ST 705 Linear models and variance components
Homework problem set 8

March 17, 2021

1. Exercise 3.9 from Monahan. Not necessary to do the "(More practice)" item.

2. Consider the model $Y_{ijk} = \mu + \alpha_i + \beta_j + \theta_{ij} + U_{ijk}$, with $k \in \{1, \ldots, m_{ij}\}$, $i \in \{1, \ldots, n\}$, $j \in \{1, \ldots, m\}$, and $E(U_{ijk}) = 0$. Find necessary and sufficient conditions for which $\lambda'\gamma$ is estimable for $\gamma = \mu, \alpha_1, \ldots, \alpha_n, \beta_1, \ldots, \beta_m, \theta_{11}, \ldots, \theta_{nm})'$.

3. Let X be an $n \times p$ matrix with $\text{rank}(X) = r$, and C be a $(p - r) \times p$ matrix with $\text{rank}(C) = p - r$, such that $\text{col}(X') \cap \text{col}(C') = \{0\}$. Show that

$$\text{rank} \begin{pmatrix} X \\ C \end{pmatrix} = p.$$

4. Let X be an $n \times p$ matrix with $\text{rank}(X) = r$, and C be a $(p - r) \times p$ matrix with $\text{rank}(C) = p - r$, such that $\text{col}(X') \cap \text{col}(C') = \{0\}$. Show that $C(X'X + C'C)^{-1}C' = I_{p-r}$.

5. Exercise 3.20 from Monahan.