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Chapter · January 2020

DOI: 10.1007/978-3-030-23756-1_110

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Risk Analysis of the Autonomous Vehicle Driving Systems by Using Pythagorean Fuzzy AHP

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Abstract. Autonomous driving system (ADS) is a combination of different components those can be composed as operations of the automobile and decision making mechanisms both in regular time and unexpected situations. These operations are performed by a virtual driver to carry out the objectives that are determined by the users. The main advantages of ADS are to free the drivers from attention states which are presented by American Automobile Association Foundation (AAAF). At the same time, there are some safety risks for these systems those need to be tested and solved. In this paper, we analyzed ADS to prioritize risks by using Pythagorean fuzzy sets (PFSs) that can be used to represent uncertainty in decision process. For this aim, a methodology based on Pythagorean analytic hierarchy process (PAHP) has been suggested. By the way, the Hazard Analysis and Risks Assessment introduced in the ISO 26262 standards are taken as a basis for risk evaluation.

Keywords: Autonomous vehicle, Driving systems, Risk Assessment, Decision making, Pythagorean fuzzy sets, AHP.

1 Introduction

Automatization, which mainly started in the industry, has become widespread in people's daily lives and has been a vital requirement of the people due to its advantages. Home appliances, smart homes, telephones, smart parking stations, smart bus stations, and computers, almost everything we use continuously develops and affects people's lives by minimizing the power and the time spent in the past for the same activity. Another way of these developments which can be included this kind of applications is automatic driving systems (ADSs). As well as the benefits that ADSs provide to the people, main question about it is does the machine implement all the required actions when faced with an undefined situation in its system. Turkey's defence industry has made many investments to the defense technologies and equipments in order to produce independent technologies and to be prepared for the regional risks. The Under Secretariat for Defence Industries efforts to promote its investments based on three

main objectives: to stimulate the economy, maintain the Turkish army, and self-sufficiency. One of its investments, automatic driving systems, is to develop ADSs for the challenging tasks such as transportation operations, intervention to terrorist attacks. For these kind of actions, autonomous vehicles' safety become one of the most important issues due to the comparison with the daily usages when the possible results are evaluated. And risk evaluation of ADSs is completely critical action. For this aim, in this study, in order to assess the risks of ADSs for vehicles an AHP methodology based Pythagorean fuzzy sets is introduced. Pythagorean fuzzy AHP method is employed to evaluate risk assessment of ADSs more effectively by providing more freedom to experts in expressing their opinions about the vagueness and impreciseness of the risk evaluation process. The possible risks are gathered by ISO 26262 standards, literature review and experts' opinions [1-5]. The rest of this paper has been organized as follows: In Section 2, main characteristics and possible risks of autonomous vehicles driving systems are presented. In Section 3, the Pythagorean fuzzy sets (PFSs) and their fundamentals are introduced. In Section 4, the proposed methodology for the risk assessment process has been presented. In Section 5, application is carried out and the obtained results are analyzed. The last section includes obtained results and further research suggestions.

2 Autonomous Vehicle Driving Systems

Autonomous vehicle systems, also known as driverless cars, are the generic name given to vehicles with the ability to act autonomously without the need for human intervention with the various sensors it has to detect its surroundings [6]. Autonomous vehicles contain automatic control systems, without the need for a driver, the road, traffic flow and the perimeter of the driver without the intervention of the driver can detect the environment. Autonomous vehicles can detect objects around them using technologies such as radar, lidar, GPS, odometry and computer vision. Driverless cars use powerful computers and a range of sensors to create a digital image of the world around them and to draw a safe path through it. In addition to this route, they are also capable of responding to unexpected dangers or uncertain road signs. The sensors used are conventional video cameras, radar, laser, or integrated sensors. In autonomous vehicles, the cruise control of the vehicle is a type of automatic pilot. This control includes many functions including keeping the vehicle in the lane and performing turn maneuvering according to the location to be followed, following the vehicle in front, adjusting the distance to follow the traffic rules. The autonomous driver has the ability to have at least the human driver to have as much vision as he, in other words, to be able to perceive the events around the traffic as human drivers at least.

3 Pythagorean Fuzzy Sets

In this section, we introduce the basic operations and characteristics of Pythagorean and interval-valued Pythagorean fuzzy sets (PFSs).

