Left to right Algorithm

```
for (int k = 0; k < n; k++) {          1 + 1 + 1
        for (int j = k; j < 2 * n - 1; j++) {
                if (disks[j] > disks[j + 1]) {   1 + 1
                        swap(disks[j], disks[j + 1]);  1
                        moves++;   1
                }
        }
}
```

$\sum_{j=k}^{2n-2} 4$ $\quad \sum_{k=0}^{n-1} 4*(2n-2-k+1)$

$$\sum_{k=0}^{n-1} 4(2n-k-1)$$

$$\sum_{k=0}^{n-1} 8n - 4k - 4$$

$$\sum_{k=0}^{n-1} 8n - \sum_{k=0}^{n-1} 4k - 4(n-1+0+1) = 8n(n-1-0+1) - 4\left(\frac{(n-1)(n-1+1)}{2}\right) - 4n$$

$$= 8n^2 - 2n^2 - 2n - 4n = 6n^2 - 6n + 3$$

$$6n^2 - 6n \in O(n^2)$$

$$\lim_{n\to\infty} \frac{6n^2-6n}{n^2} \Rightarrow \lim_{n\to\infty} \frac{6 - 6/n}{1} = 6 > 0 \text{ and a constant}$$

therefore $6n^2 - 6n \in O(n^2)$

Lawn Mower Algorithm

```
// loop to push dark ones before light ones
        for (int k = 0; k < n; k++) {
                //Start from the left
                if (k % 2 == 0) {
                        for (int j = 0; j < 2 * n - 1; j++) {
                                if (disks[j] > disks[j + 1]) {
                                        swap(disks[j], disks[j + 1]);
                                        moves++;
                                }
                        }
                }
                //start from right
                else {
                        for (int j = 2 * n - 1; j > 0; j--) {
                                if (disks[j-1] > disks[j]) {
                                        swap(disks[j-1], disks[j]);
                                        moves++;
                                }
                        }
                }
        } // end lawn mower algorithm loop
```

$1 + \sum_{j=0}^{2n-2} (4) = 1 + (2n-2-0+1)*4 = 8n-4$

$\min(8n-4, 8n-4)$

$\sum_{j=0}^{2n-2} (4) = 4(2n-2-0+1) = 8n-4$

$3 + \sum_{k=0}^{n-1} 1 + (8n-4) = 3 + \sum_{k=0}^{n-1} 8n-3 \Rightarrow 3 + \sum_{k=0}^{n-1} 8n - \sum_{k=0}^{n-1} 3$

$3 + 8n(n-1-0+1) - 3(n-1-0+1) = 3 + 8n^2 - 3n$

$8n^2 - 3n + 3 \in O(n^2)$

$8n^2 - 3n + 3 \leq cn^2 \quad \forall n > n_0$

let $c = 14$

$8n^2 - 3n + 3 \leq 14n^2$

$-3n + 3 \leq 6n^2 \quad \forall n > n_0$

let $n = 1$

$-3(1) + 3 \leq 6(1^2) \quad \forall n > 1$

$0 \leq 6 \checkmark \quad \forall n > 1$

therefore $8n^2 - 3n + 3 \in O(n^2)$