Comparison of Several Volatility Forecasting Models

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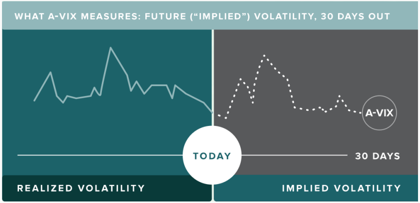
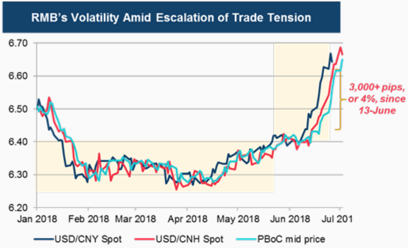
Introduction

Volatility forecasting of financial assets has important implications for option pricing, portfolio selection, risk-management and volatility trading strategies.

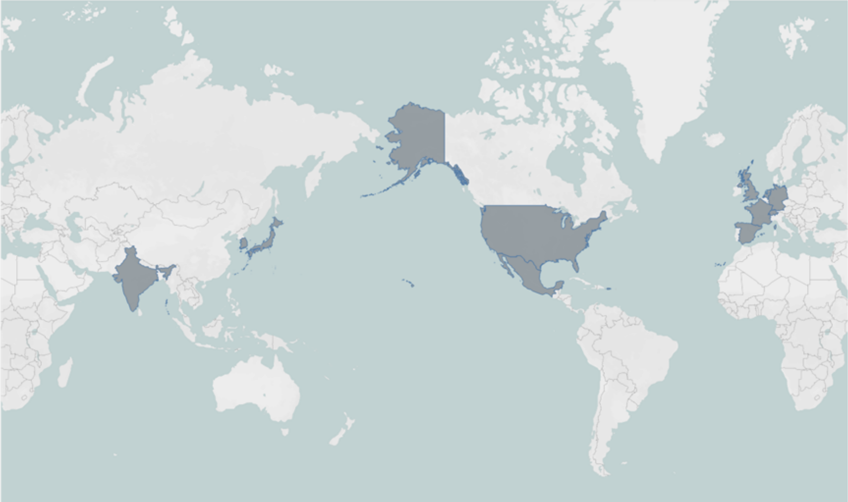




Source: Bloomberg (3 Jul 2018)

Data Selection

We select the stock indexes from markets all over the world.



Realized measures

1) realized variance (**RV**)

$$RV_t = \sum_{j=1}^{M} (r_{t,j})^2,$$

where $r_{t,j}$ is the jth intraday return of the day and M is the total number of intraday returns for the day.

2) subsampled RV measure (**RVS**)
3) realized bipower variance (**BV**)

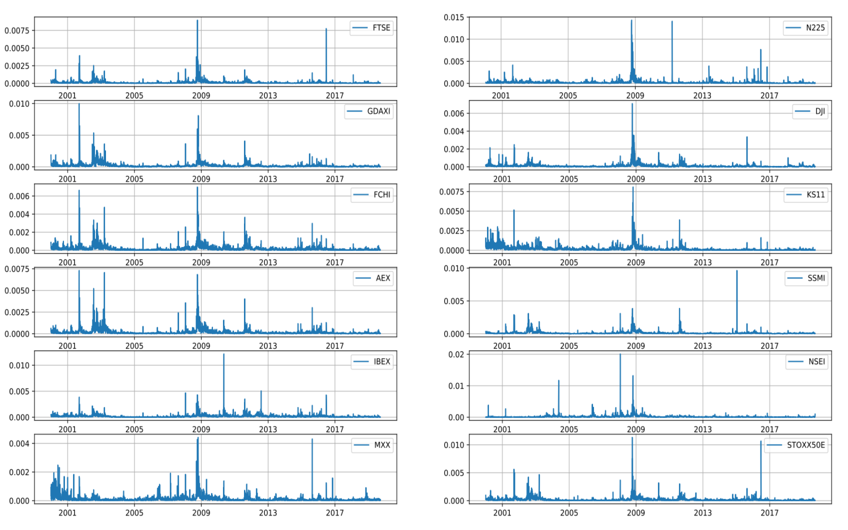
$$BV_t = \frac{\pi}{2} \sum_{j=2}^{M} |r_{t,j}||r_{t,j-1}|.$$

4) subsampled BV measure(**BVS**)
5) realized kernel estimator (**RK**)

$$RK_t = \sum_{h=-H}^{H} \kappa(\frac{h}{H+1})\gamma_h,$$

where $\gamma_h = \sum_{j=|h|+1}^{M} r_{t,j} r_{t,j-|h|}$ and $\kappa(x)$ is a kernel weight function.

