FE 5101:
FIXED INCOME AND DERIVATIVES
EXERCISES – LECTURE 4

Exercise 1: Inter-bank Deposit Trade

11am Monday, Sep 12, 201X

DBS:	Your offer on 3-mo JPY Deposit in 2 billion of yen?
BTMU:	0.56
DBS:	That's done. I take funds at 0.56% p.a. actual/360.
BTMU:	OK, Sep 14 Funding of ¥2,000,000,000. Dec 14 201X Maturity with interest of ¥2,831,111
DBS:	Agreed.

Read the conversation above. Then answer the following questions:

1. Who is lending and who is borrowing? How? BTMU lending by placing a deposit with DBS

2. This deposit has a stated rate of interest. Is it a zero-coupon instrument? Why or why not? It is because it only has 2 cash flows – one funding of P and one repayment of P+i. Even though it has a stated coupon rate

3. How are the funding and maturity dates determined, and how many days are in the calculation period? Accrual from and including Funding, to but excluding Maturity

4. Do you agree the interest amount? What formula was applied? P x (rate x days/day-count) = i

5. How does the borrower (deposit-taker) receive funds? When? DBS receives yen into a Japan based bank account (their own bank, or a correspondent bank)

Exercise 2: Calculating Implied Forward USD LIBORs

Today is Monday 11 Sep 201X. The following are the benchmarked spot-staring LIBORs for today, corresponding to Wed 13 Sep 201X funding:

Tenor	$LIBOR	Funding	Maturity	Accrual Days
3-month	2.25%	13 Sep 201X	13 Dec 201X	91
6-month	2.55%	13 Sep 201X	13 Mar 201Y	181
9-month	2.80%	13 Sep 201X	13 Jun 201Y	273
12-month	3.05%	13 Sep 201X	13 Sep 201Y	365

1. Calculate the following Implied Forward Rates, based on their real day-count. The first one is done for you. Here's the formula using the correct day-count ("dayct")

Forward Rate (short x long) =

[([1+(long rate x days/dayct)] ÷ [1+(short rate x days/dayct)]) – 1] x [Dayct/(Longdays-Shortdays)]

3 x 6 = [([1+(.0255 x 181/360)] ÷ [1+(0.225 x 91/360)])-1] x [360/(181-91)] = 2.8372%

Funding 13 Dec 201X, Maturity Mar 201Y, Frequency Quarterly, Day-count (181-91)/360=90/360

6 x 9 = [([1+(.0280 x 273/360)] ÷ [1+(0.0255 x 181/360)])-1] x [360/(273-181)] = 3.25%

Funding 13Mar1Y, Maturity 13Jun1Y Frequency quarterly Day-count 273-181 = 92 actual days/360

9 x 12 = [([1+(.0305 x 365/360)] ÷ [1+(0.0280 x 273/360)])-1] x [360/(365-273)] = 3.713%

Funding 13Jun1Y, Maturity 13Sep1Y

2. Now calculate the following semi-annual actual/360 rates, by either using the same formula above or by compounding the corresponding forward quarterly rates

3 x 9 = (1+ [3x6 rate x 90/360]) x (1+ [6x9 rate x 92/360]) x 360/182 =

(1+ [0.028372 x 90/360]) x (1+ [0.0325 x 92/360]) -1]x 360/182 = 3.0575 *using compounding*

Or… [([1+(.0280 x 273/360)] ÷ [1+(0.0225 x 91/360)])-1] x [360/(273-91)] = 3.0575 *using implication ("implication" = "inference")*

Funding 13Dec1X Maturity 13Jun1Y Frequency semi-annual Day-count (273-91)/360 = 182/360

6 x 12 = [1+ ([0.0325 x 92/360] x (1+ [0.03713 x 92/360]) -1] x 360/184 = 3.497 *using compounding*

Or… [([1+(.0305 x 365/360)] ÷ [1+(0.0255 x 181/360)])-1] x [360/(365-181)] = 3.497 *using implication*

Funding 13 Mar 201Y, Maturity 13 Sep 201Y Frequency s/a Day-count 184/360

Exercise 4: Calculating Long-dated Zero LIBOR Rates

Inter-bank deposit rates are typically quoted spot x 1 day to spot x 1yr. These by the nature of money market deposits are natural zero-coupon instruments. There are inter-bank deposits quoted to 18mo or even 2yr, but these conventionally require an interim payment of interest at the one-year anniversary of funding (i.e. the 18mo deposit pays interest at 12mo and at maturity)

As we know, yield-curve analytics and various fixed income products require a zero-coupon yield curve for various use-cases, including risk analysis and valuation of market-traded instruments.

With LIBOR futures, there is immense activity in these contracts, thereby establishing a highly visible and tradable forward curve. These contracts allow a user to lock in forward LIBOR rates by buying or selling futures contracts.

In the government bonds, you have already used bootstrapping to synthesize long-dated zero rates, and from these you were able to calculate implied forward rates. Now you'll "go the other way" and start with the forward rates to determine the zero rates.

