FE5101: Lecture 3
Bond Risk Analysis and Management

Duration
Repo Markets
Trading

Lecture 3 Topics:

1. Bond Risk Categories
2. Duration
   - MacAuley Duration
   - Modified Duration
   - Convexity
3. Delta Value of an 01 (DV01)
4. Going short of bonds and using Repo
5. Long-short Trading Strategy Homework
6. Supplementary slides on Convexity and Repo

Lecture 3, Topic 1

Fixed Income Risk Classification

Risk Definitions

- Market Risk – risk of losses from adverse price conditions in market
- Credit Risk – risk of losses from failure of counter-parties (financial or commercial) to make good on their obligations
  - Credit Default Risk – e.g. Bond issuer cannot make a coupon payment or par repayment due to insolvency
  - Credit Migration Risk – when credit quality has deteriorated by some objective measure (e.g. bond credit premium (yield spread) increases due to Agency rating downgrade from AA to A+)

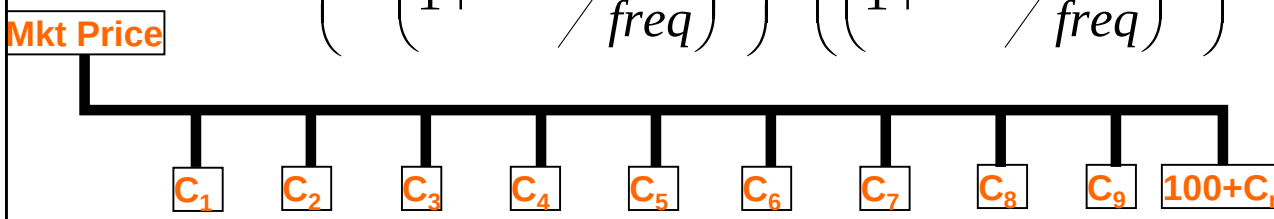
Market Risk – Interest Rates

- There are winners and losers when interest rates move, for example…
  - Variable rate lenders earn more (less) if rates go up (down), but
  - Variable rate borrowers pay more if rates go higher and less if they go lower
- Two forms of interest rate risk
  - Cash-flow based, associated with variable market rates
  - Valuation-based, associated with fixed rates
    - Market values of bonds change as yields fluctuate, even though their cash flows remain fixed.

Review: Bond Valuation

- Market Price = PV of Bond's Cashflows:

$$\text{Price}_{Bond} = \left(\sum_{n=1}^{N} \frac{Coupon_n}{\left(1 + Yield/freq\right)^n} \right) + \left(\frac{100}{\left(1 + Yield/freq\right)^N} \right)$$

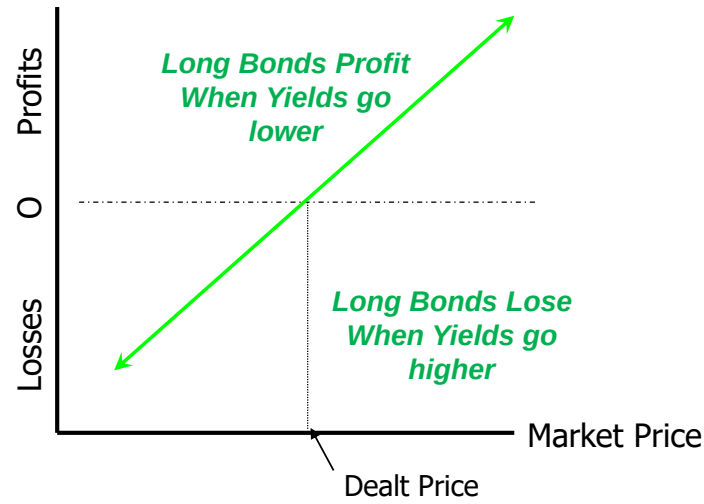


Now----------→Time →---------- Maturity

"Long" Position

Long = having something in excess of need
(Means you eventually have to sell it)
Buying bonds "long":

- Gain if price goes higher
- Lose if price goes lower

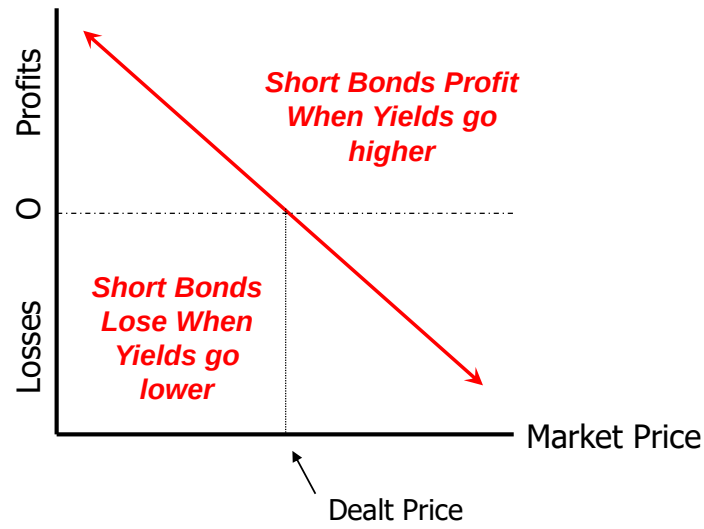


Slide 7

"Short" or Short-sold Position

Short = Needing something in excess of what you have
(means you eventually have to buy it)
Selling bonds "short":

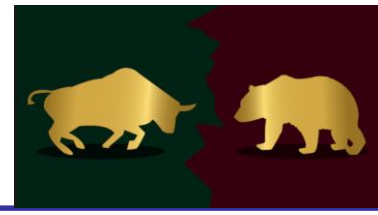
- Gain if price goes lower
- Lose if price goes higher



Animal Spirits – Bulls and Bears

Jargon	General Meaning	Fixed Income Markets	What this means for rates	Who and How?
Bull Market Bull Run Bull Rally	Prices of financial assets going higher	Bonds and Bills Prices/PVs higher	Rates and Yields LOWER	Bulls go long when conditions are "bullish"
Bear Market Bear Rout Bear Sell-off	Prices of financial assets going lower	Bonds and Bills Prices/PVs lower	Rates and Yields HIGHER	Bears go short when conditions are "bearish"

Keep in mind that certain traders and others will use terms like "I'm bullish rates" which actually means that they think rates are going higher/bond prices going lower. This is loose terminology and often requires context and clarification to understand how it's being used.



Mark-to-Market

- Valuation of an open position using available market prices as an objective and accurate estimate of prices at which deal could be struck to offset or otherwise liquidate said open position
- *Marking-to-market is at least the daily valuation of positions at readily available close out prices that are sourced independently. Examples of readily available close out prices include exchange prices, screen prices, or quotes from several independent reputable brokers.* – Basel II

Lecture 3, Topic 2

MacAuley and Modified Duration

What are Duration and Convexity?

