1

2

3

4 LK Formula

[1] $\Omega=-kT\sum\ln\left(1+\mathrm{e}^{(\zeta-\varepsilon)/kT}\right)$

$$\Omega=-kT\int_{-\infty}^{\infty}\mathrm{d}\kappa\left(\frac{eHV}{2\pi^2ch}\right)\sum_r\ln\left(1+\mathrm{e}^{(\zeta-\varepsilon_r)/kT}\right) \tag{1}$$

κ T=0

[2]
$$\delta\Omega=\delta\kappa\left(\frac{eHV}{2\pi^2ch}\right)\sum_{r=0}^{n}(\varepsilon_r-\zeta)\equiv D\sum_{r=0}^{n}(\varepsilon_r-\zeta) \tag{2}$$

r Euler-Maclaurin ,

$$\sum_0^n f(r)=\int_0^n f(r)\mathrm{d}r+\frac{1}{2}[f(n)+f(0)]+\frac{1}{12}\left[f'(n)-f'(0)\right] \tag{3}$$

$o(\frac{1}{n^2})$

$$\begin{aligned}\frac{\delta\Omega}{D}=&\int_0^n(\varepsilon_r-\zeta)\,\mathrm{d}r+\frac{1}{2}(\varepsilon_n-\zeta)+\frac{1}{2}(\varepsilon_0-\zeta)\\&+\frac{1}{12}\left[\left(\frac{\partial\varepsilon}{\partial r}\right)_{r=n}-\left(\frac{\partial\varepsilon}{\partial r}\right)_{r=0}\right]\end{aligned} \tag{4}$$

F-D n r $\varepsilon_r=\zeta$ x $r+\frac{1}{2}$ X $\varepsilon(X)=\zeta$ $\left(\frac{\partial\varepsilon}{\partial x}\right)_\kappa=\frac{(\partial a/\partial x)_\kappa}{(\partial a/\partial\varepsilon)_\kappa}=\frac{2\pi eH/ch}{2\pi m/\hbar^2}=\beta H$ [3]

$$\delta\tilde{\Omega}=\delta\kappa\frac{e\beta H^2V}{4\pi^2ch}\left\{\left[X-\left(n+\frac{1}{2}\right)\right]^2-\left[\left(X-\left(n+\frac{1}{2}\right)\right]+\frac{1}{6}\right\} \tag{5}$$

$\left(n+\frac{1}{2}\right)\leqslant X\leqslant\left(n+\frac{3}{2}\right)$ X n X

$$\delta\tilde{\Omega}=\frac{\delta\kappa e\beta H^2V}{4\pi^2ch}\sum_{p=1}^{\infty}\frac{1}{\pi^2p^2}\cos2\pi p\left(X-\frac{1}{2}\right) \tag{6}$$

κ

$$\tilde{\Omega}=\frac{eH^2V}{4\pi^2ch}\int\beta\mathrm{d}\kappa\sum_{p=1}^{\infty}\frac{1}{\pi^2p^2}\cos\left\{2\pi p\left[X(\kappa)-\frac{1}{2}\right]\right\} \tag{7}$$

---

[1]a k $a(\varepsilon,\kappa)=(r+\gamma)2\pi eH/ch$

[2] T=0

[3] x

κ κ X κ $\kappa = 0$ [4]

$$\begin{aligned} I_p &= \int_{-\infty}^{\infty} \mathrm{d}\kappa \cos\left[2\pi p\left(X(\kappa) - \frac{1}{2}\right)\right] \\ &= 2\int_{0}^{\infty} \mathrm{d}\kappa \cos\left[2\pi p\left(X_0 \pm \frac{1}{2}X''\kappa^2 - \frac{1}{2}\right)\right] \\ &= (pX'')^{-1/2}\cos\left[2\pi p\left(X_0 - \frac{1}{2}\right) \pm \frac{1}{4}\pi\right] \end{aligned} \tag{8}$$

p

$$\tilde{\Omega} = \left(\frac{e}{2\pi ch}\right)^{3/2} \frac{\beta H^{5/2}}{\pi^2 (A'')^{1/2}} \sum_{p=1}^{\infty} \frac{1}{p^{5/2}} \cos\left[2\pi p\left(\frac{F}{H} - \frac{1}{2}\right) \pm \frac{\pi}{4}\right] \tag{9}$$

$F = (ch/2\pi e)A = X_0 H, A'' = \left|\partial^2 \mathscr{A}/\partial\kappa^2\right|_{\kappa=0} = (2\pi eH/ch)X''$ “ ” **(??)** F

$\psi = 2\pi p\left(\frac{F}{H} - \frac{1}{2}\right) \pm \frac{\pi}{4}$ F $\cos(\psi)$

$$\begin{aligned} I &= \int_{-\infty}^{\infty} \cos(\psi + \phi) D(\phi/\lambda)\mathrm{d}\phi / \int_{-\infty}^{\infty} D(\phi/\lambda)\mathrm{d}\phi \\ &= \mathscr{R}\mathrm{e}^{\mathrm{i}\psi} \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}\phi} D(\phi/\lambda)\mathrm{d}\phi / \int_{-\infty}^{\infty} D(\phi/\lambda)\mathrm{d}\phi \\ &= \mathscr{R}\mathrm{e}^{\mathrm{i}\psi} \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}\lambda z} D(z)\mathrm{d}z / \int_{-\infty}^{\infty} D(z)\mathrm{d}z \\ &= \mathscr{R}\left\{[f(\lambda)/f(0)]\mathrm{e}^{\mathrm{i}\psi}\right\} \end{aligned} \tag{10}$$

$z = \phi/\lambda, f(\lambda) = \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}\lambda z} D(z)\mathrm{d}z$

“ ” $R = |f(\lambda)|/f(0)$ $D(z)$ z $f(\lambda)$

F-D

$$-\frac{\mathrm{d}f(\mu)}{\mathrm{d}\mu} = \frac{1}{2kT[1 + \cosh(\mu - \zeta)/kT]} \tag{11}$$

$R_T = \frac{\pi\lambda}{\sinh \pi\lambda} = \frac{2\pi^2 pkT/\beta H}{\sinh(2\pi^2 pkT/\beta H)}$ 1 $R_T = \frac{4\pi^2 pkT}{\beta H} \exp\left(-2\pi^2 pkT/\beta H\right)$

Dingle Lorentzian $\epsilon \sim \epsilon + d\epsilon$ $\frac{d\epsilon}{(\epsilon - \epsilon_r)^2 + (h/2\tau)^2}$ τ μ ζ “ ”

$$R_{\mathrm{D}} = \mathrm{e}^{-\pi ph/\beta H\tau} = \mathrm{e}^{-\pi p/\omega_c\tau} \tag{12}$$

Dingle

$$R_{\mathrm{D}} = \exp\left(-2\pi^2 pkx/\beta H\right) \tag{13}$$

$x = \hbar/2\pi k\tau$ Dingle

5 Bethe

6

$$H = H_0 + V = \sum_\sigma \int d\mathbf{r}\Psi_\sigma^\dagger(\mathbf{r})H_0(\mathbf{r})\Psi_\sigma(\mathbf{r}) + \sum_\sigma \int d\mathbf{r}\Psi_\sigma^\dagger(\mathbf{r})V_\sigma(\mathbf{r})\Psi_\sigma(\mathbf{r}) \tag{14}$$

[4] X

H_0 Exact $\mathcal{G}(b,a) = -\left\langle T_\tau \Psi(b)\Psi^\dagger(a)\right\rangle$

$$\mathcal{G}(b,a) = \mathcal{G}^0(b,a) + \int d1 \mathcal{G}(b,1)V(1)\mathcal{G}^0(1,a) \tag{15}$$

$$\mathcal{G}\left(\mathbf{r}_b, \mathbf{r}_a; ik_n\right) = \mathcal{G}^0\left(\mathbf{r}_b, \mathbf{r}_a; ik_n\right) + \int d\mathbf{r}_1 \mathcal{G}^0\left(\mathbf{r}_b, \mathbf{r}_1; ik_n\right) V(1) \mathcal{G}\left(\mathbf{r}_1, \mathbf{r}_a; ik_n\right) \tag{16}$$

H_0 [5]

$$\mathcal{G}_{\nu\nu'} \equiv \int d\mathbf{r} d\mathbf{r}' \langle \nu \mid \mathbf{r}\rangle \mathcal{G}\left(\mathbf{r}, \mathbf{r}'\right)\left\langle \mathbf{r}' \mid \nu'\right\rangle \quad \Leftrightarrow \quad \mathcal{G}\left(\mathbf{r}, \mathbf{r}'\right) = \sum_{\nu\nu'} \langle \mathbf{r} \mid \nu\rangle \mathcal{G}_{\nu\nu'}\left\langle \nu' \mid \mathbf{r}'\right\rangle \tag{17}$$

$\mathcal{G}^0_{\nu,\nu'}\left(ik_n\right) = \frac{1}{ik_n - \xi_\nu}\delta_{\nu,\nu'}$

$$\mathcal{G}\left(\nu_b \nu_a; ik_n\right) = \delta_{\nu_b,\nu_a} \mathcal{G}^0\left(\nu_a \nu_a; ik_n\right) + \sum_{\nu_c} \mathcal{G}^0\left(\nu_b \nu_b; ik_n\right) V_{\nu_b \nu_c} \mathcal{G}\left(\nu_c \nu_a; ik_n\right) \tag{18}$$

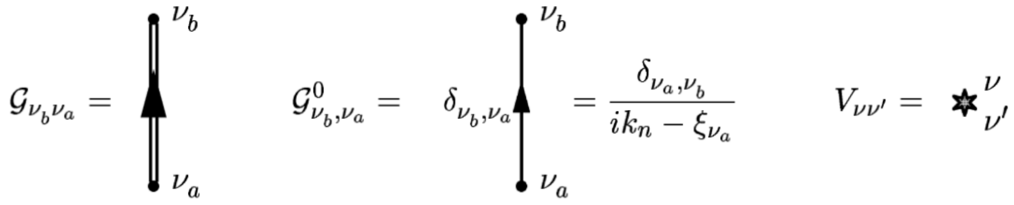


Figure 1:

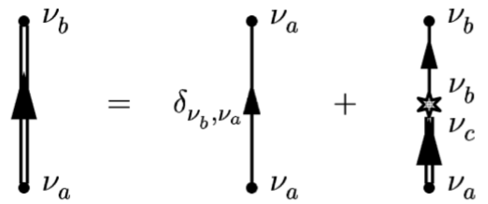


Figure 2:

$$V(\mathbf{r}) = \sum_{j=1}^{N_{\text{imp}}} u\left(\mathbf{r} - \mathbf{P}_j\right), \quad \mathbf{P}_j \text{ is randomly distributed.} \tag{19}$$

u [6] n

$$\begin{aligned} \mathcal{G}^{(n)}\left(\mathbf{r}_b, \mathbf{r}_a\right) = & \sum_{j_1}^{N_{\text{imp}}} \cdots \sum_{j_n}^{N_{\text{imp}}} \int d\mathbf{r}_1 \ldots \int d\mathbf{r}_n \\ & \times \mathcal{G}^0\left(\mathbf{r}_b - \mathbf{r}_n\right) u\left(\mathbf{r}_n - \mathbf{P}_{j_n}\right) \ldots u\left(\mathbf{r}_2 - \mathbf{P}_{j_2}\right) \mathcal{G}^0\left(\mathbf{r}_2 - \mathbf{r}_1\right) u\left(\mathbf{r}_1 - \mathbf{P}_{j_1}\right) \mathcal{G}^0\left(\mathbf{r}_1 - \mathbf{r}_a\right) \end{aligned} \tag{20}$$

[5]

[6]

$$\mathcal{G}^{(n)}\left(\mathbf{r}_b,\mathbf{r}_a\right)=\sum_{j_1\ldots j_n}^{N_{\text{imp}}}\frac{1}{\mathcal{V}^n}\sum_{\mathbf{q}_1\ldots\mathbf{q}_n}\frac{1}{\mathcal{V}^2}\sum_{\mathbf{k}_a\mathbf{k}_b}\frac{1}{\mathcal{V}^{n-1}}\sum_{\mathbf{k}_1\ldots\mathbf{k}_{n-1}}\int d\mathbf{r}_1\ldots\int d\mathbf{r}_n$$
$$\times\,\mathcal{G}^0_{\mathbf{k}_b}u_{\mathbf{q}_n}\mathcal{G}^0_{\mathbf{k}_{n-1}}u_{\mathbf{q}_{n-1}}\ldots u_{\mathbf{q}_2}\mathcal{G}^0_{\mathbf{k}_1}u_{\mathbf{q}_1}\mathcal{G}^0_{\mathbf{k}_a}e^{-i\left(\mathbf{q}_n\cdot\mathbf{P}_{j_n}+\ldots+\mathbf{q}_2\cdot\mathbf{P}_{j_2}+\mathbf{q}_1\cdot\mathbf{P}_{j_1}\right)}$$
$$\times\,e^{i\mathbf{k}_b\cdot(\mathbf{r}_b-\mathbf{r}_n)}e^{i\mathbf{q}_n\cdot\mathbf{r}_n}e^{i\mathbf{k}_{n-1}\cdot(\mathbf{r}_n-\mathbf{r}_{n-1})}\ldots e^{i\mathbf{q}_2\cdot\mathbf{r}_2}e^{i\mathbf{k}_1\cdot(\mathbf{r}_2-\mathbf{r}_1)}e^{i\mathbf{q}_1\cdot\mathbf{r}_1}e^{i\mathbf{k}_a\cdot(\mathbf{r}_1-\mathbf{r}_a)} \tag{21}$$

delta

$$\mathcal{G}^{(n)}_{\mathbf{k}_b\mathbf{k}_a}=\sum_{j_1\ldots j_n}^{N_{\text{imp}}}\frac{1}{\mathcal{V}^{n-1}}\sum_{\mathbf{k}_1\ldots\mathbf{k}_{n-1}}e^{-i\left[(\mathbf{k}_b-\mathbf{k}_{n-1})\cdot\mathbf{P}_{j_n}+\ldots+(\mathbf{k}_1-\mathbf{k}_a)\cdot\mathbf{P}_{j_1}\right]}$$
$$\times\,\mathcal{G}^0_{\mathbf{k}_b}u_{\mathbf{k}_b-\mathbf{k}_{n-1}}\mathcal{G}^0_{\mathbf{k}_{n-1}}\cdots u_{\mathbf{k}_2-\mathbf{k}_1}\mathcal{G}^0_{\mathbf{k}_1}\cdots u_{\mathbf{k}_1-\mathbf{k}_a}\mathcal{G}^0_{\mathbf{k}_a}. \tag{22}$$

k

N

$$\frac{1}{\mathcal{V}}\left\langle\mathcal{G}_{\mathbf{k}_b\mathbf{k}_a}\right\rangle_{\text{imp}}\equiv\delta_{\mathbf{k}_b,\mathbf{k}_a}\overline{\mathcal{G}}_{\mathbf{k}_a}\equiv\frac{\delta_{\mathbf{k}_b,\mathbf{k}_a}}{N_{\text{sys}}}\sum_{i=1}^{N_{\text{sys}}}\mathcal{G}^{\text{sys}_i}_{\mathbf{k}_a}\sim\delta_{\mathbf{k}_b,\mathbf{k}_a}\frac{1}{\mathcal{V}}\int d\mathbf{P}_1\frac{1}{\mathcal{V}}\int d\mathbf{P}_2\cdots\frac{1}{\mathcal{V}}\int d\mathbf{P}_{N_{\text{imp}}}\mathcal{G}_{\mathbf{k}_a} \tag{23}$$

n n Leading oder

$$\sum_{j_1,\ldots,j_n}^{N_{\text{imp}}}e^{i\sum_{l=1}^{n}\mathbf{q}_l\cdot\mathbf{P}_{j_l}}=\sum_{h_1}^{N_{\text{imp}}}e^{i\left(\sum_{\mathbf{q}_{j_1}\in Q}\mathbf{q}_{j_1}\right)\cdot\mathbf{P}_{h_1}}$$
$$+\sum_{Q_1\cup Q_2=Q}\sum_{h_1}^{N_{\text{imp}}}\sum_{h_2}^{N_{\text{imp}}}e^{i\left(\sum_{\mathbf{q}_{l_1}\in Q_1}\mathbf{q}_{l_1}\right)\cdot\mathbf{P}_{h_1}}e^{i\left(\sum_{\mathbf{q}_{l_2}\in Q_2}\mathbf{q}_{l_2}\right)\cdot\mathbf{P}_{h_2}}$$
$$+\sum_{Q_1\cup Q_2\cup Q_3=Q}\sum_{h_1}^{N_{\text{imp}}}\sum_{h_2}^{N_{\text{imp}}}\sum_{h_3}^{N_{\text{imp}}}e^{i\left(\sum_{\mathbf{q}_{l_1}\in Q_1}\mathbf{q}_{l_1}\right)\cdot\mathbf{P}_{h_1}}e^{i\left(\sum_{\mathbf{q}_{l_2}\in Q_2}\mathbf{q}_{l_2}\right)\cdot\mathbf{P}_{h_2}}e^{i\left(\sum_{\mathbf{q}_{l_3}\in Q_3}\mathbf{q}_{l_3}\right)\cdot\mathbf{P}_{h_h}}$$
$$+\ldots \tag{24}$$

