RUB

Linear Finite Element Methods

Homework 3

STATIK DYNAMIK

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Name: _____ **Matr.-Nr.:** _____ □ □ □ □

To be submitted latest on **07.02.2024**.

W X Y Z

This homework is parameterised by the last 4 digits of your student ID W X Y Z.

The given 2D structure has to be analyzed using the linear Finite Element Method. The structure consists of a TRUSS element (3) and four BERNOULLI beam elements, see Fig. 1.
Beam elements 1 and 2 are connected rigidly at node ②, and the same applies to elements 4 and 5 which are connected at node ⑤. At node ③, where the origin of the coordinate system lies, elements 2-3-4 are connected by a hinge. Node ④ is modelled as a fixed support, and node ① and ⑥ are modelled as clamped supports.
Beam elements 1 and 5 have a square cross section that linearly changes in size along their length, with the cross section at nodes ①/⑥ given by view Ⓐ in Fig. 2 and the cross section at nodes ②/⑤ given by view Ⓑ. Finally, a spring with stiffness k_s is placed along elements 1 and 5, at a distance of a third of the element's length from node ① and ⑥, respectively. Beam elements 1 and 5 have the E-modulus E_b; EA and EI for elements 2 and 4 are given below. Element 2 and 4 are each loaded with a constant distributed load p. Additionally, a force Q^* acts at the middle of element 2 and 4, respectively.

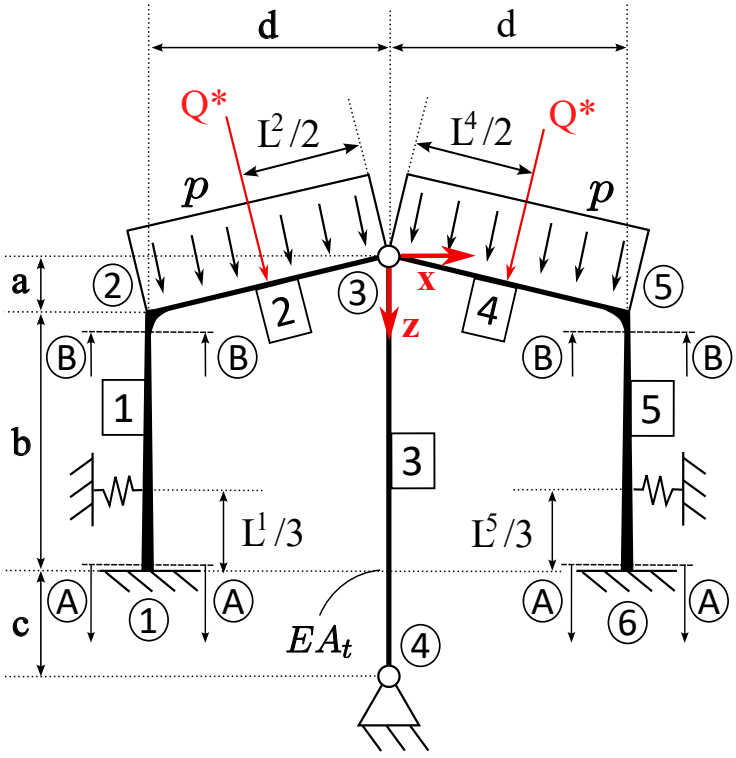

Fig. 1: 2D Structure

Geometry:
$a = 1 + 0.05 \cdot X$ m
$b = 4 + 0.1 \cdot Z$ m
$c = 0.1 + 0.2 \cdot Y$ m
$d = 3 + 0.1 \cdot W$ m
$t_1 = 0.75 + 0.05 \cdot X - 0.04 \cdot Z$ m
$t_2 = \frac{t_1}{1.5}$

Material:
$E_b = (4 + 0.25 \cdot X) \cdot 10^6$ kN/m^2
$EA_{2/4} = E_b \cdot t_2^2$
$EI_{2/4} = E_b \cdot \frac{t_2^4}{12}$
$EA_t = (3 + 0.15 \cdot W) \cdot 10^5$ kN

Loading:
$p = 80 + 2 \cdot Y$ kN/m
$Q^* = 210 + 5 \cdot Z$ kN

Spring:
$k_s = (5 + 0.5 \cdot Y) \cdot 10^3$ kN/m

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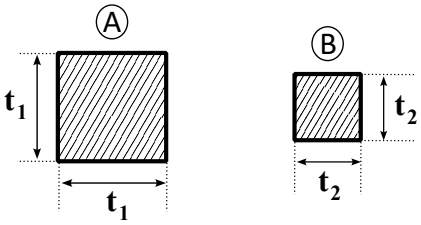

Fig. 2: Cross-Sectional Views (A) and (B)

Tasks:

1. Sketch the assumed deformed shape of the structure.

2. Determine all unknown displacements **u** of the system. You can simplify the system if possible.

   Hint: $\delta W_{ext} = \delta w \cdot Q^*$

3. Calculate the bending moment M in element $\boxed{2}$ at $\xi = -1/(2+Z)$, with Z as given by your Mat.-ID.

   Hint: $M = EI\kappa$

Please note:

- Always use at least four decimals in your calculation.
- Use the appropriate number of GAUSS points where applicable.
- Make sure to use linear FEM for every step of the solution of this task. Other methods are not accepted. This also refers to using other types of beam elements, as are taught in other classes.