FE5208 Term Structure and Interest Rate Derivatives

Section 6. Managing the Interest Rate Risks

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6.1. IR PV01

The bucket-by-bucket sensitivity of portfolio value to money market interest rates and swap rates.

$$Delta_{port}\,(T_i) = \sum_{trade} Delta_{trade}(T_i)$$

$$= \sum_{trade} \frac{\partial NPV(trade)}{\partial R(T_i)}.$$

where $R(T_i)$ are the interest rates for tenor T_i on the market yield curve. The summation runs over all trades in the portfolio.

IR PV01

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6.2. Cap/Floor (Caplet/Floorlet) Vegas

Cap/Floor Volatilities

The interest rate option market quotes volatility smiles for caps/floors and swaptions.

Cap Volatility Smile

<HELP> for explanation.
Click on Actions -> Copy Cube to edit the cube.

P216 Equity VCUB

91) Actions | 92) Charts | 94) Refresh | 95) Asset | Interest Rate Volatility Cube

Ref Currency US | Name USD Bloomberg Cube | Privilege BLP | Factor 1.00 | Spread 0.00

Swap Curve 11) (23) USD Swaps(30/360,S/A)

Market View

Caps - Black Volatility | BBIR | Bid | 04/28/12

Term	Strike	ATM	1.00%	1.50%	2.00%	3.00%	3.50%	4.00%	5.00%	6.00%	7.00%	8.00%	9.00
1 YR	1.23	58.30	69.25	80.50	88.95	101.00	105.60	109.55	116.25	121.70	126.30	130.30	133.8
2 YR	0.56	62.25	63.90	66.35	68.30	71.45	72.75	74.00	76.20	78.15	79.95	81.60	83.15
3 YR	0.68	67.45	67.65	67.45	66.85	65.50	64.90	64.35	63.45	62.75	62.20	61.80	61.55
4 YR	0.90	67.20	66.65	64.25	62.10	58.75	57.45	56.40	54.65	53.35	52.30	51.50	50.85
5 YR	1.13	62.80	64.00	59.65	56.30	51.95	50.50	49.45	47.95	47.00	46.35	45.95	45.70
6 YR	1.36	55.20	59.50	53.90	49.85	44.90	43.45	42.40	41.15	40.50	40.20	40.05	40.05
7 YR	1.59	51.20	58.10	52.10	47.85	42.70	41.20	40.10	38.75	38.05	37.70	37.55	37.55
8 YR	1.78	47.15	56.15	49.80	45.30	39.85	38.25	37.05	35.60	34.85	34.45	34.30	34.25
9 YR	1.92	44.45	55.00	48.45	43.90	38.35	36.65	35.45	33.90	33.10	32.70	32.50	32.45
10 YR	2.07	41.30	53.10	46.45	41.85	36.15	34.45	33.20	31.65	30.85	30.45	30.30	30.20
12 YR	2.29	38.70	52.05	45.35	40.75	35.10	33.35	32.10	30.45	29.60	29.10	28.85	28.70
15 YR	2.52	34.95	50.45	43.25	38.45	32.50	30.70	29.40	27.80	27.05	26.70	26.55	26.50
20 YR	2.69	32.90	51.60	43.15	37.70	31.25	29.30	27.95	26.30	25.50	25.10	24.95	24.90
25 YR	2.77	32.25	53.25	43.70	37.80	31.00	29.00	27.60	25.90	25.10	24.75	24.60	24.55
30 YR	2.81	31.35	53.90	43.55	37.35	30.35	28.35	26.95	25.35	24.65	24.35	24.20	24.20

LIBOR Index: US0003M

20) Cube | 21) Configuration

Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
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SN 811301 H189-617-2 28-Apr-12 16:37:13 CDT GMT-5:00

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At-the-Money Swaption Volatilities

