Métodos Computacionais para Equações Diferenciais
Tarefa 2
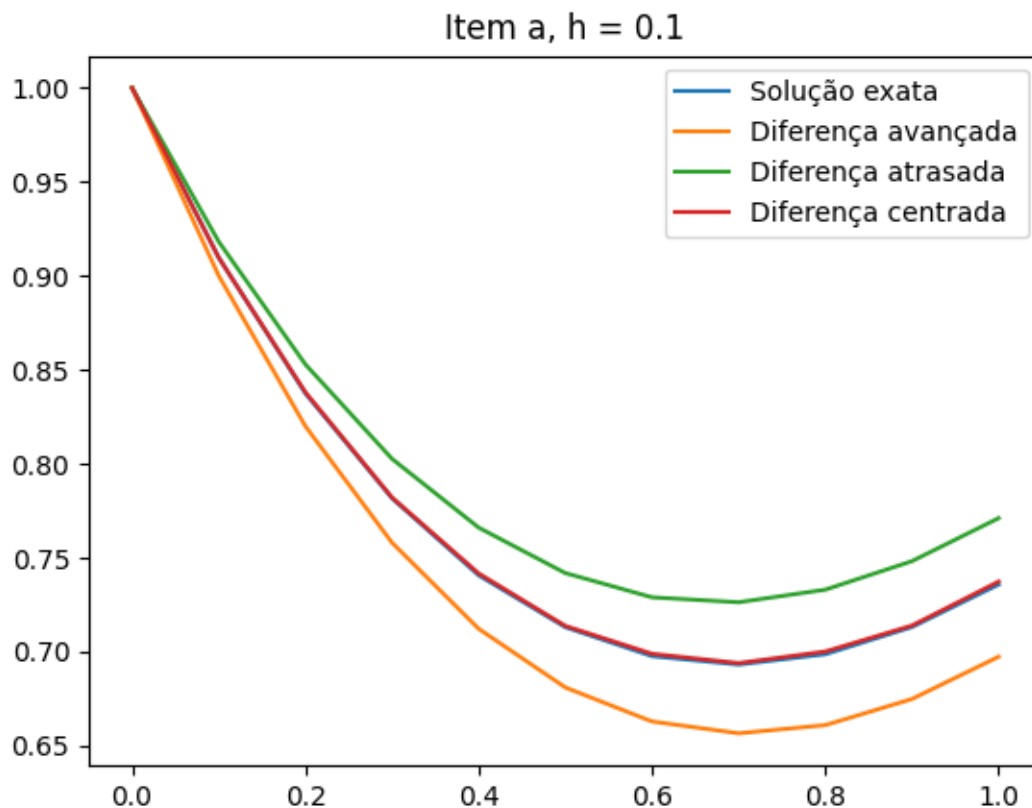
Daniel Serezane

2024, 1º Semestre

**a)**

$$\begin{cases} y' = x - y, x \in [0, 1] \\ y(0) = 1 \end{cases}$$

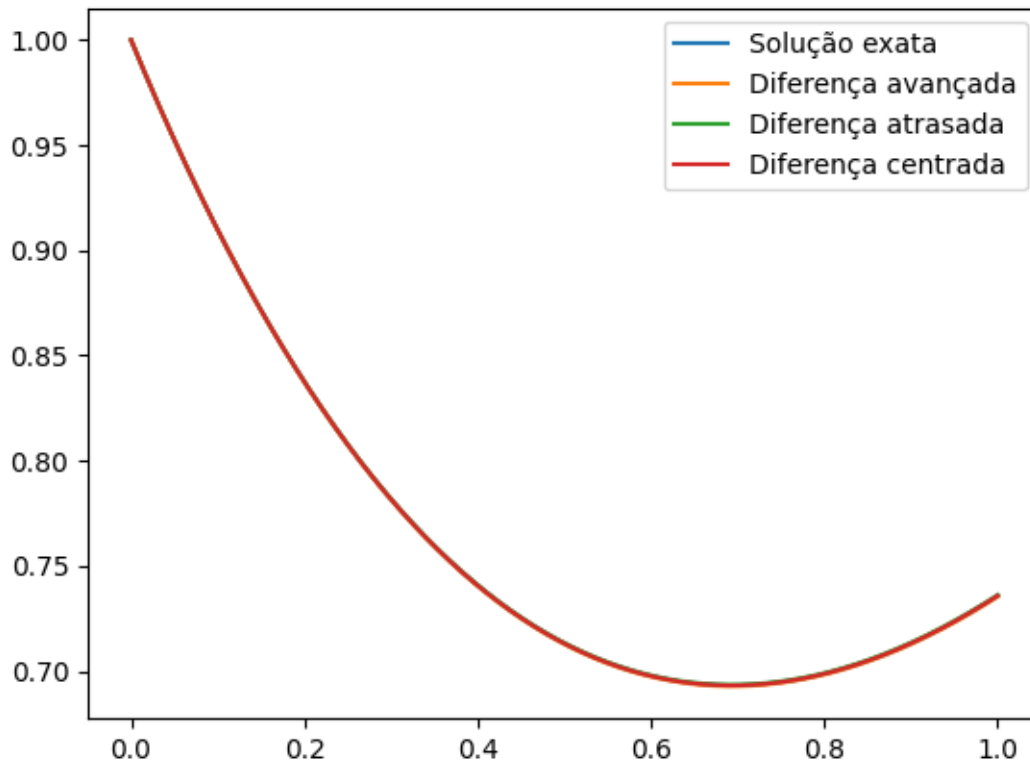Solução exata: $y(x) = x - 1 + 2e^{-x}$

Item a, h = 0.1

x	y(x)	y_avn	err_loc_avn	err_rel_avn	y_atr	err_loc_atr	err_rel_atr	y_cen	err_loc_cen	err_rel_cen
0	1	1	0	0	1	0	0	1	0	0
0.1	0.909675	0.9	0.00967484	0.0106355	0.918182	-0.00850698	0.00935167	0.909675	0	0
0.2	0.837462	0.82	0.0174615	0.0208505	0.852893	-0.0154311	0.018426	0.838065	-0.000603527	0.000720662
0.3	0.781636	0.758	0.0236364	0.0302397	0.80263	-0.0209932	0.026858	0.782062	-0.000425388	0.000544228
0.4	0.74064	0.7122	0.0284401	0.0383993	0.766027	-0.0253868	0.0342769	0.741653	-0.00101257	0.00136716
0.5	0.713061	0.68098	0.0320813	0.044991	0.741843	-0.0287813	0.040363	0.713731	-0.000669977	0.000939578
0.6	0.697623	0.662882	0.0347413	0.0497995	0.728948	-0.0313246	0.0449019	0.698906	-0.00128314	0.0018393
0.7	0.693171	0.656594	0.0365768	0.0527674	0.726316	-0.0331456	0.0478174	0.69395	-0.000779407	0.00112441
0.8	0.698658	0.660934	0.0377235	0.0539942	0.733015	-0.0343568	0.0491755	0.700116	-0.00145848	0.00208754
0.9	0.713139	0.674841	0.0382983	0.0537039	0.748195	-0.0350559	0.0491572	0.713927	-0.000787414	0.00110415
1	0.735759	0.697357	0.038402	0.0521937	0.771087	-0.0353277	0.0480153	0.737331	-0.00157218	0.00213681

Item a, h = 0.01
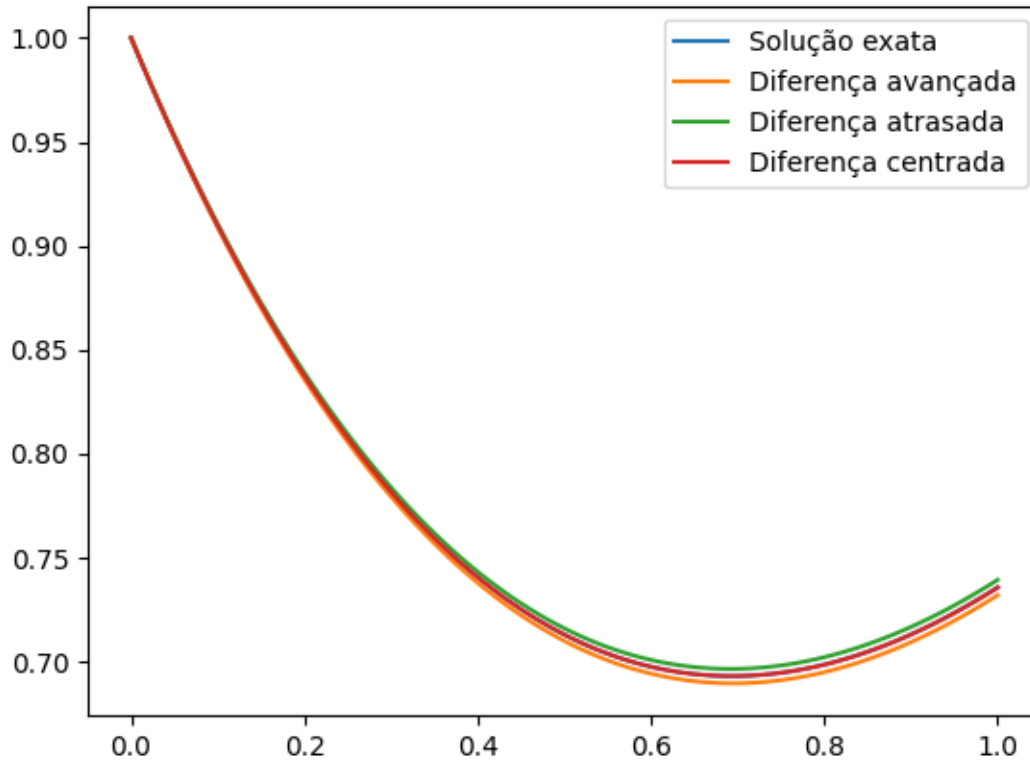
x	y(x)	y_avn	err_loc_avn	err_rel_avn	y_atr	err_loc_atr	err_rel_atr	y_cen	err_loc_cen	err_rel_cen
0	1	1	0	0	1	0	0	1	0	0
0.1	0.909675	0.908764	0.000910686	0.00100111	0.910574	-0.000899073	0.000988346	0.909678	-3.04904e-06	3.3518e-06
0.2	0.837462	0.835814	0.00164763	0.00196741	0.839089	-0.00162743	0.00194329	0.837467	-5.52441e-06	6.59661e-06
0.3	0.781636	0.779401	0.00223569	0.00286027	0.783846	-0.00220939	0.00282663	0.781644	-7.50835e-06	9.60594e-06
0.4	0.74064	0.737944	0.00269657	0.00364087	0.743306	-0.00266619	0.00359984	0.740649	-9.07277e-06	1.22499e-05
0.5	0.713061	0.710012	0.00304919	0.00427619	0.716078	-0.00301633	0.00423011	0.713072	-1.02804e-05	1.44172e-05
0.6	0.697623	0.694313	0.00330999	0.00474466	0.700899	-0.00327596	0.00469589	0.697634	-1.11859e-05	1.60343e-05
0.7	0.693171	0.689677	0.00349329	0.00503958	0.69663	-0.00345911	0.00499027	0.693182	-1.18369e-05	1.70764e-05
0.8	0.698658	0.695046	0.0036115	0.0051692	0.702236	-0.00357795	0.00512118	0.69867	-1.22747e-05	1.7569e-05
0.9	0.713139	0.709464	0.00367537	0.0051538	0.716782	-0.00364305	0.00510847	0.713152	-1.25354e-05	1.75777e-05
1	0.735759	0.732065	0.0036942	0.00502094	0.739422	-0.00366354	0.00497927	0.735772	-1.265e-05	1.71931e-05

Item a, h = 0.001

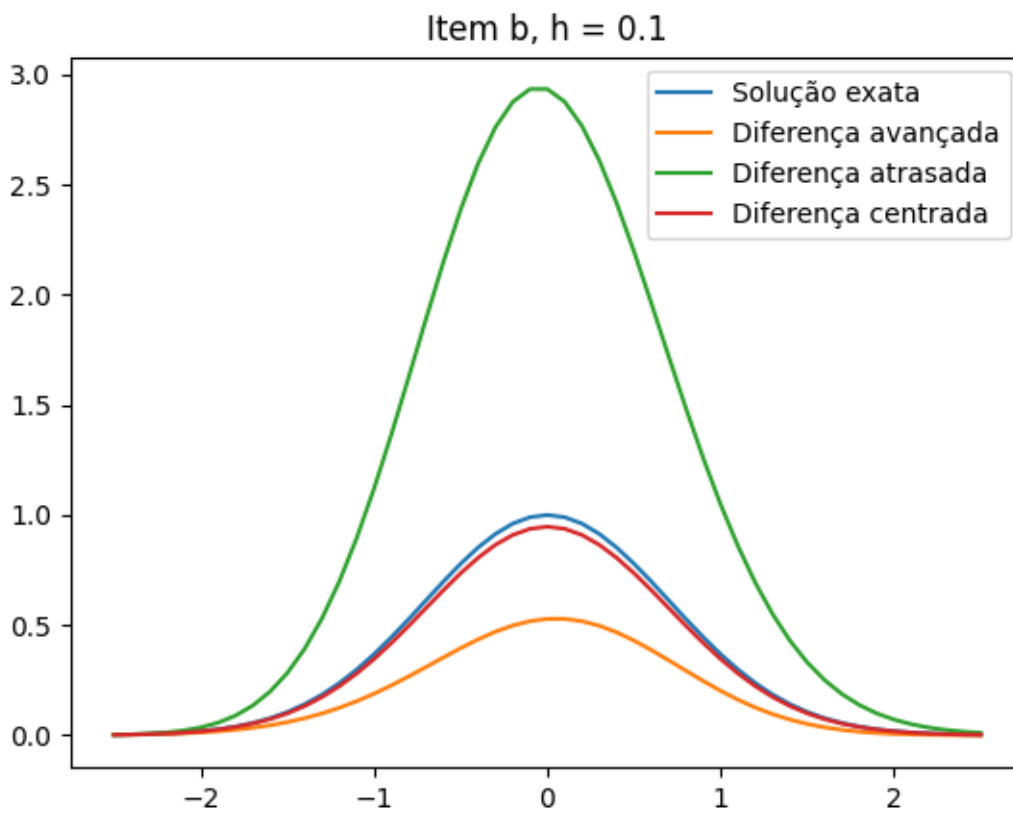| x	y(x)	y_avn	err_loc_avn	err_rel_avn	y_atr	err_loc_atr	err_rel_atr	y_cen	err_loc_cen	err_rel_cen
0	1	1	0	0	1	0	0	1	0	0
0.1	0.909675	0.909584	9.05418e-05	9.95321e-05	0.909765	-9.04257e-05	9.94044e-05	0.909675	-3.01946e-08	3.31927e-08
0.2	0.837462	0.837298	0.000163847	0.000195647	0.837625	-0.000163645	0.000195406	0.837462	-5.46491e-08	6.52556e-08
0.3	0.781636	0.781414	0.000222377	0.000284502	0.781859	-0.000222114	0.000284165	0.781637	-7.41832e-08	9.49075e-08
0.4	0.74064	0.740372	0.00026828	0.000362227	0.740908	-0.000267976	0.000361817	0.74064	-8.95127e-08	1.20859e-07
0.5	0.713061	0.712758	0.00030343	0.000425531	0.713364	-0.000303101	0.00042507	0.713061	-1.01262e-07	1.4201e-07
0.6	0.697623	0.697294	0.000329457	0.000472257	0.697952	-0.000329117	0.000471769	0.697623	-1.09974e-07	1.57641e-07
0.7	0.693171	0.692823	0.000347781	0.000501725	0.693518	-0.000347439	0.000501231	0.693171	-1.16122e-07	1.67524e-07
0.8	0.698658	0.698298	0.000359631	0.000514745	0.699017	-0.000359296	0.000514265	0.698658	-1.20117e-07	1.71925e-07
0.9	0.713139	0.712773	0.000366074	0.000513328	0.713505	-0.000365751	0.000512875	0.713139	-1.22313e-07	1.71513e-07
1	0.735759	0.735391	0.000368033	0.000500208	0.736127	-0.000367726	0.000499792	0.735759	-1.23018e-07	1.67198e-07

