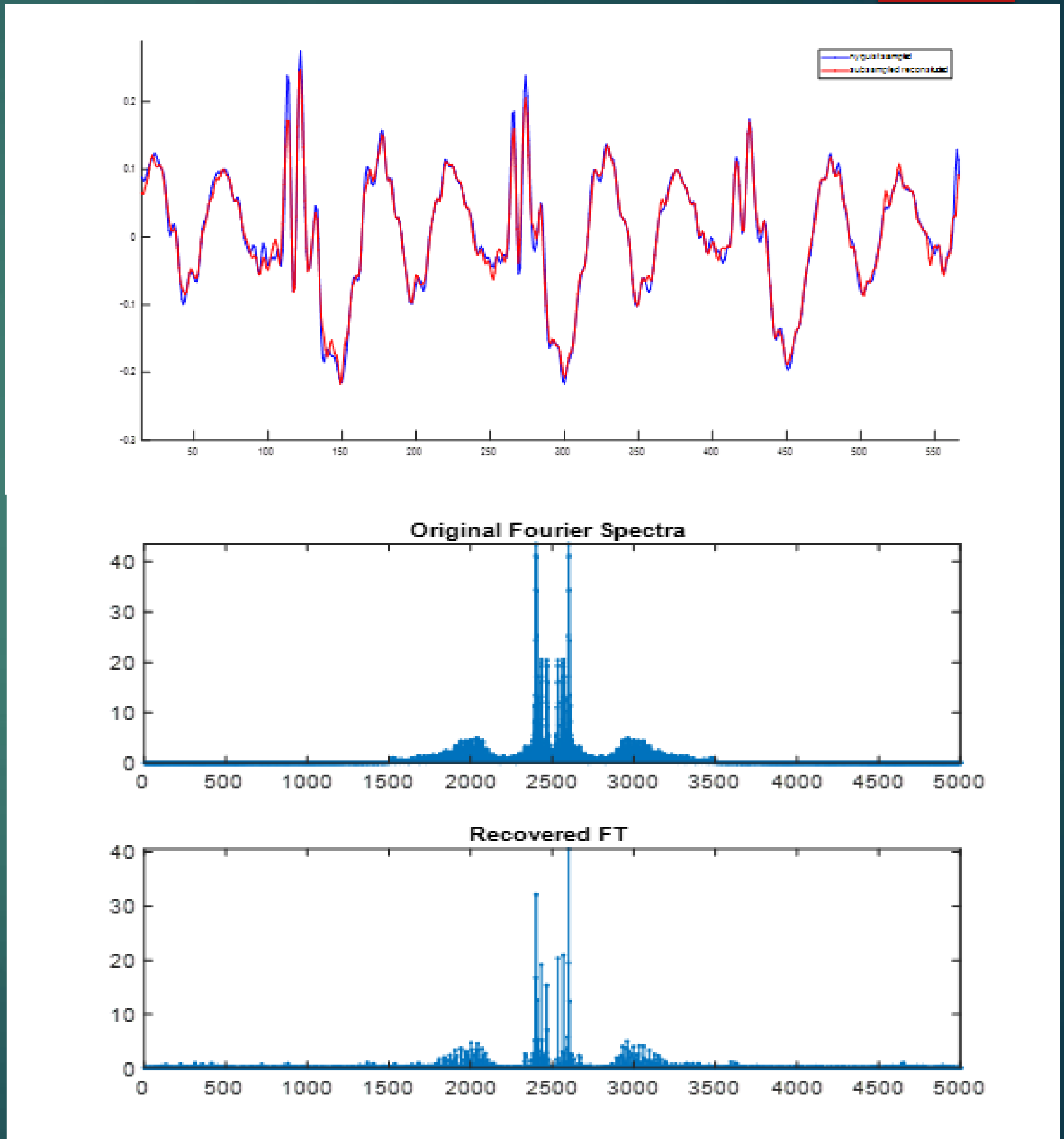
We used a practical audio signal as input for our program in an attempt to reconstruct it.

F1 = 3300, F2 = 4650, F3 = 7530, Fc = 22050 Hz

Sub-sampled Signal Length = 696

Input Signal Length = 1000

SNR = 15.81 dB



Signal Reconstruction of a practical signal (Voice Signal)

Full Signal Reconstruction:

