Legendrian Knots

A new approach to calculating non-classical invariants

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Abstract

One of the main goals in knot theory is to determine when two Knots are isotopic. We focus on Legendrian knots, which lie in the standard contact structure, and invariants which distinguish them. Traditionally, computing these invariants has been infeasible for large knots, but we have found and implemented a more direct approach which makes computing on higher crossing knots more practical. Using this we determined the cup product for n-linked unknots, and found interesting patterns in the non-commutative products of knots.

Introduction

Knots are embeddings of a circle into $\mathbb{R}^3$. We say two knots are equivalent if one can be deformed into the other without cutting the knot at any point.

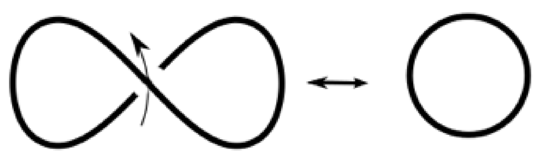

Figure 1: Two projections of the unknot

There is no general method of distinguishing knots, however there are invariants of knots which stay the same no matter how much you tangle the knot up.

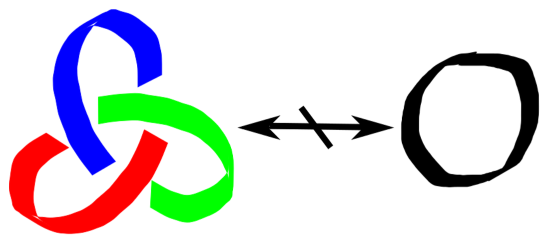

Figure 2: The trefoil and unknot are distinguished by the 3 coloring invariant

Our research focuses on *Legendrian* knots, which are knots which lie in a contact structure. When we look at a front projection of Legendrian Knots, the knot will have no vertical tangents.

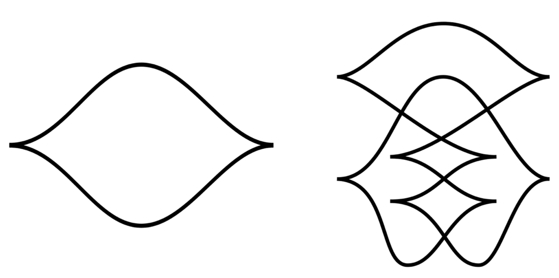

Figure 3: A legendrian unknot (left) and figure-eight knot (right)

Main Objectives

Our main objective was to write a program to calculate a series of invariants of Legendrian knots, each of which are built upon the previous invariants.