Item a, h = 0.0001
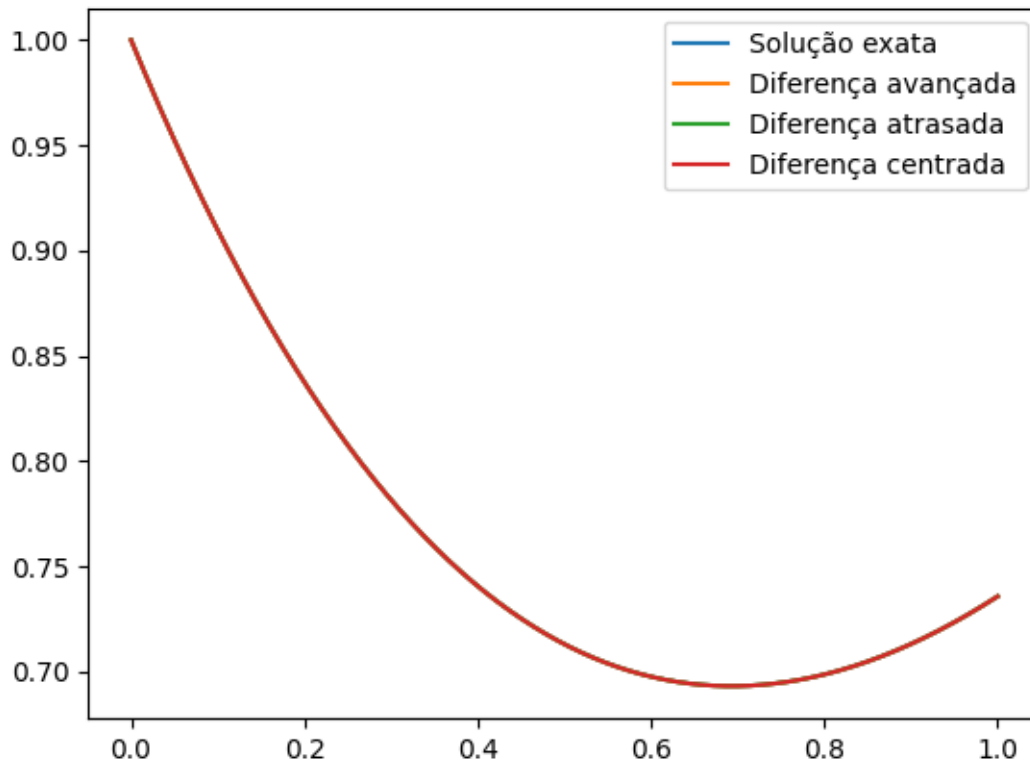
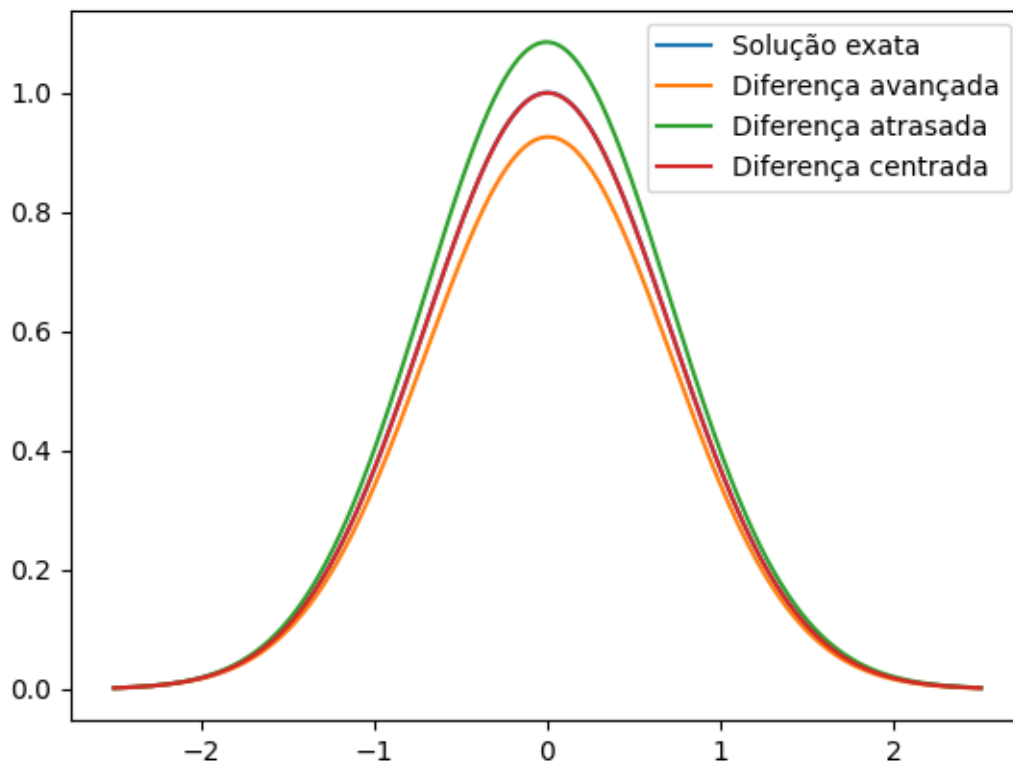| x	y(x)	y_avn	err_loc_avn	err_rel_avn	y_atr	err_loc_atr	err_rel_atr	y_cen	err_loc_cen	err_rel_cen
0	1	1	0	0	1	0	0	1	0	0
0.1	0.909675	0.909666	9.04895e-06	9.94746e-06	0.909684	-9.04779e-06	9.94618e-06	0.909675	-3.01646e-10	3.31598e-10
0.2	0.837462	0.837445	1.63756e-05	1.95539e-05	0.837478	-1.63736e-05	1.95515e-05	0.837462	-5.45888e-10	6.51836e-10
0.3	0.781636	0.781614	2.22259e-05	2.8435e-05	0.781659	-2.22232e-05	2.84317e-05	0.781636	-7.40922e-10	9.47911e-10
0.4	0.74064	0.740613	2.68143e-05	3.62043e-05	0.740667	-2.68113e-05	3.62002e-05	0.74064	-8.93898e-10	1.20693e-09
0.5	0.713061	0.713031	3.03282e-05	4.25324e-05	0.713092	-3.03249e-05	4.25277e-05	0.713061	-1.01106e-09	1.41791e-09
0.6	0.697623	0.69759	3.29304e-05	4.72037e-05	0.697656	-3.2927e-05	4.71988e-05	0.697623	-1.09784e-09	1.57368e-09
0.7	0.693171	0.693136	3.47627e-05	5.01503e-05	0.693205	-3.47593e-05	5.01453e-05	0.693171	-1.15895e-09	1.67196e-09
0.8	0.698658	0.698622	3.5948e-05	5.14529e-05	0.698694	-3.59446e-05	5.14481e-05	0.698658	-1.19851e-09	1.71544e-09
0.9	0.713139	0.713103	3.65929e-05	5.13124e-05	0.713176	-3.65897e-05	5.13079e-05	0.713139	-1.22005e-09	1.71082e-09
1	0.735759	0.735722	3.67895e-05	5.00021e-05	0.735796	-3.67864e-05	4.99979e-05	0.735759	-1.22666e-09	1.6672e-09

b)

$$\begin{cases} y' = -2xy, x \in [-2.5, 2.5] \\ y(-2.5) = 0.0019304541362277093 \end{cases}$$

Solução exata: $y(x) = e^{-x^2}$

x	y(x)	y_avn	err_loc_avn	err_rel_avn	y_atr	err_loc_atr	err_rel_atr	y_cen	err_loc_cen	err_rel_cen
-2.5	0.00193045	0.00193045	0	0	0.00193045	0	0	0.00193045	0	0
-2.4	0.00315111	0.00289568	0.00025543	0.0810604	0.00371241	-0.0005613	0.178128	0.00315111	0	0
-2.3	0.00504176	0.00428561	0.000756152	0.149978	0.00687484	-0.00183308	0.363579	0.00495552	8.6239e-05	0.0171049
-2.2	0.00790705	0.00625699	0.00165007	0.208683	0.0122765	-0.00436944	0.5526	0.00771019	0.000196863	0.0248971
-2.1	0.0121552	0.00901006	0.00314512	0.258747	0.0211664	-0.00901119	0.741346	0.0117405	0.000414689	0.0341162
-2	0.0183156	0.0127943	0.00552135	0.301455	0.0352773	-0.0169616	0.926074	0.0175722	0.000743437	0.0405903
-1.9	0.0270518	0.017912	0.00913984	0.337864	0.0568988	-0.029847	1.10333	0.0257983	0.0012536	0.0463405
-1.8	0.0391639	0.0247186	0.0144453	0.368843	0.0889044	-0.0497405	1.27006	0.0371789	0.00198502	0.050685
-1.7	0.0555762	0.0336173	0.021959	0.395114	0.134704	-0.0791275	1.42377	0.052567	0.00300917	0.054145
-1.6	0.0773047	0.0450471	0.0322576	0.417279	0.198094	-0.120789	1.5625	0.0729245	0.00438028	0.0566625
-1.5	0.105399	0.0594622	0.045937	0.435838	0.282991	-0.177592	1.68494	0.0992387	0.00616053	0.0584495
-1.4	0.140858	0.0773008	0.0635576	0.451216	0.393043	-0.252185	1.79034	0.132468	0.00839074	0.0595686
-1.3	0.18452	0.0989451	0.0855744	0.463769	0.531139	-0.34662	1.8785	0.173421	0.0110989	0.0601504
-1.2	0.236928	0.124671	0.112257	0.473802	0.698867	-0.46194	1.94971	0.222646	0.0142814	0.0602773
-1.1	0.298197	0.154592	0.143605	0.481579	0.895984	-0.597787	2.00467	0.280291	0.0179064	0.0600489
-1	0.367879	0.188602	0.179277	0.487327	1.11998	-0.7521	2.04442	0.345974	0.0219051	0.0595442
-0.9	0.444858	0.226322	0.218536	0.491248	1.36583	-0.920971	2.07026	0.418681	0.0261775	0.0588445
-0.8	0.527292	0.26706	0.260232	0.493525	1.62599	-1.09869	2.08365	0.496699	0.030593	0.0580191
-0.7	0.612626	0.30979	0.302836	0.494325	1.89068	-1.27806	2.08619	0.577624	0.035002	0.0571343
-0.6	0.697676	0.353161	0.344516	0.493804	2.1485	-1.45083	2.07951	0.658434	0.0392421	0.0562469
-0.5	0.778801	0.39554	0.383261	0.492117	2.38723	-1.60842	2.06526	0.735649	0.0431522	0.0554085
-0.4	0.852144	0.435094	0.41705	0.489412	2.59481	-1.74267	2.04504	0.805564	0.0465798	0.054662
-0.3	0.913931	0.469902	0.44403	0.485846	2.76044	-1.8465	2.0204	0.864539	0.0493923	0.0540438
-0.2	0.960789	0.498096	0.462694	0.481577	2.87545	-1.91466	1.9928	0.909309	0.0514808	0.0535818
-0.1	0.99005	0.518019	0.47203	0.476774	2.93414	-1.94409	1.96363	0.937284	0.0527663	0.0532966
0	1	0.52838	0.47162	0.47162	2.93414	-1.93414	1.93414	0.9468	0.0532001	0.0532001
0.1	0.99005	0.52838	0.46167	0.46631	2.87661	-1.88656	1.90552	0.937284	0.0527663	0.0532966
0.2	0.960789	0.517812	0.442977	0.461055	2.76597	-1.80518	1.87885	0.909309	0.0514808	0.0535818
0.3	0.913931	0.4971	0.416831	0.456086	2.6094	-1.69547	1.85514	0.864539	0.0493923	0.0540438
0.4	0.852144	0.467274	0.38487	0.451649	2.41611	-1.56397	1.83534	0.805564	0.0465798	0.054662
0.5	0.778801	0.429892	0.348909	0.448008	2.19647	-1.41767	1.82032	0.735649	0.0431522	0.0554085
0.6	0.697676	0.386903	0.310774	0.445441	1.96113	-1.26345	1.81095	0.658434	0.0392421	0.0562469
0.7	0.612626	0.340474	0.272152	0.444238	1.72029	-1.10766	1.80806	0.577624	0.035002	0.0571343
0.8	0.527292	0.292808	0.234484	0.444695	1.48301	-0.955716	1.8125	0.496699	0.030593	0.0580191
0.9	0.444858	0.245959	0.198899	0.447108	1.25679	-0.811929	1.82514	0.418681	0.0261775	0.0588445
1	0.367879	0.201686	0.166193	0.45176	1.04732	-0.679443	1.84692	0.345974	0.0219051	0.0595442
1.1	0.298197	0.161349	0.136848	0.458919	0.858461	-0.560264	1.87884	0.280291	0.0179064	0.0600489
1.2	0.236928	0.125852	0.111076	0.468816	0.692307	-0.45538	1.92202	0.222646	0.0142814	0.0602773
1.3	0.18452	0.0956476	0.0888719	0.48164	0.54945	-0.364931	1.97774	0.173421	0.0110989	0.0601504
1.4	0.140858	0.0707792	0.0700792	0.497515	0.429258	-0.2884	2.04744	0.132468	0.00839074	0.0595686
1.5	0.105399	0.0509611	0.0544382	0.516495	0.330198	-0.224799	2.13284	0.0992387	0.00616053	0.0584495
1.6	0.0773047	0.0356727	0.041632	0.538544	0.25015	-0.172846	2.2359	0.0729245	0.00438028	0.0566625
1.7	0.0555762	0.0242575	0.0313188	0.563528	0.186679	-0.131103	2.35898	0.052567	0.00300917	0.054145
1.8	0.0391639	0.0160099	0.023154	0.591207	0.137264	-0.0981004	2.50487	0.0371789	0.00198502	0.050685
1.9	0.0270518	0.0102464	0.0168055	0.621233	0.0994668	-0.072415	2.6769	0.0257983	0.0012536	0.0463405
2	0.0183156	0.00635274	0.0119629	0.653152	0.0710477	-0.0527321	2.87908	0.0175722	0.000743437	0.0405903
2.1	0.0121552	0.00381164	0.00834354	0.686418	0.0500336	-0.0378784	3.11624	0.0117405	0.000414689	0.0341162
2.2	0.00790705	0.00221075	0.0056963	0.720408	0.0347456	-0.0268385	3.39425	0.00771019	0.000196863	0.0248971
2.3	0.00504176	0.00123802	0.00380374	0.754447	0.0237983	-0.0187566	3.72024	0.00495552	8.6239e-05	0.0171049
2.4	0.00315111	0.000668532	0.00248258	0.787843	0.01608	-0.0129288	4.10295	0.00315111	8.88178e-16	2.81862e-13
2.5	0.00193045	0.000347636	0.00158282	0.81992	0.01072	-0.00878952	4.55308	0.00193045	-1.5567e-15	8.06389e-13

