

Article

A Survey of Path Planning Algorithms for Mobile Robots

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Abstract: Path planning algorithms are used by mobile robots, unmanned aerial vehicles, and autonomous cars in order to identify safe, efficient, collision-free, and least-cost travel paths from an origin to a destination. Choosing an appropriate path planning algorithm helps to ensure safe and effective point-to-point navigation, and the optimal algorithm depends on the robot geometry as well as the computing constraints, including static/holonomic and dynamic/non-holonomically-constrained systems, and requires a comprehensive understanding of contemporary solutions. The goal of this paper is to help novice practitioners gain an awareness of the classes of path planning algorithms used today and to understand their potential use cases—particularly within automated or unmanned systems. To that end, we provide broad, rather than deep, coverage of key and foundational algorithms, with popular algorithms and variants considered in the context of different robotic systems. The definitions, summaries, and comparisons are relevant to novice robotics engineers and embedded system developers seeking a primer of available algorithms.



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Autonomous Vehicles (AVs) have the potential to dramatically reduce the contribution of human driver error and negligence as the cause of vehicle collisions. These AVs must move from point A to point B safely and efficiently with respect to time, distance, energy, and other factors. Path planning is key in determining and evaluating plausible trajectories in support of these goals.

While performing navigational tasks, robotic systems, such as AVs, make use of capabilities that involve modeling the environment and localizing the system's position within the environment, controlling motion, detecting and avoiding obstacles, and moving within dynamic contexts ranging from simple to highly complicated environments. The four general problems of navigation are perception, localization, motion control, and path planning [1–3].

Among these areas, it may be argued that path planning is the most important for navigation processes. Path planning is the determination of a collision-free path in a given environment, which may often be cluttered in a real world situation [4]. To best take advantage of these capabilities while accomplishing the system's design and use objectives, an appropriate path planning technique must be identified and implemented. Often, the best-performing technique will vary with the system type and operating environment.

As mobile AVs have proliferated and increasingly require path planning or path finding, these capabilities have become a recent area of focus in the field of autonomous control. Since mobile robots are used in a range of applications, researchers have developed methods for effectively fitting their requirements to overcome some of the significant challenges faced while implementing fully or partially autonomous navigation in cluttered environments.

In order to simplify the path planning problem and ensure that the robot runs/moves smoothly while avoiding obstacles in a cluttered environment, the configuration space must be matched with the algorithm being used. Multiple path planning and path finding

algorithms exist with varied applicability determined by the system's kinematics, the environment's dynamics, robotic computation capabilities, and sensor- and other-sourced information availability. Algorithm performance and complexity trade-offs also depend on the use case.

For a novice practitioner, selecting just one approach is daunting—particularly for those individuals without a background in controls. Yet, as AVs become increasingly widespread, engineers are forced into such roles without adequate training. This document aims to provide a summary of commonly used foundational techniques for path planning suitable for individuals with limited expertise in controls and seeking to identify areas of exploration in support of new AV functionality.

We consider the applicability of algorithms in static and dynamic environmental contexts and review common path planning algorithms used in autonomous vehicles and robotics to serve as a primer for novice practitioners in the fast-evolving field of autonomy. Representative algorithms are explored, though the coverage is not exhausted and not intended to reflect the state-of-the-art in AV path planning.

1. Path Planning

Path planning is a non-deterministic polynomial-time (“NP”) hard problem [5] with the task of finding a continuous path connecting a system from an initial to a final goal configuration. The complexity of the problem increases with an increase in degrees of freedom of the system. The path to follow (the optimal path) will be decided based on a constraints and conditions, for example, considering the shortest path between end points or the minimum time to travel without any collisions. Sometimes constraints and goals are mixed, for example seeking to minimize energy consumption without causing the travel time to exceed some threshold value.

Path planning has attracted attention since the 1970s, and, in the years since, it has been used to solve problems across fields from simple spatial route planning to the selection of an appropriate action sequence that is required to reach a certain goal. Path planning can be used in fully known or partially known environments, as well as in entirely unknown environments where information is received from system-mounted sensors and update environmental maps to inform the desired motion of the robot/AV.

Path planning algorithms are differentiated based upon available environmental knowledge. Often, the environment is only partially known to the robot/AV. Path planning may be either local or global. Global path planning seeks an optimal path given largely complete environmental information and is best performed when the environment is static and perfectly known to the robot. In this case, the path planning algorithm produces a complete path from the start point to the final end point before the robot starts following the planned trajectory [6]. Global motion planning is the high level control for environment traversal.

By contrast, local path planning is most typically performed in unknown or dynamic environments. Local path planning is performed while the robot is moving, taking data from local sensors. In this case, the robot/AV has the ability to generate a new path in response to the changes within the environment. Obstacles, if any exist, may be static (when its position and orientation relative to a known fixed coordinate frame is invariant in time), or dynamic (when the position, orientation, or both change relative to the fixed coordinate frame) [6].

An effective path planning algorithm needs to satisfy four criteria. First, the motion planning technique must be capable of always finding the optimal path in realistic static environments. Second, it must be expandable to dynamic environments. Third, it must remain compatible with and enhance the chosen self-referencing approach. Fourth, it must minimize the complexity, data storage, and computation time [7]. In this paper, we present an overview of the most frequently used path planning algorithms applicable to robots/AVs and discuss which algorithm is the best suited for a static/dynamic environment.

2. Path Planning Algorithms

The paper begins with Dijkstra algorithm [8] and its variants, which are commonly used in applications, such as Google Maps [9] and other traffic routing systems. To overcome Dijkstra's computational-intensity doing blind searches, A* [10] and its variants are presented as state of the art algorithms for use within static environments.