$Q=\{\mathbf{q}_1,\mathbf{q}_2,\ldots,\mathbf{q}_n\}$ n Q_i [7]

delta

$$\left\langle e^{i\left(\sum_{\mathbf{q}_{h_i}\in Q_i}\mathbf{q}_{h_i}\right)\cdot\mathbf{P}_{h_i}}\right\rangle_{\text{imp}}=\frac{1}{\mathcal{V}}\int d\mathbf{P}_{h_i}e^{i\left(\sum_{\mathbf{q}_{h_i}\in Q_h}\mathbf{q}_{h_i}\right)\cdot\mathbf{P}_{h_i}}=\delta_{0,\sum_{\mathbf{q}_{h_i}\in Q_h}\mathbf{q}_{h_i}}. \tag{25}$$

n

$$\left\langle\mathcal{G}^{(n)}_{\mathbf{k}}\right\rangle_{\text{imp}}=\frac{1}{\mathcal{V}^{n-1}}\sum_{\mathbf{k}_1\ldots\mathbf{k}_{n-1}}\sum_{p=1}^{n}\sum_{\bigcup_{h=1}^{p}Q_h=Q}\prod_{h=1}^{p}\left(N_{\text{imp}}\delta_{0,\Sigma_{Q_h}\left(\mathbf{k}_{h_i}-\mathbf{k}_{(h_i-1)}\right)}\right)$$
$$\times\,\mathcal{G}^0_{\mathbf{k}}u_{\mathbf{k}-\mathbf{k}_1}\mathcal{G}^0_{\mathbf{k}_1}u_{\mathbf{k}_1-\mathbf{k}_2}\mathcal{G}^0_{\mathbf{k}_2}\ldots u_{\mathbf{k}_{n-1}-\mathbf{k}}\mathcal{G}^0_{\mathbf{k}}. \tag{26}$$

delta n-1-p p N_{imp}

$$\left\langle\mathcal{G}_{\mathbf{k}}\left(ik_n\right)\right\rangle_{\text{imp}}=\frac{\mathcal{G}^0_{\mathbf{k}}}{1-\mathcal{G}^0_{\mathbf{k}}\Sigma_{\mathbf{k}}}=\frac{1}{\left(\mathcal{G}^0_{\mathbf{k}}\right)^{-1}-\Sigma_{\mathbf{k}}}=\frac{1}{ik_n-\xi_{\mathbf{k}}-\Sigma_{\mathbf{k}}\left(ik_n\right)} \tag{27}$$

$\Sigma^{\text{LOA}}_{\mathbf{k}}\left(ik_n\right)\equiv n_{\text{imp}}u_0=n_{\text{imp}}\int d\mathbf{r}u(\mathbf{r}),$

[7] $1/N_{\text{imp}}$

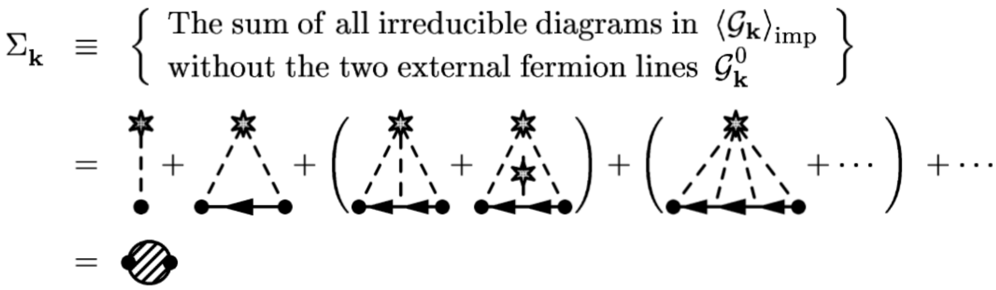


Figure 3: <caption>

[8]

$$\Sigma^{\text{1BA}}_{\mathbf{k}} \left(ik_n \right) \equiv n_{\text{imp}} \sum_{\mathbf{k}'} \left| u_{\mathbf{k}-\mathbf{k}'} \right|^2 \frac{1}{ik_n - \xi_{\mathbf{k}'}} \tag{28}$$

[9]

$$\begin{aligned} \Sigma^{\text{1BA}}_{\mathbf{k}} \left(\omega + i \,\text{sgn}\left(k_n \right) \eta \right) &= n_{\text{imp}} \sum_{\mathbf{k}'} \left| u_{\mathbf{k}-\mathbf{k}'} \right|^2 \frac{1}{\left(\omega - \xi_{\mathbf{k}'} \right) + i \,\text{sgn}\left(k_n \right) \eta} \\ &= \sum_{\mathbf{k}'} n_{\text{imp}} \left| u_{\mathbf{k}-\mathbf{k}'} \right|^2 \left[\frac{\omega - \xi_{\mathbf{k}'}}{\left(\omega - \xi_{\mathbf{k}'} \right)^2 + \eta^2} - i \,\text{sgn}\left(k_n \right) \pi \delta \left(\omega - \xi_{\mathbf{k}'} \right) \right]. \end{aligned} \tag{29}$$

$$|\mathbf{k}| \sim k_{\text{F}} \quad \text{and} \quad \left| ik_n \to \omega + i \,\text{sgn}\left(k_n \right) \eta \right| \ll \varepsilon_{\text{F}}. \tag{30}$$

u_k

$$\Sigma^{\text{1BA}}_{\mathbf{k}} \left(ik_n \right) = -i\pi \,\text{sgn}\left(k_n \right) \sum_{\mathbf{k}'} n_{\text{imp}} \left| u_{\mathbf{k}-\mathbf{k}'} \right|^2 \delta \left(\xi_{\mathbf{k}} - \xi_{\mathbf{k}'} \right) = -i \,\text{sgn}\left(k_n \right) \frac{1}{2\tau_{\mathbf{k}}}, \tag{31}$$

$\frac{1}{\tau_{\mathbf{k}}} \equiv 2\pi \sum_{\mathbf{k}'} n_{\text{imp}} \left| u_{\mathbf{k}-\mathbf{k}'} \right|^2 \delta \left(\xi_{\mathbf{k}} - \xi_{\mathbf{k}'} \right)$

$$\mathcal{G}^{\text{1BA}}_{\mathbf{k}} \left(ik_n \right) = \frac{1}{ik_n - \xi_{\mathbf{k}} + i \frac{\text{sgn}\left(k_n \right)}{2\tau_{\mathbf{k}}}} \underset{ik_n \to z}{\longrightarrow} \mathcal{G}^{\text{1BA}}_{\mathbf{k}}(z) = \begin{cases} \frac{1}{z - \xi_{\mathbf{k}} + \frac{i}{2\tau_{\mathbf{k}}}}, \text{Im}\, z > 0 \\ \frac{1}{z - \xi_{\mathbf{k}} - \frac{i}{2\tau_{\mathbf{k}}}}, \text{Im}\, z < 0. \end{cases} \tag{32}$$

n_{imp} t t [10]

$$\begin{aligned} \text{Im}\, \Sigma^{\text{FBA}}_{\mathbf{k}} \left(ik_n \right) &= \text{Im}\, t_{\mathbf{k},\mathbf{k}} \left(ik_n \right) = \text{Im} \sum_{\mathbf{k}'} \frac{\left| t_{\mathbf{k},\mathbf{k}'} \right|^2}{ik_n - \xi_{\mathbf{k}'}} \\ &\underset{ik_n \to \omega + i \,\text{sgn}(k_n)\eta}{\longrightarrow} - \text{sgn}\left(k_n \right) \pi \sum_{\mathbf{k}'} \left| t_{\mathbf{k},\mathbf{k}'} \right|^2 \delta \left(\omega - \xi_{\mathbf{k}'} \right). \end{aligned} \tag{33}$$

$n_{\text{imp}} \left| u_{\mathbf{k}-\mathbf{k}'} \right|^2 \to \left| t_{\mathbf{k},\mathbf{k}'} \right|^2$

[8] u(r)

[9] delta $\delta(x) = \lim_{\varepsilon \to 0} \frac{1}{\pi} \frac{\varepsilon}{\varepsilon^2 + x^2}$.

[10] $t = u + u\mathcal{G}^0 t$ u $t = u + \left(t^\dagger \mathcal{G}^0 t - t^\dagger \left(\mathcal{G}^0 \right)^\dagger u \mathcal{G}^0 t \right)$ $\text{Im}\, t_{\mathbf{k},\mathbf{k}} = \text{Im} \left\langle \mathbf{k} \left| t^\dagger \mathcal{G}^0 t \right| \mathbf{k} \right\rangle = \text{Im} \sum_{\mathbf{k}'} t^\dagger_{\mathbf{k},\mathbf{k}'} \mathcal{G}^0_{\mathbf{k}'} t_{\mathbf{k}',\mathbf{k}}$

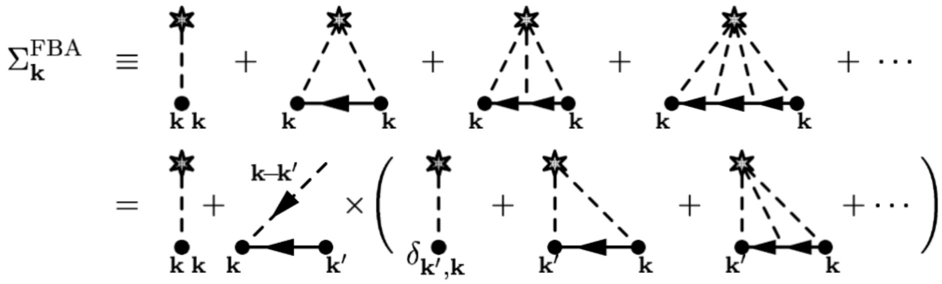
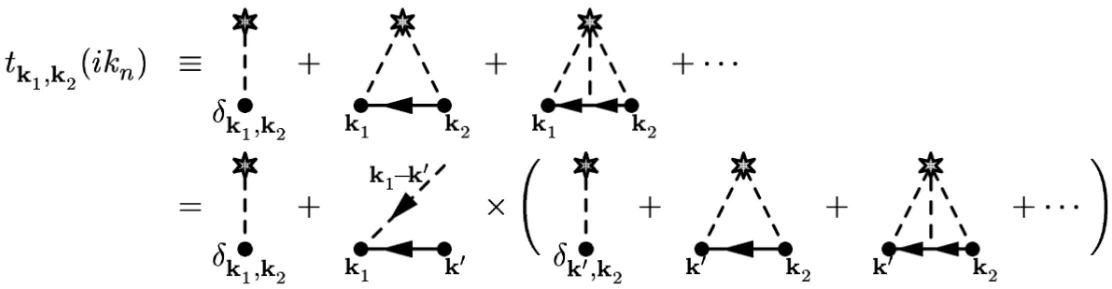


Figure 4:



$$= \quad n_{\mathrm{imp}}\Big[u_0\delta_{\mathbf{k}_1,\mathbf{k}_2} + \sum_{\mathbf{k}'} u_{\mathbf{k}_1-\mathbf{k}'}\,\mathcal{G}^0_{\mathbf{k}'}\,t_{\mathbf{k}',\mathbf{k}_2}\Big]. \qquad (1$$

Figure 5: t

$$t_{\mathbf{k}}^{\mathrm{SCBA}} \equiv n_{\mathrm{imp}}\left[u_0\delta_{\mathbf{k},\mathbf{k}} + \sum_{\mathbf{k}'} u_{\mathbf{k}-\mathbf{k}'}\mathcal{G}_{\mathbf{k}'}t_{\mathbf{k}',\mathbf{k}}\right] \tag{34}$$

$$\Sigma_{\mathbf{k}}^{i} = \mathrm{Im}\sum_{\mathbf{k}'}\frac{\left|t_{\mathbf{k},\mathbf{k}'}^{\mathrm{SCBA}}\right|^2}{ik_n - \xi_{\mathbf{k}'} - i\Sigma_{\mathbf{k}'}^{i}} \tag{35}$$

(??)

Dayson $G_p \equiv \frac{1}{-i\omega_n + \frac{\mathbf{p}^2}{2m} - \mu}$

7 Peierls

8 Quantum-Classical Correspondence

9 Wick

9.1 QFT Wick

9.2 "wick"

10 Hartree-Fock

Hartree-Fock $n_{\vec{k},\alpha}$

10.1

$$H = \sum_{\mathbf{k}}\xi_{\mathbf{k}}c_{\alpha\mathbf{k}}^{\dagger}c_{\alpha\mathbf{k}} + \frac{1}{2N_{site}}\sum_{\mathbf{k},\mathbf{k}',\mathbf{q}}c_{\alpha\mathbf{k}}^{\dagger}c_{\alpha,\mathbf{k}-\mathbf{q}}V_{\mathbf{q}}c_{\beta,\mathbf{k}'}^{\dagger}c_{\beta,\mathbf{k}'+\mathbf{q}} \tag{36}$$

$|\Psi_0\rangle$

$$\begin{aligned}\langle\Psi_0|H|\Psi_0\rangle = \langle\Psi_0\left|H_0\right|\Psi_0\rangle + \frac{1}{2}\sum_{\mathbf{i},\mathbf{j}}\left\langle c_{\alpha}^{\dagger}(\mathbf{i})c_{\alpha}(\mathbf{i})\right\rangle V(\mathbf{i}-\mathbf{j})\left\langle c_{\beta}^{\dagger}(\mathbf{j})c_{\beta}(\mathbf{j})\right\rangle \\ + \frac{1}{2}\sum_{\mathbf{i},\mathbf{j}}\left\langle c_{\alpha}^{\dagger}(\mathbf{i})c_{\beta}(\mathbf{j})\right\rangle V(\mathbf{i}-\mathbf{j})\left\langle c_{\alpha}(\mathbf{i})c_{\beta}^{\dagger}(\mathbf{j})\right\rangle\end{aligned} \tag{37}$$

Hatree $\frac{1}{2}\sum_{i,j}\rho(i)V(i-j)\rho(j)$, Fock :

$$\frac{1}{2N_s}\sum_{\mathbf{k},\mathbf{k}',\mathbf{q}}V_{\mathbf{q}}\left\langle c_{\alpha\mathbf{k}}^{\dagger}c_{\beta\mathbf{k}'+\mathbf{q}}\right\rangle\left\langle c_{\alpha\mathbf{k}-\mathbf{q}}c_{\beta\mathbf{k}'}^{\dagger}\right\rangle = -\frac{1}{2N_s}\sum_{\mathbf{k},\mathbf{q},\alpha}V_{\mathbf{q}}n_{\mathbf{k},\alpha}n_{\mathbf{k}-\mathbf{q},\alpha} + \frac{1}{2}V(0)N$$

Hatree

$$\begin{aligned}&\left\langle\Psi_{\{n_{\mathbf{k},\alpha}\}}|H|\Psi_{\{n_{\mathbf{k},\alpha}\}}\right\rangle \\ &= \sum_{\mathbf{k},\alpha}n_{\mathbf{k},\alpha}\left(\epsilon_{\mathbf{k}} - \mu + \frac{V_0}{2}\right) + \frac{V_0}{2N_s}\left(\sum_{\mathbf{k},\alpha}n_{\mathbf{k},\alpha}\right)^2 - \frac{1}{N_s}\sum_{\mathbf{k},\mathbf{q},\alpha}\frac{V_{\mathbf{q}}}{2}n_{\mathbf{k},\alpha}n_{\mathbf{k}-\mathbf{q},\alpha}\end{aligned} \tag{38}$$

$$
\begin{aligned}
n_{\mathbf{k},\alpha} = 1, \quad \epsilon_{\mathbf{k}} - \mu' + \sum_{\mathbf{k},\alpha} < 0 \\
n_{\mathbf{k},\alpha} = 0, \quad \epsilon_{\mathbf{k}} - \mu' + \sum_{\mathbf{k},\alpha} > 0
\end{aligned} \tag{39}
$$

$\sum_{\mathbf{k},\alpha} = -\frac{1}{N_s}\sum_{\mathbf{k},\alpha} V_{\mathbf{q}} n_{\mathbf{k}-\mathbf{q},\alpha}$ $\mu' = \mu - \rho_0 V_0 - \frac{1}{2}V(0)$ $\sum_{\mathbf{k},\alpha}$ k q
q k k

10.2

$n_{\mathbf{k},\alpha} + \delta n_{\mathbf{k},\alpha}$

$$
\delta E = \sum_{\mathbf{k},\alpha} \delta n_{\mathbf{k},\alpha} \left(\epsilon_{\mathbf{k}} - \mu' + \sum_{\mathbf{k},\alpha} \right) \tag{40}
$$

Hatree-Fock

-
-
- $\xi^*_{k,\alpha} = \epsilon_{\mathbf{k}} - \mu' + \sum_{\mathbf{k},\alpha}$ $\sum_{\mathbf{k},\alpha}$

$$
\sum_{k,\alpha} = -\frac{e^2 k_{F\alpha}}{\pi}\left(1 + \frac{1-y^2}{2y}\ln\left|\frac{1+y}{1-y}\right|\right), \quad y = \frac{k}{k_{F\alpha}} \tag{41}
$$

$y \to 1$ $v^*_{F\alpha}(k) = v_{F\alpha}(k) + \frac{\partial \sum_{k,\alpha}}{\partial k}$ Hatree—Fock

11

11.1

11.2

ref:AltlandP573

12 Product State

Ising

$$
H = -J\sum_{\langle ij\rangle} s_i s_j - h\sum_{i=1}^{N} s_i \tag{42}
$$

$$
\mathcal{Z}(T,h) = \sum_{\{s_i\}} e^{-\beta H} \equiv \sum_{s_1=\pm 1}\sum_{s_2=\pm 1}\cdots\sum_{s_N=\pm 1} \exp\left[\beta J\sum_{\langle ij\rangle} s_i s_j + \beta h\sum_i s_i\right] \tag{43}
$$

$$m = \langle s_i \rangle \equiv \frac{\sum_{\{s_j\}} e^{-H/T} s_i}{\sum_{\{s_j\}} e^{-H/T}} \tag{44}$$

$s_i = m + \delta s_i$

$$s_i s_j = m^2 + m\left(\delta s_i + \delta s_j\right) + \delta s_i \delta s_j = -m^2 + m\left(s_i + s_j\right) + \delta s_i \delta s_j \tag{45}$$

$$\begin{aligned} H_{\mathrm{MF}} &= \frac{m^2}{2} \sum_{ij} J_{ij} - \sum_i \left(h + \sum_j J_{ij} m \right) s_i \\ &= N \frac{zJ}{2} m^2 - \sum_i (h + zJm) s_i, \end{aligned} \tag{46}$$

