ST 705 PRACTICE MIDTERM

February 24, 2020

NAME:

STUDENT ID:

- You have **75 minutes** to complete this exam.
- This is a **closed book, closed notes** exam.

1. (3 points) Let $X \in \mathbb{R}^{n \times p}$ and $u \in \text{column}(X)$. Show that

$$\{\beta : X\beta = u\} = \{\beta : \beta = X^g u + (I_p - X^g X)z \text{ for some } z \in \mathbb{R}^p\}.$$

2. (3 points) Let S be a nonempty subset of an inner product space V. The orthogonal complement to the set S is defined as

$$S^\perp := \{x \in V : \langle x, y \rangle = 0 \text{ for every } y \in S\}.$$

Let $W \subseteq V$ be a finite dimensional subspace, and let $y \in V$. Show that there exist **unique** vectors $u \in W$ and $z \in W^\perp$ such that $y = u + z$.

3. Assume that $Y = X\beta + U$, where X is an $n \times p$ matrix with $\text{rank}(X) = k < p$, and assume $\lambda'\beta$ is estimable.

   (a) (3 points) Construct an argument to determine the rank of the matrix $\begin{pmatrix} X \\ \lambda' \end{pmatrix}$.

   (b) (3 points) Construct an argument to determine the rank of the matrix $\begin{pmatrix} X \\ \lambda'(I - P_{X'}) \end{pmatrix}$.

4. (3 points) In the simple linear regression model $y_i = \beta_0 + x_i\beta_1 + u_i$ for $i \in \{1, \ldots, n\}$, show that β_0 is estimable **by finding** a vector a and scalar c such that $E(c + a'y) = \beta_0$.

5. (3 points) Let $Q = X(X'V^{-1}X)^g X'V^{-1}$, with $V > 0$ and symmetric, and show that Q is a projection onto $\text{column}(X)$.