Item b, h = 0.01

Solução exata
Diferença avançada
Diferença atrasada
Diferença centrada

x	y(x)	y_avn	err_loc_avn	err_rel_avn	y_atr	err_loc_atr	err_rel_atr	y_cen	err_loc_cen	err_rel_cen
-2.5	0.00193045	0.00193045	0	0	0.00193045	0	0	0.00193045	0	0
-2.4	0.00315111	0.00311765	3.34591e-05	0.0106182	0.00318712	-3.60088e-05	0.0114273	0.00315066	4.4714e-07	0.000141899
-2.3	0.00504176	0.00493979	0.000101968	0.0202248	0.00515246	-0.000110699	0.0219564	0.00504042	1.34124e-06	0.000266026
-2.2	0.00790705	0.00767868	0.000228376	0.0288825	0.00815693	-0.000249879	0.031602	0.00790411	2.94275e-06	0.000372167
-2.1	0.0121552	0.0117097	0.000445492	0.0366504	0.012646	-0.000490861	0.0403828	0.0121496	5.60827e-06	0.000461389
-2	0.0183156	0.0175174	0.000798262	0.0435836	0.0192007	-0.000885031	0.0483211	0.0183058	9.80154e-06	0.000535146
-1.9	0.0270518	0.0257064	0.00134541	0.0497346	0.0285517	-0.00149981	0.0554422	0.0270358	1.60952e-05	0.000594975
-1.8	0.0391639	0.0370039	0.00215999	0.0551526	0.0415832	-0.00241935	0.0617749	0.0391387	2.51588e-05	0.000642398
-1.7	0.0555762	0.052248	0.00332818	0.059885	0.0593193	-0.00374308	0.0673503	0.0555385	3.77294e-05	0.000678876
-1.6	0.0773047	0.0723591	0.00494568	0.0639765	0.0828863	-0.00558158	0.0722023	0.0772502	5.45618e-05	0.000705802
-1.5	0.105399	0.0982879	0.00711132	0.0674703	0.113448	-0.008049	0.0763668	0.105323	7.63608e-05	0.000724491
-1.4	0.140858	0.130941	0.00991756	0.070408	0.15211	-0.011252	0.0798815	0.140755	0.000103697	0.000736181
-1.3	0.18452	0.171081	0.0134385	0.0728298	0.199795	-0.0152756	0.082786	0.184383	0.000136919	0.000742029
-1.2	0.236928	0.219212	0.0177162	0.0747747	0.257095	-0.0201675	0.0851211	0.236752	0.000176064	0.000743114
-1.1	0.298197	0.275451	0.0227467	0.0762809	0.324119	-0.0259219	0.0869288	0.297976	0.000220796	0.000740435
-1	0.367879	0.339411	0.0284686	0.0773857	0.400346	-0.0324661	0.088252	0.367609	0.000270358	0.00073491
-0.9	0.444858	0.410103	0.034755	0.078126	0.48451	-0.0396522	0.0891345	0.444534	0.000323581	0.00072738
-0.8	0.527292	0.48588	0.0414126	0.0785382	0.574549	-0.0472562	0.0896204	0.526914	0.000378914	0.000718603
-0.7	0.612626	0.564438	0.0481881	0.0786583	0.667612	-0.0549859	0.0897544	0.612192	0.000434511	0.000709259
-0.6	0.697676	0.642893	0.054783	0.0785221	0.760175	-0.0624986	0.0895811	0.697188	0.000488337	0.000699948
-0.5	0.778801	0.717925	0.0608754	0.0781656	0.848227	-0.0694265	0.0891454	0.778262	0.000538299	0.00069119
-0.4	0.852144	0.785997	0.0661472	0.0776244	0.927552	-0.0754081	0.0884922	0.851561	0.000582376	0.000683424
-0.3	0.913931	0.843618	0.070313	0.0769347	0.994052	-0.0801209	0.0876662	0.913312	0.000618741	0.000677011
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-0.1	0.99005	0.915545	0.0745052	0.075254	1.07487	-0.0848207	0.0856731	0.989387	0.000662621	0.000669281
0	1	0.925664	0.0743359	0.0743359	1.08459	-0.0845942	0.0845942	0.999332	0.000668285	0.000668285
0.1	0.99005	0.917365	0.0726845	0.073415	1.07274	-0.0826874	0.0835185	0.989387	0.000662621	0.000669281
0.2	0.960789	0.891105	0.0696847	0.0725286	1.04004	-0.0792547	0.0824892	0.960144	0.000645871	0.000672229
0.3	0.913931	0.84839	0.0655417	0.071714	0.988461	-0.0745303	0.0815492	0.913312	0.000618741	0.000677011
0.4	0.852144	0.791634	0.0605099	0.071009	0.920947	-0.0688029	0.080741	0.851561	0.000582376	0.000683424
0.5	0.778801	0.723933	0.0548677	0.0704515	0.841188	-0.0623874	0.080107	0.778262	0.000538299	0.00069119
0.6	0.697676	0.648783	0.048893	0.0700797	0.753274	-0.0555973	0.0796892	0.697188	0.000488337	0.000699948
0.7	0.612626	0.569784	0.0428421	0.0699318	0.661348	-0.048722	0.0795297	0.612192	0.000434511	0.000709259
0.8	0.527292	0.490358	0.0369348	0.0700461	0.569302	-0.0420096	0.0796704	0.526914	0.000378914	0.000718603
0.9	0.444858	0.413513	0.0313451	0.0704609	0.480515	-0.0356568	0.0801533	0.444534	0.000323581	0.00072738
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1.1	0.298197	0.276625	0.0215726	0.0723434	0.322743	-0.0245462	0.0823153	0.297976	0.000220796	0.000740435
1.2	0.236928	0.219422	0.0175057	0.0738862	0.256849	-0.0199209	0.08408	0.236752	0.000176064	0.000743114
1.3	0.18452	0.170518	0.0140012	0.0758791	0.200454	-0.0159349	0.0863589	0.184383	0.000136919	0.000742029
1.4	0.140858	0.129821	0.0110374	0.078358	0.153422	-0.0125641	0.0891964	0.140755	0.000103697	0.000736181
1.5	0.105399	0.0968242	0.00857504	0.0813577	0.115163	-0.00976403	0.0926385	0.105323	7.63608e-05	0.000724491
1.6	0.0773047	0.0707406	0.00656409	0.0849119	0.0847826	-0.00747786	0.0967322	0.0772502	5.45618e-05	0.000705802
1.7	0.0555762	0.050627	0.00494922	0.0890529	0.0612187	-0.00564244	0.101526	0.0555385	3.77294e-05	0.000678876
1.8	0.0391639	0.0354899	0.00367402	0.0938114	0.0433572	-0.00419332	0.107071	0.0391387	2.51588e-05	0.000642398
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2	0.0183156	0.0163871	0.00192852	0.105294	0.020525	-0.00220935	0.120626	0.0183058	9.80154e-06	0.000535146
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2.2	0.00790705	0.00696166	0.00094539	0.119563	0.00899705	-0.00109	0.137852	0.00790411	2.94275e-06	0.000372167
2.3	0.00504176	0.00439745	0.000644312	0.127795	0.00578792	-0.000746159	0.147996	0.00504042	1.34124e-06	0.000266026
2.4	0.00315111	0.0027201	0.000431013	0.136781	0.00365293	-0.00050182	0.159252	0.00315066	4.4714e-07	0.000141899
2.5	0.00193045	0.00164758	0.000282877	0.146534	0.0022619	-0.000331445	0.171693	0.00193045	7.83835e-15	4.06036e-12
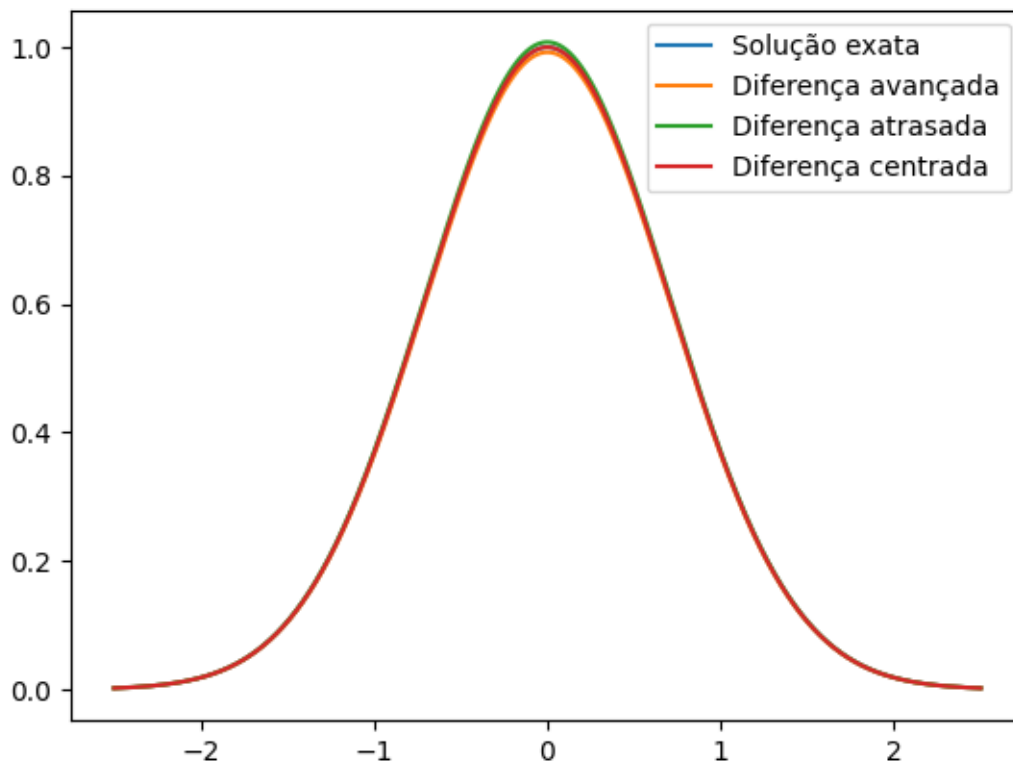
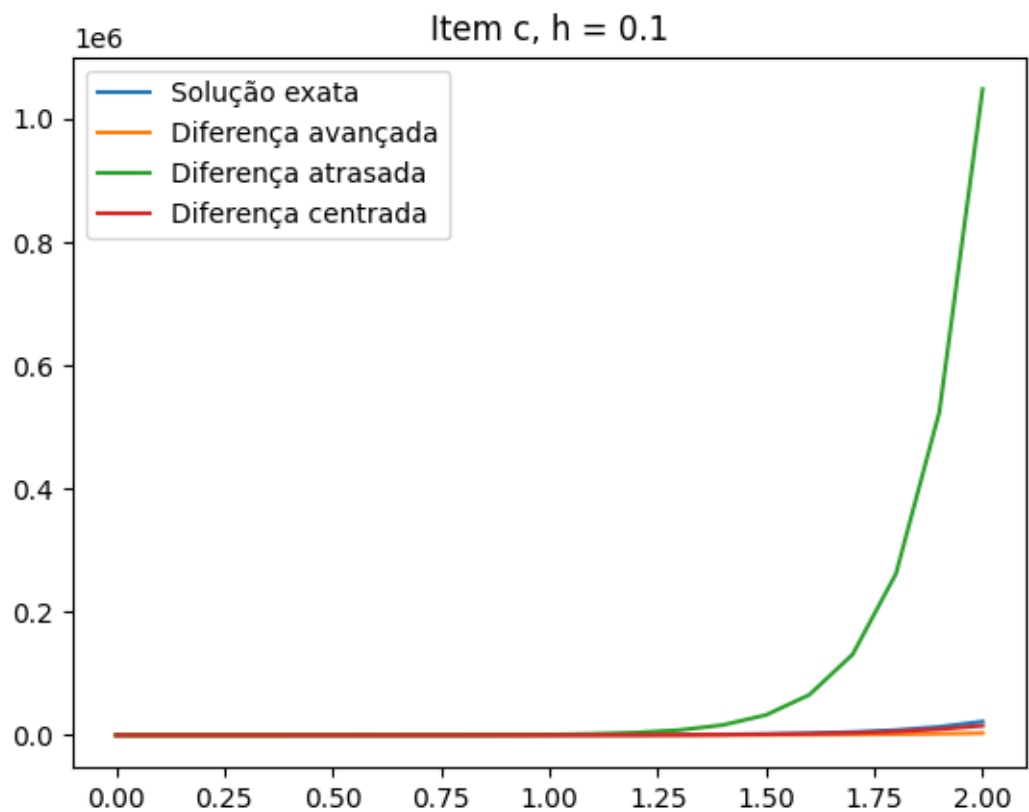| x	y(x)	y_avn	err_loc_avn	err_rel_avn	y_atr	err_loc_atr	err_rel_atr	y_cen	err_loc_cen	err_rel_cen
-2.5	0.00193045	0.00193045	0	0	0.00193045	0	0	0.00193045	0	0
-2.4	0.00315111	0.00314766	3.45565e-06	0.00109664	0.00315459	-3.4811e-06	0.00110472	0.00315111	4.62159e-09	1.46666e-06
-2.3	0.00504176	0.00503119	1.05712e-05	0.00209673	0.00505242	-1.06583e-05	0.00211401	0.00504175	1.37438e-08	2.72599e-06
-2.2	0.00790705	0.0078833	2.37567e-05	0.00300449	0.00793103	-2.39713e-05	0.00303163	0.00790702	3.00056e-08	3.79479e-06
-2.1	0.0121552	0.0121087	4.64831e-05	0.00382414	0.0122021	-4.69357e-05	0.00386138	0.0121551	5.7012e-08	4.69034e-06
-2	0.0183156	0.0182321	8.35161e-05	0.00455982	0.0184	-8.43816e-05	0.00460708	0.0183155	9.94465e-08	5.42959e-06
-1.9	0.0270518	0.0269108	0.000141094	0.00521568	0.0271945	-0.000142634	0.00527261	0.0270517	1.63091e-07	6.02884e-06
-1.8	0.0391639	0.0389369	0.000226987	0.00579582	0.0393935	-0.000229573	0.00586186	0.0391636	2.54708e-07	6.50365e-06
-1.7	0.0555762	0.0552258	0.00035037	0.00630432	0.0559307	-0.000354507	0.00637876	0.0555758	3.81742e-07	6.8688e-06
-1.6	0.0773047	0.0767833	0.000521437	0.00674522	0.0778325	-0.000527778	0.00682724	0.0773042	5.51824e-07	7.1383e-06
-1.5	0.105399	0.104649	0.000750713	0.00712257	0.106159	-0.000760062	0.00721127	0.105398	7.72084e-07	7.32533e-06
-1.4	0.140858	0.13981	0.00104804	0.00744038	0.14192	-0.00106134	0.00753483	0.140857	1.04831e-06	7.44229e-06
-1.3	0.18452	0.183098	0.00142129	0.00770266	0.185959	-0.00143961	0.00780191	0.184518	1.38404e-06	7.50079e-06
-1.2	0.236928	0.235053	0.0018749	0.0079134	0.238827	-0.00189934	0.00801654	0.236926	1.77971e-06	7.51162e-06
-1.1	0.298197	0.295789	0.00240841	0.00807657	0.300637	-0.00244006	0.00818271	0.298195	2.23194e-06	7.48479e-06
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-0.9	0.444858	0.441176	0.0036817	0.00827612	0.448589	-0.00373052	0.00838586	0.444855	3.27154e-06	7.35412e-06
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-0.7	0.612626	0.607521	0.00510503	0.00833302	0.617799	-0.00517279	0.00844363	0.612622	4.39424e-06	7.17279e-06
-0.6	0.697676	0.691873	0.00580318	0.00831787	0.703556	-0.00588009	0.00842811	0.697671	4.93929e-06	7.07963e-06
-0.5	0.778801	0.772353	0.00644764	0.00827893	0.785334	-0.00653288	0.00838838	0.778795	5.44537e-06	6.99199e-06
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-0.3	0.913931	0.906487	0.00744442	0.00814549	0.921473	-0.00754219	0.00825247	0.913925	6.26054e-06	6.85013e-06
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-0.1	0.99005	0.982165	0.00788512	0.00796437	0.998038	-0.00798795	0.00806823	0.990043	6.7054e-06	6.77279e-06
0	1	0.992134	0.00786583	0.00786583	1.00797	-0.00796809	0.00796809	0.999993	6.76283e-06	6.76283e-06
0.1	0.99005	0.98236	0.00768998	0.00776727	0.99784	-0.0077897	0.00786798	0.990043	6.7054e-06	6.77279e-06
0.2	0.960789	0.953418	0.00737179	0.00767264	0.968257	-0.00746719	0.00777193	0.960783	6.53557e-06	6.80229e-06
0.3	0.913931	0.906998	0.00693302	0.00758593	0.920954	-0.00702262	0.00768397	0.913925	6.26054e-06	6.85013e-06
0.4	0.852144	0.845743	0.00640054	0.00751111	0.858627	-0.00648321	0.00760812	0.852138	5.89197e-06	6.91429e-06
0.5	0.778801	0.772997	0.00580374	0.00745215	0.784679	-0.0058787	0.0075484	0.778795	5.44537e-06	6.99199e-06
0.6	0.697676	0.692504	0.00517191	0.00741305	0.702915	-0.00523874	0.00750884	0.697671	4.93929e-06	7.07963e-06
0.7	0.612626	0.608094	0.00453207	0.00739777	0.617217	-0.00459068	0.00749345	0.612622	4.39424e-06	7.17279e-06
0.8	0.527292	0.523385	0.0039074	0.00741031	0.53125	-0.00395799	0.00750625	0.527289	3.83146e-06	7.26629e-06
0.9	0.444858	0.441542	0.00331625	0.00745464	0.448217	-0.00335924	0.00755126	0.444855	3.27154e-06	7.35412e-06
1	0.367879	0.365108	0.00277187	0.00753474	0.370687	-0.00280784	0.0076325	0.367877	2.73316e-06	7.42949e-06
1.1	0.298197	0.295915	0.00228258	0.00765459	0.300509	-0.00231222	0.00775399	0.298195	2.23194e-06	7.48479e-06
1.2	0.236928	0.235075	0.00185234	0.00781815	0.238804	-0.00187641	0.00791977	0.236926	1.77971e-06	7.51162e-06
1.3	0.18452	0.183038	0.00148158	0.0080294	0.18602	-0.00150086	0.00813387	0.184518	1.38404e-06	7.50079e-06
1.4	0.140858	0.13969	0.00116804	0.0082923	0.142042	-0.00118326	0.00840034	0.140857	1.04831e-06	7.44229e-06
1.5	0.105399	0.104492	0.000907569	0.00861077	0.106319	-0.000919421	0.00872322	0.105398	7.72084e-07	7.32532e-06
1.6	0.0773047	0.0766099	0.000694874	0.00898877	0.0780087	-0.000703983	0.00910659	0.0773042	5.51824e-07	7.1383e-06
1.7	0.0555762	0.0550521	0.000524095	0.0094302	0.0561072	-0.000531005	0.00955453	0.0555758	3.81742e-07	6.8688e-06
1.8	0.0391639	0.0387746	0.000389249	0.00993898	0.0395583	-0.000394425	0.0100711	0.0391636	2.54708e-07	6.50365e-06
1.9	0.0270518	0.0267673	0.000284558	0.010519	0.0273402	-0.000288387	0.0106605	0.0270517	1.63091e-07	6.02884e-06
2	0.0183156	0.018111	0.00020466	0.011174	0.0185231	-0.000207459	0.0113269	0.0183155	9.94465e-08	5.42959e-06
2.1	0.0121552	0.0120104	0.000144744	0.011908	0.0123019	-0.000146765	0.0120743	0.0121551	5.7012e-08	4.69034e-06
2.2	0.00790705	0.00780644	0.000100615	0.0127248	0.00800911	-0.000102057	0.012907	0.00790702	3.00056e-08	3.79479e-06
2.3	0.00504176	0.00497305	6.87089e-05	0.013628	0.00511148	-6.97239e-05	0.0138293	0.00504175	1.37438e-08	2.726e-06
2.4	0.00315111	0.00310504	4.60736e-05	0.0146214	0.00319789	-4.67793e-05	0.0148453	0.00315111	4.62162e-09	1.46666e-06
2.5	0.00193045	0.00190013	3.0325e-05	0.0157088	0.00196126	-3.0809e-05	0.0159595	0.00193045	3.70899e-14	1.9213e-11

