Métodos Computacionais para Equações Diferenciais
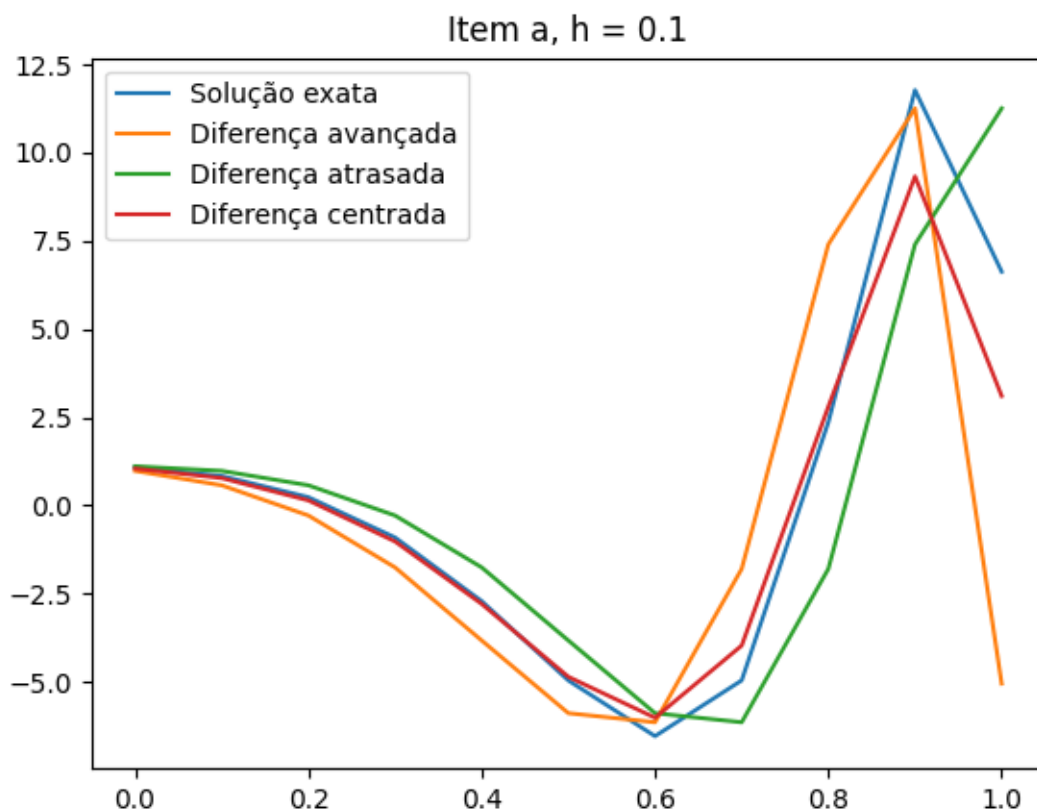## Tarefa 1

Daniel Serezane

2024, 1º Semestre
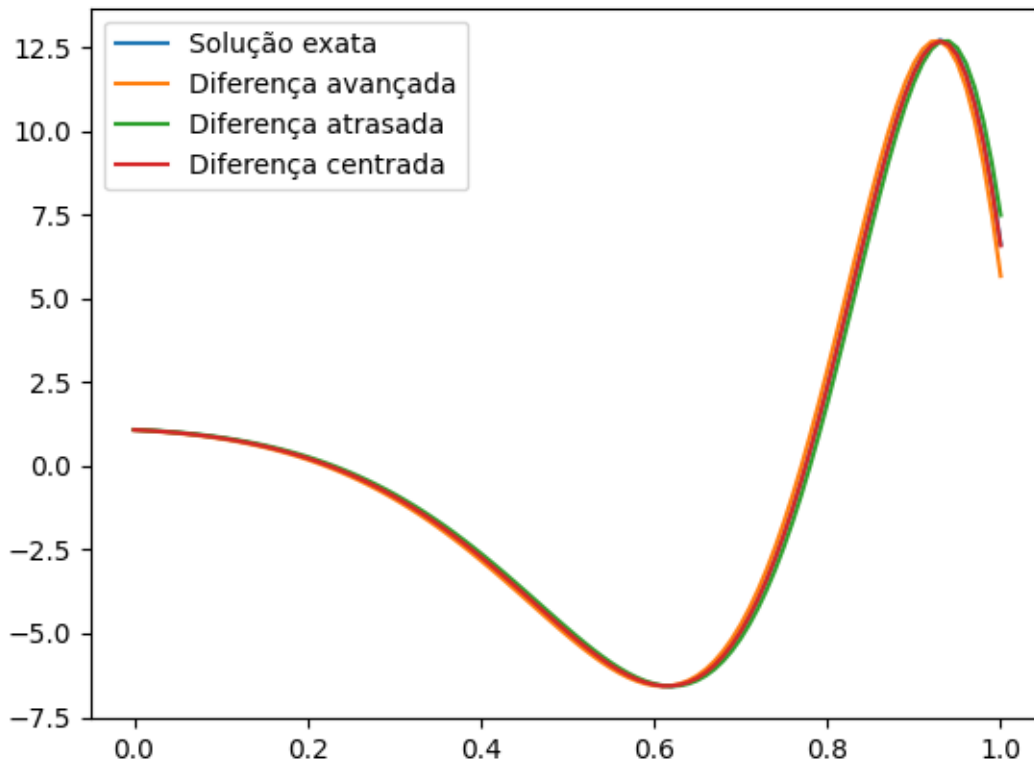
a) $f(x) = \sin(e^{2x}), \quad x \in [0,1]$
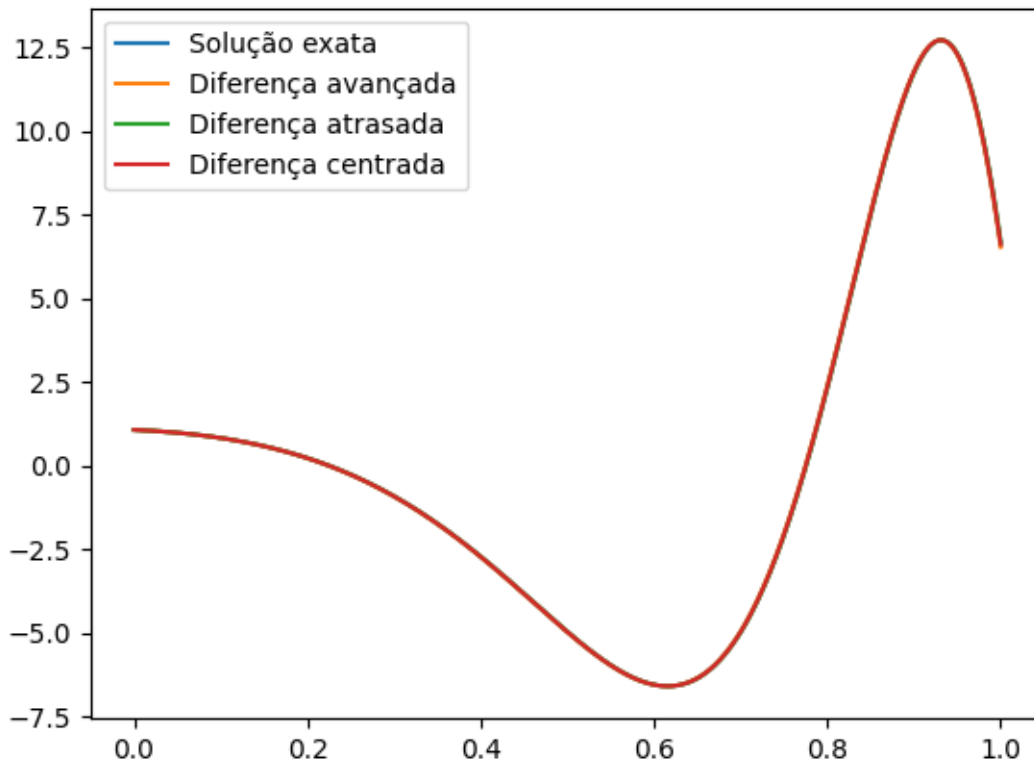**Solução exata:** $f'(x) = 2(e^{2x})\cos(e^{2x})$

Item a, h = 0.1

x	y'(x)	avn	err_loc_avn	err_rel_avn	atr	err_loc_atr	err_rel_atr	cen	err_loc_cen	err_rel_cen
0	1.0806	0.981095	0.0995096	0.092087	1.11192	-0.0313119	0.0289763	1.04651	0.0340989	0.0315554
0.1	0.836241	0.573029	0.263212	0.314756	0.981095	-0.144854	0.173221	0.777062	0.059179	0.0707679
0.2	0.235379	-0.28299	0.518369	2.20227	0.573029	-0.33765	1.4345	0.14502	0.0903593	0.383889
0.3	-0.906268	-1.75381	0.847541	0.9352	-0.28299	-0.623278	0.687742	-1.0184	0.112132	0.123729
0.4	-2.71052	-3.82422	1.11371	0.410883	-1.75381	-0.956707	0.352961	-2.78902	0.0784995	0.0289611
0.5	-4.9567	-5.88359	0.926888	0.186997	-3.82422	-1.13248	0.228474	-4.8539	-0.102795	0.0207385
0.6	-6.5347	-6.14135	-0.393349	0.0601938	-5.88359	-0.651112	0.0996391	-6.01247	-0.52223	0.0799165
0.7	-4.95459	-1.79472	-3.15987	0.637766	-6.14135	1.18676	0.239527	-3.96804	-0.986556	0.19912
0.8	2.36089	7.39764	-5.03675	2.13341	-1.79472	4.15561	1.76019	2.80146	-0.440571	0.186612
0.9	11.7708	11.2528	0.518087	0.0440145	7.39764	4.3732	0.371528	9.3252	2.44565	0.207771
1	6.62586	-5.04654	11.6724	1.76164	11.2528	-4.6269	0.698309	3.10311	3.52275	0.531667

Item a, h = 0.01

x	y'(x)	avn	err_loc_avn	err_rel_avn	atr	err_loc_atr	err_rel_atr	cen	err_loc_cen	err_rel_cen
0	1.0806	1.07424	0.00636517	0.00589038	1.0863	-0.00569191	0.00526734	1.08027	0.000336633	0.000311523
0.1	0.836241	0.815974	0.0202669	0.0242357	0.855332	-0.0190906	0.0228291	0.835653	0.000588133	0.000703306
0.2	0.235379	0.192446	0.042933	0.1824	0.276499	-0.0411197	0.174696	0.234472	0.000906646	0.00385186
0.3	-0.906268	-0.980795	0.074527	0.0822351	-0.834032	-0.0722352	0.0797063	-0.907414	0.00114591	0.00126442
0.4	-2.71052	-2.81703	0.106516	0.0392974	-2.60571	-0.104803	0.0386655	-2.71137	0.000856471	0.000315981
0.5	-4.9567	-5.066	0.109302	0.0220513	-4.8456	-0.111099	0.0224139	-4.9558	-0.000898458	0.000181261
0.6	-6.5347	-6.55561	0.0209106	0.00319994	-6.50349	-0.0312114	0.00477626	-6.52955	-0.00515039	0.00078816
0.7	-4.95459	-4.73337	-0.221227	0.0446509	-5.15521	0.200617	0.0404911	-4.94429	-0.010305	0.00207988
0.8	2.36089	2.86643	-0.505539	0.214131	1.86697	0.493916	0.209208	2.3667	-0.00581174	0.00246168
0.9	11.7708	12.0324	-0.261528	0.0222183	11.4602	0.310624	0.0263893	11.7463	0.0245481	0.0020855
1	6.62586	5.67378	0.952083	0.143692	7.49171	-0.865856	0.130678	6.58275	0.0431134	0.00650683

Item a, h = 0.001

x	y'(x)	avn	err_loc_avn	err_rel_avn	atr	err_loc_atr	err_rel_atr	cen	err_loc_cen	err_rel_cen
0	1.0806	1.08	0.000605708	0.000560527	1.0812	-0.000598977	0.000554298	1.0806	3.36589e-06	3.11482e-06
0.1	0.836241	0.834268	0.00197303	0.0023594	0.838202	-0.00196126	0.00234533	0.836235	5.88095e-06	7.0326e-06
0.2	0.235379	0.231168	0.00421091	0.0178899	0.239572	-0.00419277	0.0178129	0.23537	9.06673e-06	3.85197e-05
0.3	-0.906268	-0.913617	0.00734936	0.00810948	-0.898941	-0.00732644	0.00808419	-0.906279	1.14615e-05	1.26469e-05
0.4	-2.71052	-2.72109	0.0105766	0.00390206	-2.69996	-0.0105594	0.00389573	-2.71052	8.57181e-06	3.16243e-06
0.5	-4.9567	-4.96772	0.0110182	0.0022229	-4.94566	-0.0110362	0.00222652	-4.95669	-8.97141e-06	1.80996e-06
0.6	-6.5347	-6.53727	0.00256813	0.000392999	-6.53203	-0.00267112	0.00040876	-6.53465	-5.14956e-05	7.88033e-06
0.7	-4.95459	-4.93341	-0.0211874	0.00427632	-4.97557	0.0209813	0.00423471	-4.95449	-0.000103092	2.08073e-05
0.8	2.36089	2.41096	-0.05007	0.0212081	2.31093	0.0499535	0.0211588	2.36095	-5.82626e-05	2.46782e-05
0.9	11.7708	11.7993	-0.0284635	0.00241814	11.7419	0.0289545	0.00245985	11.7706	0.000245469	2.0854e-05
1	6.62586	6.53445	0.0914109	0.0137961	6.71641	-0.090547	0.0136657	6.62543	0.000431959	6.51929e-05

Item a, h = 0.0001
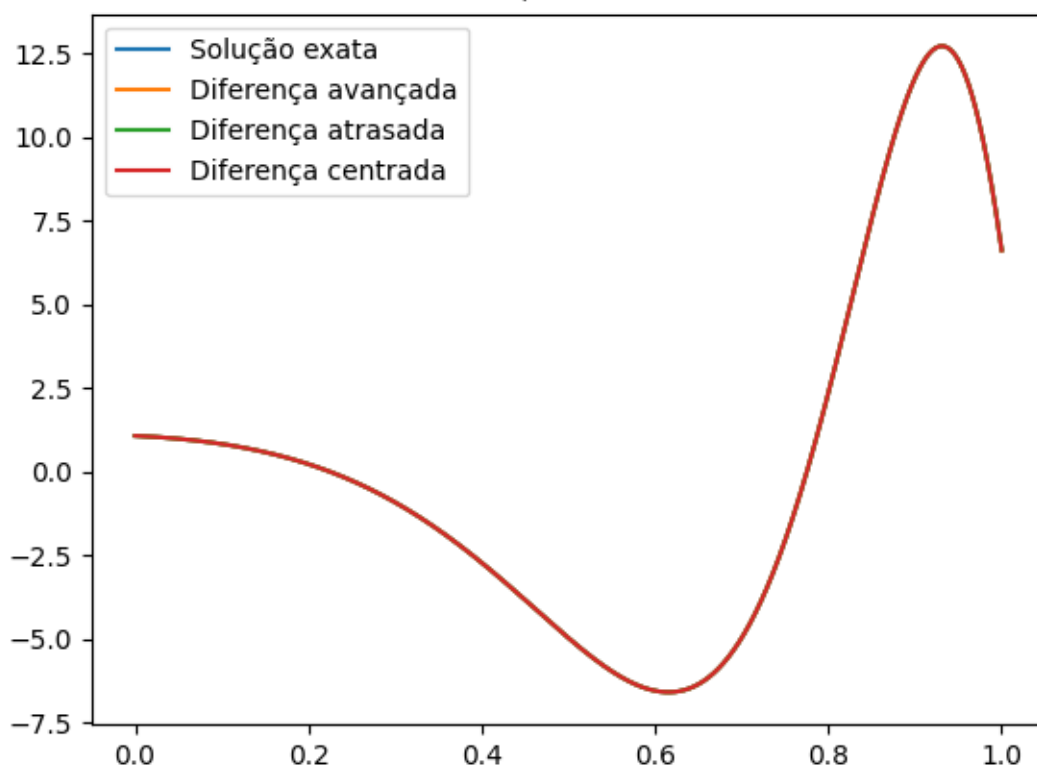
x	y'(x)	avn	err_loc_avn	err_rel_avn	atr	err_loc_atr	err_rel_atr	cen	err_loc_cen	err_rel_cen
0	1.0806	1.08054	6.02674e-05	5.57719e-05	1.08066	-6.02001e-05	5.57096e-05	1.0806	3.36593e-08	3.11485e-08
0.1	0.836241	0.836044	0.000196773	0.000235306	0.836438	-0.000196655	0.000235165	0.836241	5.88089e-08	7.03253e-08
0.2	0.235379	0.234959	0.000420274	0.00178552	0.235799	-0.000420092	0.00178475	0.235379	9.06677e-08	3.85199e-07
0.3	-0.906268	-0.907002	0.000733904	0.000809809	-0.905534	-0.000733675	0.000809557	-0.906268	1.14615e-07	1.26469e-07
0.4	-2.71052	-2.71157	0.00105689	0.000389922	-2.70946	-0.00105672	0.000389859	-2.71052	8.57185e-08	3.16244e-08
0.5	-4.9567	-4.9578	0.00110264	0.000222454	-4.9556	-0.00110282	0.00022249	-4.9567	-8.97128e-08	1.80993e-08
0.6	-6.5347	-6.53496	0.000261461	4.00112e-05	-6.53444	-0.000262491	4.01688e-05	-6.5347	-5.14956e-07	7.88032e-08
0.7	-4.95459	-4.95248	-0.00210946	0.000425758	-4.9567	0.0021074	0.000425342	-4.95459	-1.03092e-06	2.08074e-07
0.8	2.36089	2.36589	-0.0050018	0.00211861	2.35589	0.00500063	0.00211811	2.36089	-5.8264e-07	2.46789e-07
0.9	11.7708	11.7737	-0.00286855	0.000243699	11.768	0.00287346	0.000244116	11.7708	2.45469e-06	2.0854e-07
1	6.62586	6.61676	0.0091023	0.00137375	6.63495	-0.00909366	0.00137245	6.62585	4.31967e-06	6.51941e-07

