Preprocessing and loading in Data

The following cells define helper functions and combines the batch files into a single dictionary that contains the images for each class as an numpy array (array of arrays).

In [1]:
```
%matplotlib inline
```

In [2]:
```
import matplotlib.pyplot as plt
import matplotlib.image as mpimg
import seaborn as sns
import numpy as np
import _pickle as cPickle
from sklearn.decomposition import PCA
from scipy.spatial.distance import sqeuclidean
from __future__ import division
```

In [3]:
```
def unpickle(file):
    fo = open(file, 'rb')
    dict = cPickle.load(fo, encoding='latin-1')
    fo.close()
    return dict
```

In [4]:
```
#Function defined for testing purposes.
def display_image(sample_image):
    output = list()
    ratio = int(len(sample_image) / 3)

    red = sample_image[0:ratio]
    green = sample_image[ratio: 2*ratio]
    blue = sample_image[2*ratio:]

    for i in range(len(red)):
        val = list([red[i], green[i], blue[i]])
        output.append(val)

    output = np.array(output).reshape((32,32,3))

    #fig1 = plt.figure()
    #fig1.add_subplot(7,7,1)
    imgplot = plt.imshow(output, aspect='auto')
```

In [5]:
```
def cmdscale(D):
    """
    Source: http://www.nervouscomputer.com/hfs/cmdscale-in-python/

    Classical multidimensional scaling (MDS)



    Parameters
```

```
    ----------

    D : (n, n) array

        Symmetric distance matrix.


    Returns

    -------

    Y : (n, p) array

        Configuration matrix. Each column represents a dimension. Only t
he
        p dimensions corresponding to positive eigenvalues of B are retu
rned.
        Note that each dimension is only determined up to an overall sig
n,
        corresponding to a reflection.


    e : (n,) array

        Eigenvalues of B.


    """
    # Number of points

    n = len(D)

    # Centering matrix

    H = np.eye(n) - np.ones((n, n))/n

    # YY^T

    B = -H.dot(D**2).dot(H)/2

    # Diagonalize

    evals, evecs = np.linalg.eigh(B)

    # Sort by eigenvalue in descending order

    idx   = np.argsort(evals)[::-1]
    evals = evals[idx]
    evecs = evecs[:,idx]

    # Compute the coordinates using positive-eigenvalued components only

    w, = np.where(evals > 0)
```

```
    L  = np.diag(np.sqrt(evals[w]))
    V  = evecs[:,w]
    Y  = V.dot(L)

    return Y, evals
```

In [6]:
```
# Read the data into python
batches = list()
column_names = unpickle("data/batches.meta")
batches.append(unpickle("data/data_batch_1"))
batches.append(unpickle("data/data_batch_2"))
batches.append(unpickle("data/data_batch_3"))
batches.append(unpickle("data/data_batch_4"))
batches.append(unpickle("data/data_batch_5"))
batches.append(unpickle("data/test_batch"))
```

In [7]:
```
# Combine the different batches into one dataset.
label_names = column_names['label_names']
merged_data = dict()
for data_batch in batches:
    raw_data  = data_batch['data']
    labels = data_batch['labels']
    for i in range(len(raw_data)):
        label = label_names[labels[i]]
        if (label not in merged_data):
            merged_data[label] = list()
        merged_data[label].append(raw_data[i])
```

In [8]:
```
#Verifying that the data has been merged correctly,
print(merged_data.keys())
c = np.array(merged_data['automobile'])
print(c.shape)
```

```
dict_keys(['frog', 'deer', 'cat', 'dog', 'truck', 'automobile', 'airpla
ne', 'horse', 'ship', 'bird'])
(6000, 3072)
```

Problem 4.10 Part A

In the problem below, we compute the mean image and the first twenty principal coordinates for each of the categories and store them into dictionaries. Afterwards, we computed the error for each of the image classes after applying PCA to the dataset. Both the squared error and the percentage of variance not explained by the twenty components were computed.

In [9]:
```
def comp_error(A, B):
    tot_err = 0
    for A_item, B_item in zip(A, B):
        tot_err += sqeuclidean(A_item, B_item)

    return tot_err
```

In [10]:
```
# Problem 4.10 Part A
errors_dict = dict()
var_err_dict = dict()
mean_image_dict = dict()
transformed_dict = dict()

for key in merged_data:
    # Gets the mean for the image class.
    item = np.array(merged_data[key], dtype=np.uint8)
    val = np.mean(item, axis=0, dtype=int)
    mean_image_dict[key] = val

    # Fits the data and applies dimensionality reduction on the data
    pca_data = item
    pca = PCA(n_components = 20)
    X_proj = pca.fit_transform(pca_data)
    cat_rest = pca.inverse_transform(X_proj)
    transformed_dict[key] = cat_rest

    # Compute the errors
    error = comp_error(item, cat_rest)
    var_err = 1 - sum(pca.explained_variance_ratio_)
    errors_dict[key] = error
    var_err_dict[key] = var_err
```

In [11]:
```
# Plot the values
plt.bar(range(len(var_err_dict)), var_err_dict.values())
plt.xticks(range(len(var_err_dict)), var_err_dict.keys())
plt.xlabel("Class")
plt.ylabel("Proportion of Variance not explained by the selected compone
nts.")
plt.title("Error for Classes")

plt.show()
```

