FE5209

NATIONAL UNIVERSITY OF SINGAPORE

FE5209 FINANCIAL ECONOMETRICS

(Semester 1: AY2017/18)

Time Allowed : Two and A Half Hours

INSTRUCTIONS TO STUDENTS

1. Please write only your student number below. **Do not write your name.**
2. This booklet contains **two (2) Sections** and comprises **Fourteen (14)** printed pages.
3. Answer **ALL** questions. This is an OPEN Book examination.
4. Graphic calculators or other calculators may be used.
5. Write legibly. A dark pencil may be used.
6. Write your answers in the boxes provided after each part of a question, except that **answers to Section A must be recorded in the table provided**.
7. Plan your answers to ensure they fit within the spaces provided. Other than this cover page and the spaces designated for providing your answers, you may do your "rough work" anywhere. Whatever you write outside of the answer spaces will be ignored.

Write your SEAT NUMBER and MATRICULATION NUMBER below.

Seat No:

Matriculation No :

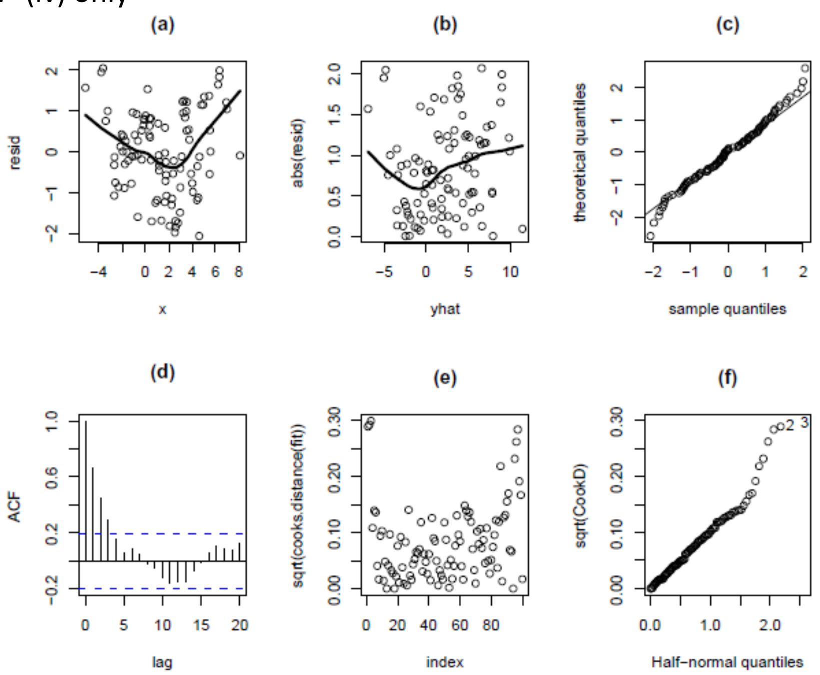
Question	Max	Marks
Section A	50	
Section B		
Question 1	25	
Question 2	25	
Total	100	

Section A (50 marks). Each question carries 5 marks. Choose the most appropriate answer and record your answer in the table below.

1. D	**2.A**	**3.C**	**4.A**	**5.B**
6. B	**7.A**	**8.B**	**9.A**	**10.B**

1. Regress the daily log returns of stock FM, denoted as Y, on the log returns of a value-weighted index, denoted as X, from January 1960 to October 2017. Residual plots and other diagnostics are shown for the regression of Y on X.
   Which of the following statements are correct?
   (i) The residual plot (a) hints a linearity of the effect of Y.
   (ii) The absolute residual plot (b) hints a GARCH effect of Y.
   (iii) The QQ plot (c) indicates right skewed distribution of Y.
   (iv) The ACF plot of the residuals (d) shows strong autocorrelations.

   A. (i) and (ii)
   B. (ii) and (iv)
   C. (iii) and (iv)
   D. (iv) only



Answer D.

2. Given 500 observations $x_1, x_2, \ldots, x_{500}$ of a time series, we estimate an AR(2) model. The sample autocovariances of the time series are as follows:

$\hat{\gamma}(0) = 4.0, \quad \hat{\gamma}(1) = 0.0, \quad \hat{\gamma}(2) = 2.5, \quad \hat{\gamma}(3) = 0.0, \quad \hat{\gamma}(4) = 1.0.$

Estimate the AR coefficients, and compute the variance of noise.

A. The AR coefficients are 0 and 0.625. The variance of noise is 2.438.
B. The AR coefficients are 0 and 2.500. The variance of noise is 2.438.
C. The AR coefficients are 0 and 0.625. The variance of noise is 2.500.
D. The AR coefficients are 0 and 2.500. The variance of noise is not computable given the information.