- Duration – First derivative of bond value with respect to YTM. Used to make, analyse, and size bond strategies in terms of their sensitivity to yield
  - MacAuley Duration
  - Modified Duration
  - Value of a Basis Point
- Convexity – Second derivative. Used by traders and portfolio managers to anticipate how much duration changes as YTMs change

MacAulay Duration

- A bond or bond portfolio's weighted-average maturity
  - Because a bond's interim cashflows come sooner than contractual maturity, a bond's duration is generally shorter than its contractual life.
  - Can be thought of a bond's maturity adjusted **for**
    - Coupon size (bigger coupon → shorter duration
    - Yield environment (higher yields → shorter duration
- Risk indicator for bond portfolio management
  - Long-only: Pension, Insurance, Mutual Funds
  - Leveraged: Hedge funds

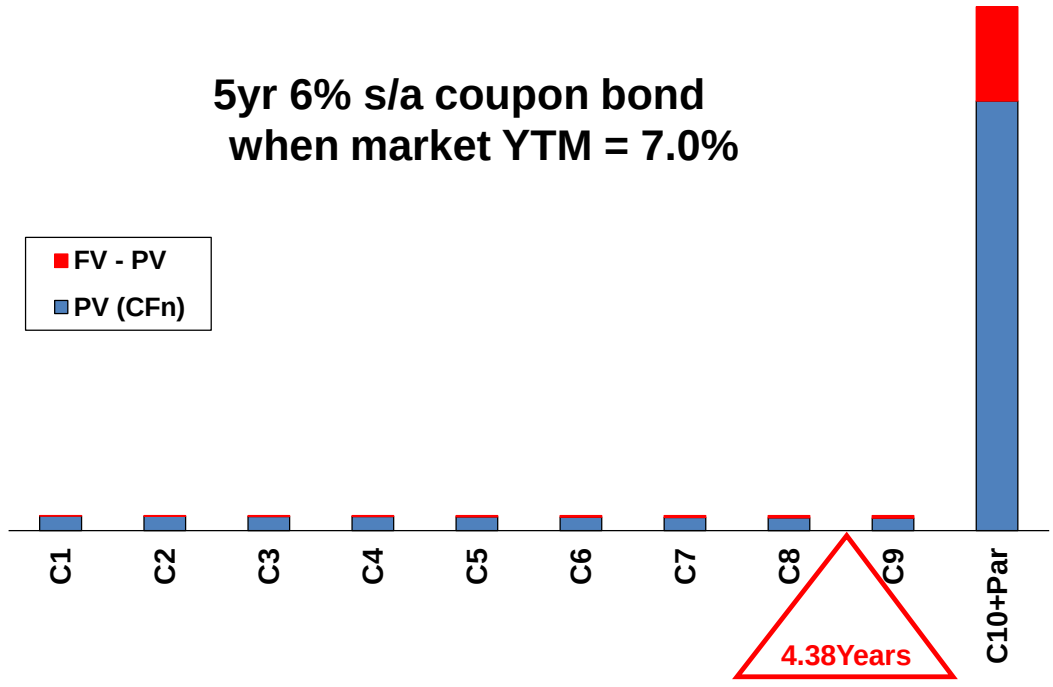
Calculating MacAuley Duration

- Method to adjust a bond to its weighted-average maturity. This makes bonds of various coupons and yields comparable for determining profitability

$$Duration_{Macauley} = \sum_{t=1}^{T} Time_t \left[\frac{PV(Cashflow_t)}{\text{Mkt Price}_{Bond}} \right]$$

- Consider the following 3 bonds, each having a 5-year maturity, and 7% s/a YTM
  - Duration of 12% s/a 5yr trading at $120.79 is 4.01 years
  - Duration of 6% s/a 5yr trading at $95.84 is 4.38 years
  - Duration of 5yr zero-coupon bond trading at $70.89 is 5.00 years

Duration Analogy → Balanced plank on fulcrum



Shortcomings of MacAuley Duration

- Time measure which facilitates analysis and management of long-only portfolios
- While longer duration = more risk, we need a standardised figure in P&L (or percentage of investment) terms
- E.g. how much will this portfolio of bonds (or single bond) lose if relevant yield(s) up by 1% p.a.

Modified Duration

- Method of predicting how much a bond's value will change for a 1% change in relevant YTM

$$\text{Duration}_{Modified} = \frac{\text{Duration}_{Macauley}}{\left(1 + Yield/freq\right)}$$

- Modified Duration of 3 bonds from earlier slide
  - 12% s/a 5yr bond trading at $120.79; **Modified Duration is 3.876%**
  - 6% s/a 5yr bond trading at $95.84; **Modified Duration is 4.229%**
  - 5yr Zero-coupon bond trading at $70.89; **Modified Duration is 4.831%**

Lecture 3, Exercise 1: Comparison of Duration for Different Debt Instruments

Exercise 1: Comparing Fixed Income Instruments

Each of these 3 Instruments has a contractual maturity of 3 years, a price of $1m, and YTM of 6.00%. But they have different cash flow structures:

Year	Instrument 1 Annuity	Instrument 2 Coupon Bond at Par	Instrument 3 0-Coupon Note
0	(1,000,000)	(1,000,000)	(1,000,000)
1	374,110	60,000	0
2	374,110	60,000	0
3	374,110	1,060,000	1,191,016

1. Their Present Values (i.e. market prices) will not change in an equal ways for a given change in yield. Give reasons for this in terms of:
    a. Duration
    b. How the yield curve moves
2. Comparing these three instruments, (without using a calculator)
    a. Which has the shortest duration?
    b. Which has the longest duration?
    c. What would happen to the Coupon Bond's duration if its coupon were higher?
    d. What would happen to the Coupon Bond's duration if its coupon were lower?
    e. Which of the three instruments has MacAulay duration = contractual maturity?
3. If I were managing a bond fund and had to stay invested in at least one of these three instruments,
    a. Which would you suggest I hold if I had a strong near-term outlook predicting lower 3-year interest rates?
    b. Why do you recommend that one?

4. If you were comparing a Zero coupon whose price is $1m and maturity is 2 years 10 months (2.83 years), with a yield of 6% and a future value of $1,179,503

   a. What would be its DV01 (you have to calculate this by taking its PV at 6.01%)?

   b. What other instrument above also has that same DV01? What can we generalise about MacAulay Duration based on these two instruments' sharing this attribute?

Exercise 1: Self-study Revisit

Using the Excel Spreadsheet from Exercise 1

1. How do the 3 duration measures change if we change the yield from 6.00% to 6.01%?

    a. Macaulay changes by _____

    b. Modified changes by _____

    c. DV01 changes by _____

2. What are the 3 duration measures now if we change the yield from 6.00% to 8.00%?

    a. Which one does NOT change? Why?

Instrument → Measure	Annuity	Coupon Bond	Zero-Coupon
Macaulay			
Modified			
DV01			

3. Now go back to YTM = 6% and 6.01%

    a. What is the DV01 of the 6% bond? What is its relationship to Modified Duration?

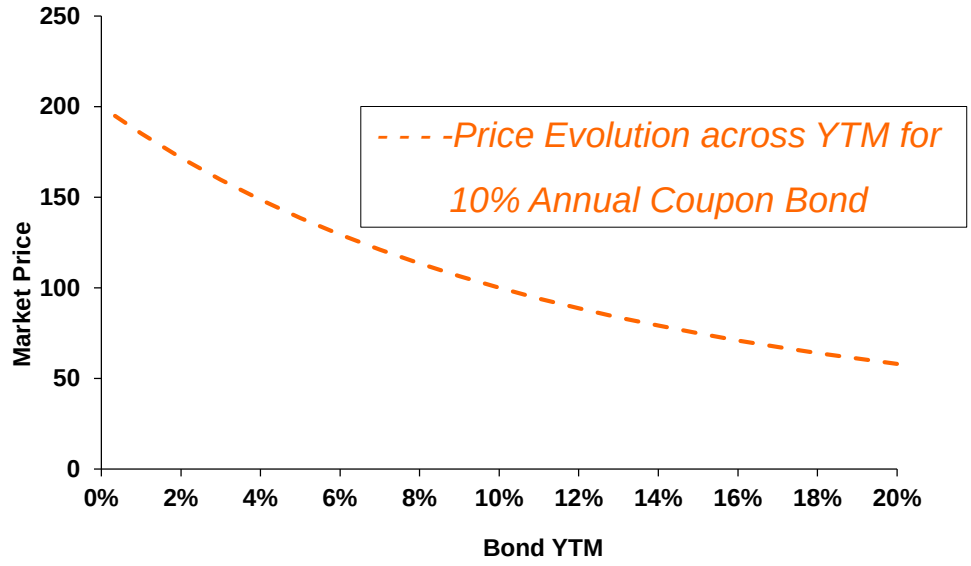
    b. What does it predict the bond's price will be if we change the yield to 7%?

    c. Changing the yield to 7%, what is the bond's new price?

    d. What accounts for the difference between the predicted and the "full valuation" price?

