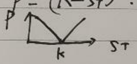
Homework I.

1. call option: $P = (S_T - K)^+$. when $K \le S_T$ $P = S_T - K$.
   put option: $P = (K - S_T)^+$. when $K > S_T$ $P = K - S_T$.
   plot :



$\therefore P = |S_T - K|$.

4. For put option.

$$(Ke^{-r(T-t)} - S_t)^+ < P_t < Ke^{-r(T-t)}$$

$\therefore$ lower $= 40 \cdot e^{-0.1 \cdot \frac{1}{12}} - 38 = \$1.01$.

5. According to the put-call parity:

$$p = c + Ke^{-r(T-t)} - S$$
$$= 1 + 20e^{-0.04 \times \frac{3}{12}} - 19 = 1.8$$

$\therefore$ the price of put option is \$1.8.

9. consider Φ_1.

$$V_T(\Phi_1) = (S_T - K)^+ + K + D \cdot e^{rT}.$$
$$= \max(S_T, K) + D \cdot e^{rT}.$$

consider Φ_2

$$V_T(\Phi_2) = (K - S_T)^+ + S_T + D \cdot e^{rT}$$
$$= \max(K, S_T) + D \cdot e^{rT}.$$

$\therefore V_T(\Phi_1) = V_T(\Phi_2)$.

according in arbitrage-condition

$\therefore V_t(\Phi_1) = V_t(\Phi_2)$

when $t = 0$

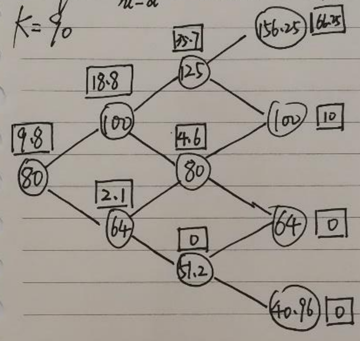
$$V_0(\Phi_1) = V_0(\Phi_2)$$

Homework II.

1. $r = 0.05$. $u = \frac{5}{4}$ $d = \frac{4}{5}$ $\rho = e^{r\delta t} = e^{0.05 \cdot \frac{2}{12}}$.

$\therefore q_u = \frac{\rho - d}{u - d} = 0.4630$. $q_d = \frac{u - \rho}{u - d} = 0.5370$.

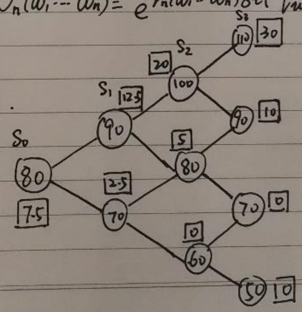
$K = 90$



2.

① $U_n(w_1 \cdots w_n) = \frac{1}{e^{r_n(w_1 \cdots w_n)\delta t}} \left(q_{u,n}(w_1 \cdots w_n) U_{n+1}(w_1 \cdots w_n H) + q_{d,n}(w_1 \cdots w_n) \cdot U_{n+1}(w_1 \cdots w_n T) \right)$

② .



$\therefore C_{t0} = \$7.5$.

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3.

considering $\Phi = -20 + \frac{1}{2}S$.

$$V_T(\Phi) = \begin{cases} u: & -20+\frac{1}{2}\cdot 80 = 20 \quad S_T > k \\ d: & -20+\frac{1}{2}\cdot 40 = 0. \quad S_T < k. \end{cases}$$

$$V_T(P) = (S_T - k)^+ = \begin{cases} 20 & S_T > k \\ 0 & S_T < k. \end{cases}$$

$\therefore\ V_T(\Phi) = V_T(P)$.

$\because\ V_0(\Phi) = V_0(P) = P_0 \Rightarrow P_0 = -20 + \frac{1}{2}S_0$.

$P_0 = -20 + 25 = 5$.

use (2.5) ~~program~~. which (2.5)? { example (2.5). maybe not, because the price of P_0 is measured in a risk-neutral world $\tilde{P}$. but ~~really~~ real world can't be that.

program 2.5 → the same as P_0.