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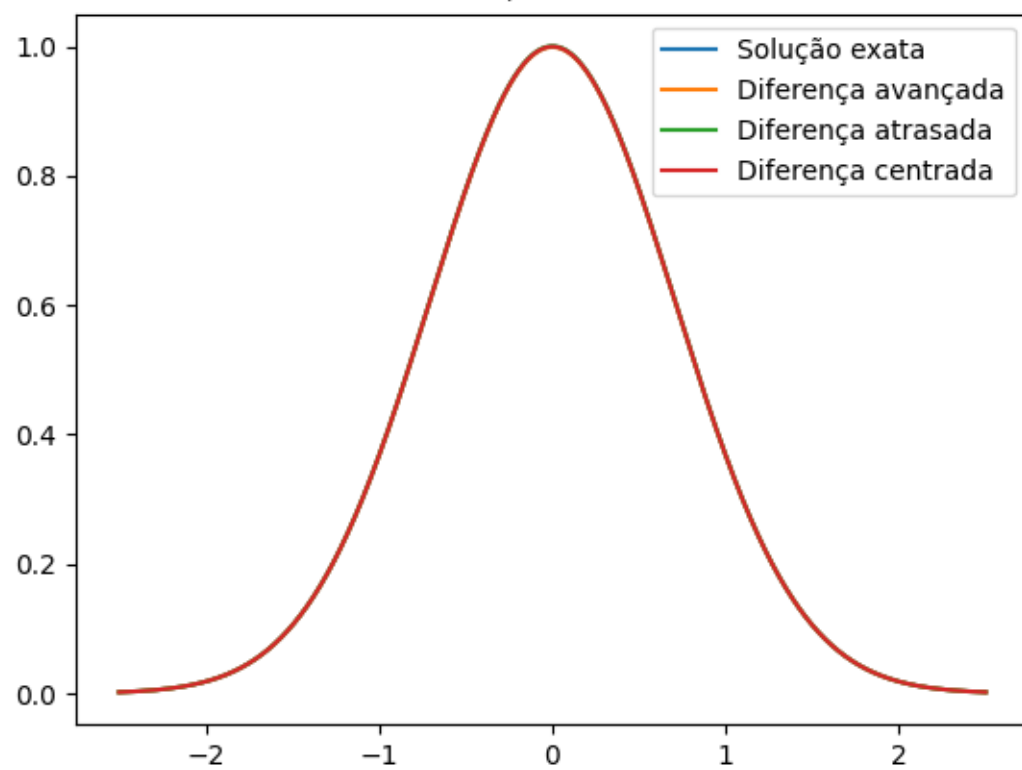
Solução exata
Diferença avançada
Diferença atrasada
Diferença centrada

x	y(x)	y_avn	err_loc_avn	err_rel_avn	y_atr	err_loc_atr	err_rel_atr	y_cen	err_loc_cen	err_rel_cen
-2.5	0.00193045	0.00193045	0	0	0.00193045	0	0	0.00193045	0	0
-2.4	0.00315111	0.00315076	3.46705e-07	0.000110026	0.00315146	-3.4696e-07	0.000110107	0.00315111	4.63572e-11	1.47114e-08
-2.3	0.00504176	0.0050407	1.06102e-06	0.000210447	0.00504282	-1.06189e-06	0.00021062	0.00504176	1.37747e-10	2.73212e-08
-2.2	0.00790705	0.00790467	2.38528e-06	0.000301664	0.00790944	-2.38742e-06	0.000301936	0.00790705	3.00591e-10	3.80155e-08
-2.1	0.0121552	0.0121505	4.66857e-06	0.000384081	0.0121599	-4.67309e-06	0.000384453	0.0121552	5.70973e-10	4.69736e-08
-2	0.0183156	0.0183072	8.39034e-06	0.000458097	0.018324	-8.399e-06	0.00045857	0.0183156	9.95769e-10	5.43672e-08
-1.9	0.0270518	0.0270377	1.41783e-05	0.000524116	0.027066	-1.41937e-05	0.000524685	0.0270518	1.63285e-09	6.03601e-08
-1.8	0.0391639	0.0391411	2.28144e-05	0.000582537	0.0391867	-2.28403e-05	0.000583197	0.0391639	2.5499e-09	6.51084e-08
-1.7	0.0555762	0.055541	3.52221e-05	0.000633761	0.0556115	-3.52634e-05	0.000634506	0.0555762	3.82143e-09	6.87601e-08
-1.6	0.0773047	0.0772523	5.24273e-05	0.00067819	0.0773572	-5.24907e-05	0.00067901	0.0773047	5.52382e-09	7.14551e-08
-1.5	0.105399	0.105324	7.54894e-05	0.000716223	0.105475	-7.55829e-05	0.00071711	0.105399	7.72844e-09	7.33254e-08
-1.4	0.140858	0.140753	0.000105399	0.000748261	0.140964	-0.000105532	0.000749206	0.140858	1.04933e-08	7.44951e-08
-1.3	0.18452	0.184377	0.000142948	0.000774704	0.184663	-0.000143131	0.000775697	0.18452	1.38537e-08	7.50801e-08
-1.2	0.236928	0.236739	0.000188583	0.000795951	0.237117	-0.000188827	0.000796983	0.236928	1.78142e-08	7.51884e-08
-1.1	0.298197	0.297955	0.000242256	0.000812403	0.29844	-0.000242573	0.000813464	0.298197	2.2341e-08	7.49201e-08
-1	0.367879	0.367576	0.000303301	0.000824459	0.368183	-0.0003037	0.000825542	0.367879	2.73581e-08	7.43671e-08
-0.9	0.444858	0.444488	0.000370353	0.000832518	0.445229	-0.000370841	0.000833616	0.444858	3.27475e-08	7.36134e-08
-0.8	0.527292	0.526851	0.000441334	0.000836981	0.527734	-0.000441916	0.000838086	0.527292	3.83527e-08	7.27351e-08
-0.7	0.612626	0.612113	0.000513532	0.000838247	0.613141	-0.00051421	0.000839353	0.612626	4.39866e-08	7.18001e-08
-0.6	0.697676	0.697093	0.000583757	0.000836716	0.698261	-0.000584526	0.000837818	0.697676	4.94432e-08	7.08684e-08
-0.5	0.778801	0.778152	0.000648575	0.000832786	0.77945	-0.000649427	0.000833881	0.778801	5.45099e-08	6.99921e-08
-0.4	0.852144	0.851439	0.000704602	0.000826859	0.852849	-0.000705526	0.000827942	0.852144	5.89812e-08	6.92151e-08
-0.3	0.913931	0.913182	0.000748813	0.000819332	0.914681	-0.000749791	0.000820402	0.913931	6.26714e-08	6.85734e-08
-0.2	0.960789	0.960011	0.000778822	0.000810606	0.961569	-0.000779835	0.000811661	0.960789	6.5425e-08	6.80951e-08
-0.1	0.99005	0.989257	0.00079311	0.000801081	0.990844	-0.000794138	0.00080212	0.99005	6.71255e-08	6.78001e-08
0	1	0.999209	0.000791156	0.000791156	1.00079	-0.000792178	0.000792178	1	6.77004e-08	6.77004e-08
0.1	0.99005	0.989276	0.000773457	0.00078123	0.990824	-0.000774454	0.000782237	0.99005	6.71254e-08	6.78001e-08
0.2	0.960789	0.960048	0.000741445	0.000771704	0.961532	-0.000742399	0.000772697	0.960789	6.5425e-08	6.80951e-08
0.3	0.913931	0.913234	0.000697308	0.000762977	0.914629	-0.000698204	0.000763957	0.913931	6.26714e-08	6.85734e-08
0.4	0.852144	0.8515	0.000643751	0.000755449	0.852788	-0.000644577	0.000756419	0.852144	5.89812e-08	6.92151e-08
0.5	0.778801	0.778217	0.000583726	0.000749519	0.779385	-0.000584476	0.000750482	0.778801	5.45099e-08	6.99921e-08
0.6	0.697676	0.697156	0.000520179	0.000745588	0.698197	-0.000520847	0.000746546	0.697676	4.94432e-08	7.08684e-08
0.7	0.612626	0.612171	0.000455828	0.000744055	0.613083	-0.000456414	0.000745012	0.612626	4.39866e-08	7.18001e-08
0.8	0.527292	0.526899	0.000393002	0.000745321	0.527686	-0.000393508	0.00074628	0.527292	3.83527e-08	7.27351e-08
0.9	0.444858	0.444525	0.000333547	0.000749784	0.445192	-0.000333977	0.00075075	0.444858	3.27475e-08	7.36134e-08
1	0.367879	0.367601	0.000278796	0.000757845	0.368159	-0.000279155	0.000758822	0.367879	2.73581e-08	7.43671e-08
1.1	0.298197	0.297968	0.000229583	0.000769903	0.298427	-0.000229879	0.000770897	0.298197	2.2341e-08	7.49201e-08
1.2	0.236928	0.236741	0.00018631	0.000786359	0.237114	-0.000186551	0.000787375	0.236928	1.78142e-08	7.51884e-08
1.3	0.18452	0.184371	0.00014902	0.000807611	0.184669	-0.000149213	0.000808656	0.18452	1.38537e-08	7.50801e-08
1.4	0.140858	0.140741	0.000117484	0.00083406	0.140976	-0.000117637	0.000835141	0.140858	1.04933e-08	7.44951e-08
1.5	0.105399	0.105308	9.12868e-05	0.000866105	0.105491	-9.14053e-05	0.000867229	0.105399	7.72844e-09	7.33254e-08
1.6	0.0773047	0.0772348	6.98947e-05	0.000904145	0.0773747	-6.99857e-05	0.000905323	0.0773047	5.52382e-09	7.14551e-08
1.7	0.0555762	0.0555235	5.27184e-05	0.000948579	0.055629	-5.27875e-05	0.000949822	0.0555762	3.82142e-09	6.87601e-08
1.8	0.0391639	0.0391247	3.91563e-05	0.000999806	0.0392031	-3.92081e-05	0.00100113	0.0391639	2.5499e-09	6.51084e-08
1.9	0.0270518	0.0270232	2.8627e-05	0.00105823	0.0270805	-2.86653e-05	0.00105964	0.0270518	1.63285e-09	6.03601e-08
2	0.0183156	0.018295	2.05911e-05	0.00112424	0.0183363	-2.06191e-05	0.00112576	0.0183156	9.95769e-10	5.43671e-08
2.1	0.0121552	0.0121406	1.45648e-05	0.00119824	0.0121698	-1.4585e-05	0.0011999	0.0121552	5.70972e-10	4.69735e-08
2.2	0.00790705	0.00789693	1.0126e-05	0.00128062	0.00791719	-1.01404e-05	0.00128245	0.00790705	3.0059e-10	3.80154e-08
2.3	0.00504176	0.00503484	6.91626e-06	0.00137179	0.00504869	-6.92641e-06	0.00137381	0.00504176	1.37746e-10	2.7321e-08
2.4	0.00315111	0.00314647	4.6389e-06	0.00147215	0.00315576	-4.64596e-06	0.00147439	0.00315111	4.63553e-11	1.47108e-08
2.5	0.00193045	0.0019274	3.05413e-06	0.00158208	0.00193351	-3.05897e-06	0.00158459	0.00193045	-3.12857e-15	1.62064e-12

