Volatility Forecasting Models

Comparison of Performance Under Different Measures

Equator Quant

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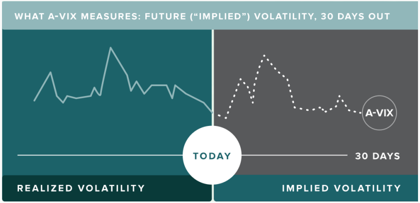
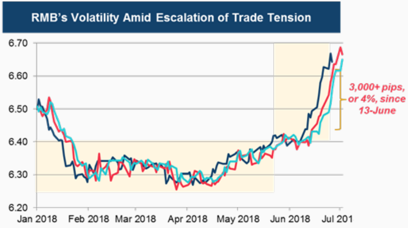
Introduction

Volatility forecasting of financial assets has important implications for option pricing, portfolio selection, risk-management and volatility trading strategies.

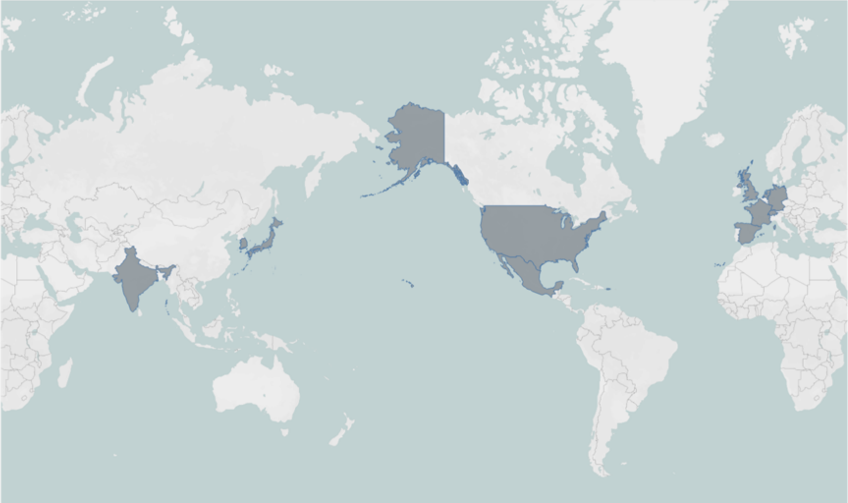




Source: Bloomberg (3 Jul 2018)

Data Selection

We select the stock indexes from markets all over the world, but mainly European markets.



Realized measures

1) realized variance (**RV**)

$$RV_t = \sum_{j=1}^{M} (r_{t,j})^2,$$

where $r_{t,j}$ is the jth intraday return of the day and M is the total number of intraday returns for the day.

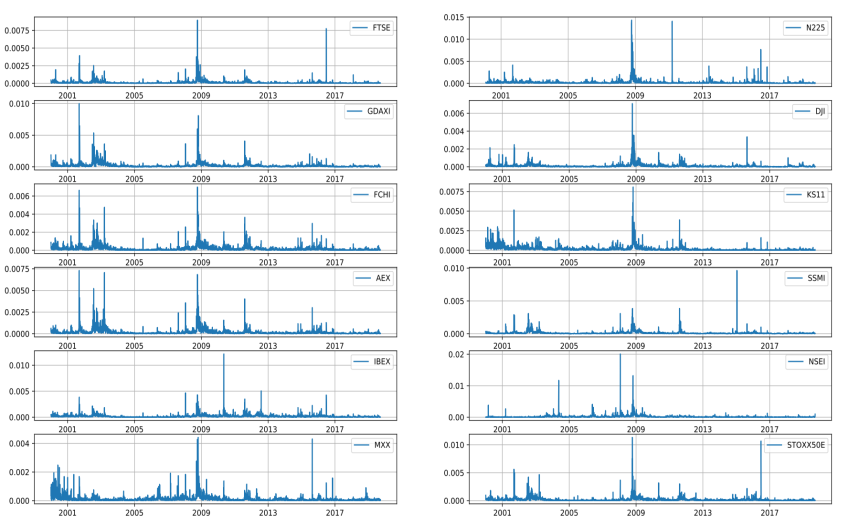
2) subsampled RV measure (**RVS**)
3) realized bipower variance (**BV**)

