

$$1. \operatorname{Res} \bar{z} f(z), -iI = \frac{-i}{(2i)^2} = \frac{i}{4}$$

$$\operatorname{Res} \bar{z} f(z), iI = \frac{i}{(2i)^2} = -\frac{i}{4}, C.$$

2. B

3. C.

$$4. B. \bar{z} f(z) = -2I.$$

$$5. u_x = v_y = x^2 - 3y^2 \Rightarrow u(x, y) = x^3 - 3xy^2 + C(y)$$

$$u_y = -v_x = -6xy = -6xy + C'(y) \Rightarrow C(y) = \text{const.}$$

$$\Rightarrow f(z) = (x^3 - 3xy^2) + i(3yx^2 - y^3) = z^3 + c. \quad (B)$$

$$6. x^2 - y^2 \rightarrow -9, \quad xy \rightarrow 0 \Rightarrow x \rightarrow 0 \Rightarrow (A)$$

$$7. \oint \frac{dz \cdot \frac{1}{z}}{a + \frac{z + 1/z}{2}}, \quad z = a \pm \sqrt{a^2 - 1}, \quad (D)$$

$$1. 4 \text{ 级极点} \quad 2. -1. \quad 3. \sqrt{2}i/2, \frac{u}{2e^2}.$$

$$4. 8\pi i, 8\pi i. \quad 5. (\sqrt{5}-1)/2.$$

$$2. \cos^2 \frac{\theta}{2} = \frac{1}{2}(1 + \cos \theta) = \frac{2 + z + \frac{1}{z}}{4}.$$

$$I = \oint_{|z|=1} f(z) \cdot \frac{z^2 + 2z + 1}{4iz^2} dz.$$

$$= \frac{2\pi i}{4i} \left( \frac{2}{-1} + \frac{4}{1} \right) = \pi \left( -2 + 4 \right) = 2\pi.$$

$$2. \operatorname{Res} \bar{z} f(z), \infty I = -\operatorname{Res} \bar{z} z^{-3} \coth \frac{1}{1-z}, 0I.$$

$$= -\frac{1}{2!} \left( \coth \frac{1}{1-z} \right)' \Big|_{z=0}.$$

$$= -\pi i (1.5e + 0.51) \quad 3 \text{ 级极点}, \quad I = 2\pi i.$$

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$$3. R = \frac{1}{\lim_{n \rightarrow \infty} \sqrt[n]{1/n}} = 1.$$

$$f(z) \triangleq \sum_{n=1}^{\infty} \frac{z^n}{n}, \quad f'(z) = \sum_{n=0}^{\infty} z^{n+1} \\ = z \cdot \frac{1}{1-z^2}.$$

$$\Rightarrow f(z) = -\frac{1}{2} \ln(1-z^2)$$

4. 本性奇点.