RUB

Linear Finite Element Methods

Homework 2

STATIK DYNAMIK

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Name: ______________________ **Matr.-Nr.:** ______________________ [ ] [ ] [ ] [ ]
W X Y Z

To be submitted latest on **11.01.2024**.

This homework is parameterised by the last 4 digits of your student ID W X Y Z.

Always show the main steps of your calculations! And use at least 4 decimals in all your calculations and results.

Task 1: A very long dam holds back a body of water on a sunny day (fig. 1.1) and has to be analyzed using the linear Finite Element Method. Since the dam is very long, the 3D structure is simplified to a 2D structure and analyzed assuming a PLANE STRAIN state. The reduced structure is discretized by a single CST element with thickness t (fig. 1.2).
Node 1 is modelled as a roller support and can only move in x-direction. Node 2 is free and can move in x- and y-direction. Node 3 is modelled as a fixed support and located at the origin of the coordinate system.
The structure is loaded by water pressure on its left side, see fig. 1.3. The load is modelled as a distributed linear load, with value 0 at node 2 and p_{max} at node 3; vertical loading due to water weight is not considered. The structure is exposed to the sun heat on its right side, resulting in the prescribed temperature θ_1 at node 1 and θ_2 at node 2.

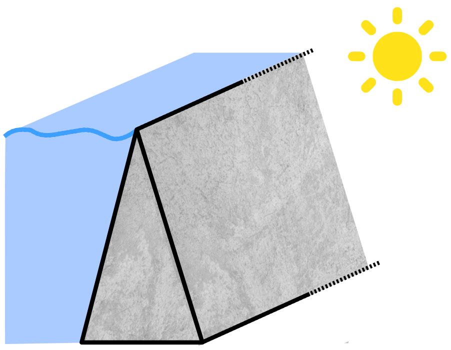

Fig. 1.1: 3D Dam Structure

Geometry:

$$a = 4 + \frac{X+Z}{10} \text{ [m]}$$

$$b = 0.5 + \frac{W+Z}{20} \text{ [m]}$$

$$c = 1 + \frac{X+Y}{20} \text{ [m]}$$

$$t = 1 \text{ [m]}$$

Material:

$E = (45 + W) \cdot 10^3$ [MN/m^2]

$\nu = 0.15 + 0.01 \cdot Y$ [-]

$\lambda = (1.33 + 0.06 \cdot Z)$ [W/mK]

$\alpha_\theta = (1 + \frac{X}{5}) \cdot 10^{-6}$ [1/K]

Loading:

$p_{max} = a \cdot 0.997 \cdot 9.81$ [MN/m^2]

$\theta_1 = 35$ [°C]

$\theta_2 = (40 + Y)$ [°C]

Other:

$\theta_{ref} = (25 - W)$ [°C]

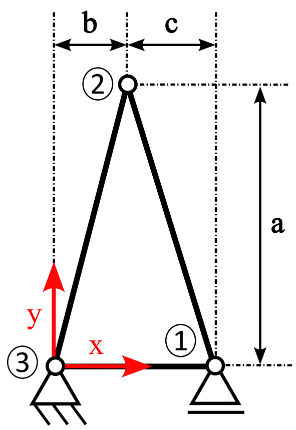

Fig. 1.2: 2D Structure

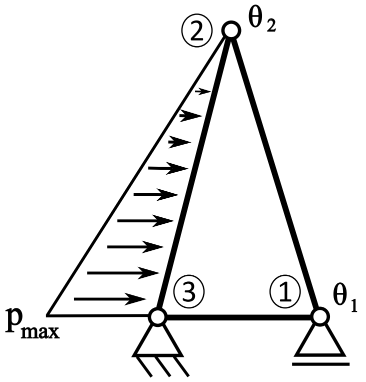

Fig. 1.3: Structure with Load

- 1.1) State all independent degrees of freedom, both thermal and mechanical.
- 1.2) Compute the reduced global conductivity matrix $\boldsymbol{K}_{\theta\theta}^{global}$, the reduced global stiffness matrix $\boldsymbol{K}_{uu}^{global}$ and the reduced global coupling matrix $\boldsymbol{K}_{u\theta}^{global}$ of the system.
- 1.3) Compute the reduced consistent load vector $\boldsymbol{r}_{red}^{global}$ of the system.
- 1.4) Compute all unknown temperatures $\boldsymbol{\theta}$ and unknown displacements $\mathbf{u}$ of the system.