Answer: A.

$$\rho_1 = 0, \quad \rho_2 = 0.625, \quad \rho_3 = 0, \quad \rho_4 = 0.25$$

$$\begin{bmatrix} 1 & \hat{\rho}_1 \\ \hat{\rho}_1 & 1 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = \begin{bmatrix} \hat{\rho}_1 \\ \hat{\rho}_2 \end{bmatrix}$$

Hence, $\begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.625 \end{bmatrix}$.

$$\gamma_0 = \phi_1^2\gamma_0 + \phi_2^2\gamma_0 + 2\phi_1\phi_2\gamma_1 + \sigma_\epsilon^2$$

Hence, $\sigma_\epsilon^2 = (1 - 0.625^2) \times 4.0 = 2.4375$

3. Estimate the risk of holding stock IBALPHA with \$1 million. We use daily log returns of the stocks, starting from January 4, 2017 for $T = 250$ observations. The R output of the fitted model is as follows:

> summary(fit_garch)

Title:
 GARCH Modelling

Call:
 garchFit(formula = ~garch(1, 1), data = BMWreturn)

	Estimate	Std. Error	t value	Pr(>|t|)
mu	4.32e-04	1.599e-04	2.704	0.00685 **
omega	8.28e-06	1.373e-06	6.034	1.6e-09 ***
alpha1	9.75e-02	1.106e-02	8.819	< 2e-16 ***
beta1	8.67e-01	1.520e-02	57.035	< 2e-16 ***

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Moreover, the estimated conditional variance and the observed return are $\hat{\sigma}_T = 0.01$ and $r_T = 4.32 \times 10^{-4}$. Compute the monthly 95% VaR and Expected shortfall at $T+1$? (The 95% quantile of normal random variable is 1.64 and the density of 1.64 is 0.103). Assume there are 25 trading days in one month.)

A. The monthly VaR is 0.016 and ES is 0.089.
B. The monthly VaR is 0.079 and ES is 0.018.
C. The monthly VaR is about 79,000 and ES is about 89,000.
D. The monthly VaR is about 16,000 and ES is about 18,000.

Answer: C.

The fitted model is

$$r_t = 4.32 \times 10^{-4} + \varepsilon_t$$
$$\varepsilon_t = \sigma_t z_t \quad z_t \sim N(0,1)$$

$$\sigma_t^2 = 8.28 \times 10^{-6} + 0.0975\varepsilon_{t-1}^2 + 0.867\sigma_{t-1}^2$$

The residual is $\hat{\varepsilon}_T = r_T - \mu = 0$, hence we have the monthly vola

$$\hat{\sigma}_{T+1} = \sqrt{25} \times \sqrt{\omega + \alpha\epsilon_T^2 + \beta\sigma_T^2} = \sqrt{25} \times 0.0097$$

The monthly mean is $\hat{\mu}_{T+1} = \mu = 4.32 \times 10^{-4} \times 25$.

The VaR is $S \times \{-\hat{\mu} + \hat{\sigma}\Phi^{-1}(1-\alpha)\} = 15903$.

The monthly VaR is 79432.

The ES is

$$S \times \{-\hat{\mu} + \hat{\sigma}\frac{\phi(\Phi^{-1}(1-\alpha))}{\alpha}\} = 10^6 \times (-4.32 \times 10^{-4} \times 25 + \sqrt{25} \times 0.0097 \times \frac{0.103}{0.05})$$
$$= 89110$$

4. Consider the monthly log stock returns of APPLEX Company $x_{1,t}$ and the log returns of a value-weighted index $x_{2,t}$ from January 1980 to October 2017. The fitted model is as follows:

$$\begin{pmatrix} x_{1,t} \\ x_{2,t} \end{pmatrix} = \begin{pmatrix} 0.77 \\ 0.27 \end{pmatrix} + \begin{pmatrix} -0.16 & 0.39 \\ 0 & 0.09 \end{pmatrix} \begin{pmatrix} x_{1,t-1} \\ x_{2,t-1} \end{pmatrix} + \begin{pmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \end{pmatrix}, \quad \Sigma_\epsilon = \begin{pmatrix} 82.98 & 21.42 \\ 21.42 & 19.74 \end{pmatrix}$$

where $\epsilon_t = (\epsilon_{1,t}, \epsilon_{2,t})'$ is a bivariate white noise process. Which of the following statements are correct?

(i) The mean of the two log return series is (0.76,0.30)'.
(ii) The variance of the series $x_{2,t}$ is 9.90.
(iii) The two series are uncoupled.
(iv) The bivariate VAR model is stationary.

A. (i) and (iv)
B. (ii) and (iv)
C. (iii) and (iv)
D. (iv) only

Answer: A

(i) $E(X_t) = (I - \Phi_1)^{-1}\Phi_0 = \begin{pmatrix} 0.76 \\ 0.30 \end{pmatrix}$
(ii) $\sigma_{x_{2,t}}^2 = 0.09^2\sigma_{X_{2,t}}^2 + \sigma_\epsilon^2 = 19.90$
(iii) The two series are unidirectional
(iv) Yes, it is stationary

5. Let $x_1, \dots, x_N$ denotes N-sample losses recorded in CS Company. Suppose that the losses follow an exponential distribution with parameter $\lambda > 0$.

$$\text{Exponential density is } f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Which of the statement is correct?

