Rough Set Theory: A True Landmark in Data Analysis

gouziwu

July 26, 2019

Contents

1 Rough Sets on Fuzzy Approximation Spaces and Intuitionistic Fuzzy Approximation Spaces

1.1 Introduction

1.1.1 Fuzzy Sets

1.1.2 Intuitionistic Fuzzy Sets

the membership and nonmembership values of an element with respect to a collection of elements from a universe may not add up to 1 in all possible cases

Definition 1.1. An *intuitionistic fuzzy set* A on a universe U is defined by two functions: *membership function* μ_A and *non-membership function* ν s.t.

$$\mu_A, \nu_A : U \to [0, 1]$$

where $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for all $x \in U$.

The *hesitation function* Π_A for an intuitionistic fuzzy set is given by

$$\Pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$$

1.1.3 Rough set

A knowledge base is also called an **approximation space**

1.1.4 Motivation

1.1.5 Fuzzy proximity relation

Definition 1.2. Let U be a universal set and $X \subseteq U$. Then a *fuzzy relation* on X is defined as any fuzzy set defined on $X \times X$

Definition 1.3. A fuzzy relation R is said to be *fuzzy reflexive* on $X \subseteq U$ if it satisfies

$$\mu_R(x, x) = 1 \quad \text{for all } x$$

Definition 1.4. A fuzzy relation R is said to be *fuzzy symmetric* on $X \subseteq U$ if it satisfies

$$\mu_R(x, y) = \mu_R(y, x) \quad \text{for all } x, y \in X$$

Definition 1.5. A fuzzy relation on $X \subseteq U$ is said to be a *fuzzy proximity relation* if it is fuzzy reflexive and fuzzy symmetric.

Definition 1.6. Let $X, Y \subseteq U$. A fuzzy relation from X to Y is a fuzzy set defined on $X \times Y$ characterized by the membership function $\mu_R : X \times Y \to [0, 1]$

Definition 1.7. For any $\alpha \in [0, 1]$, the *α-cut* of R, denoted by R_α is a subset of $X \times Y$ given by $R_\alpha = \big\{(x, y) : \mu_R(x, y) \geq \alpha\big\}$

Let R be a fuzzy proximity relation on U. Then for any $\alpha \in [0, 1]$ the elements of R_α are said to be *α-similar* to each other. $xR_\alpha y$.

Two elements x and y in U are said to be *α-identical* w.r.t. R $(xR(\alpha)y)$ if either x and y are α-similar or x and y are *transitively α-similar*, that is, there exists a sequence of elements $u_1, u_2, \ldots, u_n$ in U s.t. $xR_\alpha u_1, u_1 R_\alpha u_2, \ldots, u_n R_\alpha y$

1.1.6 Intuitionistic fuzzy proximity relation

Definition 1.8. An *intuitionistic fuzzy relation* on a universal set U is an intuitionistic fuzzy set defined on $U \times U$

Definition 1.9. An intuitionistic fuzzy relation R on a universal set U is said to be *intuitionstic fuzzy reflexive* if

$$\mu_R(x, x) = 1 \text{ and } \nu_R(x, x) = 0 \quad \text{for all } x \in X$$

Definition 1.10. An intuitionistic fuzzy relation R on a universal set U is said to be *intuitionistic fuzzy symmetric* if

$$\mu_R(x, y) = \mu_R(y, x) \text{ and } \nu_R(x, y) = \nu_R(y, x) \quad \text{for all } x, y \in X$$

Definition 1.11. *intuitionistic fuzzy proximity*

Define

$$J = \big\{(m, n) \mid m, n \in [0,1] \text{ and } 0 \leq m + n \leq 1\big\}$$

Definition 1.12. Le R be an IF-proximity relation on U. Then for any $(\alpha, \beta) \in J$ the *(α, β)-cut* of R, denoted by $\mathrm{R}_{\alpha,\beta}$ is

$$R_{\alpha,\beta} = \big\{(x, y) \mid \mu_R(x, y) \leq \alpha \text{ and } \nu_R(x, y) \leq \beta\big\}$$

The relation $R(\alpha, \beta)$ is an equivalence relation.

