RUB	**Linear Finite Element Methods** **Homework 1**	STATIK DYNAMIK **Prof.Dr.techn. G. Meschke** **M.Sc. N. Gottardi**

Name: ____________ **Matr.-Nr.:** ____________ ☐☐☐☐

To be submitted latest on **30.11.2023**. ↑W ↑X ↑Y ↑Z

This homework is parameterised by the last 4 digits of your student ID W X Y Z.

1. Task: Consider the system of equations given below:

$$\begin{aligned} 1 \cdot l + (2+Z) \cdot m - 7 \cdot n &= r \\ (2+Z) \cdot l - (1+X) \cdot m + 5 \cdot n &= 3+Y \\ -7 \cdot l + 5 \cdot m + (1+W) \cdot n &= 0 \end{aligned} \tag{1}$$

1.1) Considering the in-homogeneous DIRICHLET boundary condition $l = W+X+Y+Z$, solve the system of equations for all unknowns using static condensation. Explicitly denote the submatrices K_{uu}, K_{rr}, and K_{ru} of the problem.

2. Task: A 3D structure discretized by three LINEAR TRUSS ELEMENTS is shown in the figures below (1 to 4). The structure has to be analyzed using the linear Finite Element Method. The three trusses of the structure are connected at node 1, which is free to move in all directions. Node 2 is constrained by a guided support, where the prescribed displacement u^*_{23} is applied in z-direction, while in x- and y-direction node 2 is fixed. Node 3 is also constrained by a guided support and can move in the x-y plane with an angle β to the x-axis, as shown in Fig. 3. Node 4 is fixed and positioned at the origin of the coordinate system. Additionally, a spring with stiffness k_s connects node 2 and 3.
The nodal force F^*, acting in the x-z plane, is applied at node 1 in x- and z-direction with an angle α. Furthermore, the partial linear load $p_l(s)$ acts on element 1. The load is acting on the first and last third of the truss element and its magnitude increases from 0 to p_{max} (first third) and decreases from p_{max} to 0 (last third), see Fig. 4.
To account for the inclined support at node 3, use the master/slave method. Always use at least four decimal places in your calculation.

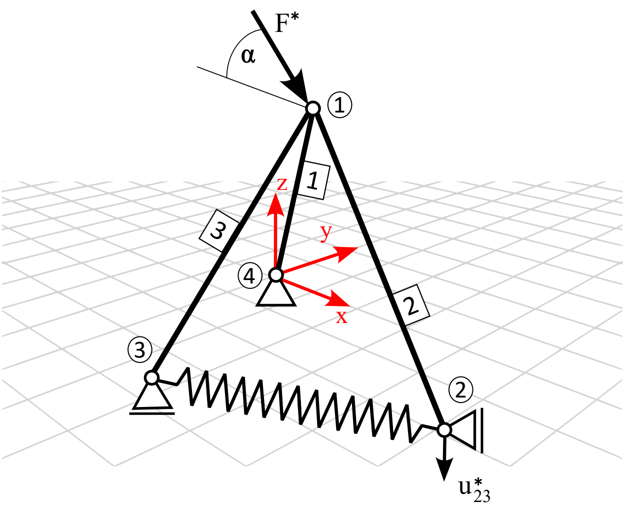

Fig. 1: 3D-View of the Structure

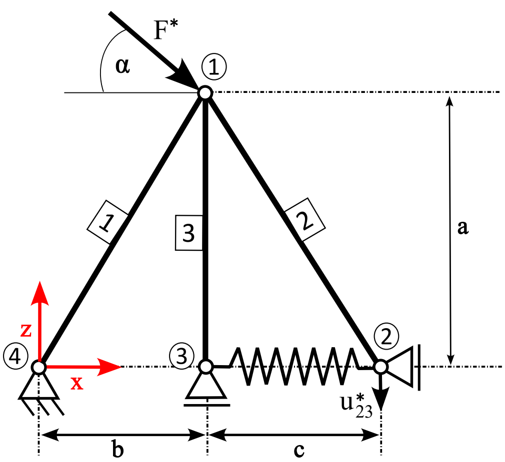

Fig. 2: View in x-z Plane

RUB

Linear Finite Element Methods

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2.1) State all independent degrees of freedom of the system.
Explain the difference between a dependent and an independent degree of freedom.

2.2) Compute the reduced stiffness matrix $\boldsymbol{K}^{global}_{red}$ of the system corresponding to the unknown degrees of freedom.

2.3) Compute the reduced load vector $\boldsymbol{r}^{global}_{red}$ of the system.

2.4) Calculate all unknown displacements $\mathbf{u}$ of the system, accounting for the in-homogeneous Dirichlet boundary condition u^*_{23} by using static condensation, similarly to task 1.

2.5) Calculate the force in the spring using the local displacements of the spring.

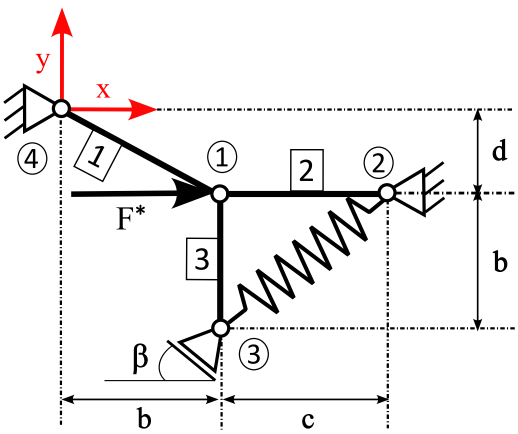

Fig. 3: View in x-y Plane

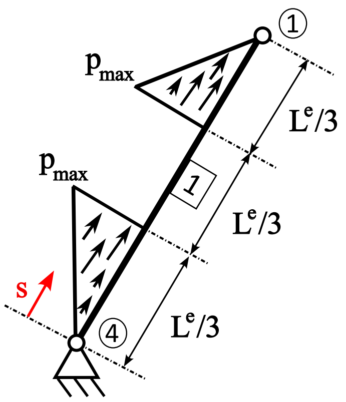

Fig. 4: Partial Load on Element $\boxed{1}$

Geometry:

$a = 6 + \dfrac{X+Y}{10}$ m

$b = 3 + \dfrac{Y+Z}{10}$ m

$c = 3 + \dfrac{W+X}{10}$ m

$d = 1 + \dfrac{W+Z}{10}$ m

$A = 2.7 \cdot 10^{-2}$ m^2

$\alpha = tan^{-1}\left(\dfrac{c}{a}\right)$

$\beta = tan^{-1}\left(\dfrac{d}{b}\right)$

Material:

$E = 200\,000$ MN/m^2

Loading:

$p_{max} = 2 + \dfrac{2W+Y}{10}$ MN/m

$F^* = 1 + \dfrac{X+Z}{10}$ MN

$u^*_{23} = -(1 + \dfrac{W}{5}) \cdot 10^{-3}$ m

Spring:

$k_s = 150 + 3Y$ MN/m