A. To test the unit root in the process of losses, the Student-t distribution is used.
B. If the losses are i.i.d., the stochastic process is weakly stationary.
C. If the empirical ACF (autocorrelation function) of the losses has a cut-off at a small lag number, it is a sign that the series obeys a pure AR process.

D. If the losses are i.i.d., the maximum likelihood estimator of exponential parameter is the sample average.

Answer: B

A. When the process has a unit root the standard t-test doesn't apply.

C. False. It is possible it follows MA.

D. Log likelihood function is $\mathrm{N}\log\lambda - \lambda\sum \mathrm{x_i}$ and thus the estimator is $\lambda = \frac{\mathrm{N}}{\sum \mathrm{x_i}}$.

6. Consider daily log returns of company FEENSTAUB. There is no autocorrelation in the time series, yet volatility clustering is likely. Implementing the following R code, we obtain an output presented blow, where some values are missing, e.g. estimate of shape.

```
>alpha = .01
>n = length(Xreturn)
>fit_garch = garchFit(~garch(1,1),Xreturn,cond.dist="std")
>summary(fit_garch)
>pred = as.numeric(predict(fit_garch,n.ahead=1))
>df = as.numeric(coef(fit_garch)[5])
>q = qstd(alpha, mean = pred[1],  sd =pred[3], nu = df )
>lambda = pred[3]/sqrt( (df)/(df-2) )
>qalpha = qt(alpha,df=df)
>es1 = dt(qalpha,df=df)/(alpha)
>es2 = (df + qalpha^2) / (df - 1)
```

```
Title:
 GARCH Modelling

Call:
 garchFit(formula = ~garch(1, 1), data = Xreturn, cond.dist = "std")

Error Analysis:
        Estimate         Std. Error  t value  Pr(>|t|)
 mu     7.147e-04        2.643e-04   2.704    0.00685   **
 omega  2.833e-06        9.819e-07   2.885    0.00392   **
 alpha1 3.287e-02        1.164e-02   2.824    0.00474   **
 beta1  9.384e-01        1.628e-02   57.633   <2e-16    ***
 shape  (missing value)  6.072e-01   7.256    4e-13     ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:
 3215.913    normalized:  3.215913

Standardised Residuals Tests:
                                Statistic   p-Value
 Jarque-Bera Test   R    Chi^2  39705.01    0
 Shapiro-Wilk Test  R    W      0.8656913   0
 Ljung-Box Test     R    Q(10)  7.88475     0.6400934
```

Ljung-Box Test	*R*	*Q(15)*	*11.5034*	*0.7161615*
Ljung-Box Test	*R*	*Q(20)*	*15.61023*	*0.7404913*
Ljung-Box Test	*R^2*	*Q(10)*	*6.557717*	*0.7664347*
Ljung-Box Test	*R^2*	*Q(15)*	*6.807427*	*0.9627747*
Ljung-Box Test	*R^2*	*Q(20)*	*7.229426*	*0.995862*
LM Arch Test	*R*	*TR^2*	*6.326875*	*0.8987163*

Information Criterion Statistics:

AIC	BIC	SIC	HQIC
-6.421825	-6.397286	-6.421875	-6.412499

Moreover, we have obtained the1-day ahead volatility forecast $\widehat{\sigma}_{n+1} = 0.0095$ and

```
> q
[1] -0.02429445
> qalpha
[1] -3.563288
> es1
[1] 0.965
> es2
[1] 5.022
```

Which of the following statements is correct?

A. The GARCH(1,1) process is stationary as the Ljung-Box Test for the squared residuals is insignificant.
B. Suppose that $1,000,000 is invested in X stock, 99% Value-at-Risk is 24,294 and Expected Shortfall is 33,300.
C. The GARCH(1,1) model is inadequate as the kurtosis of the real distribution is bigger than 3.
D. None of the above.

Answer: B

A. GARC(1,1) is stationary as alpha1+beta1<1
B. VaR = -1000000*q= 24,294

$$\widehat{\sigma}_{n+1} = 0.0095, \mathrm{df} = 7.256 * 0.6072 = 4.406$$

lambda = $\widehat{\sigma}_{n+1}$*/sqrt((df)/(df-2))=0.0070201*

es3 = -mu+lambda*es1*es2=0.0333067792

ES = 1000000*es3=33,306

7. Consider the following AR(1) model:

$$x_t = 0.2 + 0.8x_{t-1} + \epsilon_t, \qquad \epsilon_t \sim IID\ N(0, \sigma_\epsilon^2)$$

Which of the following statements are correct?

(i) $E[x_t|x_{t-1}] = 0.2 + 0.8x_{t-1}$.

(ii) $E[x_t] = 1$.

(iii) $Var[x_t|x_{t-1}] = \sigma_\epsilon^2$.
(iv) $Var[x_t] = 5\sigma_\epsilon^2$.
(v) $Var[x_t|x_{t-2}] = 5\sigma_\epsilon^2$.

A. (i), (ii) and (iii)
B. (i), (ii), (iii) and (v)
C. (ii), (iii), (iv) and (v)
D. None of the above.

Answer: A.

