

ST 705 Linear models and variance components

Lab practice problem set 6

February 10, 2020

1. Prove that if $\lambda^{(1)'}\beta, \dots, \lambda^{(k)'}\beta$ are estimable, then so is

$$\sum_{j=1}^k d_j \lambda^{(j)'} \beta,$$

for any scalar constants d_1, \dots, d_k .

2. Given an example using the one-way ANOVA model from Section 3.4 to show that if $\lambda'\beta$ is not estimable, then $\lambda'\widehat{\beta}$ is not unbiased.
3. Consider the model $Y_{ij} = \mu + \alpha_i + \beta_i x_{ij} + U_{ij}$, for $i \in \{1, \dots, n\}$ and $j \in \{1, \dots, m\}$. Further, assume that $\sum_{j=1}^m (x_{ij} - \bar{x}_{i.})^2 > 0$ for all $i \in \{1, \dots, n\}$. Derive the necessary and sufficient conditions for an estimable function $\lambda'\gamma$, where $\gamma := (\mu, \alpha_1, \dots, \alpha_n, \beta_1, \dots, \beta_n)'$.