ST 705 Linear models and variance components

Lab practice problem set 8

March 17, 2021

1. Give an example using the one-way ANOVA model from Section 3.4 to show that if $\lambda'\beta$ is not estimable, then $\lambda'\widehat{\beta}$ is not unbiased.

2. Consider the model $Y_{ij} = \mu + \alpha_i + \beta_i x_{ij} + U_{ij}$, for $i \in \{1,\dots,n\}$ and $j \in \{1\dots,m\}$. Further, assume that $\sum_{j=1}^{m}(x_{ij} - \bar{x}_{i\cdot})^2 > 0$ for all $i \in \{1,\dots,n\}$. Derive the necessary and sufficient conditions for an estimable function $\lambda'\gamma$, where $\gamma := (\mu, \alpha_1, \dots, \alpha_n, \beta_1, \dots, \beta_n)'$.

3. Maximize the function $f(x) := \sum_{i=1}^{k} n_i \log(x_i)$ subject to the constraints

$$\sum_{i=1}^{k} x_i = 1,$$
$$x_i \geq 0,$$

with $\{n_i\}$ a set of fixed positive scalars.