Correct, $E[x_t|x_{t-1}] = E[0.2 + 0.8x_{t-1} + \epsilon_t|x_{t-1}] = 0.2 + 0.8x_{t-1}$ as $E[\epsilon_t|x_{t-1}] = 0$.
Correct, $E[x_t] = 0.2/(1 - 0.8) = 1$.
Correct, $Var[x_t|x_{t-1}] = Var[0.2 + 0.8x_{t-1} + \epsilon_t|x_{t-1}] = \sigma_\epsilon^2$.
False, $Var[x_t] = \sigma_\epsilon^2/(1 - 0.8^2) = (1/0.36)\sigma_\epsilon^2$.
Correct, $Var[x_t|x_{t-2}] = Var[0.2 + 0.8x_{t-1} + \epsilon_t|x_{t-2}] = 0.64Var[x_{t-1}|x_{t-2}] + \sigma_\epsilon^2 = 1.64\sigma_\epsilon^2$.

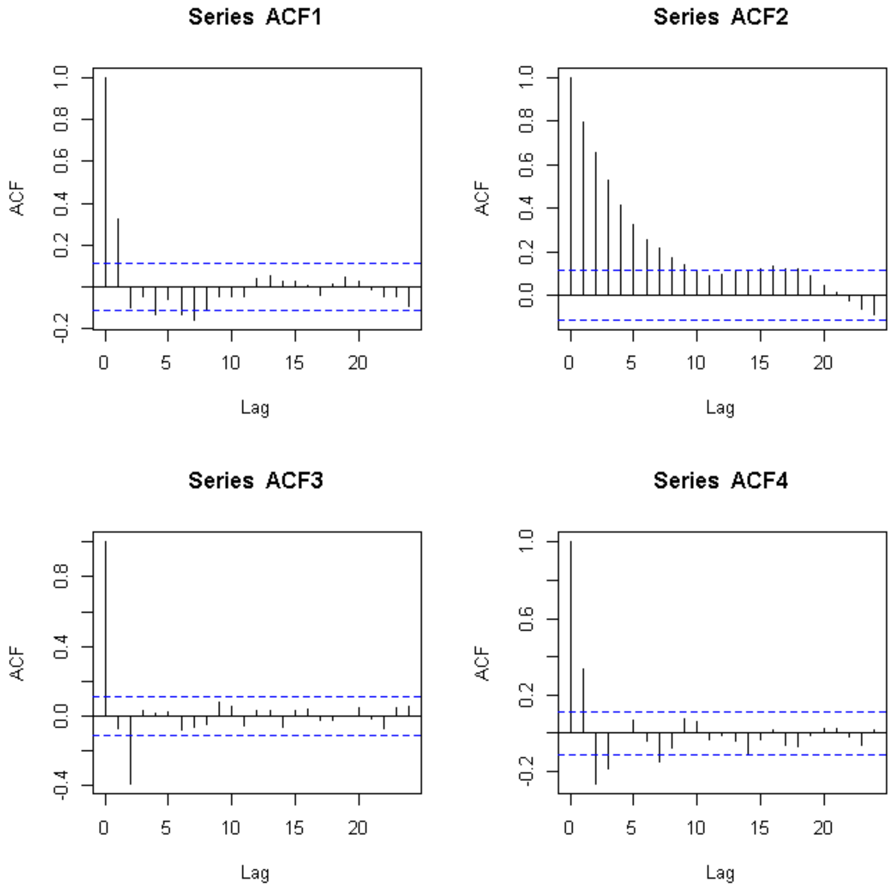
8. Suppose that $X_t = (X_{1,t}, X_{2,t})^T$ follows a bivariate VAR(1) process:
$$\Delta X_{1,t} = -0.5X_{1,t-1} - X_{2,t-1} + \epsilon_{1,t}$$
$$\Delta X_{2,t} = -0.25X_{1,t-1} - 0.5X_{2,t-1} + \epsilon_{2,t}$$
Where $\epsilon_{1,t} \sim IID(0{,}0.25)$ and $\epsilon_{2,t} \sim IID(0{,}0.81)$.

A. X_t is stationary. $X_{1,t} + X_{2,t}$ is stationary.
B. X_t is not stationary. $X_{1,t} + 2X_{2,t}$ is stationary.
C. X_t is stationary. $X_{1,t} + 2X_{2,t}$ is not stationary.
D. X_t is not stationary. $X_{1,t} + X_{2,t}$ is not stationary.

Answer. B.
In AR(1) form its "matrix" has an eigenvalue equal to 1, so the process is not stationary.
$0.5X_{1,t} + X_{2,t}$ is stationary.