1.2 Rough Sets on Fuzzy Approximation

1.2.1 Preliminaries

Definition 1.13. For any set of fuzzy proximity relation $K = (U, \mathfrak{R})$ is called a *fuzzy approximation space*

For any fixed $\alpha \in [0, 1]$, $\mathfrak{R}$ generates a set of equivalence relation $\mathfrak{R}(\alpha)$ and we call the associated space $\boldsymbol{K}(\alpha) = (U, \mathfrak{R}(\alpha))$ as the *generated approximation space* corresponding to $\boldsymbol{K}$ and α

1.2.2 Properties

1.2.3 Reduction of Knowledge in Fuzzy Approximation Spaces

Definition 1.14. Let $\mathfrak{R}$ be a family of fuzzy proximity relations on U and $\alpha \in [0, 1]$. For any $R \in \mathfrak{R}$, we say that R is *α-dispensable* or *α-superfluous* in $\mathfrak{R}$ if and only if $IND(\mathfrak{R}(\alpha)) = IND(\mathfrak{R}(\alpha) - R(\alpha))$

Consider $U = \{x_1, \ldots, x_n\}$. Define the fuzzy proximity relations P, Q, R and S over U corresponding to the attributes a, b, c and d respectively.

Table 1: Fuzzy proximity relation for attribute R

P	x_1	x_2	x_3	x_4	x_5
x_1	1	0.3	0.6	0.8	0.5
x_2	0.3	1	0.7	0.4	0.4
x_3	0.6	0.7	1	0.2	0.8
x_4	0.8	0.4	0.2	1	0.5
x_5	0.5	0.4	0.8	0.5	1

Table 2: Fuzzy proximity relation for attribute Q

P	x_1	x_2	x_3	x_4	x_5
x_1	1	0.3	0.4	0.2	0.5
x_2	0.3	1	0.8	0.6	0.6
x_3	0.4	0.8	1	0.3	0.9
x_4	0.2	0.6	0.3	1	0.7
0.5	0.2	0.2	0.9	0.7	1

Table 3: Fuzzy proximity relation for attribute R

R	x_1	x_2	x_3	x_4	x_5
x_1	1	0.3	0.2	0.8	0.7
x_2	0.3	1	0.5	0.3	0.5
x_3	0.2	0.5	1	0.6	0.4
x_4	0.8	0.3	0.6	1	0.9
x_5	0.7	0.5	0.4	0.9	1

Table 4: Fuzzy proximity relation for attribute S

S	x_1	x_2	x_3	x_4	x_5
x_1	1	0.3	0.2	0.2	0.5
x_2	0.3	1	0.5	0.3	0.2
x_3	0.2	0.5	1	0.2	0.4
x_4	0.2	0.3	0.2	1	0.5
x_5	0.5	0.4	0.4	0.5	1

Table 5: Fuzzy proximity relation for $IND(\mathfrak{R}(\alpha))$

$IND(\mathfrak{R}(\alpha))$	x_1	x_2	x_3	x_4	x_5
x_1	1	0.3	0.2	0.2	0.5
x_2	0.3	1	0.3	0.3	0.2
x_3	0.2	0.3	1	0.2	0.4
x_4	0.2	0.3	0.2	1	0.4
x_5	0.5	0.2	0.4	0.4	1

Suppose $\alpha = 0.6$, then we get

$$U/P(\alpha) = \{\{x_1, x_2, x_3, x_4, x_5\}\}$$
$$U/Q(\alpha) = \{\{x_1\}, \{x_2, x_3, x_4, x_5\}\}$$
$$U/R(\alpha) = \{\{x_1, x_3, x_4, x_5\}, \{x_2\}\}$$
$$U/S(\alpha) = \{\{x_1\}, \{x_2\}, \{x_3\}, \{x_4\}, \{x_5\}\}$$

1.2.4 Relative reducts and relative core of knowledge in fuzzy approximation spaces

Definition 1.15. Let P and Q be two fuzzy proximity relations over the universe U. For every fixed $\alpha \in [0, 1]$, the *α-positive region* of P w.r.t. Q can be defined as

$$\alpha\text{-}POS_P Q = \bigcup_{X_\alpha \in U/Q} \underline{P} X_\alpha$$

Definition 1.16. Let $\boldsymbol{P}$ and $\boldsymbol{Q}$ be two families of fuzzy proximity relations on U. For every fixed $\alpha \in [0, 1]$ and $R \in \boldsymbol{P}$,R is $(\boldsymbol{Q}, \alpha)$-*dispensable* in $\boldsymbol{P}$ if

$$\alpha\text{-}POS_I ND(\boldsymbol{Q}) = \alpha\text{-}POS_{IND(\boldsymbol{P}-\{R\})} IND(\boldsymbol{Q})$$

1.2.5 Dependency of knowledge in fuzzy approximation spaces

1.2.6 partial dependency of knowledge in fuzzy approximation spaces