

ST 705 PRACTICE MIDTERM

March 2, 2020

NAME:

STUDENT ID:

- You have **75 minutes** to complete this exam.
- This is a **closed book, closed notes** exam.

1. (3 points) Let $A \in \mathbb{R}^{n \times p}$ with $\text{rank}(A) = p$. Further, suppose $X \in \mathbb{R}^{n \times q}$ with $\text{column}(X) = \text{column}(A)$. Show that there exists a unique matrix S so that $X = AS$.
2. (3 points) If the least squares estimator $\lambda' \hat{\beta}$ is the same for all solutions $\hat{\beta}$ to the normal equations, then $\lambda' \beta$ is estimable.
3. Consider the least squares line $y = c \cdot t + d$ corresponding to the m observations $(t_1, y_1), \dots, (t_m, y_m)$.
 - (a) (3 points) Show that the normal equations take the form

$$\left\{ c \left(\sum t_i^2 \right) + d \left(\sum t_i \right) = \sum t_i y_i \right\} \cap \left\{ c \left(\sum t_i \right) + md = \sum y_i \right\}.$$

- (b) (3 points) Show that the least squares line must pass through the point (\bar{t}, \bar{y}) , where \bar{t} and \bar{y} are the averages of the t_i and y_i , respectively.
4. (3 points) Suppose that the $m \times n$ matrix A has the form

$$A = \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}$$

where A_1 is an $n \times n$ nonsingular matrix, and $m > n$. Define $A^+ := (A'A)^{-1}A'$, and prove that $\|A^+\|_2 \leq \|A_1^{-1}\|_2$.

5. (3 points) Suppose that there exists a solution to the system of equations $Ax = c$. Then the general form of a solution is

$$x_z = Gc + (I - GA)z,$$

where z is an arbitrary vector of appropriate dimension and $G := (A'A)^{-1}A'$ (do NOT need to show). Find the z that minimizes the Euclidean norm of x_z .