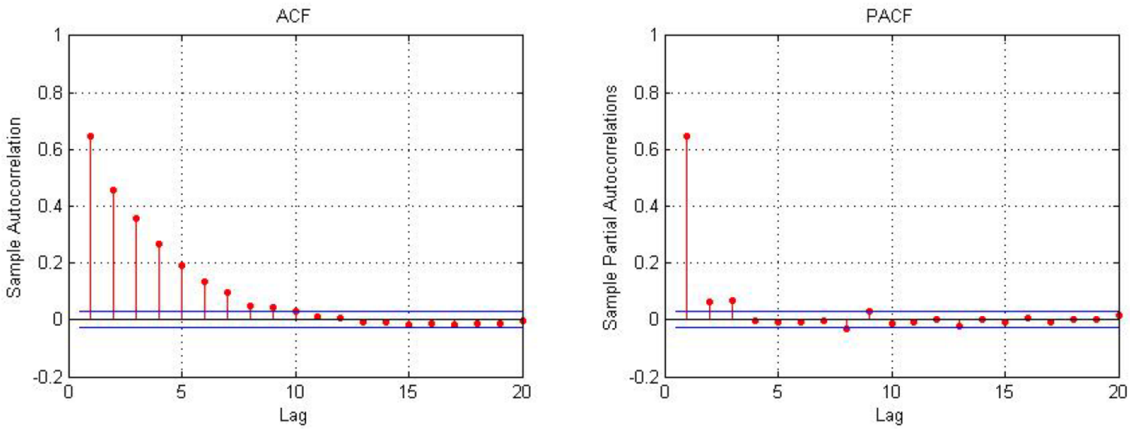
9. Suppose that the monthly log-earnings of a company, denoted as x_t, follows the model
$$(1 - 0.7B - 0.1B^2)x_t = (1 - 2.5B - 0.7B^2)\epsilon_t, \quad \epsilon_t \sim IID\ N(0, \sigma_\epsilon^2)$$
Which of the following statement is correct?

A. The process is stationary, because its two roots are larger than 1 in magnitude. Its sample autocorrelations are similar to graph ACF1.
B. The process is non-stationary, because one root equals 1. Its sample autocorrelations follow graph ACF2.
C. The process is non-stationary, because its two roots are larger than 1 in magnitude. Its sample autocorrelations follow graph ACF3.
D. The process is stationary, because one root equals 1. Its sample autocorrelations follow graph ACF4.



Answer: A
The characteristic equation is $1 - 0.7\lambda - 0.1\lambda^2 = 0$. There are two roots: 1.2 and -8.2.

10. The following graphs show the sample autocorrelation function and partial autocorrelation function of a simulated series. Horizontal lines correspond to 95% confidence bounds. Which of the following statement is correct?



A. To test the null hypothesis that the underlying process has a unit root, one may use the Student distribution.
B. An AR(3) model might have been used to simulate this series.
C. Ljung-Box test on residuals allows you to determine if the underlying process has GARCH effect.

D. None of the above.

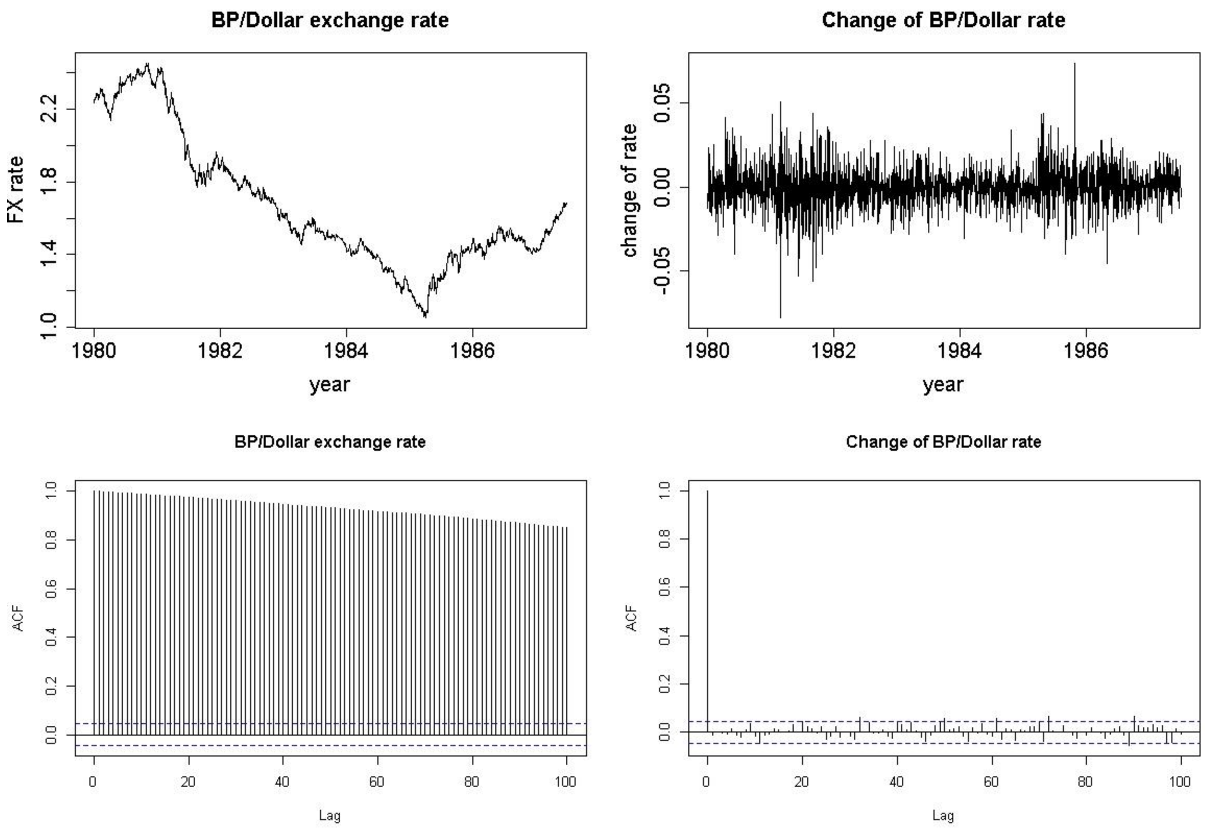
Answer: B.