Remember - Price-Yield Relationship

- Value (Price) inverse to market YTM, and a bit non-linear ("convex") too.....

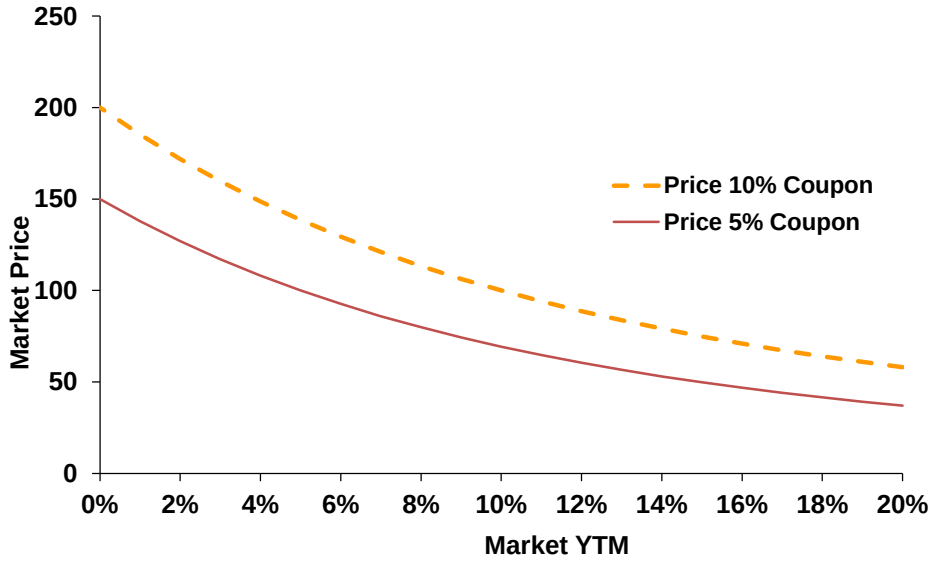


Price to Remaining Maturity Relationship

- Longer duration = More sensitivity to rates

Price-Coupon Relationship

- A bond's coupon influences its price change across market rates (YTMs)

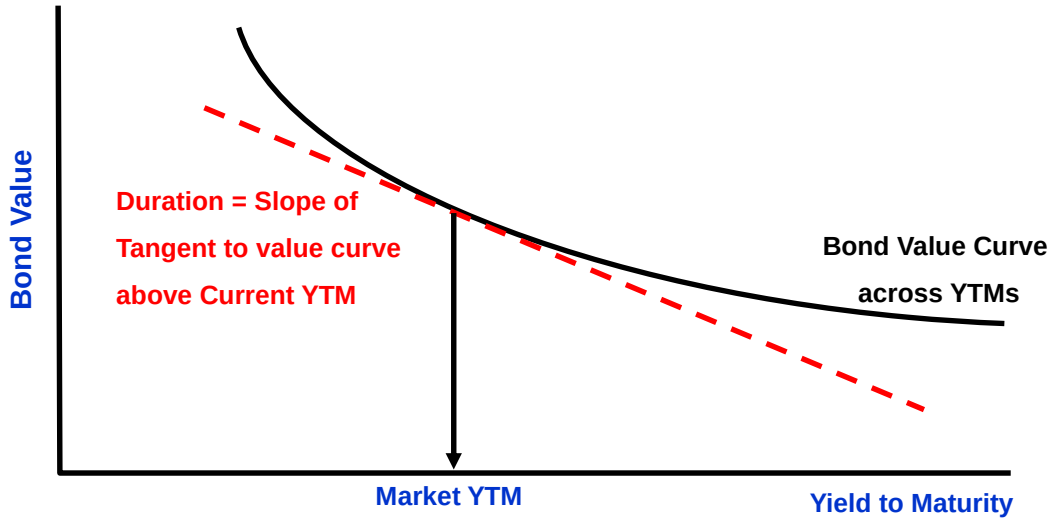


Convexity → Second Derivative

- Bond YTM vs. mkt value is inverse, but not linear
- At relatively low yields, a change in YTM changes the bond's market value more than the same YTM change would cause when yields are high
- *Duration* measures the change in price, but this change is local to the current level of yields
  - *Convexity* measures the *non-linearity* of the bond price sensitivity to interest rates
- Convexity
  - 2nd derivative of bond value with respect to YTM
  - The 1st derivative of Duration with respect to YTM

Convexity for Straight Bonds

- Standard Fixed income instruments have convexity



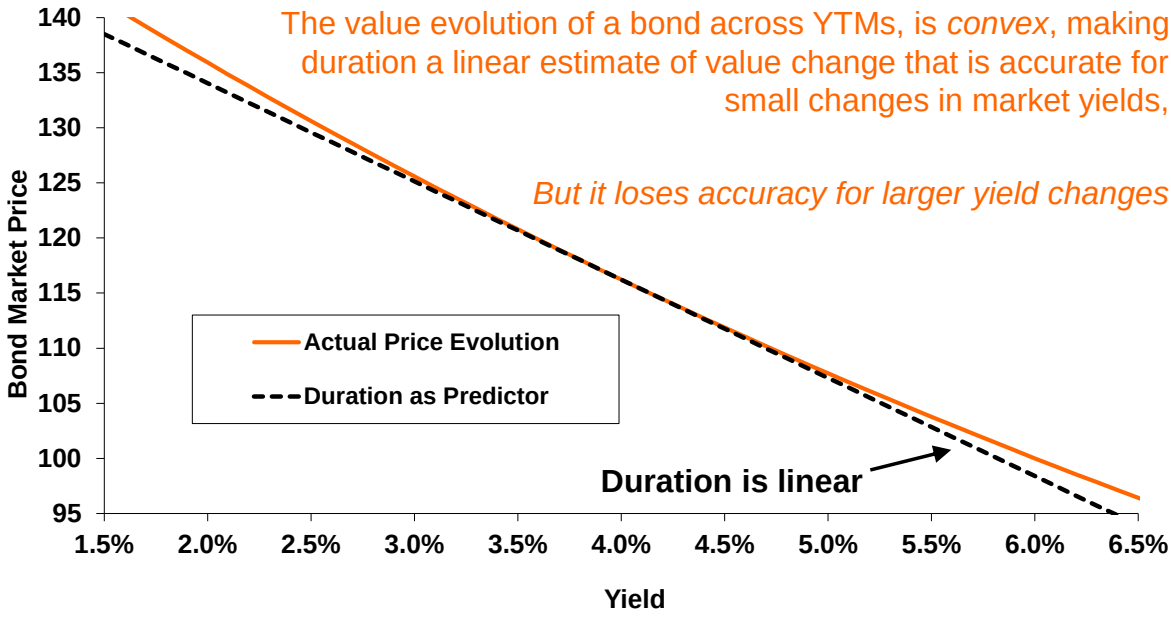
Convexity

- *Duration* measures the change in price or *slope* given current interest rates
- Convexity
  - *Convexity* measures the *non-linearity* of the bond price sensitivity to interest rates
  - Measures how fast duration changes (2nd derivative)
  - When modified duration is adjusted by convexity, we have more accuracy in predicting bond price sensitivity to yield change