Item a, h = 1e-05
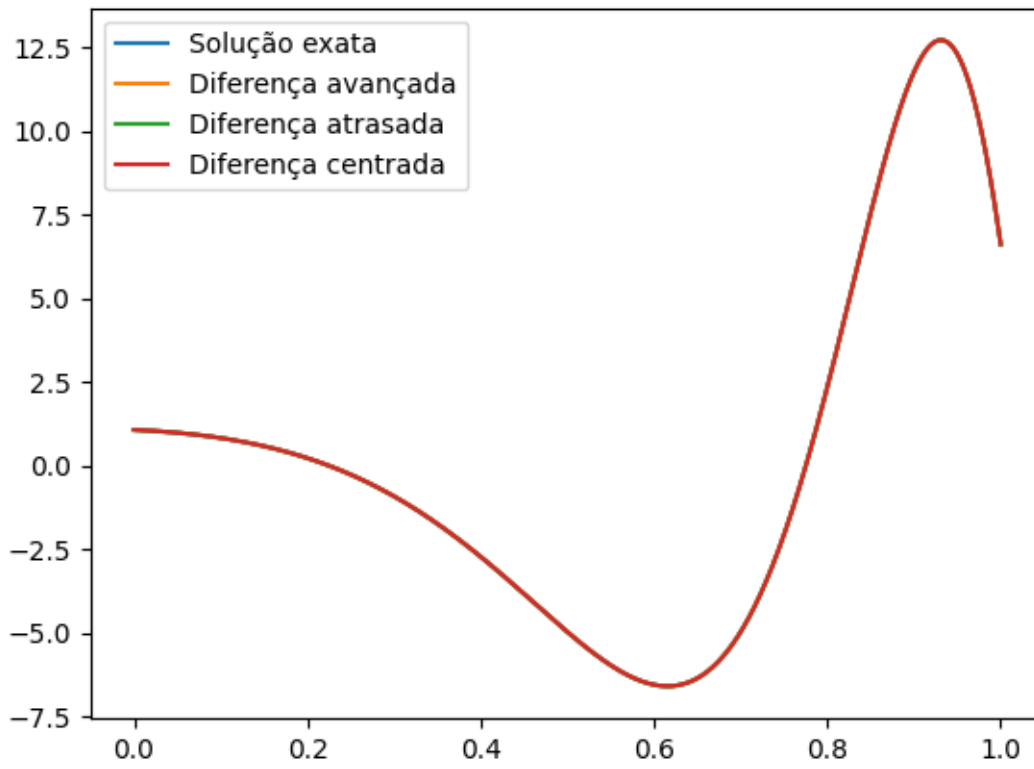
x	y'(x)	avn	err_loc_avn	err_rel_avn	atr	err_loc_atr	err_rel_atr	cen	err_loc_cen	err_rel_cen
0	1.0806	1.0806	6.02371e-06	5.57439e-06	1.08061	-6.02304e-06	5.57377e-06	1.0806	3.38695e-10	3.13431e-10
0.1	0.836241	0.836221	1.9672e-05	2.35243e-05	0.836261	-1.96708e-05	2.35229e-05	0.836241	5.90471e-10	7.06102e-10
0.2	0.235379	0.235337	4.20192e-05	0.000178517	0.235421	-4.20174e-05	0.00017851	0.235379	9.05624e-10	3.84752e-09
0.3	-0.906268	-0.906341	7.33801e-05	8.09696e-05	-0.906194	-7.33778e-05	8.0967e-05	-0.906268	1.14652e-09	1.2651e-09
0.4	-2.71052	-2.71062	0.000105681	3.89893e-05	-2.71041	-0.00010568	3.89887e-05	-2.71052	8.55226e-10	3.15522e-10
0.5	-4.9567	-4.95681	0.000110272	2.2247e-05	-4.95659	-0.000110274	2.22474e-05	-4.9567	-8.96378e-10	1.80842e-10
0.6	-6.5347	-6.53473	2.61925e-05	4.00821e-06	-6.53467	-2.62028e-05	4.0098e-06	-6.5347	-5.17651e-09	7.92157e-10
0.7	-4.95459	-4.95438	-0.000210853	4.25571e-05	-4.9548	0.000210832	4.25529e-05	-4.95459	-1.03393e-08	2.08682e-09
0.8	2.36089	2.36139	-0.000500127	0.000211839	2.36039	0.000500115	0.000211834	2.36089	-5.81236e-09	2.46194e-09
0.9	11.7708	11.7711	-0.000287076	2.43887e-05	11.7706	0.000287125	2.43929e-05	11.7708	2.46198e-08	2.09159e-09
1	6.62586	6.62495	0.000909841	0.000137317	6.62677	-0.000909755	0.000137304	6.62586	4.31976e-08	6.51955e-09

Item a, h = 1e-06
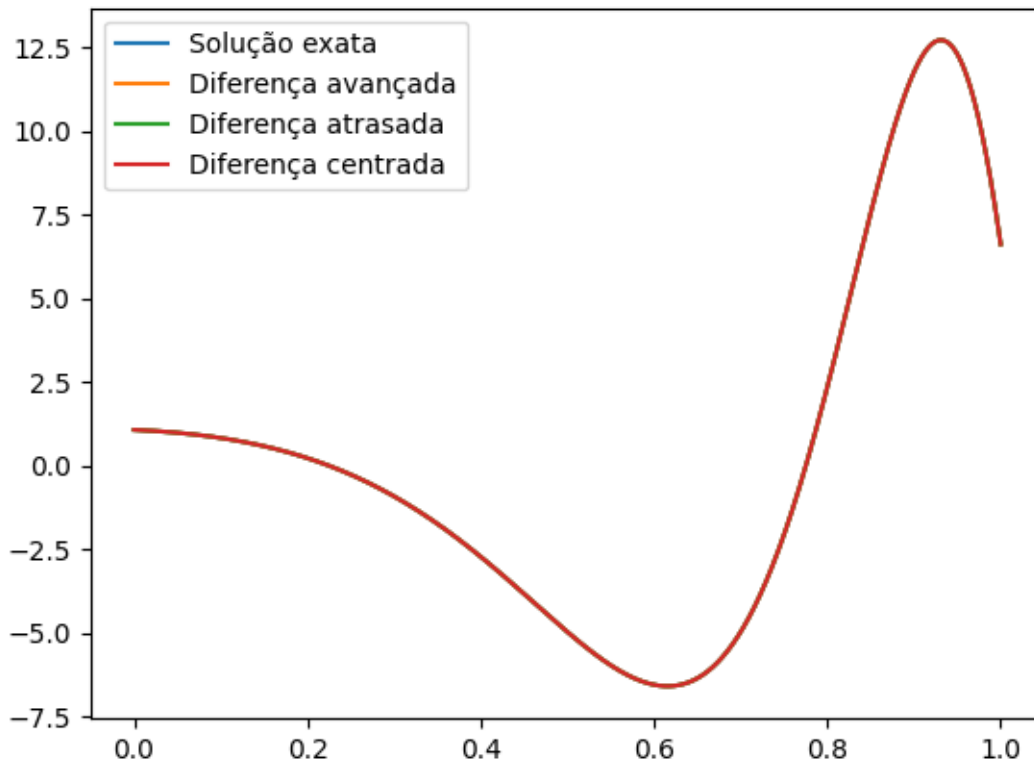
x	y'(x)	avn	err_loc_avn	err_rel_avn	atr	err_loc_atr	err_rel_atr	cen	err_loc_cen	err_rel_cen
0	1.0806	1.0806	6.02352e-07	5.57421e-07	1.08061	-6.02351e-07	5.57421e-07	1.0806	7.72715e-14	7.15077e-14
0.1	0.836241	0.836239	1.96713e-06	2.35235e-06	0.836243	-1.96717e-06	2.35239e-06	0.836241	-2.01514e-11	2.40976e-11
0.2	0.235379	0.235375	4.20182e-06	1.78513e-05	0.235383	-4.20179e-06	1.78512e-05	0.235379	1.18945e-11	5.05336e-11
0.3	-0.906268	-0.906275	7.33792e-06	8.09685e-06	-0.90626	-7.3379e-06	8.09684e-06	-0.906268	8.54206e-12	9.42553e-12
0.4	-2.71052	-2.71053	1.05679e-05	3.89886e-06	-2.71051	-1.05681e-05	3.89891e-06	-2.71052	-6.07079e-11	2.23972e-11
0.5	-4.9567	-4.95671	1.10274e-05	2.22474e-06	-4.95669	-1.10276e-05	2.22479e-06	-4.9567	-1.19222e-10	2.40527e-11
0.6	-6.5347	-6.5347	2.61992e-06	4.00924e-07	-6.5347	-2.61964e-06	4.00882e-07	-6.5347	1.35912e-10	2.07985e-11
0.7	-4.95459	-4.95457	-2.10846e-05	4.25556e-06	-4.95461	2.10845e-05	4.25554e-06	-4.95459	-3.09264e-11	6.24196e-12
0.8	2.36089	2.36094	-5.00122e-05	2.11836e-05	2.36084	5.00121e-05	2.11836e-05	2.36089	-3.92011e-11	1.66044e-11
0.9	11.7708	11.7709	-2.87099e-05	2.43907e-06	11.7708	2.87099e-05	2.43907e-06	11.7708	1.03206e-11	8.76796e-13
1	6.62586	6.62577	9.09808e-05	1.37312e-05	6.62595	-9.09796e-05	1.3731e-05	6.62586	6.20582e-10	9.36606e-11

b) $f(x) = \sin(x)/\ln(x), \quad x \in [6, 7]$
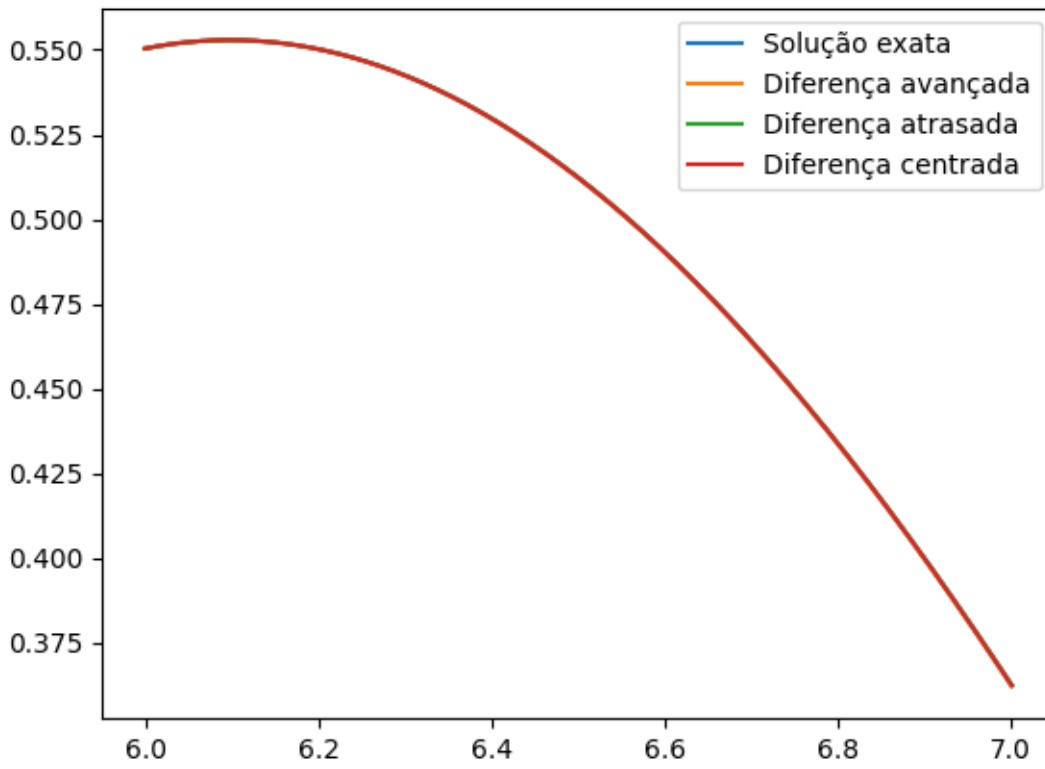**Solução exata:** $f'(x) = (\cos(x)\ln(x) - \sin(x)/x)/(\ln(x))^2$
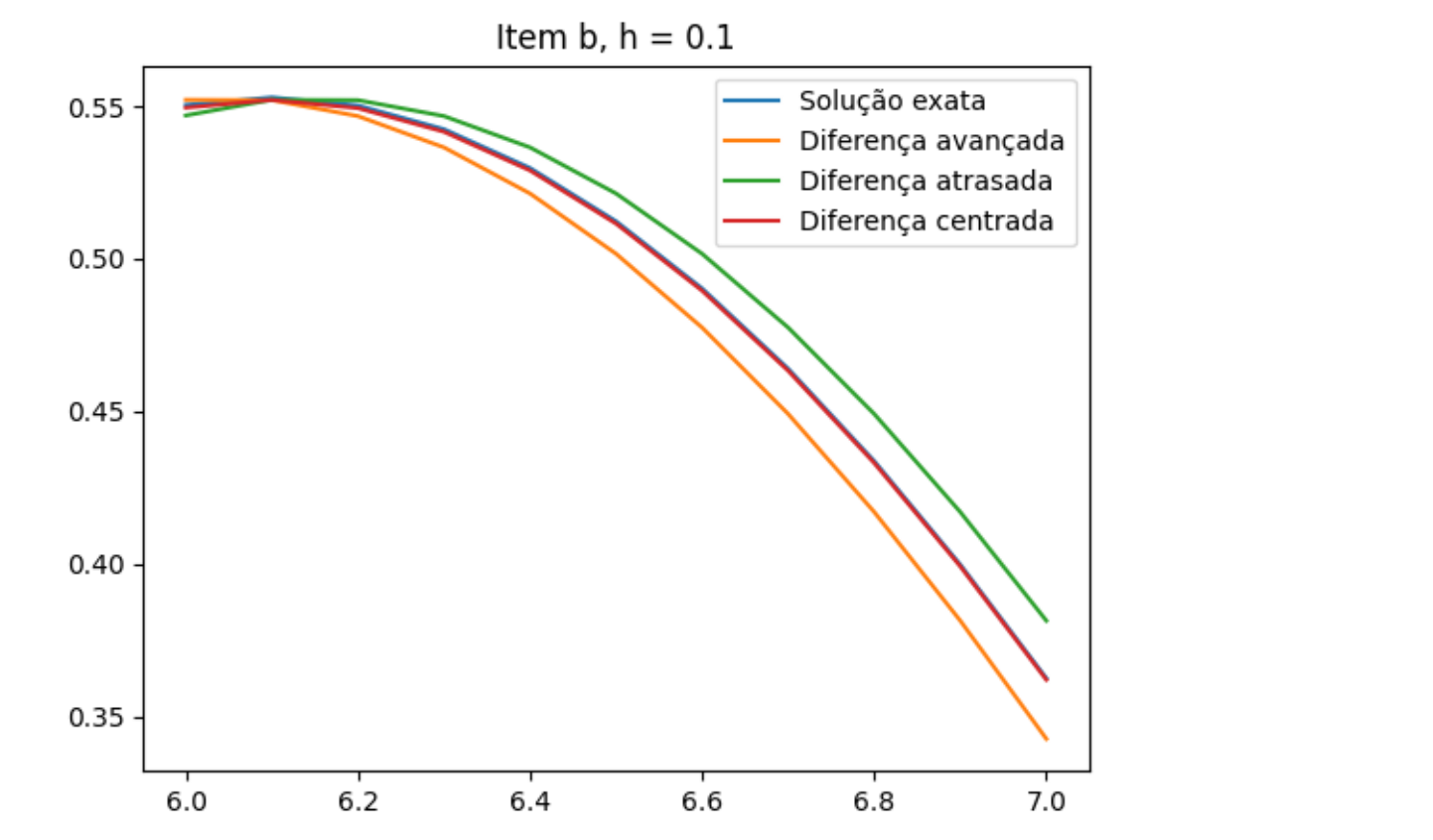
Item b, h = 0.1

x	y'(x)	avn	err_loc_avn	err_rel_avn	atr	err_loc_atr	err_rel_atr	cen	err_loc_cen	err_rel_cen
6	0.550387	0.552073	-0.00168553	0.00306245	0.546956	0.00343061	0.00623309	0.549514	0.000872541	0.00158532
6.1	0.552889	0.551978	0.000910684	0.00164714	0.552073	0.000816388	0.00147659	0.552025	0.000863536	0.00156186
6.2	0.550211	0.546749	0.00346159	0.00629139	0.551978	-0.00176715	0.00321177	0.549364	0.000847222	0.00153981
6.3	0.542451	0.536506	0.00594566	0.0109607	0.546749	-0.00429814	0.00792355	0.541628	0.000823761	0.00151859
6.4	0.52975	0.521408	0.008342	0.015747	0.536506	-0.00675525	0.0127518	0.528957	0.000793377	0.00149764
6.5	0.512291	0.50166	0.0106306	0.0207511	0.521408	-0.00911784	0.0177982	0.511534	0.000756362	0.00147643
6.6	0.490294	0.477501	0.0127923	0.0260912	0.50166	-0.0113662	0.0231824	0.489581	0.000713067	0.00145437
6.7	0.46402	0.44921	0.0148095	0.0319156	0.477501	-0.0134817	0.0290541	0.463356	0.0006639	0.00143076
6.8	0.433763	0.417098	0.0166655	0.0384207	0.44921	-0.0154469	0.0356112	0.433154	0.000609323	0.00140473
6.9	0.399852	0.381507	0.0183455	0.0458806	0.417098	-0.0172458	0.0431304	0.399302	0.000549845	0.00137512
7	0.362643	0.342807	0.019836	0.0546986	0.381507	-0.018864	0.0520181	0.362157	0.00048602	0.00134022

