FE 5101:
FIXED INCOME AND DERIVATIVES
EXERCISES – LECTURE 3

Exercise 1: Comparing Fixed Income Instruments

Each of these 3 Instruments have a contractual maturity of 3 years, a price of $1m, and a YTM of 6.00%. But they have different cash flow structures:

Year	Instrument 1 Annuity	Instrument 2 Coupon Bond at Par	Instrument 3 0-Coupon Note
0	(1,000,000)	(1,000,000)	(1,000,000)
1	374,110	60,000	0
2	374,110	60,000	0
3	374,110	1,060,000	1,191,016

1. Their Present Values (i.e. market prices) will not change in an equal way for a given change in yield. Give reasons for this in terms of:

   a. Duration **the different structure of cash flows gives them different modified durations and thus different sensitivities to a given change in YTM or the yield curve.**

   b. How the yield curve moves – **a given change in YTM implies that the yield-curve is moving in parallel fashion, but if the yield curve moves in non-parallel fashion, steepening or flattening, then only Instrument 3, the zero-coupon will change as predicted by modified duration or DV01. The others will have different changes in value due to having interim cash flows.**

2. Comparing these three instruments, (without using a calculator)

   a. Which has the shortest duration? **Instrument 1**

   b. Which has the longest duration? **Instrument 3**

   c. What would happen to the Coupon Bond's duration if its coupon were higher? **It would shorten because more cash is coming back to the investor sooner**

   d. What would happen to the Coupon Bond's duration if its coupon were lower? **It would lengthen, because it would look more like a zero-coupon instrument as coupon ~> 0.**

   e. Which of the three instruments has MacAulay duration = contractual maturity? **No. 3 – the zero-coupon**

3. If I were managing a bond fund and had to stay invested in at least one of these three instruments,

   a. Which would you suggest I hold if I had a strong near-term outlook predicting lower 3-year interest rates? **In such case, move all into the instruments with longest duration to get the biggest profit on a drop in interest rates**

   b. Why do you recommend that one?

      It has the highest DV01 and modified duration

4. If you were comparing a Zero coupon whose price is $1m and maturity is 2 years 10 months (2.8334 years to be exact), with a yield of 6% and a future value of $1,179,510.03 = ($1m x (1.06^2.8334)

   a. What would be its DV01 (you have to calculate this by taking its PV at 6.01%)?

      Re-valuing $1,179,510 by dividing it by 1.0601^2.8334 gives a new PV = $999,732.75

      $267.28 – the same as Instrument no 2, which has the same MacAuley duration of 2.8334 years (except for rounding).

   b. What other instrument above also has that same DV01? What can we generalise about MacAulay Duration based on these two instruments' sharing this attribute?
      That is to say that a zero-coupon of maturity = to the MacAuley duration of a non-zero instrument (with interim cashflows like coupons) has

      The same macauley duration in years
      The same sensitivity to a change in yield (DV01)

Exercise 1: Self-study Revisit

Using the Excel Spreadsheet from Exercise 1

1. How do the 3 duration measures change if we change the yield from 6.00% to **6.01%?**

    a. Macaulay changes **by almost nothing, but is getting a very minimum amount shorter for instruments 1 and 2**

    b. Modified changes **by almost nothing, but is getting a very minimum amount smaller for all 3 instruments**

    c. DV01 changes **by almost nothing, but is getting a very minimum amount getting smaller for all 3 instruments**

2. What are the 3 duration measures now if we change the yield from 6.00% to 8.00%?

    a. Which one does NOT change? Why?

Instrument → Measure	Annuity	Coupon Bond	Zero-Coupon
Macaulay	1.949yrs	2.829yrs	No change For Zero, Mac Duration = Maturity
Modified	1.804%	2.619%	2.778%
DV01	$173.99	$248.46	$262.68

3. Now go back to YTM = 6% and 6.01%

    a. What is the DV01 of the 6% bond? What is its relationship to Modified Duration? **$267 for a 1bp change in YTM. It's approximately 1/100 of Modified Duration of 2.673% of market value**

    b. What does it predict the bond's price will be if we change the yield to 7%? **It predicts a 2.67% change from $1,000,000 to 973,300. That is (100% - 2.67%) X $1m**

    c. Changing the yield to 7%, what is the bond's new price? **The price changes to $973,757**

d. What accounts for the difference between the predicted and the “full valuation” price? **Full valuation encompasses the second-derivative effect of convexity, whereas using the DV01 to predict a 100-bp YTM change’s effect is predicting with only the first derivative.**

Exercise 2b: DV01-Neutral Curve Trade:

Following is a step-wise discussion of yield-curve trading strategies and the answers to Lecture 3 Exercise 2b Homework.