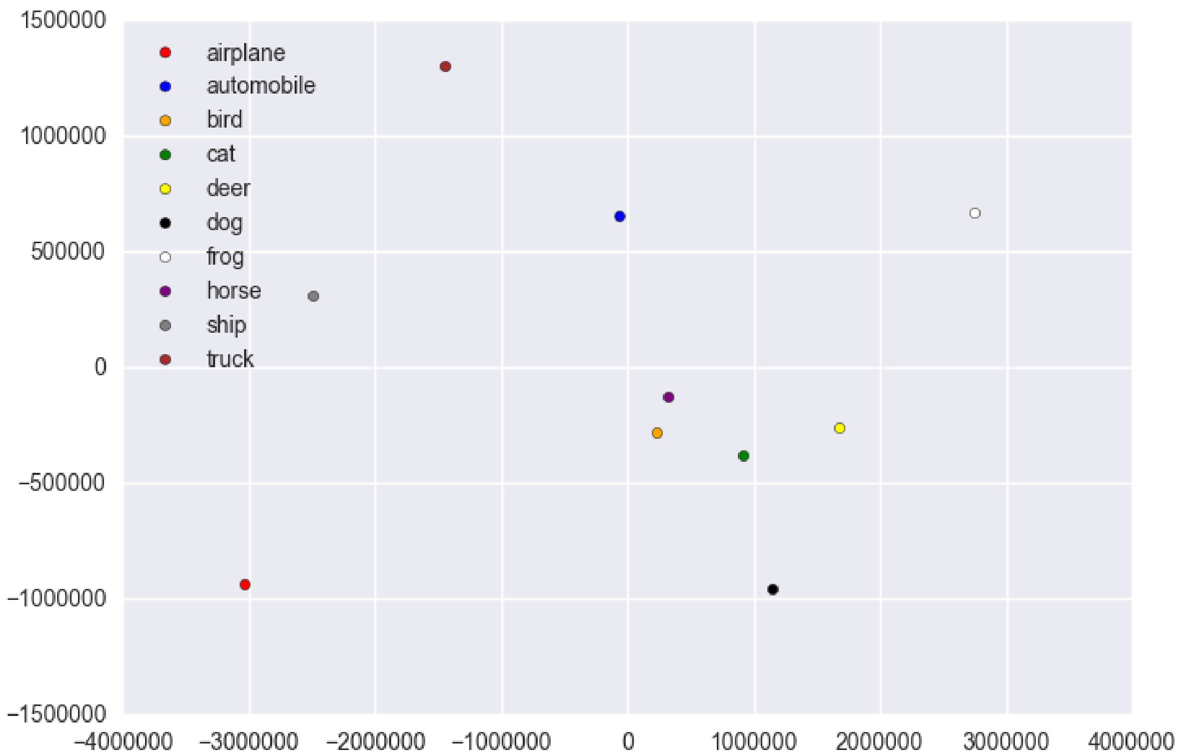
In [12]:
```
# Plot the values
plt.bar(range(len(errors_dict)), errors_dict.values())
plt.xticks(range(len(errors_dict)), errors_dict.keys())
plt.xlabel("Class")
plt.ylabel("Squared Error")
plt.title("Error for Classes")

plt.show()
```

Problem 4.10 Part B

In this problem, for each class A and B, we computed the distances between the mean images calculate in Part A to make a 2D map of the means of each categories using PCoA.

In [13]:
```
# Problem 4.10 Part B
keys = sorted(mean_image_dict.keys())

# Construct the distance matrix.
pair_distances = np.empty((10,10))
for i in range(10):
    for j in range(10):
        a = mean_image_dict[keys[i]]
        b = mean_image_dict[keys[j]]
        pair_distances[i][j] = sqeuclidean(a,b)

# Apply PCoA to the computed Distance Matrix and take the 2-D plots
coords = cmdscale(pair_distances)[0]

x_data = list()
y_data = list()

for i in coords:
    x_data.append(i[0])
    y_data.append(i[1])

fig, ax = plt.subplots()
labels = list()

col = ['Red', 'Blue', 'Orange', 'Green', 'Yellow', 'Black', 'White', 'Pu
rple', 'Grey', 'Brown']

for i in range(len(keys)):
    labels.append(ax.scatter(x_data[i], y_data[i], c= col[i],
label=keys[i]))

ax.legend(loc='upper left', handles = labels)
ax.plot()
```

Out[13]: []



Problem 4.10 Part C

Here, we define two helper functions. e_func(A,B) computes the average error from obtained by representing all the images of class A using the mean image of class A and the principal components of class B. We apply it to the similarity function:

$$(1/2)(E(A->B)+E(B->A))$$

The similarity matrix constructed is then passed in as a parameter to the PCoA function to construct another 2D map of the classes. The results are outlined below.

In [14]:
```
def e_func(A, B):
    # A is the mean of class A and B is the first 20 principal component
s of class B.

    A = mean_image_dict[A]
    B = transformed_dict[B]
    tot_sum = 0
    for row in B:
        tot_sum += sqeuclidean(A,row)

    return (tot_sum / len(A))
```

In [15]:
```
def similarity_A_B(a, b):
    e_a_b = e_func(a,b)
    e_b_a = e_func(b,a)
    return (1/2) * (e_a_b + e_b_a)
```

In [16]:

```
sim_matrix = np.empty([10,10])
for i in range(10):
    for j in range(10):
        a = keys[i]
        b = keys[j]

        sim_matrix[i][j] = similarity_A_B(a, b)
```

In [17]:

```
coords = cmdscale(sim_matrix)[0]

x_data = list()
y_data = list()

for i in coords:
    x_data.append(i[0])
    y_data.append(i[1])

fig, ax = plt.subplots()
labels = list()

col = ['Red', 'Blue', 'Orange', 'Green', 'Yellow', 'Black', 'White', 'Pu
rple', 'Grey', 'Brown']

for i in range(len(keys)):
    labels.append(ax.scatter(x_data[i], y_data[i], c= col[i],
label=keys[i], ))

ax.legend(loc='lower left', handles = labels)
ax.set_xlabel('X-Axis')
ax.set_ylabel('Y-Axis')
ax.plot()
```

Out[17]: []