Lecture 13 and 14 - Numerical Differentiation

January 28, 2019

Learning objectives:

- Learn about finite difference approximations to derivatives.
- Be able to implement forward and central difference methods.
- Calculate higher-order derivatives.

Table of Contents

1 Taylor Series Method

We can use a Taylor series expansion to estimate the accuracy of the method. Recall that Taylor series in one dimention tells us that we can expand an increment to the evaluation point of a function as follows:

$$\begin{aligned} f(x_0 + h) &= f(x_0) + hf'(x_0) + \frac{h^2}{2!}f''(x_0) + \frac{h^3}{3!}f'''(x_0) + \dots \\ &= f(x_0) + hf'(x_0) + O(h^2) \end{aligned}$$

where $O(h^2)$ represents the collection of terms that are second-order in h or higher.

If we rearrange this expression to isolate the gradient term $f'(x_0)$ on the left hand side, we find:

$$hf'(x_0) = f(x_0 + h) - f(x_0) + O(h^2)$$

and therefore, by dividing through by h,

Numerical Analysis

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