The slides are included, and additional discussion of logic and how the answers are arrived at in the notes following some of the slides.

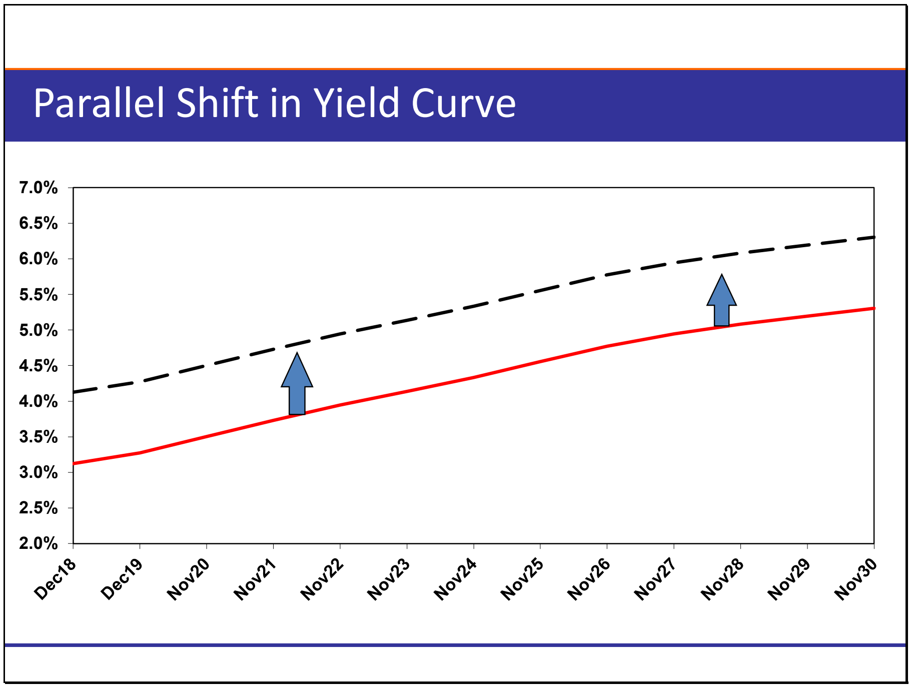
Yield-curve motivated Trading Strategies

- Possible Ways to trade:
- Outright Directionality Trading Positions
  - Long or Short position in a Bond of specified issuer and maturity
    - Long Bonds – if you think market yields going lower
    - Short Bonds if you think market yields going higher
  - E.g. buy $1m of 10yr bonds with $790 DV01
    - And profit if rates to go lower
    - And lose money if rates go higher
  - Called "outright" because its P&L depends on absolute change in yields
- Relative Value Trading Positions
  - Long of a specific bond, and
  - Short of a different bond
  - Looking for a relative change in the bonds' interest rates
  - Very often trades of this type are for changes in the yield curve

Generally, outrights will make profit or loss depending on how the yield most closely related to the bond in use has changed. For example, if the 10-yr yield goes lower by 5bp, the $1m long position in the 10yr bond above can be expected to make a profit of 5 x $790 = $3,950.

Keep in mind that the DV01 gives a pretty accurate estimate of the P&L, but the actual outcome P&L will be mildly influenced by:

Convexity – the DV01 changes as yields move
Time – all things being equal, the DV01 gets smaller as time passes and the bond gets closer to maturity