However, A* and A* re-planner are used for shortest path evaluation based on the information regarding the obstacles present in the environment, and the shortest path evaluation for the known static environment is a two-level problem, which comprises a selection of feasible node pairs and shortest path evaluation based on the obtained feasible node pairs [11]. Both of the above mentioned criteria are not available in a dynamic environment, which makes the algorithm inefficient and impractical in dynamic environments.

To support path planning in dynamic environments, D* [12] and its variants are discussed as efficient tools for quick re-planning in cluttered environments. As D* and its variants do not guarantee solution quality in large dynamic environments, we also explore Rapidly-exploring Random Trees (RRTs) [13] and a hybrid approach combining Relaxed A* (RA*) [14] and one meta-heuristic algorithm. The hybrid approach comprises two phases: the initialization of the algorithm using RA* and a post-optimization phase using one heuristic method that improves the quality of solution found in the previous phase. Three meta-heuristic algorithms are also discussed: namely, the Genetic, Ant colony, and Firefly algorithms. These are aimed at providing effective features in pursuit of a hybrid approach to path planning.

We chose these algorithms because they represent the foundational algorithms used within contemporary real time path planning solutions. Novel research builds on these algorithms to find additional performance and efficiency. The list of covered algorithms is, therefore, not exhaustive, but provides coverage of several common algorithms used in the context of path planning for autonomous vehicles and robotic systems. Other path planning algorithms are discussed in [15].

2.1. Dijkstra Algorithm

The Dijkstra algorithm works by solving sub-problems computing the shortest path from the source to vertices among the closest vertices to the source [8]. It finds the next closest vertex by maintaining the new vertices in a priority-min queue and stores only one intermediate node so that only one shortest path can be found.

Dijkstra finds the shortest path in an acyclic environment and can calculate the shortest path from starting point to every point. We find many versions of Improved Dijkstra's algorithm, this is based on specific applications we find around us. Every Improved Dijkstra's algorithm is different, to reflect the diversity of use cases and applications for the same. Variants of the improved Dijkstra's algorithm are discussed in this article.

The traditional Dijkstra algorithm relies upon a greedy strategy for path planning. It is used to find the shortest path in a graph. It is concerned with the shortest path solution without formal attention to the pragmatism of the solution. The modified Dijkstra's algorithm aims to find alternate routes in situations where the costs of generating plausible shortest paths are significant. This modified algorithm introduces another component to the classical algorithm in the form of probabilities that define the status of freedom along each edge of the graph [16]. This technique helps to overcome computational shortcomings in the reference algorithm, supporting its use in novel applications.

Another improved Dijkstra algorithm reserves all nodes with the same distance from the source node as intermediate nodes, and then searches again from all the intermediate nodes until traversing successfully through to the goal node, see Figure 1. Through iteration, all possible shortest paths are found [17] and may then be evaluated.

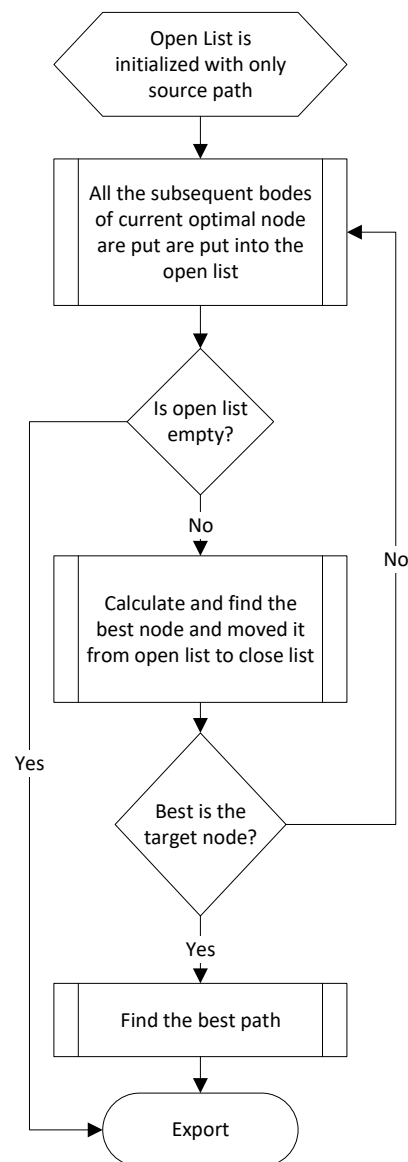


Figure 1. Flowchart of the improved Dijkstra algorithm used in path planning.

The Dijkstra algorithm cannot store data apart from the nodes that have previously been traversed. To overcome this disadvantage, a storage scheme is introduced that implements a multi-layer dictionary [18] for the storage of data, which consists of two dictionaries and a list of data structure organized in hierarchical order. The first dictionary maps each and every node to its neighboring nodes. The second dictionary stores the path information of each neighboring pathway [18].

A multi-layer dictionary provides a comprehensive data structure for Dijkstra algorithm in an indoor environment application where Global Navigation Satellite System (GNSS) coordinates and compass orientation are not reliable. The path information in the data structure helps to determine the degree of rotational angle the robot needs to execute at each node or junction. The proposed algorithm produces the shortest path in terms of length while also providing the most navigable path in terms of the lowest necessary total angle of turns in degrees, which is infeasible to compute within the traditional Dijkstra algorithm [18].

The Floyd algorithm is a popular graph algorithm for finding the shortest path in a positive or negative weighted graph [19], whereas Dijkstra works best for finding the

single-source (finding shortest paths from a source vertex to all other vertices present in the graph) shortest path in a positive weighted graph [19].