$$BV_t = \frac{\pi}{2} \sum_{j=2}^{M} |r_{t,j}||r_{t,j-1}|.$$

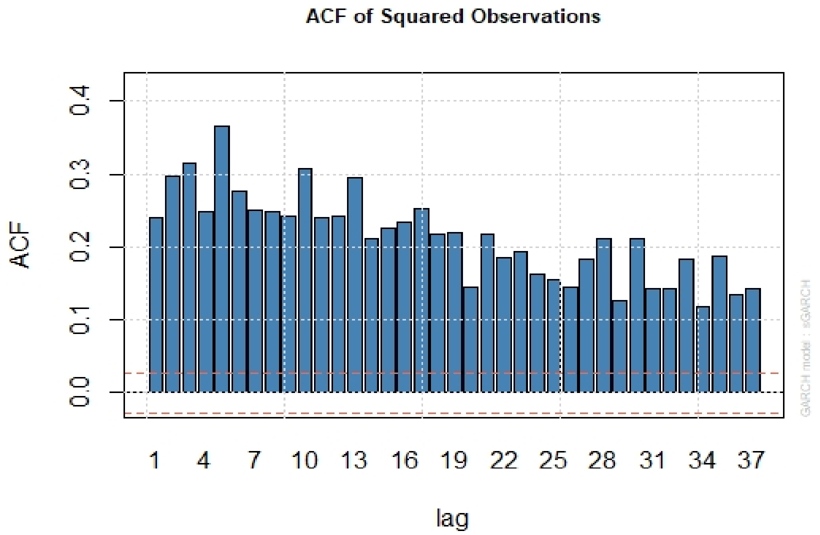
4) subsampled BV measure(**BVS**)
5) realized kernel estimator (**RK**)

$$RK_t = \sum_{h=-H}^{H} \kappa(\frac{h}{H+1})\gamma_h, \text{ where } \gamma_h = \sum_{j=|h|+1}^{M} r_{t,j} r_{t,j-|h|}.$$

Realized measures



GARCH effect



GARCH models

1) specification for modeling the conditional mean

$$\textbf{AR1: } r_t = c + \phi_1 r_{t-1} + \varepsilon_t, \quad \varepsilon_t = \sigma_t z_t, \quad z_t \sim i.i.d. N(0,1)$$

2) conditional variance equations for the various GARCH models

$$\textbf{GARCH(1,1): } \sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$

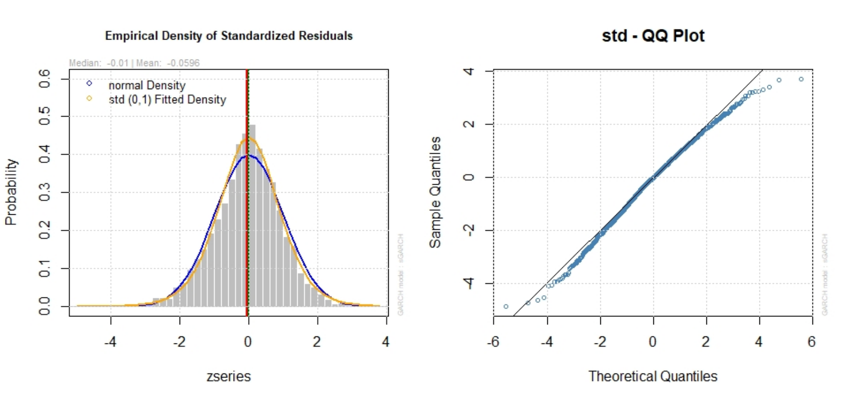
$$\textbf{EGARCH(1,1): } \log \sigma_t^2 = \omega + \beta \log \sigma_{t-1}^2 + \tau_1 z_{t-1} + \tau_2 (|z_{t-1}| - E|z_{t-1}|)$$

$$\textbf{Realized GARCH(1,1): } \log \sigma_t^2 = \omega + \beta \log \sigma_{t-1}^2 + \gamma \log x_{t-1},$$

$$\log x_t = \xi + \varphi \log \sigma_t^2 + \delta(z_t) + u_t, \text{ where } u_t \sim i.i.d N(0, \sigma_u^2)$$

Realized GARCH

Why do we choose standard residual assumption for GARCH models?



EW, RW and MA

The random walk model (RW), the moving average model (MA), and the exponentially weighted moving average model (EW)

RW: ${}^{2}_{t+1} = \sigma_t^2$

Under a random walk model, the observed scaled close-to-close variance at the end of day t is used as the best one-step ahead variance forecast for day t+1.