1. Calculate all rulings of a knot, and the associated ruling polynomial

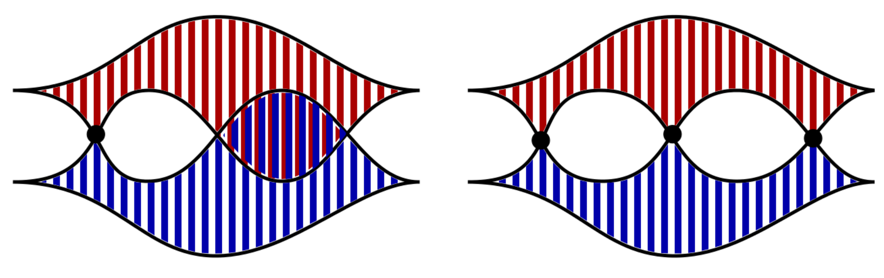

Figure 4: Two rulings on a trefoil, with switches marked

2. For each ruling find the associated augmentations

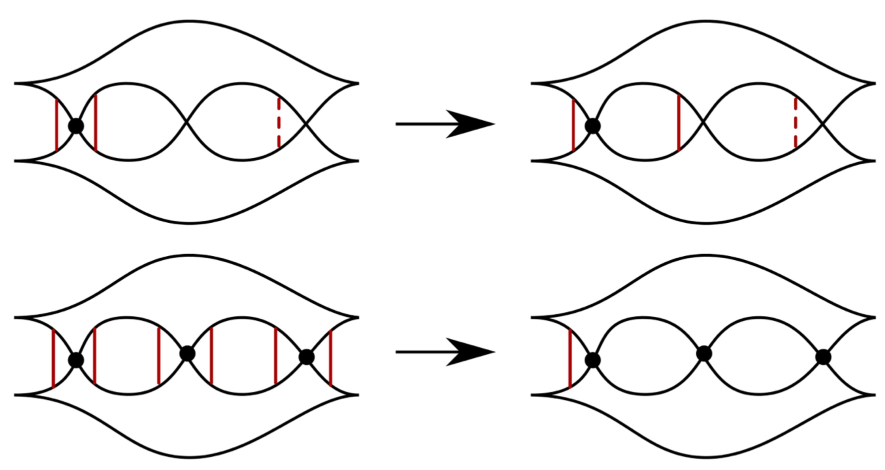

Figure 5: Switch return forms (left) and the associated a forms (right)

3. Calculate the linearized contact homology of the knot
4. Calculate the cup products of the contact homology
5. Examine non-commutative products of knots for underlying patterns

Results

For n-copies of the unknot we found that the product followed a simple pattern. If we label each column of crossings from left to right, splitting at the center, we see that each successive column has 1 less non-commutative product. We also note there is a pattern in the non-commutative products.

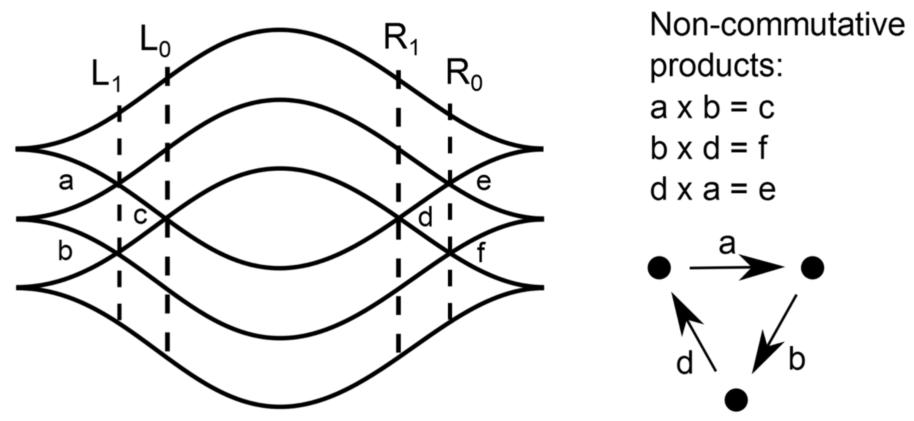

Figure 6: 3-copy of the unknot, with the cup product

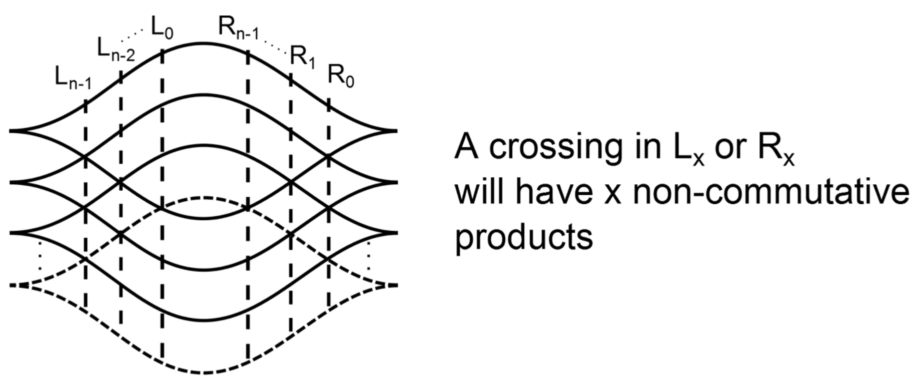

Figure 7: n-copy of the unknot

We also calculated the double product for all 10 crossing knots.

Conclusions

Our new program to compute linearized homologies and double products is significantly faster than the old methods on higher crossing knots. This allows us to examine more knots and at higher crossings for interesting properties. One such property found is that in all cases we have examined the non-commutative products of knots form loops such as ab, bc, cd, da.

Forthcoming Research

With this program we can now examine higher order knots to see if the pattern in non-commutative products still holds and further probe the products. We can also examine what happens when we take n-copies of knots other than the unknot, possibly adding twists between the crossings.

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