

**NATIONAL UNIVERSITY OF SINGAPORE**

**FINAL EXAMINATION  
(SEMESTER 1: AY2012/2013)**

**FE5101/ FE5101D DERIVATIVES AND FIXED INCOME**

**Time Allowed: 2.5 Hours**

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**INSTRUCTIONS TO CANDIDATES**

1. This examination paper contains **SIX (06)** printed pages, including this page.
2. This examination paper contains **TWO (02)** parts. Please use separate answer books for each part.
3. This examination paper contains **SIX (06)** questions.
4. Answer **ALL** questions.
5. This is an **OPEN BOOK** examination. You can refer to any materials. You are allowed to use any university approved programmable calculators.
6. The total marks for this paper is 100 marks.
7. You are not allowed to take this examination paper away from the examination hall.

**Part 1****Question 1-1 [30 marks]**

The *continuously compounded* spot rates on the default-free government bond and credit spread for a publicly listed firm ABC Entertainment are given below

Term (in year)	Spot rates (Government) (percent)	Credit Spread for ABC Entertainment (in basis point)
1	3.0	100
2	3.5	150
3	4.0	200

Assume that the recovery rate for the bonds issued by ABC Entertainment is 50 percent.

- (a) Find the risk-neutral probabilities that the ABC Entertainment would default within a year, within two years and within three years. Note that within two years means ABC would default any time from today to two years from today. What is the probability that ABC would default in the third year?  
(8 marks)
- (b) Disregard the spread given in table above and assume that the 2-year-zero coupon bonds issued by ABC Entertainment is selling with 2-year spot rate equal to 3 percent (continuously compounded) and the 2-year spot rate on the Government bond is still 3.5 percent (continuously compounded) as indicated in table above. Can you make arbitrage profit in this situation? Explain your arbitrage strategy. Assume that you can buy or sell 2-year zero-coupon bonds with face value \$100 issued by both the Government and ABC Entertainment. Assume that there are no transaction costs and short selling is allowed. In your arbitrage strategy, you need to clearly indicate which of the two bonds you buy or sell and at what prices.  
(6 marks)
- (c) Consider a 3-year CDS on bonds issued by ABC Entertainment such that the annual premium is paid at the beginning of each year so long as the ABC has not defaulted. So the first CDS premium is paid today and the next premium would be paid in one year if the ABC has not defaulted. The final premium would be paid in two years if ABC has not defaulted. In the event of default, the settlement is made at the end of the year of default. Compute the CDS premium. Assume that the notional amount is \$1. For partial credit, show your calculations.  
(16 marks)

**Question 1-2 [20 marks]**

A bond portfolio manager is holding Treasury bonds portfolio currently worth \$200 million where the total face value of the bonds is equal to \$185 million. The portfolio manager is supposed to keep the duration of the portfolio at around 6.0. However, the manager has flexibility of temporarily changing the duration to take advantage of predicted interest rate movements. Currently, the modified duration of the bond portfolio is 6. The market believes that the Federal Reserve is going to implement an open market operation such that the interest rates on Treasuries of all maturities are going to increase. Therefore, the portfolio

manager wants to temporarily decrease the modified duration of his/her portfolio to 4.0 using T-Bond futures. The following information may be helpful.

The current T-Bond futures price is 90-30 and the modified duration of the CTD T-Bond is 8.5. The estimated regression of the AAA rated corporate yield ( $r_{AAA}$ ) on the Treasury yield ( $r_{Treasury}$ ) is given below:

$$r_{AAA} = 0.0109 + 1.05r_{Treasury}, \quad R^2 = 0.845$$

Similarly, the estimated regression of AA rated corporate yield ( $r_{AA}$ ) on the Treasury yield is given below:

$$r_{AA} = 0.01 + 1.15r_{Treasury}, \quad R^2 = 0.884$$

- (a) How many T-Bond futures contract does the manager needs to sell or buy to reduce the modified duration from 6 to 4? Clearly indicate the number of contract as well as buy or sell.  
(12 marks)
- (b) Give one alternate way of achieving the reduction of modified duration from 6 to 4. Discuss the advantages and disadvantages of the alternate method compared to T-Bond futures.  
(8 marks)

**Part 2****Question 2-1 [8 marks]**

What would be the price of the asymmetric butterfly constructed from the following three options? All options share the same contract details apart from their strike. Prices are in \$ per base (denominator) units. Assume a notional of +1 base units for the 108.00 strike.

