

ST 705 Linear models and variance components

Lab practice problem set 4

February 10, 2021

1. Let $X \in \mathbb{R}^{n \times p}$ and $u \in \text{col}(X)$. Show that

$$\{\beta : X\beta = u\} = \{\beta : \beta = X^g u + (I_p - X^g X)z \text{ for some } z \in \mathbb{R}^p\}.$$

2. Let $X = QR$ where Q has orthonormal columns. Prove that if $\text{rank}(X) = \text{rank}(Q)$, then $P_X = QQ'$.