N

$$\begin{aligned} \mathcal{Z}_{\mathrm{MF}}(T,h) &= e^{-\beta N z J m^2/2} \sum_{\{s_i\}} e^{\beta(h+zJm)\sum_i s_i} \\ &= e^{-\beta N z J m^2/2} \prod_i \left[e^{\beta(h+zJm)} + e^{-\beta(h+zJm)} \right] \\ &= e^{-\beta N z J m^2/2} [2\cosh[\beta(h+zJm)]]^N \\ &= e^{-\beta N \mathcal{L}_{\mathrm{MF}}(T,h;m)} \end{aligned} \tag{47}$$

m

$$\left. \frac{\partial \mathcal{L}_{\mathrm{MF}}(T,h;m)}{\partial m} \right|_{m_0} = 0. \tag{48}$$

Ising

$$\mathcal{Z} = \sum_{\{s_i\}} \exp\left[\frac{\beta}{2} \sum_{ij} J_{ij} s_i s_j + \beta h \sum_i s_i \right] = \sum_{\{s_i\}} \exp\left[\frac{1}{2} s^T \tilde{\mathbf{J}} \boldsymbol{s} + \tilde{\mathbf{h}}^T s \right] \tag{49}$$

$\left(\prod_{i=1}^N \int_{-\infty}^{\infty} \frac{dx_i}{\sqrt{2\pi}}\right) e^{-\frac{1}{2}\boldsymbol{x}^T \mathbf{A} \boldsymbol{x} + \boldsymbol{x}^T \boldsymbol{s}} = [\det \mathbf{A}]^{-1/2} e^{\frac{1}{2} s^T \mathbf{A}^{-1} \boldsymbol{s}} \qquad \int \mathcal{D}[x] \equiv \prod_{i=1}^N \int_{-\infty}^{\infty} \frac{dx_i}{\sqrt{2\pi}}$

$$\mathcal{Z} = \frac{\int \mathcal{D}[x] \exp\left[-\frac{1}{2}\boldsymbol{x}^T \tilde{\mathbf{J}}^{-1} \boldsymbol{x}\right] \sum_{\{s_i\}} \exp\left[(\tilde{\mathbf{h}} + \boldsymbol{x})^T \boldsymbol{s}\right]}{\int \mathcal{D}[x] \exp\left[-\frac{1}{2}\boldsymbol{x}^T \tilde{\mathbf{J}}^{-1} \boldsymbol{x}\right]} \tag{50}$$

x H-S

$$\begin{aligned} \sum_{\{s_i\}} \exp\left[(\tilde{\mathbf{h}} + \boldsymbol{x})^T \boldsymbol{s}\right] &= \prod_{i=1}^N \left[\sum_{s_i = \pm 1} e^{(\beta h + x_i) s_i} \right] = \prod_{i=1}^N \left[2\cosh\left(\beta h + x_i\right) \right] \\ &= \exp\left[\sum_{i=1}^N \ln\left[2\cosh\left(\beta h + x_i\right)\right] \right]. \end{aligned} \tag{51}$$

$$\mathcal{Z} = \frac{\int \mathcal{D}[x] e^{-\tilde{S}[x]}}{\int \mathcal{D}[x] \exp\left[-\frac{1}{2}\boldsymbol{x}^T \tilde{\mathbf{J}} - 1\boldsymbol{x}\right]} = \frac{1}{\sqrt{\det \tilde{\mathbf{J}}}} \int \mathcal{D}[x] e^{-\tilde{S}[x]} \tag{52}$$

$\tilde{S}[\boldsymbol{x}] = \frac{1}{2}\boldsymbol{x}^T\tilde{\mathbf{J}}^{-1}\boldsymbol{x} - \sum_{i=1}^{N}\ln\left[2\cosh\left(\beta h + x_i\right)\right]$ x

$$
\begin{aligned}
\langle x_i\rangle_{\tilde{S}} &= \lim_{\boldsymbol{y}\to 0}\frac{\partial}{\partial y_i}\frac{\int\mathcal{D}[x]\exp\left[-\frac{1}{2}\boldsymbol{x}^T\tilde{\mathbf{J}}^{-1}\boldsymbol{x}\right]\sum_{\{s_i\}}\exp\left[(\tilde{\mathbf{h}}+\boldsymbol{x})^T\boldsymbol{s}+\boldsymbol{x}^T\boldsymbol{y}\right]}{\int\mathcal{D}[x]e^{-\tilde{S}[x]}}\\
&= \lim_{\boldsymbol{y}\to 0}\frac{\partial}{\partial y_i}\frac{\sum_{\{s_i\}}\exp\left[\frac{1}{2}(\boldsymbol{s}+\boldsymbol{y})^T\tilde{\mathbf{J}}(\boldsymbol{s}+\boldsymbol{y})+\tilde{\mathbf{h}}^T\boldsymbol{s}\right]}{\sum_{\{s_i\}}\exp\left[\frac{1}{2}\boldsymbol{s}^T\tilde{\mathbf{J}}\boldsymbol{s}+\tilde{\mathbf{h}}^T\boldsymbol{s}\right]}\\
&= \frac{\sum_{\{s_i\}}e^{-\beta H}[\tilde{\mathbf{J}}]_i}{\sum_{\{s_i\}}e^{-\beta H}} = \left\langle[\tilde{\mathbf{J}}s]_i\right\rangle
\end{aligned}
\tag{53}
$$

$\langle\boldsymbol{x}\rangle_{\tilde{S}} = \tilde{\mathbf{J}}\langle\boldsymbol{s}\rangle$

$$
\varphi = \tilde{\mathbf{J}}^{-1}\boldsymbol{x}
\tag{54}
$$

[11]

$$
\mathcal{Z} = \frac{\int\mathcal{D}[\varphi]e^{-S[\varphi]}}{\int\mathcal{D}[\varphi]\exp\left[-\frac{1}{2}\boldsymbol{\varphi}^T\tilde{\mathbf{J}}\varphi\right]} = \sqrt{\det\tilde{\mathbf{J}}}\int\mathcal{D}[\varphi]e^{-S[\varphi]}
\tag{55}
$$

$$
S[\varphi] = \frac{\beta}{2}\sum_{ij}J_{ij}\varphi_i\varphi_j - \sum_{i=1}^{N}\ln\left[2\cosh\left[\beta\left(h+\sum_{j=1}^{N}J_{ij}\varphi_j\right)\right]\right]
\tag{56}
$$

ϕ ϕ

$$
\begin{aligned}
S[\varphi] =& -N\ln 2 + \frac{\beta}{2}\sum_{ij}J_{ij}\varphi_i\varphi_j - \frac{\beta^2}{2}\sum_i\left[h+\sum_j J_{ij}\varphi_j\right]^2\\
&+\frac{\beta^4}{12}\sum_i\left[h+\sum_j J_{ij}\varphi_j\right]^4 + \mathcal{O}\left(\varphi_i^6\right).
\end{aligned}
\tag{57}
$$

$$
\varphi_i = \frac{1}{\sqrt{N}}\sum_{\boldsymbol{k}}e^{i\boldsymbol{k}\cdot\boldsymbol{r}_i}\varphi_{\boldsymbol{k}}
\tag{58}
$$

k $0 < k\frac{2\pi}{a}$,a

$$
\begin{aligned}
\frac{\beta}{2}\sum_{ij}J_{ij}\varphi_i\varphi_j =& \frac{\beta}{2}\sum_{\boldsymbol{k}}J_{\boldsymbol{k}}\varphi_{-\boldsymbol{k}}\varphi_k,\\
\frac{\beta^2}{2}\sum_i\left[\sum_j J_{ij}\varphi_j\right]^2 =& \frac{\beta^2}{2}\sum_{\boldsymbol{k}}J_{-\boldsymbol{k}}J_{\boldsymbol{k}}\varphi_{-\boldsymbol{k}}\varphi_{\boldsymbol{k}},\\
\frac{\beta^4}{12}\sum_i\left[\sum_j J_{ij}\varphi_j\right]^4 =& \frac{\beta^4}{12N}\sum_{\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4}\delta_{\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3+\boldsymbol{k}_4,0}\\
&\times J_{\boldsymbol{k}_1}J_{\boldsymbol{k}_2}J_{\boldsymbol{k}_3}J_{\boldsymbol{k}_4}\varphi_{\boldsymbol{k}_1}\varphi_{\boldsymbol{k}_2}\varphi_{\boldsymbol{k}_3}\varphi_{\boldsymbol{k}_4},
\end{aligned}
\tag{59}
$$

$J_{\boldsymbol{k}} = \sum_i e^{-i\boldsymbol{k}\cdot\boldsymbol{r}_i}J\left(\boldsymbol{r}_i\right)$

$$
\begin{aligned}
S[\varphi] =& -N\ln 2 - \beta^2 J_{k=0}h\sqrt{N}\varphi_{\boldsymbol{k}=0} + \frac{\beta}{2}\sum_{\boldsymbol{k}}J_k\left(1-\beta J_{\boldsymbol{k}}\right)\varphi_{-\boldsymbol{k}}\varphi_{\boldsymbol{k}}\\
&+\frac{\beta^4}{12N}\sum_{\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4}\delta_{\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3+\boldsymbol{k}_4,0}J_{\boldsymbol{k}_1}J_{\boldsymbol{k}_2}J_{\boldsymbol{k}_3}J_{\boldsymbol{k}_4}\varphi_{\boldsymbol{k}_1}\varphi_{\boldsymbol{k}_2}\varphi_{\boldsymbol{k}_3}\varphi_{\boldsymbol{k}_4}\\
&+\mathcal{O}\left(\varphi_i^6, h^2, h\varphi_i^3\right).
\end{aligned}
\tag{60}
$$

[11] Jacobi

$J_k = J\left[z - \boldsymbol{k}^2 a^2\right] + \mathcal{O}\left(k^4\right) = T_c\left[1 - \frac{\boldsymbol{k}^2 a^2}{z}\right] + \mathcal{O}\left(k^4\right)$ $\beta J_{\boldsymbol{k}}\left(1 - \beta J_k\right) = a^2\left(r_0 + c_0 \boldsymbol{k}^2\right) + \mathcal{O}\left(k^4\right)$ $\varphi(\boldsymbol{k}) = a\sqrt{V}\varphi_{\boldsymbol{k}}$

$$
\begin{aligned}
S_{\Lambda_0}[\varphi] &= V f_0 - h_0 \varphi(\boldsymbol{k}=0) + \frac{1}{2}\int_{\boldsymbol{k}}\left[r_0 + c_0 \boldsymbol{k}^2\right]\varphi(-\boldsymbol{k})\varphi(\boldsymbol{k}) \\
&+ \frac{u_0}{4!}\int_{\boldsymbol{k}_1}\int_{\boldsymbol{k}_2}\int_{\boldsymbol{k}_3}\int_{\boldsymbol{k}_4}(2\pi)^D \delta\left(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3+\boldsymbol{k}_4\right)\varphi\left(\boldsymbol{k}_1\right)\varphi\left(\boldsymbol{k}_2\right)\varphi\left(\boldsymbol{k}_3\right)\varphi\left(\boldsymbol{k}_4\right)
\end{aligned}
\tag{61}
$$

Γ_0 Ginzberg-Landau-Wilson D Ising

$$
\varphi(\boldsymbol{r}) = \int_{\boldsymbol{k}} e^{i\boldsymbol{k}\cdot\boldsymbol{r}}\varphi(\boldsymbol{k}) \tag{62}
$$

$$
S_{\Lambda_0}[\varphi] = \int d^D r\left[f_0 + \frac{r_0}{2}\varphi^2(\boldsymbol{r}) + \frac{c_0}{2}[\nabla\varphi(\boldsymbol{r})]^2 + \frac{u_0}{4!}\varphi^4(\boldsymbol{r}) - h_0\varphi(\boldsymbol{r})\right] \tag{63}
$$

$$
\mathcal{Z} \approx \int_{-\infty}^{\infty}\frac{d\bar{\varphi}}{\sqrt{2\pi}} e^{-S_{\Lambda_0}[\bar{\varphi}]} \tag{64}
$$

$$
S_{\Lambda_0}[\bar{\varphi}] = V\left[f_0 + \frac{r_0}{2}\bar{\varphi}^2 + \frac{u_0}{4!}\bar{\varphi}^4 - h_0\bar{\varphi}\right] \tag{65}
$$

$$
\left.\frac{\partial S_{\Lambda_0}[\bar{\varphi}]}{\partial\bar{\varphi}}\right|_{\bar{\varphi}_0} = r_0\bar{\varphi}_0 + \frac{u_0}{6}\bar{\varphi}_0^3 - h_0 = 0 \tag{66}
$$

= +

Gaussian

$$
\varphi(\boldsymbol{r}) = \bar{\varphi}_0 + \delta\varphi(\boldsymbol{r}) \tag{67}
$$

$\varphi(\boldsymbol{k}) = (2\pi)^D\delta(\boldsymbol{k})\bar{\varphi}_0 + \delta\varphi(\boldsymbol{k})$

$$
\begin{aligned}
S_{\Lambda_0}\left[\bar{\varphi}_0 + \delta\varphi\right] \approx V&\left[f_0 + \frac{r_0}{2}\bar{\varphi}_0^2 + \frac{u_0}{4!}\bar{\varphi}_0^4\right] \\
&+ \left[r_0\bar{\varphi}_0 + \frac{u_0}{6}\bar{\varphi}_0^3\right]\delta\varphi(\boldsymbol{k}=0) \\
&+ \frac{1}{2}\int_{\boldsymbol{k}}\left[r_0 + \frac{u_0}{2}\bar{\varphi}_0^2 + c_0\boldsymbol{k}^2\right]\delta\varphi(-\boldsymbol{k})\delta\varphi(\boldsymbol{k}).
\end{aligned}
\tag{68}
$$

D>4

13 Kondo

Anderson

Anderson

$$
H = \sum_{\mathbf{k},\sigma}\epsilon_{\mathbf{k}} n_{\mathbf{k}\sigma} + \sum_{\mathbf{k},\sigma}\left[V(\mathbf{k}) c_{\mathbf{k}\sigma}^{\dagger} f_\sigma + V^*(\mathbf{k}) f_\sigma^{\dagger} c_{\mathbf{k}\sigma}\right] + \underbrace{E_f n_f + U n_f n_{f\downarrow}}_{H_{\text{atomic}}}, \tag{69}
$$

U $U = \frac{e^2}{4\pi\epsilon_0}\int_{\mathbf{r},\mathbf{r}'}\frac{1}{|\mathbf{r}-\mathbf{r}'|}\rho_f(\mathbf{r})\rho_f(\mathbf{r}')$ $f_\sigma^{\dagger} = \int_{\mathbf{r}}\Psi_f(\mathbf{r})\hat{\psi}_\sigma^{\dagger}(r)$ $\epsilon_{\mathbf{k}} \in [-D, D]$ $V(\mathbf{k}) = \langle\mathbf{k}\left|V_{ion}\right| f\rangle = \int d^3 r e^{-i\mathbf{k}\cdot\mathbf{r}} V_{ion}(r)\Psi_f(\vec{r})$

$U=0$ [12]

$$\Delta = \pi \sum_{\vec{k}} |V(\mathbf{k})|^2 \delta\left(\epsilon_{\mathbf{k}} - E_f\right) \tag{70}$$

$\rho(\epsilon) = \sum_{\mathbf{k}} \delta\left(\omega - \epsilon_{\mathbf{k}}\right)$

$$\Delta(\epsilon) = \pi \sum_{\vec{k}} |V(\mathbf{k})|^2 \delta\left(\epsilon_{\mathbf{k}} - \epsilon\right) = \pi \overline{\rho(\epsilon) V^2(\epsilon)} \tag{71}$$

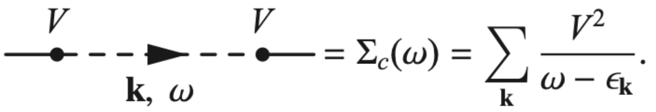


Figure 6: Kondo

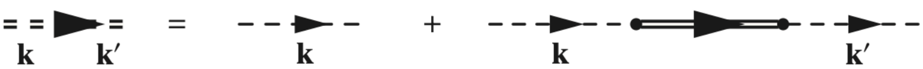
$$\Sigma_c(\omega) = \int \frac{d\epsilon}{\pi} \rho(\epsilon) \frac{\pi V^2}{\omega - \epsilon} = \int \frac{d\epsilon}{\pi} \frac{\Delta(\epsilon)}{\omega - \epsilon} \tag{72}$$

$$\operatorname{Im} \Sigma_c(\omega \pm i\delta) = \int \frac{d\epsilon}{\pi} \Delta(\epsilon) \operatorname{Im} \frac{1}{\omega - \epsilon \pm i\delta} = \mp \Delta(\omega) \tag{73}$$

$$\Sigma(\omega \pm i\delta) = \frac{\Delta}{\pi} \int_{-D}^{D} \frac{d\epsilon}{\omega - \epsilon \pm i\delta} = \frac{\Delta}{\pi} \ln\left[\frac{\omega \pm i\delta + D}{\omega \pm i\delta - D}\right] \tag{74}$$

$\omega = \pm D$ D ( ) $\Sigma_c\left(\omega + i\omega'\right) = -i\Delta\,\mathrm{sgn}\left(\omega'\right)$ [13]

$t(\omega) = V^2 G_f(\omega)$ S $S = 1 - 2\pi i \rho t(\omega + i\eta)$ S $S(\omega) = e^{2i\delta(\omega)}$



$$G(\mathbf{k}', \mathbf{k}, \omega) = \delta_{\mathbf{k}', \mathbf{k}} G^{(0)}(\mathbf{k}, \omega) + G^{(0)}(\mathbf{k}, \omega) V^2 G_f(\omega) G^{(0)}(\mathbf{k}', \omega).$$

Figure 7:

$$\delta_f(\omega) = \cot^{-1}\left(\frac{E_f - \omega}{\Delta}\right) = \tan^{-1}\left(\frac{\Delta}{E_f - \omega}\right) \tag{75}$$

f [14]

$$n_f = 2 \int_{-\infty}^{0} d\omega \rho_f(\omega) = 2 \int_{-\infty}^{0} \frac{d\omega}{\pi} \frac{\Delta}{\left(\omega - E_f\right)^2 + \Delta^2} = \frac{2}{\pi} \cot^{-1}\left(\frac{E_f}{\Delta}\right) \equiv 2 \times \frac{\delta_f}{\pi} \tag{76}$$

Friedel

$$\Delta n = \sum_{\lambda} \frac{\delta_\lambda}{\pi} \tag{77}$$

---

[12] f

[13]

[14] F-D

$$Un_\uparrow n_\downarrow \to Un_\uparrow \langle n_\downarrow \rangle + U \langle n_\uparrow \rangle n_\downarrow - U \langle n_\uparrow \rangle \langle n_\downarrow \rangle + O\left(\delta n^2\right). \tag{78}$$

f

$$E_f \to E_{f\sigma} = E_f + U \langle n_{f-\sigma} \rangle \tag{79}$$

Friedel (**??**)

$$\langle n_{f\sigma} \rangle = \frac{\delta_{f\sigma}}{\pi} = \frac{1}{\pi} \cot^{-1}\left(\frac{E_f + U \langle n_{f-\sigma} \rangle}{\Delta}\right) \tag{80}$$

$n_f = \sum_\sigma \langle n_{f\sigma} \rangle, M = \langle n_{f\uparrow} \rangle - \langle n_{f\downarrow} \rangle$

$$\begin{aligned} n_f &= \frac{1}{\pi} \sum_{\sigma=\pm 1} \cot^{-1}\left(\frac{E_f + U/2\,(n_f - \sigma M)}{\Delta}\right) \\ M &= \frac{1}{\pi} \sum_{\sigma=\pm 1} \sigma \cot^{-1}\left(\frac{E_f + U/2\,(n_f - \sigma M)}{\Delta}\right). \end{aligned} \tag{81}$$

$M \to 0^+$ $U_c = \pi\Delta$.

