# Term Structure and Interest Rate Derivatives Part II

Liu Xiaoqing

# Section 1. Interest Rate Products

(1) **Zero Coupon Bond**

$$P(T) = e^{-R(T)T} \cdot 1.$$

**(2) Coupon Bond (Straight Bonds)**

$$P(T_N) = e^{-R(T_N)T_N} \cdot 1 + \sum_{n=1}^{N} e^{-R(T_n)T_n} C(T_N) \triangle t_n,$$

$$\triangle t_n = T_n - T_{n=1}.$$

## (3) Floating Rate Note（FRN， Floater)

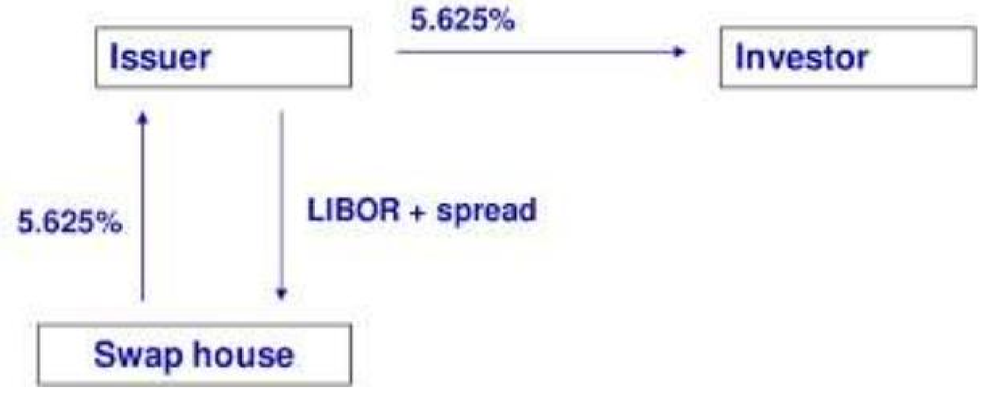
$$P(T_N) = e^{-R(T_N)T_N} \cdot 1 + \sum_{n=1}^{N} e^{-R(T_n)T_n}(LIBOR_{n-1} + b_n)\triangle t_n,$$

## (4) Interest Rate Swap (IRS)

**Swap Payer**:

$$\sum_{n=1}^{N} e^{-R(T_n)T_n} LIBOR_{n-1} \triangle t_n - \sum_{n=1}^{N} e^{-R(T_n)T_n} IRS(T_N) \triangle t_n = 0.$$

