ST 705 Linear models and variance components
Homework problem set 3

January 21, 2020

1. (2 points) Let $G(\beta) = (y - X\beta)'W(y - X\beta)$. Derive $\partial G/\partial\beta$.

2. (2 points) Let $A \in \mathbb{R}^{n\times p}$

   (a) Prove that if A^g is a generalized inverse of A (i.e., only satisfying $AA^gA = A$), then $(A^g)'$ is a generalized inverse of A'. Conclude from this fact that $P_X := X(X'X)^gX'$ is symmetric (this finishes the proof of a theorem about P_X from lecture).

   (b) Prove the existence **and** uniqueness of the Moore-Penrose generalized inverse, usually denoted A^+, of A.

   (c) Show that if A has full column rank, then $A^+ = (A'A)^{-1}A'$.

   (d) Show that if A has full row rank, then $A^+ = A(AA')^{-1}$.

3. (2 points) Let S be a nonempty subset of an inner product space V. The orthogonal complement to the set S is defined as

$$S^\perp := \{x \in V : \langle x, y\rangle = 0 \text{ for every } y \in S\}.$$

   (a) Show that $S^\perp$ is a subspace of V for any $S \subseteq V$.

   (b) Let $W \subseteq V$ be a finite dimensional subspace, and let $y \in V$. Show that there exist **unique** vectors $u \in W$ and $z \in W^\perp$ such that $y = u + z$.

   (c) Let $X \in \mathbb{R}^{n\times p}$. Verify that column$(X)$ and null(X') are orthogonal complements.

4. (2 points) Exercise 2.8 from Monahan.

5. (2 points) Exercise 2.9 from Monahan.

6. (2 points) Exercise 2.11 from Monahan.