NATIONAL UNIVERSITY OF SINGAPORE

FE5209 FINANCIAL ECONOMETRICS

(Semester 1: AY2016/17)

Time Allowed : Two and A Half Hours

INSTRUCTIONS TO STUDENTS

1. Please write only your student number below. **Do not write your name.**
2. This booklet contains **two (2) Sections** and comprises **Thirteen (13)** printed pages.
3. Answer **ALL** questions. This is an OPEN Book examination.
4. Graphic calculators or other calculators may be used.
5. Write legibly. A dark pencil may be used.
6. Write your answers in the spaces provided after each part of a question, except that **answers to Section A must be recorded in the table provided**.
7. Plan your answers to ensure they fit within the spaces provided. Other than this cover page and the spaces designated for providing your answers, you may do your "rough work" anywhere. Whatever you write outside of the answer spaces will be ignored.

Write your SEAT NUMBER and MATRICULATION NUMBER below.

Seat No:

Matriculation No :

Question	Max	Marks
Section A	50	
Section B		
Question 1	30	
Question 2	20	
Total	100	

Section A (50 marks). Each question carries 5 marks. Choose the most appropriate answer and record your answer in the table below.

1.	**2.**	**3.**	**4.**	**5.**
6.	**7.**	**8.**	**9.**	**10.**

1. A linear regression model with three predictor variables was fit to a data set with 50 observations. The correlation between Y and $\hat{Y}$ was 0.5. The total sum of squares was 10. What is the value of R^2 and the regression SS?

   A. $R^2 = 0.42$ and the regression SS is 42.
   B. $R^2 = 0.25$ and the regression SS is 7.5.
   C. $R^2 = 0.42$ and the regression SS is 58.
   D. $R^2 = 0.25$ and the regression SS is 2.5.

   R^2 = 0.5^2=RSS/TSS

2. To assess multicollinearity of linear regression models, a naïve but frequently used procedure to is to compute correlations among the predictors. While these simple correlations convey some information about the level of dependence, it is poor to detect multicollinearity jointly existing among multiple predictors. Hence alternative approaches have been proposed. Which of the following statements is correct?

