# ST 705 Linear models and variance components
# Lab practice problem set 11

April 7, 2021

1. Recall the definition of the multivariate normal distribution:

   **Definition 1** *The p-dimensional random vector $Y$ is said to follow the multivariate normal distribution with mean $\mu$ and covariance matrix $\Sigma$ if for every p-dimensional vector $v$ such that $v'\Sigma v \neq 0$,*

   $$v'Y \sim N(v'\mu, v'\Sigma v).$$

   *Denote $Y \sim N_p(\mu, \Sigma)$.* ■

   Prove that if $\Sigma$ is nonsingular, then $Y \sim \mathrm{N}_p(\mu, \Sigma)$ if and only if $Y$ has density,

   $$f(y) = \det(2\pi\Sigma)^{-\frac{1}{2}} e^{-\frac{1}{2}(y-\mu)'\Sigma^{-1}(y-\mu)}.$$