Dijkstra is a reliable algorithm for path planning. It is also memory heavy [20] as it has to compute all the possible outcomes in order to determine the shortest path, and it cannot handle negative edges. Its computation complexity is $O(n^2)$ with n being the number of nodes in the network [21]. Due to its limitations, many improved variants arose based on the applications. As we discussed earlier for a memory drawback, a new memory scheme was introduced [18], there was also a solution to map with a huge cost factor [16].

We can generalize that the Dijkstra algorithm is best suited for a static environment and/or global path planning as most of the data required are pre-defined for computation of shortest path; however, there are applications where the Dijkstra algorithm has been used for dynamic environments. In this case, the environment is partially known or completely unknown, and thus the node information with respect to obstacles is computed on the fly; this is called local planning, and the Dijkstra algorithm is run for evaluation of the shortest path computation. Please see classification Table 1 for Dijkstra and its variants. However, we cannot use the Dijkstra algorithm alone within dynamic environments [22]

Table 1. Suitability of the Dijkstra algorithm and its variants as classified based upon static and dynamic environmental constraints.

Algorithm	Static Constraints	Dynamic Constraints
Dijkstra	x	
Improved Dijkstra	x	
Multi-layer dictionary	x	x
Floyd and Dijkstra	x	

2.2. A* Algorithm

The A* Algorithm is a popular graph traversal path planning algorithm. A* operates similarly to Dijkstra's algorithm except that it guides its search towards the most promising states, potentially saving a significant amount of computation time [23]. A* is the most widely used for approaching a near optimal solution [24] with the available data-set/node.

It is widely used in static environments; there are instances where this algorithm is used in dynamic environments [25]. The base function can be tailored to a specific application or environment based on our needs. A* is similar to Dijkstra in that it works based on the lowest cost path tree from the initial point to the final target point. The base algorithm uses the least expensive path and expands it using the function shown below

$$f(n) = g(n) + h(n) \quad (1)$$

where $g(n)$ is the actual cost from node n to the initial node, and $h(n)$ is the cost of the optimal path from the target node to n .

The A* algorithm is widely used in the gaming industry [26], and with the development of artificial intelligence, the A* algorithm has since been improved and tailored for applications, including robot path planning, urban intelligent transportation, graph theory, and automatic control [27–31].

It is simpler and less computationally-heavy than many other path planning algorithms, with its efficiency lending itself to operation on constrained and embedded systems [32,33]. The A* algorithm is a heuristic algorithm that uses heuristic information to find the optimal path. The A* algorithm needs to search for nodes in the map and set appropriate heuristic functions for guidance, such as the Euclidean distance, Manhattan distance, or Diagonal distance [34,35]. An algorithm is governed by two factors for efficiency resources used for performance of the task and response time or computation time taken for performance of the task.

There is a trade off between speed and accuracy when the A* algorithm is used. We can decrease the time complexity of the algorithm in exchange for greater memory, or

consume less memory in exchange for slower executions. In both cases, we find the shortest path. One simple application for the A* algorithm is to find the shortest path to an empty space within a crowded parking lot [33].

In the hierarchical A* algorithm, path searching is divided into few processes, the optimal solution for each process is found, and then the global optimal solution is obtained, which is the shortest path to the empty spot. Another improvement is done on this algorithm, with the improved hierarchical A* algorithm using the optimal time as an evaluation index, and, introducing a variable, a weighted sum is used along with the heuristic [33]:

$$f_w(n) = (1 - w)g(n) + w.h(n) \quad (2)$$

where w is a weighted coefficient.

The weighted co-efficient is defined with respect to each application and in most cases it might be used to represent linear distance (e.g., Euclidean distance). Improved hierarchical A* is claimed to have improved efficiency and precision [36]. Another variation of this algorithm is used in valet parking where the ego-vehicle (cars that usually focus only on their local environment and do not take into account environmental context) has a direction of motion d that depends on the speed, gear, steering angle, and other parameters of the vehicle kinematics [34,37].

This algorithm is called the Hybrid A* Algorithm. The Hybrid A* search algorithm improves the normal A* algorithm when implemented on a non-holonomic robot (e.g., autonomous vehicles). This algorithm uses the kinematics of an ego vehicle to predict the motion of the vehicle depending on the speed, gear, and steering angle, and other parameters of the vehicle will add costs to the heuristic function. An improved version of Hybrid A* was introduced to the same application with the help of a visibility diagram path planner.

A visibility diagram was one of the very first graph-based search algorithms used in path planning in robotics [38,39]. This method guarantees finding the shortest path from the start to the final end position. The start, end, and obstacle positions are fed into as input to the Hybrid A* algorithm where A* is run on the results of the visibility diagram, which then provides the optimum distance. This is called the Guided Hybrid A* algorithm [37]. Common heuristic functions appear in Table 2.

Table 2. Most Common Types of Heuristic Functions Used In Path Planning Algorithms.

Function	Equation
Euclidean distance	$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
Manhattan distance	$ x_1 - x_2 + y_1 - y_2 $
Octile distance	$\max x_1 - x_2 + y_1 - y_2 $

The A* algorithm is computationally simple relative to other path planning algorithms. With adjustments for vehicle kinematics, steering angle, and vehicle size, A* is suitable for automotive applications. New cost functions can be created with respect to equal step sampling, as in the case of Equation (3). The shown cost function includes both distance and steering costs, penalizing each step by using the cost on every movement. The above considerations avoid sudden turns and in this manner improve the path smoothness [40].

$$f(n) = K_1g(n) + K_2h(n) + K_3p(n) \quad (3)$$

where K_1, K_2, K_3 are weights with a positive value, $g(n)$ and $h(n)$ are the same as the base A* function, and $p(n)$ is the penalty factor based on the turning cost.