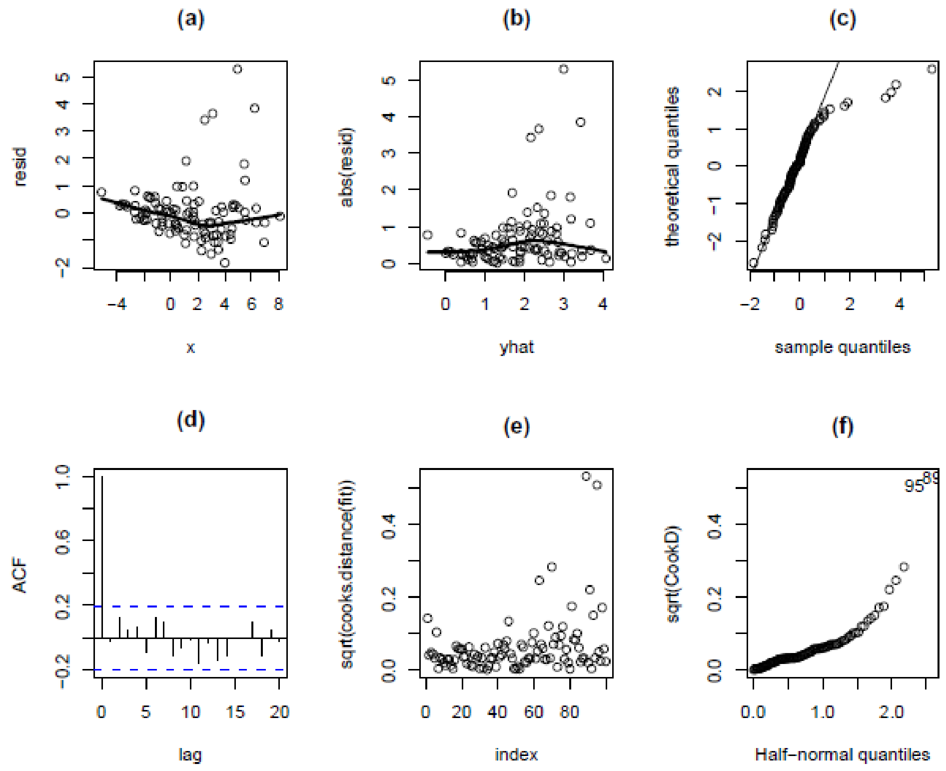
   vif: variance inflation

   A. The VIF provides an index of the variance reduction on each regression coefficient when all the predictor variables are uncorrelated.
   B. Drop either one of two highly correlated predictor variables can lessen the effect of multicollinearity without losing goodness of fit. Not sure about this
   C. Residual plot against the fitted values shows whether there is severe multicollinearity. Should be Partial residual plot, not residual plot
   D. Partial residual plot illustrates the multicollinear relationship given that other independent variables are also included in the model.

3. Following Jensen (1968), you test whether a given mutual fund provides consistently higher returns than those implied by the Capital Asset Pricing Model (CAPM). To do so, you estimate the linear equation:

   $$R_{it} - R_{ft} = \alpha_i + \beta_i(R_{mt} - R_{ft}) + \epsilon_{it}$$

   where R_{it}, R_{mt}, and R_{ft} are the returns of the mutual fund, the market return, and the risk-free rate respectively. For the residuals $\widehat{\epsilon_{it}}$, we observe:



Which of the following statements is correct?

(i) The null hypothesis of the test is H_0: $\beta_i = 1$ corresponding to no abnormal returns above those prescribed by the CAPM. Should be α, not β

(ii) The Fama-French model is less appropriate than the CAPM as it includes non-market factors such as value and size. FF is be more appropriate than CAPM

(iii) The test of no abnormal returns must be rejected given the non-linear effect in residuals. Hard to say, 2 different concepts

(iv) The sample size has no influence on the test results.

A. (i) only
B. (i) and (ii)
C. (iii) and (iv)
D. None of the above.

4. Let Y_t be a stationary mean-zero ARMA(1,1) process, $Y_t = \rho_1 Y_{t-1} + \epsilon_t + \alpha_1 \epsilon_{t-1}$, where $\epsilon_t \sim IID(0, \sigma_\epsilon^2)$. Which of the following statements is correct?

(i) $E(Y_t \epsilon_t) = \sigma_\epsilon^2$ E(use the formula to represent Yt)

(ii) $E(Y_t \epsilon_{t-1}) = \sigma_\epsilon^2(\rho_1 + \alpha_1)$ Use the formula to represent Yt expectation: in the first term * ε(t-1), you get the term

(iii) Y_t is a white-noise process if $\rho_1 = -\alpha_1$ Use Yule walker equation, plug in

(iv) The Yule-Walker equation in terms of lag order 1: $E(Y_t Y_{t-1}) = \rho_1 \sigma_Y^2 + \alpha_1 \sigma_\epsilon^2$
Use higher order YW equation σk = ργ(K-1) (k>=2)

A. (i) only
B. (i), (ii) and (iii)
C. (i), (ii), (iii) and (iv)
D. None of the above

5. Consider the monthly log stock returns of Ford Motor Company $x_{1,t}$ and the log returns of a value-weighted index $x_{2,t}$ from January 1960 to December 2015. The fitted model is as follows:

$$\begin{pmatrix} x_{1,t} \\ x_{2,t} \end{pmatrix} = \begin{pmatrix} 0.27 \\ 0.67 \end{pmatrix} + \begin{pmatrix} -0.16 & 0.39 \\ 0 & 0.09 \end{pmatrix} \begin{pmatrix} x_{1,t-1} \\ x_{2,t-1} \end{pmatrix} + \begin{pmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \end{pmatrix}, \quad \Sigma_\epsilon = \begin{pmatrix} 82.98 & 21.42 \\ 21.42 & 19.74 \end{pmatrix}$$

where $\epsilon_t = (\epsilon_{1,t}, \epsilon_{2,t})'$ is a bivariate white noise process. Which of the following statements are correct?

