ST 705 Linear models and variance components
Homework problem set 5

February 17, 2021

1. Show that $I_n - P_X$ is the unique symmetric projection matrix onto $\text{null}(X')$.

2. Let $X \in \mathbb{R}^{n\times p}$ with full column rank, $\Phi \in \mathbb{R}^{p\times m}$, and $U \in \mathbb{R}^{p\times p}$ be positive definite. Show that,

$$\|\Phi' X'(P_X - X(X'X+U)^{-1}X')^2 X\Phi\| \leq \|\Phi' X'(P_X - X(X'X+U)^{-1}X')X\Phi\|,$$

   where $P_X = X(X'X)^{-1}X'$.

3. Let V be a finite-dimensional inner product space over $\mathbb{C}$, and let $\{v_1, \ldots, v_n\}$ be an orthonormal basis for V.

   (a) Show that for any $x, y \in V$,

$$\langle x, y\rangle = \sum_{i=1}^{n} \langle x, v_i\rangle \overline{\langle y, v_i\rangle}.$$

   This is called Parseval's identity.

   (b) For $V = \mathbb{R}^2$ use Parseval's identity to prove the Pythagorean theorem. Generalize this result to $\mathbb{R}^n$.

4. Exercise A.34 from Monahan.

5. Exercise A.35 from Monahan.

6. Exercise 2.12 from Monahan.