LogNormal Vols for strike offset 0							
0	1Y	2Y	5Y	10Y	15Y	20Y	30Y
1M	12,4559%	15,6906%	18,5884%	18,7795%	18,0630%	17,7101%	17,5578%
3M	14,1064%	17,1848%	19,5244%	19,6338%	18,9183%	18,5667%	18,4021%
6M	15,3294%	18,1170%	20,7427%	20,7399%	19,9287%	19,5322%	19,3833%
1Y	17,7833%	20,1802%	22,1637%	21,9932%	21,0658%	20,5908%	20,3785%
2Y	22,1572%	23,2539%	23,9005%	23,0334%	21,9114%	21,2860%	21,0047%

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Swaption Volatility Smile

LogNormal Vols for strike offset -25

-25	1Y	2Y	5Y	10Y	15Y	20Y	30Y
1M	16,1224%	18,7759%	21,1707%	21,2160%	20,4002%	20,0122%	19,8596%
3M	16,9077%	19,5302%	21,4938%	21,4442%	20,6626%	20,2832%	20,1363%
6M	17,2860%	19,7721%	#N/A	#N/A	#N/A	#N/A	#N/A
1Y	19,1239%	21,4288%	#N/A	#N/A	#N/A	#N/A	#N/A
2Y	23,3314%	24,4099%	#N/A	#N/A	#N/A	#N/A	#N/A

LogNormal Vols for strike offset 25

25	1Y	2Y	5Y	10Y	15Y	20Y	30Y
1M	12,1184%	15,1527%	18,5501%	19,1003%	18,4222%	18,0859%	17,9512%
3M	13,1751%	16,3615%	19,1536%	19,5366%	18,8505%	18,5098%	18,3513%
6M	14,3446%	17,3007%	#N/A	#N/A	#N/A	#N/A	#N/A
1Y	16,8889%	19,3389%	#N/A	#N/A	#N/A	#N/A	#N/A
2Y	21,2621%	22,3582%	#N/A	#N/A	#N/A	#N/A	#N/A

LogNormal Vols for strike offset -50

-50	1Y	2Y	5Y	10Y	15Y	20Y	30Y
1M	21,5883%	23,4515%	25,5379%	25,6015%	24,6235%	24,1794%	24,0454%
3M	20,7284%	22,9278%	24,7304%	24,6516%	23,7625%	23,3528%	23,2295%
6M	19,9622%	22,1458%	24,2594%	23,9487%	23,0723%	22,6602%	22,5841%
1Y	20,9003%	23,0879%	24,6450%	24,2814%	23,3367%	22,8855%	22,7549%
2Y	24,8170%	25,8622%	26,1768%	25,1529%	24,0049%	23,3974%	23,2039%

LogNormal Vols for strike offset 50

50	1Y	2Y	5Y	10Y	15Y	20Y	30Y
1M	14,8878%	17,2127%	20,2511%	21,0660%	20,3587%	20,0071%	19,8616%
3M	14,0414%	16,9108%	19,9357%	20,5695%	19,8882%	19,5514%	19,4056%
6M	14,2883%	17,2236%	20,3530%	20,7470%	19,9838%	19,6021%	19,4513%
1Y	16,4050%	18,8483%	21,2240%	21,2809%	20,3786%	19,9112%	19,6697%
2Y	20,6163%	21,6925%	22,6565%	21,9672%	20,8778%	20,2487%	19,9330%

LogNormal Vols for strike offset -100

-100	1Y	2Y	5Y	10Y	15Y	20Y	30Y
1M	36,1115%	36,4761%	37,5157%	37,4117%	36,0021%	35,4300%	35,4006%
3M	30,5621%	31,8620%	33,6734%	33,6621%	32,4248%	31,8830%	31,8150%
6M	27,0085%	28,6728%	30,6225%	30,1681%	29,1172%	28,6659%	28,6915%
1Y	25,7271%	27,7032%	28,9725%	28,4795%	27,4723%	27,0426%	27,0514%
2Y	28,8589%	29,8057%	29,8406%	28,6508%	27,4550%	26,8733%	26,8093%