Item b, h = 0.01
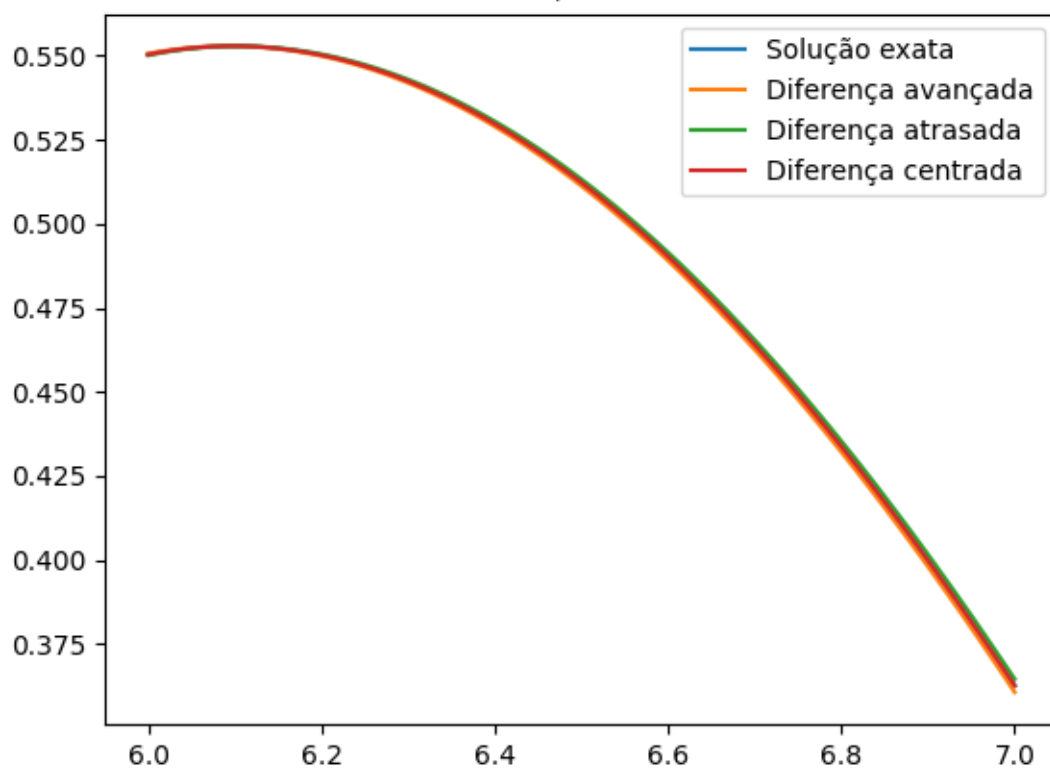
x	y'(x)	avn	err_loc_avn	err_rel_avn	atr	err_loc_atr	err_rel_atr	cen	err_loc_cen	err_rel_cen
6	0.550387	0.550634	-0.000246947	0.000448678	0.550123	0.000264405	0.000480398	0.550378	8.72908e-06	1.58599e-05
6.1	0.552889	0.552875	1.36677e-05	2.47205e-05	0.552885	3.61028e-06	6.52985e-06	0.55288	8.63898e-06	1.56252e-05
6.2	0.550211	0.549941	0.000270406	0.000491459	0.550465	-0.000253454	0.000460649	0.550203	8.47576e-06	1.54046e-05
6.3	0.542451	0.54193	0.000521099	0.000960637	0.542956	-0.000504617	0.000930252	0.542443	8.24103e-06	1.51922e-05
6.4	0.52975	0.528987	0.000763635	0.0014415	0.530498	-0.000747761	0.00141153	0.529742	7.93706e-06	1.49826e-05
6.5	0.512291	0.511295	0.000995983	0.00194418	0.513271	-0.000980849	0.00191464	0.512283	7.56674e-06	1.47704e-05
6.6	0.490294	0.489078	0.00121621	0.00248057	0.491496	-0.00120194	0.00245147	0.490287	7.13358e-06	1.45496e-05
6.7	0.46402	0.462597	0.00142248	0.00306557	0.465429	-0.0014092	0.00303694	0.464013	6.64168e-06	1.43134e-05
6.8	0.433763	0.43215	0.00161313	0.00371891	0.435364	-0.00160094	0.0036908	0.433757	6.09566e-06	1.40529e-05
6.9	0.399852	0.398066	0.00178659	0.00446813	0.401628	-0.00177559	0.00444062	0.399847	5.5006e-06	1.37566e-05
7	0.362643	0.360701	0.00194149	0.00535374	0.364575	-0.00193177	0.00532692	0.362638	4.86206e-06	1.34073e-05

Item b, h = 0.001

x	y'(x)	avn	err_loc_avn	err_rel_avn	atr	err_loc_atr	err_rel_atr	cen	err_loc_cen	err_rel_cen
6	0.550387	0.550412	-2.54801e-05	4.6295e-05	0.550361	2.56547e-05	4.66122e-05	0.550387	8.72909e-08	1.58599e-07
6.1	0.552889	0.552888	5.89573e-07	1.06635e-06	0.552889	-4.16794e-07	7.53847e-07	0.552889	8.639e-08	1.56252e-07
6.2	0.550211	0.550185	2.62783e-05	4.77603e-05	0.550237	-2.61088e-05	4.74523e-05	0.550211	8.47578e-08	1.54046e-07
6.3	0.542451	0.5424	5.13689e-05	9.46976e-05	0.542503	-5.1204e-05	9.43938e-05	0.542451	8.24105e-08	1.51922e-07
6.4	0.52975	0.529675	7.565e-05	0.000142803	0.529826	-7.54913e-05	0.000142503	0.52975	7.93707e-08	1.49827e-07
6.5	0.512291	0.512192	9.89183e-05	0.00019309	0.512389	-9.8767e-05	0.000192795	0.51229	7.56675e-08	1.47704e-07
6.6	0.490294	0.490173	0.00012098	0.000246749	0.490415	-0.000120837	0.000246459	0.490294	7.1336e-08	1.45496e-07
6.7	0.46402	0.463878	0.000141652	0.000305271	0.464161	-0.000141519	0.000304985	0.46402	6.64169e-08	1.43134e-07
6.8	0.433763	0.433603	0.000160765	0.000370629	0.433924	-0.000160644	0.000370348	0.433763	6.09567e-08	1.4053e-07
6.9	0.399852	0.399674	0.000178166	0.000445579	0.40003	-0.000178056	0.000445303	0.399852	5.5006e-08	1.37566e-07
7	0.362643	0.362449	0.000193713	0.000534171	0.362836	-0.000193616	0.000533903	0.362643	4.86207e-08	1.34073e-07

Item b, h = 0.0001
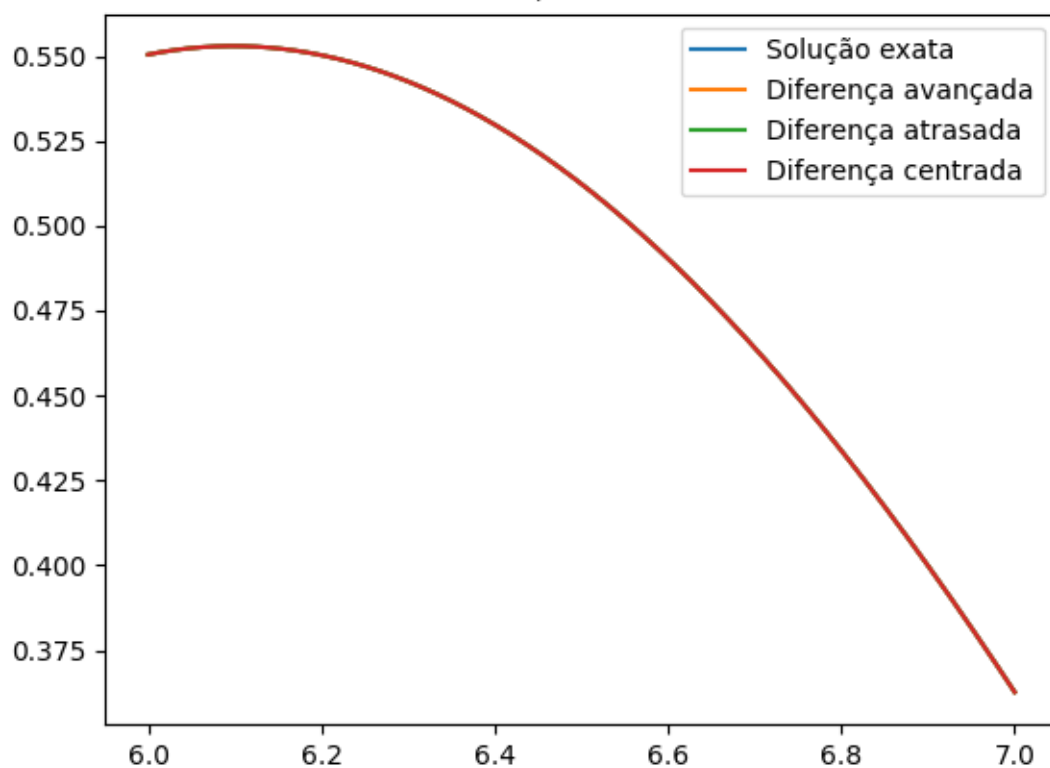
x	y'(x)	avn	err_loc_avn	err_rel_avn	atr	err_loc_atr	err_rel_atr	cen	err_loc_cen	err_rel_cen
6	0.550387	0.55039	-2.55587e-06	4.64377e-06	0.550384	2.55762e-06	4.64694e-06	0.550387	8.74056e-10	1.58808e-09
6.1	0.552889	0.552889	5.11838e-08	9.25752e-08	0.552889	-4.94536e-08	8.94458e-08	0.552889	8.65108e-10	1.56471e-09
6.2	0.550211	0.550208	2.6202e-06	4.76217e-06	0.550214	-2.6185e-06	4.75909e-06	0.550211	8.48871e-10	1.54281e-09
6.3	0.542451	0.542446	5.12947e-06	9.45609e-06	0.542456	-5.12782e-06	9.45305e-06	0.542451	8.2537e-10	1.52156e-09
6.4	0.52975	0.529743	7.55786e-06	1.42668e-05	0.529758	-7.55627e-06	1.42638e-05	0.52975	7.94893e-10	1.50051e-09
6.5	0.512291	0.512281	9.88502e-06	1.92957e-05	0.5123	-9.88351e-06	1.92928e-05	0.512291	7.57898e-10	1.47943e-09
6.6	0.490294	0.490282	1.20916e-05	2.46619e-05	0.490306	-1.20901e-05	2.46589e-05	0.490294	7.14493e-10	1.45728e-09
6.7	0.46402	0.464006	1.41592e-05	3.05143e-05	0.464034	-1.41579e-05	3.05114e-05	0.46402	6.65311e-10	1.4338e-09
6.8	0.433763	0.433747	1.60711e-05	3.70503e-05	0.43378	-1.60698e-05	3.70475e-05	0.433763	6.10425e-10	1.40728e-09
6.9	0.399852	0.399834	1.78116e-05	4.45455e-05	0.39987	-1.78105e-05	4.45427e-05	0.399852	5.5097e-10	1.37793e-09
7	0.362643	0.362623	1.9367e-05	5.34051e-05	0.362662	-1.9366e-05	5.34024e-05	0.362643	4.86968e-10	1.34283e-09

Item b, h = 1e-05
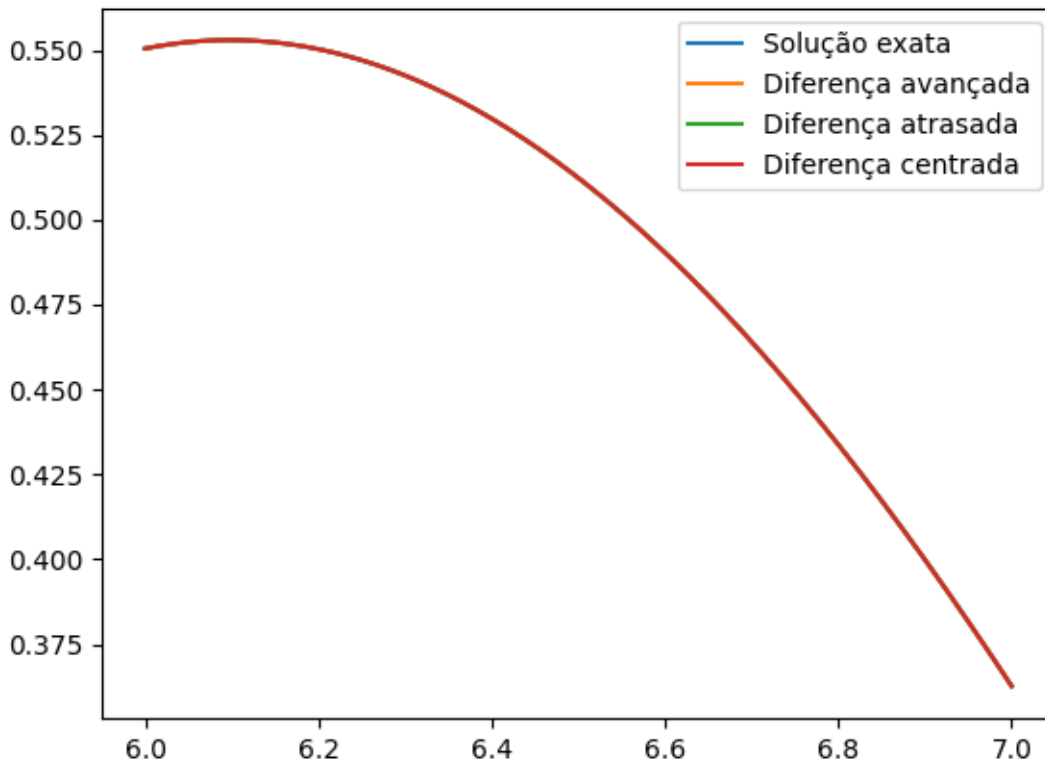
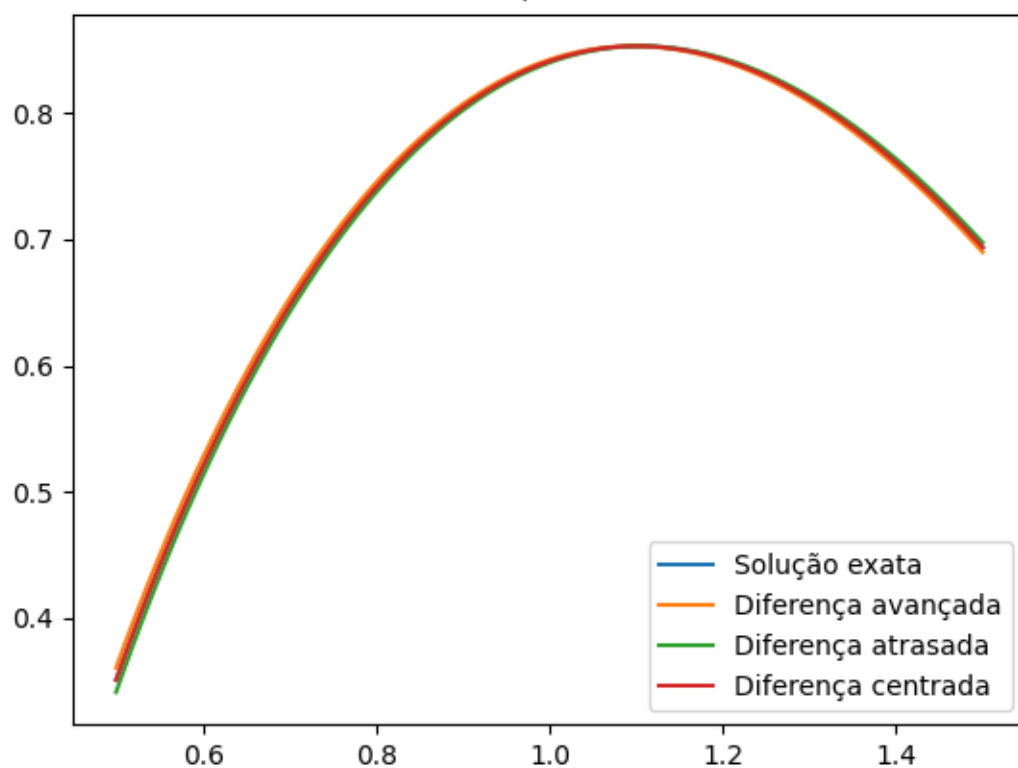| x	y'(x)	avn	err_loc_avn	err_rel_avn	atr	err_loc_atr	err_rel_atr	cen	err_loc_cen	err_rel_cen
6	0.550387	0.550387	-2.55643e-07	4.64478e-07	0.550387	2.55701e-07	4.64584e-07	0.550387	2.9315e-11	5.32625e-11
6.1	0.552889	0.552889	5.06224e-09	9.15598e-09	0.552889	-5.00193e-09	9.04691e-09	0.552889	3.01513e-11	5.45341e-11
6.2	0.550211	0.550211	2.61965e-07	4.76117e-07	0.550211	-2.61906e-07	4.76009e-07	0.550211	2.97002e-11	5.39797e-11
6.3	0.542451	0.542451	5.12893e-07	9.4551e-07	0.542452	-5.12836e-07	9.45404e-07	0.542451	2.88456e-11	5.31764e-11
6.4	0.52975	0.52975	7.55734e-07	1.42659e-06	0.529751	-7.55679e-07	1.42648e-06	0.52975	2.75906e-11	5.20823e-11
6.5	0.512291	0.51229	9.88454e-07	1.92948e-06	0.512292	-9.884e-07	1.92937e-06	0.512291	2.70941e-11	5.28882e-11
6.6	0.490294	0.490293	1.20911e-06	2.46609e-06	0.490295	-1.20906e-06	2.46598e-06	0.490294	2.72653e-11	5.56101e-11
6.7	0.46402	0.464018	1.41587e-06	3.05132e-06	0.464021	-1.41583e-06	3.05123e-06	0.46402	2.27699e-11	4.9071e-11
6.8	0.433763	0.433762	1.60707e-06	3.70494e-06	0.433765	-1.60702e-06	3.70483e-06	0.433763	2.39496e-11	5.52135e-11
6.9	0.399852	0.39985	1.78113e-06	4.45446e-06	0.399854	-1.78108e-06	4.45434e-06	0.399852	2.36138e-11	5.90564e-11
7	0.362643	0.362641	1.93667e-06	5.34042e-06	0.362645	-1.93662e-06	5.34031e-06	0.362643	2.06746e-11	5.7011e-11