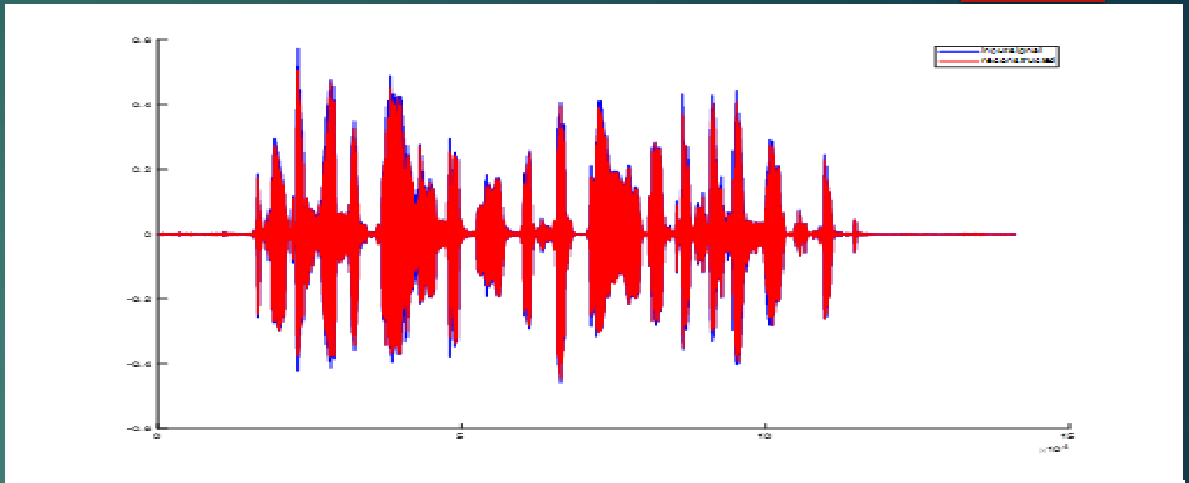
Sub-sampled Signal Length = 98414

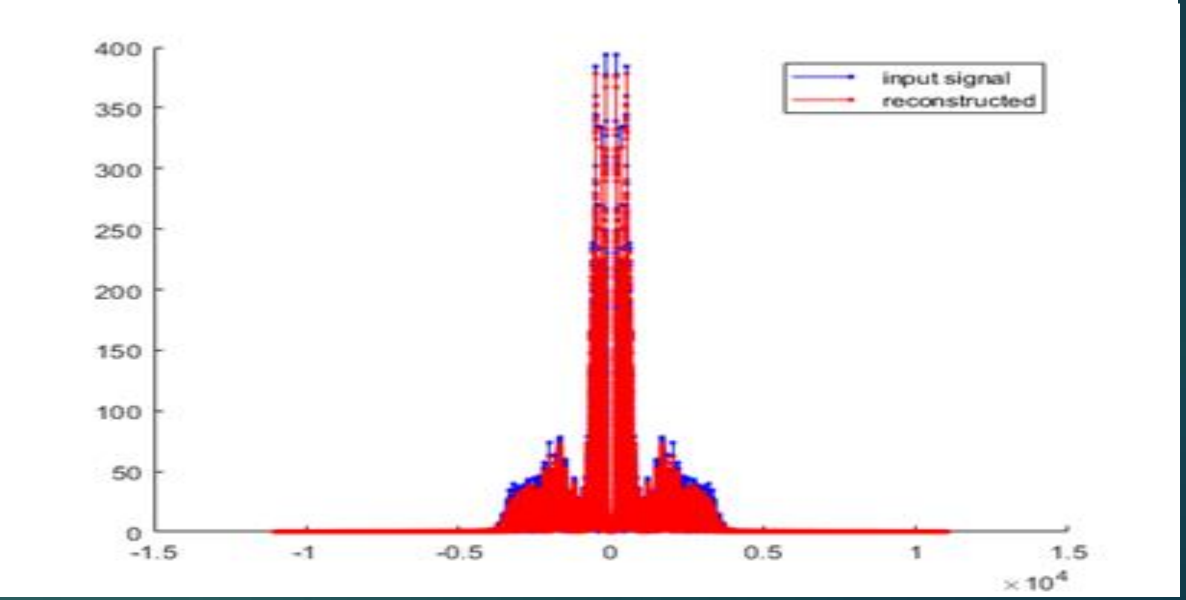
Input Signal Length = 141120

Full Signal SNR = 16.23 dB

Input Signal

Reconstructed Signal





Limitations and Adjustments

- Spectral Leakage contributes to Noise and Distortion.
- Signals without Sparse Representation gets distorted and have significant noise.
- If Delay and Sub sampling frequencies are not appropriate, Compressive Sensing may fail to properly reconstruct signal, and much noise is introduced.
- Adjust Sensing Matrix size to minimize spectral leakage.
- Increasing the equivalent/target sampling frequency.
- Adjust Sub sampler Frequency and Delay emulate pseudo-randomness to allow Compressive Sensing to execute.

Advantages and Application

- Application Fields
  - Cognitive radio
  - Ultra-wideband communication for info-stations
  - High frequency electric field measurement
  - Wideband radar

With the advancement of communication technologies, the demand of higher sampling rate is increasing day by day. But it's difficult to construct Analog to Digital Converter with higher bits per sample at high sampling rate. So sub-Nyquist rate creates an alternative approach in this way.

So, desired tasks can be completed with feasible ADCs with lower sampling rate. Also, this type of sampling is less complex than RES method

Less Bandwidth is required as sampling is done at lower frequencies

Low memory consumption for Digital Signal Storage as it requires fewer number of data points.

Drawbacks

Noise

Noise increases when smaller sub sampling frequencies are used. A post filter may be required for noise removal in practical cases.

Hardware Requirements

Multiple Sub samplers are needed instead of one, as well as any other accessories.

Computationally Expensive

Compressive Sensing and Measurement matrices can get large while reconstructing practical signals.

THANK YOU ALL FOR YOUR ATTENTION