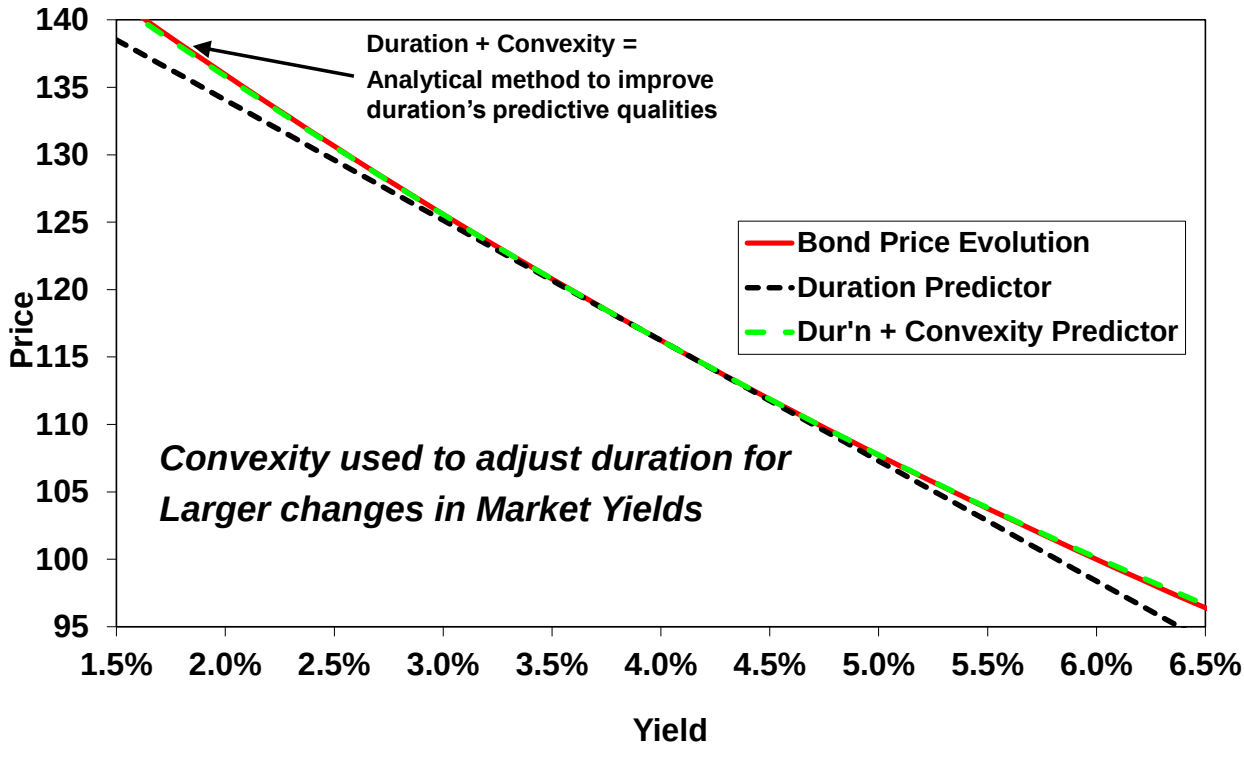
Duration as a Predictor of bond price change…

The value evolution of a bond across YTMs, is *convex*, making duration a linear estimate of value change that is accurate for small changes in market yields,

But it loses accuracy for larger yield changes



Adjusting for Convexity



Uses of Convexity

- Adjustment to duration formula to make it more accurate for predicting bond (portfolio) value changes under yield-change scenarios
- Trading strategies
  - E.g. long convex instrument (bond), short linear futures contracts, profit from price volatility
  - Positioning in high-convexity bonds against other asset (e.g. convertible bond arbitrage)
- *See Supplementary Slides on Convexity at end of this lecture slide-pack*

Users of Duration Methodologies/Applications

- Bond investment managers
  - To measure and manage average maturity of bond portfolios
  - Investment strategies
  - Risk quantification
- Risk Management
  - Open position interest rate risk quantification
  - Limit allocation to traders
- Bond Traders
  - Long-short strategies
  - Trading book risk measurement/limit usage

Lecture 3, Topic 3

Delta Value of an 01 (DV01)

Delta Value of a Yield Basis Point (DV01)

- Value of a 1 yield basis point (0.01%) change in the relevant discount rate
  - AKA
    - Delta Value of an 01, DV01, ΔV01
    - Basis Point Value, BPV
    - Present Value of a Basis Point, PVBP
    - Present Value of an 01, PV01
    - Dollar Volatility
    - Tick Value
    - Value at Risk (although this term has evolved to refer to a method of risk management)
- To calculate (full valuation method)
  - Determine the price of the bond at the current yield
  - Subtract the price of the bond valued at 1bp more than the current yield
- Why do we use DV01?
  - For a dollar-value for a bond's or portfolio of bonds' price sensitivity to yield
    - Risk Quantification
    - Trade-size specification
  - Position directionality and sizing
    - If the DV01 = $125, and YTM goes up 6bp, then the long position loses appx $750

Calculating and Describing DV01

- Using the 5 year 6% semi-annual bond valued to a 4% market YTM
- Bond Price
  - @ market yield of 4.01%, price = $108.935
  - @ market yield of 4.00%, price = $108.983
  - @ market yield of 3.99%, price = $109.030
- For a 0.01% fall in yield from 4.00% the bond will gain $0.0475 per hundred dollars of par
  - Market professional can say the sensitivity is “4.75 basis points per 01”
  - Might also say it “is 4.75 cents per $100.00 of par”
  - Or can say it is “$475 per million”
- Is modified duration ÷ 100 = DV01?

How to Use DV01

- For Bond Investment Management
  - As a portfolio risk parameter
  - E.g. "The bond portfolio's DV01 is $300,000, down from $400,000. This is because of the manager's recent shortening of average maturity"
- For Risk Control
  - To limit open-position risk
  - "The Bond-trading Desk is net long of bonds, with a net DV01 of $45,000" versus a limit of $50,000"
- For Trading
  - DV01 is used to size trading strategy components
  - "I have bought 10-year bonds and sold a larger amount of 2-year bonds in a long-short value trade. The DV01 of each of the two bonds is $15,000."