Item b, h = 1e-06
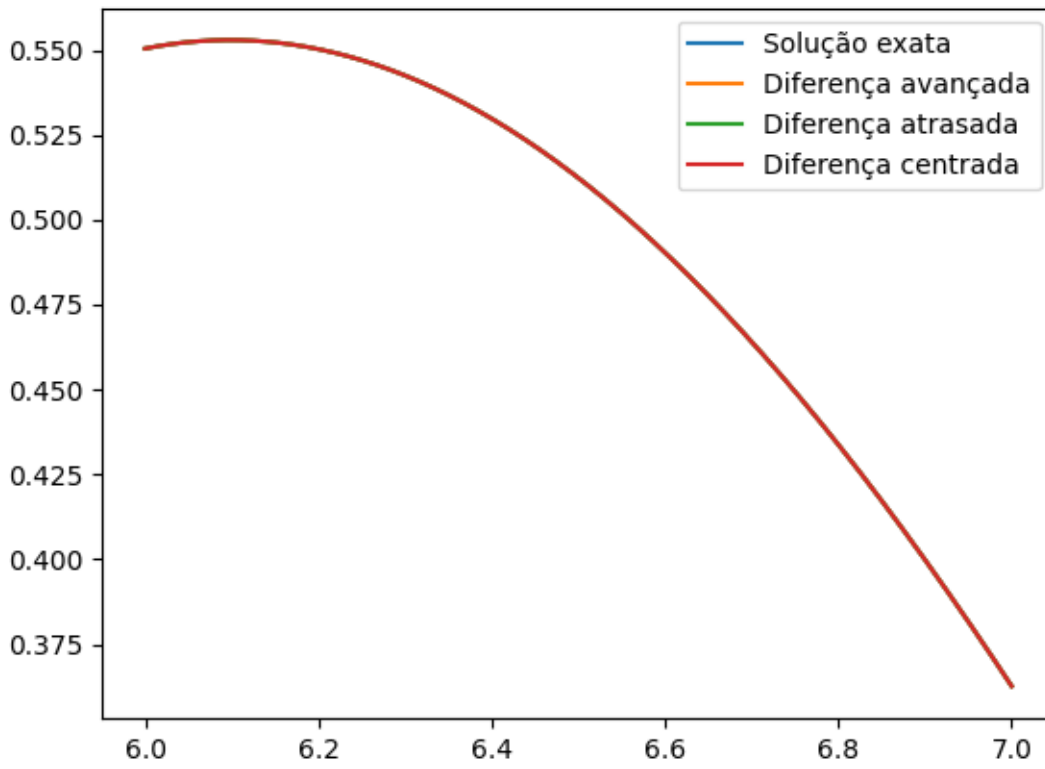
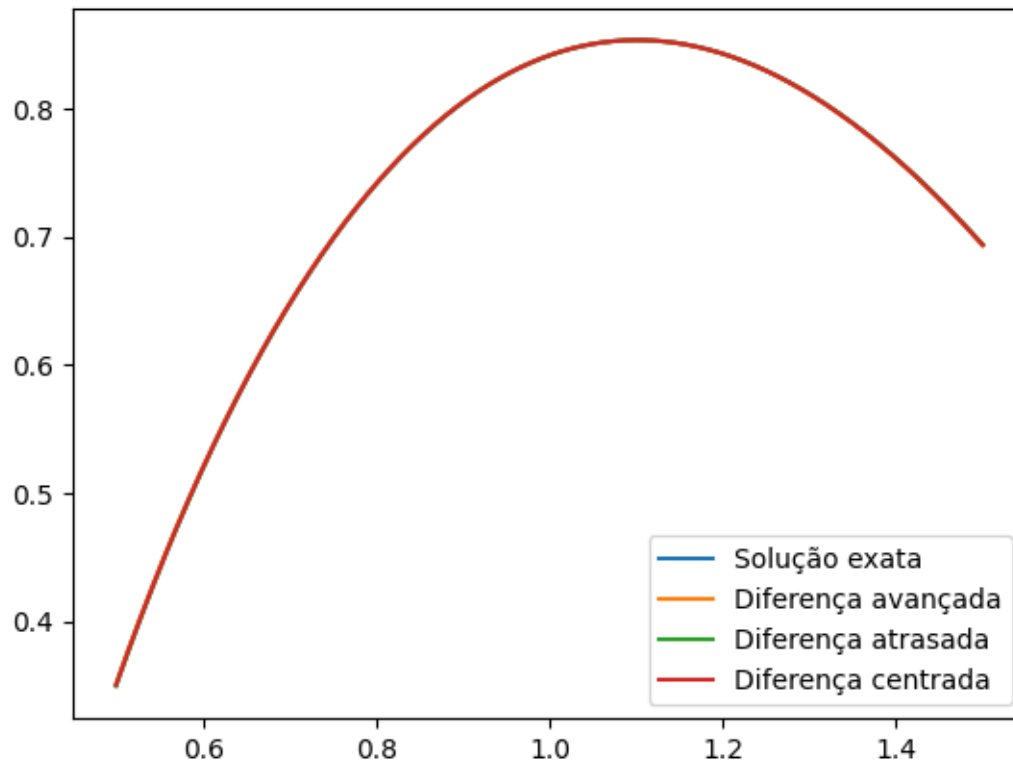| x	y'(x)	avn	err_loc_avn	err_rel_avn	atr	err_loc_atr	err_rel_atr	cen	err_loc_cen	err_rel_cen
6	0.550387	0.550387	-2.56349e-08	4.65761e-08	0.550387	2.54631e-08	4.62641e-08	0.550387	-8.58706e-11	1.56019e-10
6.1	0.552889	0.552889	4.2428e-10	7.67388e-10	0.552889	-5.88798e-10	1.06495e-09	0.552889	-8.22588e-11	1.4878e-10
6.2	0.550211	0.550211	2.61154e-08	4.74644e-08	0.550211	-2.62732e-08	4.77512e-08	0.550211	-7.88934e-11	1.43388e-10
6.3	0.542451	0.542451	5.12113e-08	9.44073e-08	0.542451	-5.13629e-08	9.46866e-08	0.542451	-7.57583e-11	1.39659e-10
6.4	0.52975	0.52975	7.54881e-08	1.42497e-07	0.52975	-7.5641e-08	1.42786e-07	0.52975	-7.64928e-11	1.44394e-10
6.5	0.512291	0.51229	9.87759e-08	1.92812e-07	0.512291	-9.89271e-08	1.93107e-07	0.512291	-7.56015e-11	1.47575e-10
6.6	0.490294	0.490294	1.20803e-07	2.46389e-07	0.490294	-1.20976e-07	2.46742e-07	0.490294	-8.65326e-11	1.76491e-10
6.7	0.46402	0.46402	1.41489e-07	3.0492e-07	0.46402	-1.41618e-07	3.05198e-07	0.46402	-6.46602e-11	1.39348e-10
6.8	0.433763	0.433763	1.60601e-07	3.7025e-07	0.433764	-1.60753e-07	3.70601e-07	0.433763	-7.59705e-11	1.75143e-10
6.9	0.399852	0.399852	1.78037e-07	4.45256e-07	0.399852	-1.78178e-07	4.4561e-07	0.399852	-7.07552e-11	1.76953e-10
7	0.362643	0.362643	1.93638e-07	5.33964e-07	0.362643	-1.93719e-07	5.34186e-07	0.362643	-4.03877e-11	1.1137e-10

c) $f(x) = \ln(x)\sin(x), \quad x \in [0.5, 1.5]$
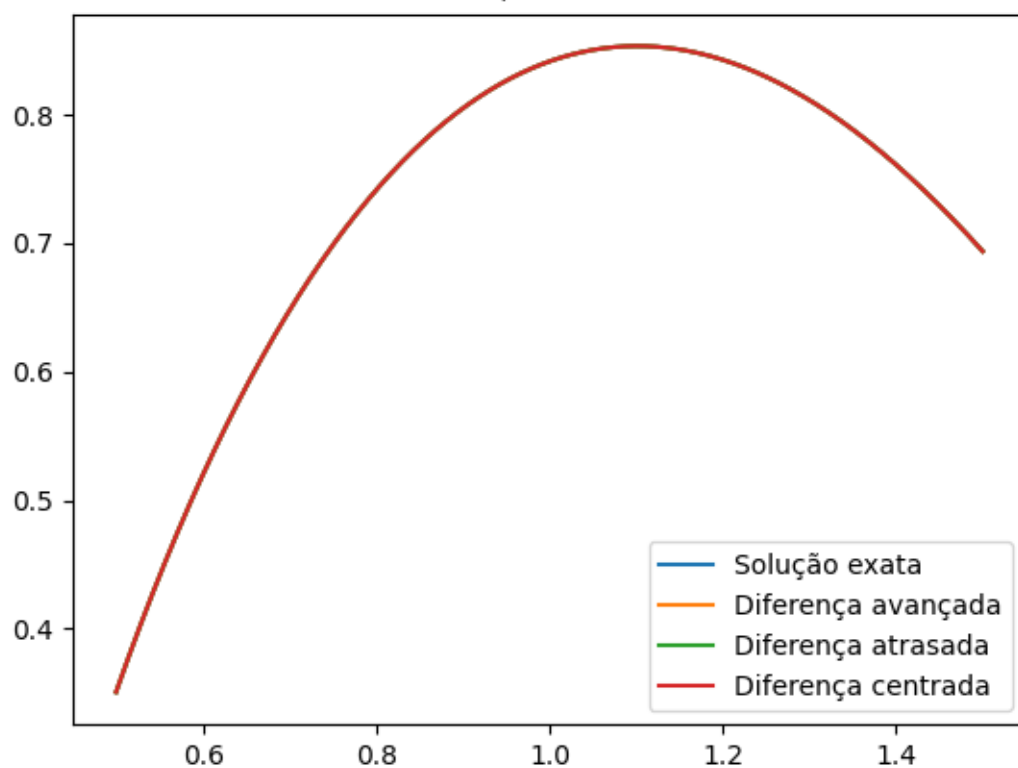
Solução exata: $f'(x) = \sin(x)/x + \ln(x)\cos(x)$
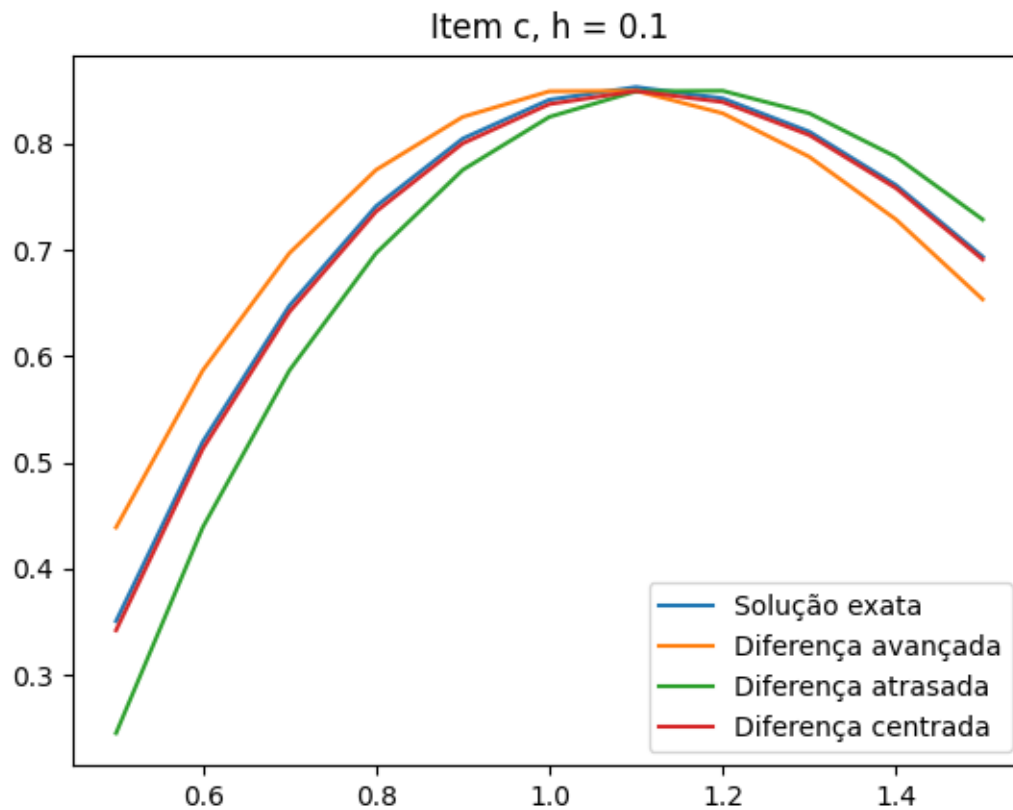
Item c, h = 0.1

x	y'(x)	avn	err_loc_avn	err_rel_avn	atr	err_loc_atr	err_rel_atr	cen	err_loc_cen	err_rel_cen
0.5	0.350557	0.438786	-0.088229	0.251682	0.24508	0.105478	0.300886	0.341933	0.00862433	0.0246018
0.6	0.519468	0.586575	-0.0671072	0.129184	0.438786	0.080682	0.155317	0.512681	0.00678744	0.0130661
0.7	0.647511	0.697029	-0.0495183	0.0764748	0.586575	0.0609356	0.0941074	0.641802	0.00570865	0.0088163
0.8	0.74123	0.775417	-0.0341871	0.0461221	0.697029	0.0442003	0.059631	0.736223	0.0050066	0.00675446
0.9	0.80487	0.825317	-0.0204472	0.0254043	0.775417	0.0294535	0.0365941	0.800367	0.00450316	0.00559489
1	0.841471	0.849411	-0.00794035	0.00943628	0.825317	0.0161537	0.019197	0.837364	0.00410668	0.00488036
1.1	0.853421	0.849897	0.003524	0.00412927	0.849411	0.0040095	0.00469815	0.849654	0.00376675	0.00441371
1.2	0.842765	0.828724	0.0140407	0.0166603	0.849897	-0.00713197	0.00846258	0.839311	0.00345436	0.00409885
1.3	0.811381	0.787732	0.0236483	0.0291458	0.828724	-0.0173434	0.0213752	0.808228	0.00315245	0.00388529
1.4	0.761082	0.728729	0.0323525	0.0425086	0.787732	-0.0266505	0.0350166	0.758231	0.002851	0.00374598
1.5	0.693678	0.653538	0.04014	0.0578655	0.728729	-0.0350513	0.0505296	0.691134	0.00254439	0.00366797

Item c, h = 0.01

x	y'(x)	avn	err_loc_avn	err_rel_avn	atr	err_loc_atr	err_rel_atr	cen	err_loc_cen	err_rel_cen
0.5	0.350557	0.360097	-0.00953982	0.0272133	0.340846	0.00971078	0.027701	0.350472	8.54816e-05	0.000243845
0.6	0.519468	0.526757	-0.00728832	0.0140303	0.512045	0.00742337	0.0142903	0.519401	6.75241e-05	0.000129987
0.7	0.647511	0.652956	-0.00544485	0.00840889	0.641952	0.00555867	0.00858467	0.647454	5.69096e-05	8.78898e-05
0.8	0.74123	0.745085	-0.00385503	0.00520086	0.737275	0.00395497	0.00533569	0.74118	4.99719e-05	6.74176e-05
0.9	0.80487	0.807309	-0.00243921	0.00303057	0.802341	0.00252917	0.00314234	0.804825	4.49807e-05	5.58856e-05
1	0.841471	0.842626	-0.00115472	0.00137226	0.840234	0.0012368	0.0014698	0.84143	4.10399e-05	4.87716e-05
1.1	0.853421	0.853399	2.13537e-05	2.50213e-05	0.853367	5.39553e-05	6.32223e-05	0.853383	3.76545e-05	4.41218e-05
1.2	0.842765	0.841664	0.00110072	0.00130608	0.843797	-0.00103164	0.00122411	0.84273	3.45388e-05	4.09827e-05
1.3	0.811381	0.809292	0.00208855	0.00257406	0.813406	-0.0020255	0.00249636	0.811349	3.15245e-05	3.88529e-05
1.4	0.761082	0.758096	0.00298617	0.00392359	0.764011	-0.00292915	0.00384866	0.761053	2.85127e-05	3.74634e-05
1.5	0.693678	0.689885	0.00379269	0.00546751	0.69742	-0.0037418	0.00539414	0.693653	2.5448e-05	3.66855e-05

Item c, h = 0.001

x	y'(x)	avn	err_loc_avn	err_rel_avn	atr	err_loc_atr	err_rel_atr	cen	err_loc_cen	err_rel_cen
0.5	0.350557	0.351519	-0.000961616	0.00274311	0.349594	0.000963326	0.00274798	0.350556	8.54741e-07	2.43824e-06
0.6	0.519468	0.520203	-0.000734876	0.00141467	0.518732	0.000736226	0.00141727	0.519468	6.75206e-07	1.2998e-06
0.7	0.647511	0.648061	-0.000549586	0.000848767	0.64696	0.000550724	0.000850525	0.64751	5.69078e-07	8.78871e-07
0.8	0.74123	0.741619	-0.000389986	0.000526134	0.740839	0.000390985	0.000527482	0.741229	4.9971e-07	6.74164e-07
0.9	0.80487	0.805118	-0.000247959	0.000308073	0.804621	0.000248858	0.000309191	0.80487	4.49802e-07	5.5885e-07
1	0.841471	0.84159	-0.000119157	0.000141605	0.841351	0.000119977	0.00014258	0.841471	4.10396e-07	4.87713e-07
1.1	0.853421	0.853422	-1.24558e-06	1.45951e-06	0.853419	1.99866e-06	2.34194e-06	0.85342	3.76544e-07	4.41217e-07
1.2	0.842765	0.842658	0.000106971	0.000126928	0.842871	-0.00010628	0.000126109	0.842765	3.45388e-07	4.09827e-07
1.3	0.811381	0.811175	0.000206025	0.000253919	0.811586	-0.000205394	0.000253142	0.81138	3.15245e-07	3.88529e-07
1.4	0.761082	0.760786	0.000296058	0.000388997	0.761377	-0.000295488	0.000388248	0.761082	2.85127e-07	3.74634e-07
1.5	0.693678	0.693301	0.000376987	0.00054346	0.694055	-0.000376478	0.000542727	0.693678	2.5448e-07	3.66856e-07

Item c, h = 0.0001

x	y'(x)	avn	err_loc_avn	err_rel_avn	atr	err_loc_atr	err_rel_atr	cen	err_loc_cen	err_rel_cen
0.5	0.350557	0.350653	-9.62385e-05	0.00027453	0.350461	9.62556e-05	0.000274579	0.350557	8.54756e-09	2.43828e-08
0.6	0.519468	0.519542	-7.35483e-05	0.000141584	0.519395	7.35618e-05	0.00014161	0.519468	6.75232e-09	1.29985e-08
0.7	0.647511	0.647566	-5.50098e-05	8.49558e-05	0.647456	5.50212e-05	8.49734e-05	0.647511	5.69083e-09	8.78879e-09
0.8	0.74123	0.741269	-3.90436e-05	5.26741e-05	0.74119	3.90536e-05	5.26875e-05	0.741229	4.99714e-09	6.74169e-09
0.9	0.80487	0.804895	-2.48363e-05	3.08576e-05	0.804845	2.48453e-05	3.08688e-05	0.80487	4.49818e-09	5.5887e-09
1	0.841471	0.841483	-1.19526e-05	1.42044e-05	0.841459	1.19608e-05	1.42141e-05	0.841471	4.10359e-09	4.87668e-09
1.1	0.853421	0.853421	-1.58438e-07	1.85651e-07	0.853421	1.65969e-07	1.94476e-07	0.853421	3.76553e-09	4.41228e-09
1.2	0.842765	0.842754	1.0666e-05	1.26559e-05	0.842776	-1.06591e-05	1.26477e-05	0.842765	3.45393e-09	4.09833e-09
1.3	0.811381	0.81136	2.05741e-05	2.53569e-05	0.811401	-2.05678e-05	2.53492e-05	0.811381	3.15248e-09	3.88533e-09
1.4	0.761082	0.761052	2.95802e-05	3.8866e-05	0.761111	-2.95745e-05	3.88585e-05	0.761082	2.85147e-09	3.7466e-09
1.5	0.693678	0.69364	3.76758e-05	5.4313e-05	0.693716	-3.76707e-05	5.43057e-05	0.693678	2.54481e-09	3.66857e-09