Strike	88.00	100.00	108.00
Price	\$3.33	\$8.45	\$13.35

(Choose one)

- A) \$0.22
- B) \$1.49
- C) \$1.85
- D) \$4.90

**Question 2-2 [8 marks]**

Identify the largest arbitrage opportunity from the following four possibilities. Assume zero interest rates; and that one unit of each option is available for purchase or sale at these prices.

(Choose one)

- A) 

Strike	93	100
Put Price	\$4	\$14



- B) 

Strike	93	100
Call Price	\$25	\$14

- C) 

Strike	100	107
Call Price	\$14	\$11
Put Price	\$14	\$12

- D) 

Strike	93	100
Call Price	\$21	\$14
Put Price	\$9	\$14

**Question 2-3 [9 marks]**

Based on the following information, what would the volatilities be for the 25 delta Put and Call components of the strangle and risk reversal?

	Market Smile	Equivalent Smile
Delta Neutral Straddle	30%	30%
25 delta Risk Reversal	-3.0%	-3.2%
25 delta Butterfly	+1.0%	+1.2%

(where the 25 delta Butterfly is strictly the volatility spread between the Delta Neutral Straddle and the 25 delta Strangle)

(Choose one)

	<u>Market Strangle</u>	<u>Market Risk Reversal</u>	<u>Equivalent Strangle</u>	<u>Equivalent Risk Reversal</u>
A) Put : Call	32.5% : 31.1%	32.8% : 29.6%	31.1% : 31.1%	31.5% : 28.5%
B) Put : Call	32.5% : 31.1%	32.5% : 31.1%	32.8% : 29.6%	32.8% : 29.6%
C) Put : Call	31.0% : 31.0%	31.5% : 28.5%	31.1% : 31.1%	31.5% : 28.5%
D) Put : Call	31.0% : 31.0%	31.5% : 28.5%	32.8% : 29.6%	32.8% : 29.6%

**Question 2-4 [25 marks]**

(a) Given the ATM Implied Volatilities:

	Tenor (years)		Implied Volatility
T0	0.00		
T1	0.50	V1	32.00%
T2	1.00	V2	
T3	1.50	V3	28.00%
T4	2.00	V4	



- (i) What would be the Forward Volatility for the period between T1 and T3?  
(2 marks)
- (ii) What would be the interpolated Implied Volatility for T2? (Note this is also the Forward Volatility for the period between T0 and T2)  
(2 marks)
- (iii) What would be the Forward Volatility for the period between T1 and T2, and between T2 and T3?  
(2 marks)
- (iv) If there were a series of major policy and political events in the period between T0 and T1, what would be the effect be on your answers to (i) – (iii) and why?  
(2 marks)
- (v) If there were a series of market holidays in the period between T2 and Y3, what would be the effect be on your answers to (i) – (iii) and why?  
(2 marks)
- (vi) Describe and enumerate the arbitrage constraints on V4.  
(2 marks)

(b) The following GARCH (1,1) parameters are provided:

$$\alpha = 0.04$$

$$\beta = 0.95$$

$$V_L = 0.00024$$

$$V(0) = 0.00012$$



- (i) Describe the meaning of each parameters. (2 marks)

Calculate the following values for the tenors  $n=1$ ,  $n=64$ ,  $n=128$ ,  $n=256$  (where  $n$  is the tenor in trading days):

- (ii) The future instantaneous variance. (2 marks)
- (iii) The average daily variance. (2 marks)
- (iv) The annualized volatility (assume 252 trading days per annum). (2 marks)
- (v) The equivalent shift on the annualized volatility of a 1% shift in the 1-day volatility. (2 marks)
- (vi) Make a representative plot of (ii) and (iii), and give an interpretation of the results. (2 marks)
- (vii) Make a representative plot of (iv) & (v), and give an interpretation of the results. (2 marks)

**- END OF EXAM -**