c)

$$\begin{cases} y' = 5y - 1, x \in [0, 2] \\ y(0) = 1.2 \end{cases}$$

Solução exata: $y(x) = e^{5x} + 0.2$

Item c, h = 0.1

x	y(x)	y_avn	err_loc_avn	err_rel_avn	y_atr	err_loc_atr	err_rel_atr	y_cen	err_loc_cen	err_rel_cen
0	1.2	1.2	0	0	1.2	0	0	1.2	0	0
0.1	1.84872	1.7	0.148721	0.0804455	2.2	-0.351279	0.190012	1.84872	0	0
0.2	2.91828	2.45	0.468282	0.160465	4.2	-1.28172	0.439203	2.84872	0.0695606	0.0238361
0.3	4.68169	3.575	1.10669	0.236387	8.2	-3.51831	0.751505	4.49744	0.184247	0.0393547
0.4	7.58906	5.2625	2.32656	0.306567	16.2	-8.61094	1.13465	7.14616	0.442892	0.0583593
0.5	12.3825	7.79375	4.58874	0.370583	32.2	-19.8175	1.60045	11.4436	0.938888	0.0758238
0.6	20.2855	11.5906	8.69491	0.428626	64.2	-43.9145	2.16482	18.3898	1.89577	0.0934541
0.7	33.3155	17.2859	16.0295	0.481144	128.2	-94.8845	2.84806	29.6334	3.68208	0.110522
0.8	54.7982	25.8289	28.9692	0.528654	256.2	-201.402	3.67534	47.8231	6.975	0.127285
0.9	90.2171	38.6434	51.5738	0.571663	512.2	-421.983	4.67741	77.2565	12.9606	0.14366
1	148.613	57.865	90.7481	0.610633	1024.2	-875.587	5.89172	124.88	23.7335	0.1597
1.1	244.892	86.6976	158.194	0.645976	2048.2	-1803.31	7.36369	201.936	42.9557	0.175407
1.2	403.629	129.946	273.682	0.678055	4096.2	-3692.57	9.14843	326.616	77.0129	0.190801
1.3	665.342	194.82	470.522	0.707189	8192.2	-7526.86	11.3128	528.352	136.99	0.205894
1.4	1096.83	292.129	804.704	0.733661	16384.2	-15287.4	13.9377	854.768	242.065	0.220695
1.5	1808.24	438.094	1370.15	0.757724	32768.2	-30960	17.1216	1382.92	425.322	0.235213
1.6	2981.16	657.041	2324.12	0.779602	65536.2	-62555	20.9835	2237.49	743.67	0.249457
1.7	4914.97	985.461	3929.51	0.799498	131072	-126157	25.668	3620.21	1294.76	0.263432
1.8	8103.28	1478.09	6625.19	0.817593	262144	-254041	31.3504	5857.5	2245.79	0.277145
1.9	13359.9	2217.04	11142.9	0.834053	524288	-510928	38.2433	9477.5	3882.42	0.290602
2	22026.7	3325.46	18701.2	0.849026	1.04858e+06	-1.02655e+06	46.6049	15334.8	6691.87	0.303808

Item c, h = 0.01
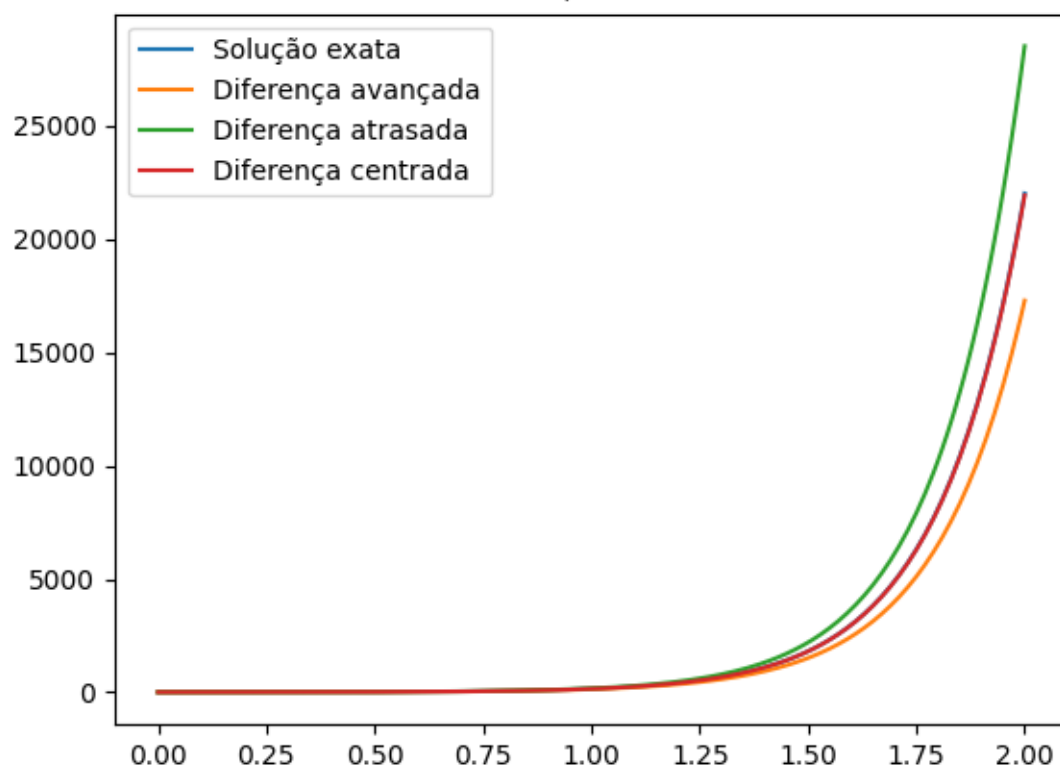
x	y(x)	y_avn	err_loc_avn	err_rel_avn	y_atr	err_loc_atr	err_rel_atr	y_cen	err_loc_cen	err_rel_cen
0	1.2	1.2	0	0	1.2	0	0	1.2	0	0
0.1	1.84872	1.82889	0.0198266	0.0107245	1.87018	-0.0214613	0.0116087	1.84839	0.000331682	0.000179411
0.2	2.91828	2.8533	0.0649841	0.0222679	2.98951	-0.071228	0.0244075	2.91718	0.00110545	0.0003788
0.3	4.68169	4.52194	0.159747	0.0341216	4.85899	-0.177302	0.0378713	4.67894	0.00275054	0.000587511
0.4	7.58906	7.23999	0.349067	0.0459962	7.98137	-0.392309	0.051694	7.58299	0.00606889	0.00079969
0.5	12.3825	11.6674	0.715094	0.0577504	13.1963	-0.813806	0.0657223	12.37	0.0125372	0.00101249
0.6	20.2855	18.8792	1.40635	0.0693278	21.9062	-1.62066	0.0798923	20.2607	0.0248445	0.00122474
0.7	33.3155	30.6264	2.68903	0.0807141	36.4533	-3.13786	0.0941862	33.2676	0.0478431	0.00143606
0.8	54.7982	49.7614	5.03671	0.0919139	60.7496	-5.95149	0.108608	54.7079	0.0902238	0.00164648
0.9	90.2171	80.9304	9.28677	0.102938	101.329	-11.1118	0.123168	90.0497	0.167453	0.00185611
1	148.613	131.701	16.9119	0.113798	169.104	-20.4907	0.137879	148.306	0.306908	0.00206514
1.1	244.892	214.402	30.4902	0.124505	282.3	-37.4083	0.152754	244.335	0.556815	0.00227372
1.2	403.629	349.112	54.5168	0.135067	471.359	-67.7301	0.167803	402.627	1.00179	0.00248195
1.3	665.342	568.541	96.8008	0.14549	787.121	-121.78	0.183033	663.552	1.78972	0.00268993
1.4	1096.83	925.967	170.866	0.155781	1314.5	-217.669	0.198452	1093.65	3.17833	0.00289773
1.5	1808.24	1508.18	300.065	0.165943	2195.32	-387.082	0.214066	1802.63	5.6153	0.00310539
1.6	2981.16	2456.54	524.622	0.175979	3666.46	-685.301	0.229877	2971.28	9.8764	0.00331294
1.7	4914.97	4001.31	913.656	0.185892	6123.52	-1208.55	0.245892	4897.67	17.3027	0.00352041
1.8	8103.28	6517.59	1585.69	0.195685	10227.3	-2123.98	0.262114	8073.08	30.2075	0.0037278
1.9	13359.9	10616.3	2743.58	0.205359	17081.3	-3721.34	0.278545	13307.4	52.5732	0.00393514
2	22026.7	17292.8	4733.88	0.214916	28528.7	-6502.03	0.295189	21935.4	91.2437	0.00414242

Item c, h = 0.001
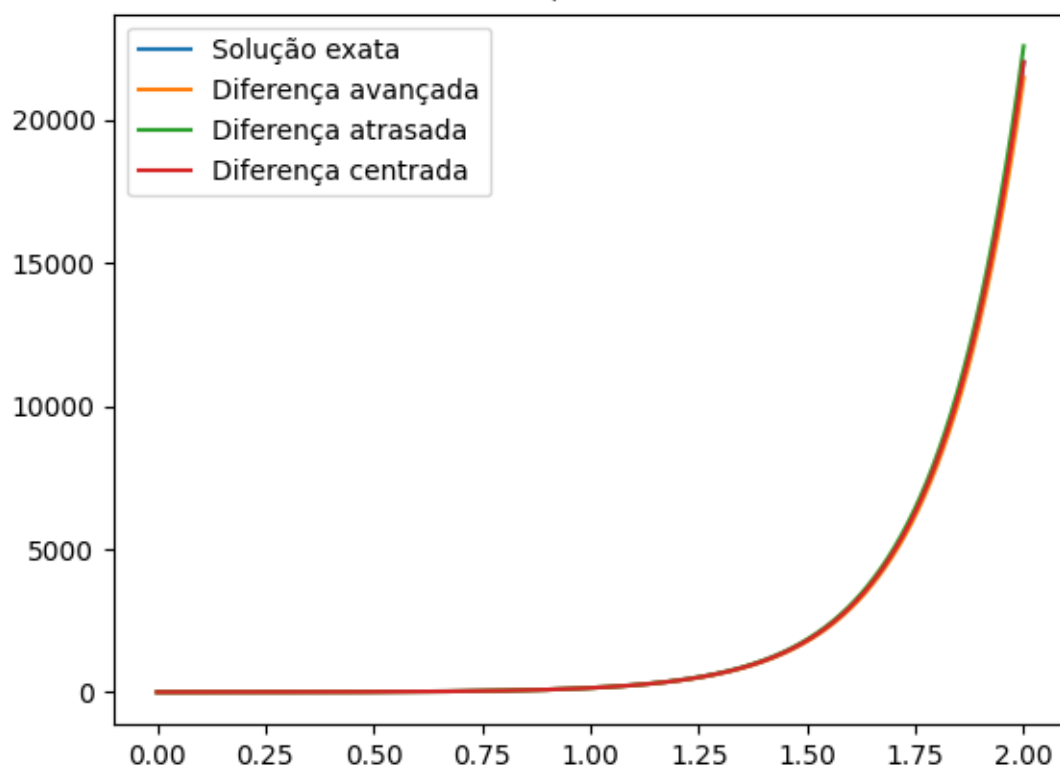
x	y(x)	y_avn	err_loc_avn	err_rel_avn	y_atr	err_loc_atr	err_rel_atr	y_cen	err_loc_cen	err_rel_cen
0	1.2	1.2	0	0	1.2	0	0	1.2	0	0
0.1	1.84872	1.84667	0.00205278	0.00111038	1.85079	-0.00206909	0.0011192	1.84872	3.42388e-06	1.85203e-06
0.2	2.91828	2.91152	0.00676471	0.00231804	2.92511	-0.006827	0.00233939	2.91827	1.13014e-05	3.87263e-06
0.3	4.68169	4.66497	0.0167193	0.0035712	4.69858	-0.0168943	0.0036086	4.68166	2.79656e-05	5.97339e-06
0.4	7.58906	7.55233	0.036731	0.00483999	7.62622	-0.037162	0.00489679	7.58899	6.14986e-05	8.10359e-06
0.5	12.3825	12.3068	0.0756519	0.00610958	12.4591	-0.0766354	0.00618901	12.3824	0.000126772	1.0238e-05
0.6	20.2855	20.136	0.149581	0.0073738	20.4373	-0.151716	0.00747901	20.2853	0.000250855	1.23662e-05
0.7	33.3155	33.0279	0.287542	0.0086309	33.6075	-0.29201	0.00876499	33.315	0.000482578	1.44851e-05
0.8	54.7982	54.2567	0.541466	0.00988109	55.3487	-0.550566	0.0100472	54.7972	0.00090938	1.65951e-05
0.9	90.2171	89.2134	1.00369	0.0111253	91.239	-1.02184	0.0113264	90.2154	0.00168684	1.86976e-05
1	148.613	146.776	1.83753	0.0123645	150.486	-1.87309	0.0126038	148.61	0.00309032	2.07944e-05
1.1	244.892	241.561	3.33047	0.0135997	248.291	-3.39916	0.0138803	244.886	0.00560483	2.2887e-05
1.2	403.629	397.642	5.98647	0.0148316	409.746	-6.1176	0.0151565	403.619	0.0100813	2.49766e-05
1.3	665.342	654.656	10.6859	0.0160608	676.275	-10.9336	0.0164331	665.324	0.0180068	2.70641e-05
1.4	1096.83	1077.87	18.9615	0.0172875	1116.26	-19.4253	0.0177104	1096.8	0.0319728	2.91501e-05
1.5	1808.24	1774.77	33.4745	0.0185122	1842.58	-34.3362	0.0189887	1808.19	0.0564809	3.12352e-05
1.6	2981.16	2922.33	58.8328	0.0197349	3041.58	-60.4229	0.0202683	2981.06	0.0993313	3.33197e-05
1.7	4914.97	4811.97	102.997	0.0209559	5020.88	-105.913	0.0215491	4914.79	0.174008	3.54037e-05
1.8	8103.28	7923.59	179.692	0.0221752	8288.29	-185.01	0.0228315	8102.98	0.303772	3.74875e-05
1.9	13359.9	13047.4	312.526	0.0233928	13682.1	-322.179	0.0241153	13359.4	0.528666	3.9571e-05
2	22026.7	21484.6	542.052	0.0246089	22586.2	-559.492	0.0254007	22025.7	0.917509	4.16545e-05