Item c, h = 1e-05
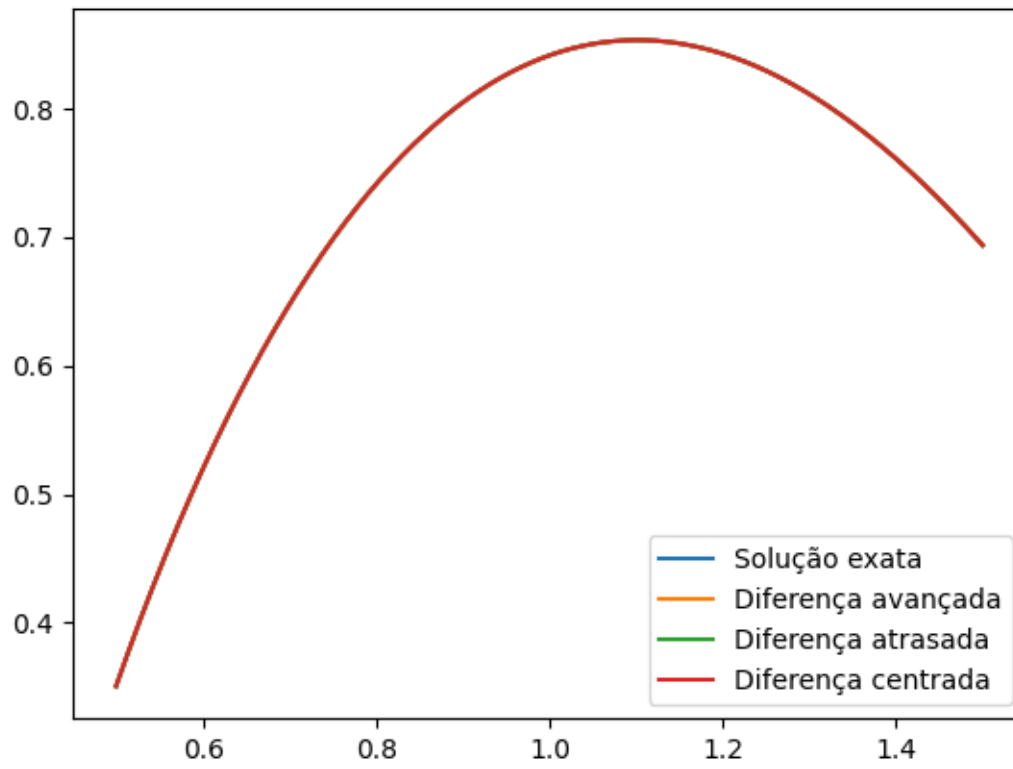
x	y'(x)	avn	err_loc_avn	err_rel_avn	atr	err_loc_atr	err_rel_atr	cen	err_loc_cen	err_rel_cen
0.5	0.350557	0.350567	-9.62462e-06	2.74552e-05	0.350548	9.62478e-06	2.74557e-05	0.350557	8.37773e-11	2.38983e-10
0.6	0.519468	0.519476	-7.35544e-06	1.41596e-05	0.519461	7.35557e-06	1.41598e-05	0.519468	6.85025e-11	1.31871e-10
0.7	0.647511	0.647516	-5.50149e-06	8.49636e-06	0.647505	5.50161e-06	8.49654e-06	0.647511	5.88123e-11	9.08283e-11
0.8	0.74123	0.741233	-3.9048e-06	5.26801e-06	0.741226	3.90491e-06	5.26815e-06	0.74123	5.37331e-11	7.24919e-11
0.9	0.80487	0.804873	-2.48404e-06	3.08626e-06	0.804868	2.48413e-06	3.08638e-06	0.80487	4.81967e-11	5.98813e-11
1	0.841471	0.841472	-1.19562e-06	1.42087e-06	0.84147	1.19571e-06	1.42098e-06	0.841471	4.48692e-11	5.33224e-11
1.1	0.853421	0.853421	-1.61868e-08	1.89669e-08	0.853421	1.62512e-08	1.90424e-08	0.853421	3.21955e-11	3.77252e-11
1.2	0.842765	0.842764	1.06628e-06	1.26522e-06	0.842766	-1.06623e-06	1.26515e-06	0.842765	2.80556e-11	3.32899e-11
1.3	0.811381	0.811379	2.05713e-06	2.53534e-06	0.811383	-2.05707e-06	2.53527e-06	0.811381	2.91436e-11	3.59185e-11
1.4	0.761082	0.761079	2.95776e-06	3.88626e-06	0.761085	-2.95771e-06	3.88619e-06	0.761082	2.5954e-11	3.41015e-11
1.5	0.693678	0.693674	3.76735e-06	5.43097e-06	0.693682	-3.7673e-06	5.4309e-06	0.693678	2.46037e-11	3.54684e-11

Item c, h = 1e-06
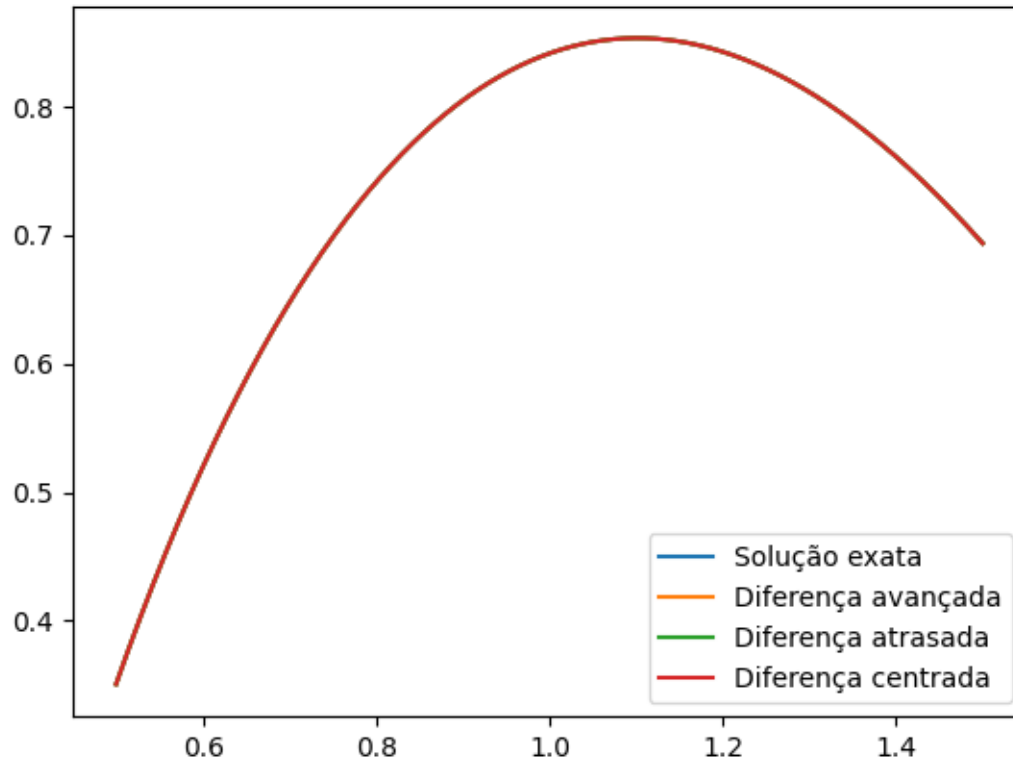
x	y'(x)	avn	err_loc_avn	err_rel_avn	atr	err_loc_atr	err_rel_atr	cen	err_loc_cen	err_rel_cen
0.5	0.350557	0.350558	-9.62441e-07	2.74546e-06	0.350556	9.62464e-07	2.74553e-06	0.350557	1.16127e-11	3.31264e-11
0.6	0.519468	0.519469	-7.3556e-07	1.41599e-06	0.519467	7.35597e-07	1.41606e-06	0.519468	1.85425e-11	3.56952e-11
0.7	0.647511	0.647511	-5.50182e-07	8.49687e-07	0.64751	5.5016e-07	8.49654e-07	0.647511	-1.05767e-11	1.63343e-11
0.8	0.74123	0.74123	-3.9052e-07	5.26854e-07	0.741229	3.90494e-07	5.2682e-07	0.74123	-1.28804e-11	1.7377e-11
0.9	0.80487	0.80487	-2.48423e-07	3.0865e-07	0.80487	2.48388e-07	3.08606e-07	0.80487	-1.77229e-11	2.20196e-11
1	0.841471	0.841471	-1.19591e-07	1.42121e-07	0.841471	1.19543e-07	1.42064e-07	0.841471	-2.37868e-11	2.82681e-11
1.1	0.853421	0.853421	-1.54293e-09	1.80794e-09	0.853421	1.69059e-09	1.98096e-09	0.853421	7.38287e-11	8.65092e-11
1.2	0.842765	0.842765	1.06686e-07	1.2659e-07	0.842765	-1.0656e-07	1.26441e-07	0.842765	6.27499e-11	7.44572e-11
1.3	0.811381	0.811381	2.05767e-07	2.53601e-07	0.811381	-2.05626e-07	2.53427e-07	0.811381	7.07768e-11	8.72301e-11
1.4	0.761082	0.761082	2.95864e-07	3.88742e-07	0.761082	-2.95774e-07	3.88622e-07	0.761082	4.53828e-11	5.96293e-11
1.5	0.693678	0.693678	3.76837e-07	5.43245e-07	0.693679	-3.76671e-07	5.43006e-07	0.693678	8.28903e-11	1.19494e-10

d) $f(x) = e^{\sin(x)}, \quad x \in [0.5, 1.5]$
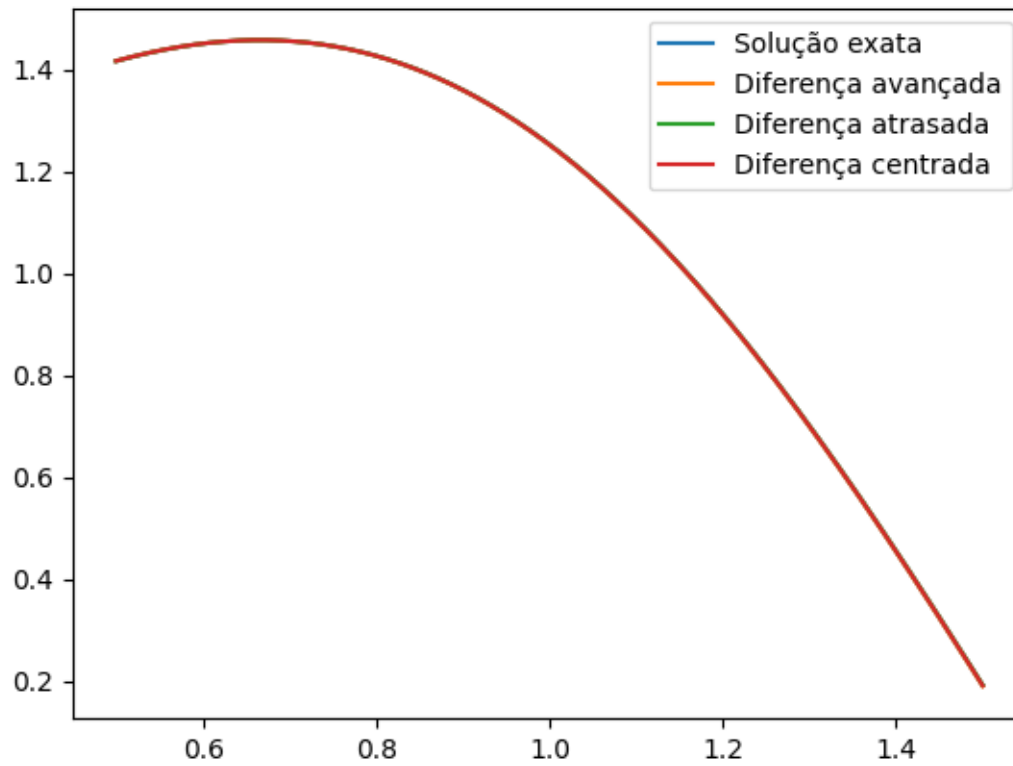**Solução exata:** $f'(x) = \cos(x)e^{\sin(x)}$
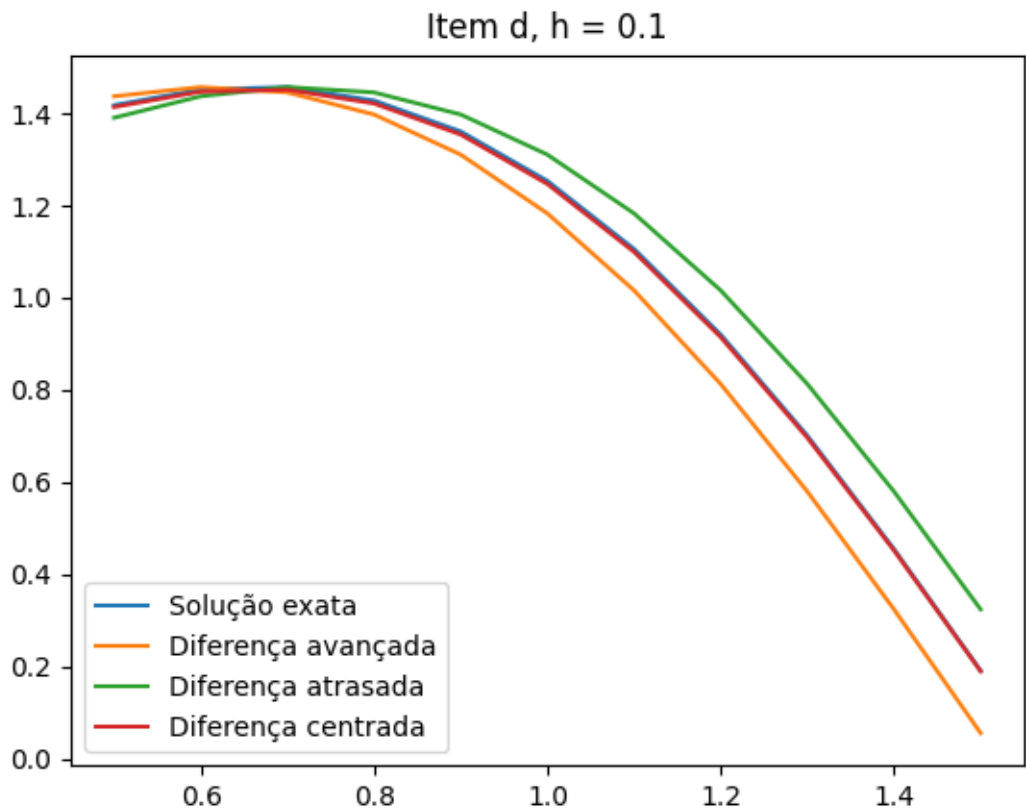
Item d, h = 0.1

x	y'(x)	avn	err_loc_avn	err_rel_avn	atr	err_loc_atr	err_rel_atr	cen	err_loc_cen	err_rel_cen
0.5	1.41742	1.43673	-0.0193013	0.0136171	1.39024	0.0271807	0.0191761	1.41348	0.00393973	0.0027795
0.6	1.45162	1.45678	-0.00516105	0.00355538	1.43673	0.0148903	0.0102578	1.44675	0.00486464	0.00335119
0.7	1.45664	1.44512	0.0115181	0.00790734	1.45678	-0.000137591	9.44579e-05	1.45095	0.00569027	0.00390644
0.8	1.42756	1.39733	0.0302254	0.0211728	1.44512	-0.0175631	0.0123029	1.42123	0.00633118	0.00443497
0.9	1.36054	1.31035	0.0501947	0.0368931	1.39733	-0.0367888	0.0270398	1.35384	0.00670292	0.00492665
1	1.25338	1.18295	0.070434	0.0561952	1.31035	-0.0569684	0.0454518	1.24665	0.0067328	0.00537172
1.1	1.1059	1.01611	0.0897895	0.0811914	1.18295	-0.077047	0.0696691	1.09953	0.00637124	0.00576114
1.2	0.920274	0.813234	0.10704	0.116313	1.01611	-0.0958366	0.104139	0.914672	0.00560155	0.00608683
1.3	0.701116	0.580105	0.121011	0.172597	0.813234	-0.112118	0.159914	0.69667	0.00444644	0.00634194
1.4	0.455345	0.324646	0.130699	0.287033	0.580105	-0.12476	0.273991	0.452375	0.00296931	0.00652102
1.5	0.191803	0.0564199	0.135383	0.705844	0.324646	-0.132843	0.692603	0.190533	0.00126978	0.00662022

Item d, h = 0.01
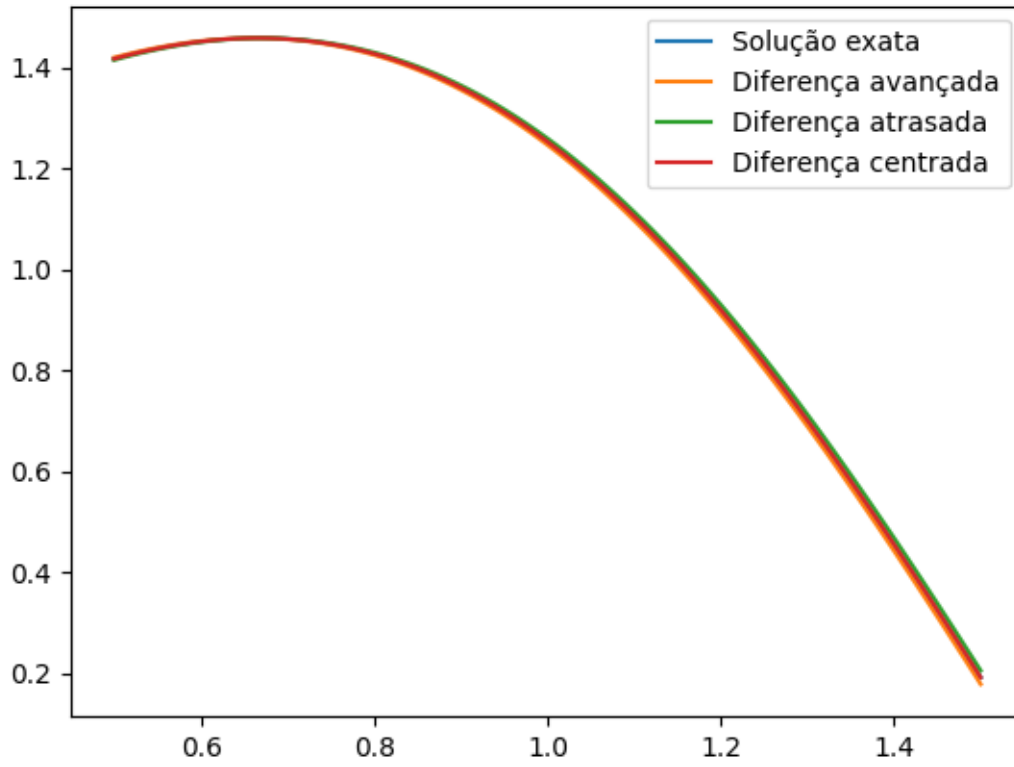
x	y'(x)	avn	err_loc_avn	err_rel_avn	atr	err_loc_atr	err_rel_atr	cen	err_loc_cen	err_rel_cen
0.5	1.41742	1.41973	-0.00230818	0.00162843	1.41504	0.00238699	0.00168403	1.41738	3.94073e-05	2.7802e-05
0.6	1.45162	1.45259	-0.000975914	0.000672295	1.45054	0.0010733	0.000739386	1.45157	4.86951e-05	3.35455e-05
0.7	1.45664	1.45602	0.000621238	0.000426487	1.45715	-0.00050725	0.000348233	1.45658	5.69942e-05	3.91272e-05
0.8	1.42756	1.42512	0.00243997	0.00170919	1.42987	-0.00231308	0.00162031	1.42749	6.34457e-05	4.44435e-05
0.9	1.36054	1.35613	0.00441112	0.00324217	1.36482	-0.00427672	0.00314339	1.36048	6.71997e-05	4.93918e-05
1	1.25338	1.24694	0.00644159	0.00513937	1.25969	-0.00630654	0.00503162	1.25331	6.75236e-05	5.38732e-05
1.1	1.1059	1.09748	0.00841975	0.00761349	1.11419	-0.00829192	0.00749789	1.10584	6.39166e-05	5.7796e-05
1.2	0.920274	0.91005	0.010224	0.0111098	0.930385	-0.0101116	0.0109876	0.920217	5.62086e-05	6.10781e-05
1.3	0.701116	0.689382	0.011734	0.0167362	0.712761	-0.0116448	0.0166089	0.701071	4.46258e-05	6.36497e-05
1.4	0.455345	0.442502	0.0128426	0.0282042	0.468128	-0.012783	0.0280732	0.455315	2.98047e-05	6.54552e-05
1.5	0.191803	0.178335	0.0134679	0.0702176	0.205245	-0.0134424	0.0700846	0.19179	1.27463e-05	6.64555e-05