Realized measures



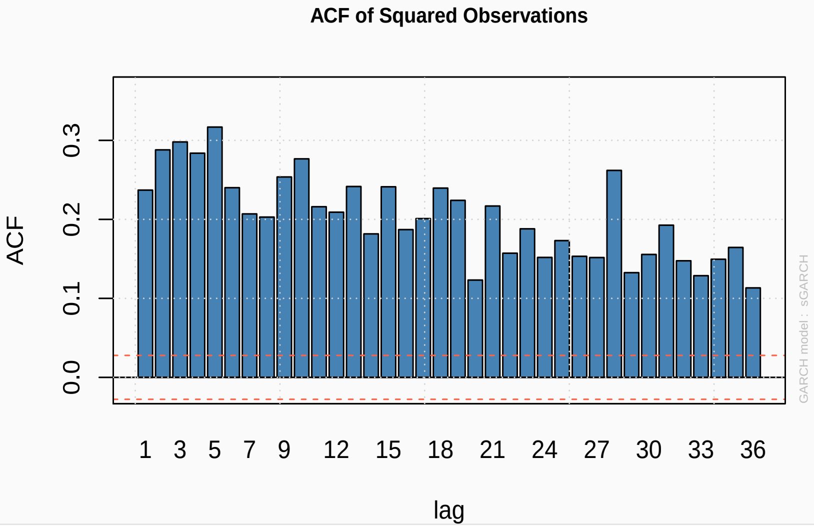
Volatility Forecasting Models

1. Standard GARCH models
2. Other time series models
3. Realized GARCH model

GARCH effect

Specification for modeling the conditional mean

$$\textbf{AR1: } r_t = c + \phi_1 r_{t-1} + \varepsilon_t.$$



GARCH models

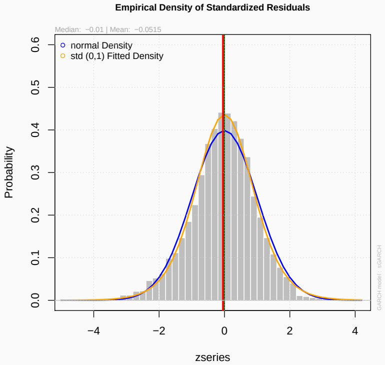
Conditional variance equations for the various GARCH models

$$
\begin{aligned}
\textbf{GARCH(1,1):}\ & \sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \\
\textbf{EGARCH(1,1):}\ & \log\sigma_t^2 = \omega + \beta\log\sigma_{t-1}^2 + \tau_1 z_{t-1} + \tau_2(|z_{t-1}| - E|z_{t-1}|) \\
\textbf{Realized GARCH(1,1):}\ & \log\sigma_t^2 = \omega + \beta\log\sigma_{t-1}^2 + \gamma\log x_{t-1}, \\
& \log x_t = \xi + \varphi\log\sigma_t^2 + \delta(z_t) + u_t, \\
& \delta(z_t) = \delta_1 z_t + \delta_2(z_t^2 - 1), \\
& \text{where } u_t \sim i.i.d.\, N(0, \sigma_u^2) \text{ and } x_t \text{ is realized volatility.}
\end{aligned}
$$

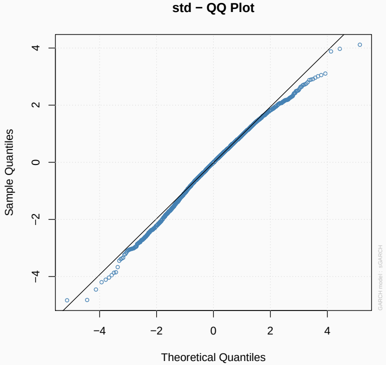
Realized GARCH

We estimate GARCH models with the Student's t-distribution.

