FE5101: Lectures 1 and 2 – Debt Securities and Interest Rates

Instrument characteristics and analytical mathematics

Lectures 1 and 2 Topics:

1. Primary and Secondary Markets for Debt Instruments
2. Profitability, Pricing, and Valuation
3. Yield Curves and Bootstrapping Zero Rates
4. Valuing Bonds to the Zero-curve
5. Calculating Implied Forward Rates

Homework as of slide 42 for next lecture – don't forget

Lecture 1, Topic 1

Bills and Bonds:

Origination and Secondary Markets

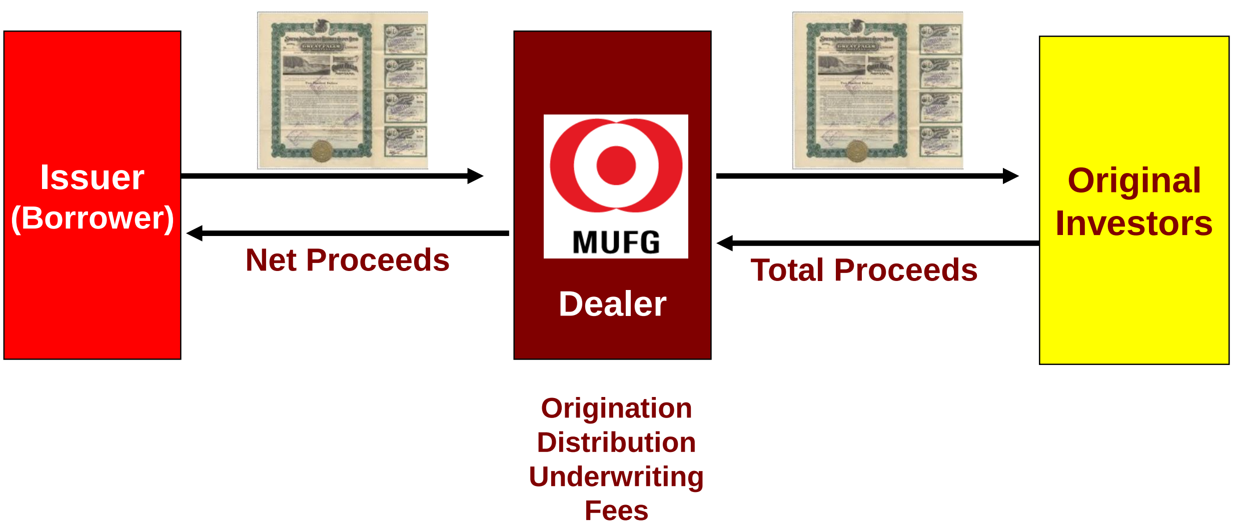
Debt Security Issuance

- Auction – used by governments
  - Dealers bid by showing what interest yield they would accept and how much they will invest in securities if their yield is equal or higher
  - Debt issuer accepts bids from lowest yield until the issued amount is exhausted
- Underwritten – used by corporates and institutions and some governments (when issuing in non-domestic currency)
  - Bond desk at bank or securities house negotiates with issuer (borrower) details of the debt securities to be issued and also finds investors to buy them
  - Yield is determined in a negotiated process leading up to the issuance
- Best Efforts – for smaller issues, shelf offerings
- Private Placement – small group of professional or related creditors buys entire issue

Primary Dealers in US Treasury Bills and Bonds

- BNP Paribas Securities Corp.
- Barclays Capital Inc.
- Cantor Fitzgerald & Co.
- Citigroup Global Markets Inc.
- Credit Suisse Securities (USA) LLC
- Daiwa Capital Markets America Inc.
- Deutsche Bank Securities Inc.
- Goldman, Sachs & Co.
- Greenwich Capital Markets, Inc.
- HSBC Securities (USA) Inc.
- J. P. Morgan Securities Inc.
- Jefferies and Company Inc
- Merrill Lynch
- Mizuho Securities USA Inc.
- Morgan Stanley & Co. Incorporated
- Nomura Securities International, Inc.
- RBC Capital Markets, LLC
- RBS Securities Inc.
- SG Americas Securities
- UBS Securities LLC.

Primary Issue of Debt Securities



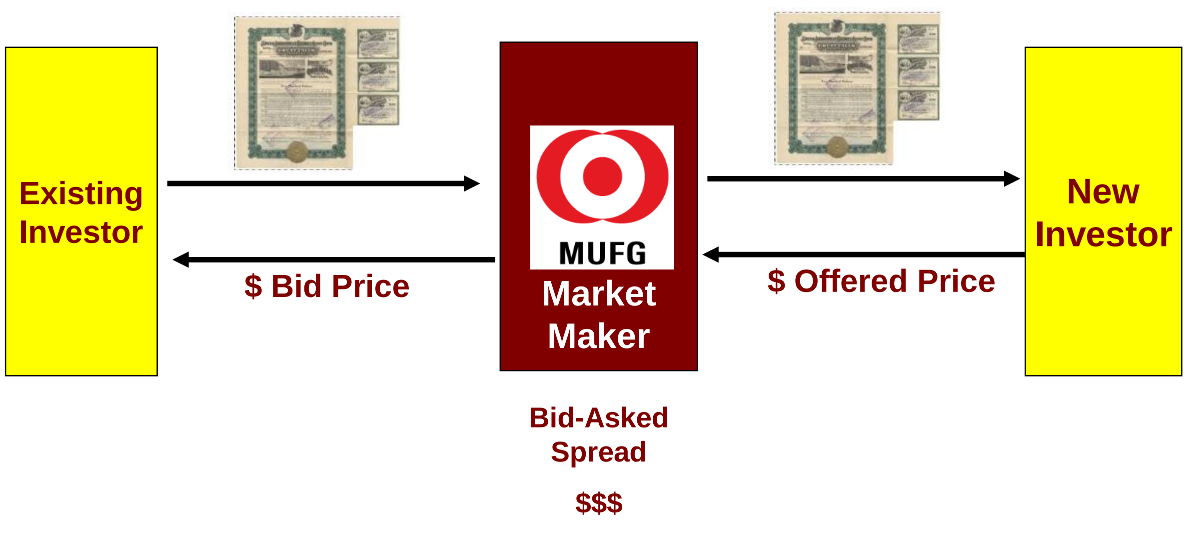
Use Case 1: Treasury Bills

• Issuer:	Government
• Currency:	Currency of issuing country
• Credit Quality:	"Riskless" General Obligation
• Form:	Zero-coupon
– Face amount refers to *future value*	
• Denominations:	Round FV amounts ("face value" e.g. US$100,000)
• Tenor upon initial issue:	≤ 1 year
• Pricing:	Discount to face value
• Distribution:	Auction

