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Question 1

(i) Calibrate the three GARCH(1,1) parameters using the Maximum Likelihood Method.

a) Start with trial value of α, β, ω

b) Calculate Likelihood = $\sum_{i=1}^{m} -\ln(v_i) - \frac{{u_i}^2}{v_i}$

1. m is the number of return data points, $m = 522$

2. u_i is return, $u_i = \ln(\frac{S_i}{S_{i-1}})$

3. v_i is daily variance estimated by GARCH, $v_I = \omega + \alpha u_{i-1}^2 + \beta v_{i-1}$

c) Use Excel Solver to search for α, β, ω that maximize Likelihood

d) The Parameters are $\alpha = 0.070028, \beta = 0.905095, \omega = 0.0000008$

GARCH(1,1)

	parameter					
alpha α	0.070028					
beta β	0.905095					
gamma γ	0.024877					
Without variance targetting:				**With variance targetting:**		
V_L	0.0000324			V_L	0.0000535	
omega ω	0.0000008			omega ω	0.0000013	
annualized volatility	9.20%			annualized volatility	11.81%	
likelihood	4746.21257	*(to be maximized)*		likelihood	#N/A	

(ii) Calibrate the GARCH(1,1) parameters using the Maximum Likelihood Method and Variance Targeting.

a) Using the same method to calculate Likelihood

b) Fix V_L, long run average variance to unweighted average daily variance

c) Use Excel Solver to search for α, β that maximize Likelihood. In this case ω is fixed, $\omega = V_L * (1 - \alpha - \beta)$

d) The Parameters are $\alpha = 0.042546, \beta = 0.954045, \omega = 0.0000002$

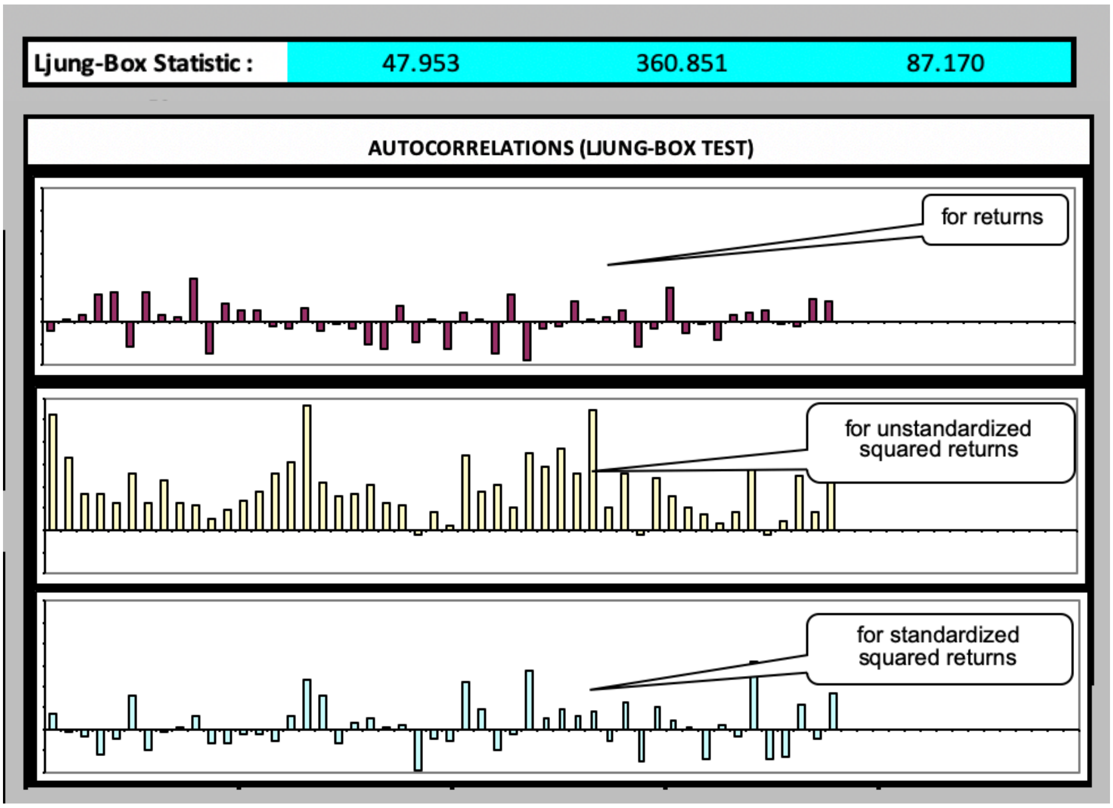
GARCH(1,1)