A. False. When the process has a unit root the standard t-test doesn't apply.

<u>**Section B (50 marks). There are 2 questions. Write down your answers in the boxes provided.**</u>

Question 1. (25 marks)

Consider the daily exchange rate USD/British Pound Sterling from 1980–01 to 1987–05. The time series plot and the ACF plot of the data are displayed below. The time series plot shows a downward trend. The sample ACF plot shows a slow decay up to lag 100, which implies the existence of ``long memory'' in the price series. The time series plot and the sample ACF of the first order difference of the data are displayed on the right panel.



1) [10p] In order to describe the trend of the exchange rate series effectively, one can consider either a process with trend stationarity or a random walk process. Denote Y_t a stationary AR(1) process with a deterministic linear trend term and X_t a random walk process:

$$Y_t = \theta\, Y_{t-1} + \gamma\, t + e_t, \quad |\theta| < 1$$
$$X_t = X_{t-1} + \varepsilon_t,$$

where e_t and ε_t are white noise processes. Use Y_0 and X_0 to denote the initial values for Y_t and X_t respectively. Show the means and variances of the trend stationary and random walk processes and discuss the influence of shock on the evolution of the two processes.

Trend stationary:

$$E(Y_t) = E\left(\theta^t\, Y_0 + \sum_{j=0}^{t-1} \theta^j \gamma t - \sum_{j=1}^{t-1} j\theta^j \gamma + \sum_{j=0}^{t-1} \theta^j e_{t-j}\right) = \theta^t\, Y_0 + \sum_{j=0}^{t-1} \theta^j \gamma t - \sum_{j=1}^{t-1} j\theta^j \gamma$$

$$var(Y_t) = var\left(\sum_{j=0}^{t-1} \theta^j e_{t-j}\right) = \sigma_e^2 \sum_{j=0}^{t-1} \theta^{2j} \to \frac{\sigma_\varepsilon^2}{1-\theta^2}, \text{ as } t \to \infty$$

Random walk:

$$E(X_t) = X_0$$

$$var(X_t) = t\sigma_\varepsilon^2$$

Shock has temporal impact on the trend stationary process, but permanent influence on the random walk process.

2) [5p] In order to select a reasonable model for the exchange rate series, we use the augmented Dickey Fuller (ADF) test:

$$\Delta S_t = \pi\, S_{t-1} + \sum_{i=1}^{p} c_i \Delta S_{t-i} + \epsilon_t$$

where S_t represents the daily foreign exchange rate of USD/British Pound Sterling. Write down the null hypothesis and the alternative hypothesis of the ADF test.
For $p = 10$, we obtained the least square estimate of π: $\hat{\pi} = -0.0012$, with standard deviation of 0.013. Compute the test statistic of ADF and draw conclusion on the hypothesis of unit root (The critical value is -2.89 at 5% significance level)

$H_0: \pi = 0$ against $H_1: \pi < 0$

The null is unit-root process nonstationary. The alternative is stationary.

The test statistic is $\frac{\hat{\pi}}{SD(\hat{\pi})} = -\frac{0.0012}{0.013} = -0.0923$. It is insignificant. We don't reject the null and conclude there is unit root.

3) [5p] The ADF test is also performed to the differenced series with $\mathrm{p} = 2$. The R output is provided below. Draw conclusion on the ADF test of the differenced series.

```
> adf.test(diffbp, k = 2)
     Augmented Dickey-Fuller Test
data:  diffbp
Dickey-Fuller = -25.2641, Lag order = 2, p-value = 0.01
```

According to the R output, we will reject the null given p value<5%, and conclude there is no evidence of unit root in the differenced series.

4) [5p] There exhibits heteroscedasticity and volatility clustering in the differenced exchange rate series. It motivates to adopt GARCH model. Under normality, derive the kurtosis of GARCH(1,1) process. Show your proof clearly.

$$K_\epsilon = \frac{E(\epsilon_t^4)}{[Var(\epsilon_t)]^2} = \frac{E(\epsilon_t^4)}{E(\epsilon_t^2)^2} = 3\,\frac{1-(a_1+b_1)^2}{1-2a_1^2-(a_1+b_1)^2}$$

Question 2. (25 marks)

Consider the daily log returns of STI from 1987/12/28 to 2011/02/11. The ACF and PACF plots of the log return series and the squared log return series are shown in Figure 2.