Kondo

$$\Sigma_I(\omega - i\eta) = \Sigma_I(0) + \left(1 - Z^{-1}\right)\omega + iA\omega^2 \tag{82}$$

f

$$G_f(\omega - i\eta) = \frac{Z}{\omega - E_f^* - i\Delta^* - iO\left(\omega^2\right)} \tag{83}$$

$A_f(0) = \frac{1}{\pi} \operatorname{Im} G_f(0 - i\eta) = \frac{\sin^2 \delta_f}{\pi\Delta}$

Kondo $U \sim 10eV$ Kondo 10meV $H(D)$ D——— $D \to D' = D/b$ $E \in [D', D]$ $\tilde{H}_L$ $H(D') = b\tilde{H}_L$

$$H = \left[\begin{array}{c|c} \frac{H_L}{V} & \frac{V^\dagger}{H_H} \end{array}\right] \tag{84}$$

“ ”

$$H(D) \to \tilde{H} = UH(D)U^\dagger = \left[\begin{array}{c|c} \frac{\tilde{H}_L}{0} & \frac{0}{\tilde{H}_H} \end{array}\right] \tag{85}$$

$\tilde{H}_L = P\tilde{H}P$

$$H(D') = b\tilde{H}_L \tag{86}$$

$b \to 1$

$$\frac{\partial g_j}{\partial \ln D} = \beta_j\left(\{g_i\}\right) \tag{87}$$

1. D
2.

Hilbert

$$D < E_f + U: \quad \left|f^0\right\rangle, \quad \left|f^1, \sigma\right\rangle \quad \left(\sigma = \pm\frac{1}{2}\right) \tag{88}$$

Hubbad

$$\begin{aligned} X_{\sigma 0} &= \left|f^1,\sigma\right\rangle\left\langle f^0\right| = Pf_\sigma^\dagger, \quad X_{0\sigma} = \left|f^0\right\rangle\left\langle f^1,\sigma\right| = f_\sigma^\dagger P, \\ X_{\sigma\sigma'} &= \left|f^1,\sigma\right\rangle\left\langle f^1,\sigma'\right|, \end{aligned} \tag{89}$$

Infinite U Anderson model

$$H = \sum_{\mathbf{k},\sigma} \epsilon_{\mathbf{k}} n_{\mathbf{k}\sigma} + \left[V(\mathbf{k}) c_{\mathbf{k}\sigma}^\dagger X_{0\sigma} + V(\mathbf{k})^* X_{\sigma 0} c_{\mathbf{k}\sigma}\right] + E_f \sum_\sigma X_{\sigma\sigma} \tag{90}$$

D Hilbert spin 1/2

$$\left|f^1,\sigma\right\rangle \quad \left(\sigma = \pm\frac{1}{2}\right) \tag{91}$$

f Kondo [15]

$$H = \sum_{k\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + J\psi^\dagger(0)\vec{\sigma}\psi(0)\cdot\vec{S}_f \tag{92}$$

Schrieffer–Wolff

Schrieffer–Wolff Anderson Kondo

$$H = H_1 + \lambda\mathcal{V} \tag{93}$$

λ H_1 f^1 f^2, f^0

$$H_1 = H_{\text{band}} + H_{atomic} = \left[\frac{H_L}{0} \mid \frac{0}{H_H}\right] \tag{94}$$

$$\mathcal{V} = H_{mix} = \sum_{\mathbf{k}\sigma}\left[V_{\vec{k}} c_{k\sigma}^\dagger f_\sigma + \text{ H.c. }\right] = \left[\frac{0}{V} \mid \frac{V^\dagger}{0}\right] \tag{95}$$

Schrieffer–Wolff

$$\mathcal{U}\left[\frac{H_L}{\lambda V} \mid \frac{\lambda V^\dagger}{H_H}\right]\mathcal{U}^\dagger = \left[\frac{H^*}{0} \mid \frac{0}{H'}\right] \tag{96}$$

$\mathcal{U} = e^S$ S S λ

$$S = \lambda S_1 + \lambda^2 S_2 + \cdots \tag{97}$$

Baker–Campbell–Hausdorff

$$e^A B e^{-A} = B + [A,B] + \frac{1}{2!}[A,[A,B]] + \cdots \tag{98}$$

(??)

$$e^S\left(H_1 + \lambda\mathcal{V}\right)e^{-S} = H_1 + \lambda\left(\mathcal{V} + [S_1, H_1]\right) + \lambda^2\left(\frac{1}{2}\left[S_1,[S_1,H]\right] + [S_1,\mathcal{V}] + [S_2,H_1]\right) + \cdots \tag{99}$$

$\mathcal{V}$ $[S_1, H_1] = -\mathcal{V}$[16]

$$e^S\left(H_1 + \lambda\mathcal{V}\right)e^{-S} = H_1 + \lambda^2\left(\frac{1}{2}\left[S_1,\mathcal{V}\right] + [S_2,H_1]\right) + \cdots \tag{100}$$

$S_2 = 0$

$$H^* = H_L + \lambda^2\Delta H \tag{101}$$

[15] $\begin{aligned} e_\uparrow + f_\downarrow^1 &\leftrightarrow f^2 \leftrightarrow e_\downarrow + f_\uparrow^1 \\ e_\uparrow + f_\downarrow^1 &\leftrightarrow e_\uparrow + e_\downarrow \leftrightarrow e_\downarrow + f_\uparrow^1 \end{aligned}$ $\epsilon_k \sim \epsilon_F = 0$ $J_{\text{eff}} = -\left|V_{\mathbf{k}d}\right|^2 \frac{U}{|\epsilon_f|(U-|\epsilon_f|)} < 0$

[16] S_1 $[S_1, \mathcal{V}]$

$\Delta H = \frac{1}{2} P_L [S_1, \mathcal{V}] P_L + \cdots$

$$S = \left[\begin{array}{c|c} 0 & -s^\dagger \\ \hline s & 0 \end{array}\right] \tag{102}$$

$V = -sH_L + H_H s$

$$s_{ab} = \frac{V_{ab}}{E_a^H - E_b^L}, \quad -s_{ab}^\dagger = \frac{V_{ab}^\dagger}{E_a^L - E_b^H}, \tag{103}$$

$$S = \sum_{H,L} \left(|H\rangle \frac{\langle H|V|L\rangle}{E_H - E_L} \langle L| - \text{ H.c. } \right) + O\left(V^3\right) \tag{104}$$

$$\Delta H_{LL'} = -\frac{1}{2}\left(V^\dagger s + s^\dagger V\right)_{LL'} = -\frac{1}{2}\sum_H \left(V_{LH}^\dagger V_{HL'}\right)\left[\frac{1}{E_H - E_L} + \frac{1}{E_H - E_{L'}}\right] \tag{105}$$

T $\Delta H_{LL'} = \frac{1}{2}\left[T\left(E_L\right) + T\left(E_{L'}\right)\right],$

$$\begin{aligned} \hat{T}(E) &= P_L \mathcal{V} \frac{P_H}{E - H_1} \mathcal{V} P_L \\ T_{LL'}(E) &= \sum_{|H\rangle} \left[\frac{V_{LH}^\dagger V_{HL'}}{E - E_H}\right] \end{aligned} \tag{106}$$

Anderson $\rightarrow$Kondo $\Delta H = T\left(E_L\right) = -\frac{1}{\Delta E_{HL}}\left(\mathcal{V} P_H \mathcal{V}\right)$

$$\begin{aligned} \Delta H &= -\frac{VP\left[f^2\right]V}{E_f + U} - \frac{VP\left[f^0\right]V}{-E_f} \\ &= -\sum_{k\alpha, k'\beta} V_{k'}^* V_k [\frac{\overbrace{\left(c_{k\alpha}^\dagger f_\alpha\right)\left(f_\beta^\dagger c_{k'\beta}\right)}^{E_f+U}}{E_f + e^- \leftrightarrow f^2} + \overbrace{\frac{f_\beta^\dagger c_{k'\beta}\left(c_{k\alpha}^\dagger f_\alpha\right)}{-E_f}}^{f^1 \leftrightarrow f^0 + e^-}\Bigg] P_{n_f=1}, \end{aligned} \tag{107}$$

$P_{n_f=1} = \left(n_{f\uparrow} - n_{f\downarrow}\right)^2$ Fierz $2\delta_{\alpha\gamma}\delta_{\eta\beta} = \delta_{\alpha\beta}\delta_{\eta\gamma} + \vec{\sigma}_{\alpha\beta}\cdot\vec{\sigma}_{\eta\gamma}$

$$\begin{aligned} \left(c_{k\alpha}^\dagger f_\alpha\right)\left(f_\beta^\dagger c_{k'\beta}\right) &= \left(c_{k\alpha}^\dagger f_\gamma\right)\left(f_\eta^\dagger c_{k'\beta}\right) \times \overbrace{\left(\delta_{\alpha\gamma}\delta_{\eta\beta}\right)}^{\frac{1}{2}\left(\delta_{\alpha\beta}\delta_{\eta\gamma} + \vec{\sigma}_{\alpha\beta}\cdot\vec{\sigma}_{\eta\gamma}\right)} \\ &= \frac{1}{2} c_{k\alpha}^\dagger c_{k'\alpha} - \left(c_{k\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{k'\beta}\right)\cdot\vec{S}_f \end{aligned} \tag{108}$$

$n_f = 1$ $\vec{S}_f \equiv f_\sigma^\dagger\left(\frac{\vec{\sigma}_{\alpha\beta}}{2}\right) f_\beta,$ $(n_f = 1)$ f $f_\eta, f_\gamma^\dagger = \delta(\eta - \gamma)$

$$\Delta H = \sum_{k\alpha, k'\beta} J_{k,k'} c_{k\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{k'\beta} \cdot \vec{S}_f + H' \tag{109}$$

Konbo

$$J_{k,k'} = V_{k'}^* V_k \left[\frac{1}{E_f + U} + \frac{1}{-E_f}\right] \tag{110}$$

(??)

$$H' = -\frac{1}{2}\sum_{k,k'\sigma} V_{k'}^* V_k \left[\frac{1}{E_f + U} + \frac{1}{E_f}\right] c_{k\sigma}^\dagger c_{k'\sigma} \tag{111}$$

Anderson

$$H = \sum_{k\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + \sum_{k,k'} J_{k,k'} c_{k\alpha}^\dagger \vec{\sigma} c_{k'\beta} \cdot \vec{S}_f \tag{112}$$

Kondo

$$H = \sum_{k\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + J\vec{\sigma}(0)\cdot\vec{S}_f \tag{113}$$

$\vec{\sigma}(0) = \psi^\dagger(0)\vec{\sigma}\psi(0)$ $\psi_\alpha(0) = \sum_k c_{k\alpha}$

“ ”

Anderson “ ” Kondo

$$H=\sum_{|\epsilon_k|<D}\epsilon_k c_{k\sigma}^{\dagger}c_{k\sigma}+H^{(I)} \tag{114}$$

T Schrieffer–Wolff T

$$T_{ab}(E)=\sum_{\lambda\in|H\rangle}\left[\frac{H_{a\lambda}^{(I)}H_{\lambda b}^{(I)}}{E-E_{\lambda}^{H}}\right] \tag{115}$$

$\lambda\in[D',D]$ T

$$\begin{aligned}\delta H_{k'\beta\sigma';k\alpha\sigma}^{int}=\hat{T}^{(I)}+\hat{T}^{(I)}&=-\frac{J^2\rho|\delta D|}{D}\left[\sigma^a,\sigma^b\right]_{\beta\alpha}S^aS^b\\&=-\frac{1}{2}\frac{J^2\rho|\delta D|}{D}\overbrace{\left[\sigma^a,\sigma^b\right]_{\beta\alpha}}^{2i\epsilon^{abc}\sigma^c}\overbrace{\left[S^a,S^b\right]}^{ibdS^d}\\&=\frac{J^2\rho|\delta D|}{D}\overbrace{\epsilon^{abc}\epsilon^{abd}\sigma_{\beta d}}^{\sigma_{\beta\alpha}^{c}S^d}\\&=2\frac{J^2\rho|\delta D|}{D}\vec{\sigma}_{\beta\alpha}\cdot\vec{S}_{\sigma'\sigma}.\end{aligned} \tag{116}$$

$$\frac{\partial J\rho}{\partial\ln D}=-2(J\rho)^2 \tag{117}$$

$g\left(D'\right)=\frac{g_0}{1-2g_0\ln\frac{D_0}{D'}}$

$$g\left(D'\right)=\frac{g_o}{1-2g_o\ln\left(D/D'\right)}=\frac{1}{2}\frac{1}{\ln\left(D'/D_0\right)+\frac{1}{2g_0}}=\frac{1}{2}\frac{1}{\ln\left[\frac{D'}{D_0\exp(-1/(2g_0))}\right]} \tag{118}$$

$D'=T_K=T_K=D_0\exp\left[-\frac{1}{2g_o}\right]$ Kondo Kondo D_0 Kondo

Kondo Wick

$$\vec{S}=f_{\alpha}^{\dagger}\left(\frac{\vec{\sigma}}{2}\right)_{\alpha\beta}f_{\beta} \tag{119}$$

Hilbert $n_f=1$ n_f f / n_f

$$\mu=-i\pi\frac{T}{2} \tag{120}$$

$$Z=\mathrm{Tr}\left[e^{-\beta\left(H+i\pi\frac{T}{2}(n_f-1)\right)}\right] \tag{121}$$

n_f Trace

$$Z=e^{i\pi/2}Z\left(f^0\right)+Z\left(f^1\right)+e^{-i\pi/2}Z\left(f^2\right) \tag{122}$$

[17] f

$$\mathcal{G}_f\left(i\tilde{\omega}_n\right)=\frac{1}{i\omega_n+\mu}=\frac{1}{i\omega_n-i\pi T/2}=\frac{1}{i2\pi T\left(n+\frac{1}{4}\right)} \tag{123}$$

Kondo Kondo Kondo

17

14

14.1

$$\hat{H}_{\mathrm{F}}=\int d^d r F_i'(\mathbf{r},t)\hat{X}_i'(\mathbf{r}) \tag{124}$$

F_i' $\hat{X}_i$ $\hat{X}_i, F_i'$

$$X_i(\mathbf{r},t)=\int d^d r'\int dt'\chi_{ij}\left(\mathbf{r},t;\mathbf{r}',t'\right)F_j'\left(\mathbf{r}',t'\right)+\mathcal{O}\left(F^2\right) \tag{125}$$

χ χ

- $\chi_{ij}\left(\mathbf{r},\mathbf{r}';t,t'\right)=0, t<t'$
-
$$X_i(\mathbf{r},\omega)=\int d^d r'\chi_{ij}\left(\mathbf{r},\mathbf{r}';\omega\right)F_j'\left(\mathbf{r}',\omega\right)+\mathcal{O}\left(F^2\right) \tag{126}$$
“ ” “ ” “ ”
-
$$X_i(\mathbf{q},\omega)=\chi_{ij}(\mathbf{q};\omega)F_j'(\mathbf{q},\omega)+\mathcal{O}\left(F'^2\right) \tag{127}$$

$X(t)$ $\hat{X}=\sum_{aa'}c_a^\dagger X_{aa'}c_{a'}$

$$X(\tau)=\sum_{aa'}\left\langle\bar{\psi}_a(\tau)X_{aa'}\psi_{a'}(\tau)\right\rangle \tag{128}$$

$$\delta S'\left[F',\bar{\psi},\psi\right]=\int d\tau\hat{H}_{F'}=\int d\tau F'(\tau)\sum\bar{\psi}_a(\tau)X_{aa'}'\psi_{a'}(\tau) \tag{129}$$