Item d, h = 0.001

x	y'(x)	avn	err_loc_avn	err_rel_avn	atr	err_loc_atr	err_rel_atr	cen	err_loc_cen	err_rel_cen
0.5	1.41742	1.41766	-0.000234388	0.000165362	1.41719	0.000235176	0.000165918	1.41742	3.94074e-07	2.78021e-07
0.6	1.45162	1.45172	-0.000101996	7.02638e-05	1.45151	0.00010297	7.09347e-05	1.45162	4.86956e-07	3.35458e-07
0.7	1.45664	1.45658	5.69757e-05	3.91145e-05	1.4567	-5.58358e-05	3.83319e-05	1.45664	5.69951e-07	3.91278e-07
0.8	1.42756	1.42732	0.000238274	0.00016691	1.4278	-0.000237005	0.000166022	1.42756	6.3447e-07	4.44445e-07
0.9	1.36054	1.36011	0.000435058	0.000319768	1.36098	-0.000433714	0.00031878	1.36054	6.72014e-07	4.9393e-07
1	1.25338	1.25274	0.000638085	0.000509091	1.25402	-0.000636735	0.000508014	1.25338	6.75256e-07	5.38748e-07
1.1	1.1059	1.10506	0.000836237	0.00075616	1.10673	-0.000834958	0.000755004	1.1059	6.39186e-07	5.77979e-07
1.2	0.920274	0.919256	0.00101737	0.00110551	0.92129	-0.00101625	0.00110429	0.920273	5.62105e-07	6.10802e-07
1.3	0.701116	0.699947	0.00116942	0.00166794	0.702285	-0.00116853	0.00166666	0.701116	4.46274e-07	6.3652e-07
1.4	0.455345	0.454063	0.00128162	0.00281461	0.456626	-0.00128102	0.0028133	0.455344	2.98058e-07	6.54577e-07
1.5	0.191803	0.190457	0.00134569	0.007016	0.193148	-0.00134543	0.00701467	0.191802	1.27468e-07	6.6458e-07

Item d, h = 0.0001
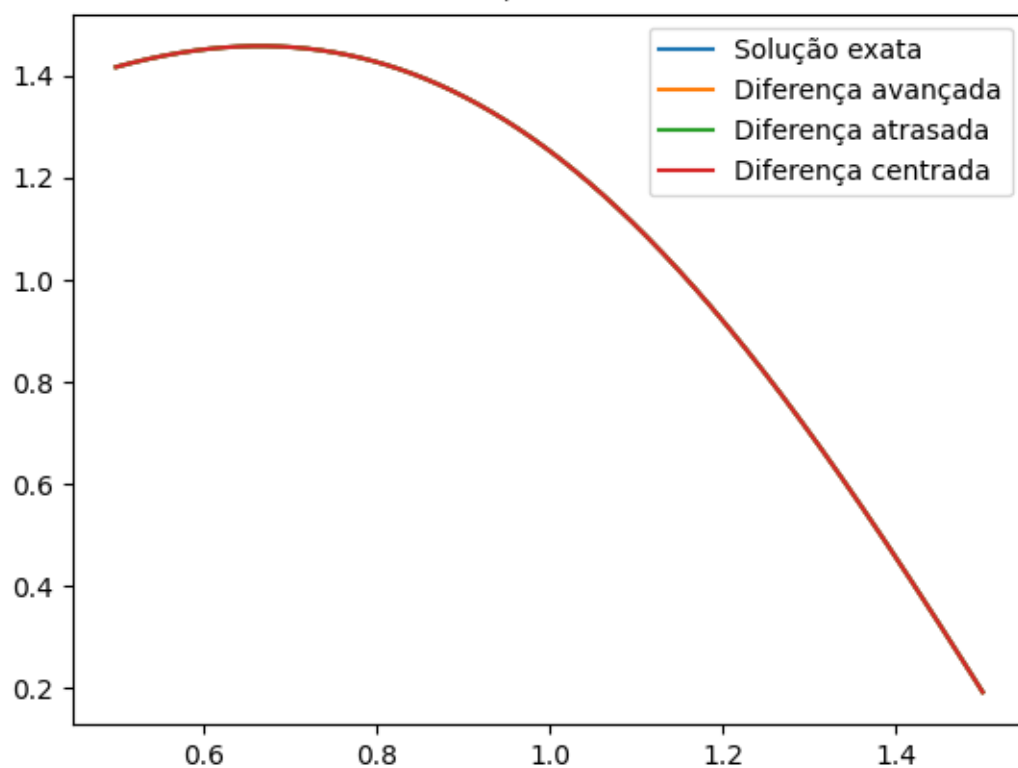
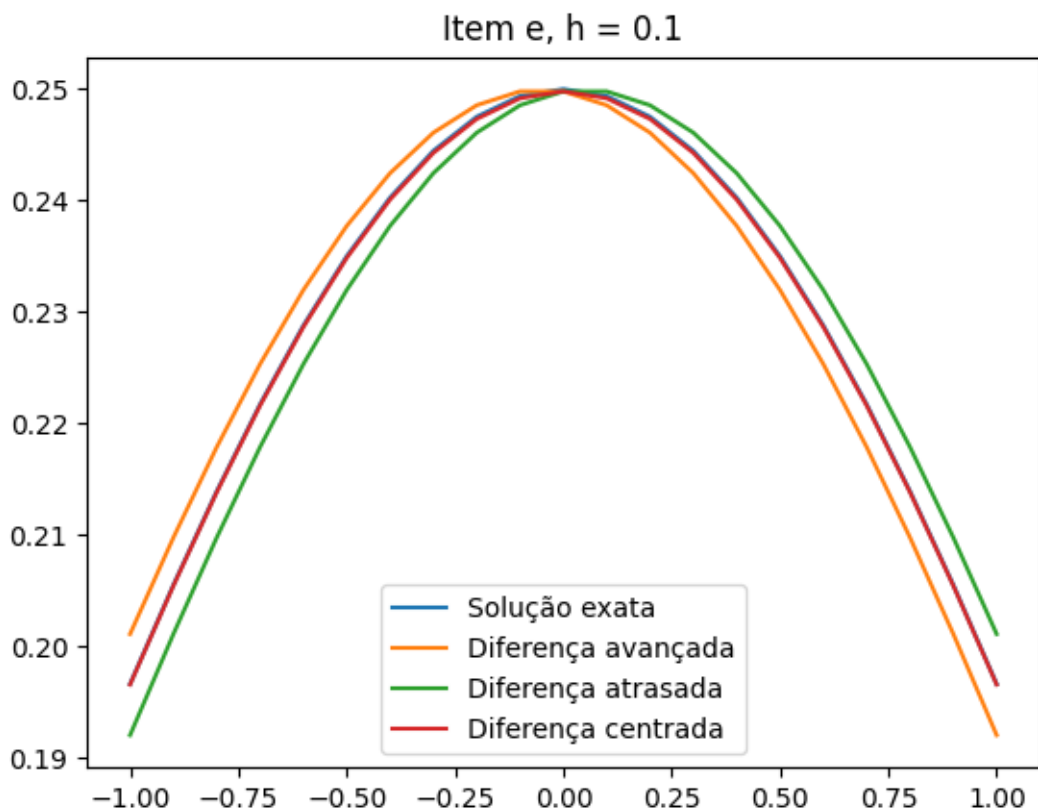| x	y'(x)	avn	err_loc_avn	err_rel_avn	atr	err_loc_atr	err_rel_atr	cen	err_loc_cen	err_rel_cen
0.5	1.41742	1.41745	-2.34743e-05	1.65612e-05	1.4174	2.34822e-05	1.65668e-05	1.41742	3.94061e-09	2.78012e-09
0.6	1.45162	1.45163	-1.02435e-05	7.05658e-06	1.45161	1.02532e-05	7.06329e-06	1.45162	4.86994e-09	3.35484e-09
0.7	1.45664	1.45663	5.64626e-06	3.87622e-06	1.45664	-5.63486e-06	3.8684e-06	1.45664	5.70048e-09	3.91345e-09
0.8	1.42756	1.42753	2.37703e-05	1.6651e-05	1.42758	-2.37576e-05	1.66421e-05	1.42756	6.34507e-09	4.4447e-09
0.9	1.36054	1.3605	4.34454e-05	3.19324e-05	1.36059	-4.34319e-05	3.19225e-05	1.36054	6.72107e-09	4.93999e-09
1	1.25338	1.25332	6.37478e-05	5.08607e-05	1.25344	-6.37343e-05	5.08499e-05	1.25338	6.75389e-09	5.38854e-09
1.1	1.1059	1.10582	8.35662e-05	7.5564e-05	1.10598	-8.35534e-05	7.55524e-05	1.1059	6.39239e-09	5.78027e-09
1.2	0.920274	0.920172	0.000101686	0.000110496	0.920375	-0.000101675	0.000110484	0.920274	5.62222e-09	6.10929e-09
1.3	0.701116	0.700999	0.000116902	0.000166737	0.701233	-0.000116893	0.000166724	0.701116	4.46241e-09	6.36473e-09
1.4	0.455345	0.455217	0.000128135	0.000281402	0.455473	-0.000128129	0.000281389	0.455345	2.98143e-09	6.54763e-09
1.5	0.191803	0.191668	0.000134557	0.000701541	0.191937	-0.000134555	0.000701528	0.191803	1.27494e-09	6.64716e-09

Item d, h = 1e-05
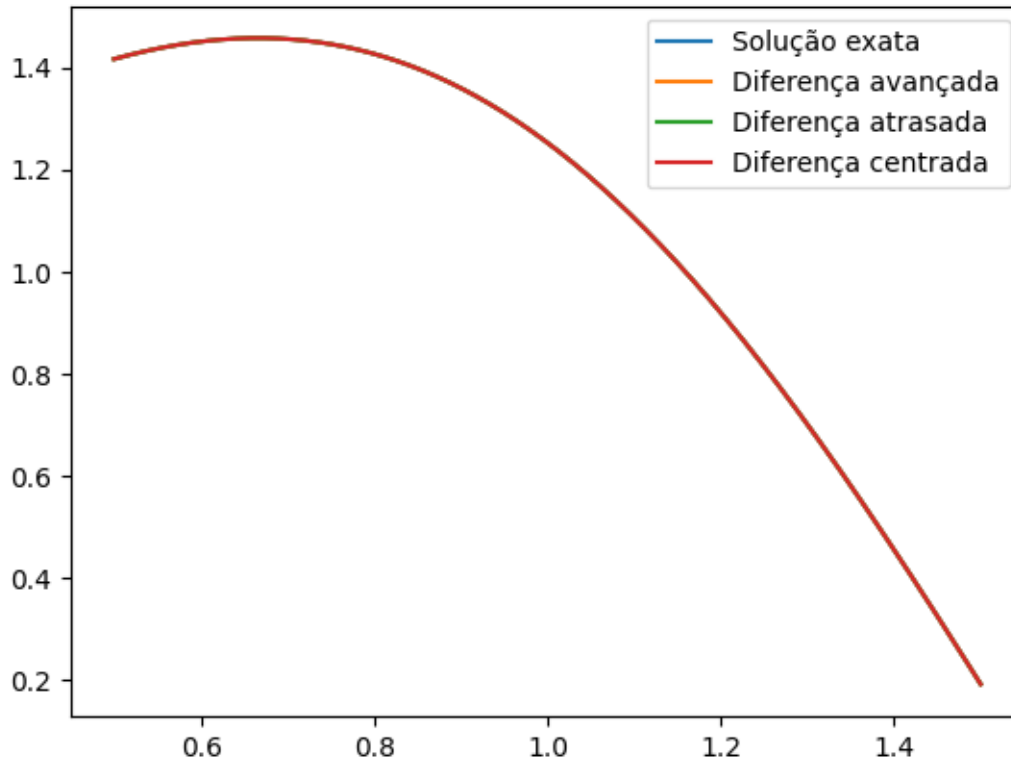
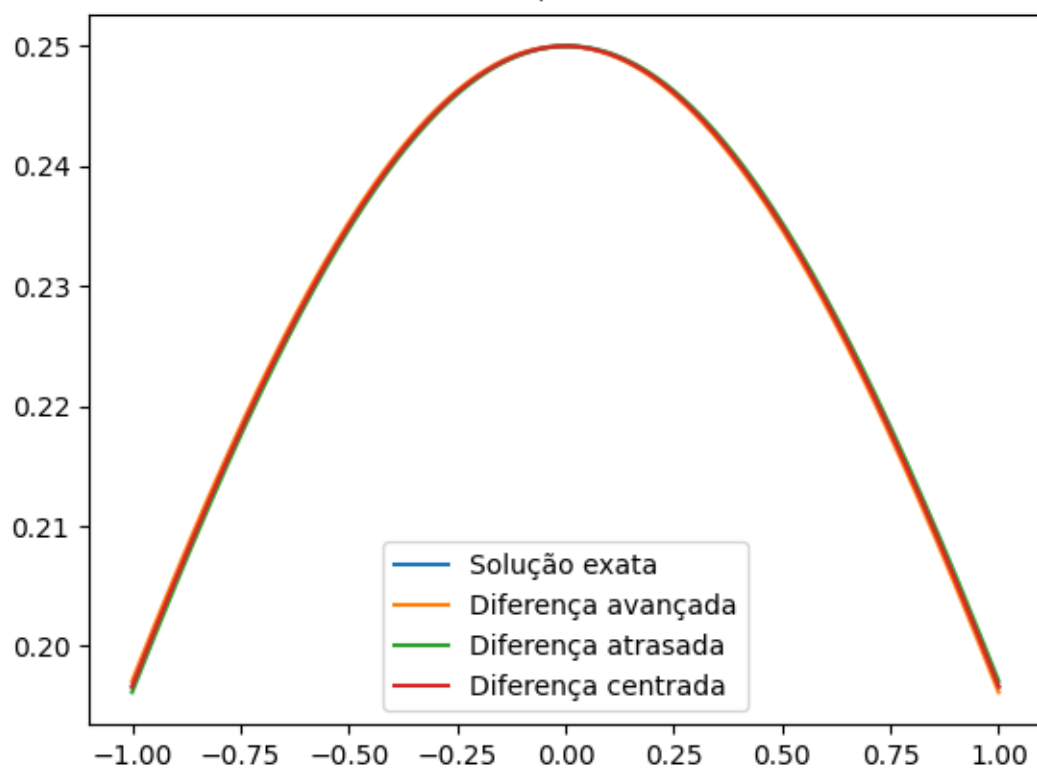| x	y'(x)	avn	err_loc_avn	err_rel_avn	atr	err_loc_atr	err_rel_atr	cen	err_loc_cen	err_rel_cen
0.5	1.41742	1.41743	-2.34777e-06	1.65636e-06	1.41742	2.34786e-06	1.65642e-06	1.41742	4.48419e-11	3.16362e-11
0.6	1.45162	1.45162	-1.02478e-06	7.05955e-07	1.45161	1.0249e-06	7.06038e-07	1.45162	6.04548e-11	4.16465e-11
0.7	1.45664	1.45664	5.64113e-07	3.8727e-07	1.45664	-5.63985e-07	3.87182e-07	1.45664	6.38798e-11	4.38542e-11
0.8	1.42756	1.42756	2.37647e-06	1.66471e-06	1.42756	-2.3763e-06	1.66459e-06	1.42756	8.34135e-11	5.84309e-11
0.9	1.36054	1.36054	4.34392e-06	3.19278e-06	1.36055	-4.34375e-06	3.19266e-06	1.36054	8.41578e-11	6.1856e-11
1	1.25338	1.25337	6.37415e-06	5.08557e-06	1.25339	-6.37405e-06	5.08548e-06	1.25338	5.25877e-11	4.19567e-11
1.1	1.1059	1.10589	8.35602e-06	7.55585e-06	1.10591	-8.35595e-06	7.55579e-06	1.1059	3.30367e-11	2.98731e-11
1.2	0.920274	0.920263	1.01682e-05	1.10491e-05	0.920284	-1.0168e-05	1.10489e-05	0.920274	6.22254e-11	6.76162e-11
1.3	0.701116	0.701104	1.16897e-05	1.66731e-05	0.701128	-1.16896e-05	1.66729e-05	0.701116	5.48264e-11	7.81987e-11
1.4	0.455345	0.455332	1.28132e-05	2.81396e-05	0.455358	-1.28132e-05	2.81395e-05	0.455345	2.15723e-11	4.73757e-11
1.5	0.191803	0.191789	1.34557e-05	7.01536e-05	0.191816	-1.34556e-05	7.01533e-05	0.191803	3.59336e-11	1.87347e-10