EW: ${}^{2}_{t+1} = \lambda\sigma_t^2 + (1 - \lambda)\hat{\sigma}_t^2$

λ, the smoothing parameter, is constrained to lie between zero and one and estimated from the data.

MA: ${}^{2}_{t+1} = p^{-1}\sum_{i=1}^{p}\sigma_{t+1-i}^2$

The moving average model (MA) predicts the variance by calculating the arithmetic mean of close-to-close variances over past p days.

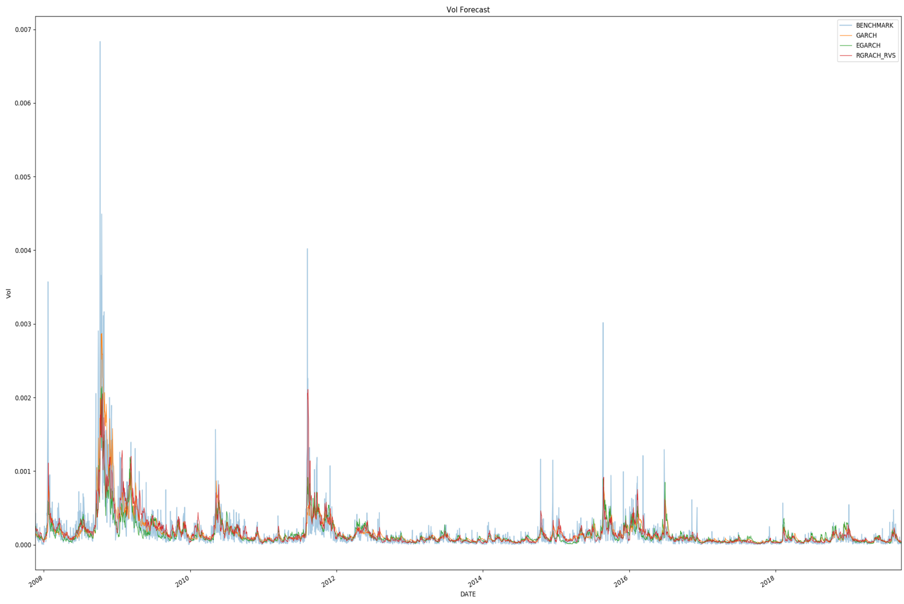
Forecast Method

Implementing rolling forecast method, we set the previous 2000 data points as obervations to estimate the parameters.

```
regarchroll = ugarchroll(spec = realgarch,data = logret,n.ahead=1,realizedVol = realize_vol,
refit.every = 100,n.start = 2000,refit.window = "moving",window.size = 1000)
```

We maintain our window length to 1000, and only predict one next point at a time. Every time we generate 100 outputs, we re-estimate the parameters according to our window length.

Forecast Results



Evaluation

Loss Functions: evaluating the accuracy of volatility forecasts

MSE = E($L_{1,k,t}$), where $L_{1,k,t} = (\sigma_t^2 - \hat{\sigma}_t^2)^2$
penalizes the forecasting errors in a symmetrical manner

QLIKE = E($L_{2,k,t}$), where $L_{2,k,t} = (\log(\hat{\sigma}_t^2) + \sigma_t^2 \hat{\sigma}_t^{-2})$
It is an asymmetric loss function that penalizes the under-prediction more heavily than the over-prediction.It is **more suitable** for the applications like risk management and VaR forecasting, where an under-prediction of volatility can be more costly than an over-prediction.

Results

The performance of models under MSE measures.

Model	FTSE	N225	GDAXI	DJI	FCHI	KS11	AEX	SSMI	IBEX	NSEI	MXX	STOXX50E	Mean
Garch	9	8	8	12	7	5	1	10	13	5	6	6	8
EGarch	1	6	13	2	8	14	5	1	7	1	1	10	5
RGarch_rvs	3	7	4	8	3	2	6	5	2	7	5	3	3
RGarch_bvs	2	3	8	6	6	3	9	8	1	4	2	4	3
RGarch_rk	4	12	6	11	4	7	10	4	5	8	3	1	7
RW_rvs	13	2	12	14	14	11	12	13	11	14	14	13	13
RW_bvs	12	1	2	9	12	10	13	12	4	2	11	8	10
RW_rk	14	14	10	13	13	9	14	14	14	13	13	14	14
MA_rvs	10	9	11	10	11	13	7	7	10	12	12	12	11
MA_bvs	7	10	9	4	9	12	8	6	8	6	10	7	10
MA_rk	11	13	14	5	10	8	11	11	12	11	8	11	12
EW_rvs	8	5	5	7	5	4	2	3	6	10	9	9	6
EW_bvs	5	4	1	1	2	6	3	2	3	3	7	2	1
EW_rk	6	11	3	3	1	1	4	9	9	9	4	5	4