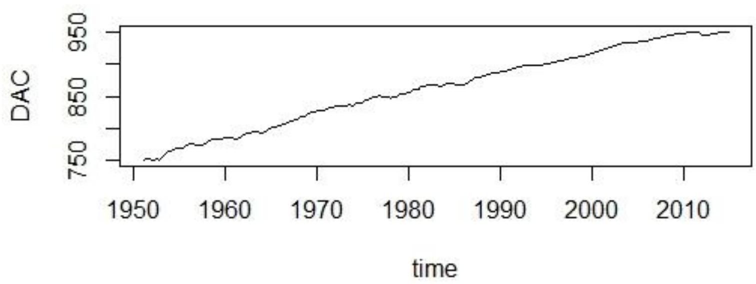
E(Xt)=(I-ϕ1)^(-1, inverse)*ϕ0
=[0.48 0.70] (column vector)

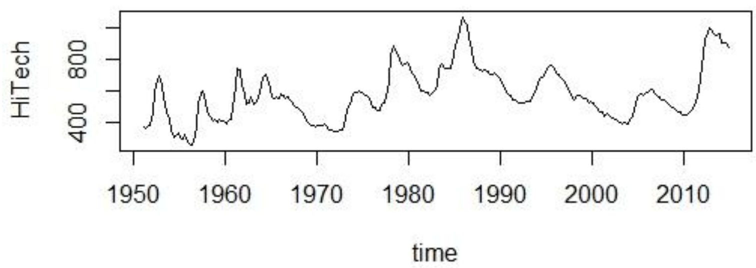
(i) The mean of the two log return series is (0.89, 0.48)'.
(ii) The variance of the series $x_{2,t}$ is 19.90. Variance of ar(1) model
(iii) The two series are uncoupled.
(iv) The bivariate VAR model is stationary. Roots > 1

Check AR polynomial, freedom: 4.4

A. (i) and (ii)
B. (ii) and (iv)
C. (iii) and (iv)
D. (iv) only

6. The historical profits of two companies, DAC and HiTech, are recorded from 1951 to 2014, see Figure 1. The growth rate of profits is the first order difference of the original time series. We use VARorder to select the order of VAR model for the growth rates.





> VARorder(dzt)
Summary table:

	p	AIC	BIC	HQ	M(p)	p-value
[1,]	0	-11.9390	-11.9390	-11.9390	0.0000	0.0000
[2,]	1	-12.5285	-12.4729	-12.5061	148.0804	0.0000
[3,]	2	-12.5585	-12.4474	-12.5138	14.5156	0.0058

[4,]	*3*	*-12.5405*	*-12.3738*	*-12.4734*	*3.1324*	$\boxed{0.5359}$
[5,]	*4*	*-12.5331*	*-12.3110*	*-12.4438*	*5.5907*	*0.2319*
[6,]	*5*	*-12.5230*	*-12.2452*	*-12.4113*	*4.8899*	*0.2988*
[7,]	*6*	*-12.5118*	*-12.1785*	*-12.3778*	*4.6154*	*0.3291*
[8,]	*7*	*-12.4942*	*-12.1054*	*-12.3378*	*3.1227*	*0.5375*
[9,]	*8*	*-12.5184*	*-12.0740*	*-12.3396*	*12.4624*	*0.0142*
[10,]	*9*	*-12.5291*	*-12.0292*	*-12.3280*	*9.3690*	*0.0525*
[11,]	*10*	*-12.5182*	*-11.9627*	*-12.2948*	*4.5213*	*0.3400*
[12,]	*11*	*-12.5404*	*-11.9293*	*-12.2946*	*11.6944*	*0.0198*
[13,]	*12*	*-12.5983*	*-11.9317*	*-12.3301*	*19.3247*	*0.0007*
[14,]	*13*	*-12.6032*	*-11.8810*	*-12.3127*	*7.7785*	*0.1000*

