From lecture 1: This BEY calculation is specific to US T-bills. Remember, they are discounted with a weird formula which not exactly the one associated bond yields. You'll also notice that it really only converts to a yield on act/365 day-count.

Bond-Equivalent Yield (BEY)

- T-bills peculiar price-quoting often is not on a yield or Internal Rate of Return basis
- With BEY, we convert a T-bill's discount rate to make it comparable to a bond's yield (in US T-bond yields are s/a actual/365)
- T-Bills' BEYs often different from their quoted or published discount rates because
  - Discount formula not correct PV formula
  - 360 Day-count
- T-bills' BEYs are often published along side of their quoted discount rates
- What is the BEY of a 6mo (182 day) bill offered at a discount rate of 5% Actual/360?

$$\text{BEY}_{\text{US T-Bill}} = \left(1 \div (1 - [\text{Rate} \times \frac{\text{Days}}{360}]) - 1\right) \times \frac{365}{\text{Days}}$$

The formula above is introduced in the notes, which is the same as the method mentioned in Smith Ch1. Notice what Smith has here: The four week BEY ends up with a frequency of monthly, the 13-week BEY is quarterly, the 26-week is s/a and the 52wk is annual. None of these is directly comparable to each other until they are all put on an equivalent frequency. Smith is showing how the financial media calc's the BEYs on US T-bills specifically.

But in fact, **the only one of Smith's BEYs that is a true bond-equivalent is the 26wk, since it already has a semi-annual frequency and act/365, which is comparable to US T-bonds**. The other 3 are comparable in day-count, but not frequency.

16 Bond Math

TABLE 1.1 T-Bill Auction Results

Term	Maturity Date	Discount Rate	Investment Rate	Price (per $100 in par value)
4 week	07-31-2008	1.850%	1.878%	99.856111
13 week	10-02-2008	1.900%	1.936%	99.519722
26 week	01-02-2009	2.135%	2.188%	98.914708
52 week	07-03-2009	2.295%	2.368%	97.679500

Now in the Exercise 2, I give you another method to calculate BEY as follows below, but notice this is a *more general one*, and it ignores day-count.

The reason is that we're dealing with long-dated zero-rates, and assuming (for the purpose of avoiding tedium) that the year is divisible into equal-period frequencies (e.g. semi-annual freq = 2 equal half-yr periods). We know this isn't really the case, and that in the real world, we'd need to take specific day-count into account, but for what we're learning, the short-cut is worth taking, so we don't side-track onto miniscule pricing differentials arising from whether a coupon period is 183 days or 182 days.

So in this exercise, we've come to a long-dated zero, for which we can calc the df = PV/FV and then (skipping the step of calculating exact day-count) we use exponents to get us quickly to s/a act/365 equivalent rates, which in any case will be within a basis point or two of what they'd be if we were 100% precise in our analysis.

4) What is the corresponding bond-equivalent yield for this bill? Use this formula:

BEY for target frequency, TF) =
TF x $[(1 \div df_{period})^{\wedge(TFperiod/period)} - 1]$

$2 \times [(1 \div df_{12\ month})^{\wedge(1/2)} - 1] =$

Your fellow student asked:
I think both formulas are reasonable, but I wonder why there can be different methods to calculate BEY and how we can tell which formula to use in specific situation. Or do we only use the second method on the bill when the time to maturity exceeds six months? I learned from Smith Ch1 that time to maturity matters.

Short answer
The way Smith shows the BEYs for T-bills is what you'll see on Bloomberg screens and in other media. But except for the 6-mo bills for which the freq = 2, this is not really correct, moreso just a lazy convention that's never been corrected.

So what I did in the exercise was to simplify by using SG govt bills which are discounted on Act/365 basis (thereby avoiding the problem of US bills) and give you a formula that makes TF=2, so you always end up with s/a yields for the exercise in question – notice how this worked in the case of the 12mo bill? You ended up with the s/a act/365 equivalent (although we assumed the year divisible into two equal halves)

For purposes of bootstrapping Sg govt zero-rates, best practice is to have all rates in s/a bond equivalent, because the bond coupons are s/a. I also argue means it saves work to calc directly from discount factors.

Now, back to the original question, which I believe can be summarised as follows

Are these BEY formulas different or are they the same?

$$\mathrm{BEY}_{\mathrm{US\,T\text{-}Bill}} = \left(1 \div (1 - [\mathrm{Rate} \times \frac{\mathrm{Days}}{360}]) - 1\right) \times \frac{365}{\mathrm{Days}}$$

But remember to avoid the issues of Act/360, I used Sg bills and bonds in the exercise, so here's the formula for SG Govt Bill BEY

$$\mathrm{BEY}_{\mathrm{SG\ govt\,Bill}} = \left(1 \div (1 - [\mathrm{Rate} \times \frac{\mathrm{Days}}{365}]) - 1\right) \times \frac{365}{\mathrm{Days}}$$

So if the 6mo bill matures in 182 days, you plug in the rate we used of 2.3% and get a df = 0.988531507, whose BEY above is 2.32668% by the above formula.

But if you use this formula that we used in the exercise, and assume the 6mo is exactly half of a year,

```
BEY for target frequency, TF) =
TF x [(1 ÷ df_period) ^(TFperiod/period)  -1]
```

Then

TF =2 or TF period = 6mo

F = 2 or F period = 6mo

And you'll get df = 0.9885 and BEY = 2.32676%

So we were incorrect by 1/100th of a bp, which is a significant error when billions are being traded each day, but not the end of the world for what we're doing in class.

By the way, this part of the BEY equation, IS the discount factor for a US T-bill

$$1 - [\mathrm{Rate} \times \frac{\mathrm{Days}}{360}]$$

And this is the discount factor for a SG govt bill

$$1 - [\mathrm{Rate} \times \frac{\mathrm{Days}}{365}])$$