Item c, h = 0.0001
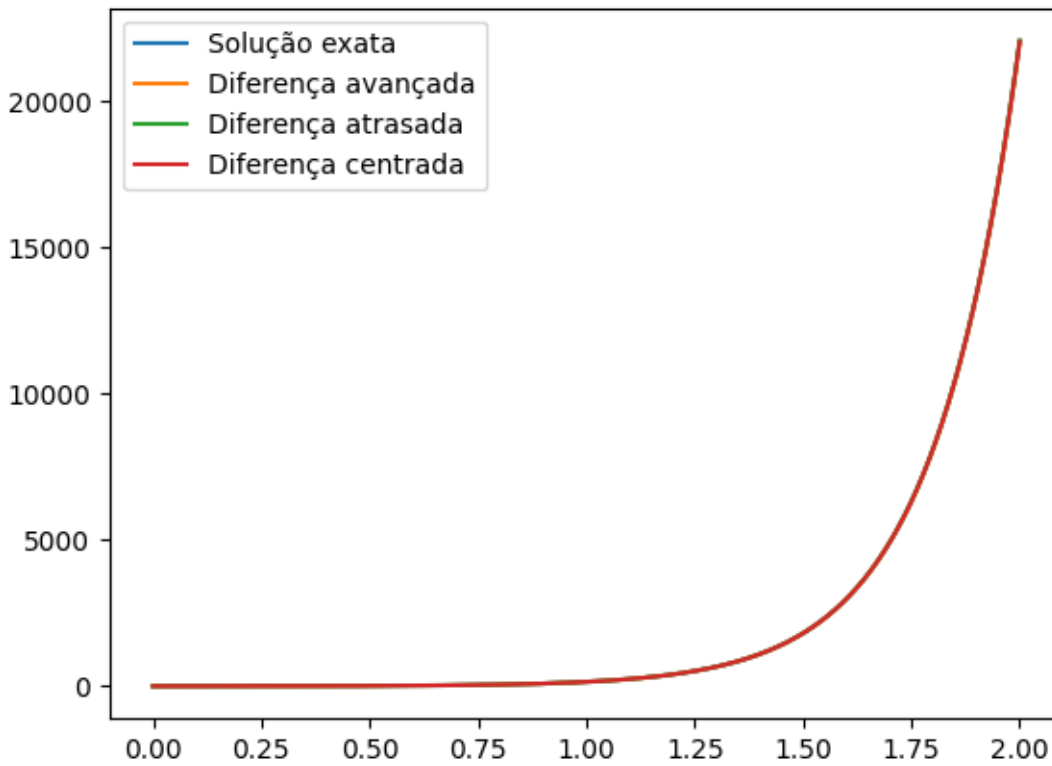
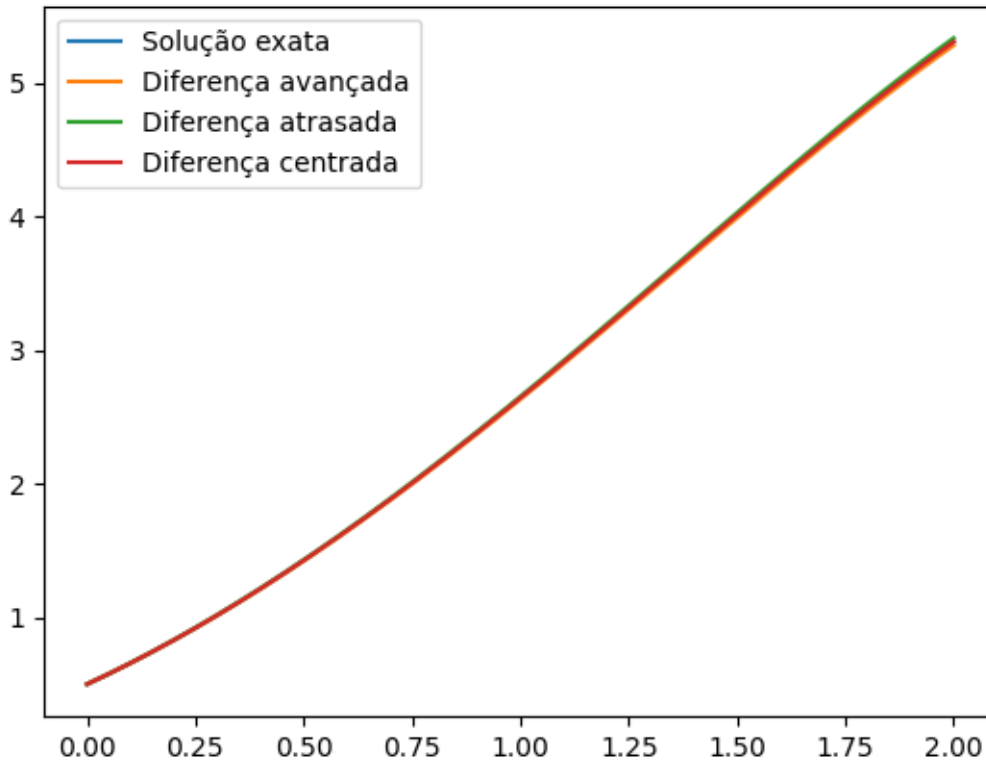| x	y(x)	y_avn	err_loc_avn	err_rel_avn	y_atr	err_loc_atr	err_rel_atr	y_cen	err_loc_cen	err_rel_cen
0	1.2	1.2	0	0	1.2	0	0	1.2	0	0
0.1	1.84872	1.84852	0.000206009	0.000111433	1.84893	-0.000206172	0.000111521	1.84872	3.43375e-08	1.85736e-08
0.2	2.91828	2.9176	0.000679259	0.00023276	2.91896	-0.000679882	0.000232973	2.91828	1.13237e-07	3.88027e-08
0.3	4.68169	4.68001	0.00167976	0.000358793	4.68337	-0.00168151	0.000359167	4.68169	2.80061e-07	5.98205e-08
0.4	7.58906	7.58536	0.00369237	0.000486539	7.59275	-0.00369668	0.000487107	7.58906	6.15679e-07	8.11272e-08
0.5	12.3825	12.3749	0.00760914	0.000614508	12.3901	-0.00761898	0.000615302	12.3825	1.26888e-06	1.02474e-07
0.6	20.2855	20.2705	0.0150535	0.00074208	20.3006	-0.0150748	0.000743132	20.2855	2.51048e-06	1.23757e-07
0.7	33.3155	33.2865	0.0289537	0.000869077	33.3445	-0.0289984	0.000870418	33.3154	4.82899e-06	1.44947e-07
0.8	54.7982	54.7436	0.0545527	0.000995521	54.8528	-0.0546437	0.000997181	54.7981	9.09912e-06	1.66048e-07
0.9	90.2171	90.116	0.101179	0.0011215	90.3185	-0.10136	0.00112351	90.2171	1.68773e-05	1.87074e-07
1	148.613	148.428	0.185339	0.00124712	148.799	-0.185694	0.00124952	148.613	3.09179e-05	2.08043e-07
1.1	244.892	244.556	0.336108	0.00137248	245.229	-0.336795	0.00137528	244.892	5.60727e-05	2.28969e-07
1.2	403.629	403.024	0.604488	0.00149763	404.235	-0.605799	0.00150088	403.629	0.000100853	2.49866e-07
1.3	665.342	664.262	1.07962	0.00162265	666.424	-1.08209	0.00162637	665.341	0.000180136	2.70741e-07
1.4	1096.83	1094.92	1.91679	0.00174757	1098.75	-1.92143	0.0017518	1096.83	0.00031984	2.91603e-07
1.5	1808.24	1804.86	3.38578	0.00187241	1811.64	-3.39439	0.00187718	1808.24	0.000564994	3.12455e-07
1.6	2981.16	2975.2	5.95398	0.0019972	2987.13	-5.96987	0.00200254	2981.16	0.000993621	3.333e-07
1.7	4914.97	4904.54	10.4293	0.00212195	4925.43	-10.4585	0.00212788	4914.97	0.0017406	3.54142e-07
1.8	8103.28	8085.08	18.2054	0.00224667	8121.54	-18.2586	0.00225323	8103.28	0.00303857	3.7498e-07
1.9	13359.9	13328.2	31.6812	0.00237136	13391.7	-31.7777	0.00237858	13359.9	0.00528808	3.95817e-07
2	22026.7	21971.7	54.9791	0.00249602	22081.8	-55.1535	0.00250394	22026.7	0.00917746	4.16652e-07

d)

$$\begin{cases} y' = y - x^2 + 1, x \in [0, 2] \\ y(0) = 0.5 \end{cases}$$
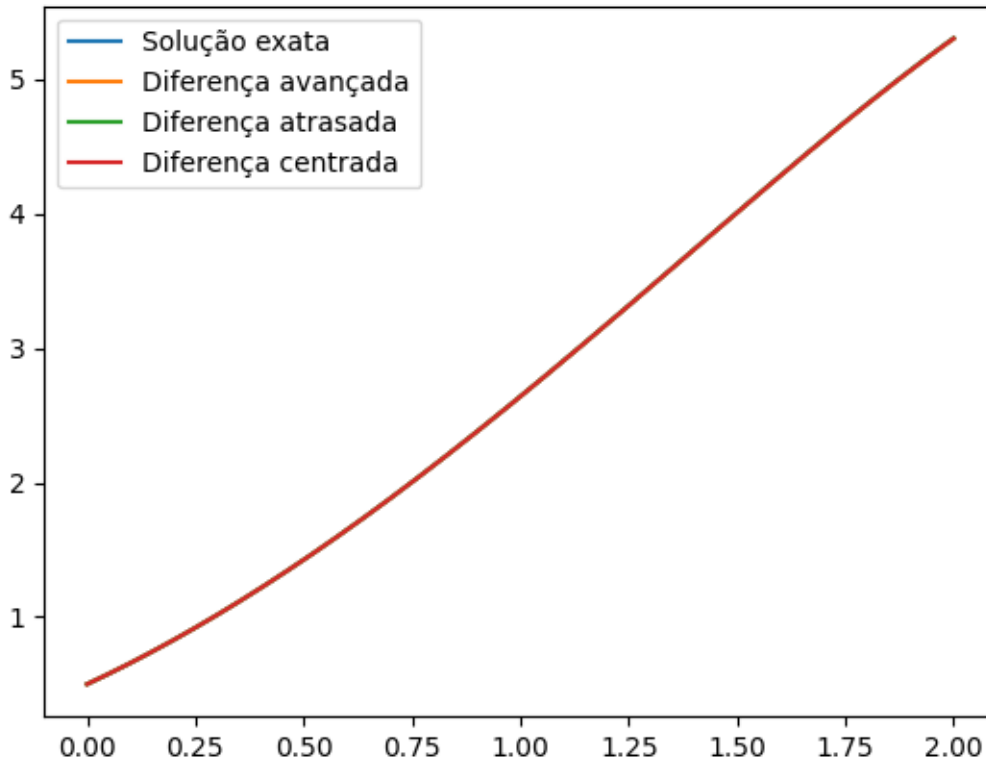
Solução exata: $y(x) = (x+1)^2 - 0.5e^x$
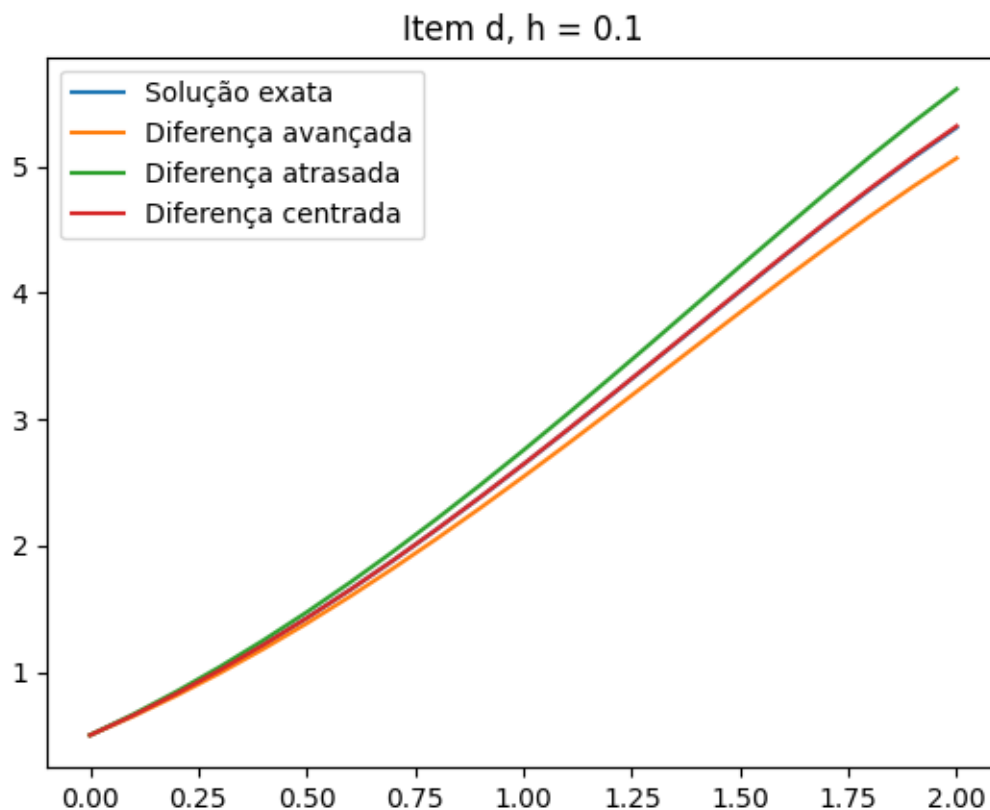
Item d, h = 0.1

x	y(x)	y_avn	err_loc_avn	err_rel_avn	y_atr	err_loc_atr	err_rel_atr	y_cen	err_loc_cen	err_rel_cen
0	0.5	0.5	0	0	0.5	0	0	0.5	0	0
0.1	0.657415	0.65	0.00741454	0.0112783	0.665556	-0.00814101	0.0123834	0.657415	0	0
0.2	0.829299	0.814	0.0152986	0.0184477	0.846173	-0.0168742	0.0203476	0.829483	-0.000184287	0.000222221
0.3	1.01507	0.9914	0.0236706	0.0233192	1.0413	-0.0262326	0.0258431	1.01531	-0.000240526	0.000236955
0.4	1.21409	1.18154	0.0325477	0.0268083	1.25034	-0.0362492	0.0298571	1.21455	-0.000457482	0.000376811
0.5	1.42564	1.38369	0.0419454	0.0294221	1.4726	-0.0469571	0.0329376	1.42622	-0.000580784	0.000407385
0.6	1.64894	1.59706	0.0518772	0.0314609	1.70733	-0.0583888	0.0354099	1.64979	-0.000848563	0.000514611
0.7	1.88312	1.82077	0.0623539	0.033112	1.9537	-0.0705757	0.037478	1.88418	-0.00105434	0.000559886
0.8	2.12723	2.05385	0.0733828	0.0344969	2.21078	-0.0835475	0.0392753	2.12862	-0.00139522	0.000655887
0.9	2.3802	2.29523	0.0849671	0.0356975	2.47753	-0.0973316	0.0408922	2.3819	-0.00170449	0.000716112
1	2.64086	2.54375	0.0971046	0.0367701	2.75281	-0.111952	0.0423923	2.64301	-0.00214626	0.000812713
1.1	2.90792	2.79813	0.109787	0.0377545	3.03535	-0.127429	0.0438213	2.9105	-0.00258701	0.000889645
1.2	3.17994	3.05694	0.122999	0.0386795	3.32372	-0.143776	0.0452134	3.18311	-0.00316461	0.000995178
1.3	3.45535	3.31864	0.136714	0.039566	3.61635	-0.161001	0.0465947	3.45913	-0.00377357	0.00109209
1.4	3.7324	3.5815	0.150899	0.0404295	3.9115	-0.179103	0.047986	3.73693	-0.00453118	0.00121401
1.5	4.00916	3.84365	0.165504	0.0412816	4.20723	-0.19807	0.0494045	4.01451	-0.00535601	0.00133594
1.6	4.28348	4.10302	0.180468	0.042131	4.50136	-0.217878	0.0508647	4.28983	-0.0063497	0.00148237
1.7	4.55303	4.35732	0.195708	0.0429843	4.79151	-0.238487	0.0523799	4.56048	-0.00745186	0.00163668
1.8	4.81518	4.60405	0.211127	0.0438461	5.07501	-0.259838	0.0539624	4.82393	-0.00875285	0.00181776
1.9	5.06705	4.84045	0.226598	0.0447199	5.34891	-0.281852	0.0556245	5.07726	-0.0102112	0.00201522
2	5.30547	5.0635	0.241972	0.045608	5.60989	-0.304423	0.057379	5.31738	-0.01191	0.00224485