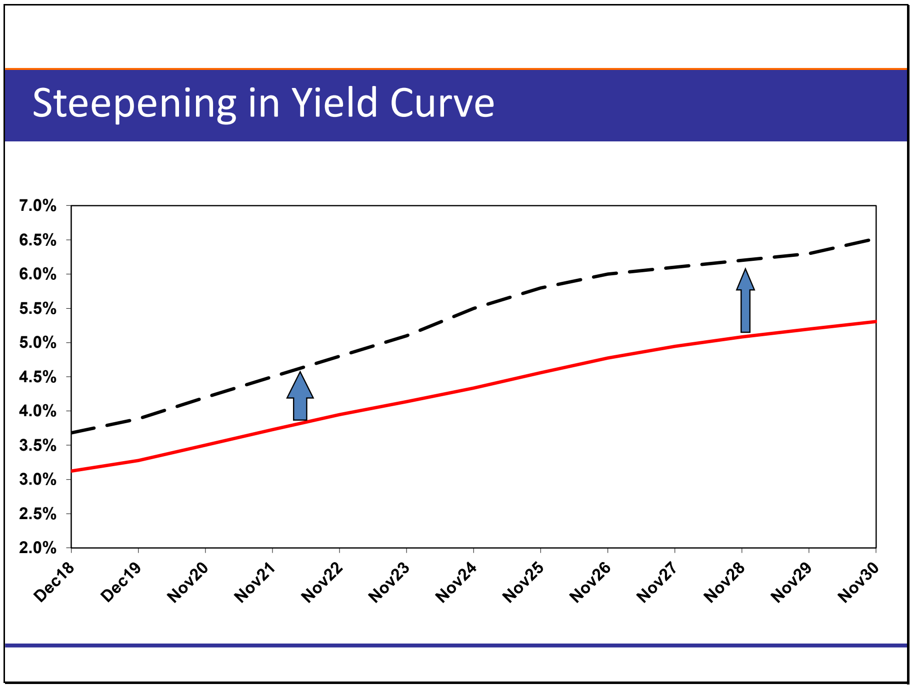


Parallel shifts in the yield curve, whereby all rates across the curve change by the same per annum yield (above a 1% shift upward), seldom occur. However, many risk management policies/procedures/departments measure the $-change in portfolio value (e.g. a dealer trading-book, or a fund manager's long-only bond portfolio) for a 1% yield-curve shift as a standard risk management parameter.

Also, when we use YTMs as a surrogate for using the entire zero-curve in analyses like MacAuley and Modified Duration, we are implicitly assuming the yield-curve shifts parallel. For certain uses-cases (e.g. apportioning risk limits to bond trading desks), such a simplifying approach and assumption does not have much impact on the analysis, but in others (e.g. pricing and valuation of a hedge-fund's complex trading position), it is better to avoid such a simplifying assumption in favor of more robust yield-curve scenario analysis and simulation.

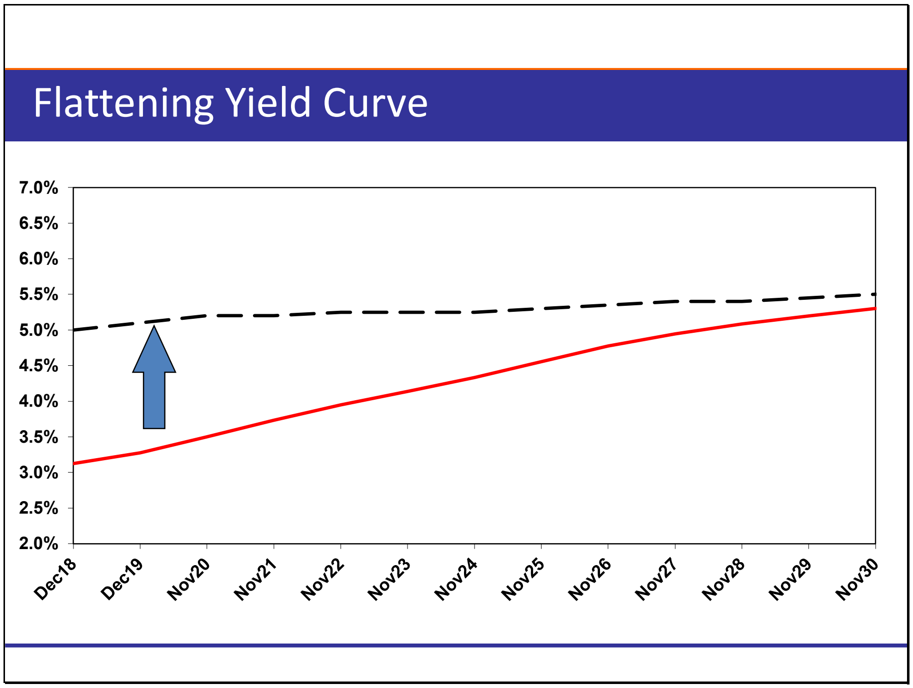
DV01 Neutral *Curve Trade*

- A "curve trade" usually consists of a long-short that will benefit from a steepening or flattening of the yield curve
  - Occur often in Bond and Interest Rate Derivatives trading operations
- Curve-trades are often sized to neutralise the position from having P&L due to parallel shifts in the yield curve
  - "DV01" Neutral → Long and short bond in curve-trade both have same $DV01



The above picture shows steepening in the context of an overall rise in yields across the curve ("bear-market steepening"). This is one of 4 possible ways a curve can steepen. The other 3 are

- Short term rates go lower and longer term ones stay the same ("steepening at the short-end of the curve")
- Long term rates go higher, while short-term ones stay the same ("steepening at the long-end")
- All rates go lower, but the longer-dated ones by a lesser amount ("bull-market steepening")



The above picture shows flattening in the context of an overall rise in yields across the curve ("bear-market flattening"). This is one of 4 possible ways a curve can flatten. The other two are