Item d, h = 1e-06
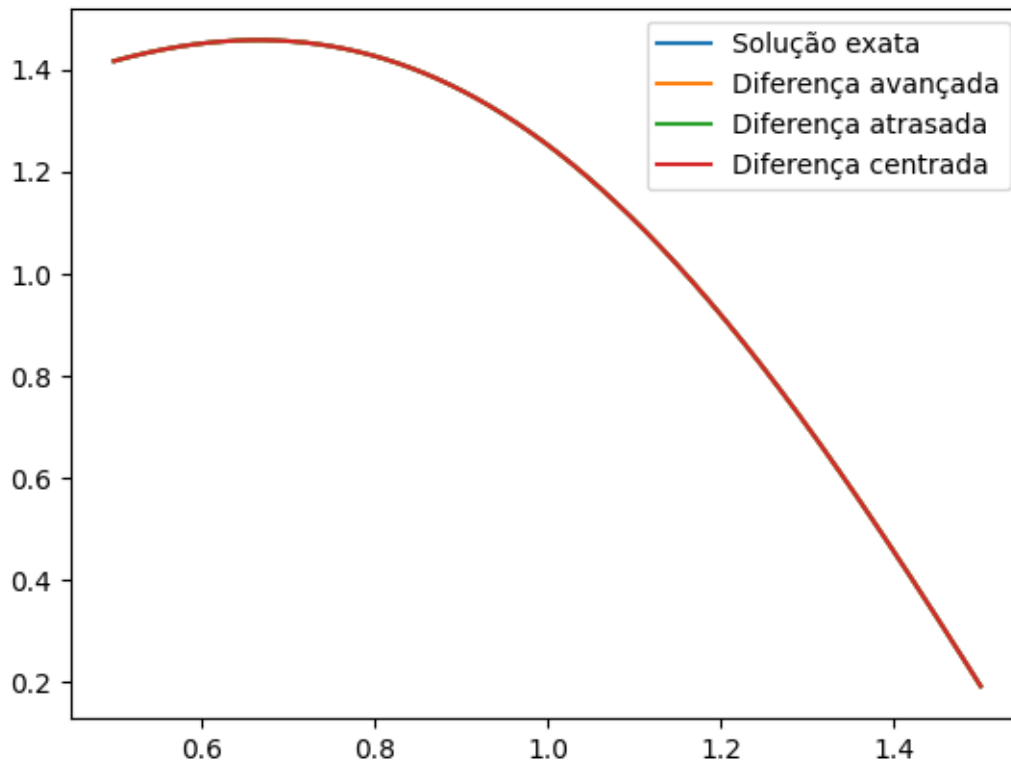
x	y'(x)	avn	err_loc_avn	err_rel_avn	atr	err_loc_atr	err_rel_atr	cen	err_loc_cen	err_rel_cen
0.5	1.41742	1.41742	-2.34923e-07	1.65739e-07	1.41742	2.34702e-07	1.65583e-07	1.41742	-1.1059e-10	7.80215e-11
0.6	1.45162	1.45162	-1.02313e-07	7.04823e-08	1.45162	1.02412e-07	7.05503e-08	1.45162	4.93523e-11	3.39982e-11
0.7	1.45664	1.45664	5.64743e-08	3.87703e-08	1.45664	-5.65464e-08	3.88198e-08	1.45664	-3.60405e-11	2.47422e-11
0.8	1.42756	1.42756	2.37693e-07	1.66503e-07	1.42756	-2.37482e-07	1.66355e-07	1.42756	1.05618e-10	7.39849e-11
0.9	1.36054	1.36054	4.34204e-07	3.1914e-07	1.36054	-4.33991e-07	3.18983e-07	1.36054	1.06362e-10	7.81761e-11
1	1.25338	1.25338	6.37143e-07	5.0834e-07	1.25338	-6.37393e-07	5.08539e-07	1.25338	-1.25048e-10	9.97685e-11
1.1	1.1059	1.1059	8.35853e-07	7.55813e-07	1.1059	-8.35254e-07	7.55271e-07	1.1059	2.9949e-10	2.70811e-10
1.2	0.920274	0.920273	1.01698e-06	1.10509e-06	0.920275	-1.01695e-06	1.10505e-06	0.920274	1.78164e-11	1.93599e-11
1.3	0.701116	0.701115	1.16892e-06	1.66723e-06	0.701117	-1.16877e-06	1.66701e-06	0.701116	7.70307e-11	1.09869e-10
1.4	0.455345	0.455343	1.28135e-06	2.81403e-06	0.455346	-1.28149e-06	2.81432e-06	0.455345	-6.72456e-11	1.47681e-10
1.5	0.191803	0.191801	1.34556e-06	7.01534e-06	0.191804	-1.34562e-06	7.01566e-06	0.191803	-3.06798e-11	1.59955e-10

e) $f(x) = 1/(1 + e^{-x}), \quad x \in [-1, 1]$
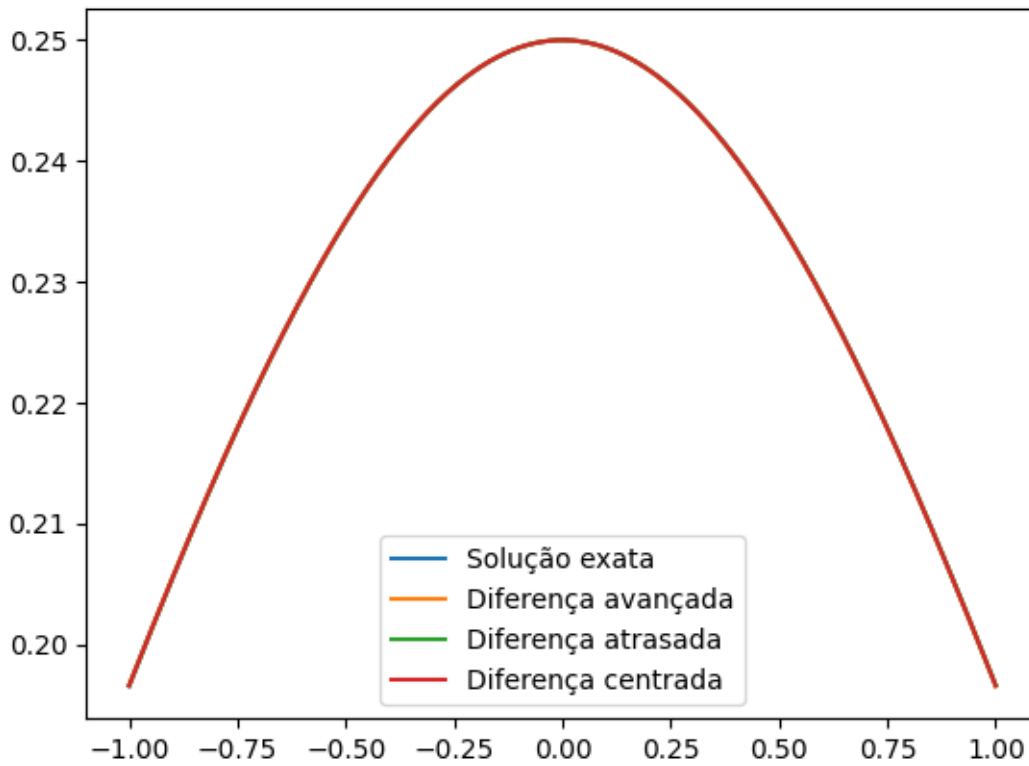**Solução exata:** $f'(x) = e^x/(e^x + 1)^2$

Item e, h = 0.1

x	y'(x)	avn	err_loc_avn	err_rel_avn	atr	err_loc_atr	err_rel_atr	cen	err_loc_cen	err_rel_cen
-1	0.196612	0.201091	-0.00447883	0.02278	0.192015	0.00459666	0.0233794	0.196553	5.89184e-05	0.000299668
-0.9	0.2055	0.20975	-0.00424991	0.0206808	0.201091	0.00440955	0.0214576	0.20542	7.98198e-05	0.000388417
-0.8	0.21391	0.217867	-0.00395739	0.0185003	0.20975	0.00415948	0.019445	0.213809	0.000101044	0.000472369
-0.7	0.221713	0.225315	-0.00360179	0.0162453	0.217867	0.00384578	0.0173458	0.221591	0.000121999	0.000550256
-0.6	0.228784	0.23197	-0.00318551	0.0139236	0.225315	0.00346958	0.0151653	0.228642	0.000142036	0.000620828
-0.5	0.235004	0.237717	-0.002713	0.0115445	0.23197	0.00303396	0.0129103	0.234843	0.000160482	0.00068289
-0.4	0.240261	0.242451	-0.00219069	0.00911796	0.237717	0.00254403	0.0105886	0.240084	0.000176674	0.000735342
-0.3	0.244458	0.246085	-0.00162688	0.00665505	0.242451	0.00200688	0.00820949	0.244268	0.000189998	0.000777219
-0.2	0.247517	0.248548	-0.00103153	0.0041675	0.246085	0.00143138	0.00578296	0.247317	0.000199926	0.000807728
-0.1	0.249376	0.249792	-0.000415835	0.0016675	0.248548	0.000827942	0.00332005	0.24917	0.000206054	0.000826277
0	0.25	0.249792	0.000208125	0.000832501	0.249792	0.000208125	0.000832501	0.249792	0.000208125	0.000832501
0.1	0.249376	0.248548	0.000827942	0.00332005	0.249792	-0.000415835	0.0016675	0.24917	0.000206054	0.000826277
0.2	0.247517	0.246085	0.00143138	0.00578296	0.248548	-0.00103153	0.0041675	0.247317	0.000199926	0.000807728
0.3	0.244458	0.242451	0.00200688	0.00820949	0.246085	-0.00162688	0.00665505	0.244268	0.000189998	0.000777219
0.4	0.240261	0.237717	0.00254403	0.0105886	0.242451	-0.00219069	0.00911796	0.240084	0.000176674	0.000735342
0.5	0.235004	0.23197	0.00303396	0.0129103	0.237717	-0.002713	0.0115445	0.234843	0.000160482	0.00068289
0.6	0.228784	0.225315	0.00346958	0.0151653	0.23197	-0.00318551	0.0139236	0.228642	0.000142036	0.000620828
0.7	0.221713	0.217867	0.00384578	0.0173458	0.225315	-0.00360179	0.0162453	0.221591	0.000121999	0.000550256
0.8	0.21391	0.20975	0.00415948	0.019445	0.217867	-0.00395739	0.0185003	0.213809	0.000101044	0.000472369
0.9	0.2055	0.201091	0.00440955	0.0214576	0.20975	-0.00424991	0.0206808	0.20542	7.98198e-05	0.000388417
1	0.196612	0.192015	0.00459666	0.0233794	0.201091	-0.00447883	0.02278	0.196553	5.89184e-05	0.000299668

Item e, h = 0.01

x	y'(x)	avn	err_loc_avn	err_rel_avn	atr	err_loc_atr	err_rel_atr	cen	err_loc_cen	err_rel_cen
-1	0.196612	0.197066	-0.000453695	0.00230757	0.196157	0.000454872	0.00231355	0.196611	5.88764e-07	2.99455e-06
-0.9	0.2055	0.205933	-0.000432699	0.00210559	0.205066	0.000434295	0.00211335	0.2055	7.98034e-07	3.88337e-06
-0.8	0.21391	0.214315	-0.000405358	0.001895	0.213502	0.000407379	0.00190444	0.213909	1.01057e-06	4.7243e-06
-0.7	0.221713	0.222085	-0.000371668	0.00167635	0.221339	0.000374109	0.00168736	0.221712	1.22044e-06	5.5046e-06
-0.6	0.228784	0.229116	-0.000331813	0.00145033	0.22845	0.000334655	0.00146275	0.228783	1.42114e-06	6.21172e-06
-0.5	0.235004	0.23529	-0.000286174	0.00121774	0.234714	0.000289386	0.00123141	0.235002	1.60593e-06	6.83366e-06
-0.4	0.240261	0.240496	-0.000235336	0.000979502	0.240022	0.000238872	0.00099422	0.240259	1.76816e-06	7.35935e-06
-0.3	0.244458	0.244638	-0.000180076	0.000736634	0.244274	0.00018388	0.000752192	0.244456	1.90166e-06	7.77909e-06
-0.2	0.247517	0.247638	-0.000121344	0.000490247	0.247391	0.000125347	0.000506417	0.247515	2.00115e-06	8.08491e-06
-0.1	0.249376	0.249436	-6.02285e-05	0.000241517	0.249312	6.43536e-05	0.000258059	0.249374	2.06255e-06	8.27086e-06
0	0.25	0.249998	2.08331e-06	8.33325e-06	0.249998	2.08331e-06	8.33325e-06	0.249998	2.08331e-06	8.33325e-06
0.1	0.249376	0.249312	6.43536e-05	0.000258059	0.249436	-6.02285e-05	0.000241517	0.249374	2.06255e-06	8.27086e-06
0.2	0.247517	0.247391	0.000125347	0.000506417	0.247638	-0.000121344	0.000490247	0.247515	2.00115e-06	8.08491e-06
0.3	0.244458	0.244274	0.00018388	0.000752192	0.244638	-0.000180076	0.000736634	0.244456	1.90166e-06	7.77909e-06
0.4	0.240261	0.240022	0.000238872	0.00099422	0.240496	-0.000235336	0.000979502	0.240259	1.76816e-06	7.35935e-06
0.5	0.235004	0.234714	0.000289386	0.00123141	0.23529	-0.000286174	0.00121774	0.235002	1.60593e-06	6.83366e-06
0.6	0.228784	0.22845	0.000334655	0.00146275	0.229116	-0.000331813	0.00145033	0.228783	1.42114e-06	6.21172e-06
0.7	0.221713	0.221339	0.000374109	0.00168736	0.222085	-0.000371668	0.00167635	0.221712	1.22044e-06	5.5046e-06
0.8	0.21391	0.213502	0.000407379	0.00190444	0.214315	-0.000405358	0.001895	0.213909	1.01057e-06	4.7243e-06
0.9	0.2055	0.205066	0.000434295	0.00211335	0.205933	-0.000432699	0.00210559	0.2055	7.98034e-07	3.88337e-06
1	0.196612	0.196157	0.000454872	0.00231355	0.197066	-0.000453695	0.00230757	0.196611	5.88764e-07	2.99455e-06

Item e, h = 0.001

x	y'(x)	avn	err_loc_avn	err_rel_avn	atr	err_loc_atr	err_rel_atr	cen	err_loc_cen	err_rel_cen
-1	0.196612	0.196657	-4.5423e-05	0.000231029	0.196566	4.54348e-05	0.000231088	0.196612	5.88761e-09	2.99453e-08
-0.9	0.2055	0.205544	-4.33422e-05	0.000210911	0.205457	4.33582e-05	0.000210988	0.2055	7.98031e-09	3.88336e-08
-0.8	0.21391	0.21395	-4.06273e-05	0.000189927	0.213869	4.06475e-05	0.000190022	0.21391	1.01057e-08	4.7243e-08
-0.7	0.221713	0.22175	-3.72772e-05	0.000168133	0.221676	3.73016e-05	0.000168243	0.221713	1.22045e-08	5.50462e-08
-0.6	0.228784	0.228818	-3.33097e-05	0.000145594	0.228751	3.33381e-05	0.000145718	0.228784	1.42115e-08	6.21175e-08
-0.5	0.235004	0.235032	-2.87623e-05	0.000122391	0.234975	2.87945e-05	0.000122528	0.235004	1.60595e-08	6.83372e-08
-0.4	0.240261	0.240284	-2.36931e-05	9.86141e-05	0.240237	2.37284e-05	9.87612e-05	0.240261	1.76818e-08	7.3594e-08
-0.3	0.244458	0.244476	-1.81791e-05	7.43647e-05	0.24444	1.82171e-05	7.45203e-05	0.244458	1.90168e-08	7.77916e-08
-0.2	0.247517	0.247529	-1.23147e-05	4.97531e-05	0.247504	1.23547e-05	4.99148e-05	0.247517	2.00117e-08	8.08498e-08
-0.1	0.249376	0.249382	-6.20858e-06	2.48965e-05	0.24937	6.24984e-06	2.50619e-05	0.249376	2.06257e-08	8.27093e-08
0	0.25	0.25	2.08334e-08	8.33335e-08	0.25	2.08333e-08	8.33332e-08	0.25	2.08333e-08	8.33333e-08
0.1	0.249376	0.24937	6.24984e-06	2.50619e-05	0.249382	-6.20858e-06	2.48965e-05	0.249376	2.06258e-08	8.27096e-08
0.2	0.247517	0.247504	1.23547e-05	4.99148e-05	0.247529	-1.23147e-05	4.97531e-05	0.247517	2.00117e-08	8.08499e-08
0.3	0.244458	0.24444	1.82171e-05	7.45203e-05	0.244476	-1.81791e-05	7.43647e-05	0.244458	1.90168e-08	7.77916e-08
0.4	0.240261	0.240237	2.37284e-05	9.87612e-05	0.240284	-2.36931e-05	9.86141e-05	0.240261	1.76818e-08	7.3594e-08
0.5	0.235004	0.234975	2.87945e-05	0.000122528	0.235032	-2.87623e-05	0.000122391	0.235004	1.60594e-08	6.8337e-08
0.6	0.228784	0.228751	3.33381e-05	0.000145718	0.228818	-3.33097e-05	0.000145594	0.228784	1.42115e-08	6.21177e-08
0.7	0.221713	0.221676	3.73016e-05	0.000168243	0.22175	-3.72772e-05	0.000168133	0.221713	1.22045e-08	5.50463e-08
0.8	0.21391	0.213869	4.06475e-05	0.000190022	0.21395	-4.06273e-05	0.000189927	0.21391	1.01057e-08	4.7243e-08
0.9	0.2055	0.205457	4.33582e-05	0.000210988	0.205544	-4.33422e-05	0.000210911	0.2055	7.98037e-09	3.88339e-08
1	0.196612	0.196566	4.54348e-05	0.000231088	0.196657	-4.5423e-05	0.000231029	0.196612	5.88766e-09	2.99456e-08