Primary Issuance – US Bill Auction

- New T-Bills
  - 3-month and 6-month auctioned every Monday – settled Thursday
  - 12-month once a month, auctioned Tuesday – settled Thursday
- Competitive Bids (to set the market interest rate)
  - Bid smaller discount rate = expect higher allotment
  - Bid higher discount rate → allotment at risk
- Non-competitive (to allow certain investors to participate)
  - Bidder accepts the weighted average discount on competitive bids
  - Way for banks and individuals to assure allotment in an auction
- The auction in any country is designed to allow interest rates for government borrowing (i.e. the riskless rate) to be set fairly by supply and demand

Secondary Trading of Debt Securities



Secondary Trading of Issued T-Bills

- Bills have no stated coupon rate of interest. Only a redemption amount and date
  - Priced at discount to FV
  - Implies a yield
- Secondary Market Maker (Dealer) quotes in discount terms
  - Bid 5.05% (buys at bigger discount)
  - Offer 4.95% (sells at smaller discount)
- The deal discount price is used to calculate the bill's PV for settlement between buyer and seller.

Conventions and Day-counts

- Most markets use mathematical formulae and day-counts based on convention
  - Slightly diverge from the "real" mathematics
  - E.g. US T-bills quoted on a "discount" basis
- Because of legacy systems and long-standing convention in interest accruals
  - Day-count is not necessarily the exact number of days you expect to see in a given year
  - E.g. US T-bills use Actual/360-day year

Settlement Pricing to Discount (act/365)

- The Singapore Government T-Bills are quoted on a *discount basis* using Actual/365.
- What is the settlement price for 1,000,000 FV of a 365 day bill dealt at 5%?
- This means the 5% rate understates the true yield (IRR)

$$\text{Price}_{\text{Singapore T-Bill}} = \text{FV} \times \left(1 - \text{Rate} \times \frac{\text{Days}}{365}\right)$$

$$\text{Price}_{\text{Singapore T-Bill}} = 1{,}000{,}000 \times \left(1 - 5.00\% \times \frac{365}{365}\right)$$

$$= 950{,}000.00$$

Settlement Pricing to Discount (act/360)

- The US Government T-Bills are quoted on a *discount* basis using Actual/360
- What is the settlement price for 1,000,000 FV of a 365 day bill dealt at 5%?
- This means the yield is understated by the rate for two reasons

$$\text{Price}_{\text{US T-Bill}} = \text{FV} \times \left(1 - \text{Rate} \times \frac{\text{Days}}{360}\right)$$

$$\text{Price}_{\text{US T-Bill}} = 1{,}000{,}000 \times \left(1 - 5.00\% \times \frac{365}{360}\right)$$

$$= 949{,}305.56$$

Using PV formula to Discount Bills

- Sertifikat Bank Indonesia (SBI)
  - Zero Coupon Bills issued by the Central Bank
    - Used for liquidity/monetary policy implementation and to provide high-quality collateral to inter-bank money market
  - Like T-bills, but not issued by Treasury
- Settlement Formula – PV equation using Act/360

$$\text{Settlement Price}_{\text{SBI}} = \frac{\text{Face Value}}{1+\text{Rate}\times\dfrac{\text{Days}}{360}}$$

$$= \frac{\text{IDR 1m}}{1+\left(.05\times\dfrac{365}{360}\right)} = 951{,}751$$

Calculation of Indian Treasury Bill Settlement Price

- Only when Settlement Price is calculated using proper PV formula and Act/365, the rate = yield
  - i.e. price exactly portrays profitability.
- Settlement price for
  - Face Value = future value = INR 1m
  - Rate = dealt interest rate = 5% Act/365
  - Days = actual days = 365

$$\text{Settlement Price}_{IndiaBill} = \frac{\text{Face Value}}{1+\left(\text{Rate}\times\dfrac{\text{Days}}{365}\right)} = \_\_\_\_\_\_\_\_$$

Bond-Equivalent Yield (BEY)

- T-bills peculiar price-quoting often is not on a yield or Internal Rate of Return basis
- With BEY, we convert a T-bill's discount rate to make it comparable to a bond's yield (in US T-bond yields are s/a actual/365)
- T-Bills' BEYs often different from their quoted or published discount rates because
  - Discount formula not correct PV formula
  - 360 Day-count
- T-bills' BEYs are often published along side of their quoted discount rates
- What is the BEY of a 6mo (182 day) bill offered at a discount rate of 5% Actual/360?

$$\text{BEY}_{\text{US T-Bill}} = \left(1 \div (1 - [\text{Rate} \times \frac{\text{Days}}{360}]) - 1\right) \times \frac{365}{\text{Days}}$$

$$= \left(1 \div (1 - [0.05 \times \frac{182}{360}]) - 1\right) \times \frac{365}{182} = \_\_\_\_\_\_$$

Caveat: Bond Equivalent Yields

- BEYs are a standardised way to show a Yield on a T-bill, specifically because its discount price may not properly reflect its profitability
- E.g. SG T-bill discount rate of 5%,
  - $95 spot buys $100 in a year's time
  - Int/principal x Day-ct/Days = $5/$95 x 365/365 =
  - a yield of about 5.26% p.a.

