ST 705 Linear models and variance components
Lab practice problem set 6

February 17, 2020

1. Let X be an $n \times p$ matrix with $\text{rank}(X) = r$, and let C be a $(p-r) \times p$ matrix. If

   (i) $\text{rank}(C) = p - r$ and

   (ii) $\text{column}(X') \cap \text{column}(C') = \{0\}$,

   then

   $$\text{rank} \begin{pmatrix} X \\ C \end{pmatrix} = p.$$

2. Denote by W a matrix with $\text{column}(W) = \text{null}(P')$, where P is a matrix with full column rank. Show that $\text{null}(W') = \text{column}(P)$.

3. Consider the restricted linear model $Y = X\beta + U$ over the constrained parameter space $\{P'\beta = \delta\}$, for some full-column rank matrix P. Set up the Langrangian function and derive the *restricted normal equations* (RNE),

   $$\begin{pmatrix} X'X & P \\ P' & 0 \end{pmatrix} \begin{pmatrix} \beta \\ \theta \end{pmatrix} = \begin{pmatrix} X'y \\ \delta \end{pmatrix}.$$

4. Prove that there exists a solution to the RNE.