$$\varepsilon_t = \sigma_t z_t, \quad z_t \sim i.i.d.t(d)$$



(a) Empirical Density of Standardized Residuals



(b) QQ Plot

Scaling Factor

The close-to-close variance for day t is estimated as $\sigma_t^2 = \eta x_t$, where η is the scaling factor and $x_t = RVS_t$, BVS_t or RK_t. The scaling factor η is calculated as

$$\eta = \frac{T^{-1} \sum_{t=1}^{T} (r_t - \mu_{cc})^2}{T^{-1} \sum_{t=1}^{T} (r_{oc,t} - \mu_{oc})^2},$$

where T is the total number of days in the sample period,
$r_{oc,t}$ is the open-to-close log return for day t,
$\mu_{cc} = T^{-1} \sum_{t=1}^{T} r_t$ and $\mu_{cc} = T^{-1} \sum_{t=1}^{T} r_{cc,t}$.

EW, RW and MA

The random walk model (RW), the moving average model (MA), and the exponentially weighted moving average model (EW)

RW: $\hat{\sigma}^2_{t+1} = \sigma^2_t$
Under a random walk model, the observed scaled close-to-close variance at the end of day *t* is used as the best one-step ahead variance forecast for day t+1.

EW: $\hat{\sigma}^2_{t+1} = \lambda\sigma^2_t + (1-\lambda)\hat{\sigma}^2_t$
where λ, the smoothing parameter, is constrained to lie between zero and one and estimated from the data.

MA: $\hat{\sigma}^2_{t+1} = p^{-1}\sum_{i=1}^{p}\sigma^2_{t+1-i}$
The moving average model (MA) predicts the variance by calculating the arithmetic mean of close-to-close variances over past *p* days.

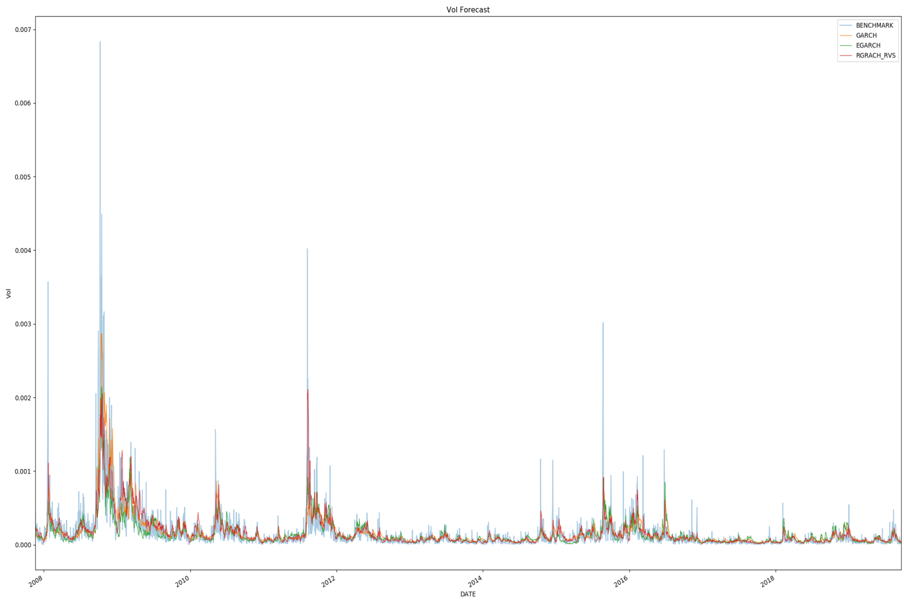
Forecast Method

Implementing rolling forecast method, we set the previous 2000 data points as obervations to estimate the parameters.

```
regarchroll = ugarchroll(spec = realgarch,data = logret,n.ahead=1,realizedVol = realize_vol,
refit.every = 100,n.start = 2000,refit.window = "moving",window.size = 1000)
```

We maintain our window length to 1000, and only predict one next point at a time. Every time we generate 100 outputs, we re-estimate the parameters according to our window length.

Forecast Results



Evaluation

Loss Functions: evaluating the accuracy of volatility forecasts

MSE = $E(L_{1,k,t})$, where $L_{1,k,t} = (\sigma_t^2 - \hat{\sigma}_t^2)^2$
penalizes the forecasting errors in a symmetrical manner

QLIKE = $E(L_{2,k,t})$, where $L_{2,k,t} = (\log(\hat{\sigma}_t^2) + \sigma_t^2 \hat{\sigma}_t^{-2})$
It is an asymmetric loss function that penalizes the under-prediction more heavily than the over-prediction.It is **more suitable** for the applications like risk management and VaR forecasting, where an under-prediction of volatility can be more costly than an over-prediction.