See Supplemental Note on Bond Equivalent Yields in IVLE

Fictional Country Bill Auction Simulation

- Auction rules
  - No one takes more than 25% of total available
  - Non-competitive bids can take up to 30% of total at weighted average of accepted prices
  - First-past-the-post allotments – treasury accepts bids from lowest to highest, awarding allotments until the supply is exhausted
  - Highest bid price accepted – allotment is what is remaining.
    - Means this allotment smaller than amount bid for
    - If 2 or more dealers tie, then proportional allotments based on Size of original bid

Auction Simulation continued

- Auction specifics:
  - $1 billion face amount of 12-mo Bills
    - Non-competitive bids total $320m
  - 6 primary dealers submitting bids
    - Citi bid 2.41% for $200m
    - Barclays bid 2.40% for $150m
    - ICBC bid 2.38% for $250m
    - BNP bid 2.40% for $250m
    - DBS bid 2.39% for $250m
    - Mandiri bid 2.42% for $100m
- Highest bid price accepted?_______

Auction Simulation - Allotments

- Citi gets _______ at 2.41% (of $200m bid)
- Barclays gets _______at 2.40% (of $150m bid)
- ICBC gets _______at 2.38% (of $250m bid)
- BNP gets _______at 2.40% (of $250m bid)
- DBS gets _______at 2.39% (of $250m bid)
- Mandiri gets _______at 2.42% (of $100m bid)
- Non-comps get ______ at ______ (of $320m bid)

Auction Sim Cont’d – some statistics

- Weighted average bid price?_____
- Highest bid accepted? _____
- “Tail” ?_____
- Ratio of total bids to auction size?______
- Ratio of competitive bids to competitive allotment?_____
- Non-comp allotment percentage?_____

Use Case 2: US Treasury Bonds

- Issuer: US Government Treasury
- Currency: Currency of issuing country
- Credit Quality: Riskless General Obligation
- Form: Coupon-bearing
- Denominations: Round *Par* amounts ("par value" e.g. US$100,000)
  - Par amount refers to principal returned at maturity
  - Par amount is used to calculate interest payments
- Tenor at issue: ≥ 2 years
- Pricing: Percentage of Par Value
- Distribution: Auction

Straight Bond in Paper Form

Bond Corpus

- **Par Amount**
- **Indenture**

Bond Coupons

(each redeemable For an interest Payment)

Definitions

- Par Amount – represents a bond's principal
  - The coupon interest rate is applied to the par amount to size the interest payments
  - The par amount is repaid by the bond issuer at maturity
- Issuer – the borrower of funds who emits the bond as a security for the borrowing
- Coupon – the bond's interest payments

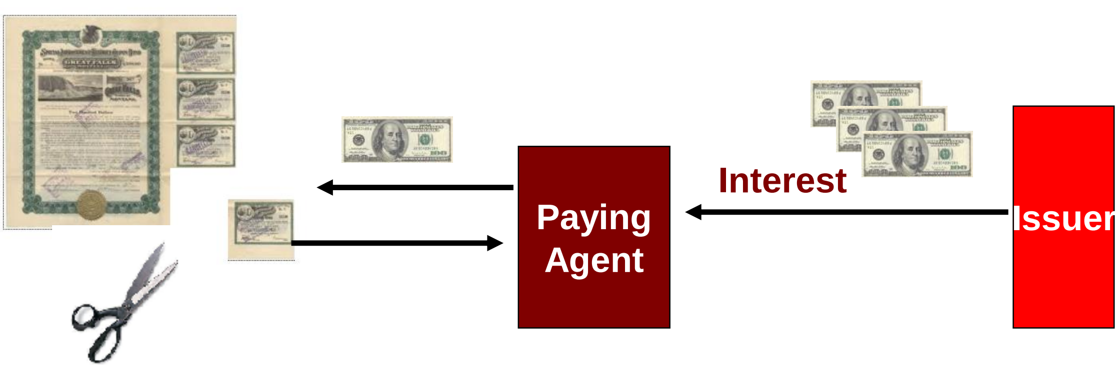
Lifecycle of a Bond

Our Sample Bond

Par Value	$10 million
Original Maturity	2 Years from Issue Date
Coupon Rate	5% per annum
Coupon Frequency	Semi-annual
Day Count	Act/Act
Coupon Amount	$250,000

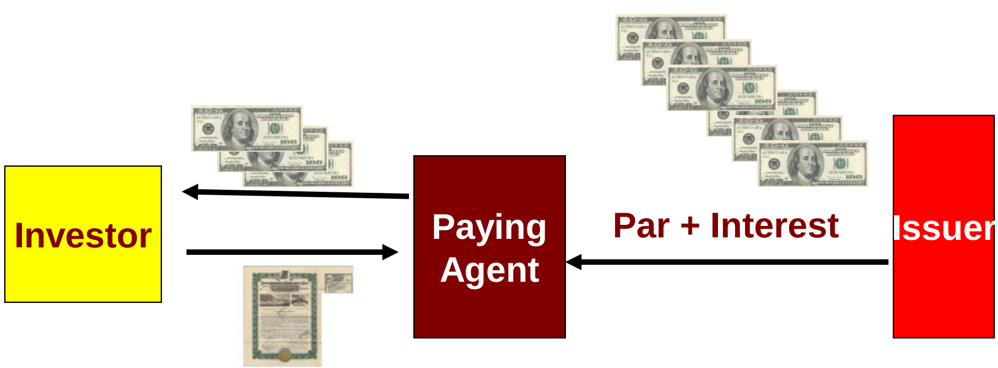
Interest Coupon Payments

- Each coupon promises a payment amount on a specific date
- When a coupon date arrives, the coupon is redeemed for cash



Bond Maturity

- Final Redemption
  - Last coupon, and
  - Par amount



Secondary Bond Markets

- Where existing bonds are traded among investors and dealers
- In most markets, price quoted in bid-offer per $100 of face value
  - For example, "101.20 / 101.25"
  - In US government bonds, the price for $100 of par value quoted in 32nds, e.g. "99-29 / 99-31"
    - Be aware, 99-29 = $99.90625 per $100
- In Australia and Malaysia, secondary prices quoted in YTM
  - E.g. "4.56% / 4.51%"
  - Once a YTM price is dealt, the PV for settlement is then calculated by applying the dealt yield to the bond's cash flows

Variations on Straight Bonds

- Straight Bond characteristics
  - *Pay interest on a fixed rate basis*
  - *Repayment Priority* equal with other senior unsecured lenders
  - Defined *Maturity* at which point the par value is repaid
  - Repay a *Fixed Cash Amount* at the Maturity
- Most bonds are straight, but there are many variations
  - See post-lecture slides for non-straight bond variants

Lecture 1, Topic 2

Bond Profitability, Pricing, and Valuation

Yield to Maturity (YTM) = Bond IRR

- Yield to Maturity (YTM): Market Interest Rate
  - The internal rate of return anticipated on a *bond if it is held until the maturity date, and all payments are made as promised*
  - The single discount rate which when applied to all future coupons and the par repayment make the bond's PV equal its market price
- YTM is
  - A Measure of profitability
    - Market Price
    - Remaining term to maturity
    - Coupon
  - A market price requirement based on driven by
    - Riskless Interest Rates
    - Pricing of Risky Credit
    - Issuer-specific Risk

