ST 705 Linear models and variance components
Lab practice problem set 3

February 3, 2021

1. If P is a symmetric and idempotent matrix, show that the following Pythagorean relationship holds.
$$\|y\|^2 = \|Py\|^2 + \|(I-P)y\|^2.$$

2. Let V be an inner product space over $\mathbb{C}$, and let $\{v_1, \ldots, v_n\} \subset V$ be orthonormal.

   (a) Prove that for any $x \in V$,
   $$\|x\|^2 \geq \sum_{i=1}^{n} |\langle x, v_i \rangle|^2.$$
   This is called Bessel's inequality.

   (b) Show that Bessel's inequality is an equality if and only if $x \in \text{span}\{v_1, \ldots, v_n\}$.