ST 705 Linear models and variance components
Lab practice problem set 6

February 24, 2021

1. Prove that if a (symmetric) matrix is positive definite, then all of its eigenvalues are greater than zero.

2. Let A be an $n \times n$ matrix. Show that if A is positive-definite, then it must be symmetric, or construct a counter example if this statement is not true. Do not simply appeal to the Cholesky factorization.

3. What is the contrapositive of the statement given in the previous problem? Think about what this contrapositive statement means.

4. Construct an $n \times n$ matrix A such that $\lambda_{\max}(A) \neq \sup_{v \neq 0} \left\{ \frac{v'Av}{v'v} \right\}$, where $\lambda_{\max}(\cdot)$ denotes the maximum eigenvalue of its argument. Why does your counter example not violate the Courant-Fischer theorem?