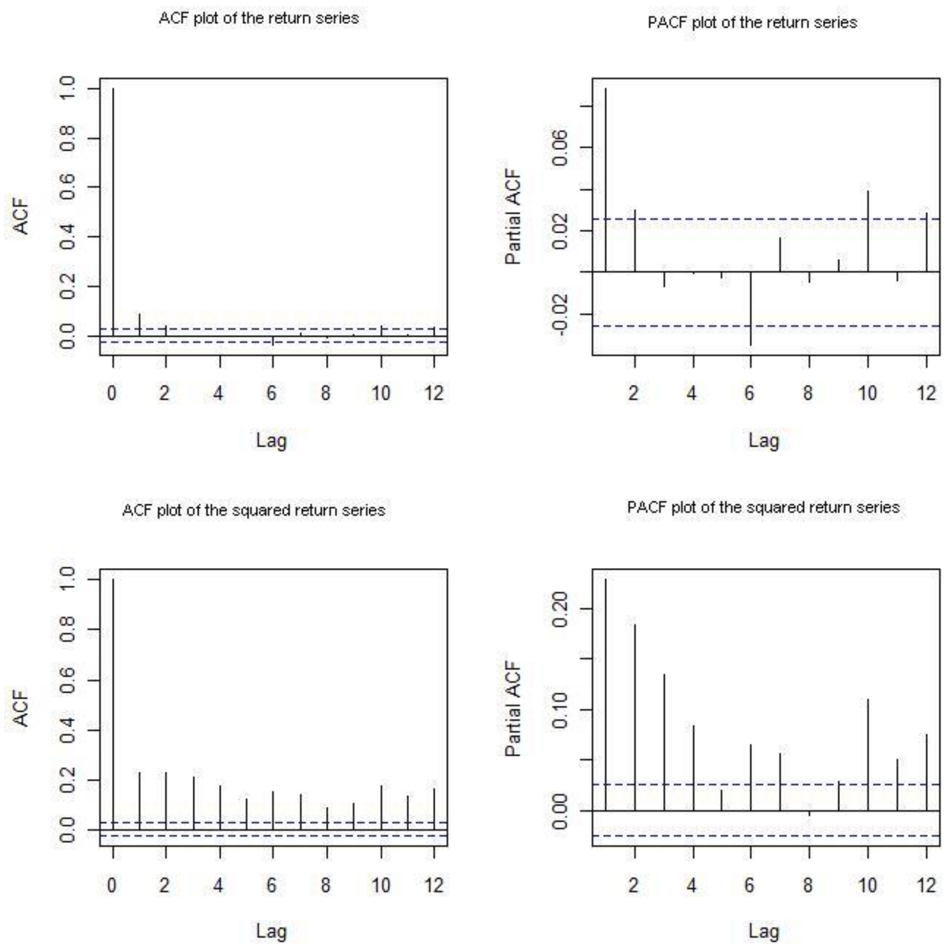


Figure 2. The ACF and PACF plots for the log returns of STI from 1987/12/28 to 2011/02/11 (upper). The ACF and PACF plots for the squared log returns of STI (bottom).

The estimated coefficients for different models are reported as follows:

	AR(3)-GARCH(1,1)		**AR(1)-GARCH(1,1)**		**AR(1)-ARCH(1)**	
	Estimate	Std. Error	Estimate	Std. Error	Estimate	Std. Error
mu	4.312e-04	1.149e-04	4.383e-04	1.146e-04	3.343e-04	1.219e-04
ar1	8.279e-02	1.387e-02	8.400e-02	1.383e-02	1.018e-01	1.529e-02
ar2	1.108e-02	1.355e-02	--	--	--	--
ar3	-1.620e-03	1.344e-02	--	--	--	--
omega	3.710e-06	6.350e-07	3.728e-06	6.373e-07	1.086e-04	5.736e-06
alpha1	1.315e-01	1.308e-02	1.321e-01	1.314e-02	4.582e-01	4.354e-02
beta1	8.492e-01	1.366e-02	8.484e-01	1.374e-02	--	--
AIC	1783		1689		1700	

5) [5p] Select a model from AR(1)-ARCH(1), AR(1)-GARCH(1,1) and AR(3)-GARCH(1,1) that fits the data well. Justify your selection and write down the fitted model.

Answer: AR(1)-GARCH(1,1) is selected, as it has the smallest value of AIC. The fitted model is:

$$r_t = 4.383e - 04 + 0.084 r_{t-1} + \epsilon_t$$
$$\epsilon_t = \sqrt{h_t} z_t, \quad z_t \sim (0,1)$$
$$h_t = 3.728e - 06 + 0.132\epsilon_t^2 + 0.848 h_{t-1}$$

6) [10p] Based on the fitted model, derive the unconditional variance and kurtosis of the residuals under the assumption of normality and compute the estimated values. Show your derivation clearly. Does the unconditional distribution of the residuals have heavy tails?