Don't reject, choose VAR 2

A. The likelihood-ratio test selects VAR(7) given the minimum of M(p).

BIC penalize more

B. BIC penalizes less on model complexity and selects a smaller model VAR(1) than AIC.

C. AIC selects VAR(1) but the selection is biased.

D. None of the above.

According to AIC, VAR(2) is better, we choose the one with the smallest AIC

7. Consider daily log returns of company X. There is no autocorrelation in the time series, yet volatility clustering is likely. Implementing the following R code, we obtain an output presented blow.

```
>alpha = .01
>n = length(Xreturn)
>fit_garch = garchFit(~garch(1,1),Xreturn,cond.dist="std")
>summary(fit_garch)
>pred = as.numeric(predict(fit_garch,n.ahead=1))
>df = as.numeric(coef(fit_garch)[5])
>q = qstd(alpha, mean = pred[1],  sd =pred[3], nu = df )
>lambda = pred[3]/sqrt( (df)/(df-2) )
>qalpha = qt(alpha,df=df)
>es1 = dt(qalpha,df=df)/(alpha)
>es2 = (df + qalpha^2) / (df - 1)
```

```
Title:
 GARCH Modelling

Call:
 garchFit(formula = ~garch(1, 1), data = Xreturn, cond.dist = "std")

Error Analysis:
         Estimate    Std. Error   t value   Pr(>|t|)
mu       7.147e-04   2.643e-04    2.704     0.00685    **
omega    2.833e-06   9.819e-07    2.885     0.00392    **
alpha1   3.287e-02   1.164e-02    2.824     0.00474    **
beta1    9.384e-01   1.628e-02    57.633    <2e-16     ***
shape    4.406e+00   6.072e-01    7.256     4e-13      ***
---
```

*Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1*

Log Likelihood:
3215.913 normalized: 3.215913

Standardised Residuals Tests:

			Statistic	p-Value
Jarque-Bera Test	R	Chi^2	39705.01	0
Shapiro-Wilk Test	R	W	0.8656913	0
Ljung-Box Test	R	Q(10)	7.88475	0.6400934
Ljung-Box Test	R	Q(15)	11.5034	0.7161615
Ljung-Box Test	R	Q(20)	15.61023	0.7404913
Ljung-Box Test	R^2	Q(10)	6.557717	0.7664347
Ljung-Box Test	R^2	Q(15)	6.807427	0.9627747
Ljung-Box Test	R^2	Q(20)	7.229426	0.995862
LM Arch Test	R	TR^2	6.326875	0.8987163

Information Criterion Statistics:

AIC	BIC	SIC	HQIC
-6.421825	-6.397286	-6.421875	-6.412499

Moreover, we have obtained the1-day ahead volatility forecast $\widehat{\sigma}_{n+1} = 0.0095$ and

> q
[1] -0.02429445
> qalpha
[1] -3.563288
> es1
[1] 0.965
> es2
[1] 5.022

Which of the following statements is correct?

Just tells whether GARCH is adequate

A. The GARCH(1,1) process is stationary as the Ljung-Box Test for the squared residuals is insignificant.

B. Suppose that $1,000,000 is invested in X stock, 99% Value-at-Risk is 24,294 and Expected Shortfall is 33,300.

VAR = initial investment * quantile (refer to notes)
Expected shortfall = -μ + λ*es1*es2 λ = σ

C. The GARCH(1,1) model is inadequate as the kurtosis of the real distribution is bigger than 3.

Not related, GARCH: use Ljng box

D. None of the above.

8. Suppose that $Y_t = (Y_{1,t}, Y_{2,t})^T$ is the bivariate AR(1) process:

$$\Delta Y_{1,t} = 0.5(Y_{1,t-1} - Y_{2,t-1}) + \epsilon_{1,t}$$
$$\Delta Y_{2,t} = 0.55(Y_{1,t-1} - Y_{2,t-1}) + \epsilon_{2,t}$$

Where $\epsilon_{1,t}$ and $\epsilon_{2,t}$are white noises.

roots
λ = 1
1+0.5*0.55 = 0.95

A. Y_t is stationary. $Y_{1,t} - Y_{2,t}$ is stationary.
$\boxed{\text{B.}}$ Y_t is not stationary. $Y_{1,t} - Y_{2,t}$ is stationary.
C. Y_t is stationary. $Y_{1,t} - Y_{2,t}$ is not stationary.
D. Y_t is not stationary. $Y_{1,t} - Y_{2,t}$ is not stationary.