Task 2: A circular slab with a hole and thickness t has to be analyzed using the linear Finite Element Method, see fig. 2.1. Due to its small thickness, the structure can be reduced to 2D by considering a PLANE STRESS state. The structure is discretized by 4 8-noded SERENDIPITY elements (fig. 2.2). The structure is furthermore loaded in radial direction with a constant load p_c acting on the outer edges of the elements, see fig. 2.3 and 2.4. The origin of the global coordinate system is placed at the center of the structure.

Note: this task has a new parametrisation, as given below.

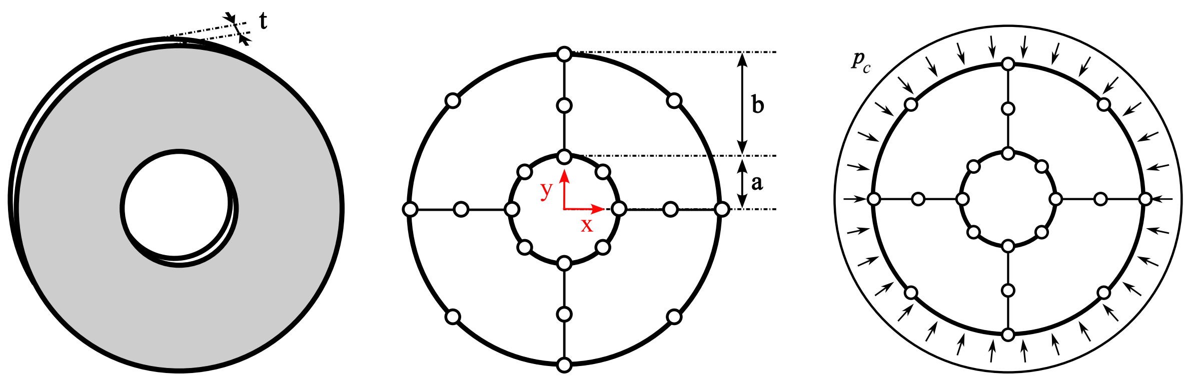

Fig. 2.1: 3D Slab Structure Fig. 2.2: 2D Structure Fig. 2.3: Structure with Load

2.1) State all independent degrees of freedom of the system considering symmetry.

2.2) Compute the reduced global stiffness matrix $\boldsymbol{K}_{red}^{global}$ of the system corresponding to all unknown degrees of freedom and considering symmetry. Use 4 GAUSS points for the 8-noded element (see ⓧ in fig. 2.4).

2.3) Compute the reduced global load vector $\boldsymbol{r}_{red}^{global}$ of the system using 3 GAUSS points.

2.4) Compute all unknown displacements $\mathbf{u}$ of the system.

2.5) Compute the stress at the GAUSS Point marked in red in fig. 2.4.

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Geometry:

$a = 1 + \frac{W + X}{10}$ [m]

$b = 5 + \frac{Y + Z}{10}$ [m]

$t = 0.03 + \frac{W}{300}$ [m]

Material:

$E = (18 + 0.2 \cdot X + 0.1 \cdot Y) \cdot 10^4$ [MN/m^2]

$\nu = 0.23 + 0.01 \cdot Z$ [-]

Loading:

$p_c = 5 + 0.3 \cdot W + 0.1 \cdot Y$ [MN/m^2]

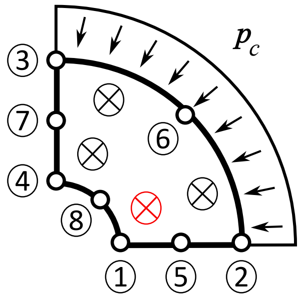

Fig. 2.4: Element with Load