ST 705 Linear models and variance components
Lab practice problem set 2

January 27, 2021

1. Let $A \in \mathbb{R}^{p \times p}$ and $\alpha \in \mathbb{R}$. Is the matrix αA similar to A (i.e., do they share the same spectrum)?

2. Use Jensen's inequality to establish the arithmetic-geometric mean inequality. That is, show that if $a_1, \ldots, a_n$ are positive constants, then

$$\frac{1}{n} \sum_{i=1}^{n} a_i \geq \left(\prod_{i=1}^{n} a_i \right)^{\frac{1}{n}}.$$

3. Let $\{a_1, \ldots, a_n\}$ and $\{b_1, \ldots, b_n\}$ be sequences of real numbers. Show that

$$\min\{a_i\} + \min\{b_i\} \leq \min\{a_i + b_i\} \leq \min\{a_i\} + \max\{b_i\}.$$

4. Let V be a convex subset of some vector space. Recall that a function $f : V \to \mathbb{R}$ is said to be *convex* if for every $x, y \in V$ and every $\lambda \in [0, 1]$,

$$f(\lambda x + (1 - \lambda) y) \leq \lambda f(x) + (1 - \lambda) f(y).$$

   Show, by definition, that the sum of squared errors function

$$Q(\beta) := \|Y - X\beta\|_2^2$$

   is convex.