The performance of models under MSE measures.

Model	FTSE	N225	GDAXI	DJI	FCHI	KS11	AEX	SSMI	IBEX	NSEI	MXX	STOXX50E	Mean
Garch	9	8	8	12	7	5	1	10	13	5	6	6	8
EGarch	1	6	13	2	8	14	5	1	7	1	1	10	5
RGarch_rvs	3	7	4	8	3	2	6	5	2	7	5	3	3
RGarch_bvs	2	3	8	6	6	3	9	8	1	4	2	4	3
RGarch_rk	4	12	6	11	4	7	10	4	5	8	3	1	7
RW_rvs	13	2	12	14	14	11	12	13	11	14	14	13	13
RW_bvs	12	1	2	9	12	10	13	12	4	2	11	8	10
RW_rk	14	14	10	13	13	9	14	14	14	13	13	14	14
MA_rvs	10	9	11	10	11	13	7	7	10	12	12	12	11
MA_bvs	7	10	9	4	9	12	8	6	8	6	10	7	10
MA_rk	11	13	14	5	10	8	11	11	12	11	8	11	12
EW_rvs	8	5	5	7	5	4	2	3	6	10	9	9	6
EW_bvs	5	4	1	1	2	6	3	2	3	3	7	2	1
EW_rk	6	11	3	3	1	1	4	9	9	9	4	5	4

Table: MSE rank

Empirical Results

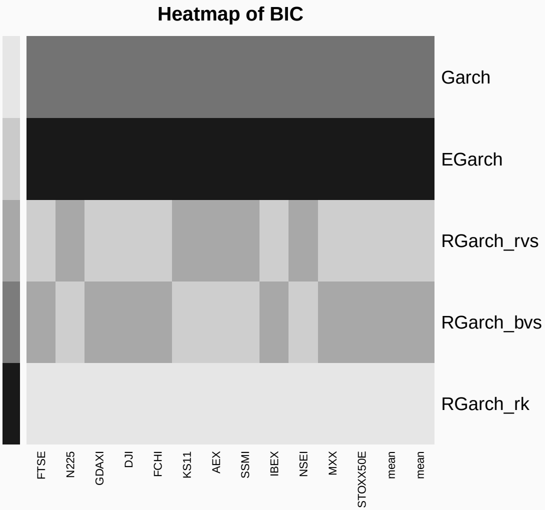
The performance of models under QLIKE measures.

Model	FTSE	N225	GDAXI	DJI	FCHI	KS11	AEX	SSMI	IBEX	NSEI	MXX	STOXX50E	Mean
Garch	8	10	10	9	10	7	10	13	11	11	8	9	10
EGarch	9	11	11	8	11	12	11	12	10	10	6	11	11
RGarch_rvs	3	4	3	4	2	2	2	2	2	1	7	7	3
RGarch_bvs	1	3	2	1	3	1	3	5	1	3	4	10	2
RGarch_rk	2	8	5	5	4	4	4	6	4	4	2	4	4
RW_rvs	11	12	12	11	12	11	12	7	12	12	12	12	12
RW_bvs	14	13	13	13	13	13	13	11	13	13	14	14	13
RW_rk	13	14	14	14	14	14	14	14	14	14	13	13	14
MA_rvs	7	2	6	6	5	5	6	8	6	7	9	2	6
MA_bvs	12	9	9	12	8	9	8	9	9	9	11	8	9
MA_rk	6	6	8	7	9	10	9	10	8	8	5	5	8
EW_rvs	5	1	1	2	1	3	1	1	3	2	3	1	1
EW_bvs	10	7	7	10	6	6	5	3	7	6	10	6	7
EW_rk	4	5	4	3	7	8	7	4	5	5	1	3	5

Table: QLIKE rank

Results

Heatmaps :The performance of models under BIC measure



Conclusion

1. Loss Function Sensitivity
2. Realized GARCH, EGARCH, GARCH
3. Model Fit and Forecast
4. Analysis of EW model: parameters, conditional long run mean variance, noise-logreturn

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Q&A Time