Market Value = PV of Bond's Cash Flows

- First coupon \$250,000 received in 6mo

- Second coupon \$250,000 received in 12mo

- Third coupon \$250,000 received in 18mo

- Fourth coupon \$250,000 received in 2 years

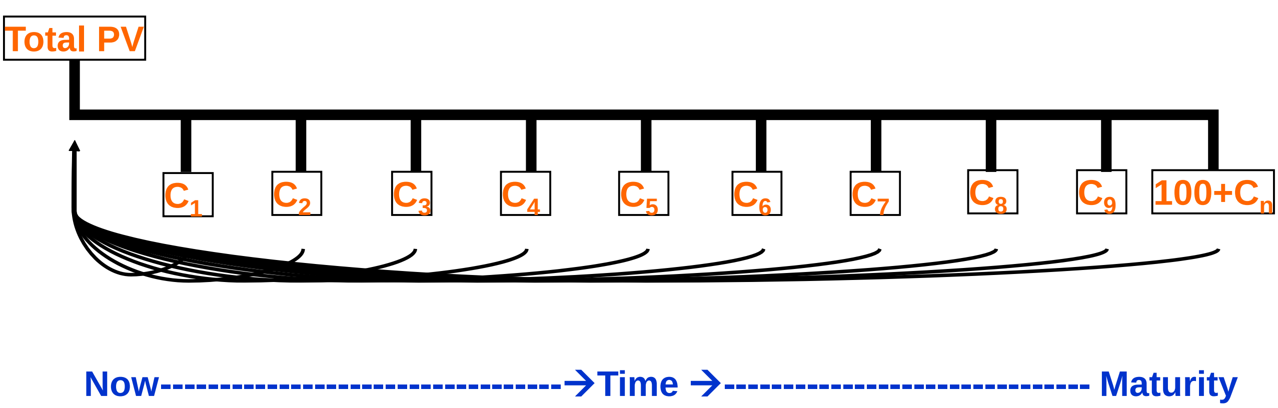
- Par Value \$10m received in 2 years

$$Price_{mkt} = \frac{100}{\left(1+\frac{YTM}{freq}\right)^n} + \sum_{i=1}^{n} \frac{Coupon_i}{\left(1+\frac{YTM}{freq}\right)^i}$$

Bond Valuation Logic

- Bond value (price) = The sum of the PVs of the contractual cash flows. The bond's price fluctuates as market interest rates fluctuate

$$\frac{Cashflow_i}{\left(1+{YTM}/{freq}\right)^i}$$



Exercise 1a: Bond Valuation – *at Discount*

- If you have to earn a YTM of 6.0%, how much will you pay for the bond on the previous slide?
- Use this formula to fill in the table:

$$\frac{Cashflow_i}{\left(1+{YTM}/{freq}\right)^i}$$

Cash Flow *for i-period*	Future Value	PV Factor PV of $1	PV(FV)
i = 1; 1st Coupon	$250,000		
i = 2; 2nd Coupon	$250,000		
i = 3; 3rd Coupon	$250,000		
i = 4; 4th Coupon	$250,000		
i = 4; Par Redemption	$10m		
Total Present Value =			

Exercise 1b: Bond Valuation – *at Premium*

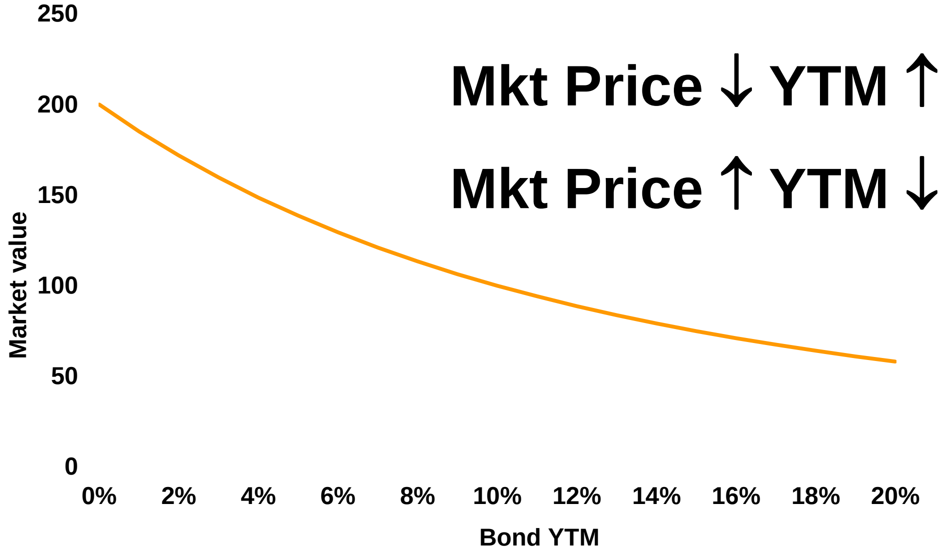
- If you have to earn a YTM of 4.0%, how much will you pay for the bond on the previous slide?
- Use this formula to fill in the table:

$$\frac{Cashflow_i}{\left(1+\frac{YTM}{freq}\right)^i}$$

Cash Flow *for i-period*	Future Value	PV Factor PV of $1	PV(FV)
i = 1; 1st Coupon	$250,000		
i = 2; 2nd Coupon	$250,000		
i = 3; 3rd Coupon	$250,000		
i = 4; 4th Coupon	$250,000		
i = 4; Par Redemption	$10m		
Total Present Value =			

Price-Yield Relationship

- A bond's market value is inverse to the market rate (i.e. YTM)



Market Value of a Bond

- A bond's market value over time will diverge from its par value
  - Coupon and payment dates are fixed
  - But YTM required by investors fluctuates
    - If YTM > Coupon, bond at discount to par, e.g. $97.85
    - If YTM < Coupon, bond at premium to par, e.g. $103.20
- Pricing Terminology
  - Par – when bond price is approximately 100.00% of par value
  - Premium when bond price is > 100% of par
  - Discount – when bond price is < 100% of par

Typical Bond Pricing

- Often bonds' prices are shown on a 100-basis
- Examples
  - "The distribution syndicate took the bonds into inventory at 99.75; the bonds were then offered onto retail at par."
  - Secondary Market quote: 102.75/102.85
- This refers to how much the market price is for every 100 currency units of par

Calculating Yield to Maturity

- Typically the bond's price is discoverable directly in the market, implying a YTM
- The YTM is calculated iteratively in this case

$$\text{Price}_{Mkt} = \frac{100}{\left(1+YTM\,?\big/ freq\right)^{n}} + \sum_{i=1}^{n} \frac{Coupon_i}{\left(1+YTM\,?\big/ freq\right)^{i}}$$

- What YTM if you bought our sample bond with 18mo (3 coupons) left at a price of $97.20?