Lecture 3, Topic 4

Repo and Creating Short Position in Bonds

Going Short Bonds

- To get short of a bond pro-actively, must borrow it from someone who has it
  - Certain clients make their securities available to lend, by providing their broker/custodian with authorising instructions to lend the shares
  - When you borrow bonds, you have to
    - Re-pay in the same bonds
    - Post cash as collateral against the "loan" of the bond to you
    - Make up any coupons to the bond-lender if it goes ex-coupon during the borrowing
- Sell the borrowed bond to someone else
- Hope for the bond's yield to go _____ so that its price goes_____

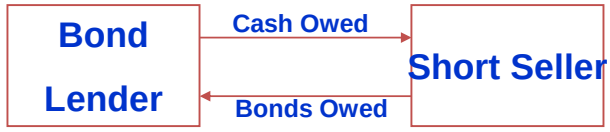
Outcomes to Short Position

- P&L outcomes
  - Mkt price goes lower?
  - Mkt price goes higher?
- How to close out a short position?
  - ____ from someone on secondary market
  - ____ bond owed to repo counter-party

Trading into Short Position

- Creating the short position:

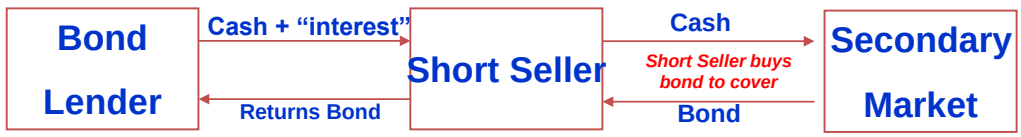
- The resulting *open* short position:



The short-seller earns interest on the cash posted as collateral. Short-seller must also make good to* bond lender *for any coupons the lender misses if the bonds pay coupon during the "loan" of the bond

Covering the Short Position

- Trading out of a short (*covering short*) position :



If the cost to* Short Seller *of "buying back" the bond is lower than the amount of* Cash + "Interest" *coming from the* Bond Lender *, the short position earns a profit. If the share price has gone higher,* Short Seller *realises a loss.

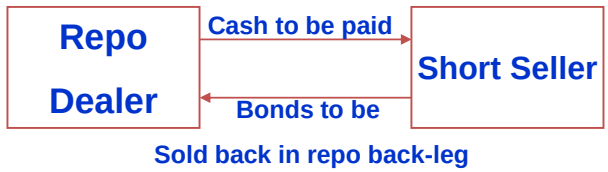
Repo Dealer = Lender or Borrower of Bonds

- A bond repo trading desk will usually be the bond lender to the short seller.
  - The short seller may also find someone who wants to lend him bonds directly, but there is low likelihood of this
- And… bond investors (long players) are also clients of repo desks
  - They lend bonds to and borrow cash from repo dealers
  - That is, they're the other side to the repo desk's business model

Using Repo Market to get Short

- Short Seller buys bond spot (e.g. t+1) from *Repo Dealer*, with agreement to **sell it back** following day:

- The resulting *open* short position:



Sold back in repo back-leg

The short-seller is paid interest by the repo dealer on the cash paid in the front leg until it is returned in the back leg.

Repo Back-leg for Short Seller

- Returning the *borrowed* bond by selling it back to repo dealer in the back-leg of the repo transaction

The net effect of a sale-and-repurchase agreement with two different bond prices in each leg, and the* Short Seller *compensating* Repo Dealer *for coupons is a collateralized short term loan of cash with high-quality collateral (bond) posted.

Repo Dealer Business

- A money-lending desk at a bank, making and taking short term cash loans, using bonds as collateral:

Repo Dealer Lends cash at Repo Offered Rate

For example:

2.20% p.a. daily Act/360

Repo Dealer Borrows cash at Repo Market Bid Rate

For example:

2.10% p.a. daily Act/360

The interest rate used is low because the repo dealer has collateralized the cash loan with a bond that is free of credit risk

Repo Market Use Cases

- Bond longs (investors) can "repo out" bonds as collateral to borrow cash very cheaply
- Short-sellers can "repo in" bonds to meet deliveries on their short-sales
- Banks use the repo market for
  - Safe short term lending of cash in surplus
  - Cheap borrowing of cash when in deficit
  - Run a repo desk as part of bond dealing operations
- Central Banks use repo for cash management and monetary policy execution
- See Supplementary Slides at end of this lecture, and read *Supplement on Bond Settlement and Repo Financing (attached in pdf)*

Lecture 3, Topic 5

DV01-Based Trading Strategy

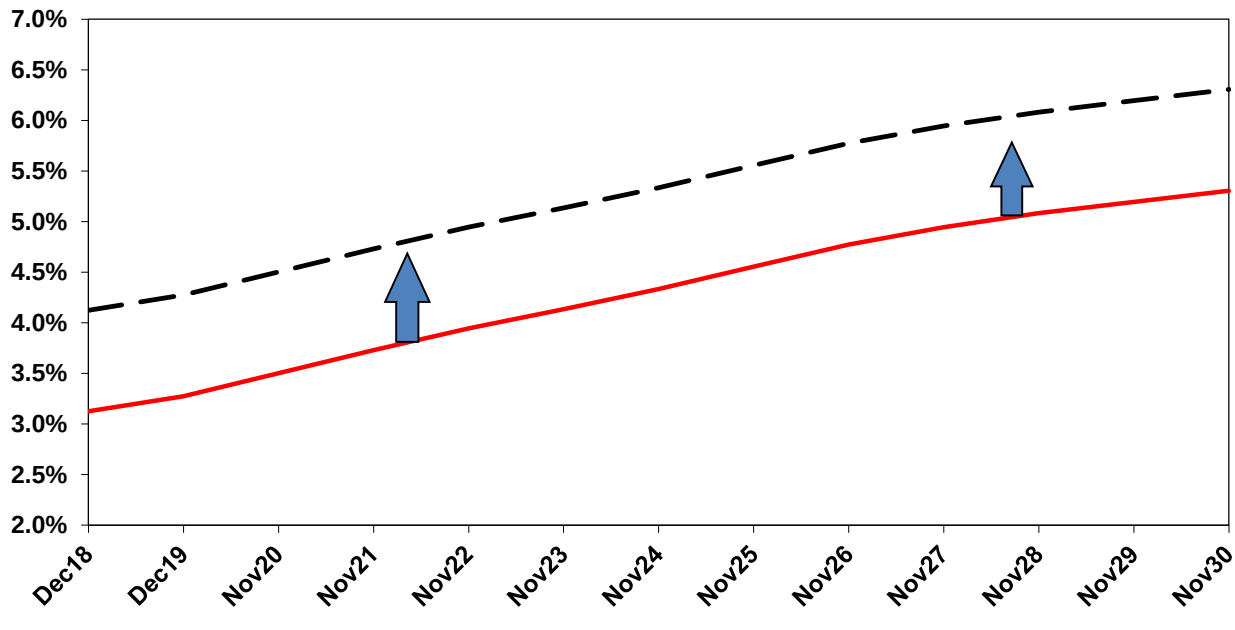
Exercise 2a: Bond Trade Sized to DV01

- Assume you want to go long of the bond with the following attributes
  - Maturity 10yr
  - Price 85.12
  - Coupon 4% s/a
  - YTM 6%
- You are told to deal in size that has DV01 = $10,000
- What is the appx par amount of the bond?