LogNormal Vols for strike offset 100

100	1Y	2Y	5Y	10Y	15Y	20Y	30Y
1M	20,7479%	22,6577%	25,0939%	25,9198%	25,0888%	24,6938%	24,5417%
3M	17,5946%	19,8349%	22,6442%	23,5797%	22,8634%	22,5117%	22,3693%
6M	15,6483%	18,3060%	21,5843%	22,2371%	21,4958%	21,1277%	20,9907%
1Y	16,3277%	18,6540%	21,2598%	21,5007%	20,6410%	20,1942%	19,9665%
2Y	19,9026%	20,9011%	22,1194%	21,5927%	20,5538%	19,9338%	19,6112%

LogNormal Vols for strike offset -200

-200	1Y	2Y	5Y	10Y	15Y	20Y	30Y
1M	87,7743%	80,7015%	76,8580%	74,8240%	#N/A	#N/A	#N/A
3M	66,2899%	62,7704%	63,8677%	63,6416%	60,6912%	59,5767%	59,8672%
6M	51,8101%	51,5045%	53,4669%	52,3187%	50,2726%	49,5548%	50,0661%
1Y	42,9437%	44,5567%	45,7249%	44,8265%	43,2665%	42,7614%	43,3413%
2Y	43,9302%	44,7234%	44,3232%	42,4099%	40,7945%	40,2185%	40,7281%

LogNormal Vols for strike offset 200

200	1Y	2Y	5Y	10Y	15Y	20Y	30Y
1M	#N/A	31,3429%	33,6882%	34,1884%	33,2720%	32,8971%	32,8391%
3M	23,6435%	25,7257%	27,9547%	29,0973%	28,2884%	27,8940%	27,7446%
6M	19,3030%	21,5516%	24,7629%	25,6238%	24,8838%	24,5240%	24,4022%
1Y	17,5553%	19,6098%	22,3782%	22,8442%	22,0528%	21,6434%	21,4468%
2Y	19,8162%	20,6097%	22,1196%	21,8365%	20,8897%	20,3047%	19,9991%

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So the data structure of vega sensitivities is multi-dimensional.

Cap/Floor (Caplet/Floorlet) Vegas

$$Vega^{CF}_{port}(T_i,K_j) = \sum_{trade} Vega^{CF}_{trade}(T_i,K_j)$$

$$= \sum_{trade} \frac{\partial NPV(trade)}{\partial \sigma^{CF}(T_i,K_j)},$$

where T_i are option maturities, K_j are strikes, and $\partial\sigma^{CF}(T_i,K_j)$ are the respective cap/floor volatilities.

$T\backslash K$	1%	2%	$\cdots$	9%
$1y$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$2y$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\vdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$10y$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

Caplet/Floorlet Vegas

$$Vega_{port}^{CLFL}(T_i,K_j) = \sum_{trade} Vega_{trade}^{CLFL}(T_i,K_j)$$

$$= \sum_{trade} \frac{\partial NPV(trade)}{\partial \sigma^{CLFL}(T_i,K_j)},$$

where T_i are option maturities, K_j are strikes, and $\partial\sigma^{CLFL}(T_i,K_j)$ are the respective caplet/floorlet volatilities.

$T\backslash K$	1%	2%	$\cdots$	9%
$6m$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$9m$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\vdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$10y$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

6.3. Swaption Vegas

$$Vega^{SW}_{port}(T_u, T_i, K_j) = \sum_{trade} Vega^{SW}_{trade}(T_u, T_i, K_j)$$

$$= \sum_{trade} \frac{\partial NPV(trade)}{\partial \sigma^{SW}(T_u, T_i, K_j)},$$

where T_u are underlying swap tenors, T_i are option maturities, K_j are strikes and $\sigma^{SW}(T_u, T_i, K_j)$ are the respective volatilities .