Table: MSE rank

Results

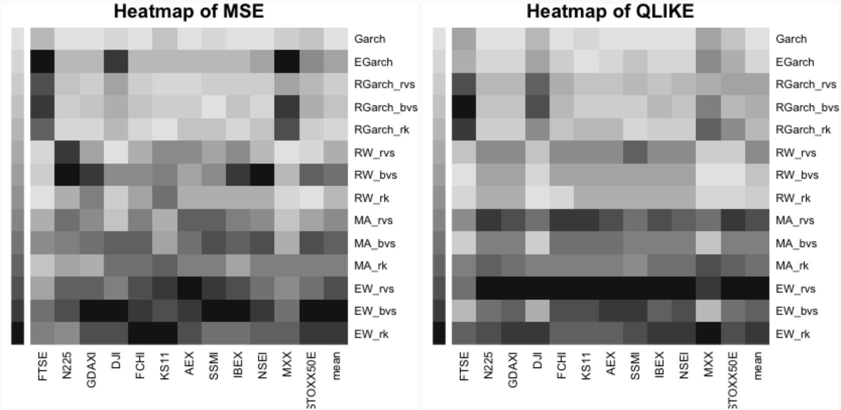
The performance of models under QLIKE measures.

Model	FTSE	N225	GDAXI	DJI	FCHI	KS11	AEX	SSMI	IBEX	NSEI	MXX	STOXX50E	Mean
Garch	8	10	10	9	10	7	10	13	11	11	8	9	10
EGarch	9	11	11	8	11	12	11	12	10	10	6	11	11
RGarch_rvs	3	4	3	4	2	2	2	2	2	1	7	7	3
RGarch_bvs	1	3	2	1	3	1	3	5	1	3	4	10	2
RGarch_rk	2	8	5	5	4	4	4	6	4	4	2	4	4
RW_rvs	11	12	12	11	12	11	12	7	12	12	12	12	12
RW_bvs	14	13	13	13	13	13	13	11	13	13	14	14	13
RW_rk	13	14	14	14	14	14	14	14	14	14	13	13	14
MA_rvs	7	2	6	6	5	5	6	8	6	7	9	2	6
MA_bvs	12	9	9	12	8	9	8	9	9	9	11	8	9
MA_rk	6	6	8	7	9	10	9	10	8	8	5	5	8
EW_rvs	5	1	1	2	1	3	1	1	3	2	3	1	1
EW_bvs	10	7	7	10	6	6	5	3	7	6	10	6	7
EW_rk	4	5	4	3	7	8	7	4	5	5	1	3	5

Table: QLIKE rank

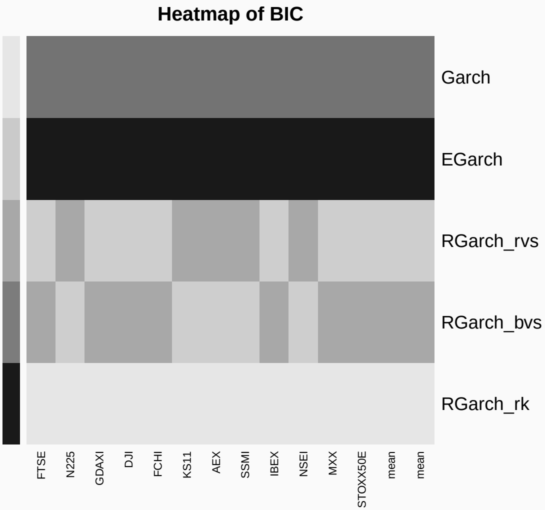
Results

Heatmaps :The performance of models under MSE and QLIKE measures. The **deeper** color means the **higher** rank of the performance among these models.



Results

Heatmaps :The performance of models under BIC measure



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Members Contribution

Name	Contribution
Zheng Hao	Literature review, data analysis, R code for GARCH models
Xiao Chao	Literature review, R code for EW and RW
Zheng Pin	Literature review, R code for MA, model evaluation
Liu Yonghao	Loss function analysis, slides compilation
Shan Changhan	R code for MSE&QLIKE rank and visualization, report writing

Q&A Time