**Application**: Bond Issuance.



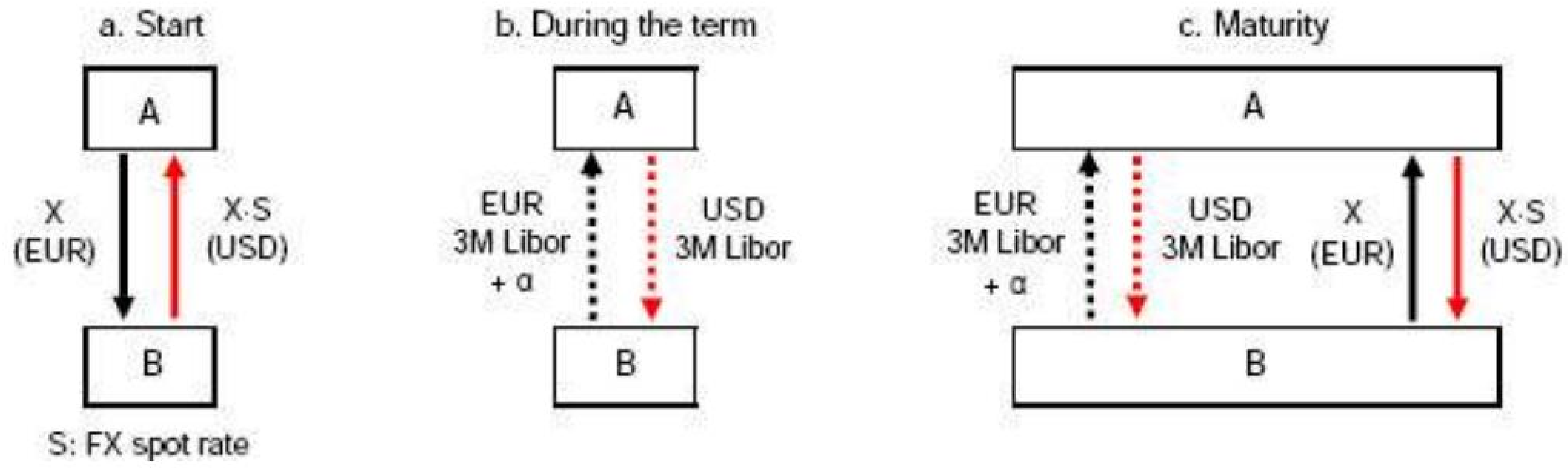
## (5) Cross Currency Swap（CCS）

$$FX_0^{EURUSD}\left[\sum_{n=1}^{N} e^{-R^{CCS}(T_n)T_n}(EURIBOR_n + b^{CCS}(T_N))\triangle t_n + e^{-R^{CCS}(T_N)T_N}\right]€1$$

$$-\left[\sum_{n=1}^{N} e^{-R^{USD}(T_n)T_n} LIBOR_n \Delta t_n + e^{-R^{USD}(T_N)T_N}\right]\$FX_0^{EURUSD} = 0.$$

**Key Point**:

$$R^{CCS}(T_N) - R^{EUR}(T_N) \approx b^{CCS}(T_N).$$




**Application**: Corporate issues a foreign currency bond.

## (6) Forward Rate Agreement

$$e^{-R(T_1)T_1} \frac{[LIBOR(T_1, T_2) - FRA](T_2 - T_1)}{1 + LIBOR(T_1, T_2)(T_2 - T_1)} = 0.$$

**(7) Callable Bond**

The bond issuer has the following optionality:

$$\max(0, PV_t - X) = \max\left(0,\ e^{-R_t(T_N-t)(T_N-t)} \cdot 1 + \sum_{\{n|t<T_{n=1}\}} e^{-R_t(T_n-t)(T_n-t)} C_{T_N} \Delta t_n - X\right) \text{ at } t.$$

Other conditions being equal, a callable bond is cheaper than a non-call bond.

**(8) Swaption**

**Swap Payer**:

$$\max\left(0, \sum_{\{n|t<T_{n=1}\}} e^{-R(T_n-t)(T_n-t)} LIBOR_t(T_{n-1}, T_n)\triangle t_n - \sum_{\{n|t<T_{n=1}\}} e^{-R(T_n-t)(T_n-t)} K\triangle t_n\right) \text{ at } t.$$

## (9) Caplet and Floorlet

$$Caplet_n = P(0, T_n)\max[0, (LIBOR_{n-1} - X)\triangle t_n],$$

$$Floorlet_n = P(0, T_n)\max[0, (X - LIBOR_{n-1})\triangle t_n].$$

**(10) Cap and Floor**

$$Cap(T_N) = \sum_{n=2}^{N} Caplet_n,$$

$$Floor(T_N) = \sum_{n=2}^{N} Floorlet_n.$$

## (11) Examples of Bespoke/Exotic Products

### (11.1) In-Arrears IRS

**Swap Payer**:

$$\sum_{n=1}^{N} e^{-R(T_n)T_n}[LIBOR(T_n, T_{n+1}) + b_n]\triangle t_n - \sum_{n=1}^{N} e^{-R(T_n)T_n} K\triangle t_n.$$

## (11.2) LIBOR Path Dependent Notes

### (11.2.1) Inverse Floater

$$(-\alpha_n \times LIBOR_{n-1} + \beta_n)\triangle t_n \text{ for } -\alpha_n < 0.$$

### (11.2.2) Range Accrual

$$\beta_n \times sign[(Max_n - \alpha_n \times LIBOR_{n_i})^+] \frac{1}{\text{yearly day count}},$$

$$\beta_n \times sign[(\alpha_n \times LIBOR_{n_i} - Min_n)^+] \frac{1}{\text{yearly day count}},$$

for day $n_i$ in the $n$-th interest period.

## (11.2.3) Target Redemption

Note knocks out at $T_M$ when accrued coupon

$$AC(M) \equiv \sum_{n}^{M} (\alpha_n \times LIBOR_{n-1} + \beta_n)\Delta t_n$$

up to $T_M$ reaches or exceeds the Target before maturity $T_N$ :

$$AC(M) \geq Target;$$

Target is guaranteed if no knockout:

$$Last\ Coupon = \min[0, Target - AC(N-1)].$$

Knockout is not an optionality, but a gap risk, for the issuer.

