

ST 705 Linear models and variance components

Lab practice problem set 4

January 27, 2020

1. In lecture, we proved a lemma that $(X'X)^gX'$ is a generalized inverse of X .
 - (a) Verify that $X(X'X)^g$ is a generalized inverse of X' .
 - (b) We proved that $P_X := X(X'X)^gX'$ is the unique projection onto $\text{column}(X)$. Is $(X'X)^gX'$ the unique generalized inverse of X ?
2. Show that $I_n - P_X$ is the unique symmetric projection matrix onto $\text{null}(X')$.
3. Let $x_i, y_i \in \mathbb{R}$ for $i \in \{1, \dots, n\}$, and show that

$$\frac{1}{n} \sum_{i=1}^n \sum_{j < i} (x_i - x_j)(y_i - y_j) = \sum_{i=1}^n (x_i - \bar{x}_n)y_i = \sum_{i=1}^n (x_i - \bar{x}_n)(y_i - \bar{y}_n).$$

Note the particular case when $x_i = y_i$ for every i .