# ST 705 MIDTERM

March 2, 2020

**NAME:**

**STUDENT ID:**

- You have **75 minutes** to complete this exam.
- This is a **closed book, closed notes** exam.

1. (3 points) Let $A \in \mathbb{R}^{n \times p}$ with $\text{rank}(A) = p$. Further, suppose $X \in \mathbb{R}^{n \times q}$ with $\text{column}(X) = \text{column}(A)$. Show that there exists a unique matrix $S$ so that $X = AS$.

2. (3 points) If the least squares estimator $\lambda'\widehat{\beta}$ is the same for all solutions $\widehat{\beta}$ to the normal equations, then $\lambda'\beta$ is estimable.

3. Consider the least squares line $y = c \cdot t + d$ corresponding to the $m$ observations $(t_1, y_1), \ldots, (t_m, y_m)$.

   (a) (3 points) Show that the normal equations take the form
   $$\left\{c\Big(\sum t_i^2\Big) + d\Big(\sum t_i\Big) = \sum t_i y_i\right\} \bigcap \left\{c\Big(\sum t_i\Big) + md = \sum y_i\right\}.$$

   (b) (3 points) Show that the least squares line must pass through the point $(\bar{t}, \bar{y})$, where $\bar{t}$ and $\bar{y}$ are the averages of the $t_i$ and $y_i$, respectively.

4. (3 points) Suppose that the $m \times n$ matrix $A$ has the form
   $$A = \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}$$
   where $A_1$ is an $n \times n$ nonsingular matrix, and $m > n$. Define $A^+ := (A'A)^{-1}A'$, and prove that $\|A^+\|_2 \leq \|A_1^{-1}\|_2$.

5. (3 points) Suppose that there exists a solution to the system of equations $Ax = c$. Then the general form of a solution is
   $$x_z = Gc + (I - GA)z,$$
   where $z$ is an arbitrary vector of appropriate dimension and $G := (A'A)^g A'$ (do NOT need to show). Find the $z$ that minimizes the Euclidean norm of $x_z$.