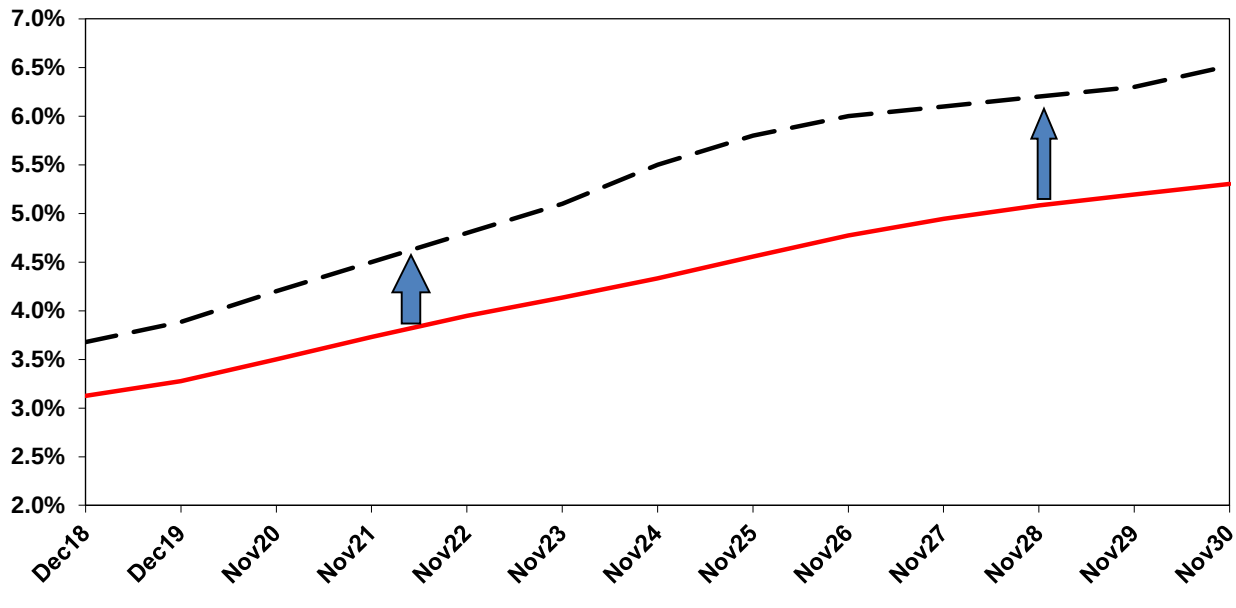
Yield-curve motivated Trading Strategies

- Possible Ways to trade:
- Outright Directionality Trading Positions
  - Long or Short position in a Bond of specified issuer and maturity
    - Long Bonds – if you think market yields going lower
    - Short Bonds if you think market yields going higher
  - E.g. buy $1m of 10yr bonds with $790 DV01
    - And profit if rates to go _____
    - And lose money if rates go _____
  - Called "outright" because its P&L depends on absolute change in yields
- Relative Value Trading Positions
  - Long of a specific bond, and
  - Short of a different bond
  - Looking for a relative change in the bonds' interest rates
  - Very often trades of this type are for changes in the yield curve

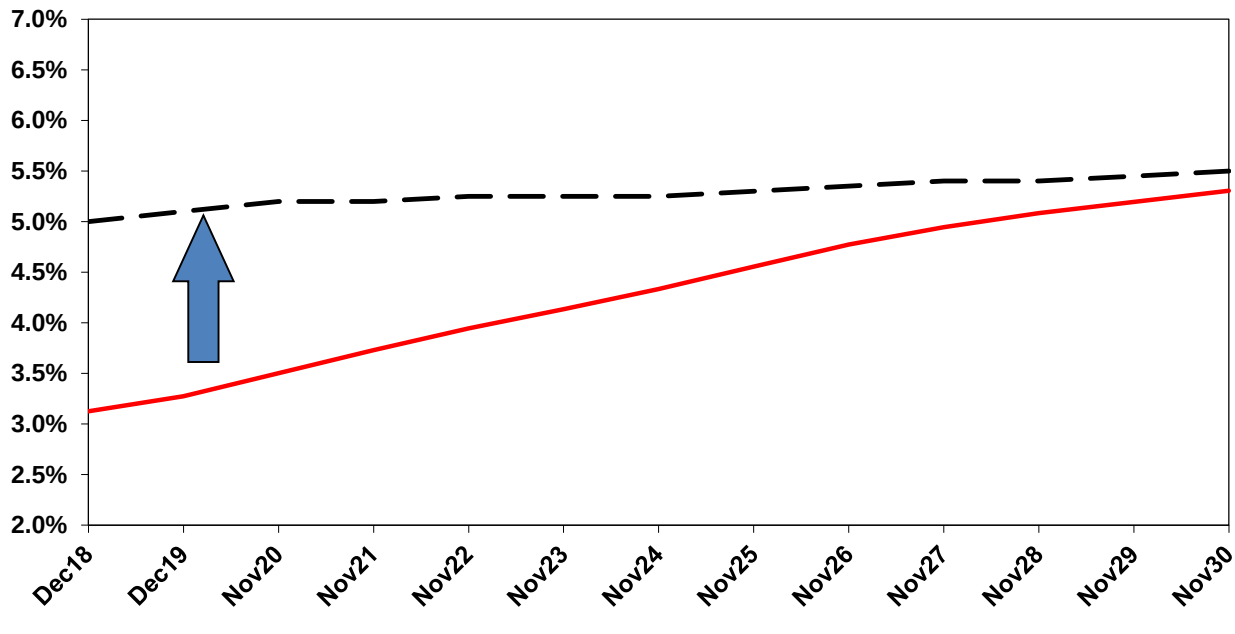
Parallel Shift in Yield Curve



Steepening in Yield Curve



Flattening Yield Curve



DV01 Neutral *Curve Trade*

- A "curve trade" usually consists of a long-short bond position that will profit from a steepening or flattening of the yield curve
  - Occur often in Bond and Interest Rate Derivatives trading operations
- Curve-trades are often sized to neutralise the position from having P&L due to parallel shifts in the yield curve
  - "DV01" Neutral → Long and short bond in curve-trade both have same $DV01

Homework: Lecture 3, Ex 2b: Trade-sizing to DV01

- Use the bond pricing spreadsheet to do the following exercise
- Below you see two points on a yield-curve situation for which the "2yr vs, 10yr" spread (10yr YTM – 2yr YTM) is at +150 bp
- A long-short portfolio which earns profit (makes losses) if the yield curve becomes flatter (steeper)
  - Go long $14.8m of the 10yr (YTM = 6.0%)
  - Go short $52.5m of the 2yr (YTM = 4.5%)
  - Size each bond at DV01 ≈ $10,000

Bond 1	Bond 2
Maturity = 10 years	Maturity = 2 years
Coupon = 4% s.a.	Coupon = 5% s.a.
Yield = 6%	Yield = 4.5%

Lecture 3, Exercise 2b continued

- What P&L from the long-short trade if the yield curve...
  - …*Flattens* with YTM_{10yr} = 5.80% and YTM_{2yr} = 4.90%
    - Now 2yr-10yr spread at 90bp (60bp lower than 150bp trade inception)
    - Approximate $P\&L_{10yr}$ = $201,740 profit
    - Approximate $P\&L_{2yr}$ = $397,876 profit
    - Position P&L = $599,616 profit ≈ 60bp x $10,000/bp
  - …*Flattens* with YTM_{10yr} = 7.20% and YTM_{2yr} = 6.10%
    - Spread now at +110bp (40bp lower than 150bp at trade inception)
    - Approximate $P\&L_{10yr}$ = +120bp x -$10,000/bp = _____Loss
    - Approximate $P\&L_{2yr}$ = +160bp x +$10,000/bp = _____Profit
    - Position P&L = _____net profit ≈ 40bp x $10,000/bp

Lecture 3, Exercise 2b (homework)

- What is the P&L if the yield curve..
  - ..*Steepens* with YTM_{10yr} = 6.00% and YTM_{2yr} = 3.90%
    - Spread now at + 210bp
    - Approximate $P\&L_{10yr}$ = _____ x _____ = _____
    - Approximate $P\&L_{2yr}$ = _____ x _____ = _____
    - Position P&L = _____ + _____ = _____
  - ..*Shifts parallel* with YTM_{10yr} = 6.50% and YTM_{2yr} = 5.00%
    - Spread stayed at 150bp
    - Approximate $P\&L_{10yr}$ = _____ x _____ = _____
    - Approximate $P\&L_{2yr}$ = _____ x _____ = _____
    - Position P&L = _____ + _____ = _____

