ST 705 Linear models and variance components
Homework problem set 4

February 10, 2021

1. Exercise 2.8 from Monahan.

2. Exercise 2.9 from Monahan.

3. Exercise 2.11 from Monahan.

4. Let A be an $m \times n$ matrix with rank m. Prove that there exists an $n \times m$ matrix B such that $AB = I_m$.

5. Let $A \in \mathbb{R}^{n \times p}$ with $\text{rank}(A) = p$. Further, suppose $X \in \mathbb{R}^{n \times q}$ with $\text{col}(X) = \text{col}(A)$. Show that there exists a unique matrix S so that $X = AS$.

6. Let A be an $m \times n$ matrix and B be an $n \times p$ matrix. Prove that AB can be written as a sum of n matrices of rank at most one. Hint: think about empirical covariance matrices.