Swap tenor $T_u = 1y$

$T\backslash K$	1%	2%	$\cdots$	9%
$1y$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$2y$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\vdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$10y$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

Swap tenor $T_u = 2y$

$T\backslash K$	1%	2%	$\cdots$	9%
$1y$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$2y$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\vdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$10y$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

$\cdots$

Swap tenor $T_u = 30y$

$T\backslash K$	1%	2%	$\cdots$	9%
$1y$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$2y$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\vdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$10y$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

Swaption volatilities are **correlated** to cap/floor volatilities.

See (4.42) in section 4.

6.4. Hedging Delta and Vega Risks

In the Black-Scholes world, dynamic hedge is only about delta hedging. However, mark-to-market of volatility has established the variation of volatilities as another risk factor to be hedged as well. It is no doubt that the interest rate vega hedging is the most complicated vega.

Step 1. Hedge the vegas of swaption sensitive book/positions.

Step 2. Hedge the vegas of cap/floor sensitive book/positions.

Step 3. Hedge the PV01 of the vega hedged positions after step 1 and step 2.

Questions

(1) Are Step 1 and Step 2 exchangeable?

(2) Can Step 3 go before any one of the first two steps?

Bucket-by-bucket hedging

Eg. Trade the following notional amount H in a vanilla option with maturity T_i and strike K_j to hedge a vega position of $Vega_{port}\,(T_i, K_j)$ for a portfolio:

$$H(T_i, K_j) = -\frac{Vega_{port}\,(T_i, K_j)}{Vega_{vanilla}\,(T_i, K_j)}.$$

Drawbacks of bucket-by-bucket vega hedging

(a) Sensitivities by bucket bumping may not be reliable for hedging.

If a bumping quantum is too small, the value obtained could be noisy.

If a bumping quantum is too big, the resulting risk factors, yield curve or vol curves, could become unreasonable.

(b) The bucketed sensitivities are hardly linearly additive.

(c) Risk scatters all over the place.

How to manage multi-bucket risk with second order P&L?

1. Find the most representative movement patterns of the risk buckets by principle component analysis.

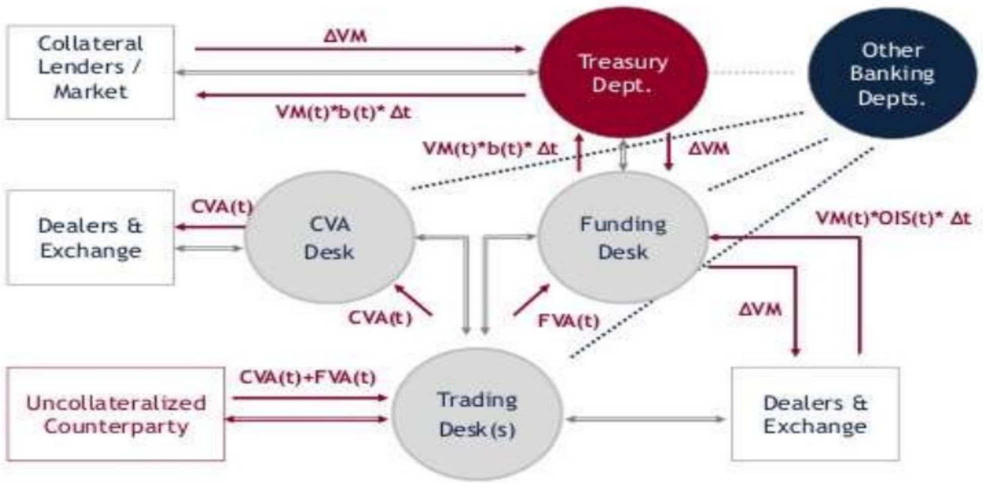
2. Compute the sensitivities to movements of such patterns.

3. Attribute the sensitivities to selected risk buckets and hedge them accordingly.

6.5. OIS Risk, CVA Risk and FVA Risk

The paradigm switch of valuation (OIS discounting) and the incorporation of valuation adjustments (XVAs) after 08 crisis has make the risk management of interest rate derivatives much more complicated and challenging than in the past.

XVA Risk Transfer



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