3.1 Pythagorean fuzzy sets (PFSs)

PFSs were introduced by Yager [7; 9]. In PFSs, the sum of membership degree and non-membership degree assigned by experts may be greater than 1 but sum of their squares is less than or equal to 1 in some practical problems [8]. Mathematical representation of the PFS is given in Definition 1 as follows:

Definition 1 [9]. Let X be a fixed set. A PFS \tilde{L} is an object having the form:

$$\tilde{L} \cong \{(x, \mu_{\tilde{L}}(x), \vartheta_{\tilde{L}}(x)); x \in X\} \quad (1)$$

where the function $\mu_{\tilde{L}}(x): X \rightarrow [0,1]$ and $\vartheta_{\tilde{L}}(x): X \rightarrow [0,1]$ defines the degree of membership and non-membership of the element $x \in X$ to \tilde{L} , respectively. For every $x \in X$, it holds that:

$$0 \leq \mu_{\tilde{L}}(x)^2 + \vartheta_{\tilde{L}}(x)^2 \leq 1 \quad (2)$$

Also, $\pi_{\tilde{L}}(x) = \sqrt{1 - \mu_{\tilde{L}}(x)^2 - \vartheta_{\tilde{L}}(x)^2}$ is the hesitation degree of element \tilde{L} in set X . In a similar way, we obtain $0 \leq \pi_{\tilde{L}}(x)^2 \leq 1$ by using Eq. (2).

Definition 2 [10]. Let $\tilde{A} \cong \langle \mu_{\tilde{A}}, \vartheta_{\tilde{A}} \rangle$ and $\tilde{B} \cong \langle \mu_{\tilde{B}}, \vartheta_{\tilde{B}} \rangle$ be PFNs, and $\lambda > 0$. The arithmetical operations of these Pythagorean fuzzy numbers (PFNs) are defined as follows:

$$\tilde{A} \oplus \tilde{B} \cong \langle \sqrt{\mu_{\tilde{A}}^2 + \mu_{\tilde{B}}^2 - \mu_{\tilde{A}}^2 \mu_{\tilde{B}}^2}, \vartheta_{\tilde{A}} \vartheta_{\tilde{B}} \rangle \quad (3)$$

$$\tilde{A} \otimes \tilde{B} \cong \langle \mu_{\tilde{A}} \mu_{\tilde{B}}, \sqrt{\vartheta_{\tilde{A}}^2 + \vartheta_{\tilde{B}}^2 - \vartheta_{\tilde{A}}^2 \vartheta_{\tilde{B}}^2} \rangle \quad (4)$$

$$\lambda \tilde{A} = \left(\sqrt{1 - (1 - \mu^2)^\lambda}, \vartheta^\lambda \right) \quad (5)$$

$$\tilde{A}^\lambda = \left(\mu^\lambda, \sqrt{1 - (1 - \vartheta^2)^\lambda} \right) \quad (6)$$

3.2 Interval-Valued PFSs

Interval-valued PFSs (IV-PFSs) are introduced by Zhang [11]. Mathematical representation of the IV-PFSs is given as follows:

Definition 3 [12]. An IV-PFS L is denoted over X is given as:

$$\tilde{L} \cong \{(x, \mu_{\tilde{L}}(x), \vartheta_{\tilde{L}}(x)); x \in X\} \quad (7)$$

where $\mu_{\tilde{L}}(x) \subseteq [0,1]$ and $\vartheta_{\tilde{L}}(x) \subseteq [0,1]$ are interval numbers such that $0 \leq \sup \mu_{\tilde{L}}(x) + \sup \vartheta_{\tilde{L}}(x) \leq 1$ for all $x \in X$.

For convenience, let $\mu_{\tilde{L}}(x) = [a, b]$ and $\vartheta_{\tilde{L}}(x) = [c, d]$ then this pair is often denoted by $\tilde{L} = \langle [a, b], [c, d] \rangle$ and called an IV PFN where

$$[a, b] \subseteq [0, 1], [c, d] \subseteq [0, 1], \text{ and } 0 \leq b^2 + d^2 \leq 1 \quad (8)$$

