Challenge Problems

1. Multiple Shortest Paths in a Weighted DAG

You are given a weighted directed acyclic graph G, a start node s in G. You are also given a positive integer k. We assume to be a topological ordering of the nodes. The topological ordering of the nodes implies that there is only an edge from i to j if i < j.
Example: We consider the directed acyclic graph G described below. Let the nodes of G be the numbers 0,1,2,3,4,5,6,7,8. We assume that s = 0. The following table gives the weight of the edge from some node i to node j.

	1	2	3	4	5	6	7	8
0	1	3	2	1	2	1	2	3
1		4	1	0	1	0	1	2
2			3	4	3	4	3	2
3				1	0	1	0	3
4					1	0	1	2
5						1	0	3
6							1	2
7								3

The problem is to find k directed paths starting at s, such that every node of G lies on at least one of those paths.
For k = 1, there is only one possible solution, namely the path (0,1,2,3,4,5,6,7,8), which has cost 15.
Suppose k = 2. The two paths could be chosen to be (0,1,3,4,5,6,7) and (0,2,8). The total cost for these two paths is 11. That is not the best solution.
The best solution for k = 2 is (0,1,4,6,8) and (0,2,3,5,7) with total cost 9.
What is the best solution for k = 3?