$X(\tau)$ QFT X

$$\delta S[F,\bar{\psi},\psi]\equiv\int d\tau F(\tau)\hat{X}(\tau)=\int d\tau F(\tau)\sum_{aa'}\bar{\psi}_a(\tau)X_{aa'}\psi_{a'}(\tau) \tag{130}$$

$$X(\tau)=-\left.\frac{\delta}{\delta F(\tau)}\right|_{F=0}\ln\mathcal{Z}\left[F,F'\right] \tag{131}$$

$G\left[F'\right]\simeq G[0]+\int d\tau'\left.\frac{\delta G\left[F'\right]}{\delta F'\left(\tau'\right)}\right|_{F'=0}F'\left(\tau'\right),$

$$X(\tau)\simeq-\int d\tau'\left(\left.\frac{\delta^2}{\delta F(\tau)\delta F'\left(\tau'\right)}\right|_{F=F'=0}\ln\mathcal{Z}\left[F,F'\right]\right)F'\left(\tau'\right) \tag{132}$$

(??),

$$\chi\left(\tau,\tau'\right)=-\left.\frac{\delta^2}{\delta F(\tau)\delta F'\left(\tau'\right)}\right|_{F=F'=0}\ln\mathcal{Z}\left[F,F'\right] \tag{133}$$

$\langle\hat{X}(\tau)\rangle_{F'=0}$

$$\chi\left(\tau,\tau'\right)=-\mathcal{Z}^{-1}\left.\frac{\delta^2}{\delta F(\tau)\delta F'\left(\tau'\right)}\right|_{F=F'=0}\mathcal{Z}\left[F,F'\right] \tag{134}$$

14.2

$F'(t)$ $X(t)$ $\tau \to it$

$$C^{\tau}_{X_1X_2}(\tau_1-\tau_2) \equiv -\left\langle T_\tau \hat{X}_1(\tau_1)\hat{X}_2(\tau_2)\right\rangle \equiv -\begin{cases}\left\langle \hat{X}_1(\tau_1)\hat{X}_2(\tau_2)\right\rangle, & \tau_1 \geq \tau_2 \\ \zeta_{\hat{X}}\left\langle \hat{X}_2(\tau_2)\hat{X}_1(\tau_1)\right\rangle, & \tau_2 > \tau_1\end{cases} \tag{135}$$

$$\hat{X}(\tau) \equiv e^{\tau(\hat{H}-\mu\hat{N})}\hat{X}e^{-\tau(\hat{H}-\mu\hat{N})} \tag{136}$$

$$C^{T}_{X_1X_2}(t_1-t_2) = -i\left\langle T_t \hat{X}_1(t_1)\hat{X}_2(t_2)\right\rangle \tag{137}$$

$$C^{+}_{X_1X_2}(t_1-t_2) = -i\Theta(t_1-t_2)\left\langle\left[\hat{X}_1(t_1),\hat{X}_2(t_2)\right]_{\zeta_X}\right\rangle \tag{138}$$

$$X(t) = \left\langle \hat{X}^{F'}(t)\right\rangle$$

$$X(t) = -i\int dt'\theta(t-t')F'(t')\quad\left[\hat{X}(t),\hat{X}'(t')\right] = \int dt' C^{+}_{XX'}(t-t')F'(t') \tag{139}$$

$$C^{-}_{X_1X_2}(t_1-t_2) = +i\Theta(t_2-t_1)\left\langle\left[\hat{X}_1(t_1),\hat{X}_2(t_2)\right]_{\zeta_X}\right\rangle \tag{140}$$

Lehmann $\{|\Psi_\alpha\rangle\}$

$$C^{T}(t) = -i\mathcal{Z}^{-1}\sum X_{1\alpha\beta}X_{2\beta\alpha}e^{it\Xi_{\alpha\beta}}\left(\Theta(t)e^{-\beta\Xi_\alpha}+\zeta_X\Theta(-t)e^{-\beta\Xi_\beta}\right) \tag{141}$$

$\Xi_\alpha \equiv E_\alpha - \mu N_\alpha, \Xi_{\alpha\beta} \equiv \Xi_\alpha - \Xi_\beta, X_{\alpha\beta} \equiv \left\langle \Psi_\alpha|\hat{X}|\Psi_\beta\right\rangle$

$$\left.\begin{matrix}C^T(\omega)\\ C^+(\omega)\\ C^-(\omega)\end{matrix}\right\} = \mathcal{Z}^{-1}\sum_{\alpha\beta}X_{1\alpha\beta}X_{2\beta\alpha}\left[\frac{e^{-\beta\Xi_\alpha}}{\omega+\Xi_{\alpha\beta}\left\{\begin{matrix}+\\+\\-\end{matrix}\right\}i\eta} - \zeta_{\hat{X}}\frac{e^{-\beta\Xi_\beta}}{\omega+\Xi_{\alpha\beta}\left\{\begin{matrix}-\\+\\-\end{matrix}\right\}i\eta}\right] \tag{142}$$

-
$$\mathrm{Re}\, C^T(\omega) = \mathrm{Re}\, C^+(\omega) = \mathrm{Re}\, C^-(\omega) \tag{143}$$

-
$$\mathrm{Im}\, C^T(\omega) = \pm\,\mathrm{Im}\, C^{\pm}(\omega)\times\begin{cases}\coth\frac{\beta\omega}{2}, & \text{bosons}\\ \tanh\frac{\beta\omega}{2}, & \text{fermions}\end{cases} \tag{144}$$

c^τ:

$$C^{\tau}(\tau) = -\mathcal{Z}^{-1}\sum_{\alpha\beta}X_{1\alpha\beta}X_{2\beta\alpha}e^{\Xi_{\alpha\beta}\tau}\left(\Theta(\tau)e^{-\beta\Xi_\alpha}+\zeta_{\hat{X}}\Theta(-\tau)e^{-\beta\Xi_\beta}\right) \tag{145}$$

$\hat{X}$ $C^\tau(\tau) = \zeta_{\hat{X}}C^\tau(\tau+\beta), \quad \tau<0$ $C^\tau(i\omega_n) = \int_0^\beta d\tau C^\tau(\tau)e^{i\omega_n\tau}$ Lehmann

$$C^{\tau}(i\omega_n) = \mathcal{Z}^{-1}\sum_{\alpha\beta}\frac{X_{1\alpha\beta}X_{2\beta\alpha}}{i\omega_n+\Xi_{\alpha\beta}}\left[e^{-\beta\Xi_\alpha}-\zeta_X e^{-\beta\Xi_\beta}\right] \tag{146}$$

$$C(z) = \mathcal{Z}^{-1}\sum_{\alpha\beta}\frac{X_{1\alpha\beta}X_{2\beta\alpha}}{z+\Xi_{\alpha\beta}}\left[e^{-\beta\Xi_\alpha}-\zeta_X e^{-\beta\Xi_\beta}\right] \tag{147}$$

$z = \omega^+, \omega^-, i\omega_n$ C^+, C^-, C^τ $C^\tau(i\omega_n)$ $i\omega_n \to \omega + i0$ C^+

[18]

$$C(z)|_{\substack{\hat{X}_1 = c_a \\ \hat{X}_2 = c_a^\dagger}} \equiv G_a(z) = \frac{1}{z - \xi_a} \tag{148}$$

$\Sigma(z) : G_a(\omega_n) \to (i\omega_n - \xi_a - \Sigma(i\omega_n))^{-1}$

$$G_a^+(\omega) = \frac{1}{\omega^+ - \xi_a - \Sigma(\omega^+)} \tag{149}$$

(??)

$$\operatorname{Re}\Sigma\left(\omega^+\right) = +\operatorname{Re}\Sigma\left(\omega^-\right), \quad \operatorname{Im}\Sigma\left(\omega^+\right) = -\operatorname{Im}\Sigma\left(\omega^-\right) < 0 \tag{150}$$

$\Sigma(z)$

14.3

Lehmann

$$A(\omega) \equiv -2\operatorname{Im} C^+(\omega) \tag{151}$$

Lehmann

$$A(\omega) = 2\pi \mathcal{Z}^{-1} \sum_{\alpha\beta} X_{1\alpha\beta} X_{2\beta\alpha} \left[e^{-\beta\Xi_\alpha} - \zeta_{\hat{X}} e^{-\beta\Xi_\beta}\right] \delta\left(\omega + \Xi_{\alpha\beta}\right) \tag{152}$$

a $A_a(\omega) = 2\pi\delta(\omega - \xi_a)$, δ $c_a^\dagger|\alpha\rangle$ $|\alpha\rangle$ $|a\rangle$ β $c_a^\dagger|\alpha\rangle$ δ N+1

$$\begin{aligned}
\int \frac{d\omega}{2\pi} A_a(\omega) &= \mathcal{Z}^{-1} \sum_{\alpha\beta} c_{a\alpha\beta} c_{a\beta\alpha}^\dagger \left[e^{-\beta\Xi_\alpha} - \zeta_c e^{-\beta\Xi_\beta}\right] \\
&= \mathcal{Z}^{-1}\left(\sum_\alpha \left\langle \alpha \left| c_a c_a^\dagger \right| \alpha \right\rangle e^{-\beta\Xi_\alpha} - \zeta_c \sum_\beta \left\langle \beta \left| c_a^\dagger c_a \right| \beta \right\rangle e^{-\beta\Xi_\beta}\right) \\
&= \mathcal{Z}^{-1} \sum_\alpha e^{-\beta\Xi_\alpha} \left\langle \alpha | \underbrace{c_a c_a^\dagger - \zeta_c c_a^\dagger c_a}_{\left[c_a, c_a^\dagger\right]_{\zeta_c} = 1} | \alpha \right\rangle = \underbrace{\mathcal{Z}^{-1} \sum_\alpha e^{-\beta\Xi_\alpha}}_{1} = 1
\end{aligned}$$

$$\begin{aligned}
\int \frac{d\omega}{2\pi} n_{\mathrm{F/B}}(\omega) A_a(\omega) &= \mathcal{Z}^{-1} \sum_{\alpha\beta} c_{a\alpha\beta} c_{a\beta\alpha}^\dagger \left[e^{-\beta\Xi_\alpha} - \zeta_c e^{-\beta\Xi_\beta}\right] \int d\omega \delta\left(\omega + \Xi_{\alpha\beta}\right) \frac{1}{e^{\beta\omega} - \zeta_c} \\
&= \mathcal{Z}^{-1} \sum_{\alpha\beta} c_{a\alpha\beta} c_{a\beta\alpha}^\dagger e^{-\beta\Xi_\beta} \left[e^{\beta\Xi_{\beta\alpha}} - \zeta_c\right] \frac{1}{e^{\beta\Xi_{\beta\alpha}} - \zeta_c} \\
&= \mathcal{Z}^{-1} \sum_\beta e^{-\beta\Xi_\beta} \left\langle \beta \left| c_a^\dagger c_a \right| \beta \right\rangle = \langle \hat{n}_a \rangle
\end{aligned}$$

$c_a^\dagger|\alpha\rangle$ N+1 $|\beta\rangle$ ω $|\alpha\rangle$ $|\beta\rangle$

$$:A(\omega) = i\left(C^+(\omega) - C^-(\omega)\right)$$

$$C(z) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{A(\omega)}{z - \omega} \tag{153}$$

[18] $\sum_\alpha n_a e^{-\beta\Xi_\alpha} = < n_a > = \frac{1}{e^{\beta\xi_a} \pm 1}$,

Justify $C^{\pm}$ ( ) $|z| \rightarrow$ inf ω^{-1} $Imz>0$ c^{+} [19]

$$\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{A(\omega)}{z-\omega} \stackrel{\operatorname{Im} z>0}{=} -\frac{1}{2\pi i} \int_{\gamma} d\omega \frac{C^{+}(\omega)}{z-\omega}=C(z) \tag{154}$$

$z=\omega^{+}$

$$C^{+}(\omega)=-\frac{1}{2\pi i} \int d\omega' \frac{C^{+}\left(\omega'\right)}{\omega-\omega'+i0} \tag{155}$$

Dirac $\lim_{\eta \searrow 0} \frac{1}{x \pm i\eta}=\mp i\pi\delta(x)+P\frac{1}{x}$

$$C^{+}(\omega)=\frac{1}{\pi i} \int d\omega' C^{+}\left(\omega'\right) \mathrm{P} \frac{1}{\omega'-\omega} \tag{156}$$

$$\begin{aligned} \operatorname{Re} C^{+}(\omega) &=\frac{1}{\pi} \int d\omega' \operatorname{Im} C^{+}\left(\omega'\right) \mathrm{P} \frac{1}{\omega'-\omega} \\ \operatorname{Im} C^{+}(\omega) &=-\frac{1}{\pi} \int d\omega' \operatorname{Re} C^{+}\left(\omega'\right) \mathrm{P} \frac{1}{\omega'-\omega} \end{aligned} \tag{157}$$

14.4

Altland Coleman

$$S\left(t-t'\right)=\left\langle A(t) A\left(t'\right)\right\rangle=\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega\left(t-t'\right)} S(\omega) \tag{158}$$

$$\begin{aligned} S(\omega) &=\int_{-\infty}^{\infty} dt e^{i\omega t} S(t) \\ &=\sum_{\lambda, \zeta} e^{-\beta\left(E_{\lambda}-F\right)}|\langle\zeta|A|\lambda\rangle|^{2} 2\pi\delta\left(E_{\zeta}-E_{\lambda}-\omega\right) \end{aligned} \tag{159}$$

$$\begin{aligned} \chi_{R}\left(t-t'\right) &=i\left\langle\left[A(t), A\left(t'\right)\right]\right\rangle \theta\left(t-t'\right) \\ &=i \sum_{\lambda, \zeta} e^{-\beta\left(E_{\lambda}-F\right)}\left\{\langle\lambda|A(t)|\zeta\rangle\left\langle\zeta\left|A\left(t'\right)\right| \lambda\right\rangle-\left\langle\lambda\left|A\left(t'\right)\right| \zeta\right\rangle\langle\zeta|A(t)|\lambda\rangle\right\} \theta\left(t-t'\right) \\ &=i \sum_{\lambda, \zeta} e^{\beta F}\left(e^{-\beta E_{\lambda}}-e^{-\beta E_{\zeta}}\right)|\langle\zeta|A|\lambda\rangle|^{2} e^{-i\left(E_{\zeta}-E_{\lambda}\right)\left(t-t'\right)} \theta\left(t-t'\right) \end{aligned} \tag{160}$$

theta

$$\chi_{R}(t)=i \int \frac{d\omega}{\pi} e^{-i\omega t} \theta(t) \chi''(\omega) \tag{161}$$

$$\chi''(\omega)=\pi\left(1-e^{-\beta\omega}\right) \sum_{\lambda, \zeta} p_{\lambda}|\langle\zeta|A|\lambda\rangle|^{2} \delta\left[\omega-\left(E_{\zeta}-E_{\lambda}\right)\right] \tag{162}$$

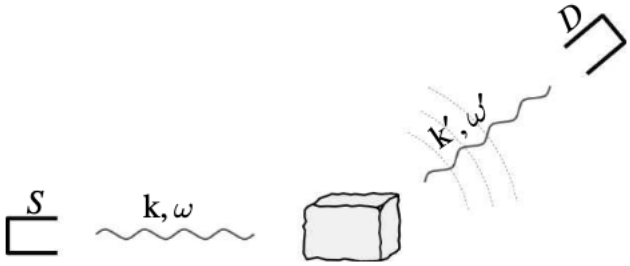
theta

$$\chi(z)=\int \frac{d\omega'}{\pi} \frac{1}{\omega'-z} \chi''\left(\omega'\right) \tag{163}$$

(??) $\chi''=A(\omega)$ **(??) (??)**

$$S(\omega)=\frac{2\hbar}{1-e^{-\beta\hbar\omega}} \chi''(\omega)=2\hbar\left[1+n_{B}(\hbar\omega)\right] \chi''(\omega). \tag{164}$$

$n_{B}(\hbar\omega) \rightarrow n_{F}(\hbar\omega)$ [20]



14.5

Hilbet Fock Hilbert $\mathcal{H}=\mathcal{F}\otimes\mathcal{H}_1$ $\hat{H}_{\text{int}}=\sum_i V\left(\hat{\mathbf{r}}_i-\hat{\mathbf{r}}\right)$ $\hat{\mathbf{r}}_i$
$\hat{\mathbf{r}}$ delta $V\left(\hat{\mathbf{r}}-\hat{\mathbf{r}}'\right)=C\delta\left(\hat{\mathbf{r}}-\hat{\mathbf{r}}'\right)$

$$\hat{H}_{\text{int}}=C\int d^d r\delta(\hat{\mathbf{r}}-\mathbf{r})c^\dagger(\mathbf{r})c(\mathbf{r}) \tag{165}$$

$$\hat{H}_{\text{int}}=C\int d^d r\int\frac{d^d q}{(2\pi)^d}e^{i\mathbf{q}(\hat{\mathbf{r}}-\mathbf{r})}c^\dagger(\mathbf{r})c(\mathbf{r})=C\int\frac{d^d q}{(2\pi)^d}e^{i\mathbf{q}\cdot\hat{\mathbf{r}}}\hat{\rho}(\mathbf{q}) \tag{166}$$