	parameter					
alpha α	0.042546					
beta β	0.954045					
gamma γ	0.003409					
Without variance targetting:				**With variance targetting:**		
V_L	0.0002425			V_L	0.0000535	
omega ω	0.0000008			omega ω	0.0000002	
annualized volatility	25.16%			annualized volatility	11.81%	
likelihood	#N/A	*(to be maximized)*		likelihood	4757.65906	

(iii) Calculate the Ljung-Box statistics for each. Comment on these results and what it means that the calibrated GARCH(1,1) model has achieved.

a) The autocorrelation for daily returns seems not significant since Ljung-Box statistics is 47.953, relatively small.
b) The autocorrelation for daily variance(squared returns) seems significant since Ljung-box test statistics is 360.851, relatively large. Lag 1, 17, 35 are at peak compared to other lags.
c) The autocorrelation for standardized variance seems not significant since Ljung-box test statistics is 87.170, relatively unremarkable. However, several lags have larger autocorrelation compared to others.

In conclusion, there is heteroskedasticity in return series and applying GARCH method is reasonable. GARCH(1,1) achieves acceptable results as the standardized squared returns generally are not autocorrelated.



Question 2

Given the ATM Implied Volatilities:

Tenor (years)	Implied Volatility
T0 = 0.00	
T1 = 0.25	30.00%
T2 = 0.50	
T3 = 1.00	28.00%
T4 = 2.00	

(a) Complete the following table of variances and volatility, in your answer sheet.

Period	Implied Volatility	Total Variance	Comment on how we calculate total variance
T0→T1	30.00%	0.0225	0.3*0.3*0.25
T0→T2	0.2867	0.0411	0.0225+0.0186
T1→T2	0.2728	0.0186	0.0559/3
T2→T3	0.2728	0.0372	0.0559/3*2
T1→T3	0.2728	0.0559	0.28*0.28-0.0225
T0→T3	28.00%	0.0784	0.28*0.28

(b) Given the volatilities provided at the start of the question (assume these are FIXED), if there were a series of major policy and political events in the period T1→T2, what would be the effect on the variances in (b) and why? Do not complete the table in the exam question sheet, make a copy on your answer sheet.

Period	Impact on expected total variance
T0→T1	Expected total variance won’t change since the major events happened after T1.
T0→T2	Expected total variance increase.
T1→T2	Expected total variance increase, and the increased amount equals that of T0-T2.
T2→T3	Expected total variance decrease, and the decreased amount equals the increased amount of T1-T2, since the total variance from T1-T3 is fixed.
T1→T3	Won’t change
T0→T3	Won’t change

(c) what is the arbitrage constraint on the volatility for the period T0→T4 ?

$$\sigma_{T0-T4}{}^2 * (T_4 - T_0) \geq \sigma_{T0-T3}{}^2 * (T_3 - T_0) = 0.28^2 * 1$$

$$\sigma_{T0-T4} \geq 0.1979$$

Given the following option prices:

Strike	$ 1.3500	$ 1.4000	$ 1.5000
Call Premium		$ 0.1800	$ 0.1400
Put Premium		$ 0.1550	$ 0.2150

(a) what is the prevailing outright forward price?

According to call-put parity, we have

$$c(0) - p(0) = s(0) - K * D(T)$$

and we have condition1: c(0)=0.18,p(0)=0.155,K=1.4 and

condition 2: c(0)=0.14,p(0)=0.215,K=1.5

Hence we get **S(0) = 1.425 , D(T)=1**

(b) what are the maximum and minimum arbitrage-free prices for the Call and Put Options strike $1.3500?

