ST 705 Linear models and variance components
Lab practice problem set 12

April 14, 2021

1. Let $X \sim \mathrm{N}_p(\mu, \Sigma)$. Show that for any partition of components, i.e.,

$$X = \begin{pmatrix} X_1 \\ \vdots \\ X_m \end{pmatrix}, \qquad \mu = \begin{pmatrix} \mu_1 \\ \vdots \\ \mu_m \end{pmatrix}, \qquad \Sigma = \begin{pmatrix} \Sigma_{11} & \cdots & \Sigma_{1m} \\ \vdots & \ddots & \vdots \\ \Sigma_{m1} & \cdots & \Sigma_{mm} \end{pmatrix},$$

   $X_1, \ldots, X_m$ are mutually independent if and only if $\Sigma_{ij} = 0$ for every $i \neq j$.

2. Show that a $p \times p$ matrix A is symmetric and idempotent with rank s if and only if there exists a $p \times s$ matrix G with orthonormal columns such that $A = GG'$. Note that G is called a *semi-orthogonal* matrix.