9. Perform a principal components analysis on changes in the stock prices. The eigenvalues are reported. How many principal components are needed to capture 85% of the data's variability?

PC1	PC2	PC3	PC4	PC5
0.697	0.195	0.055	0.029	0.024

A. One PC is enough.
$\boxed{\text{B.}}$ Two PCs.
C. Five PCs.
D. The answer is not computable.

10. Which of the following statements are true concerning the Box-Jenkins approach to diagnostic testing for ARMA models?
(i) The Ljung-Box test tells whether the identified model is either too large or too small.
(ii) The Ljung-Box test checks the model's residuals for autocorrelation.
(iii) If the model is appropriate, the residuals are autocorrelated.
(iv) If the model is appropriate, an over-fitted model with additional variables will have statistically insignificant coefficients.

$\boxed{\text{A.}}$ (ii) and (iv) only
B. (i) and (iii) only
C. (i), (ii), and (iii) only
D. (i), (ii), (iii), and (iv)

<u>Section B (50 marks). There are 2 questions.</u>

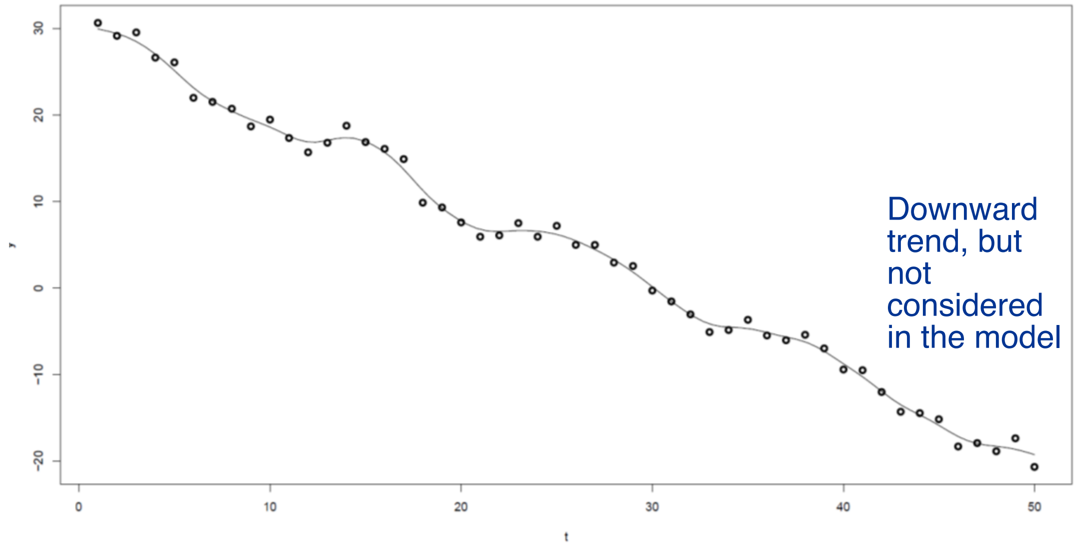
Question 1. (30 marks)

Figure 1 displays the time plot of a time series data (dots) and the regression based on a nonparametric method (line). We suggest a parametric model to fit the data:

$$Y \;=\; X\beta \;+\epsilon$$

or $$y_t \;=\beta_0 \;+\beta_1\, sin(2\pi\, t/12) \;+\epsilon_t \quad (1.1)$$

where $t \;=\; 1,\cdots,50$ and $\epsilon_t \sim\; N(0,\sigma^2)$.