Answer to the previous slide

- Contractual Details
  - Par Value — $10 million
  - Original Maturity — 2 years from Issue
  - Coupon Rate — 5% per annum
  - Coupon Frequency — Semi-annual
  - Day Count — Act/Act
  - Coupon Amount — $250,000
- Market Details (Now, 6mo later)
  - Remaining life — 1.5 years
  - Remaining Coupon payments — 3
  - Market Price — $97.20 per $100 par
  - **Yield to Maturity** — 7%

$$\$9.72\text{m} = \frac{\$10m}{\left(1+7.0\%/2\right)^3} + \sum_{i=1}^{3} \frac{\$250{,}000}{\left(1+7.00\%/2\right)^i}$$

Don’t Confuse HPR with YTM

- *Holding period return*, HPR is the ex-post IRR measure of profitability based on the buyer’s actual holding period (including selling the bond prior to maturity)
- E.g. you buy a 5% annual 10-year coupon bond at 100.00 (par)
  - YTM = ____
- You sell it one year later (also collecting 1 coupon) at a market price of 101.00
  - Cashflow out at Spot = - ______
  - Cash flows back at Spot + 1yr
    - Coupon amount + ______
    - Bond Sale Proceeds + _______
  - What is your HPR? _____
  - Why is this different from YTM?
  - When is HPR = YTM?

Homework to be done in advance of Lecture 2

- Exercise 2 is a step-wise instructional on using market price information to:
  - Construct Yield Curves
  - Derive Zero-coupon Rates
  - Derive Forward Rates
  - Value existing securities
- Spend sufficient time on it to prepare for lecture 2, as we'll move quickly through reviewing it and further into related similar analytics
- *It won't be graded; do not submit it*
- And bring a financial calculator to Lecture 2!

End of Lecture 1

Final Q&A

L1 Supplement: Variations on Straight Bonds

- Floating Rate Notes (FRNs) – coupon re-prices each time to money-market rate (e.g. 3-mo LIBOR + 1.2%)
- Zero-coupon Bonds – long dated bond paying a future value, but no interim coupons.
- Perpetual Bonds – bonds that pay interest forever, never repaying principal
- Subordinated Bonds – specifically giving up senior creditor status for a higher coupon rate
- Collateralised Bonds – backed by specific asset pledge
- Dual-currency Bonds – principal can be repaid in alternative currency
- Callable bonds – bonds that the issuer can retire early
- Putable bonds – bonds that the investor can redeem early
- Convertible bonds – principal repayment can be in another asset (e.g. shares in the company)

Lecture 1, Exercise 2: Homework

Exercise 2:
Deriving Zero Coupon Rates

Observe the following market prices for Singapore Government bills and bonds:

6-month bill:	2.3% discount
1-year bill:	2.5% discount
Old 2.5% bond with 18mo to maturity:	$99.50

(99.50 implies a YTM of 2.8429% s/a)

To keep day-count from being a distraction, assume the exact half-year, i.e.

- The 6-month period is exactly half a year, 182.5 days/365
- The 12- and 18-month securities have exactly 2 and 3 half-years to maturity

Using the above pricing information and assumptions, give the correct series of discount factors and zero rates expressed on a semi-annual bond-equivalent yield basis:

1) Using market convention for discounting an SGS bill, what is the discount factor of the 6-month bill? (This is easy – it's simply the settlement price factor for $1 of face value) Use this formula:

$$df_{6\ month} = SettlePriceFactor_{6\ month\ bill} =$$
$$1 - (\text{discount rate} \times \text{days}/365) =$$

2) What is the corresponding bond-equivalent yield for this bill? Use this formula:

$$\text{BEY (target freq, TF)} = [(1 \div df_{period})^{\wedge(Freq/TF)} - 1] \times TF$$

3) Using market convention for discounting an SGS bill, what is the discount factor of the 12-month bill? Use this formula:

$$df_{12\ month\ govt\ bill} = 1 - (\text{discount rate} \times \text{days}/365) =$$

4) What is the corresponding bond-equivalent yield for this bill? Use this formula:

Hint - Target frequency is what you want = 2
Frequency is what you have = 1

BEY for target frequency, TF) =
TF x $[(1 \div df_{period})^{\wedge(Freq/TF)} - 1]$

$$2 \times [(1 \div df_{12mo})^{\wedge(1/2)} - 1] =$$

You can also use this formula:
TF x $[(1 \div df_{period})^{\wedge(TFperiod/Fperiod)} - 1]$

$$2 \times [(1 \div df_{12mo})^{\wedge(6mo/12mo)} - 1] =$$

5) Using the discount factors for the 6-month and 1-year bills, calculate the PVs of the cash flows of the 18-month bond in the table below.

- Subtract the PV of the first 2 coupons from the price to get the PV of the 18-month payment (PV of CF_3).

- How do you attain the discount factor from this PV?

- What allows us to use the bill rates to discount the bond's coupons?

Don't forget, with single cash-flow debt instuments, FV x df = PV, which means df = PV/FV

Cash flow	Future Value (CF)	Discount Factor (df)	Present Value of CF_n
Price $100 par 18-mo bond at t0	**-99.50**	**1.0000**	-99.50
First Coupon at t0 + 6mo	+ 1.25	**x 0.9885 =**	**+**
Second Coupon at t0 + 12mo	+ 1.25	**x =**	**+ 1.21875**
Last Coupon + Par at t0 + 18mo	101.25 FV		**=**

In this process we've synthetically created a 18-month zero-coupon instrument from a coupon-bearing instrument with f=0.66667 (i.e. 2/3 of a payment per year) but TF = 2 (payment frequency of semi-annual). We want this rate on s/a basis equiv, so on to question 6…

6) What is the 18-month zero rate in semi-annual act/365 equivalent? Use either of these formulas:

TF x $[(1 \div df_{period})^{\wedge(Freq/TF)} - 1]$ = yield s/a act/365

$2 \times [(1 \div df_{18\ month})^{\wedge 0.6666/2} - 1] =$

TF x $[(1 \div df_{18mo})^{\wedge(6mo/18mo)} - 1] =$

7) Now, if you saw a new 2-year SGS bond with a 3.0% coupon trading at 99.90

Calculate YTM in excel or on a financial calculator. Did you get 3.0519% s/a?),

Now use the above calculated information to figure the 2-year zero rate

Cash flow	Future Value (CF_n)	Discount Factor (df_n)	Present Value of CF_n
24-month bond. Buy at t0	-99.9	1.0	**-99.90**
First Coupon at t0 + 6mo	**1.50**	**0.9885**	**1.48275**
Second Coupon at t0 + 12mo			
Third Coupon at t0 + 18mo			
Last Coupon + Par at t0 + 24mo	**101.50**		

Just to finish up, review this table and see if you understand how these numbers were attained and if you have attained the same ones. *The BEY rates were convertible from the price information given you. The Zero Coupon discount factors and rates were derivable in a process generally known as* yield-curve boot-strapping.

Maturity	ZC Disc.Factor	S/A ZC BEY	S/A YTM (BEY)*
½ Year	**0.9885**	2.3267%	2.3267%
1 Year	**0.9750**	2.5479%	2.5479%
1½ Years	**0.95848**	2.8475%	2.8429%
2 Years	**0.94105**	3.0611%	3.0519%

You may have slightly different answers due to rounding

*All BEYs are on semi-annual Act/365 in this analysis. These are the BEYs and YTMs (same freq and day-count) from the visible prices in the market for bills and bonds

Lecture 2, Topic 1

Issuers and Credit Quality

Traded Debt Issuers

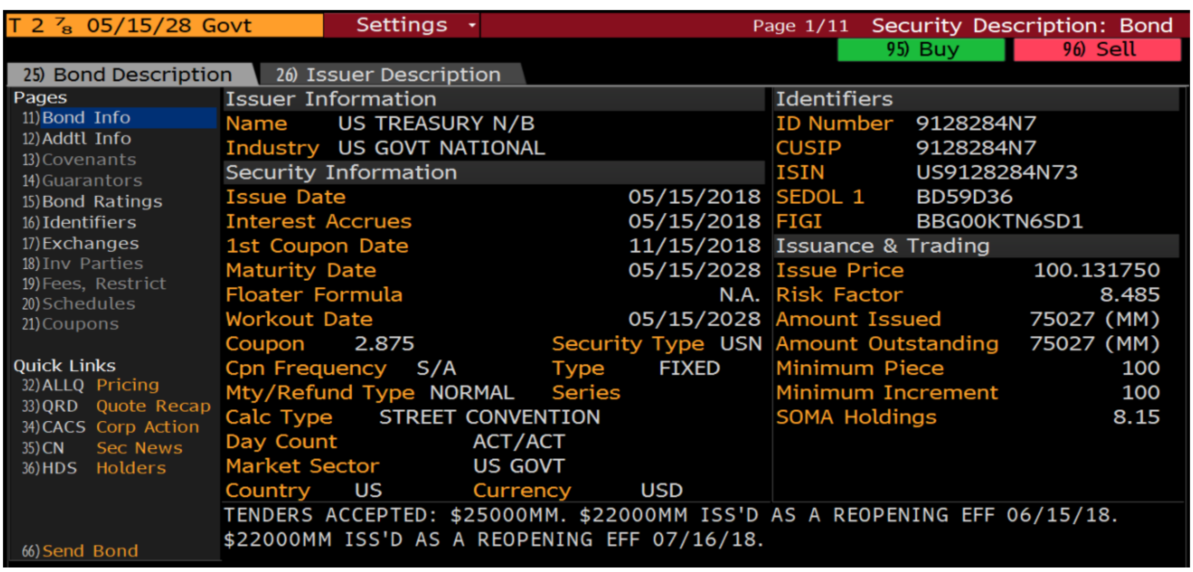
- Governments
  - Treasury (General Obligation)
  - Agency and Revenue (Repaid out of business or infrastructure income)
  - State/Provincial and Local (If have borrowing authority)
- Banks
  - Medium Term Notes (MTNs) and FRNs for medium term
  - Bonds for long term funding
- Corporations
  - Publicly held corporations often issue bonds
  - Closely held (less common, but possible)
- Supra-nationals
  - World Bank
  - Asian Development Bank

Government Bonds

- Best Credit Quality =
  - National Government Treasury (General Obligation)
  - Denominated in National Currency
- Backed By Full
  - Faith and Credit
  - Taxing Authority
  - Ability to Borrow
  - Right to Print Money
- State and Local (“municipal” in US) depend on
  - Credit, taxing, borrowing authority, because…
  - No right to print money!

Sample US On-the-run 10yr Treasury

- Bloomberg Screenshot from 2018:



Main Sources of Bond Risk

- Credit Risk: regards losing money due to borrower's inability or unwillingness to pay
  - The risk/return profile to lending (buying bonds) is asymmetric
    - Small upside (interest payments and getting your principal back)
    - Large downside (partial or full loss on default)
  - Credit risk has a few different forms, but the most common is *default risk*
- So what aspects of debt induce investors to lend?
  - Seniority, ability to declare a default, and acceleration, mitigate the risks of lending (or buying bonds)

- Market Risk: regards losing money due to secondary market pricing of higher YTM than you're earning on a bond you hold
  - E.g. bought 5% coupon 5yr bond at $100, so YTM to you is 5%
  - Now market YTM is at 6%, so bond market price appx $95.5

Analytical Credit Risk Measurement

- Banks' internal credit analysis and history with the borrower (issuer)
  - A bank's internal rating system often tacitly subject to regulator approval
- External Credit rating
  - Specific Quality Rating by recognised agency
  - Based on their *Probability of Default* and *Loss Severity* estimates

Ratings Agencies

	Moody's	Standard & Poor's	Fitch Ratings	Definition
High grade	Aaa	AAA	AAA	Highest quality, exceptionally strong capacity to pay principal and interest.
	Aa1 Aa2 Aa3	AA+ AA AA-	AA+ AA AA-	High quality, very low credit/default risk.
Medium grade	A1 A2 A3	A+ A A-	A+ A A-	Low expectation of credit/default risk but more vulnerable to adverse economic conditions.
Lowest investment – grade category	Baa1 Baa2 Baa3	BBB+ BBB BBB-	BBB+ BBB BBB-	Adequate protection parameters but more subject to adverse economic conditions, moderate credit risk.
Speculative	Ba1 Ba2 Ba3	BB+ BB BB-	BB+ BB BB-	Substantial credit risk if exposed to adverse economic conditions over time.
Speculative	B1 B2 B3	B+ B B-	B+ B B-	Highly speculative, significant credit risk, financial commitments currently being met.
	Caa1 Caa2 Caa3	CCC+ CCC CCC-	CCC	Poor quality, very high credit risk.
	Ca	CC	CC	Highly speculative, default probable.
Default		D	D	In default, no income being paid on income bonds.