$\mathcal{A}(\mathbf{q})=\left\langle\beta,\mathbf{k}-\mathbf{q}\left|\hat{H}_{\text{int}}\right|0,\mathbf{k}\right\rangle\propto\left\langle\beta\left|\hat{\rho}_{\mathbf{q}}\right|0\right\rangle$ Ferimi Golden Rule

$$\mathcal{P}(q)=2\pi\sum_\beta|\langle\beta|\hat{\rho}(\mathbf{q})|0\rangle|^2\delta\left(\omega-\Xi_{\beta0}\right), \tag{167}$$

$$\begin{aligned}\mathcal{P}(q)&=\int dt\sum_\beta|\langle\beta|\hat{\rho}(\mathbf{q})|0\rangle|^2e^{+it(\omega-\Xi_\beta)}\\&=\int dte^{+i\omega t}\sum_\beta\left\langle 0\left|e^{i(\hat{H}-\mu\hat{N})t}\hat{\rho}(-\mathbf{q})e^{-i(\hat{H}-\mu\hat{N})t}\right|\beta\right\rangle\langle\beta|\hat{\rho}(\mathbf{q})|0\rangle\\&=\int dte^{+i\omega t}\left\langle 0\left|e^{i(\hat{H}-\hat{\mu}N)t}\hat{\rho}(-\mathbf{q})e^{-i(\hat{H}-\mu\hat{N})t}\hat{\rho}(\mathbf{q})\right|0\right\rangle=\int dte^{+i\omega t}\langle 0|\hat{\rho}(-\mathbf{q},t)\hat{\rho}(\mathbf{q},0)|0\rangle\end{aligned} \tag{168}$$

delta **(??)**

$$\begin{aligned}\mathcal{P}(q)&=-2\operatorname{Im}\sum_\beta\frac{\rho(\mathbf{q})_{\beta0}\rho(-\mathbf{q})_{0\beta}}{\omega^++\Xi_{0\beta}}=-2\lim_{T\to0}\operatorname{Im}\mathcal{Z}^{-1}\sum_{\alpha\beta}\frac{\rho(\mathbf{q})_{\beta\alpha}\rho(-\mathbf{q})_{\alpha\beta}e^{-\beta\Xi_\alpha}}{\omega^++\Xi_{\alpha\beta}}\\&=-2\lim_{T\to0}\operatorname{Im}\mathcal{Z}^{-1}\sum_{\alpha\beta}\frac{\rho(\mathbf{q})_{\beta\alpha}\rho(-\mathbf{q})_{\alpha\beta}\left(e^{-\beta\Xi_\alpha}-e^{-\beta\Xi_\beta}\right)}{\omega^++\Xi_{\alpha\beta}}\\&=-2\lim_{T\to0}\operatorname{Im}C^+(\omega)=A(\mathbf{q},\omega),\end{aligned} \tag{169}$$

(??) **(??)**

14.6

1+d $A^\mu(x)=(\phi(x),\mathbf{A}(x))$ $j^\mu=(\rho,\mathbf{j})$

[19] γ Cauchy $f\left(z_0\right)=\frac{1}{2\pi i}\oint_c\frac{f(z)}{z-z_0}dz$
[20] $\zeta e^{-\beta\omega}$ p110

$$j_\mu(x)=\int_{t'<t} dx' K_{\mu\nu}\left(x,x'\right) A^\nu\left(x'\right) \tag{170}$$

$$0 \stackrel{!}{=} \int_{t'<t} dx' K_{\mu\nu}\left(x,x'\right) \partial^\nu f\left(x'\right)=-\int_{t'<t} dx'\left(\partial_{x'}^\nu K_{\mu\nu}\left(x,x'\right)\right) f\left(x'\right) \tag{171}$$

f $0=K_{\mu\nu}\overleftarrow{\partial}$ $\partial^\mu j_\mu=0$

$$0 \stackrel{!}{=} \int_{t'<t} dx' \partial_x^\mu K_{\mu\nu}\left(x,x'\right) A^\nu\left(x'\right) \tag{172}$$

A_μ $\overrightarrow{\partial^\mu} K_{\mu\nu}=0$

$$j_\mu=\frac{\delta S_c[A]}{\delta A_\mu} \tag{173}$$

$$K_{\mu\nu}\left(x,x'\right)=\mathcal{Z}^{-1} \frac{\delta^2}{\delta A_\mu(x) \delta A_\nu\left(x'\right)} \mathcal{Z}[A]\bigg|_{A=0} \tag{174}$$

$K_{\mu\nu}\left(x,x'\right)=K_{\nu\mu}\left(x',x\right)$

15 JW

16 -

17 H-S

Fractionalization FQHE

18

18.1

18.2

19

20 Goldstone ( )

$\phi^a(x)$

$$\mathcal{L}=(\text{ terms with derivatives })-V(\phi) \tag{175}$$

ϕ_a^0 V

$$\frac{\partial}{\partial \phi^a} V\bigg|_{\phi^a(x)=\phi_0^a}=0.$$

V

$$V(\phi)=V\left(\phi_0\right)+\frac{1}{2}\left(\phi-\phi_0\right)^a\left(\phi-\phi_0\right)^b\left(\frac{\partial^2}{\partial \phi^a \partial \phi^b} V\right)_{\phi_0}+\cdots.$$

Goldstone

$$\phi^a \longrightarrow \phi^a+\alpha \Delta^a(\phi) \tag{176}$$

α Δ^a

$$\Delta^a(\phi) \frac{\partial}{\partial \phi^a} V(\phi)=0$$

ϕ^b $\phi = \phi_0$:

$$0 = \left(\frac{\partial \Delta^a}{\partial \phi^b}\right)_{\phi_0} \left(\frac{\partial V}{\partial \phi^a}\right)_{\phi_0} + \Delta^a(\phi_0) \left(\frac{\partial^2}{\partial \phi^a \partial \phi^b} V\right)_{\phi_0}$$

ϕ_0 V 0 ϕ_0 $\Delta^a(\phi_0) = 0$

21 Multipartite Entanglement

22

23 Dirac,Weyl and Majorana fermions

[?] Dirac fermion Dirac Majorana fermion Dirac " " Weyl fermion Dirac

23.1 Dirac

Dirac

$$(i\gamma^\mu \partial_\mu - m)\,\Psi = 0 \tag{177}$$

$$H = \gamma^0 \left(\gamma^i p^i + m\right) \tag{178}$$

γ

$$\begin{aligned} &[\gamma^\mu, \gamma^\nu]_+ = 2g^{\mu\nu}, \\ &\gamma_0 \gamma_\mu \gamma_0 = \gamma_\mu^\dagger \end{aligned} \tag{179}$$

γ Dirac Majorana

$$\begin{aligned} \widetilde{\gamma}^0 &= \begin{bmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{bmatrix}, \quad \widetilde{\gamma}^1 = \begin{bmatrix} i\sigma^1 & 0 \\ 0 & i\sigma^1 \end{bmatrix}, \\ \tilde{\gamma}^2 &= \begin{bmatrix} 0 & \sigma^2 \\ -\sigma^2 & 0 \end{bmatrix}, \quad \tilde{\gamma}^3 = \begin{bmatrix} i\sigma^3 & 0 \\ 0 & i\sigma^3 \end{bmatrix}, \end{aligned} \tag{180}$$

σ^i Pauli Dirac

$$\widetilde{\psi} = \tilde{\psi}^\star \tag{181}$$

Majorana fermion

$$\gamma^\mu = U \tilde{\gamma}^\mu U^\dagger \tag{182}$$

$\tilde{\Psi}$ Majorana

$$\Psi = U \tilde{\Psi} \tag{183}$$

(??)

$$\psi = U U^\top \psi^\star \tag{184}$$

U

$$U U^\top = \gamma_0 C \tag{185}$$

$$\hat{\Psi} \equiv \gamma_0 C \Psi^\star \tag{186}$$

(??)

$$\hat{\psi} = \psi \tag{187}$$

23.2 Fourier

Majorana fermion Fourier

$$\psi(x) = \sum_s \int_p \left(a_s(p) u_s(p) e^{-ip\cdot x} + a_s^\dagger(p) v_s(p) e^{+ip\cdot x} \right) \tag{188}$$

v u $v_s(p) = \gamma_0 C u_s^\star(p)$

23.3 C

Majorana C C " "
C

$$C^{-1}\gamma_\mu C = -\left(U\tilde{\gamma}_\mu U^\dagger\right)^\top = -\gamma_\mu^\top \tag{189}$$

C C (**??**) C

23.4

(**??**) Lorentz $\sigma^{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$ Pauli fermion

$$\Psi'(x') = \exp\left(-\frac{i}{4}\omega^{\mu\nu}\sigma_{\mu\nu}\right)\Psi(x) \tag{190}$$

$\gamma_0 C$

$$\widehat{\Psi}'(x') = \exp\left(-\frac{i}{4}\omega^{\mu\nu}\sigma_{\mu\nu}\right)\widehat{\Psi}(x) \tag{191}$$

23.5

helicity chirality

Dirac

$$h_p = \frac{\cdot \mathbf{p}}{\mathrm{p}} \tag{192}$$

h_p ± 1 " " " " h_p boost

γ_5

$$[\gamma_5, \gamma_\mu]_+ = 0 \quad \forall \mu \tag{193}$$

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 \tag{194}$$

γ_5

$$\gamma_5^\dagger = \gamma_5, \quad (\gamma_5)^2 = 1 \tag{195}$$

$$L = \frac{1}{2}(1 - \gamma_5), \quad R = \frac{1}{2}(1 + \gamma_5) \tag{196}$$

" " " " Dirac γ_5 , γ_5

23.6 Wyle fermion

γ_5 Schur Dirac Lorentz Wyle fermion $\frac{1}{2}\oplus\frac{1}{2}$, Wyle Wyle

23.7 Wyle Majorana Dirac

Majorana fermion Wyle

$$\psi(x)=\chi(x)+\widehat{\chi}(x) \tag{197}$$

Dirac fermion

$$\Psi(x)=\chi_1(x)+\widehat{\chi}_2(x) \tag{198}$$

24 Schwinger–Dyson

24.1

$$Z[j]=\int[D\phi(x)]e^{i\mathcal{S}[\phi]+i\int j\phi} \tag{199}$$

$\phi(x)\rightarrow\phi(x)+\delta\phi(x)$[21]

$$0=\delta Z[j]=i\int[D\phi(x)]e^{i\delta[\phi]+i\int j\phi}\left\{\int d^4x\delta\phi(x)\left(j(x)+\frac{\delta\delta}{\delta\phi(x)}\right)\right\} \tag{200}$$

n $j=0$

$$\begin{aligned}0=\int[D\phi(x)]e^{i\delta[\phi]}\int d^4x\delta\phi(x)&\left\{i\phi\left(x_1\right)\cdots\phi\left(x_n\right)\frac{\delta\mathcal{S}}{\delta\phi(x)}\right.\\&\left.+\sum_{i=1}^{n}\delta\left(x-x_i\right)\prod_{j\neq i}\phi\left(x_j\right)\right\}.\end{aligned} \tag{201}$$

$@d\phi(x)$

$$\begin{aligned}0=\int[D\phi(x)]e^{i\mathcal{S}[\phi]}\{&i\phi\left(x_1\right)\cdots\phi\left(x_n\right)\frac{\delta\mathcal{S}}{\delta\phi(x)}\\&\left.+\sum_{i=1}^{n}\delta\left(x-x_i\right)\prod_{j\neq i}\phi\left(x_j\right)\right\}\end{aligned} \tag{202}$$

Schwinger-Dyson

24.2 Noether

$$\frac{\delta\mathcal{S}}{\delta\phi(x)}\delta\phi(x)=-\partial_\mu(\underbrace{\frac{\partial\mathcal{L}}{\partial\left(\partial_\mu\phi(x)\right)}\delta\phi(x))}_{J^\mu(x)} \tag{203}$$

$J^\mu(x)$ Noether $\partial_\mu J^\mu=0$

Schwinger-Dyson , **(??)** **(??)**,

$$\begin{aligned}&\partial_\mu\left\langle 0_{\text{out}}\left|\mathrm{T}J^\mu(x)\phi\left(x_1\right)\cdots\phi\left(x_n\right)\right|0_{\text{in}}\right\rangle\\&+i\sum_{i=1}^{n}\delta\left(x-x_i\right)\left\langle 0_{\text{out}}\left|\mathrm{T}\delta\phi(x)\prod_{j\neq i}\phi\left(x_j\right)\right|0_{\text{in}}\right\rangle=0\end{aligned} \tag{204}$$

Ward LSZ [22]

$$\langle f\mid i\rangle=i\varepsilon^\mu\int d^4xe^{-ikx}\left(-\partial^2\right)\ldots\langle 0\,|\,\mathrm{T}A_\mu(x)\ldots|\,0\rangle \tag{205}$$

[21]

[22] LSZ LSZ

$-Z_3\partial^2 A_\mu = \frac{\partial \mathcal{L}}{\partial A^\mu} = Z_1 j^\mu$

$$\langle f \mid i\rangle = i Z_3^{-1} Z_1 \varepsilon^\mu \int d^4 x e^{-ikx} \ldots [\langle 0 \mid \mathrm{T} j_\mu(x) \ldots \mid 0\rangle + \text{ contact terms }] \tag{206}$$

$\delta(x-x_j)$ $\quad -\partial_j^2 + m_j^2$ $\quad \epsilon^\mu \quad k^\mu$ $\quad$ **(??)** $\quad k^\mu M_\mu = 0$ $\quad$ Ward

25 Pology and LZS formula

25.1 K-L

Heisenberg $\quad \langle\Omega|T\phi(x)\phi(y)|\Omega\rangle$

$$\mathbf{1} = |\Omega\rangle \left\langle \Omega \middle| + \sum_\lambda \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2E_{\mathbf{p}}(\lambda)} \middle| \lambda_{\mathbf{p}} \right\rangle \langle \lambda_{\mathbf{p}}| \tag{207}$$

λ

$$\langle\Omega|\phi(x)\phi(y)|\Omega\rangle = \sum_\lambda \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2E_{\mathbf{p}}(\lambda)} \langle\Omega|\phi(x)|\lambda_{\mathbf{p}}\rangle \langle\lambda_{\mathbf{p}}|\phi(y)|\Omega\rangle \tag{208}$$

$$\begin{aligned} \langle\Omega|\phi(x)|\lambda_{\mathbf{p}}\rangle &= \left\langle \Omega \left| e^{iP\cdot x}\phi(0) e^{-iP\cdot x} \right| \lambda_{\mathbf{p}} \right\rangle \\ &= \left. \langle\Omega|\phi(0)|\lambda_{\mathbf{p}}\rangle e^{-ip\cdot x} \right|_{p^0 = E_{\mathbf{p}}} \\ &= \left. \langle\Omega|\phi(0)|\lambda_0\rangle e^{-ip\cdot x} \right|_{p^0 = E_{\mathbf{p}}} \end{aligned} \tag{209}$$

$\langle\Omega|, \phi(0)$ Lorentz $\quad$ Boost $\quad U^{-1}U$

$$i\theta\left(x^0 - y^0\right) \int \widetilde{dp} e^{ip(x-y)} + i\theta\left(y^0 - x^0\right) \int \widetilde{dp} e^{-ip(x-y)} = \int \frac{d^4 k}{(2\pi)^4} \frac{e^{ip(x-y)}}{p^2 + m^2 - i\varepsilon} \tag{210}$$

$$\langle\Omega|T\phi(x)\phi(y)|\Omega\rangle = \int_0^\infty \frac{dM^2}{2\pi} \rho\left(M^2\right) D_F\left(x-y; M^2\right) \tag{211}$$

$\rho(M^2)$

$$\rho\left(M^2\right) = \sum_\lambda (2\pi)\delta\left(M^2 - m_\lambda^2\right) \left|\langle\Omega|\phi(0)|\lambda_0\rangle\right|^2 \tag{212}$$

Delta $\quad M > 2m$

$$\rho\left(M^2\right) = 2\pi\delta\left(M^2 - m^2\right) \cdot Z + \left(\text{ nothing else until } M^2 \gtrsim (2m)^2\right) \tag{213}$$

Z $\quad$ m

$$\begin{aligned} \int d^4 x e^{ip\cdot x} \langle\Omega|T\phi(x)\phi(0)|\Omega\rangle &= \int_0^\infty \frac{dM^2}{2\pi} \rho\left(M^2\right) \frac{i}{p^2 - M^2 + i\epsilon} \\ &= \frac{iZ}{p^2 - m^2 + i\epsilon} + \int_{\sim 4m^2}^\infty \frac{dM^2}{2\pi} \rho\left(M^2\right) \frac{i}{p^2 - M^2 + i\epsilon} \end{aligned} \tag{214}$$

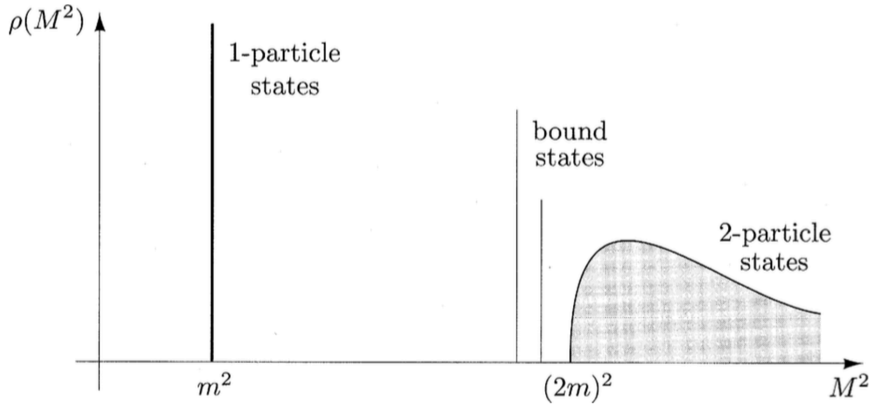


Figure 8:

25.2 LSZ

p^2

$$\int d^4x e^{ip\cdot x}\langle\Omega|T\phi(x)\phi(0)|\Omega\rangle \underset{p^2\to m^2}{\sim} \frac{iZ}{p^2-m^2+i\epsilon} \tag{215}$$

S $2\to n$ p (**??**) n+2 S

n+2

$$\int d^4x e^{ip\cdot x}\left\langle\Omega\left|T\left\{\phi(x)\phi\left(z_1\right)\phi\left(z_2\right)\cdots\right\}\right|\Omega\right\rangle \tag{216}$$

p^0

$$\int dx^0=\int_{T_+}^{\infty}dx^0+\int_{T_-}^{T_+}dx^0+\int_{-\infty}^{T_-}dx^0 \tag{217}$$