Lifelong Planning A* (LPA*) [41] combines the best techniques of both incremental and heuristic search methods. It promises to find the shortest paths while the edge costs of a graph change or vertices are added or deleted due to addition or deletion of obstacles. The

algorithm initializes the g-values (start distances) of the vertices encountered to infinity and their rhs-values (one-step look ahead values) according to the Equation (4) and performs A* search in the beginning.

In the subsequent stages, it updates the rhs-values and keys (5) of the vertices affected by changed edge costs as well as decides their membership based on the local consistency in the priority queue and ultimately recalculates the shortest path from start to goal upon expanding locally inconsistent (when g-value not equals to rhs-value) vertices in order of their priorities, until the goal vertex is locally consistent and the key of the next vertex to expand is no less than the goal key. However, when people use the LPA* algorithm, they need to re-plan the route from the starting point to the target point [42].

The algorithm is faster than the two search methods individually because it reuses parts of the previous search tree that are identical to the new tree and applies heuristic knowledge to focus the search.

$$rhs(s) = \begin{cases} 0 & \text{if } s = s_{start} \\ \min_{s' \in pred(s)} (g(s') + c(s, s')) & \text{otherwise} \end{cases} \quad (4)$$

$$K(s) = [\min(g(s), rhs(s)) + h(s); \min(g(s), rhs(s))] \quad (5)$$

To summarize, see Table 3 for a list of A* and its variants used in path planning within static and dynamic contexts.

Table 3. Classification of the A* algorithm and its variants based on static and dynamic constraints.

Algorithm	Static Constraints	Dynamic Constraints
A*	x	
Hierarchical A*	x	
Improved Hierarchical A*	x	
Hybrid A*		x
Guided Hybrid A*		x
A* with equal step sampling	x	
Diagonal A*	x	
A* with smart heuristics		x
Lifelong Planning A*	x	x

Importantly, the A* algorithm is computationally efficient [43,44]. This makes it suitable for applications deployed in static environments. The computational speed and efficiency of A* and its variants depends on the accuracy of the heuristic function.

2.3. D* Algorithm

Path planning in partially known and dynamic environments in an efficient manner is increasingly critical, e.g., for automated vehicles. To solve this problem, the D* (or Dynamic A*) algorithm is used to generate a collision-free path amidst moving obstacles. D* is an informed incremental search algorithm that repairs the cost map partially and the previously calculated cost map.

The D* algorithm processes a robot's state until it is removed from the open list, and, at the same time, the states sequence is computed along with back pointers to either direct the robot to goal position or update the cost due to a detected obstacle and place the affected states on the open list. The states in the open list are processed until the path cost from the current state to the goal is less than a minimum threshold, the cost changes are propagated to next state, and the robot continues to follow back pointers in the new sequence towards the goal [45]. D* is over 200-times faster than an optimal re-planner [45,46]. The main drawback of the D* algorithm is its high memory consumption when compared with other D* variants [47].

The D* Lite algorithm [48] builds upon LPA* by exchanging the start and goal vertex and reversing all edges algorithmically. The algorithm finds the shortest path from the goal node to the start node in an unknown dynamic environment by minimizing rhs values calculated using Equation (4). The key values of vertices are calculated and updated using Equation (5) when a connection changes with variation in its connecting edge weights. With these changes, the heuristics are updated from the estimated cost from the goal to the original start to the estimated cost from the goal node to the new start node.

Since the heuristic value decreases by a value (start original and start new), the same value is added to all the new calculated keys. This method avoids traversing the priority queue every time connections change. Therefore, the D* Lite algorithm has a lower computational cost compared to LPA* as it avoids reordering of the priority queue. This algorithm is suitable for path planning of autonomous vehicles in cluttered environments, where the algorithm can reach a quick re-planning result when unexpected obstacles are encountered [49].

The Enhanced D* Lite algorithm [50] is an improvement of the D* Lite algorithm, indeed, it keeps the same path finding principle but overcomes its problems through enhancements that emphasize avoiding complicated obstacles, preventing the robot from traversing between two obstacles and across obstacles' sharp corners, creating virtual walls if necessary and removing unnecessary pathways yielding the shortest path from the goal position towards the starting position. The algorithm was implemented in a real-world system using a Team AmigoBot equipped with sonar sensors for detecting obstacles [50].

The idea of using interpolation to produce better value functions for discrete samples over a continuous state space is not new. This approach has been used in dynamic programming to compute the value of successors that are not in the set of samples [38,51,52]. The algorithm Field D* [53] is an extension of the widely-used D* family of algorithms that uses linear interpolation to produce globally smooth, low-cost paths.

As with D* and D* Lite, our approach focuses its search towards the most relevant areas of the state space during both initial planning and re-planning. It is effective in practice and is currently employed as the path planner in a wide range of fielded robotic systems, such as Pioneers [54] that are used in indoor environments and Automated All Terrain Vehicles [55] for outdoor extreme environments.

The algorithm D* and its variants can be employed for any path cost optimization problem where the path cost changes during the search for the optimal path to the goal. D* is most efficient when these changes are detected closer to the current node in the search space. The D* algorithm has wide range of applications, including planetary rover mission planning [56]. Please see classification Table 4 for D* and its variants.

Table 4. Summary of D* and its variants.