Investment Grade ↑

↓ "Junk"

Empirical Pricing of Credit Risk

- Market-based *Yield-spread*
- *Credit risk premium* over the riskless rate
  - E.g. 5yr US T-bond YTM = 3.0%
    - US Gov't considered credit *riskless* in it USD debts
  - 5yr Exxon AA-rated Bond YTM = 3.45%
    - While not likely, Exxon can default on its debt
    - So there is credit risk
  - Yield Spread = Credit Risk Premium = 0.45% p.a.
- Credit risk premium also discoverable through a derivative type known as Single-name Credit Default Swap

Sample Yield-spreads ("Credit Risk Premium")

Bond Quality	YTM p.a.	Yield Spread
US 5yr Treasury	2.30%	N/A
AAA 5yr $ Bond	2.55%	+ 25bp
AA 5yr $ Bond	2.85%	+ 55bp
A 5yr $ Bond	3.20%	+ 90bp
BBB 5yr $ Bond	3.70%	+140bp
BB 5yr $ Bond	5.20%	+290bp

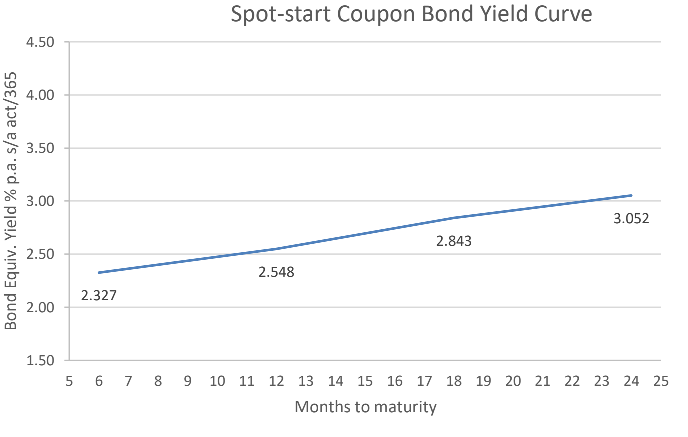
Lecture 2, Topic 2

Yield Curve Analytics

Yield Curve

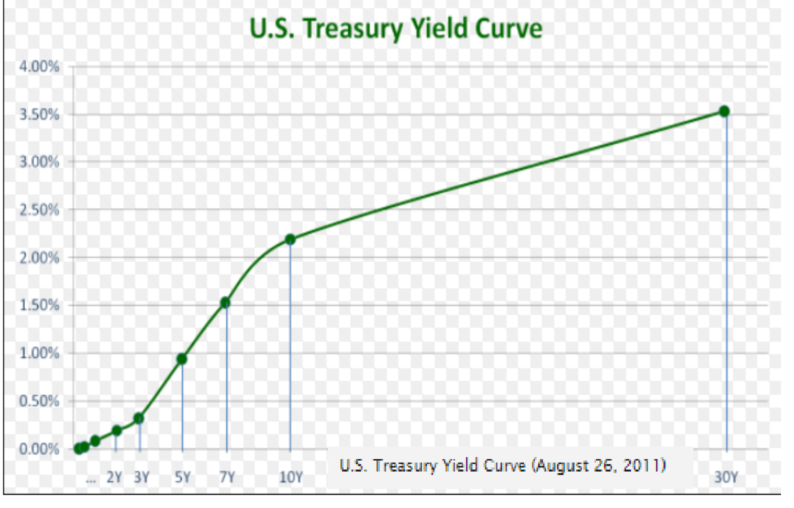
- Graphical depiction of an issuer's debt securities, showing their YTMs vs contractual maturity
- Usually this is taken from market data at a specific moment in time
- Ideally, the YTMs (y-axis) are all expressed in same frequency and day-count

- Here's a simple spot-starting yield curve with 6mo, 12mo, 18mo and 2yr YTMs:



Par Yield Curve – US Govt Bond Market

- Consists of BEYs YTMs s/a Act/365 of the most recently issued:
  - 3mo bill
  - 6mo bill
  - 12mo bill
  - 2yr note
  - 3yr note
  - 5yr note
  - 7yr note
  - 10yr note
  - 30yr bond



- Yields of on-the-run (OTR) Treasury Securities

Yield Curve – Unbiased Expectations Hypothesis

- A long-term rate will give the same income (on a PV basis) as investing and consecutively re-investing in equal credit quality short term instruments for an equal period of time
- Long rate ${}_0R_T$ is a sort of average of the current spot-starting Short Term rate ${}_0R_1$ and expected subsequent Short Term *forward rates* ${}_mf_n$:

$$(1+{}_0R_T)^T = (1+{}_0R_1) \times (1+{}_1f_2) \times .... \times (1+{}_{T-1}f_T)$$

Implicit Yield Curves

- From easily discoverable price data, we can quickly and cheaply ascertain yield curves like the *US Gov't Par Curve*
- From such curves is possibility to extract analytical data key to pricing and valuation
  - Zero-coupon rate Curve
  - Forward Implied Rate Curve

Questions before the review of Exercise 2

- You are offered a choice
  - to buy a US T-bill paying $10,000 in 6 months, or
  - to buy a coupon (cut away and sold separately) from a US T-bond, which pays $10,000 in 6 months
  - And you can discount them at the same rate (e.g. 5%)
- Do you care which you get?
  - Why or why not?
- Should there be a difference in the discount rate?
  - Why or why not?

Lecture 2: Topic 3

(Review of Homework Lex 1 Ex 2 on) Yield Curve Bootstrapping

Questions to ponder before Exercise 3

- You buy a 2yr US T-bond and cut away its 4 coupons, offering to sell them to investors. The bond's YTM = 3.5%.
- Will you discount the coupons all at 3.5%?
  - Why or why not?
  - What rate should you use to discount the coupon due in 6 months?
- Should there be a difference in the discount rate across the coupons of various maturities?
  - Why or why not?