Assume today is 18Sep1X and you face the following market, and you will invest $100m spot-3mo, and roll P+I over into new subsequent 3mo deposits at each maturity. You will use the futures market to lock in the reinvestment rates for each forward 3mo period (assume you can reinvest all odd amounts of interest at the hedged forward rate even though the min contract size is $1m).

Here are the market prices and rates

3mo Calc Period	Accrual Period Start Date	Market Price	Implied Rate	Act Days
Spot-3mo	20-Sep-1X	2.25%	2.25%	91
3 X 6	20-Dec-1X	97.50	2.50%	90
6 X 9	20-Mar-1Y	97.40	2.60%	92
9 x 12	20-Jun-1Y	97.20	2.80%	91
12 x 15	19-Sep-1Y	97.05	2.95%	91
15 x 18	19-Dec-1Y	96.85	3.15%	91
18 x 21	20-Mar-1Z	96.60	3.40%	91
21 x 24	19-Jun-1Z	96.50	3.50%	91
	18-Sep-1Z			

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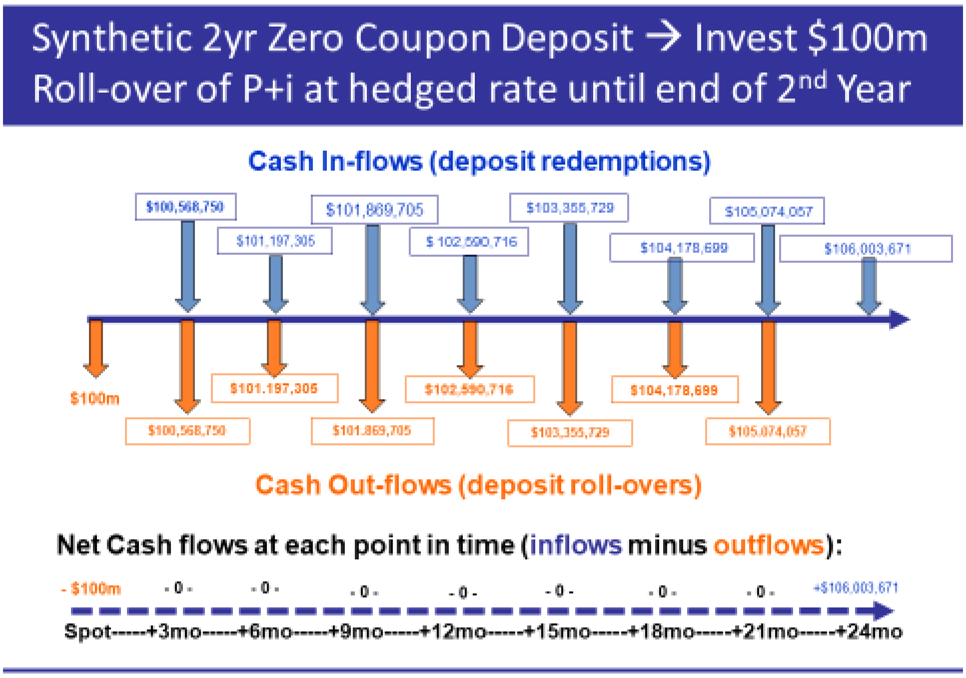

Cash out at Spot? $100m . Cash Back at Spot + 2yrs? $106,003,671

Any net cash flows occuring in between? No, they all net to zero

2yr Discount Factor? $100m ÷ 106,003,671 = 0.94336

Approximate Quarterly Equivalent Act/360 Rate per annum?

F = 0.5 (i.e. the CF at the end of 2yrs comes only once so has a frequency of ½ times per annum)

TF = 4 (we want to express this rate in quarterly equivalent)

TF x [(1 ÷ df_{period})^$^{(F/TF)}$ -1]

4 x [(1 ÷ 0.94336)^$^{(0.5/4)}$ -1] = 2.926% quarterly Act/365

2.296% quarterly Act/365 x 360/365 = 2.886% quarterly Act/360 equiv.

Hey, don't forget: if you have or end up with an Act/365 rate, which you want to express as an Act/360 equivalent, you first convert it to the target frequency, and then multiply it by 360/365.

1yr df and zero rate in quarterly act/360? df_{1yr} = 0.97475, F = 1

4 x [(1 ÷ 0.97475)^$^{(1/4)}$ -1] x 360/365 = 2.5304% quarterly Act/360

15mo df and zero? df_{15mo} = 0.96753, F = 0.8

4 x [(1 ÷ 0.96753)^$^{(0.8/4)}$ -1] x 360/365 = 2.6131% quarterly Act/360

18mo df and zero? df_{18mo} = 0.95989, F = .6666

4 x [(1 ÷ 0.95989)^$^{(0.6666/4)}$ -1] x 360/365 = 2.7001% quarterly Act/360