Algorithm	Static Constraints	Dynamic Constraints
D*		x
D* Lite		x
Enhanced D* Lite		x
Field D*		x

2.4. Rapidly-Exploring Random Trees

We have discussed algorithms like A*, which are static in nature and require a path specified to them upfront. Let us now discuss dynamic and online algorithms like RRT [13,38], which do not require a path to be specified upfront. Rather, they expand in all regions, and, based on weights assigned to each node, create a path from start to goal. RRT's were introduced to handle broad classes of path planning problems. They were specifically designed to handle non-holonomic constraints (constraints that are non-integrable into positional constraints).

RRT's and Probabilistic Road Maps (PRMs) share the same desirable properties, and both were designed with few heuristics and arbitrary parameters. This gives better performance and consistency in the results. PRM's may require connections of thousands of configurations or states to find a solution, whereas RRT's do not require any connections to be made between states to find a solution. This helps in applying RRT's to non-holonomic and kinodynamic planning [57].

RRT's expand by rapidly sampling the space, grow from the starting point, and expand until the tree is sufficiently close to goal point. In every iteration, the tree expands to the nearest vertex of the randomly generated vertex. This nearest vertex is selected in terms of a distance metric. It can be Euclidean, Manhattan, or any other distance metric.

RRT expands heavily in unexplored portions of the configuration space of the robot when compared to naive random trees, who tend to expand heavily in places that are already explored (see Figures 2 and 3). Thus, we can say that RRT is biased to unexplored regions. The vertices of RRT follow a uniform distribution. The algorithm is relatively simple and RRTs always stay connected even though the number of edges is minimal.



Figure 2. Exploration of naive random tree selects a node at random from the tree and adds an edge in a random direction [58].

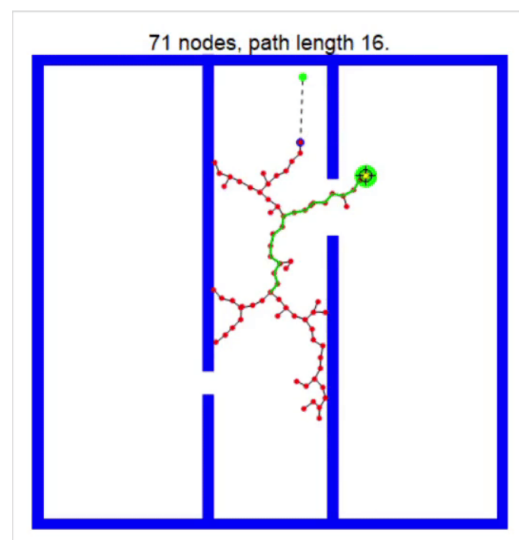


Figure 3. Exploration of rapidly-exploring random tree are designed to efficiently explore paths in a high-dimensional space [58].

Since RRT algorithms are able to cope with non-holonomic constraints, they can be applied to almost any wheeled system. Depending on the user's needs, they can select a variant of RRTs [59].

RRT Connect or bi-directional RRTs combine two RRTs, one at the starting position and a second at the goal position, connecting them using a heuristic. This approach is suitable for problems not involving differential constraints. During each iteration, one tree is extended, and the new vertex is connected to the nearest vertex of the other tree. Then, each tree's role is reversed with both the trees exploring the free configuration space. This algorithm is suitable to plan motions for a robotic arm with high degrees of freedom [60].

In Sensor-based Random Trees (SRTs), a road map of the explored area is created with an associated Safe Region (SR) [61,62]. The Local Safe Area (LSR) is detected by the sensors. Each node of the SRT consists of a free configuration with associated Local Safe Region. The safe region consists of all local safe regions. It is an estimate of the free space surrounding the robot at a given configuration. The shape of the LSR depends on the sensor characteristics (e.g., the angular resolution) of the robot. The LSR shape can be a ball or a star.

The tree is expanded towards randomly-selected directions such that the new configuration and the path reaching towards the test robot are contained in the local safe region of the node. During each iteration, sensor data is collected and analyzed to generate a plausible region estimating the free space surrounding the robot at the current configuration. Then, a new node containing the current configuration and LSR is added to the tree. Depending on the robot's sensors, different perception techniques can be used. The star-shaped LSR exploration strategy is more accurate as proven experimentally [61].

RRT* extends RRT to find the optimal path from a start to a goal node using triangle inequality (see Figure 4). As the number of nodes increases, lower-cost (more optimal) paths are discovered. RRT-connect is particularly useful for robotic arms, like Programmable Universal Machine for Assembly (PUMA) robots [60]. The connect heuristic works most effectively when one can expect relatively open spaces for a majority of planning queries. The connect heuristic was originally developed with this kind of problem in mind [63]. Please see classification in Table 5 for RRT and its variants.

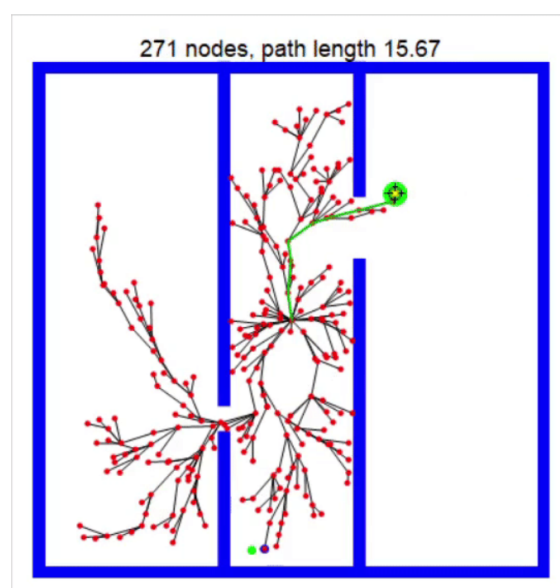


Figure 4. Exploration of RRT*, which improves on the existing tree by rewiring the tree to form the shortest paths [58].