To be arbitrage-free, the prices of options need to meet following conditions:

1. C(1.35)-P(1.35)+1.35 = 1.4250
2. 0≤(C(1.35)-C(1.4))/(1.4-1.35) ≤1
3. 0≤(C(1.35)-C(1.4))/(1.4-1.35)- (C(1.4)-C(1.5))/(1.5-1.4) ≤1
4. 0≤(P(1.4)-P(1.35))/(1.4-1.35) ≤1
5. 0≤(P(1.5)-P(1.4))/(1.5-1.4)- (P(1.4)-P(1.35))/(1.4-1.35) ≤1

Hence we have

$$0.2 \leq C(1.35) \leq 0.23$$
$$0.125 \leq P(1.35) \leq 0.155$$

(c) if the price of the Put strike $1.3500 is $0.1300, what is the arbitrage-free price of the Call strike $1.3500?

$$c = p + s(0) - K * D(T) = 0.205$$

(d) assume the prices for the Call and Put Options strike $1.3500 are $0.2350 and $0.1200 respectively. Identify the three possible arbitrages, explaining in each case:

1. call-out parity (K=1.35)

   c – p =0.115 > s(0) – K * D(T) = 0.075

   The price of the call option is too high in respect of the put option. Consider portfolio

   V = P(1.35) – C(1.35) + S

   V(0) = P(t0) - C(t0) + S(t0) =0.12 – 0.2350 + 1.425 = 1.31

   V(T) = $(K - S(T))^{+} - (S(T) - K)^{+} - 1.31 + S(T)$ = K – 1.31 =0.04

   Hence the **notional amount is 1.31 and P&L is +0.04.** There is no risk in the arbitrage strategy, no matter what S(T) is, we can lock in the P&L +0.04.

2. call spread

   consider portfolio V= - C(1.35) + C(1.4)

V(t0) = -0.235 + 0.18 = -0.055 **notional amount = - 0.055**

If S(T) ≤1.35, V(T) = 0. P&L = 0.055 **(Best)**

If 1.35 <S(T) <1.4, V(T) = 1.35- S(T) P&L = 1.405- S(T)

If S(T) > =1.4, V(T) = (1.35 - S(T)) + (S(T) - 1.4) = - 0.05 P&L = 0.005 **(Worst)**

3. put butterfly

consider portfolio V= P(1.35) – 1.5*P(1.4) + 0.5 * P(1.5)

V(t0) = 0.12 – 1.5 * 0.155 + 0.5*0.215= -0.005 **notional amount = - 0.005**

If S(T) ≤1.35, V(T) = 0. P&L = 0.005 **(Worst)**

If 1.35 < S(T) <1.4, V(T) = S(T) – 1.35 P&L = S(T) – 1.345

If 1.4 ≤S(T) <1.5, V(T) = 0.75 – 0.5 * S(T) P&L = 0.755 – 0.5 * S(T) **（Best when S(T)=1.4，P&L=0.055）**

If S(T) ≥1.5, V(T)=0, P&L = 0.005 **(Worst)**

Question 3

(i) Using the volatilities above : for each of the three market structures, calculate the volatility, strike and price of each of its components, and the net cost of buying the structure.

Delta Neutral Straddle:

Straddle Strike for Delta Neutral (DN)

P		C	
35.00%	vol	35.00%	
99.4978	strike	99.4978	**total**
3.7453	price	4.2474	**7.993**

The both volatility of put and call are exactly σ_{ATM} which is equal to **35%**

The strike price we can use the formula: $K = F \cdot e^{-1/2 \cdot \sigma^2 (T-t)}$ to calculate and is **99.4978**

And we can use BSM to calculate the price of each components which is shown above and the total cost buying the structure is **7.993** DEN per NUM payable.

