

ST 705 Linear models and variance components

Homework problem set 3

January 22, 2020

1. (2 points) Let $G(\beta) = (y - X\beta)'W(y - X\beta)$. Derive $\partial G/\partial\beta$.
2. (2 points) Let $A \in \mathbb{R}^{n \times p}$
 - (a) Prove that if A^g is a generalized inverse of A (i.e., only satisfying $AA^gA = A$), then $(A^g)'$ is a generalized inverse of A' . Conclude from this fact that $P_X := X(X'X)^gX'$ is symmetric (this finishes the proof of a theorem about P_X from lecture).
 - (b) Prove the existence **and** uniqueness of the Moore-Penrose generalized inverse, usually denoted A^+ , of A .
 - (c) Show that if A has full column rank, then $A^+ = (A'A)^{-1}A'$.
 - (d) Show that if A has full row rank, then $A^+ = A'(AA')^{-1}$.
3. (2 points) Let S be a nonempty subset of an inner product space V . The orthogonal complement to the set S is defined as
$$S^\perp := \{x \in V : \langle x, y \rangle = 0 \text{ for every } y \in S\}.$$
 - (a) Show that S^\perp is a subspace of V for any $S \subseteq V$.
 - (b) Let $W \subseteq V$ be a finite dimensional subspace, and let $y \in V$. Show that there exist **unique** vectors $u \in W$ and $z \in W^\perp$ such that $y = u + z$.
 - (c) Let $X \in \mathbb{R}^{n \times p}$. Verify that $\text{column}(X)$ and $\text{null}(X')$ are orthogonal complements.
4. (2 points) Exercise 2.8 from Monahan.
5. (2 points) Exercise 2.9 from Monahan.
6. (2 points) Exercise 2.11 from Monahan.