6.

① for call option.

$$\Omega = \frac{(P^u - P^d)/P_0}{u-d}$$

又 $\because\ P_0 = \frac{1}{\rho}\left(\frac{\rho-d}{u-d}P^u + \frac{u-\rho}{u-d}P^d\right)$.

$$\Rightarrow \Omega = \frac{P^u - P^d}{P^u - P^d - (dP^u - uP^d)}.$$

$\therefore$ if we want to prove $\Omega \geq 1 \Rightarrow dP^u - uP^d \geq 0$.

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$\because\ P^u = \max(uS_0 - k, 0)$.

$P^d = \max(dS_0 - k, 0)$

$\therefore\ dP^u = \max(udS_0 - dk, 0)$

$uP^d = \max(udS_0 - uk, 0)$

$\because\ d < u \quad \therefore\ dP^u \geq uP^d$.

$\therefore$ for call option. $\Omega \geq 1$.

② for put option.

$P^u = \max(k - uS_0, 0)$

$P^d = \max(k - dS_0, 0)$

$\because\ u > d. \quad \therefore\ P^u - P^d \leq 0. \Rightarrow \Omega \leq 0$.

10.

$$Sk = E\left[\left(\frac{X - E[X]}{\sigma}\right)^3\right].$$

$$= E\left[\frac{X^3 - 3X^2E(X) + 3XE^2(X) - E^3(X)}{\sigma^3}\right].$$

$$= E\left[\frac{X^3 - 3X^2np + 3Xn^2p^2 - n^3p^3}{np(1-p)^{\frac{3}{2}}}\right].$$

又 $\because\ E(X^3) = npE[(Z+1)^2]$

$= npE[Z^2 + 2Z + 1]$

$= np(n-1)p[(n-2)p+1] + 2np(n-1)p + np$

$= np - 3np^2 + 2np^3 + 3n^2p^2 - 3n^2p^3 + n^3p^3$.

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So

$$Sk = \frac{np-3np^2+2np^3+3n^2p^2-3n^2p^3+n^3p^3-3np((n-1)p+1)np+3n^3p^3-n^3p^3}{np(1-p)^{\frac{3}{2}}}$$

$$= \frac{np(1-p)(1-2p)}{np(1-p)^{\frac{3}{2}}}$$

$$= \frac{1-2p}{\sqrt{np(1-p)}}.$$

15. $J_n = \sum_{j=0}^{n-1} e^{\sigma M_j}(M_{j+1}-M_j)$, $n = \phi, 2 \cdots$

①

$$E_n[J_{n+1}] = q_u J_{n+1}(w_1 \cdots w_n H) + q_d J_{n+1}(w_1 \cdots w_n T)$$

$$= \frac{1}{2}\left(J_n + e^{\sigma M_n} X_{n+1}(H)\right)(w_1 \cdots w_n) + \frac{1}{2}\left(J_n + e^{\sigma M_n} X_{n+1}(T)\right)(w_1 \cdots w_n)$$

$$= \frac{1}{2}(J_n + e^{\sigma M_n}) + \frac{1}{2}(J_n - e^{\sigma M_n}) = J_n$$

$\Rightarrow$ $E_n(J_{n+1})(w_1 \cdots w_n) = J_n$.

$\therefore$ $J_0; J_1 \cdots J_N$ is a martingale.

②. $K_n = \sum_{j=0}^{n-1} M_{j+1}(M_{j+1}-M_j)$, $n = 1, 2 \cdots$.

$\because M_{j+1}(M_{j+1}-M_j) = \frac{1}{2}\left[M_{j+1}^2 - M_j^2 + (M_{j+1}-M_j)^2\right]$.

$$= \frac{1}{2}\left[M_{j+1}^2 - M_j^2 + 1\right].$$

$\therefore K_n = \sum_{j=0}^{n-1} M_{j+1}(M_{j+1}-M_j) = \frac{1}{2}M_n^2 + \frac{1}{2}$.