Lecture 2, Exercise 3:
Valuing a bond using the Zero-coupon rates

Exercise 3:
Valuing Bonds Using Zero Coupon Rates

We use the Zero Coupon Rates and Discount Factors we derived in Exercise 2 for this exercise:

Maturity	ZC Rate	Discount Factor*
½ Year	2.3267%	0.9885
1 Year	2.5479%	0.9750
1½ Years	2.8475%	0.958475
2 Years	3.0611%	0.94105

* To keep it simplest, don't worry about leap (366 days) years and assume ½ a year is exactly 0.5 of a year (182½ days)

1) Find the value of (based on $100 of par) the following 2-year Singapore government securities (s.a. act/act) to the above zero-curve. Follow the example of pricing the 10% coupon bond, and then do the same for the 0% and 6% bonds:

Value of 10% coupon (i.e. $100 of par & $5 coupons)
= $\Sigma(CF_n \times df_n)$
= (5 x .9885)+(5 x .9750)+(5 x .958475)+(105 x .94105)
= $113.371 per $100 of par
YTM (-113.42 PV at t0, s/a coupon = $5, Par repayment of $100) = 3.0336% s/a

Coupon	Value	YTM(IRR)
0%		
6%		
10%	113.42	3.0336%

2. Can you think of any reason that the 3 securities of equal contractual maturity and obligations of the same borrower would offer 3 different yields?

YTM Drawbacks

- YTM is an imperfect indicator of profitability about bonds
- It gives a concise single-figure interest rate understandable in context, but
- It assumes
  - Investor holds bond to maturity
  - All interim coupons are re-investable at the same YTM
  - No default or interruption to contractual payment calendar
- Therefore it is descriptive of bond profitability, but not comprehensive
  - The investor may sell prior to maturity
  - The coupons may NOT be re-investable at the same rate
  - There can be a default (uni-lateral) or re-structuring (bi-lateral)

Upshot of YTM being Imperfect

- Still used a lot for profitability analysis and comparison among alternatives
- Most bigger players on buy- and sell-side now do pricing and valuation of bonds using a series of credit-adjusted zero-rates corresponding to the cashflow dates
  - This is also DCF methodology
  - But uses as many discount rates as there are cashflow dates

Question to ponder before Exercise 4

- We have been calculating discount factors in the previous exercises, but do we really need them?
  – Can’t we always discount using the correct rate?
- Or is there something inherently scalable about discount factors, making them better in a lot of cases then their corresponding interest rates?

Lecture 2, Exercise 4:
Deriving Implied Forward Rates

Exercise 4:
Deriving Forward Rates from Zero Rates

We use the Zero Coupon Rates and Discount Factors we derived in Exercise 1 for this exercise:

Maturity	ZC Disc.Factor	S/A ZC Rate
½ Year	**0.9885**	2.3268%
1 Year	**0.9750**	2.5479%
1½ Years	**0.958475**	2.8475%
2 Years	**0.94105**	3.0611%

1) Each of the zero-coupon rates you see above is one for which interest accruals begin immediately (spot, usually T+1), and end at their respective maturities. But what can the 1yr zero and 6mo zero tell you about what a forward-starting funding should cost? With the above information we can calculate the so-called forward implied rates. Follow this procedure to calculate the first one.

   Implied Forward Rate =
   $[(df_{shortdate} \div df_{longdate})-1] \times 1/(\text{date diff. in years})$

By the way, the other (more common) way to calculate implied forward rates is to use the spot-starting zero rates in the following equation. F = Frequency = 2 in this case.

$$Rate_{ShortxLong}\% = \left(\frac{(1+Rate_{0xLong}/F)^{nlong}}{1+Rate_{0xShort}/F)^{nshort}} - 1 \right) \times F$$

Where both rates already on same frequency F and day-count
And nLong = number of periods from spot to long maturity (0 x Long)
And nShort = number of periods from spot to short maturity (0 X Short)

*You should get **2.7692% s/a**, but may get a slight difference in answer between the two formulas (if you try both), due to rounding. Keep in mind we've simplified this away from day-count, by assuming even 6-mo periods.*

2. Now do the same for the 12 x 18mo and the 18 x 24mo forward rates and complete the following table:

Forward Period	Implied Forward 6-mo Rate
0mo X 6mo (spot-6mo)	**2.3267%**
6mo X 12mo	**2.7692%**
12mo X 18mo	
18mo X 24mo	

End of Lecture 2

Final Q&A

Lecture 2 Self-Study: Practicalities of Bond Settlement in the Secondary Markets

Not optional – please understand how this works

Practicality of Bond Settlement in Secondary Mkt

- Assume today is 23Sep202X, and you're trading for T+1 settlement (i.e. 24Sep202X)
  - The bond pays coupons at 5% Act/Act semi-annually on 15th of Jan and July.
- You buy $10m face amount of the bond, agreeing to pay a *Clean Price* of $101.00
- What is the full amount you pay to the bond seller on 24Sep?
  - Remember, there is a s/a coupon payment that you as buyer will receive on 15Jan. How do you compensate the bond seller for the interest accrued since the last coupon payment 15Jul until 24Sep?

Settlement (“*Invoice*” or “*Dirty*”) Bond Pricing

- Remember…
  - Bonds Trade on a *Clean Price,*
  - But they settle on a *Dirty Price*
- Clean Price = Dealt Price x Par Amount

  ($101 ÷ $100) x $10m = __________
- Accrued Interest (Acc Int) =

  Coupon $ for Period x days elapsed/days in period

  = $250,000 x 71/184 = ____________
- Dirty Price = Clean + Acc Int = ________________

Answers to Self-Study

- Coupon amount = ½ x .05 x \$10m = \$250,000
- Coupon period = from and incl 15Jul to but excl 15Jan = 184 days
- Elapsed days since last coupon = 71 days
  (i.e. from and incl 15Jul to but excl 24Sep)
- Clean Price = 101% of \$10m = \$10.1m
- Acc Int = \$250,000 x 71/184 = \$96,467.39
- Settlement Price = \$10,196,467.39 due 24Sep
- *The dirty settlement compensates seller for accrued interest since the last coupon period*