STGL

Market price of market STGL

25dP		25dC	
38.000%	vol	38.000%	
25%	raw delta	25%	
93.4683	strike	108.2652	**total**
1.718	price	1.540	**3.258**

The both volatility of call and put can be calculate as $\boldsymbol{\sigma_{ATM} + \sigma_{BF} = 35\% + 3\% = 38\%}$

The Strike price of put and call can be calculate by the definition of delta

The 25DP. $-N\left(-\frac{\ln\left(\frac{100}{K}\right)+0.5*0.38^2*\left(\frac{30}{365}\right)}{0.38*\sqrt{\frac{30}{365}}}\right) = -25\%$ we get strike of put option **93.4683**

The 25DC. $N\left(\frac{\ln\left(\frac{100}{K}\right)+0.5*0.38^2*\left(\frac{30}{365}\right)}{0.38*\sqrt{\frac{30}{365}}}\right) = 25\%$ we get the strike of call option is **108.2652**

The price of put option is **1.718, of call option is 1.540**

And we will cost **3.258** DEN per NUM payable to buy this structure.

Risk Reversal

Market price of market RR

25dP		25dC	
36.000%	vol	40.000%	
25%	raw delta	25%	
93.7735	strike	108.7545	**total**
1.623	price	1.617	**-0.006**

The volatility of call and put option can be calculate by using $\boldsymbol{\sigma_{ATM} + \sigma_{BF} \pm 1/2\sigma_{RR}}$**=40%/36%**

The 25DP. $-N\left(-\frac{\ln\left(\frac{100}{K}\right)+0.5*0.36^2*\left(\frac{30}{365}\right)}{0.36*\sqrt{\frac{30}{365}}}\right) = -25\%$ we get strike of put option **93.7735**

The 25DC. $N\left(\frac{\ln\left(\frac{100}{K}\right)+0.5*0.4^2*\left(\frac{30}{365}\right)}{0.4*\sqrt{\frac{30}{365}}}\right) = 25\%$ we get the strike of call option is **108.7545**

Use BSM, we can solve the price of put option is **1.623, call option is 1.617**

We will cost **-0.006** DEN per NUM payable to buy this structure.

(ii) Using the smile function : for each of the market structures, calculate the volatility and price of each of its components, and the net cost of buying the structure

We use the smile function as the following:

Smile Function σ(K)

25dP	ATM	25dC	
93.7735	99.4978	108.7545	strike
-6.43E-02	-5.03E-03	8.39E-02	ln K/F
36.000%	35.000%	40.000%	vol
0.3507	0.1731	4.9311	polynomial

value the market structures using the smile function σ(K), three points are first required, and then a polynomial function is solved to pass through these points.

We give 25% delta put option 36% volatility, 25% delta call option 40% and call or put option at 35% volatility.

Using the smile function above we can calculate the three structure as the followings:

Straddle Strike for Delta Neutral (DN)

P		C	
35.00%	vol	35.00%	
99.4978	strike	99.4978	total
3.7453	price	4.2474	**7.993**

we cost 7.993 DEN per NUM for DN

Smile price of market RR (as two separate options)

93.7735	strike	108.7545	
-0.0643	log strike	0.0839	
36.000%	smile vol	40.000%	total
1.623	price	1.617	**-0.006**

we cost -0.006 DEN per NUM for RR

Smile price of market STGL (as 2 separate options)

93.4683	strike	108.2652	
-0.0675	log strike	0.0794	
36.155%	smile vol	39.559%	total
1.552	price	1.684	**3.235**

we cost 3.235 DEN per NUM for STGL

(iii) Solve for (calibrate) the smile function. Show the calibrated 25 delta Put option and 25 delta Call option volatilities and explain the steps you made in calibration.

What we solve is as the followings:

Smile Function σ(K)

25dP	ATM	25dC	
93.7735	99.4978	108.7545	strike
-6.43E-02	-5.03E-03	8.39E-02	ln K/F
36.128%	35.000%	40.128%	vol
0.3507	0.1683	5.1734	polynomial

Actually is near what we guess at start.

The 25DP volatility is **36.128%**

The 25DC volatility is **40.128%**

Frist we guess a volatility for put and call option, you can just guess which are equal to the RR structure, and you will find that the error of STGL is not nil, means we should slight amend our volatility to make the error of STGL is nil. And use the linear programming to solve the vol.

(iv) Construct a vega-neutral butterfly as a ratio spread of delta-neutral straddles and market strangles.