Item d, h = 0.01

x	y(x)	y_avn	err_loc_avn	err_rel_avn	y_atr	err_loc_atr	err_rel_atr	y_cen	err_loc_cen	err_rel_cen
0	0.5	0.5	0	0	0.5	0	0	0.5	0	0
0.1	0.657415	0.656643	0.000771825	0.00117403	0.658194	-0.000779055	0.00118503	0.657415	-9.12503e-07	1.38802e-06
0.2	0.829299	0.827703	0.00159554	0.00192396	0.83091	-0.00161122	0.00194287	0.829301	-2.01863e-06	2.43414e-06
0.3	1.01507	1.0126	0.00247354	0.00243682	1.01757	-0.00249902	0.00246192	1.01507	-3.34886e-06	3.29914e-06
0.4	1.21409	1.21068	0.00340816	0.00280717	1.21753	-0.00344496	0.00283749	1.21409	-4.93794e-06	4.0672e-06
0.5	1.42564	1.42124	0.00440159	0.00308745	1.43009	-0.0044514	0.00312239	1.42565	-6.82548e-06	4.78766e-06
0.6	1.64894	1.64348	0.00545592	0.00330874	1.65446	-0.00552061	0.00334797	1.64895	-9.05656e-06	5.49235e-06
0.7	1.88312	1.87655	0.00657296	0.00349046	1.88978	-0.00665461	0.00353382	1.88314	-1.16825e-05	6.20376e-06
0.8	2.12723	2.11948	0.0077543	0.00364526	2.13508	-0.0078552	0.00369269	2.12724	-1.47614e-05	6.93927e-06
0.9	2.3802	2.3712	0.00900111	0.00378166	2.38932	-0.00912379	0.00383321	2.38022	-1.83597e-05	7.7135e-06
1	2.64086	2.63054	0.0103141	0.0039056	2.65132	-0.0104614	0.00396136	2.64088	-2.25522e-05	8.53973e-06
1.1	2.90792	2.89622	0.0116936	0.00402128	2.91979	-0.0118684	0.00408142	2.90794	-2.74243e-05	9.4309e-06
1.2	3.17994	3.1668	0.0131389	0.00413179	3.19329	-0.0133447	0.00419653	3.17997	-3.30723e-05	1.04003e-05
1.3	3.45535	3.4407	0.0146487	0.00423941	3.47024	-0.0148892	0.00430902	3.45539	-3.96056e-05	1.14621e-05
1.4	3.7324	3.71618	0.0162206	0.00434589	3.7489	-0.0164998	0.00442069	3.73245	-4.71477e-05	1.2632e-05
1.5	4.00916	3.9913	0.0178511	0.00445259	4.02733	-0.0181733	0.00453295	4.00921	-5.58387e-05	1.39278e-05
1.6	4.28348	4.26395	0.0195352	0.00456059	4.30339	-0.0199051	0.00464694	4.28355	-6.58367e-05	1.53699e-05
1.7	4.55303	4.53176	0.0212662	0.00467078	4.57472	-0.0216889	0.00476361	4.5531	-7.73203e-05	1.69822e-05
1.8	4.81518	4.79214	0.0230353	0.00478389	4.83869	-0.0235163	0.00488378	4.81527	-9.04917e-05	1.8793e-05
1.9	5.06705	5.04222	0.0248315	0.00490058	5.09243	-0.0253767	0.00500819	5.06716	-0.000105579	2.08363e-05
2	5.30547	5.27883	0.0266411	0.00502143	5.33273	-0.0272569	0.00513751	5.30559	-0.000122838	2.31531e-05

Item d, h = 0.001

x	y(x)	y_avn	err_loc_avn	err_rel_avn	y_atr	err_loc_atr	err_rel_atr	y_cen	err_loc_cen	err_rel_cen
0	0.5	0.5	0	0	0.5	0	0	0.5	0	0
0.1	0.657415	0.657337	7.75055e-05	0.000117894	0.657492	-7.75778e-05	0.000118004	0.657415	-9.2014e-09	1.39963e-08
0.2	0.829299	0.829138	0.000160254	0.000193241	0.829459	-0.000160411	0.00019343	0.829299	-2.03399e-08	2.45266e-08
0.3	1.01507	1.01482	0.000248492	0.000244803	1.01532	-0.000248747	0.000245054	1.01507	-3.37211e-08	3.32204e-08
0.4	1.21409	1.21375	0.000342458	0.000282071	1.21443	-0.000342826	0.000282374	1.21409	-4.96932e-08	4.09305e-08
0.5	1.42564	1.4252	0.000442382	0.000310304	1.42608	-0.00044288	0.000310654	1.42564	-6.86532e-08	4.81561e-08
0.6	1.64894	1.64839	0.000548478	0.000332624	1.64949	-0.000549125	0.000333017	1.64894	-9.10528e-08	5.5219e-08
0.7	1.88312	1.88246	0.000660938	0.00035098	1.88379	-0.000661755	0.000351413	1.88312	-1.17406e-07	6.23462e-08
0.8	2.12723	2.12645	0.000779929	0.000366641	2.12801	-0.000780938	0.000367115	2.12723	-1.48295e-07	6.97129e-08
0.9	2.3802	2.37929	0.00090558	0.000380464	2.38111	-0.000906806	0.000380979	2.3802	-1.84385e-07	7.7466e-08
1	2.64086	2.63982	0.00103798	0.000393045	2.6419	-0.00103945	0.000393602	2.64086	-2.26425e-07	8.57393e-08
1.1	2.90792	2.90674	0.00117715	0.000404808	2.9091	-0.0011789	0.000405409	2.90792	-2.7527e-07	9.46624e-08
1.2	3.17994	3.17862	0.00132305	0.000416062	3.18127	-0.00132511	0.00041671	3.17994	-3.31886e-07	1.04368e-07
1.3	3.45535	3.45388	0.00147557	0.00042704	3.45683	-0.00147798	0.000427736	3.45535	-3.97365e-07	1.15e-07
1.4	3.7324	3.73077	0.00163449	0.000437918	3.73404	-0.00163728	0.000438666	3.7324	-4.72948e-07	1.26714e-07
1.5	4.00916	4.00736	0.00179945	0.000448834	4.01096	-0.00180267	0.000449638	4.00916	-5.60033e-07	1.39689e-07
1.6	4.28348	4.28151	0.00196997	0.0004599	4.28546	-0.00197367	0.000460763	4.28348	-6.60206e-07	1.54128e-07
1.7	4.55303	4.55088	0.00214541	0.000471205	4.55518	-0.00214964	0.000472133	4.55303	-7.75255e-07	1.70272e-07
1.8	4.81518	4.81285	0.0023249	0.000482828	4.81751	-0.00232971	0.000483827	4.81518	-9.07201e-07	1.88405e-07
1.9	5.06705	5.06455	0.00250737	0.000494838	5.06957	-0.00251282	0.000495914	5.06705	-1.05833e-06	2.08864e-07
2	5.30547	5.30278	0.00269145	0.000507298	5.30817	-0.00269761	0.000508458	5.30547	-1.23121e-06	2.32063e-07

Item d, h = 0.0001
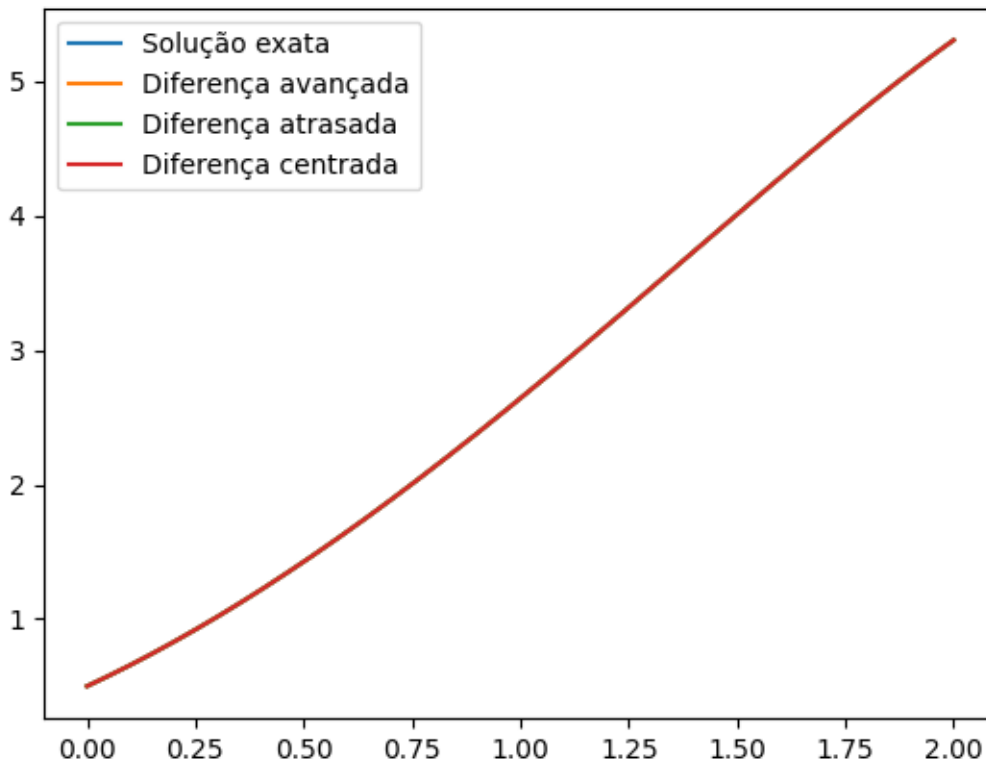
x	y(x)	y_avn	err_loc_avn	err_rel_avn	y_atr	err_loc_atr	err_rel_atr	y_cen	err_loc_cen	err_rel_cen
0	0.5	0.5	0	0	0.5	0	0	0.5	0	0
0.1	0.657415	0.657407	7.7538e-06	1.17944e-05	0.657422	-7.75453e-06	1.17955e-05	0.657415	-9.20892e-11	1.40078e-10
0.2	0.829299	0.829283	1.60325e-05	1.93326e-05	0.829315	-1.6034e-05	1.93345e-05	0.829299	-2.03551e-10	2.45449e-10
0.3	1.01507	1.01505	2.48607e-05	2.44916e-05	1.0151	-2.48632e-05	2.44941e-05	1.01507	-3.37441e-10	3.32431e-10
0.4	1.21409	1.21405	3.42624e-05	2.82207e-05	1.21412	-3.42661e-05	2.82237e-05	1.21409	-4.97243e-10	4.09561e-10
0.5	1.42564	1.4256	4.42606e-05	3.10462e-05	1.42568	-4.42656e-05	3.10496e-05	1.42564	-6.86926e-10	4.81837e-10
0.6	1.64894	1.64889	5.48769e-05	3.32801e-05	1.649	-5.48833e-05	3.3284e-05	1.64894	-9.11011e-10	5.52483e-10
0.7	1.88312	1.88306	6.61305e-05	3.51175e-05	1.88319	-6.61387e-05	3.51218e-05	1.88312	-1.17463e-09	6.23767e-10
0.8	2.12723	2.12715	7.80382e-05	3.66854e-05	2.12731	-7.80483e-05	3.66901e-05	2.12723	-1.48363e-09	6.97447e-10
0.9	2.3802	2.38011	9.06131e-05	3.80696e-05	2.38029	-9.06254e-05	3.80747e-05	2.3802	-1.84463e-09	7.74989e-10
1	2.64086	2.64076	0.000103864	3.93295e-05	2.64096	-0.000103878	3.93351e-05	2.64086	-2.26515e-09	8.57732e-10
1.1	2.90792	2.9078	0.000117793	4.05078e-05	2.90803	-0.000117811	4.05138e-05	2.90792	-2.75372e-09	9.46972e-10
1.2	3.17994	3.17981	0.000132398	4.16353e-05	3.18007	-0.000132418	4.16418e-05	3.17994	-3.32e-09	1.04405e-09
1.3	3.45535	3.4552	0.000147666	4.27353e-05	3.4555	-0.00014769	4.27423e-05	3.45535	-3.97494e-09	1.15037e-09
1.4	3.7324	3.73224	0.000163574	4.38254e-05	3.73256	-0.000163602	4.38329e-05	3.7324	-4.73093e-09	1.26753e-09
1.5	4.00916	4.00898	0.000180089	4.49196e-05	4.00934	-0.000180122	4.49276e-05	4.00916	-5.60196e-09	1.39729e-09
1.6	4.28348	4.28329	0.000197163	4.60288e-05	4.28368	-0.0001972	4.60374e-05	4.28348	-6.60387e-09	1.54171e-09
1.7	4.55303	4.55281	0.000214731	4.71622e-05	4.55324	-0.000214773	4.71715e-05	4.55303	-7.75456e-09	1.70317e-09
1.8	4.81518	4.81494	0.000232707	4.83277e-05	4.81541	-0.000232755	4.83377e-05	4.81518	-9.07425e-09	1.88451e-09
1.9	5.06705	5.0668	0.000250982	4.95322e-05	5.0673	-0.000251037	4.95429e-05	5.06705	-1.05857e-08	2.08913e-09
2	5.30547	5.3052	0.000269422	5.07819e-05	5.30574	-0.000269484	5.07935e-05	5.30547	-1.23148e-08	2.32115e-09

e)