Table 5. Classification of the RRT algorithm and its variants based on static and dynamic constraints.

Algorithm	Static Constraints	Dynamic Constraints
RRT		x
RRT Connect		x
Sensor-based Random Tree		x
RRT*		x

2.5. Genetic Algorithm

Discrete path planning algorithms, such as grid based algorithms and potential fields, require substantial CPU performance and/or require significant memory. In this section, we introduce genetic algorithms (GA), which help to overcome such limitations. For example, genetic algorithms can be applied with the advantage that such algorithms cover a large search space and use a minimal memory and CPU resources. They are also able to adapt to changing environments.

One disadvantage is that the solution found for the optimization problem may not always be a global minimum (e.g., the overall shortest path). One of the interesting applications of genetic algorithms is in small size humanoid robots that can play football [64]. We note that the robot needs to know the goal post in-order to kick the ball to the goal with the opposite team being obstacles, as the robot needs to avoid collisions and head towards the goal post to kick the ball into the goal. Thus, dynamic path planning is required in this case for the robot.

Since the algorithm is also applicable to dynamic environments, the optimal solution that finds a collision-free path between two points needs to be constantly updated according to change in surroundings. In this case, evolutionary methods converge upon an optimal solution. All possible solutions are represented as individuals of a population, with each gene representing a parameter. Individuals are formed by a complete set of genes.

A new generation is formed by selecting the best individuals from the parent generation and applying genetic operators, like crossover and mutation, to explore additional solutions. Each offspring from the new generation is tested with a fitness function devised for that problem. From all offspring, the best individuals are chosen as the parents of the next generation.

A fitness function is used to guide the simulation towards optimal design solutions. The fitness function must consider not only the shortest path but also the smoothness and clearance for dynamic path planning [65].

$$eval(p) = w_d dist(p) + w_s smooth(p) + w_c clear(p) \quad (6)$$

where w_d , w_s , and w_c represent the weights on the total cost and $dist(p)$, $smooth(p)$, and $clear(p)$ are defined as follows:

$$dist(p) = \sum_{i=1}^{n-1} d(s_i)$$

where $d(s_i)$ is the distance between two adjacent nodes

$$smooth(p) = \sum_{i=2}^{n-1} e^{a(\theta_i - \alpha)}$$

where θ_i is the angle between the extension of the two line segments connecting the i th knot point. α is the desired steering angle, and a is a coefficient

$$clear(p) = \sum_{i=1}^{n-2} e^a (g_i - \tau)$$

where g_i is the smallest distance from the i th segment to all the obstacles, and τ is the desired clearance distance.

Genetic operators are used to evolve plausible paths for selected parents. The selection operator chooses the fittest individuals and lets them pass their genes to the next generation. The crossover operator selects crossover point chosen at random from the genes for each pair of parents to be mated. The mutation operator performs the flipping of some bits in the bit string to maintain diversity. The GA produces the optimal path and takes advantage of the GA's optimization ability [66]. Al-Taharwa et al. [67] described how to employ the GA approach in solving the path planning problem in a non-dynamic environment.

Their results showed that the hired method was efficient in handling the path planning problem in different static fields. The important characteristic that made GA suitable for solving these kinds of problem is that GA is an inherently parallel search method and has the capability to search for the optimal path in a given environment [66,68]. These are useful for path planning in complex environments containing U-shaped obstacles [69].

2.6. Ant Colony Algorithm

Computer Scientists and biologists have been inspired from nature for the design of path planning optimization algorithms. The ant colony optimization (ACO) algorithm, which is based on a heuristic approach inspired by the collective behavior of trail-laying ants to find the shortest and collision-free path, is one such derivative algorithm. This algorithm was first proposed by Marco Dorigo in his doctoral thesis "Ant system: by colony of cooperating agents" in 1992 [70] to simulate the ant foraging for food in Ant System (AS) theory.

According to the different characteristics of the problem, a variety of algorithms have been derived, which are probabilistic heuristic algorithms to find the shortest path. When ants forage for foods, they release a substance along the road, called pheromones, which are chemical substances. The following ants will then choose a suitable route according to the pheromone concentration left by the previous ants, and the probability of any ant to select the path is the same. When the concentration of existing pheromone is higher, the probability of ants to choose the path becomes higher as well.

However, the pheromone concentration becomes lower by evaporation. When the number of searches for paths increases, the shorter path will have a higher pheromone concentration because more ants have visited this path, while the other paths' concentration will be lowered because less ants visit and there is a natural evaporation of the pheromones, which means that ants can search according to the pheromone information to move toward a path with shorter distances [71].

However, the ACO algorithm has a simple modeling process attributed to collective outcomes corresponding to many actions. The striking features of ACO algorithm are positive feedback helping in discovering a goal rapidly, distributed computation avoiding premature convergence, and greedy heuristics aiding in finding a goal in the early stages. The algorithm [70] is based on pheromone deposits, which determine the probability of an artificial ant to go from one node to another node in the path. The transition probability from node i to j is given by:

$$p_{ij}^k = \begin{cases} \frac{(\tau_{ij}^k)^\alpha (\eta_{ij}^k)^\beta}{\sum_{l \in N_i^k} (\tau_{il}^k)^\alpha (\eta_{il}^k)^\beta} & \text{if } j \in N_i^k \\ 0 & \text{if } j \notin N_i^k \end{cases} \quad (7)$$

where τ_{ij}^k is the pheromone level indicating the number of ants choosing the same edge as previously, η_{ij}^k is a heuristic indicating the closeness of the goal position, and α and β are weights determining the importance of the pheromones and heuristics.