We can build the vega-neutral butterfly like the following:

+VNBF = $w * DNSTDL(\sigma_{ATM}) - STGL(\sigma = \sigma_{ATM} + \sigma_{BF})$

And the Vega of DNSTDL is : 11.380+11.380 = 22.76

The Vega of STGL is : 8.805+8.690 = 17.495

Thus the riato w = 17.495/22.76 = 0.76867

So we can **long 0.76867 shares DNSTDL and short 1 share STGL to construct the VNBF.**

Or equivalently, **long 1 share DNSTDL and short 1.3010 STGL.**

(v) Calculate the vega, vanna and volga of each of the delta neutral straddle, the market risk reversal, and the vega-neutral butterfly from (v). Evaluate the 'Black-Scholes price' for each using the atm volatility for all components

We use the atm volatility to calculate the vega, Volga, vanna and BS price as the followings:

	DNSTDL	25DRR	VNBF
BS VOL (BSV)	35.00%	35.00%	35.00%
BSV VEGA	22.760	-0.612	0.000
BSV VOLGA	0.000	6.220	-34.899
BSV VANNA	0.000	1.272	-0.237
BSV PRICE	7.993	-0.354	4.451

(vi) Calculate the weights of each of the three structures in (v) required, to build three portfolios that have:

Only Vega, no Vanna, Volga:

DNSTDL	25DRR	VNBF
1.000	0	0

Only Vanna, no Vega and Volga:

DNSTDL	25DRR	VNBF
0.0269	1.000	0.1782

Only Volga, no Vega and Vanna:

DNSTDL	25DRR	VNBF
0.0050	0.1863	1.000

(vii) For each of these three portfolios, calculate the differences between their respective weighted prices and weighted Black-Scholes prices. Calculate the implied change in price per unit of each of vega, vanna and volga.

For portfolios only have Vega:

	DNSTDL	25DRR	VNBF	Total

weight	1.000	0	0	
Weight BSV PRICE	7.9927	0	0	
Weight MKT PRICE	7.9927	0	0	
W MKT ADJ	0	0	0	0
W BSV Vega	22.760	0	0	22.760
W BSV Volga	0	0	0	0
W BSV Vanna	0	0	0	0
ADJUST PER UNIT RISK(W MKT ADJ/W BSV Vega)				nil

For portfolios only have Vanna:

	DNSTDL	25DRR	VNBF	Total
weight	0.0269	1.000	0.1782	
Weight BSV PRICE	0.2151	-0.3538	0.7933	
Weight MKT PRICE	0.2151	-0.0057	0.6690	
W MKT ADJ	0	0.3480	-0.1243	0.2237
W BSV Vega	0.6125	-0.6125	0	0
W BSV Volga	0	6.2198	-6.2198	0
W BSV Vanna	0	1.2720	-0.0422	1.2298
ADJUST PER UNIT RISK(W MKT ADJ/W BSV Vanna)				0.1819

For portfolios only have Volga:

	DNSTDL	25DRR	VNBF	Total
weight	0.0050	0.1863	1.000	
Weight BSV PRICE	0.0401	-0.0659	4.4511	
Weight MKT PRICE	0.0401	-0.0011	3.7536	
W MKT ADJ	0	0.0648	-0.6975	-0.6326
W BSV Vega	0.1141	-0.1141	0	0
W BSV Volga	0	1.1589	-34.8991	-33.7402
W BSV Vanna	0	0.2370	-0.2370	0
ADJUST PER UNIT RISK(W MKT ADJ/W BSV Volga)				0.0187

(viii) For a Call option of tenor 30 days and strike 103.10, calculate the smile price using (a) the polynomial smile function, and (b) the adjustments from (vii).

(a) use polynomial smile function:

	25dP	ATM	25dC	(Call)
vol	36.13%	35.00%	40.13%	36.125%
strike	93.7735	99.4978	108.7545	103.1000
call price	7.8609	4.2474	1.6286	2.8246

The price is **2.8246**

(b) use the adjustments from(vii)

use the atm volatility 35% give the BS price is 2.70, and BS Volga is 2.849, BS Vanna is 0.391

so the adj price is :

2.7+ 2.849*0.0187 + 0.391*0.1819 = **2.8244**

And we can see the methods give the very similar answer for the price.