ST 705 Linear models and variance components
Lab practice problem set 2

January 27, 2020

1. In lecture, we proved a lemma that $(X'X)^g X'$ is a generalized inverse of X.

   (a) Verify that $X(X'X)^g$ is a generalized inverse of X'.

   (b) We proved that $P_X := X(X'X)^g X'$ is the unique projection onto column(X). Is $(X'X)^g X'$ the unique generalized inverse of X?

2. Show that $I_n - P_X$ is the unique symmetric projection matrix onto null(X').

3. Let $x_i, y_i \in \mathbb{R}$ for $i \in \{1, \ldots, n\}$, and show that

$$\frac{1}{n} \sum_{i=1}^{n} \sum_{j<i}^{n} (x_i - x_j)(y_i - y_j) = \sum_{i=1}^{n} (x_i - \bar{x}_n) y_i = \sum_{i=1}^{n} (x_i - \bar{x}_n)(y_i - \bar{y}_n).$$

   Note the particular case when $x_i = y_i$ for every i.