

## FE5208: problem set 2

Each group of  $\leq 5$  people submits a copy on LumiNUS

Due on 3 April

### Part 1:

1. Establish Equation (6) on Slide 15 of Lecture 4.
2. Calculate the Varsicek bond price following the steps on Slide 25 of Lecture 5
3. Verify the bond price solution to the Ho-Lee model on Slide 30 of Lecture 5 satisfies the bond PDE with the boundary condition.
4. Verify the bond price solution to the Varsicek model on Slide 31 of Lecture 5 satisfies the bond PDE with the boundary condition.
5. Establish the two Facts stated on Slide 19 of Lecture 6.
6. Derive the first-order condition for the second-price auction. Verify that the bidding strategy  $\beta^{II}$  defined on Slide 14 of Lecture 6 satisfies the first-order condition.
7. Consider a common-value first-price auction where  $X_i$  is uniformly distributed in  $[0, 1]$  and independently across the bidders. Assume that the common value  $V = \frac{1}{n} \sum_{i=1}^n x_i$  where  $x_i$  is the realization of  $X_i$ . Again each bidder  $i$  knows only the realization of his signal  $X_i$  but not that of the other bidders' signals. Consider a symmetry equilibrium where  $\beta(x) = \alpha x$  with  $\alpha > 0$ . Solve  $\alpha$ . What is the value of  $\alpha$  approaching when  $n \rightarrow \infty$ ?

## Part 2:

This is a mini-project. Prepare a write-up of no more than two pages with the following components:

1. Pick up a treasury auction adopted by a country of your choice within the past three years.<sup>1</sup> Summarize how the auction is being conducted and the related bond markets (secondary, when-issue, etc..) in place. Quote evidence to back up your claim of the institutional details (which can be the web links, research papers, etc.).
2. Highlight all practices and institutional details described in the part 1 which are inconsistent with those in the early 90s US market described in Bikhchandani and Huang (1993). The more the better.
3. For the treasury auction you choose in part 1, conjecture the revenue ranking of the three auction formats: discriminatory auction, uniform-price auction, and English auction. Justify your answer from what you learned from Lecture 6, your reading of Bikhchandani and Huang (1993), and your answers in the previous two parts.

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<sup>1</sup>You may consult the table from 2005 on Slide 4 of Lecture 6 for your choice, as the auction format may well be the same now.