- Short term rates go higher and longer term ones stay the same ("flattening at the short-end")
- Long term rates go lower, while short-term ones stay the same ("flattening at the long-end")
- All rates go lower, but the longer-dated ones by a great amount ("bull-market flattening")

Homework: Lecture 3, Ex 2: Trade-sizing to DV01

- Use the bond pricing spreadsheet to do the following exercise
- Below you see two points on a yield-curve situation for which the "2yr vs, 10yr" spread (10yr YTM – 2yr YTM) is at +150 bp
- A long-short portfolio which earns profit (makes losses) if the yield curve becomes flatter (steeper)
  - Go long \$14.8m of the 10yr (YTM = 6.0%)
  - Go short \$52.5m of the 2yr (YTM = 4.5%)
  - Size each bond at DV01 ≈ \$10,000

Bond 1	Bond 2
Maturity = 10 years	Maturity = 2 years
Coupon = 4% s.a.	Coupon = 5% s.a.
Yield = 6%	Yield = 4.5%

YOU ARE ADVISED TO USE THE BOND-PRICING SPREADSHEET TO FOLLOW ALONG WITH THIS SET OF ANSWERS

Sizing of Bonds to get appx \$10,000 DV01 on each of long and short. The 10yr bond is trading at a discount, so we must take this into account to get correct sizing of its par amount, as follows:

Bond 1 = 10yr. Price = 85.12; mod dur = 7.931%, and DV01 = \$10,000 if PAR size is appx \$14.8m;

Mod dur x price ÷ 10000 ≈ appx DV01/\$100par = .07931 x \$0.8512 = \$0.0675/\$100par

0.0675 x 14.8m ÷ 100 =\$10,000 appx

Bond 2 = 2yr. Price = 100.946; mod dur = 1.886% DV01 = \$10,000 when PAR size is \$52.5m; DV01 = 1.904bp

0.01904 x 52.5m ÷ 100 = \$10,000 appx

Short-cut appx for ratio ≈ (Mod Dur x price of BIG DV01) ÷ (Mod Dur x Price of SMALL DV01) = (7.931% x 85.12) ÷ (1.886% x 100.95) = 3.547X ratio of par amount of the 2yr to the par amount of the 10yr (i.e. 52.5m ÷ 14.8m = 3.547X)

Lecture 3, Exercise 2 continued

- What P&L from the long-short trade if the yield curve…
  - …*Flattens* with YTM_{10yr} = 5.80% and YTM_{2yr} = 4.90%
    - Now 2yr-10yr spread at 90bp (60bp lower than 150bp trade inception)
    - Approximate $P\&L_{10yr}$ = $201,740 profit
    - Approximate $P\&L_{2yr}$ = $397,876 profit
    - Position P&L = $599,616 profit ≈ 60bp x $10,000/bp
  - …*Flattens* with YTM_{10yr} = 7.20% and YTM_{2yr} = 6.10%
    - Spread now at +110bp (40bp lower than 150bp at trade inception)
    - Approximate $P\&L_{10yr}$ = +120bp x -$10,000/bp = -$1.2m Loss
    - Approximate $P\&L_{2yr}$ = +160bp x +$10,000/bp = +$1.6m Profit
    - Position P&L = $400,000 net profit ≈ 40bp x $10,000/bp

Bond 1 pos'n DV01 = $10,000. YTM up 120bp so losses = appx $1.2m
Bond 2 pos'n DV01 = $10,000. YTM up 160bp so gains on short position of appx $1.6m

Try this out using the bond-pricing spreadsheet and you should find your actual P&L numbers pretty close to these estimated amounts

Lecture 3, Exercise 2 (homework)

- What is the P&L if the yield curve..
  - ..*Steepens* with YTM_{10yr} = 6.00% and YTM_{2yr} = 3.90%
    - Spread now at + 210bp
    - Approximate $P\&L_{10yr}$ = 0 x $10,000 = 0
    - Approximate $P\&L_{2yr}$ = -60 x $10,000 = - $600,000 Loss
    - Position P&L = 0 + - $600,000 = - $600,000 Loss
  - ..*Shifts parallel* with YTM_{10yr} = 6.50% and YTM_{2yr} = 5.00%
    - Spread stayed at 150bp
    - Approximate $P\&L_{10yr}$ = -50 x $10,000 = -$500,000 Loss
    - Approximate $P\&L_{2yr}$ = +50 x $10,000 = +$500,000 Gain
    - Position P&L = +$500,000 + -$500,000 = no P&L

If it steepens the trade loses, so in this case from 150bp to 210 an adverse change of 60bp for a loss of appx $600k

Parallel shift, except for rounding and convexity, we expect flat net P&L on the strategy