Based on the application, the values of α , β , and η are chosen to determine which neighboring nodes will be selected in the path to the goal. The more pheromone is present on a path, higher the probability for an ant to take that path. The ACO algorithm is used for

robot dynamic path planning and exploited to find the pre-optimized path for the vehicle routing problem [72]. The traditional ant colony algorithm has a slow convergence speed due to the limitations of heuristic searches. In the application of vehicle path planning, it is easy to fall into a local optimal solution at the initial stage of optimization, resulting in stagnation.

The Enhanced Ant Colony Algorithm (EACA) [73] overcomes the shortcomings of the traditional ant colony algorithm, as ants move randomly in the process of searching for food and returning to the nest using state transition probability function (7). η_{il}^k is the inspiration function of the node i from the node l . The purpose of modifying the inspiration function is to enhance the ant's perceptions of the target node at the current node, which will guide ant to move to, thus, reduce the search time and avoid falling into a local optimum [74]. This algorithm can speed up the convergence and effectively generate reasonable solutions even in complex scenarios, which inspired people to use this in collision avoidance system with some more improvements in the ACO algorithm [75–77].

The Improved Ant Colony Optimization algorithm [78] executes a pheromone update method after each cycle of target search process, and the optimal path is planned using an improved cost function. The algorithm enhances the calculation ability of ACO and converges to optimal path much faster. It is used to solve large-scale fleet assignment problem and path planning for multiple automated guided vehicles (AGV) in a dynamic environment [78]. It is used for the optimal path planning of unmanned aerial vehicles (UAVs) [79] as it has better effectiveness in dynamic environments and more efficient use of computing power.

The Adaptive Ant Colony (AAC) algorithm [80] improves the ACO algorithm by adaptively updating the value of the evaporation rate, ρ using Equation (8) and updating pheromone on each path, which is then fed to the transition probability Equation (7) to find the path direction.

$$\rho(t) = \begin{cases} 0.95\rho(t-1) & \text{if } 0.95\rho(t-1) \geq \rho_{min} \\ \rho_{min} & \text{if others} \end{cases} \quad (8)$$

where ρ is a random number uniformly distributed in $(0, 1)$.

The AAC algorithm finds the global optimal path, better stability, convergence, and operational speed overcoming the defects of prematurity and stagnation in the ACO algorithm. To solve the vehicle routing problem, AAC has been used in conjunction with the Adaptive memory programming algorithm (AMP), where the algorithm is run in two iterations. The first iteration AMP is run so that it can cover the drawback of AAC falling into local minimum and on the result of AMP, i.e., a second iteration of AAC is run. By doing this, we have seen that the path length is minimized for vehicle path planning used in logistics [81].

The Bidirectional Ant Colony algorithm generates a global optimal path from two sets of paths generated by positive ants N and negative ants M from the original to the terminal and the terminal to the original node, respectively. The positive ant selects the nodes that have stronger pheromones whereas negative ants prefer weaker pheromone consistency using the improved transition probability formula [82].

As one group of ants is always towards the better nodes and another is always towards the poorer nodes, thus the poorer nodes will have the chance to be chosen. Secondly, in order to speed up the convergence of the latter part of the algorithm, the descending strategy is taken to reduce the scale of reverse ants and increase the scale of positive ants. Thereby, the positive ants search paths mainly, and the positive feedback is enhanced. Finally, each path can be optimized locally. The algorithm overcomes the problem of premature convergence and low convergence speed posed by the ACO algorithm.

Bio-inspired optimization methods resulted in the slime mold-based optimization algorithm (SLIMO). The SLIMO algorithm [83] mimics a true slime mold that connects food sources and distributes nutrients through a self-assembled resource distribution network

of tubes with varying diameters evolving with changing environmental conditions. The advantage of SLIMO path planning is that only a partial re-growth followed by short tube dynamics needs to be conducted to find the optimal path automatically in response to a dynamic environment. The application ranges from evacuation planning in uncertain or hazardous environments to path planning for real-world infrastructure networks—e.g., the Tokyo rail system [84].

ACO algorithm and its variants are types of parallel optimization algorithms, and thus they have good convergence and global optimization performance. The adaptive ant colony algorithm has a good global optimization ability, and it exhibits better optimization performance compared to other variants of the ACO algorithm. Please see classification in Table 6 for ACO and its variants.

Table 6. Classification of the ant colony algorithm and its variants based on static and dynamic constraints.

Algorithm	Static Constraints	Dynamic Constraints
Ant Colony Optimization		x
Slime mold based optimization	x	x
Improved ant colony optimization		x
Adaptive ant colony		x
Enhanced Ant Colony Algorithm		x
Bidirectional ant colony		x

2.7. Firefly Algorithm

The Firefly algorithm is a meta-heuristic algorithm based on Firefly mating behavior. It is a promising swarm-intelligence-based algorithm inspired by the collective behavior of insects or animals who cooperate in order to solve complex problems. Colonies of insects act as decentralized, self-organized systems that prevent a single insect from acting. The algorithm is used for solving continuous and discrete optimization problems [85]. In order to tackle optimization problems efficiently, many variants of the Firefly algorithm have been developed, see Figure 5.

The Firefly algorithm works on huge data sets and can be applied on path planning. Path planning can widely be divided into two branches: Global Path Planning [86] and Local Path Planning. Global planning is done on a larger set, and calculation of the shortest path is done from the start node to the goal node. The Local Planner begins when an obstacle is found to move around, and this is done on previous nodes and near nodes, which are available in the global set.

