

# ST 705 Linear models and variance components

## Lab practice problem set 8

March 17, 2021

1. Given an example using the one-way ANOVA model from Section 3.4 to show that if  $\lambda'\beta$  is not estimable, then  $\lambda'\hat{\beta}$  is not unbiased.
2. Consider the model  $Y_{ij} = \mu + \alpha_i + \beta_j x_{ij} + U_{ij}$ , for  $i \in \{1, \dots, n\}$  and  $j \in \{1, \dots, m\}$ . Further, assume that  $\sum_{j=1}^m (x_{ij} - \bar{x}_{i.})^2 > 0$  for all  $i \in \{1, \dots, n\}$ . Derive the necessary and sufficient conditions for an estimable function  $\lambda'\gamma$ , where  $\gamma := (\mu, \alpha_1, \dots, \alpha_n, \beta_1, \dots, \beta_m)'$ .
3. Maximize the function  $f(x) := \sum_{i=1}^k n_i \log(x_i)$  subject to the constraints

$$\begin{aligned}\sum_{i=1}^k x_i &= 1, \\ x_i &\geq 0,\end{aligned}$$

with  $\{n_i\}$  a set of fixed positive scalars.