End of Lecture 3

Final Q&A

Supplementary Slides

1. A closer look at Convexity

Convexity

- As the yield to maturity decreases, the price of the security increases at an increasing rate
- *Duration* measures the change in price or *slope* under current interest rates
  - *Convexity* measures the *curvature* of the bond price sensitivity to interest rates
- Convexity
  - Measures how fast Duration changes (2nd derivative)
  - The 1st derivative of Duration with respect to interest rate
- Convexity, when added to the modified duration, provides a better degree of accuracy of a security's price sensitivity to yield change

Estimating Convexity

- Do full-valuation re-calc for 1bp yield differential
  - 5yr s/a 6% coupon, $108.98 (YTM = 4%, Mod Dur = 4.34%)

$$Convexity \approx \frac{\text{Price}_{+0.01\%} + \text{Price}_{-0.01\%} - 2 \times \text{Price}_{\pm 0}}{\text{Price}_{\pm 0} \times (0.0001)^2}$$

$$\approx \frac{108.9353 + 109.0299 - 2 \times 108.9826}{108.9826 \times (0.0001)^2}$$

$$\approx 22.3917$$

Price Change Attributable to Convexity

- If bond's yield falls 50bp from 4% to 3.50%, the market price is estimated to go up 2.198% from $108.98 to $111.375

$$4.34 \times 0.50\% = 2.17\% \text{ due to Duration}$$

and

$$0.5 \times 22.39 \times 0.50\%^2 = 0.028\% \text{ due to Convexity}$$

$$\text{Total} = 2.17\% + 0.028\% = 2.198\%$$

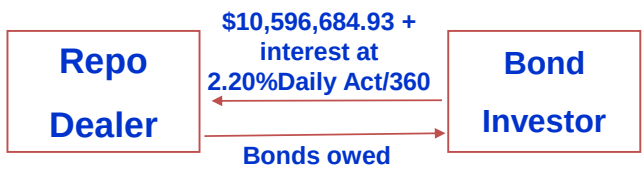
Supplementary Slides

2. More info on Repo

Using Repo to Finance Long Bond Position

- Bond-investor sells bond spot (e.g. t+1) to *Repo Dealer*, with agreement to buy it back following day:

- The resulting *open* long position:



The bond investor pays interest to the repo dealer on the cash dealer paid in the front leg, until bonds and cash are re-exchanged in the repo back leg.

Repo Back-leg – Earning Net Carry

- Bond Investor gets full financing of bond at low interest rate by putting out on repo
- At End of repo period (one day in this case), the assets are re-exchanged:

From the perspective of the bond investor, he has fully financed the dirty price of the bond, at a cost of 2.20% (appx $648 per day) and earns the coupon accrual of $1,369. Thus for bearing the market and credit risk, the bond investor net earns about $721 per day

Repo Transaction Structure

- Bonds in many markets are lent and borrowed by means of a combination of buying the bond for immediate value and selling it in a forward contract
- The transaction used is called a “sale and repurchase” or “repo” agreement
  - Financial equivalent to lending cash and securities
  - But with legal advantages using the repo arrangement

Repo Dealer Quotes

- A repo legally involves no borrowing or lending
- But because it's the financial equivalent of a collateralised loan, the pricing is shown as an interest rate
- The rate is applied and added to the original sale price and the resulting amount is the repurchase price
- Sample price quote: 2.20/2.10 Daily Act/360

Bond Repo - General and Special Collateral

- General refers to govt bonds for which there is ample supply by lenders
  - Most bonds, at a standard repo rate is standardised, and among the lowest of money market rates because of the short-term and high quality collateral.
- Special collateral refers to bonds for which there is excessive demand to borrow relative to the supply to be lent.
  - Specials have rates that are lower than general and sometimes even negative
  - E.g. while US treasury bond repo "general" are being quoted 2.11/2.07%, a bond repo for the new 3yr notes is being quoted 1.50/1.44%

[***This*** *supplement is best read* ***immediately after*** *attending Lecture 3*]

Lecture 3 Supplement on Bond Settlement and Repo Financing

I hope you are enjoying *FE 5101: Fixed Income and Derivatives*. We have covered a considerable amount of content on debt trading and investing, and the practical rate maths that underlie it.

The following is supplementary reading to lecture 3, and begins with a review of the practicalities of invoice pricing of dealt bonds to include accrued interest (i.e. *dirty settlement pricing*), as well as how this figures into Repo dealing. This is almost always a simple straight-line apportionment in line with the bond in questions actual coupon accrual since the last coupon payment date, and fairly apportions the next coupon amount between secondary market seller and buyer of a bond.

This is followed by an example of how the repo-market works. We will have spent time on repo in lecture 3, but thus far only from the perspective of a bond short-seller. Here we explore the long bond position in a repo context.

The benefit here of presenting these topics in written form is that they are better explained in writing, and this way avoid using too much class-time compared to the other topics we are discussing. However it is very important to understand settlement and repo together from a business stand-point, since they can often have significant inter-dependencies.

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Secondary Bond Market Quoting and Dealing:

QUOTE AND DEAL ON A *CLEAN PRICE*

Bonds are most-frequently quoted by dealers on a clean-price basis:

- In percentage of the bond's par value
- A typical bid-offer could be quoted 102.23/102.25
- Often this is stylised to currency-units and fractions per 100 currency units, so the above quote would be £102.23/£102.25, meaning pounds and pence per £100 of par value

However, because coupons are paid only to the owner of the bond as of the discrete date it goes ex-coupon, this means that the buyer and seller of a bond who are most likely going to settle the trade on a day between coupon payments, must pro-rate the coupon. Practically this means the bond buyer adds an amount to the settlement price of the bond to compensate the seller for a fair portion of the next coupon to be paid, based on how long the seller has waited since the last coupon. Remember, the bond's buyer will be owner of record for the next coupon payment in even though the buyer has owned the bond for less than the full coupon period.

CALCULATING ACCRUED COUPON INTEREST

A bond's coupon accrues on a straight-line basis. For example, if you are holding $10m par of a **5yr 5% semi-annual Actual/365** bond that pays its next two semi-annual coupons on 15JanX9 and 15JulX9, then the coupon amounts of are slightly different.

Coupon Date	Acc days	$Coupon	.
15JanX9	184	184/365 x 5% x $10m =	$252,054.80
15JulX9	181	181/365 x 5% x $10m =	$247,945.20
Total Coupons received in 20X9 =			$500,000.00

Because of the Act/365 day-count, the daily accrual is consistently $**1,369.86** for both coupon periods.

The general procedure for apportioning coupons pro-rata is to:

1. Figure the coupon amount for the period, based on the frequency and day-count.

2. Divide that coupon amount by the accrual ("actual") days in the period, to get the daily coupon accrual for this coupon period.

3. Pro-rate the coupon between buyer and seller based on how many of the accrual days of that coupon period each of seller and buyer will hold the bond.

For example, if the seller and buyer trade on 20AugX9 for t+1 (21AugX9) value, then the seller would have held the bond for 37 *actual* days from (and including) 15July to (but excluding) 21Aug. Therefore the seller's accrued interest portion of the coupon is 37 x $1,369.86 = $50,684.93.