(11.2.4) **CMS Spread**

Multiplier: $\alpha_n \times \left(CMS_n^{30y} - CMS_n^{2y}\right)\triangle t_n;$

Range Accrual: $\alpha_n \times sign\left[\left(CMS_{n_i}^{30y} - CMS_{n_i}^{2y}\right)^+\right]\dfrac{1}{\text{yearly day count}}$

for day $n_i$ in the $n$-th interest period.

(11.2.5) Snowball

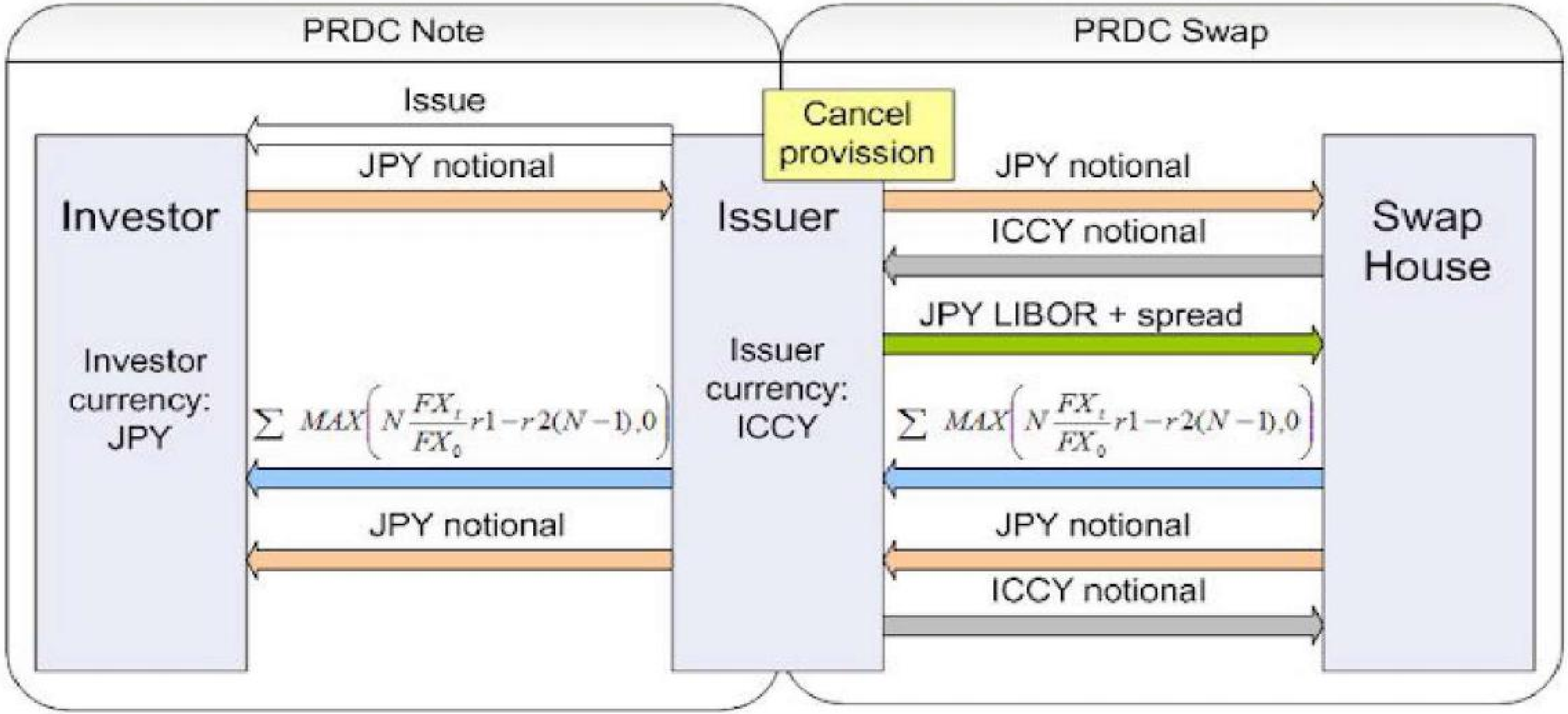
$$\left[\text{Previous Coupon} + \alpha_n \times (\text{Strike} - LIBOR_{n-1})\triangle t_n\right]^+.$$

(11.3) **Bermudan swaptions**:

Option to enter a swap, typically of a fixed tenor, at one and only one of a series of exercise dates.

Equivalent to a swap and a callabe back-to-back swap.

## (11.4) Power Reverse Dual Currency Note (PRDC)



**(12) Note and swap**

As is reflected in the diagram on PRDC, interest rate derivatives are structured in two forms: Note and swap.

From the funding perspective, they are funded and unfunded respectively.