Similar to PFSs, the hesitancy degree of this IV PFN is given as:

$$\tilde{\pi}_L = [\sqrt{1 - b^2 - d^2}, \sqrt{1 - a^2 - c^2}] \quad (9)$$

Definition 4. Let $\tilde{A} \cong \langle [\mu_{\tilde{A}_L}, \mu_{\tilde{A}_U}], [\vartheta_{\tilde{A}_L}, \vartheta_{\tilde{A}_U}] \rangle$ and $\tilde{B} \cong \langle [\mu_{\tilde{B}_L}, \mu_{\tilde{B}_U}], [\vartheta_{\tilde{B}_L}, \vartheta_{\tilde{B}_U}] \rangle$ be IV-PFNSs, and $\lambda > 0$. The arithmetical operations of these IV PFNs are defined as follows:

$$\tilde{A} \oplus \tilde{B} \cong \langle [\sqrt{\mu_{\tilde{A}_L}^2 + \mu_{\tilde{B}_L}^2 - \mu_{\tilde{A}_L}^2 \mu_{\tilde{B}_L}^2}, \sqrt{\mu_{\tilde{A}_U}^2 + \mu_{\tilde{B}_U}^2 - \mu_{\tilde{A}_U}^2 \mu_{\tilde{B}_U}^2}], [\vartheta_{\tilde{A}_L} \vartheta_{\tilde{B}_L}, \vartheta_{\tilde{A}_U} \vartheta_{\tilde{B}_U}] \rangle \quad (10)$$

$$\tilde{A} \otimes \tilde{B} \cong \langle [\mu_{\tilde{A}_L} \mu_{\tilde{B}_L}, \mu_{\tilde{A}_U} \mu_{\tilde{B}_U}], [\sqrt{\vartheta_{\tilde{A}_L}^2 + \vartheta_{\tilde{B}_L}^2 - \vartheta_{\tilde{A}_L}^2 \vartheta_{\tilde{B}_L}^2}, \sqrt{\vartheta_{\tilde{A}_U}^2 + \vartheta_{\tilde{B}_U}^2 - \vartheta_{\tilde{A}_U}^2 \vartheta_{\tilde{B}_U}^2}] \rangle \quad (11)$$

$$\lambda \tilde{A} = \langle [\sqrt{1 - (1 - \mu_{\tilde{A}_L})^\lambda}, \sqrt{1 - (1 - \mu_{\tilde{A}_U})^\lambda}], [\vartheta_L^\lambda, \vartheta_U^\lambda] \rangle \quad (12)$$

$$\tilde{A}^\lambda = \langle [\mu_{\tilde{A}_L}^\lambda, \mu_{\tilde{A}_U}^\lambda], [\sqrt{1 - (1 - \vartheta_{\tilde{A}_L})^\lambda}, \sqrt{1 - (1 - \vartheta_{\tilde{A}_U})^\lambda}] \rangle \quad (13)$$

Definition 5 [13]. Let $\tilde{A}_i = \langle [\mu_{\tilde{A}_i L}, \mu_{\tilde{A}_i U}], [\vartheta_{\tilde{A}_i L}, \vartheta_{\tilde{A}_i U}] \rangle$, $i = (1, 2, \dots, n)$ be a collection of IV-PFNs and $w_i = (w_1, w_2, \dots, w_n)^T$ be the weight vector of \tilde{A}_i where $\sum_{i=1}^n w_i = 1$. Then, Pythagorean fuzzy ordered weighted averaging (PFWA) operator of dimension n is a mapping $PFWA: L^n \rightarrow L$ that:

$$\begin{aligned} PFWA(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n) &= w_1 \cdot \tilde{A}_1 \otimes w_2 \cdot \tilde{A}_2 \otimes \dots \otimes w_n \cdot \tilde{A}_n \\ &= \langle [\sqrt{1 - \prod_{i=1}^n (1 - \mu_{\tilde{A}_i L})^{w_i}}, \sqrt{1 - \prod_{i=1}^n (1 - \mu_{\tilde{A}_i U})^{w_i}}], [\prod_{i=1}^n \vartheta_{\tilde{A}_i L}^{w_i}, \prod_{i=1}^n \vartheta_{\tilde{A}_i U}^{w_i}] \rangle \end{aligned} \quad (14)$$

Definition 6. Let $\tilde{A} \cong \langle [\mu_{\tilde{A}_L}, \mu_{\tilde{A}_U}], [\vartheta_{\tilde{A}_L}, \vartheta_{\tilde{A}_U}] \rangle$ be an interval-valued PFN and π_L, π_U are the hesitancy degree of the lower and upper points of \tilde{A} , then the defuzzified value of this IV PFN is obtained by Eq. (15):

$$\mathfrak{S}(\tilde{A}) = \frac{\mu_{\tilde{A}_L}^2 + \mu_{\tilde{A}_U}^2 + (1 - \pi_L^4 - \vartheta_{\tilde{A}_L}^2) + (1 - \pi_U^4 - \vartheta_{\tilde{A}_U}^2) + \mu_{\tilde{A}_L} \mu_{\tilde{A}_U} + \sqrt{(1 - \pi_L^4 - \vartheta_{\tilde{A}_L}^2)(1 - \pi_U^4 - \vartheta_{\tilde{A}_U}^2)}}{6} \quad (15)$$

4 The Proposed Fuzzy Based Methodology for Risk Analysis of Autonomous Vehicle Driving Systems

In this section, the proposed methodology based on PFSs used in analyzing the risks of autonomous vehicle driving systems is presented step by step [14].