T_+,T_- / z_i^0 I,II,III, II p^0 $\exp(ip^0x^0)$ p^0 I III

I $\phi(x)$

$$\begin{aligned}\int_{T_+}^{\infty}dx^0\int d^3x e^{ip^0x^0}e^{-i\mathbf{p}\cdot\mathbf{x}}\sum_\lambda\int\frac{d^3q}{(2\pi)^3}\frac{1}{2E_{\mathbf{q}}(\lambda)}\langle\Omega|\phi(x)|\lambda_{\mathbf{q}}\rangle\\ \times\left\langle\lambda_{\mathbf{q}}\left|T\left\{\phi\left(z_1\right)\phi\left(z_2\right)\cdots\right\}\right|\Omega\right\rangle\end{aligned} \tag{218}$$

(**??**) $e^{-\epsilon x^0}$

$$\begin{aligned}&\sum_\lambda\int_{T_+}^{\infty}dx^0\int\frac{d^3q}{(2\pi)^3}\frac{1}{2E_{\mathbf{q}}(\lambda)}e^{ip^0x^0}e^{-iq^0x^0}e^{-\epsilon x^0}\langle\Omega|\phi(0)|\lambda_0\rangle(2\pi)^3\delta^{(3)}(\mathbf{p}-\mathbf{q})\\ &\times\left\langle\lambda_{\mathbf{q}}\left|T\left\{\phi\left(z_1\right)\cdots\right\}\right|\Omega\right\rangle\\ &=\sum_\lambda\frac{1}{2E_{\mathbf{p}}(\lambda)}\frac{ie^{i\left(p^0-E_{\mathbf{p}}+i\epsilon\right)T_+}}{p^0-E_{\mathbf{p}}(\lambda)+i\epsilon}\langle\Omega|\phi(0)|\lambda_0\rangle\left\langle\lambda_{\mathbf{p}}\left|T\left\{\phi\left(z_1\right)\cdots\right\}\right|\Omega\right\rangle.\end{aligned} \tag{219}$$

$$\begin{aligned}&\int d^4x e^{ip\cdot x}\left\langle\Omega\left|T\left\{\phi(x)\phi\left(z_1\right)\cdots\right\}\right|\Omega\right\rangle\\ &\underset{p^0\to+E_{\mathbf{p}}}{\sim}\frac{i}{p^2-m^2+i\epsilon}\sqrt{Z}\left\langle\mathbf{p}\left|T\left\{\phi\left(z_1\right)\cdots\right\}\right|\Omega\right\rangle\end{aligned} \tag{220}$$

III

$$\begin{aligned}&\int d^4x e^{ip\cdot x}\langle\Omega|T\left\{\phi(x)\phi\left(z_1\right)\cdots\right\}|\Omega\rangle\\ &\underset{p^0\to-E_{\mathbf{p}}}{\sim}\left\langle\Omega\left|T\left\{\phi\left(z_1\right)\cdots\right\}\right|-\mathbf{p}\right\rangle\sqrt{Z}\frac{i}{p^2-m^2+i\epsilon}\end{aligned} \tag{221}$$

$$\int d^4x e^{ip^0x^0} e^{-i\mathbf{p}\cdot\mathbf{x}} \to \int \frac{d^3k}{(2\pi)^3} \int d^4x e^{ip^0x^0} e^{-i\mathbf{k}\cdot\mathbf{x}} \varphi(\mathbf{k}) \tag{222}$$

$\varphi(\mathbf{k})$ p

$$\begin{aligned} &\sum_\lambda \int \frac{d^3k}{(2\pi)^3} \varphi(\mathbf{k}) \frac{1}{2E_{\mathbf{k}}(\lambda)} \frac{i}{p^0 - E_{\mathbf{k}}(\lambda) + i\epsilon} \langle\Omega|\phi(0)|\lambda_0\rangle \langle\lambda_{\mathbf{k}} |T\{\phi(z_1)\cdots\}|\Omega\rangle \\ &\underset{p^0\to+E_{\mathbf{p}}}{\sim} \int \frac{d^3k}{(2\pi)^3} \varphi(\mathbf{k}) \frac{i}{\tilde{p}^2 - m^2 + i\epsilon} \sqrt{Z} \langle \mathbf{k} |T\{\phi(z_1)\cdots\}|\Omega\rangle \end{aligned} \tag{223}$$

$\varphi(\mathbf{k})$

$$\left(\prod_i \int \frac{d^3k_i}{(2\pi)^3} \int d^4x_i e^{i\tilde{p}_i\cdot x_i} \varphi_i(\mathbf{k}_i)\right) \langle\Omega |T\{\phi(x_1)\phi(x_2)\cdots\}|\Omega\rangle \tag{224}$$

T_+, T_- I III x=0 1 2

$$\begin{aligned} &\sum_\lambda \int \frac{d^3K}{(2\pi)^3} \frac{1}{2E_{\mathbf{K}}} \left(\prod_{i=1,2} \int \frac{d^3k_i}{(2\pi)^3} \int d^4x_i e^{i\tilde{p}_i\cdot x_i} \varphi_i(\mathbf{k}_i)\right) \\ &\times \langle\Omega |T\{\phi(x_1)\phi(x_2)\}|\lambda_{\mathbf{K}}\rangle \langle\lambda_{\mathbf{K}} |T\{\phi(x_3)\cdots\}|\Omega\rangle \end{aligned} \tag{225}$$

$\lambda_{\mathbf{K}}$

$$\begin{aligned} &\sum_\lambda \int \frac{d^3K}{(2\pi)^3} \frac{1}{2E_{\mathbf{K}}} \langle\Omega |T\{\phi(x_1)\phi(x_2)\}|\lambda_{\mathbf{K}}\rangle \langle\lambda_{\mathbf{K}}| \\ &= \sum_{\lambda_1\lambda_0} \int \frac{d^3q_1}{(2\pi)^3} \frac{1}{2E_{\mathbf{q}_1}} \int \frac{d^3q_2}{(2\pi)^3} \frac{1}{2E_{\mathbf{q}_2}} \langle\Omega |\phi(x_1)|\lambda_{\mathbf{q}_1}\rangle \langle\Omega |\phi(x_2)|\lambda_{\mathbf{q}_2}\rangle \langle\lambda_{\mathbf{q}_1}\lambda_{\mathbf{q}_2}| \end{aligned} \tag{226}$$

x_1, x_0

$$\left(\prod_{i=1,2} \int \frac{d^3k_i}{(2\pi)^3} \varphi_i(\mathbf{k}_i) \frac{i}{\tilde{p}_i^2 - m^2 + i\epsilon} \cdot \sqrt{Z}\right) \langle \mathbf{k}_1\mathbf{k}_2 |T\{\phi(x_3)\cdots\}|\Omega\rangle \tag{227}$$

$$\left(\prod_{i=1,2} \frac{i}{{p_i}^2 - m^2 + i\epsilon} \cdot \sqrt{Z}\right) \left(\prod_{i=3,\ldots..} \frac{i}{{p_i}^2 - m^2 + i\epsilon} \cdot \sqrt{Z}\right)_{\text{out}} \langle \mathbf{p}_1\mathbf{p}_2 \mid -\mathbf{p}_3\cdots\rangle_{\text{in}} \tag{228}$$

S LSZ

$$\begin{aligned} &\prod_1^n \int d^4x_i e^{ip_i\cdot x_i} \prod_1^m \int d^4y_j e^{-ik_j\cdot y_j} \langle\Omega |T\{\phi(x_1)\cdots\phi(x_n)\phi(y_1)\cdots\phi(y_m)\}|\Omega\rangle \\ &\underset{\substack{\text{each } p_i^0\to+E_{\mathbf{p}_i} \\ \text{each } k_j^0\to+E_{\mathbf{k}_j}}}{\sim} \left(\prod_{i=1}^n \frac{\sqrt{Z}i}{p_i^2-m^2+i\epsilon}\right) \left(\prod_{j=1}^m \frac{\sqrt{Z}i}{k_j^2-m^2+i\epsilon}\right) \langle \mathbf{p}_1\cdots\mathbf{p}_n|S|\mathbf{k}_1\cdots\mathbf{k}_m\rangle . \end{aligned} \tag{229}$$

LSZ S Amputated S Amputated

26 Noether local

27 :QED, ,

28

28.1

Helmholtz

$$Z(H) = e^{-\beta F(H)} = \int \mathcal{D}s \exp\left[-\beta \int dx(\mathcal{H}[s] - Hs(x))\right] \tag{230}$$

H Helmholtz

$$
\begin{aligned}
-\left.\frac{\partial F}{\partial H}\right|_{\beta \text{ fixed}} &= \frac{1}{\beta}\frac{\partial}{\partial H}\log Z \\
&= \frac{1}{Z}\int dx \int \mathcal{D}s s(x) \exp\left[-\beta \int dx (\mathcal{H}[s] - Hs)\right] \\
&= \int dx \langle s(x)\rangle \equiv M.
\end{aligned}
$$

Legendre

$$G = f + MH$$

$$
\begin{aligned}
\frac{\partial G}{\partial M} &= \frac{\partial F}{\partial M} + M\frac{\partial H}{\partial M} + H \\
&= \frac{\partial H}{\partial M}\frac{\partial F}{\partial H} + M\frac{\partial H}{\partial M} + H \\
&= H
\end{aligned}
\tag{231}
$$

$H = 0$ $G(M)$ QFT

28.2

$$Z[J] = e^{-iE[J]} = \int \mathcal{D}\phi \exp\left[i\int d^4x(\mathcal{L}[\phi] + J\phi)\right] \tag{232}$$

$E(J)$, $W(J)$, $E(J)$ - J J
$E(J)$ J

$$\frac{\delta}{\delta J(x)}E[J] = i\frac{\delta}{\delta J(x)}\log Z = -\frac{\int \mathcal{D}\phi e^{i\int(\mathcal{L}+J\phi)}\phi(x)}{\int \mathcal{D}\phi e^{i\int(\mathcal{L}+J\phi)}} \tag{233}$$

$$\frac{\delta}{\delta J(x)}E[J] = -\langle\Omega|\phi(x)|\Omega\rangle_J \tag{234}$$

J ϕ

$$\phi_{\text{cl}}(x) = \langle\Omega|\phi(x)|\Omega\rangle_J \tag{235}$$

QFT , $E(J)$ Legendre [23]

$$\Gamma\left[\phi_{\text{cl}}\right] \equiv -E[J] - \int d^4y J(y)\phi_{\text{cl}}(y) \tag{237}$$

$$
\begin{aligned}
\frac{\delta}{\delta\phi_{\text{cl}}(x)}\Gamma\left[\phi_{\text{cl}}\right] &= -\frac{\delta}{\delta\phi_{\text{cl}}(x)}E[J] - \int d^4y \frac{\delta J(y)}{\delta\phi_{\text{cl}}(x)}\phi_{\text{cl}}(y) - J(x) \\
&= -\int d^4y \frac{\delta J(y)}{\delta\phi_{\text{cl}}(x)}\frac{\delta E[J]}{\delta J(y)} - \int d^4y \frac{\delta J(y)}{\delta\phi_{\text{cl}}(x)}\phi_{\text{cl}}(y) - J(x) \\
&= -J(x).
\end{aligned}
\tag{238}
$$

QFT **??**

$$\frac{\delta}{\delta\phi_{\text{cl}}(x)}\Gamma\left[\phi_{\text{cl}}\right] = 0 \tag{239}$$

x
Lorentz Γ

$$\Gamma\left[\phi_{\text{cl}}\right] = -(VT)\cdot V_{\text{eff}}\left(\phi_{\text{cl}}\right) \tag{240}$$

$V_{\text{eff}}\left(\phi_{\text{cl}}\right)$ $\Gamma\left[\phi_{\text{cl}}\right]$ $V_{\text{eff}}\left(\phi_{\text{cl}}\right)$

---

[23] Weinberg Legendre J Legendre J J_ϕ ϕ^r Legendre

$$\Gamma[\phi] \equiv -\int \mathrm{d}^4x\phi^r(x)J_{\phi r}(x) + W\left[J_\phi\right] \tag{236}$$

Magnetic System	Quantum Field Theory
$\mathbf{x}$	$x=(t,\mathbf{x})$
$s(\mathbf{x})$	$\phi(x)$
H	$J(x)$
$\mathcal{H}(s)$	$\mathcal{L}(\phi)$
$Z(H)$	$Z[J]$
$F(H)$	$E[J]$
M	$\phi_{\mathrm{cl}}(x)$
$G(M)$	$-\Gamma[\phi_{\mathrm{cl}}]$

Figure 9: QFT

28.3

$Z[J]$ $\Gamma\left[\phi_{\mathrm{cl}}\right]$

$$\begin{aligned}
\frac{\delta^2 E[J]}{\delta J(x)\delta J(y)} &= -\frac{i}{Z}\int \mathcal{D}\phi e^{i\int(\mathcal{L}+J\phi)}\phi(x)\phi(y) \\
&+\frac{i}{Z^2}\int \mathcal{D}\phi e^{i\int(\mathcal{L}+J\phi)}\phi(x)\cdot\int \mathcal{D}\phi e^{i\int(\mathcal{L}+J\phi)}\phi(y) \\
&= -i[\langle\phi(x)\phi(y)\rangle-\langle\phi(x)\rangle\langle\phi(y)\rangle].
\end{aligned} \tag{241}$$

(link cluster theorem)

$$\frac{\delta^n E[J]}{\delta J\left(x_1\right)\cdots\delta J\left(x_n\right)} = (i)^{n+1}\left\langle\phi\left(x_1\right)\cdots\phi\left(x_n\right)\right\rangle_{\text{conn}} \tag{242}$$

γ, **(??)** $J(y)$

$$\frac{\delta}{\delta J(y)}\frac{\delta\Gamma}{\delta\phi_{\mathrm{cl}}(x)} = -\delta(x-y)$$

$$\begin{aligned}
\delta(x-y) &= -\int d^4z\frac{\delta\phi_{\mathrm{cl}}(z)}{\delta J(y)}\frac{\delta^2\Gamma}{\delta\phi_{\mathrm{cl}}(z)\delta\phi_{\mathrm{cl}}(x)} \\
&= \int d^4z\frac{\delta^2 E}{\delta J(y)\delta J(z)}\frac{\delta^2\Gamma}{\delta\phi_{\mathrm{cl}}(z)\delta\phi_{\mathrm{cl}}(x)} \\
&= \left(\frac{\delta^2 E}{\delta J\delta J}\right)_{yz}\left(\frac{\delta^2\Gamma}{\delta\phi_{\mathrm{cl}}\delta\phi_{\mathrm{cl}}}\right)_{zx}
\end{aligned} \tag{243}$$

$$\left(\frac{\delta^2 E}{\delta J\delta J}\right) = \left(\frac{\delta^2\Gamma}{\delta\phi_{\mathrm{cl}}\delta\phi_{\mathrm{cl}}}\right)^{-1} \tag{244}$$

$$\widetilde{D}^{-1}(p) = -i\left(p^2-m^2-M^2\left(p^2\right)\right)$$

$M^2\left(p^2\right)$

$$\frac{\delta}{\delta J(z)} = \int d^4w\frac{\delta\phi_{\mathrm{cl}}(w)}{\delta J(z)}\frac{\delta}{\delta\phi_{\mathrm{cl}}(w)} = i\int d^4w D(z,w)\frac{\delta}{\delta\phi_{\mathrm{cl}}(w)}$$

$$\frac{\partial}{\partial\alpha}M^{-1}(\alpha) = -M^{-1}\frac{\partial M}{\partial\alpha}M^{-1}$$

(??)

$$
\begin{aligned}
\frac{\delta^3 E[J]}{\delta J_x \delta J_y \delta J_z} &= i \int d^4 w D(z,w) \frac{\delta}{\delta \phi_w^{\mathrm{cl}}} \left(\frac{\delta^2 \Gamma}{\delta \phi_x^{\mathrm{cl}} \delta \phi_y^{\mathrm{cl}}} \right)^{-1} \\
&= i \int d^4 w D_{zw} (-1) \int d^4 u \int d^4 v \, (-i D_{xu}) \, \frac{\delta^3 \Gamma}{\delta \phi_u^{\mathrm{cl}} \delta \phi_v^{\mathrm{cl}} \delta \phi_w^{\mathrm{cl}}} \, (-i D_{vy}) \\
&= i \int d^4 u d^4 v d^4 w D_{xu} D_{yv} D_{zw} \frac{\delta^3 \Gamma}{\delta \phi_u^{\mathrm{cl}} \delta \phi_v^{\mathrm{cl}} \delta \phi_w^{\mathrm{cl}}}
\end{aligned}
$$

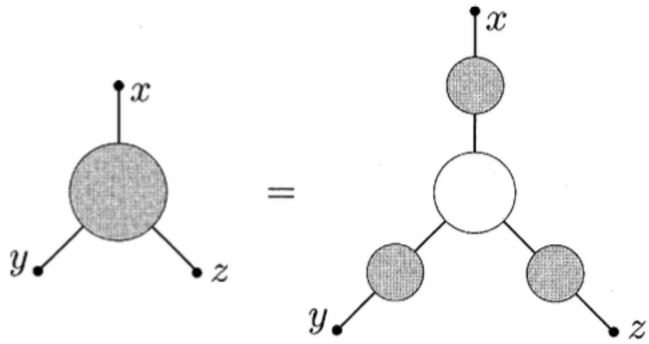


Figure 10:

$$
\frac{i \delta^3 \Gamma}{\delta \phi_{\mathrm{cl}}(x) \phi_{\mathrm{cl}}(y) \phi_{\mathrm{cl}}(z)} = \langle \phi(x) \phi(y) \phi(z) \rangle_{\mathrm{1PI}}
$$

$$
\frac{\delta^n \Gamma \left[\phi_{\mathrm{cl}} \right]}{\delta \phi_{\mathrm{cl}} \left(x_1 \right) \cdots \delta \phi_{\mathrm{cl}} \left(x_n \right)} = -i \left\langle \phi \left(x_1 \right) \cdots \phi \left(x_n \right) \right\rangle_{\mathrm{1PI}} \tag{245}
$$