Answer: The unconditional variance of the residuals is

$$\frac{\omega}{1 - \alpha_1 - \beta_1} = 0.019\%$$

The unconditional kurtosis is

$$\frac{3[1 - (\alpha_1 + \beta_1)^2]}{1 - (\alpha_1 + \beta_1)^2 - 2\alpha_1^2} = 31.1539 > 3$$

(*if ARCH(1) is selected, unconditional kurtosis $=3\frac{(1-\alpha_1^2)}{1-3\alpha_1^2} = 6.4031 > 3$)

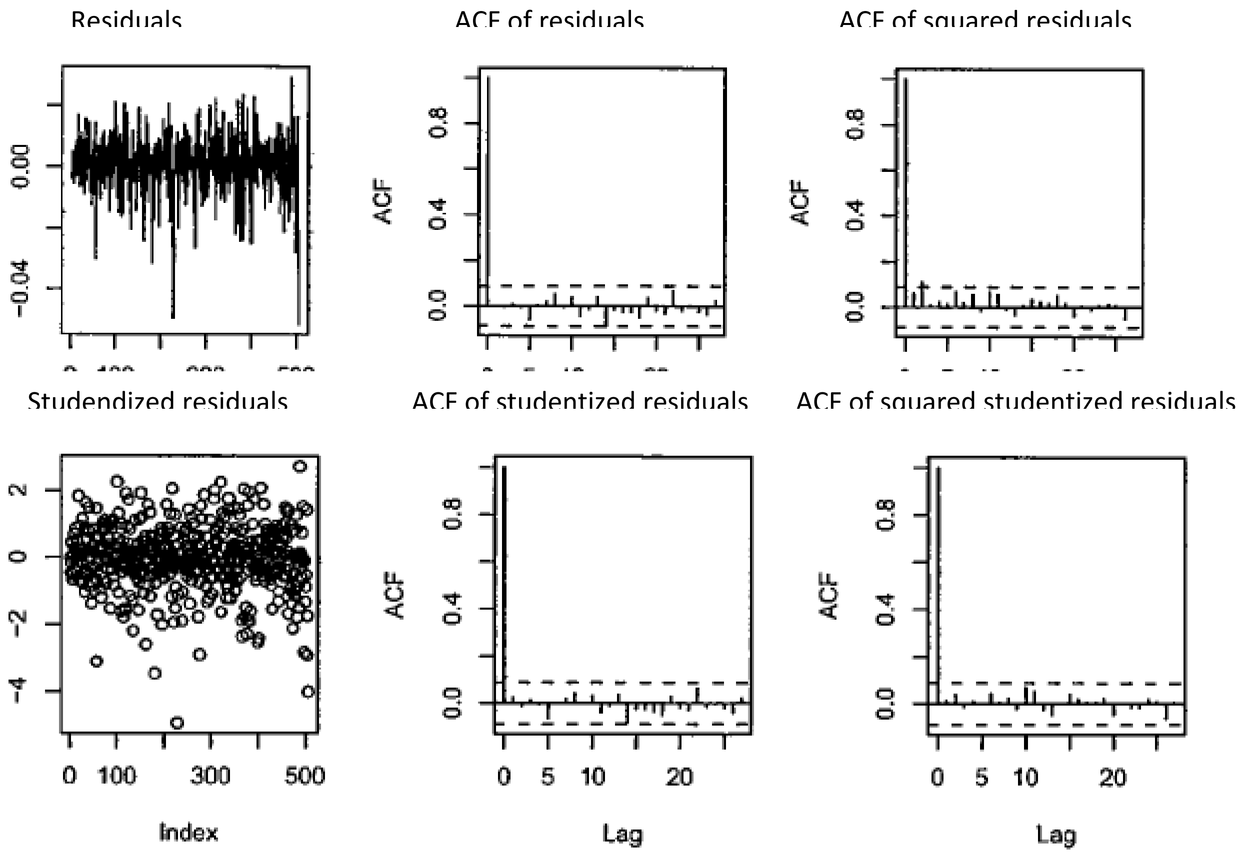
The kurtosis is larger than 3. It indicates that the residual series have heavy tails.

7) [5p] Represent the fitted model using the ARMA representation of the squared residuals.

Answer:

$$\epsilon_t^2 = \omega + (a_1 + b_1)\epsilon_{t-1}^2 + u_t - b_1 u_{t-1}, \quad u_t = h_t(z_t^2 - 1)$$
$$= 3.728e - 06 + 0.9805\epsilon_{t-1}^2 + u_t - 0.8484 u_{t-1}$$

8) [5p] The residuals and studendized residuals of the selected fitted model are displayed below, together with the ACF plot of the residuals and squared residuals respectively? Is the fitted model adequate? Justify your answer?



Answer:
Yes.
ACF of residuals shows no left-over ACF, implying adequacy of ARMA
ACF of squared studendized residuals indicates no GARCH effect left over.

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