Downward trend, but not considered in the model

Figure 1 Time plot of a time series

We obtain the following results:

- The critical values for the standard normal distribution are: $z_{0.975} \;=\; 1.96,\; z_{0.95} \;=\; 1.645$.
- $(X'X)^{-1} \;= \begin{pmatrix} 0.0200 & -0.0011 \\ -0.0011 & 0.0401 \end{pmatrix}$ and $X'Y = \begin{pmatrix} 226.2420 \\ 106.8887 \end{pmatrix}$
- The R output for fitting the model (with missing values):

```
Call:
lm(formula = y ~ sin(2 * pi * t/12))
Residuals:
   Min          1Q          Median        3Q          Max
-28.541      -13.058      -1.038        10.744      24.203
Coefficients:
                      Estimate    Std. Error    t value    Pr(>|t|)
(Intercept)            ??            ??         2.149      0.0367 *
sin(2 * pi * t/12)     ??            ??         1.388      0.1714
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

variance of noise

Residual standard error: 14.52 on 48 degrees of freedom Multiple R-squared: 0.03861, Adjusted R-squared: 0.01859 F-statistic: 1.928 on 1 and 48 DF, p-value: 0.1714

Little variance explained

1) [10p] Estimate the unknown parameters β_0 and β_1. Show the 95% confidence intervals for the parameters.

2) [5p] Give the prediction for t=60 based on the fitted model. Discuss the difference on variance of forecast errors at time t=60 and t=100.

3) [5p] According to the R output, does the fitted model well explain the variation in the data? Justify your argument. If not, please suggest one more explanatory variable.

Add time as a predictor for the downward trend

4) [5p] Suppose that we have fitted the data to a new model:

$$y_t = \beta_0 + \beta_1 \sin\left(2\pi \frac{t}{12}\right) + \beta_2(?) + \epsilon_t \qquad (1.2)$$

The R output is as follows:

Call:

lm(formula = ...)

Residuals:

Min	1Q	Median	3Q	Max
-2.419785	-0.592666	-0.001889	0.669250	2.127929

Coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	29.70497	0.29416	100.981	< 2e-16 ***
sin(2 * pi * t/12)	1.79097	0.20494	8.739	2.06e-11 ***
?	-0.98938	0.01003	-98.598	< 2e-16 ***

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*Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1*

Residual standard error: 1.018 on 47 degrees of freedom Multiple R-squared: 0.9954, Adjusted R-squared: 0.9952 F-statistic: 5057 on 2 and 47 DF, p-value: < 2.2e-16

Calculate the value of a modified Akaike information criterion (AIC) for the new model (1.2) and the original model (1.1):

$$\text{Modified AIC} = \ log\ \hat{\sigma}_k^2 \ \ + \frac{n+2k}{n}$$

where $\hat{\sigma}_k^2$ denotes the estimated variance of residuals for a model with k parameters.

Plug in the value

(1,1) K=2, n = 50, σk = 14.5^2
(1,2) K=3, n = 50, σk = 10.18

5) [5p] Which model is preferred? Justify your choice.

(1,2)

Question 2. (20 marks)
Consider two time series:

$$X_t \;=\; 0.3\,X_{t-1} \;+\varepsilon_t$$
$$Y_t \;= X_t \;+\; 0.5\varepsilon_t$$

where X_t is a weakly stationary (covariance stationary) process and ε_t is a white noise with $\varepsilon_t \sim\; WN(0,1)$.

1) [5p] Explain the difference between a white noise process and a covariance stationary process.

White noise: AR / ACF = 0

Covariance Stationary

ACF independent on time

2) [5p] Is the series Y_t weakly stationary? Justify your answer.

stationary

3) [10p] Compute the first 2 auto-covariance (not autocorrelation) coefficients of Y_t: $\gamma_Y(1)$ and $\gamma_Y(2)$.

E(YtYt-1)=E(0.3Xt-1+1.5εt)
E(0.3Xt-k-1+1.5εt-k-1)

END OF PAPER