$W(J)$ - $\Gamma[\phi_{cl}]$

$I[\phi]$ $\Gamma[\phi]$

$$
Z_\Gamma[j] = e^{W_\Gamma[j]} = \int [D\phi(x)] \exp \left[i \Gamma[\phi(x)] + i \int d^4 x j(x) \phi(x) \right] \tag{246}
$$

- $I[\phi]$

$$
Z_r[j;\hbar] = e^{w_r[j\hbar]} = \int [D\phi(x)] \exp \left[\frac{i}{\hbar} \left(\Gamma[\phi(x)] + \int d^4 x j(x) \phi(x) \right) \right] \tag{247}
$$

$\hbar$ $\hbar^{-1}$ n_L I V $n_L = I - V + 1$ L $\hbar^{n_L - 1}$

$$
W_\Gamma[j;\hbar] = \sum_{n_L = 0}^{\infty} \hbar^{n_L - 1} \underbrace{W_{\Gamma, n_L}[j]}_{n_L \text{ loops}} \tag{248}
$$

$\hbar \to 0$

28.4

(??)

$$
\mathcal{L} = \mathcal{L}_1 + \delta \mathcal{L} \tag{249}
$$

$J(x) = J_1(x) + \delta J(x)$

$$
\left. \frac{\delta \mathcal{L}_1}{\delta \phi} \right|_{\phi = \phi_{\mathrm{cl}}} + J_1(x) = 0 \tag{250}
$$

$\delta J(x)$ $\phi_{cl}(x)$ $\langle\phi(x)\rangle_J = \phi_{\text{cl}}(x)$

$$Z[J] = \int \mathcal{D}\phi e^{i\int d^4x(\mathcal{L}_1[\phi]+J_1\phi)} e^{i\int d^4x(\delta\mathcal{L}[\phi]+\delta J\phi)} \tag{251}$$

$\phi(x) = \phi_{\text{cl}}(x) + \eta(x)$

$$\begin{aligned}\int d^4x\left(\mathcal{L}_1 + J_1\phi\right) = &\int d^4x\left(\mathcal{L}_1\left[\phi_{\text{cl}}\right] + J_1\phi_{\text{cl}}\right) + \int d^4x\eta(x)\left(\frac{\delta\mathcal{L}_1}{\delta\phi} + J_1\right)\\ &+ \frac{1}{2}\int d^4xd^4y\eta(x)\eta(y)\frac{\delta^2\mathcal{L}_1}{\delta\phi(x)\delta\phi(y)}\\ &+ \frac{1}{3!}\int d^4xd^4yd^4z\eta(x)\eta(y)\eta(z)\frac{\delta^3\mathcal{L}_1}{\delta\phi(x)\delta\phi(y)\delta\phi(z)} + \cdots\end{aligned} \tag{252}$$

(??)

$$\begin{aligned}&\int\mathcal{D}\eta\exp\left[i\left(\int\left(\mathcal{L}_1\left[\phi_{\text{cl}}\right] + J_1\phi_{\text{cl}}\right) + \frac{1}{2}\int\eta\frac{\delta^2\mathcal{L}_1}{\delta\phi\delta\phi}\eta\right)\right]\\ &\quad = \exp\left[i\int\left(\mathcal{L}_1\left[\phi_{\text{cl}}\right] + J_1\phi_{\text{cl}}\right)\right]\cdot\left(\det\left[-\frac{\delta^2\mathcal{L}_1}{\delta\phi\delta\phi}\right]\right)^{-1/2}.\end{aligned} \tag{253}$$

Feynman $-i\left(\frac{\delta^2\mathcal{L}_1}{\delta\phi\delta\phi}\right)^{-1}$ Feynman ϕ_{cl}

$$\left(\delta\mathcal{L}\left[\phi_{\text{cl}}\right] + \delta J\phi_{\text{cl}}\right) + \left(\delta\mathcal{L}\left[\phi_{\text{cl}} + \eta\right] - \delta\mathcal{L}\left[\phi_{\text{cl}}\right] + \delta J\eta\right) \tag{254}$$

Taylor Feynman

$$\begin{aligned}\Gamma\left[\phi_{\text{cl}}\right] = &\int d^4x\mathcal{L}_1\left[\phi_{\text{cl}}\right] + \frac{i}{2}\log\det\left[-\frac{\delta^2\mathcal{L}_1}{\delta\phi\delta\phi}\right]\\ &- i\cdot(\text{ connected diagrams }) + \int d^4x\delta\mathcal{L}\left[\phi_{\text{cl}}\right]\end{aligned} \tag{255}$$

\- δJ
$\langle\phi\rangle = \phi_{\text{cl}}$ Feynman “ ” $\delta J\eta$ $\delta J\eta$

28.5

, , $I[\phi]$ $\Gamma[\phi]$,

$$\chi^n(x) \rightarrow \chi^n(x) + \epsilon F^n[x;\chi] \tag{256}$$

$$\begin{gathered}I[\chi + \epsilon F] = I[\chi]\\ \prod_{n,x}\mathrm{d}\left(\chi^n(x) + \epsilon F[x;\chi]\right) = \prod_{n,x}\mathrm{d}\chi^n(x)\end{gathered} \tag{257}$$

$$\begin{aligned}Z[J] = &\int\left[\prod_{n,x}\mathrm{d}\left(\chi^n(x) + \epsilon F^n[x;\chi]\right)\right]\\ &\times\exp\left\{\mathrm{i}I[\chi + \epsilon F] + \mathrm{i}\int\mathrm{d}^4x\left(\chi^n(x) + \epsilon F^n[x;\chi]\right)J_n(x)\right\}\\ = &\int\left[\prod_{n,x}\mathrm{d}\chi^n(x)\right]\exp\left\{\mathrm{i}I[\chi] + \mathrm{i}\int\mathrm{d}^4x\left(\chi^n(x) + \epsilon F^n[x;\chi]\right)J_n(x)\right\}\\ = &Z[J] + \mathrm{i}\epsilon\int\left(\prod_{n,x}\mathrm{d}\chi^n(x)\right)\int F^n(y;\chi)J_n(y)\mathrm{d}^4y\\ &\times\exp\left\{\mathrm{i}I[\chi] + \mathrm{i}\int\mathrm{d}^4x\chi^n(x)J_n(x)\right\}\end{aligned} \tag{258}$$

Taylor

$$\int \mathrm{d}^4 y \left\langle F^n(y) \right\rangle_J J_n(y) = 0 \tag{259}$$

$$J_{n,\chi}(y) = -\frac{\delta \Gamma[\chi]}{\delta \chi^n(y)} \tag{260}$$

$$0 = \int \mathrm{d}^4 y \left\langle F^n(y) \right\rangle_{J_\chi} \frac{\delta \Gamma[\chi]}{\delta \chi^n(y)} \tag{261}$$

$\Gamma[\chi]$

$$\chi^n(y) \to \chi^n(y) + \epsilon \left\langle F^n(y) \right\rangle_{J_\chi} \tag{262}$$

Slavnov-Taylor

29 Goldstone

QFT Goldstone .

29.1

$$\left(\frac{\delta \Gamma[\phi]}{\delta \phi(x)} \right)_{\phi = \langle \Omega | \phi | \Omega \rangle} = 0 \tag{263}$$

$|\Psi >$ Lagrange

$$\langle \Psi | H | \Psi \rangle - A \langle \Psi \mid \Psi \rangle - \int d^3 x B(\mathbf{x}) \langle \Psi | \phi(\mathbf{x}) | \Psi \rangle \tag{264}$$

$$H|\Psi\rangle = A|\Psi\rangle + \int d^3 x B(\mathbf{x}) \phi(\mathbf{x}) |\Psi\rangle \tag{265}$$

$$\left(H - \int d^3 x J(\mathbf{x}) \phi(\mathbf{x}) \right) |\Psi_J\rangle = E[J] \, |\Psi_J\rangle \tag{266}$$

A B $\phi_0 x 0$ A B

$$\left(H - \int d^3 x J(\mathbf{x}) \phi(\mathbf{x}) \right) |\Psi_J\rangle = E[J] \, |\Psi_J\rangle \tag{267}$$

$B = J_0 := J_{\phi_0}, A = E\left[J_{\phi_0}\right]$ $|\Psi_J\rangle$
$-\infty \to +\infty$ $J(\vec{x})$ T
-

$$\langle \Omega, \infty \mid \Omega, -\infty \rangle_J = \exp(-iE[J]T) \tag{268}$$

$W[J] = -E[J]T$

$$\begin{aligned} H \left|\Psi_{J_0}\right\rangle &= \left(E[J_0] + \int d^3 x J_0(\mathbf{x}) \phi_0(\mathbf{x}) \right) \left|\Psi_{J_0}\right\rangle \\ &= \frac{1}{T} \left(-W\left[J_0\right] + \int d^4 x J_0(x) \phi_0(x) \right) \left|\Psi_{J_0}\right\rangle \\ &= -\frac{\Gamma\left[\phi_0\right]}{T} \left|\Psi_{J_0}\right\rangle \end{aligned} \tag{269}$$

ϕ_0

Poincare V_{eff} $\Gamma\left[\phi_0\right] = -VTV_{\text{eff}}\left(\phi_0\right)$

29.2

$\phi \to -\phi$ $|\mathrm{VAC},+\rangle$ $|\mathrm{VAC},-\rangle$ $|\mathrm{VAC},\rangle \pm |\mathrm{VAC},+\rangle$

$$\begin{aligned}\langle \mathrm{VAC},+|H|\mathrm{VAC},+\rangle = \langle \mathrm{VAC},-|H|\mathrm{VAC},-\rangle \equiv a \\ \langle \mathrm{VAC},+|H|\ \mathrm{VAC}\ ,-\rangle = \langle \mathrm{VAC},-|H|\mathrm{VAC},+\rangle \equiv b\end{aligned} \tag{270}$$

l

$$\left\langle \Omega_+ \left| e^{iHt} \right| \Omega_- \right\rangle \approx e^{-S_E} = e^{-V\int_0^t \mathcal{L}_E(\phi_{\mathrm{cl}})dt} \tag{271}$$

S_E wick $\pm$ VAC, $\pm\rangle$ “ ”

29.3 Goldstone

Proof 1

$$\phi_n(x) \to \phi_n(x) + \mathrm{i}\epsilon \sum_m t_{nm}\phi_m(x) \tag{272}$$

$$\sum_{n,m} \int \frac{\delta\Gamma[\phi]}{\delta\phi_n(x)} t_{nm}\phi_m(x)\mathrm{d}^4x = 0$$

$\Gamma[\phi] = -\mathscr{V}V(\phi)$

$$\sum_{n,m} \frac{\partial V(\phi)}{\partial \phi_n} t_{nm}\phi_m = 0$$

$$\sum_{n,m} \left.\frac{\partial^2 V(\phi)}{\partial\phi_n\partial\phi_\ell}\right|_{\phi=\bar{\phi}} t_{nm}\bar{\phi}_m = 0$$

$$\frac{\partial^2 V(\phi)}{\partial\phi_n\partial\phi_\ell} = \Delta^{-1}_{n\ell}(0)$$

$$\sum_{n,m} \Delta^{-1}_{n\ell}(0) t_{nm}\bar{\phi}_m = 0$$

$\sum_m t_{nm}\bar{\phi}_m$ $\Delta^{-1}_{n\ell}(0)$ $\phi_{Gm} = U_{nm}\phi_m$ U_{mn} n $\Delta^{-1}_{n\ell}(q)$ $q^2 = 0$

Goldstone ϕ_α D $\phi'_\alpha = D_{\alpha\beta}(\Lambda)\phi_\beta$ Goldstone U Lorentz $D^{-1}\otimes$ D^{-1} Lorentz D^{-1} Goldstone

Proof 2 J^μ Q Q

$$Q = \int \mathrm{d}^3x J^0(\mathbf{x}, 0)$$

$$[Q, \phi_n(x)] = -\sum_m t_{nm}\phi_m(x)$$

$$\left\langle \left[J^\lambda(y), \phi_n(x)\right]\right\rangle_{\mathrm{VAC}} = (2\pi)^{-3} \int \mathrm{d}^4p \left[\rho_n^\lambda(p)\mathrm{e}^{\mathrm{i}p\cdot(y-x)} - \tilde{\rho}_n^\lambda(p)\mathrm{e}^{\mathrm{i}p\cdot(x-y)}\right] \tag{273}$$

$$\begin{aligned}(2\pi)^{-3}\mathrm{i}\rho_n^\lambda(p) = \sum_N \left\langle \mathrm{VAC}\left|J^\lambda(0)\right| N\right\rangle \langle N\left|\phi_n(0)\right| \mathrm{VAC}\rangle \delta^4(p - p_N), \\ (2\pi)^{-3}\mathrm{i}\tilde{\rho}_n^\lambda(p) = \sum_N \langle \mathrm{VAC}\left|\phi_n(0)\right| N\rangle \left\langle N\left|J^\lambda(0)\right| \mathrm{VAC}\right\rangle \delta^4(p - p_N).\end{aligned} \tag{274}$$

N ρ Lorentz p^μ

$$\begin{aligned}\rho_n^\lambda(p) &= p^\lambda \rho_n\left(-p^2\right)\theta\left(p^0\right)\\ \tilde{\rho}_n^\lambda(p) &= p^\lambda \tilde{\rho}_n\left(-p^2\right)\theta\left(p^0\right)\end{aligned} \tag{275}$$

$$\begin{aligned}\left\langle\left[J^\lambda(y),\phi_n(x)\right]\right\rangle_{\text{VAC}} =& \frac{\partial}{\partial y_\lambda}\int \mathrm{d}\mu^2\left[\rho_n\left(\mu^2\right)\Delta_+\left(y-x;\mu^2\right)\right.\\ &\left.+\tilde{\rho}_n\left(\mu^2\right)\Delta_+\left(x-y;\mu^2\right)\right]\end{aligned} \tag{276}$$

$\Delta_+\left(z;\mu^2\right)=(2\pi)^{-3}\int \mathrm{d}^4 p\theta\left(p^0\right)\delta\left(p^2+\mu^2\right)\mathrm{e}^{\mathrm{i}p\cdot z}$

$z^2>0$ Lorentz $\Delta_+\left(z;\mu^2\right)$ z^2 μ^2 $\Delta_+\left(z;\mu^2\right)$ $(x-y)$

$$\left\langle\left[J^\lambda(y),\phi_n(x)\right]\right\rangle_{\text{VAC}}=\frac{\partial}{\partial y_\lambda}\int \mathrm{d}\mu^2\left[\rho_n\left(\mu^2\right)+\tilde{\rho}_n\left(\mu^2\right)\right]\Delta_+\left(y-x;\mu^2\right) \tag{277}$$

$$\rho_n\left(\mu^2\right)=-\tilde{\rho}_n\left(\mu^2\right) \tag{278}$$

x,y

$$\left\langle\left[J^\lambda(y),\phi_n(x)\right]\right\rangle_{\text{VAC}}=\frac{\partial}{\partial y_\lambda}\int \mathrm{d}\mu^2\rho_n\left(\mu^2\right)\left[\Delta_+\left(y-x;\mu^2\right)-\Delta_+\left(x-y;\mu^2\right)\right] \tag{279}$$

Y^λ

$$\left(\square_y-\mu^2\right)\Delta_+\left(y-x;\mu^2\right)=0$$

x y

$$0=\int \mathrm{d}\mu^2\mu^2\rho_n\left(\mu^2\right)\left[\Delta_+\left(y-x;\mu^2\right)-\Delta_+\left(x-y;\mu^2\right)\right] \tag{280}$$

$$\mu^2\rho_n\left(\mu^2\right)=0 \tag{281}$$

$rho_n\left(\mu^2\right)=0$ $rho_n\left(\mu^2\right)\propto\delta(\mu^2)$ $\lambda=0, x^0=y^0=0$

$$\begin{aligned}\left\langle\left[J^0(\mathbf{y},t),\phi_n(\mathbf{x},t)\right]\right\rangle_{\text{VAC}} =& 2\mathrm{i}(2\pi)^{-3}\int \mathrm{d}\mu^2\rho_n\left(\mu^2\right)\\ &\times\int \mathrm{d}^4 p\sqrt{\mathbf{p}^2+\mu^2}\mathrm{e}^{\mathrm{i}\mathbf{p}\cdot(\mathbf{y}-\mathbf{x})}\theta(p_0)\delta\left(p^2+\mu^2\right)\\ =& \mathrm{i}\delta^3(\mathbf{y}-\mathbf{x})\int \mathrm{d}\mu^2\rho_n\left(\mu^2\right).\end{aligned} \tag{282}$$

$\int_{-\infty}^{+\infty} dk^0\delta\left(k^2+m^2\right)\theta\left(k^0\right)=\frac{1}{2\omega}$ y Q

$$-\sum_m t_{nm}\left\langle\phi_m\right\rangle_{\text{VAC}}=\mathrm{i}\int \mathrm{d}\mu^2\rho_n\left(\mu^2\right) \tag{283}$$

$$\rho_n\left(\mu^2\right)=\mathrm{i}\delta\left(\mu^2\right)\sum_m t_{nm}\left\langle\phi_m(0)\right\rangle_{\text{VAC}} \tag{284}$$

$\rho_n\left(\mu^2\right)$ $\delta\left(\mu^2\right)$ $\phi_n(0)|\text{VAC}\rangle$ [24] $\langle N\left|\phi_n(0)\right|\text{ VAC }\rangle$ J_0 N $\langle VA$
Goldstone J_0

[24] ϕ