$$\begin{cases} y' = 2e^x - 10x, x \in [0,1] \\ y(0) = 11 \end{cases}$$

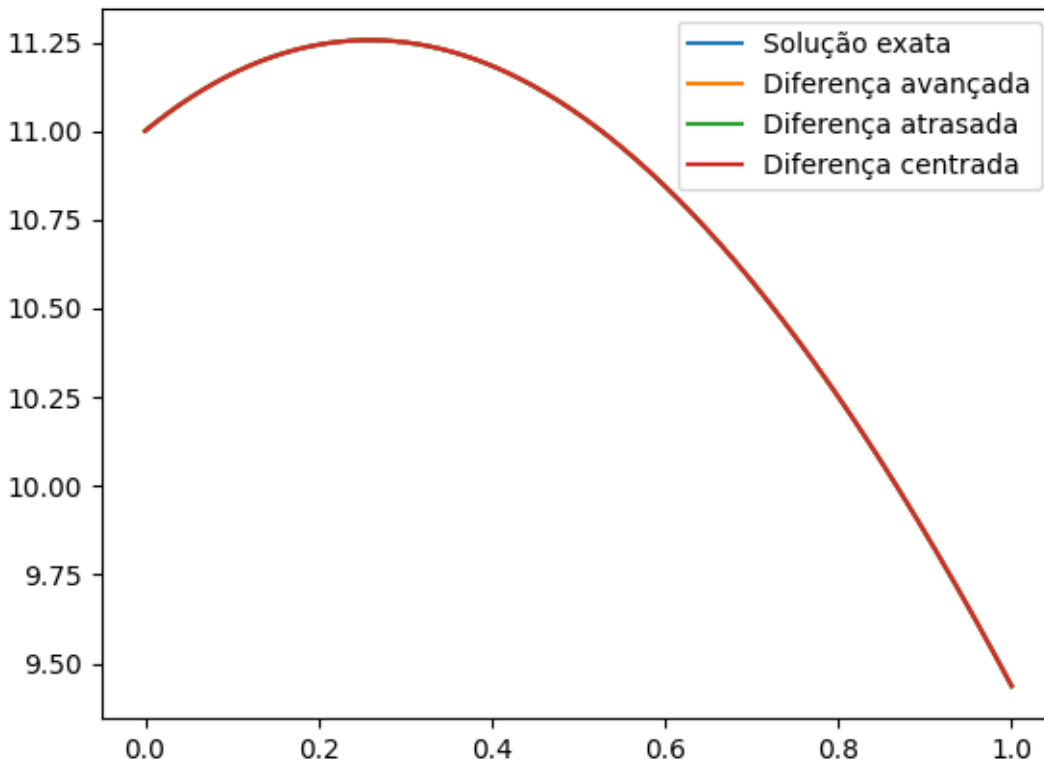Solução exata: $y(x) = 2e^x - 5x^2 + 9$
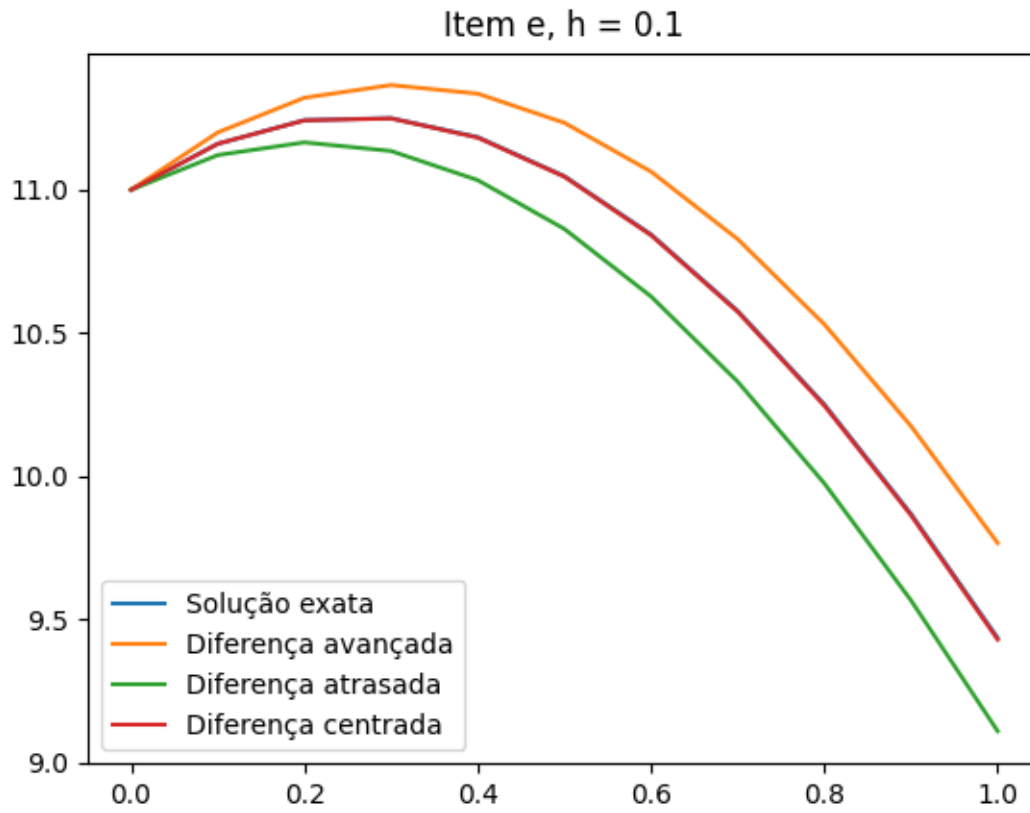
Item e, h = 0.1

x	y(x)	y_avn	err_loc_avn	err_rel_avn	y_atr	err_loc_atr	err_rel_atr	y_cen	err_loc_cen	err_rel_cen
0	11	11	0	0	11	0	0	11	0	0
0.1	11.1603	11.2	-0.0396582	0.00355349	11.121	0.0393077	0.00352208	11.1603	0	0
0.2	11.2428	11.321	-0.0782287	0.00695811	11.1653	0.0774908	0.00689248	11.2421	0.000737149	6.55663e-05
0.3	11.2497	11.3653	-0.115597	0.0102756	11.1353	0.114431	0.0101719	11.2489	0.000814676	7.24174e-05
0.4	11.1836	11.3353	-0.151637	0.0135588	11.0337	0.149998	0.0134123	11.182	0.00163751	0.00014642
0.5	11.0474	11.2337	-0.186209	0.0168554	10.8634	0.184047	0.0166597	11.0456	0.00180972	0.000163814
0.6	10.8442	11.0634	-0.219158	0.0202096	10.6278	0.216418	0.019957	10.8415	0.0027372	0.000252411
0.7	10.5775	10.8278	-0.250314	0.0236648	10.3306	0.246935	0.0233453	10.5745	0.00302508	0.000285991
0.8	10.2511	10.5306	-0.279488	0.0272643	9.97568	0.275404	0.0268658	10.247	0.00408038	0.000398043
0.9	9.86921	10.1757	-0.306472	0.0310534	9.5676	0.301607	0.0305605	9.8647	0.00450951	0.000456928
1	9.43656	9.7676	-0.331035	0.0350801	9.11126	0.325308	0.0344732	9.43084	0.00572093	0.000606252

Item e, h = 0.01
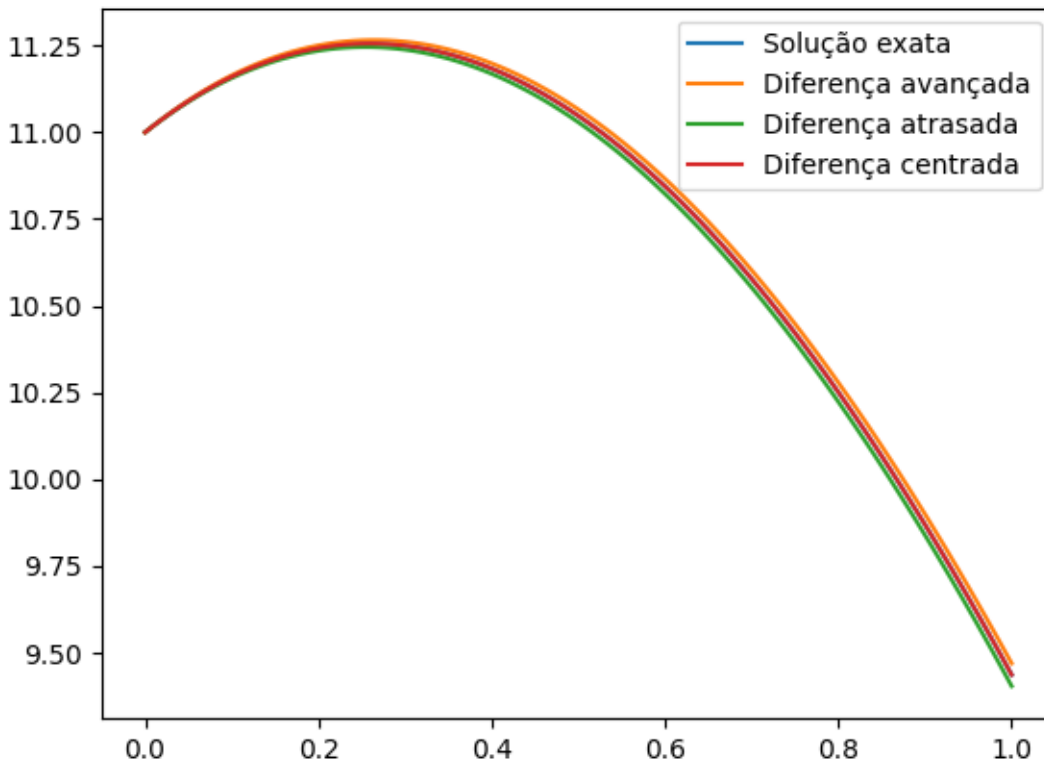
x	y(x)	y_avn	err_loc_avn	err_rel_avn	y_atr	err_loc_atr	err_rel_atr	y_cen	err_loc_cen	err_rel_cen
0	11	11	0	0	11	0	0	11	0	0
0.1	11.1603	11.1643	-0.00395004	0.000353936	11.1564	0.00394654	0.000353622	11.1603	3.50566e-06	3.14117e-07
0.2	11.2428	11.2506	-0.00778966	0.000692858	11.235	0.00778228	0.000692201	11.2428	7.38001e-06	6.5642e-07
0.3	11.2497	11.2612	-0.0115072	0.00102289	11.2382	0.0114956	0.00102186	11.2497	1.16618e-05	1.03663e-06
0.4	11.1836	11.1987	-0.01509	0.00134929	11.1686	0.0150736	0.00134782	11.1836	1.6394e-05	1.46589e-06
0.5	11.0474	11.066	-0.0185236	0.00167673	11.0289	0.018502	0.00167477	11.0474	2.16238e-05	1.95736e-06
0.6	10.8442	10.866	-0.0217925	0.00200959	10.8225	0.0217651	0.00200707	10.8442	2.74036e-05	2.52702e-06
0.7	10.5775	10.6024	-0.0248794	0.0023521	10.5527	0.0248456	0.00234891	10.5775	3.37914e-05	3.19464e-06
0.8	10.2511	10.2788	-0.027765	0.0027085	10.2234	0.0277242	0.00270451	10.251	4.08509e-05	3.98503e-06
0.9	9.86921	9.89963	-0.0304283	0.00308316	9.83883	0.0303796	0.00307823	9.86916	4.86529e-05	4.92977e-06
1	9.43656	9.46941	-0.0328458	0.0034807	9.40378	0.0327885	0.00347463	9.43651	5.72754e-05	6.06952e-06

Item e, h = 0.001
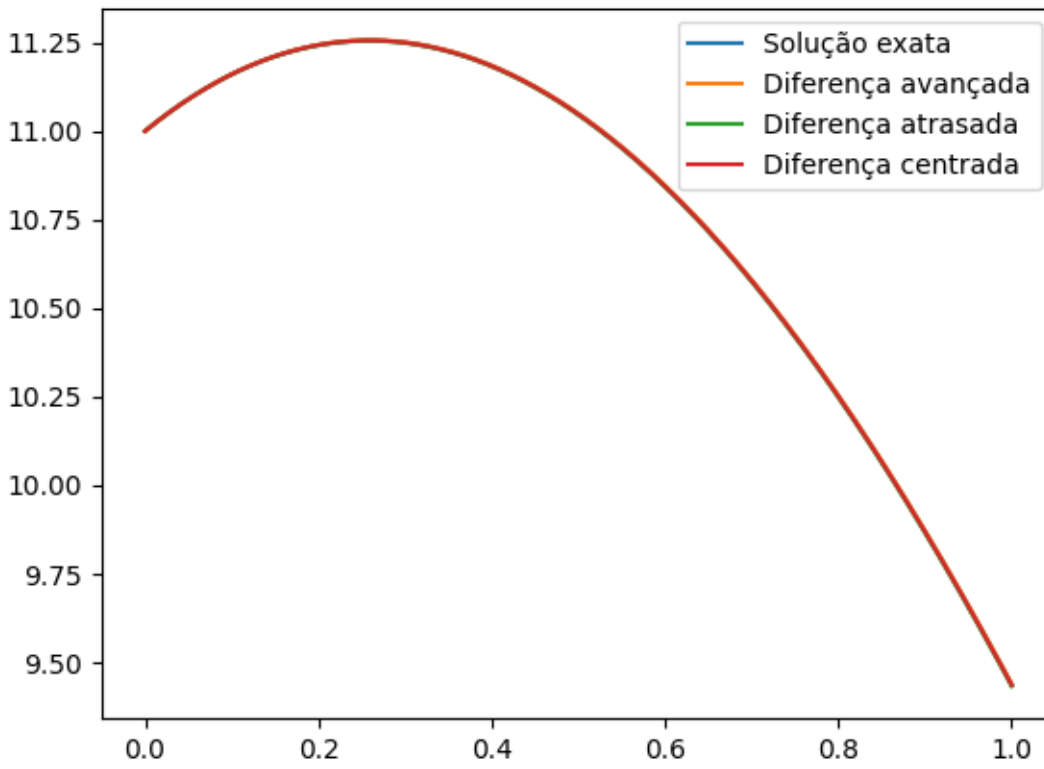
x	y(x)	y_avn	err_loc_avn	err_rel_avn	y_atr	err_loc_atr	err_rel_atr	y_cen	err_loc_cen	err_rel_cen
0	11	11	0	0	11	0	0	11	0	0
0.1	11.1603	11.1607	-0.000394847	3.53794e-05	11.1599	0.000394812	3.53763e-05	11.1603	3.5057e-08	3.14121e-09
0.2	11.2428	11.2436	-0.000778634	6.92562e-05	11.242	0.00077856	6.92496e-05	11.2428	7.38009e-08	6.56428e-09
0.3	11.2497	11.2509	-0.0011502	0.000102243	11.2486	0.00115008	0.000102232	11.2497	1.1662e-07	1.03664e-08
0.4	11.1836	11.1852	-0.00150826	0.000134863	11.1821	0.00150809	0.000134848	11.1836	1.63942e-07	1.4659e-08
0.5	11.0474	11.0493	-0.00185139	0.000167585	11.0456	0.00185117	0.000167566	11.0474	2.1624e-07	1.95738e-08
0.6	10.8442	10.8464	-0.00217802	0.000200846	10.8421	0.00217774	0.00020082	10.8442	2.7404e-07	2.52705e-08
0.7	10.5775	10.58	-0.00248642	0.000235066	10.575	0.00248608	0.000235034	10.5775	3.37918e-07	3.19468e-08
0.8	10.2511	10.2539	-0.00277466	0.00027067	10.2483	0.00277425	0.00027063	10.2511	4.08514e-07	3.98508e-08
0.9	9.86921	9.87225	-0.00304064	0.000308094	9.86617	0.00304015	0.000308044	9.86921	4.86534e-07	4.92982e-08
1	9.43656	9.43985	-0.003282	0.000347797	9.43328	0.00328143	0.000347736	9.43656	5.72761e-07	6.06959e-08

Item e, h = 0.0001

x	y(x)	y_avn	err_loc_avn	err_rel_avn	y_atr	err_loc_atr	err_rel_atr	y_cen	err_loc_cen	err_rel_cen
0	11	11	0	0	11	0	0	11	0	0
0.1	11.1603	11.1604	-3.94831e-05	3.5378e-06	11.1603	3.94827e-05	3.53777e-06	11.1603	3.50568e-10	3.14119e-11
0.2	11.2428	11.2429	-7.78601e-05	6.92533e-06	11.2427	7.78594e-05	6.92526e-06	11.2428	7.38005e-10	6.56424e-11
0.3	11.2497	11.2498	-0.000115015	1.02238e-05	11.2496	0.000115014	1.02237e-05	11.2497	1.16619e-09	1.03664e-10
0.4	11.1836	11.1838	-0.000150818	1.34856e-05	11.1835	0.000150817	1.34855e-05	11.1836	1.63939e-09	1.46589e-10
0.5	11.0474	11.0476	-0.000185129	1.67576e-05	11.0473	0.000185127	1.67574e-05	11.0474	2.16239e-09	1.95737e-10
0.6	10.8442	10.8445	-0.000217789	2.00834e-05	10.844	0.000217787	2.00832e-05	10.8442	2.74038e-09	2.52704e-10
0.7	10.5775	10.5778	-0.000248626	2.35052e-05	10.5773	0.000248623	2.35049e-05	10.5775	3.37917e-09	3.19467e-10
0.8	10.2511	10.2514	-0.000277448	2.70652e-05	10.2508	0.000277444	2.70648e-05	10.2511	4.08511e-09	3.98505e-10
0.9	9.86921	9.86951	-0.000304042	3.08072e-05	9.8689	0.000304037	3.08067e-05	9.86921	4.86528e-09	4.92976e-10
1	9.43656	9.43689	-0.000328175	3.47769e-05	9.43624	0.000328169	3.47763e-05	9.43656	5.72755e-09	6.06953e-10