ST 705 Linear models and variance components
Homework problem set 6

February 24, 2021

1. Exercise 2.14 from Monahan.

2. Suppose that $v_1, \ldots, v_p \in \mathbb{R}^n$ are a set of linearly independent vectors, and $w_1, \ldots, w_p \in \mathbb{R}^n$ are the orthogonal vectors obtained from $v_1, \ldots, v_p$ by the Gram-Schmidt process. Furthermore, denote by $u_1, \ldots, u_p$ the normalized vectors corresponding to $w_1, \ldots, w_p$, and define the matrix $R \in \mathbb{R}^{p \times p}$ by

$$R_{ij} := \begin{cases} \|w_j\| & \text{if } i = j \\ \langle v_j, u_i \rangle & \text{if } i < j \\ 0 & \text{if } i > j \end{cases} .$$

   Prove that $V = UR$, where V is the matrix with columns $v_1, \ldots, v_p$ and U is the matrix with columns $u_1, \ldots, u_p$.

3. In the notation of the previous problem, suppose that $p = n$. Further, assume that $V = U_1 R_1 = U_2 R_2$, where U_1 and U_2 are orthogonal matrices and R_1 and R_2 are upper triangular. Prove that the matrix $R_2 R_1^{-1}$ is orthogonal and diagonal.

4. Exercise 2.22 from Monahan.

5. Exercise 2.23 from Monahan.

6. Exercise 2.24 from Monahan.