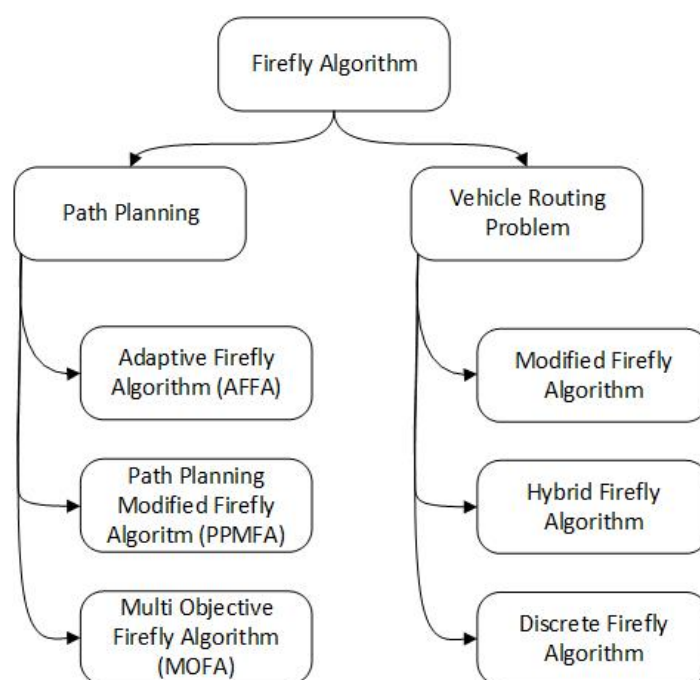


Figure 5. Variants of Firefly algorithm usage, adapted from [87].

The Modified Firefly Algorithm (MFA) is appropriate for global path planning and has yielded better results [88] as Modified Firefly uses a Gaussian random walk to replace the fixed-size step of the Standard Firefly Algorithm (SFA). Multiple checking methods enhance the success of the Firefly movement during the iteration process. The Modified Firefly algorithm overcomes one of the weaknesses of the conventional Firefly algorithm, which is slow convergence.

The number of iterations that are required is based on the distance between two fireflies and the intensity of the next brightest Firefly, enabling faster convergence during the final stages of the search. The complexity of three-dimensional (3D) path planning for Autonomous Underwater Vehicles (AUVs) requires optimization algorithms. The required optimization algorithm must have a fast convergence speed. The modified Firefly algorithm overcomes the convergence slowness of the basic Firefly algorithm. This is helpful in 3D underwater planning as we can achieve faster optimization [87].

The Adaptive Firefly Algorithm (AFA) was proposed [89] by utilizing the composite benefit of Firefly Algorithm (FA) for global exploration and Temporal Difference Q-Learning (TDQL) for local tuning of the absorption coefficient. A relative comparison AFA technique with classical FA and Particle Swarm Optimization (PSO) demonstrated that the AFA algorithm outperformed all its competitors with respect to accuracy and the run-time required for convergence [89].

The Developed Firefly Algorithm (DFA) provides a shorter path length and path smoothness in multi-objective path planning [90]. In SFA, the attraction of the fireflies is depended by the Euclidean distance. However in DFA, the Pareto-based method (here, the objective function is no longer a scalar value, but rather a vector) is used to estimate the brightness of the fireflies. By using this approach it helps to balance the optimized solution, yielding a smooth passable path as the result.

The Firefly algorithm is often used where large data sets exist and is suitable for dynamic environments. As discussed above, the Firefly algorithm is very popular in AUVs [89] where a 3D optimal path is found from the start point to the destination with the shortest path length and with collision free navigation with any obstacles in the working space. Please see classification in Table 7 for Firefly and its variants.

Table 7. Classification of the Firefly algorithm and its variants based on static and dynamic constraints.

Algorithm	Static Constraints	Dynamic Constraints
Firefly	x	
Adaptive Firefly		x
Developed Firefly		x
Modified Firefly		x

3. Conclusions

Recent years have seen the rapid growth of artificial intelligence in driverless vehicles and other automated mobile systems. As a result, the amount of research into path planning has increased dramatically. In the last decade, many new methods of path planning have been developed; however, it is difficult to find a comprehensive survey on the most popular path planning algorithms suitable for the novice practitioner.

In this paper, we presented several widely used path planning algorithms and their variants, categorized based primarily upon whether the environment in which the robot operates is static or dynamic. In each section, we covered the methodology of all the algorithms as well as their advantages and disadvantages, creating a comparative table in each section for where each algorithm was suited best based on a static or dynamic environment. The advances in computing and perception hardware have enabled us to use complex algorithms for path planning.

As both D* Lite and Enhanced D* Lite are based on A* it could be concluded that the added complexity tends to cost computational time [91]. Field D* is one such algorithm that stands out as one of the best algorithms suited for a dynamic environment as this is shown to decrease the computational time significantly. This helps in the presence of dynamic obstacles where the reaction time is very important, particularly with faster moving dynamic obstacles.

We conclude that, for a given map the algorithm A* is a strong-performing option for static environments because of the low memory usage, computational speed, less implementation complexity, and efficiency making it suitable also for use in embedded system deployment. One of the main limitations of bio-inspired algorithms is that they do not have excellent results in real-time path planning as they have a possibility of becoming stuck in local minima.

It is our hope that the reader, having witnessed a brief survey of the path planning landscape for AVs and other mobile robotic systems, will gain an awareness of the vocabulary needed and design considerations to take into account in designing, comparing, and implementing a path planning system with particular goals and constraints.

Though not exhaustive, this coverage provides a simplified summary for these techniques and their use cases and trade-offs to help readers become up-to-speed quickly such that a system might be implemented rapidly and safely and be optimized over time. The field of path planning continues to grow, and we encourage readers to seek out the latest and greatest algorithms suitable for their respective domains after reading this primer text.

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