# Project Topics

October 11, 2019

## 1 Requirement

1. Each group is required to submit a final report together with source code for your project, stating in details the derivation of pricing method (if not discussed in class), the choice of numerical algorithm, test results and analysis of results.
2. **Deadline:** November 31, 2019

## 2 Spread Option

Let $S_1(t)$ and $S_2(t)$ be the prices of two stocks such that

$$\frac{dS_i(t)}{S_i(t)} = rdt + \sigma_i dW_i(t), \qquad i = 1, 2 \tag{1}$$

and

$$dW_1(t)dW_2(t) = \rho dt \tag{2}$$

where $W_1(t)$ and $W_2(t)$ are two Brownian motions with instantaneous correlation $\rho$ under the risk neutral measure $\mathbb{P}$. Assume that $\sigma_1, \sigma_2$ and $\rho$ are constant. A spread call option with expiry $T$ and strike $K$ has the payoff

$$\max\{S_1(T) - S_2(T) - K, 0\} \tag{3}$$

Spread option is often priced using Kirk's approximation (see Kirk (1995)) for small $K$

$$C(S_1, S_2, T) = S_1\mathcal{N}(d_1) - (S_2 + Ke^{-rT})\mathcal{N}(d_2) \tag{4}$$

where $\mathcal{N}$ is the distribution function of a standard normal distribution

$$d_{1,2} = \frac{\ln\left(S_1/(S_2 + Ke^{-rT})\right) \pm \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}} \tag{5}$$

and

$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2\left(\frac{S_2}{S_2 + Ke^{-rT}}\right)^2 - 2\rho\sigma_1\sigma_2\left(\frac{S_2}{S_2 + Ke^{-rT}}\right)} \tag{6}$$

In this project, you shall

- Derive Kirk's approximation formula.

  Hint: Let $Y(t) = S_2(t) + Ke^{-r(T-t)}$. For $K << S_2$, assume $Y(t)$ also follows log-normal. Derive the SDE for $Y(t)$ and then apply the formula for exchange option we derived in class.

- Compare Kirk's approximation with Monte Carlo simulation for different scenarios of stock prices, volatilities and correlations. For example, you can investigate the price difference between Kirk's approximation and MC simulation for the following scenarios

  1. Price: $30, 50, 70, 90, 100, 110, 130, 150, 200$
  2. Volatility: $10\%, 20\%, 50\%, 70\%, 90\%$
  3. Correlation: $-0.9, -0.5, 0.2, 0, 0.2, 0.5, 0.9$
  4. Strike: $10\%, 50\%, 100\%, 200\%$ of stock price,
  5. Combinations of different price, volatility and correlation levels.

  Note that these values are just for your references. You may choose any levels and combinations you would like to.

  Comment on the accuracy of Kirk's approximation based on the results you have obtained.

## 3 Binomial Tree and Its Improvements

In class we have discussed two methods for improving a binomial tree pricing method for American option, namely Binomial Black Scholes method (BBS) and BBS with Richardson extrapolation (BBSR). In this project, you are asked to investigate the effectiveness of these two improvements.

1. Implement Binomial Tree, BBS and BBSR
2. Investigate the convergence of these three methods by increasing the number of time steps. Plot a graph to show price against number of steps (same as the two graphs on pp 53 to 54 in the lecture notes) for each method.
3. Discuss whether BBS and BBSR have improved the efficiency of pricing American option based on your results. If not, what are the possible reason(s)?

## 4 Pricing American Option

Implement least square Monte Carlo method for pricing an American option. Investigate how the number of Monte Carlo sample paths affect the pricing accuracy by plotting a graph of price vs. number of sample path.

## 5 Discrete Hedging P&L

Suppose we want to hedge a European call option with one year expiry. The stock price follows GBM with

$$\frac{dS(t)}{S(t)} = \mu dt + \sigma_r dW(t)$$

with $S(0) = 100$. Assume the riskless interest rate is $r = 5\%$. Let $\sigma_i = 20\%$ be the implied volatility for the option and we use this volatility in hedging. Let $\sigma_r$ be the realized volatility and $n$ be the number of hedging steps. Use Monte Carlo simulation to investigate the P&L with different hedging strategies and frequencies.

1. $\sigma_r = 20\%$, $\mu = 5\%$, $n = 250$.
2. $\sigma_r = 20\%$, $\mu = 5\%$, $n = 1000$.
3. $\sigma_r = 40\%$, $\mu = 5\%$, $n = 250$.
4. $\sigma_r = 40\%$, $\mu = 5\%$, $n = 1000$.
5. $\sigma_r = 20\%$, $\mu = 0\%$, $n = 250$.
6. $\sigma_r = 20\%$, $\mu = 0\%$, $n = 1000$.
7. $\sigma_r = 40\%$, $\mu = 0\%$, $n = 250$.
8. $\sigma_r = 40\%$, $\mu = 0\%$, $n = 1000$.

Plot histogram of simulated P&L and discuss the results.

## References

[Kirk] Kirk E. (1995); Correlation in the Energy Markets, in: Managing Energy Price Risk, Risk Publications and Enron, London, pp. 7178