DEAL AT *CLEAN PRICE* – SETTLE AT *DIRTY PRICE*

Let's say that for this $10m 5% s/a act/365 bond, buyer and seller have agreed a clean price of $105.46, for value 21AugX9. The clean price portion of the seller's "invoice" to the buyer is going to be:

105.46% of $10m = $10,546,000

To this amount is added the 37 days accrued interest (37 x $1,369.86) of $50,684.93, to compensate him for the 37 days he's held the bond since the last coupon payment, to make the full invoice amount or "dirty price" of:

$10,546,000 + $50,684.93 = $10,596,684.93

The buyer, upon reaching the coupon date of 15JanX9, collects a full half-year of coupon interest, thereby recovering 37 days' accrued coupon she paid the seller on the 21AugX9 settlement day.

COUPON ACCRUALS UNDER OTHER DAY-COUNTS

There are about a dozen or so day-count conventions. Most of the bonds we're looking at in this class are Actual/Actual or Actual/365. Of the other 10 or so, none is particularly hard to understand, but it is crucial to remember that there are several possible day-counts, and to always use the correct day-count for accruals, pricing, and valuation. The key thing to remember, regarding bond trade settlement, is that different day-counts will lead to different dirty prices, even if the bonds trade on the same clean price.

Actual/Actual example:

In the example above, because the bond had Act/365 and there was no leap-year in question, the daily accrual was the same in both half-year coupon periods. But this is not the case under Act/Act.

Coup Date	Act days	Act/Act $Coupon	Daily Accrual
15JanX9	184	½ x 5% x 10m = $250,000.00	$1,358.70
15JulX9	181	½ x 5% x 10m = $250,000.00	$1,381.22
Total Coupons received in 20X9 =		$500,000.00	

The same CLEAN price (from above example) of 105.46 but using Act/Act day-ct would lead to a dirty settlement price of:

$10,546,000 + (37 x $1,382.22) = $10,597,105.14

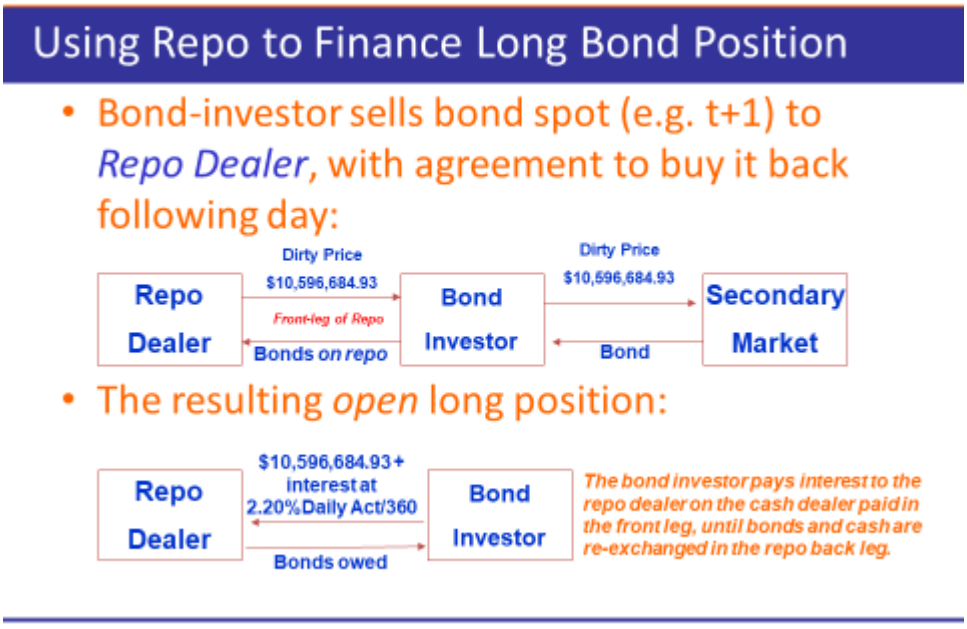
Again, for FE5101's purposes, it is not necessary to memorise the rules of all possible day-counts. It is important to understand how different day-counts lead to different settlement amounts and different yield profitability, and of course it is imperative to apply day-counts correctly in a professional dealing context.

REPO MARKETS, BOND FINANCE, AND CARRY

In session 3, you were introduced to the use of sale-and-repurchase ("repo") agreements. Here you can see how repo allows bond players from both the short and long-sides to finance their positions.

We'll use the same bond from above, being $10m par of a 5% s/a act/365 bond with coupons on 15Jan and 15Jul. Assume it's a government bond, issued in the currency of the government, so free of credit risk.

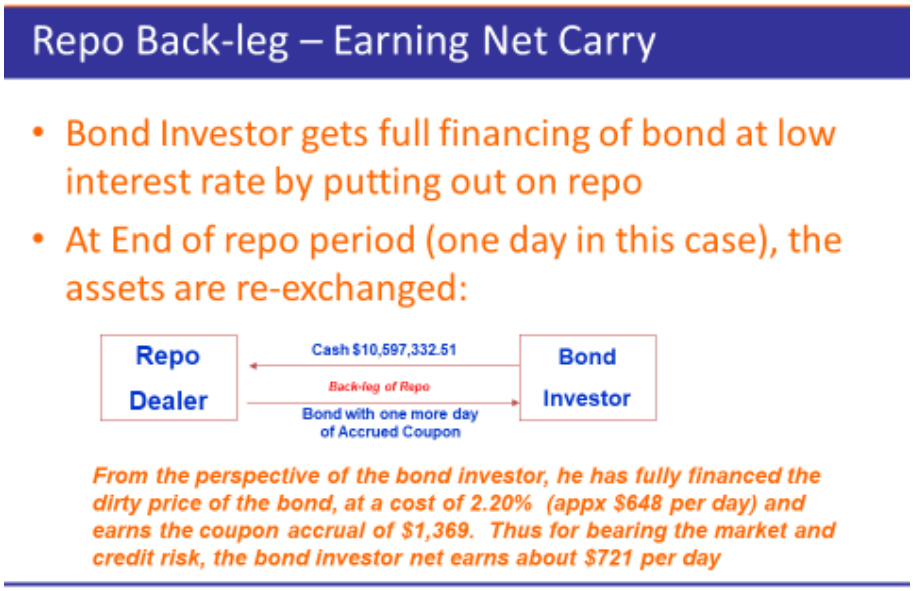
Repo Use-case Example 1: Long Player (Bond Investor) buys bond with 100% financing at a clean price of 105.46 with 37 days accrued interest (value 21Aug) and puts it out on repo.



Notice how the bond investor has a great incentive to put the bond out on repo? Doing so frees up substantially all his cash, while he

still fundamentally owns the bond and earns the daily difference between the repo interest rate of 2.20% and the bond yield to maturity of approximately 4%. Of course he bears the risks of being long, which exposes him to losses if the bond price goes lower.

Here's what happens on the following day when the repo back-leg exchange occurs:



Notice how the Repo dealer has simply financed the dirty price of the bond at a low rate of interest? This is because the collateral, a government bond in this case, is of such high quality as to make both parties of the repo deal essentially free of credit risk.

Notice also how the bond investor in this case is earning a **net positive carry** on the transaction of the difference between the daily interest (which is effectively paid daily) and the bond coupon interest which is collected each half-year.

If by chance, a bond is repo'd out on the coupon date, the counter-party borrowing the bond (the repo dealer in the above examle) compensates the bond's lender (bond investor above) for the coupon payment they've missed. Because bonds pay coupons with frequencies of quarterly, semi-annually, or annually in most cases, compensation for missed coupons occurs infrequently. In most repo transactions, the bond is returned the following day with another day of accrued coupon interest.

By the way, if the long position (investor) in the bond earns the net carry of appx $721, think about the short-player's position on the other side. If the short seller is on the other side of the repo dealer, lending cash and borrowing the bond, what do you think the $721 per day is to her?