Item e, h = 0.0001
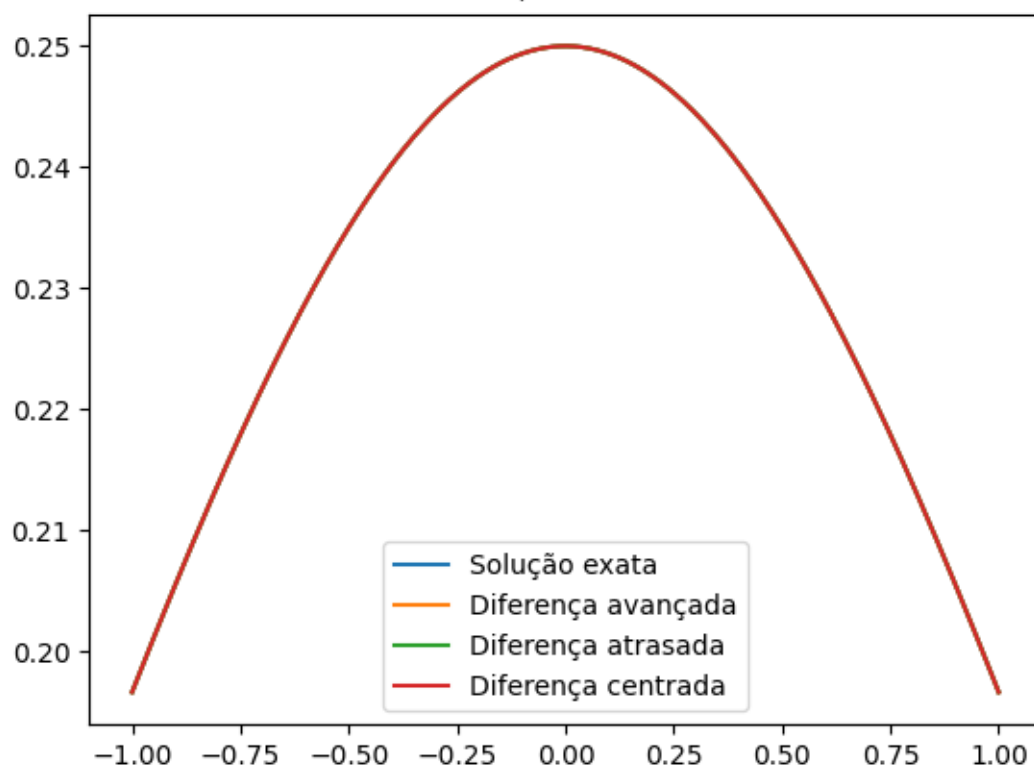
x	y'(x)	avn	err_loc_avn	err_rel_avn	atr	err_loc_atr	err_rel_atr	cen	err_loc_cen	err_rel_cen
-1	0.196612	0.196616	-4.54283e-06	2.31056e-05	0.196607	4.54295e-06	2.31062e-05	0.196612	5.86599e-11	2.98354e-10
-0.9	0.2055	0.205505	-4.33494e-06	2.10946e-05	0.205496	4.3351e-06	2.10953e-05	0.2055	7.99116e-11	3.88864e-10
-0.8	0.21391	0.213914	-4.06364e-06	1.8997e-05	0.213906	4.06384e-06	1.89979e-05	0.21391	1.00971e-10	4.72027e-10
-0.7	0.221713	0.221717	-3.72882e-06	1.68182e-05	0.221709	3.72906e-06	1.68193e-05	0.221713	1.22089e-10	5.50661e-10
-0.6	0.228784	0.228788	-3.33224e-06	1.4565e-05	0.228781	3.33253e-06	1.45662e-05	0.228784	1.42129e-10	6.21237e-10
-0.5	0.235004	0.235007	-2.87768e-06	1.22453e-05	0.235001	2.878e-06	1.22466e-05	0.235004	1.60546e-10	6.83163e-10
-0.4	0.240261	0.240263	-2.3709e-06	9.86803e-06	0.240258	2.37125e-06	9.8695e-06	0.240261	1.7726e-10	7.37782e-10
-0.3	0.244458	0.24446	-1.81962e-06	7.44347e-06	0.244456	1.82e-06	7.44503e-06	0.244458	1.90251e-10	7.78255e-10
-0.2	0.247517	0.247518	-1.23327e-06	4.98259e-06	0.247515	1.23367e-06	4.98421e-06	0.247517	1.99986e-10	8.07968e-10
-0.1	0.249376	0.249377	-6.22715e-07	2.49709e-06	0.249375	6.23128e-07	2.49875e-06	0.249376	2.06299e-10	8.27261e-10
0	0.25	0.25	2.09305e-10	8.37218e-10	0.25	2.07084e-10	8.28337e-10	0.25	2.08194e-10	8.32777e-10
0.1	0.249376	0.249375	6.23128e-07	2.49875e-06	0.249377	-6.22715e-07	2.49709e-06	0.249376	2.06577e-10	8.28374e-10
0.2	0.247517	0.247515	1.23367e-06	4.98421e-06	0.247518	-1.23327e-06	4.98259e-06	0.247517	1.99986e-10	8.07968e-10
0.3	0.244458	0.244456	1.82e-06	7.44503e-06	0.24446	-1.81962e-06	7.44347e-06	0.244458	1.89973e-10	7.7712e-10
0.4	0.240261	0.240258	2.37125e-06	9.8695e-06	0.240263	-2.3709e-06	9.86803e-06	0.240261	1.7726e-10	7.37782e-10
0.5	0.235004	0.235001	2.878e-06	1.22466e-05	0.235007	-2.87768e-06	1.22453e-05	0.235004	1.60268e-10	6.81982e-10
0.6	0.228784	0.228781	3.33253e-06	1.45663e-05	0.228788	-3.33224e-06	1.4565e-05	0.228784	1.42407e-10	6.22451e-10
0.7	0.221713	0.221709	3.72906e-06	1.68193e-05	0.221717	-3.72882e-06	1.68182e-05	0.221713	1.22366e-10	5.51912e-10
0.8	0.21391	0.213906	4.06384e-06	1.89979e-05	0.213914	-4.06364e-06	1.8997e-05	0.21391	1.00971e-10	4.72027e-10
0.9	0.2055	0.205496	4.3351e-06	2.10953e-05	0.205505	-4.33494e-06	2.10946e-05	0.2055	8.01892e-11	3.90215e-10
1	0.196612	0.196607	4.54295e-06	2.31062e-05	0.196616	-4.54283e-06	2.31056e-05	0.196612	5.89375e-11	2.99765e-10

Item e, h = 1e-05
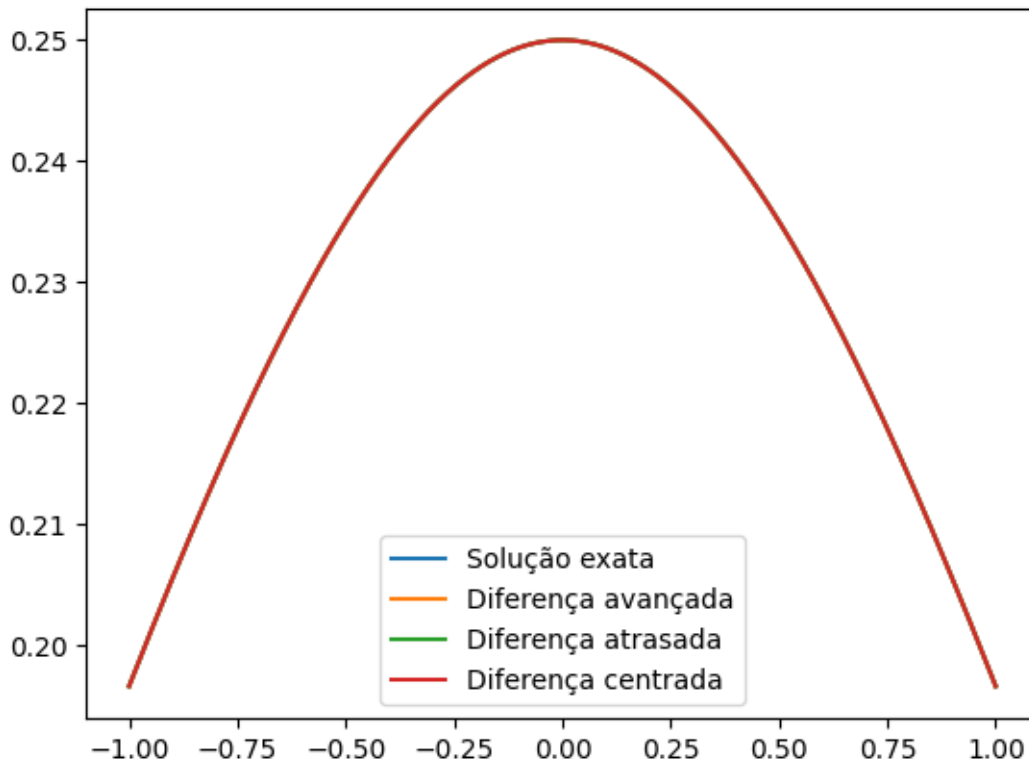
x	y'(x)	avn	err_loc_avn	err_rel_avn	atr	err_loc_atr	err_rel_atr	cen	err_loc_cen	err_rel_cen
-1	0.196612	0.196612	-4.5429e-07	2.31059e-06	0.196611	4.54288e-07	2.31058e-06	0.196612	-1.01458e-12	5.1603e-12
-0.9	0.2055	0.205501	-4.33504e-07	2.10951e-06	0.2055	4.33508e-07	2.10952e-06	0.2055	1.91847e-12	9.33558e-12
-0.8	0.21391	0.21391	-4.06372e-07	1.89973e-06	0.213909	4.06378e-07	1.89977e-06	0.21391	3.27161e-12	1.52943e-11
-0.7	0.221713	0.221713	-3.72891e-07	1.68187e-06	0.221713	3.72895e-07	1.68188e-06	0.221713	1.90695e-12	8.60097e-12
-0.6	0.228784	0.228785	-3.33229e-07	1.45652e-06	0.228784	3.33238e-07	1.45656e-06	0.228784	4.18415e-12	1.82886e-11
-0.5	0.235004	0.235004	-2.87781e-07	1.22458e-06	0.235003	2.87787e-07	1.22461e-06	0.235004	3.1718e-12	1.34968e-11
-0.4	0.240261	0.240261	-2.37106e-07	9.86871e-07	0.240261	2.37103e-07	9.86858e-07	0.240261	-1.48581e-12	6.18416e-12
-0.3	0.244458	0.244458	-1.81979e-07	7.44419e-07	0.244458	1.81991e-07	7.44465e-07	0.244458	5.67627e-12	2.32198e-11
-0.2	0.247517	0.247517	-1.23344e-07	4.98326e-07	0.247516	1.23348e-07	4.98341e-07	0.247517	1.81075e-12	7.31566e-12
-0.1	0.249376	0.249376	-6.22862e-08	2.49768e-07	0.249376	6.22919e-08	2.49791e-07	0.249376	2.85069e-12	1.14313e-11
0	0.25	0.25	9.46446e-12	3.78578e-11	0.25	-7.18892e-12	2.87557e-11	0.25	1.13778e-12	4.55114e-12
0.1	0.249376	0.249376	6.22919e-08	2.49791e-07	0.249376	-6.22973e-08	2.49813e-07	0.249376	-2.70042e-12	1.08287e-11
0.2	0.247517	0.247516	1.23353e-07	4.98363e-07	0.247517	-1.2335e-07	4.98349e-07	0.247517	1.81072e-12	7.31554e-12
0.3	0.244458	0.244458	1.81985e-07	7.44442e-07	0.244458	-1.81979e-07	7.44419e-07	0.244458	2.90079e-12	1.18662e-11
0.4	0.240261	0.240261	2.37098e-07	9.86835e-07	0.240261	-2.37101e-07	9.86847e-07	0.240261	-1.48584e-12	6.18428e-12
0.5	0.235004	0.235003	2.87792e-07	1.22463e-06	0.235004	-2.87781e-07	1.22458e-06	0.235004	5.94733e-12	2.53074e-11
0.6	0.228784	0.228784	3.33249e-07	1.45661e-06	0.228785	-3.3324e-07	1.45657e-06	0.228784	4.18421e-12	1.82889e-11
0.7	0.221713	0.221713	3.72901e-07	1.68191e-06	0.221713	-3.72891e-07	1.68187e-06	0.221713	4.68248e-12	2.11196e-11
0.8	0.21391	0.213909	4.06384e-07	1.89979e-06	0.21391	-4.06377e-07	1.89976e-06	0.21391	3.27166e-12	1.52946e-11
0.9	0.2055	0.2055	4.33502e-07	2.1095e-06	0.205501	-4.33504e-07	2.10951e-06	0.2055	-8.57037e-13	4.17049e-12
1	0.196612	0.196611	4.54288e-07	2.31058e-06	0.196612	-4.54285e-07	2.31057e-06	0.196612	1.76101e-12	8.95677e-12

Item e, h = 1e-06
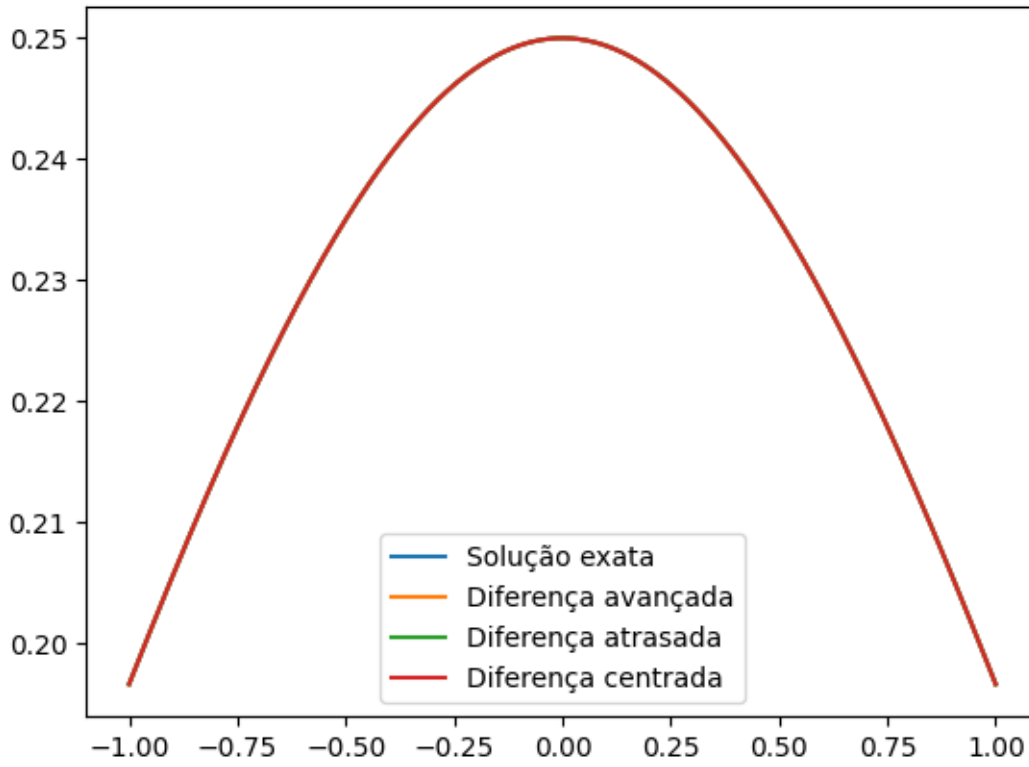
x	y'(x)	avn	err_loc_avn	err_rel_avn	atr	err_loc_atr	err_rel_atr	cen	err_loc_cen	err_rel_cen
-1	0.196612	0.196612	-4.54563e-08	2.31198e-07	0.196612	4.54709e-08	2.31273e-07	0.196612	7.31207e-12	3.71904e-11
-0.9	0.2055	0.2055	-4.33939e-08	2.11162e-07	0.2055	4.337e-08	2.11046e-07	0.2055	-1.19594e-11	5.81963e-11
-0.8	0.21391	0.21391	-4.06253e-08	1.89918e-07	0.21391	4.05875e-08	1.89741e-07	0.21391	-1.89329e-11	8.85088e-11
-0.7	0.221713	0.221713	-3.72933e-08	1.68205e-07	0.221713	3.72582e-08	1.68047e-07	0.221713	-1.7522e-11	7.90301e-11
-0.6	0.228784	0.228784	-3.33469e-08	1.45757e-07	0.228784	3.32665e-08	1.45405e-07	0.228784	-4.02248e-11	1.7582e-10
-0.5	0.235004	0.235004	-2.8821e-08	1.22641e-07	0.235004	2.87441e-08	1.22313e-07	0.235004	-3.84616e-11	1.63664e-10
-0.4	0.240261	0.240261	-2.36492e-08	9.84315e-08	0.240261	2.37018e-08	9.86502e-08	0.240261	2.62697e-11	1.09338e-10
-0.3	0.244458	0.244458	-1.82214e-08	7.45379e-08	0.244458	1.83049e-08	7.48795e-08	0.244458	4.17585e-11	1.7082e-10
-0.2	0.247517	0.247517	-1.22939e-08	4.9669e-08	0.247517	1.22975e-08	4.96837e-08	0.247517	1.81072e-12	7.31554e-12
-0.1	0.249376	0.249376	-6.23105e-09	2.49866e-08	0.249376	6.20345e-09	2.48759e-08	0.249376	-1.38027e-11	5.53489e-11
0	0.25	0.25	7.60778e-11	3.04311e-10	0.25	-3.49445e-11	1.39778e-10	0.25	2.05667e-11	8.22666e-11
0.1	0.249376	0.249376	6.25896e-09	2.50985e-08	0.249376	-6.28656e-09	2.52092e-08	0.249376	-1.38027e-11	5.53489e-11
0.2	0.247517	0.247517	1.2353e-08	4.99079e-08	0.247517	-1.22939e-08	4.9669e-08	0.247517	2.95663e-11	1.19452e-10
0.3	0.244458	0.244458	1.80829e-08	7.39712e-08	0.244458	-1.82214e-08	7.45379e-08	0.244458	-6.92637e-11	2.83336e-10
0.4	0.240261	0.240261	2.36463e-08	9.84192e-08	0.240261	-2.36492e-08	9.84315e-08	0.240261	-1.48587e-12	6.18439e-12
0.5	0.235004	0.235004	2.87996e-08	1.22549e-07	0.235004	-2.8821e-08	1.22641e-07	0.235004	-1.0706e-11	4.55569e-11
0.6	0.228784	0.228784	3.3322e-08	1.45648e-07	0.228784	-3.32914e-08	1.45514e-07	0.228784	1.52864e-11	6.68158e-11
0.7	0.221713	0.221713	3.73137e-08	1.68298e-07	0.221713	-3.72933e-08	1.68205e-07	0.221713	1.02336e-11	4.61568e-11
0.8	0.21391	0.21391	4.0643e-08	1.90001e-07	0.21391	-4.06253e-08	1.89918e-07	0.21391	8.82275e-12	4.12452e-11
0.9	0.2055	0.2055	4.34255e-08	2.11316e-07	0.2055	-4.33939e-08	2.11162e-07	0.2055	1.57963e-11	7.68674e-11
1	0.196612	0.196612	4.54709e-08	2.31273e-07	0.196612	-4.53453e-08	2.30634e-07	0.196612	6.28232e-11	3.19529e-10