Step 1. Construct the compromised pairwise comparison matrix $R = (r_{ij})_{m \times m}$ with respect to decision makers' judgements. Before applying the next steps, we calculated the consistency ratios of the pairwise comparison matrices based on the Saaty's method (Saaty, 2008). To illustrate Saaty's consistency procedure, we matched the linguistic terms with Saaty's AHP scale as follows: "1=Average Importance—AI; 3=Above Average Importance—AAI; 5=High Importance—HI; 7=Very High Importance—VHI; 9=Certainly High Importance—CHI." For the reciprocal terms, we took reverse of the numbers. For example, if the linguistic term is Below Average Importance, the corresponding value is equal to 0.33. The used scale for the linguistic terms is presented in Table 1:

Table 1. Linguistic Scale for the IVPF-AHP method

Linguistic Terms	IV PFNs
Certainly Low Importance-SLS	$\langle [0, 0.15], [0.8, 0.95] \rangle$
Very Low Importance-VLS	$\langle [0.1, 0.25], [0.7, 0.85] \rangle$
Low Importance-LS	$\langle [0.2, 0.35], [0.6, 0.75] \rangle$
Below Average Importance-BSS	$\langle [0.3, 0.45], [0.5, 0.65] \rangle$
Average Importance-SSS	$\langle [0.4, 0.55], [0.4, 0.55] \rangle$
Above Average Importance-ASS	$\langle [0.5, 0.65], [0.3, 0.45] \rangle$
High Importance-HS	$\langle [0.6, 0.75], [0.2, 0.35] \rangle$
Very High Importance-VHS	$\langle [0.7, 0.85], [0.1, 0.25] \rangle$
Certainly High Importance-SHS	$\langle [0.8, 0.95], [0, 0.15] \rangle$

Step 2. Calculate the differences matrix $D = (d_{ij})_{m \times m}$ between lower and upper points of the membership and non-membership functions by using Eq. (16) and Eq. (17):

$$d_{ijL} = \mu_{ijL}^2 - \vartheta_{ijU}^2 \quad (16)$$

$$d_{ijU} = \mu_{ijU}^2 - \vartheta_{ijL}^2 \quad (17)$$

Step 3. Calculate the interval multiplicative matrix $S = (s_{ij})_{m \times m}$ by using Eq. (18) and Eq. (19):

$$s_{ijL} = \sqrt{1000^{d_{ijL}}} \quad (18)$$

$$s_{ijU} = \sqrt{1000^{d_{ijU}}} \quad (19)$$

Step 4. Obtain the indeterminacy value (h_{ij}) by using Eq. (20):

$$h_{ij} = 1 - (\mu_{ijU}^2 - \mu_{ijL}^2) - (\vartheta_{ijU}^2 - \vartheta_{ijL}^2) \quad (20)$$

Step 5. Multiply the indeterminacy degrees with $S = (s_{ij})_{m \times m}$ matrix for finding the matrix of unnormalized weights $T = (t_{ij})_{m \times m}$ using Eq. (21):

$$t_{ij} = \left(\frac{s_{ijL} + s_{ijU}}{2} \right) h_{ij} \quad (21)$$

Step 6. Find the priority weights (w_i) by using Eq. (22):

$$w_i = \frac{\sum_{j=1}^m w_{ij}}{\sum_{i=1}^m \sum_{j=1}^m w_{ij}} \quad (22)$$

5 Application

A consensus consists of academicians agreed to develop a roadmap for ADSs vehicles' safety prioritization for the military operations. Before constructing it, several existing ADS systems are analyzed and a review on reliability and safety conditions is conducted from the several data such as ISO 26262-1:2018 and related literature to create a framework. After the investigations, several risk factors both for prioritizing them and for measuring their risk magnitudes have been determined. As a result of it, the consensus aims to introduce a roadmap to select the riskiest factor and for further studies take appropriate precautions to reduce its risk. The determined risk factors are given as shown in Table 2:

Table 2. Determined risk groups and related main and sub criteria

Hardware Requirements – C1	Hardware Integration – C2
C11 - Hardware safety requirements	C21 - Functional testing under normal conditions
C12 - Designing hardware for safety concerns	C22 - Worst case testing
C13 - Safety lifecycle steps for hardware	C23 - Mechanical endurance test
C14 - Assessment of architectural constraints	C24 - Accelerated life test
C15 – Incorrect specifications of hardware	C25 - Over limit testing
Supporting Processes – C3	Others – C4
C31 - Update management	C41 - Correct implementation of the functionality
C32 - Qualification of hardware components	C42 - Robustness
C33 - Qualification of software components	C43 - Sufficiency of the resources to support the functionality
C34 - Qualification of software tools	C44 – Human errors
C35 - Configuration management	C45 – Loss of energy supply or disturbances

The consensus decided to compare the sub-criteria by using Pythagorean AHP methodology and constructed the pairwise comparison matrices as shown in Table 3:

Table 3. Pairwise matrices with respect to main criteria

C1	C11	C12	C13	C14	C15	C2	C21	C22	C23	C24	C25
C11	SSS	HS	VHS	ASS	BSS	C21	SSS	VHS	SHS	HS	ASS
C12	LS	SSS	ASS	BSS	VLS	C22	VLS	SSS	ASS	BSS	LS
C13	VLS	BSS	SSS	LS	SLS	C23	SLS	BSS	SSS	LS	VLS
C14	BSS	ASS	HS	SSS	LS	C24	LS	ASS	HS	SSS	BSS
C15	ASS	VHS	SHS	HS	SSS	C25	BSS	HS	VHS	ASS	SSS
Consistency Ratio= 0.05						Consistency Ratio= 0.05					
C3	C31	C32	C33	C34	C35	C4	C41	C42	C43	C44	C45
C31	SSS	ASS	VLS	BSS	LS	C41	SSS	ASS	HS	BSS	LS
C32	BSS	SSS	SLS	LS	VLS	C42	BSS	SSS	ASS	LS	VLS
C33	VHS	SHS	SSS	HS	ASS	C43	LS	BSS	SSS	VLS	SLS
C34	ASS	HS	LS	SSS	BSS	C44	ASS	HS	VHS	SSS	BSS
C35	HS	VHS	BSS	ASS	SSS	C45	HS	VHS	SHS	ASS	SSS
Consistency Ratio= 0.054						Consistency Ratio= 0.054					

By applying the steps of the Pythagorean AHP as detailed above, we determined the riskiest sub-criteria for each main criterion as shown in Table 4:

Table 4. Results of the application

The Risk Weight of Criteria							
C11	0.26	C21	0.5	C31	0.07	C41	0.13
C12	0.07	C22	0.07	C32	0.04	C42	0.07
C13	0.04	C23	0.04	C33	0.5	C43	0.04
C14	0.13	C24	0.13	C34	0.13	C44	0.26
C15	0.5	C25	0.26	C35	0.26	C45	0.5

According to the obtained results as shown in Table 4, “C15 – *Incorrect specifications of hardware*”; “C21 - *Functional testing under normal conditions*”; “C33 - *Qualification of software components*”; and “C45 – *Loss of energy supply or disturbances*” are determined as the riskiest sub-criteria for each main criterion, respectively.

6 Conclusions

Autonomous vehicles are the vehicles that can detect the environment of the driver without the need for drivers with the automatic control systems, without the need for driver, traffic flow and driver intervention. There are many benefits provided by these vehicles carried out with ADSs. In addition to the benefits provided ADSs, the basic question is whether the vehicles perform all necessary actions when it encounters an undefined situation in the system. Turkey's defense industry invests the automatic driving system in transportation operations, for coping the challenges such as transportation operations, intervention to terrorist attacks. Therefore, safety in autonomous vehicles is also a strategic issue and should be carefully considered. In this paper, Pythagorean fuzzy AHP methodology is applied to evaluate the risks of ADSs for vehicles. After determining the possible risk groups with the literature review and the opinions of the experts, the proposed fuzzy based methodology is used for obtaining

of the rank of risks. As a result of calculations, the risk groups such as “C15 – *Incorrect specifications of hardware*”; “C21 – *Functional testing under normal conditions*”; “C33 – *Qualification of software components*”; and “C45 – *Loss of energy supply or disturbances*” are determined as the riskiest sub-criteria for each main criterion, respectively. For further research, the data can be extended by using R&D manager’s judgements and opinions. Also, an integrated decision making process consists of fuzzy MCDM method and fuzzy inference system can be used and the obtained results can be compared. Additionally, a sensitivity analysis can be also conducted to check the robustness of the decision making process.

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