Homework #1: Impulse Response Measurement
Due Date: April 12, 2011

Problem 1. [**20 Points**]

1(a). [**5 points**] Show that the sequences $A(t) = [1, 1]$ and $B(t) = [1, -1]$ form a Golay pair, that is, show that the sum of their autocorrelations is a scaled impulse.

1(b). [**15 points**] Show that if $A(t)$ and $B(t)$ are Golay, then so are the concatenated sequences $[A(t), B(t)]$ and $[A(t), -B(t)]$.

Problem 2. [**40 points**]

The Matlab function `RESPONSE = hmeasure(SIGNAL)` returns the noisy response of an unknown system $h(t)$ to an input signal, clipped to have a maximum absolute level of 1.0,

$$r(t) = s(t) * h(t) + n(t). \tag{1}$$

2(a). [**10 points**] Using the recursion above, write a Matlab script to generate Golay codes of length 2^n, starting with $A(t) = 1$, $B(t) = 1$.

2(b). [**20 points**] Write a Matlab script to measure the system $h(t)$ using Golay codes of length 2^n, for n in the range $n = [0, 1, \ldots, 10]$. Estimate the impulse response using length 1, length 32 and length 1024 Golay codes. Turn in plots of your impulse response estimates.

2(c). [**10 points**] Using the fact that the impulse response $h(t)$ begins with a series of zeros, estimate the signal-to-noise ratio improvement using length 32 and length 1024 Golay codes relative to using length 1 Golay codes.

Problem 3. [**40 Points**]

Consider the cascade of n identical first-order allpass filters

$$G_n(z) = \left(\frac{\rho + z^{-1}}{1 + \rho z^{-1}} \right)^n. \tag{2}$$

3(a). [**10 points**] Write a Matlab script to generate $g_n(t)$, the impulse response of the allpass cascade $G_n(z)$. Turn in a plot of the impulse response for $\rho = 0.5$ and $n = 64$.

3(b). **[10 points]** Find a value of ρ which approximately minimizes the maximum absolute impulse response tap level for $n = 64$,

$$\rho^* = \text{Argmin}_\rho\{\max_t |g_{64}(t)|\}. \tag{3}$$

You can do this numerically.

3(c). **[15 points]** Write a script which uses $g_{64,\rho^*}(t)$ to measure the system $h(t)$ from Problem 1 above. Remember that the maximum absolute input level can be at most one.

3(d). **[5 points]** Compute the measurement signal-to-noise ratio improvement compared with that of using a unit pulse to measure the system. How does the measurement signal-to-noise ratio improvement compare to that of the Golay codes?