ST 705 Linear models and variance components
Homework problem set 5

February 3, 2020

1. (2 points) Prove that a (square) matrix that is both orthogonal and upper triangular must be a diagonal matrix.

2. (2 points) Suppose that $v_1, \ldots, v_p \in \mathbb{R}^n$ are a set of linearly independent vectors, and $w_1, \ldots, w_p \in \mathbb{R}^n$ are the orthogonal vectors obtained from $v_1, \ldots, v_p$ by the Gram-Schmidt process. Furthermore, denote by $u_1, \ldots, u_p$ the normalized vectors corresponding to $w_1, \ldots, w_p$, and define the matrix $R \in \mathbb{R}^{p \times p}$ by

$$R_{ij} := \begin{cases} \|w_j\| & \text{if } i = j \\ \langle v_j, u_i \rangle & \text{if } i < j \\ 0 & \text{if } i > j \end{cases} .$$

   Prove that $V = UR$, where V is the matrix with columns $v_1, \ldots, v_p$ and U is the matrix with columns $u_1, \ldots, u_p$.

3. (2 points) In the notation of the previous problem, suppose that $p = n$. Further, assume that $V = U_1 R_1 = U_2 R_2$, where U_1 and U_2 are orthogonal matrices and R_1 and R_2 are upper triangular. Prove that the matrix $R_2 R_1^{-1}$ is orthogonal and diagonal.

4. (2 points) Exercise 2.22 from Monahan.

5. (2 points) Exercise 2.23 from Monahan.

6. (2 points) Exercise 2.24 from Monahan.

7. (2 points) Exercise 3.7 from Monahan.