ST 705 Linear models and variance components
Homework problem set 4

January 30, 2020

1. (2 points) Let $X \in \mathbb{R}^{n \times p}$ and $u \in \text{column}(X)$. Show that

$$\{\beta : X\beta = u\} = \{\beta : \beta = X^g u + (I_p - X^g X)z \text{ for some } z \in \mathbb{R}^p\}.$$

2. (2 points) Let $X = QR$ where Q has orthonormal columns. Prove that if $\text{rank}(X) = \text{rank}(Q)$, then $P_X = QQ'$.

3. (2 points) Exercise 2.12 from Monahan.

4. (2 points) Exercise 2.13 from Monahan.

5. (2 points) Let A be an $m \times n$ matrix with rank m. Prove that there exists an $n \times m$ matrix B such that $AB = I_m$.

6. (2 points) Let $A \in \mathbb{R}^{n \times p}$ with $\text{rank}(A) = p$. Further, suppose $X \in \mathbb{R}^{n \times q}$ with $\text{column}(X) = \text{column}(A)$. Show that there exists a unique matrix S so that $X = AS$.

7. (2 points) Exercise 2.14 from Monahan.

8. (2 points) Let A be an $m \times n$ matrix and B be an $n \times p$ matrix. Prove that AB can be written as a sum of n matrices of rank at most one. Hint: think about empirical covariance matrices.