FE5101 – ASSIGNMENT FOR PART 2
Oct/Nov 2019

INSTRUCTIONS

Precision should be to two decimal places for bond prices, interest rates, option prices and pv's (e.g. 101.15 or 8.12%); and to 5 significant figures for foreign exchange rates and share prices, and to the nearest dollar for notional amounts.

SHOW YOUR WORKING
EXPLAIN ANY ASSUMPTIONS YOU HAVE MADE
INCLUDE AN EXPLICIT REFERENCE AND QUOTATION MARKS FOR ANY OTHER WORK YOU QUOTE

SUBMISSIONS TO BE ON A **GROUP BASIS (UP TO FIVE STUDENTS PER GROUP)**

PLEASE WRITE LEGIBLY – ATTACH MORE PAGES AS NECESSARY
ONLY PDF FILES SUBMITTED ONLINE TO LUMINUS WILL BE ACCEPTED
IF YOU DO NOT HAVE ACROBAT, USE PRIMOPDF (FREE) TO CONVERT WORD/EXCEL FILES
DO NOT ATTACH WORD, EXCEL OR OTHER DOCUMENTS

DEADLINE FOR SUBMISSION IS 19:00 (SG TIME) ON FRIDAY 8TH NOV 2019

NAMES :	STUDENT IDS:

DO NOT SUBMIT ANSWERS BY JUST COPYING SCREENDUMPS OF THE SPREADSHEETS WITHOUT ANY WORKINGS OR EXPLANATION – YOU WILL BE HEAVILY PENALIZED

Question 1 (25 marks)

Refer to the market data sheet posted with this document on LUMINUS.

(i) Calibrate the three GARCH(1,1) parameters using the Maximum Likelihood Method. Explain the steps you took.

(8 marks)

(ii) Calibrate the GARCH(1,1) parameters using the Maximum Likelihood Method and Variance Targeting. Explain the steps you took.

(8 marks)

(iii) Calculate and show graphically, for lags of 1 – 50 trading days, the autocorrelations for:

a. the daily returns
b. the squared returns (or actual variance)
c. the actual variance standardized to the GARCH(1,1) variance estimates based on (i).

Calculate the Ljung-Box statistics for each. Comment on these results and what it means that the calibrated GARCH(1,1) model has achieved.

(9 marks)

Question 2 (25 marks)

This question is about the constraints on the implied volatility surface. Show your working.

All the options in this question are European Options with the same tenor and same underlying (whose numerator unit is $). Assume zero interest rates. Answers should be to 4 decimal places.

Given the ATM Implied Volatilities:

Tenor (years)	Implied Volatility
T0 = 0.00	
T1 = 0.25	30.00%
T2 = 0.50	
T3 = 1.00	28.00%
T4 = 2.00	

(a) Complete the following table of variances and volatility, in your answer sheet. Show your working.

(5 marks)

Period	Implied Volatility	Total Variance
T0→T1	30.00%	
T0→T2		
T1→T2		
T2→T3		
T1→T3		
T0→T3	28.00%	

(b) Given the volatilities provided at the start of the question (assume these are FIXED), if there were a series of major policy and political events in the period T1→T2, what would be the effect on the variances in (b) and why? Do not complete the table in the exam question sheet, make a copy on your answer sheet.

(5 marks)

Period	Impact on expected total variance
T0→T1	
T0→T2	
T1→T2	
T2→T3	
T1→T3	
T0→T3	

(c) what is the arbitrage constraint on the volatility for the period T0→T4 ?

(2 marks)

Given the following option prices:

Strike	$1.3500	$1.4000	$1.5000
Call Premium		$0.1800	$0.1400
Put Premium		$0.1550	$0.2150

(a) what is the prevailing outright forward price?

(1 mark)

(b) what are the maximum and minimum arbitrage-free prices for the Call and Put Options strike $1.3500?

(4 marks)

(c) if the price of the Put strike $1.3500 is $0.1300, what is the arbitrage-free price of the Call strike $1.3500?

(2 mark)

(d) assume the prices for the Call and Put Options strike $1.3500 are $0.2350 and $0.1200 respectively. Identify the three possible arbitrages, explaining in each case:

- the source of the arbitrage (i.e. what price is too high or low in respect of which constraints)
- what trade(s) you would do to monetize the arbitrage, specifying notionals and premiums paid/received
- what the best and worst outcomes would be in cash terms and under which scenarios

(6 marks)

Question 3 (25 marks)

The following volatilities are observed for the three market structures, all with tenor 30 days :

Structure	**Volatility**
Delta Neutral Straddle	35.00%
25 delta Risk Reversal	+4.00%
25 delta Butterfly 25 delta Strangle	+3.00% +38.00%

Assume the Spot Rate = 100.00 and interest rates are zero.

(i) Using the volatilities above : for each of the three market structures, calculate the volatility, strike and price of each of its components, and the net cost of buying the structure.

(5 marks)

(ii) Using the smile function : for each of the market structures, calculate the volatility and price of each of its components, and the net cost of buying the structure.

(5 marks)

(iii) Solve for (calibrate) the smile function. Show the calibrated 25 delta Put option and 25 delta Call option volatilities and explain the steps you made in calibration.

(3 marks)

(iv) Construct a vega-neutral butterfly as a ratio spread of delta-neutral straddles and market strangles.

(1 mark)

(v) Calculate the vega, vanna and volga of each of the delta neutral straddle, the market risk reversal, and the vega-neutral butterfly from (v). Evaluate the 'Black-Scholes price' for each using the atm volatility for all components.

(3 marks)

(vi) Calculate the weights of each of the three structures in (v) required, to build three portfolios that have:

(3 marks)

(vii) For each of these three portfolios, calculate the differences between their respective weighted prices and weighted Black-Scholes prices. Calculate the implied change in price per unit of each of vega, vanna and volga.

(3 marks)

(viii) For a Call option of tenor 30 days and strike 103.10, calculate the smile price using (a) the polynomial smile function, and (b) the adjustments from (vii).

(2 mark)