Music 320
Autumn 2010–2011
Homework #6
DFT, Convolution, Correlation, Spectrogram
140 points
Due in one week (11/05/2009)

Theory Problems

1. (20 pts) [Convolution] For $x = [1, 2, 3, 2]$ and $h = [3, -1, 2, 1]$, find $(x * h)_n$ and $(x \star h)_n$. Note that they are both cyclic, not linear. Hint: knowing x is even can help you.

   Solution: $x = [1, 2, 3, 2]$, $h = [3, -1, 2, 1]$.
   $(x * h)_n = \sum_{m=0}^{3} x(m)h(n - m)$

$$
\begin{aligned}
(x * h)_0 &= \sum_{m=0}^{3} x(m)h(0 - m) = [1, 2, 3, 2] \cdot [3, 1, 2, -1] = 9 \\
(x * h)_1 &= \sum_{m=0}^{3} x(m)h(1 - m) = [1, 2, 3, 2] \cdot [-1, 3, 1, 2] = 12 \\
(x * h)_2 &= \sum_{m=0}^{3} x(m)h(2 - m) = [1, 2, 3, 2] \cdot [2, -1, 3, 1] = 11 \\
(x * h)_3 &= \sum_{m=0}^{3} x(m)h(3 - m) = [1, 2, 3, 2] \cdot [1, 2, -1, 3] = 8
\end{aligned}
$$

$(x \star y)_n = \sum_{m=0}^{3} \overline{x(m)}h(m + n)$

$$
\begin{aligned}
(x \star h)_0 &= \sum_{m=0}^{3} \overline{x(m)}h(m + 0) = [1, 2, 3, 2] \cdot [3, -1, 2, 1] = 9 \\
(x \star h)_1 &= \sum_{m=0}^{3} \overline{x(m)}h(m + 1) = [1, 2, 3, 2] \cdot [-1, 2, 1, 3] = 12 \\
(x \star h)_2 &= \sum_{m=0}^{3} \overline{x(m)}h(m + 2) = [1, 2, 3, 2] \cdot [2, 1, 3, -1] = 11 \\
(x \star h)_3 &= \sum_{m=0}^{3} \overline{x(m)}h(m + 3) = [1, 2, 3, 2] \cdot [1, 3, -1, 2] = 8
\end{aligned}
$$

2. (20 pts) You are given a signal $y(n) = [1, 0, -0.75, 0, 0.5, 0, -0.25, 0]$, which corresponds to the autocorrelation of an 8-sample long signal $x(n)$, zero-padded with a zfp of 2 (note that you are given only one half of the autocorrelation).

   (a) Assuming that $x(n) = \alpha \cos(\omega_0 n)$, find α and ω_0

   Solution:

   First, note the linearly decaying envelope in the correlation. This is explained because we are autocorrelating a time limited signal. Therefore, omitting the envelope (or better yet, correcting by dividing by the number of overlapping samples), we see that there's a correlation of -1 at a lag of 2 samples and a correlation of 1 at a lag of 4 samples. Therefore, there are exactly 4 samples per period $\Rightarrow \omega_0 = \frac{\pi}{2}$. To compute α, first compute the autocorrelation of $\cos(\frac{\pi}{2}n)$ and then find the scaling factor required ($\alpha = \sqrt{2}$)

   (b) Can you find another signal (other than $-x(n)$ or $x(-n)$) with the same autocorrelation? If you answer yes, give an example. If you answer no, explain why.

   Solution: Some possibilities are $x(n) = \sin(\omega_0 n), x(n) = \cos(\omega_0 n + \phi), x(n) = j\cos(\omega_0 n)$, etc. In general, any sinusoid (real, imaginary or complex) of the same frequency and any phase.

   (c) Without explicitly computing it, can you find the autocorrelation of $x_{1/2}(n) = \cos(\frac{\omega_0}{2}n)$? And $x_2(n) = \cos(2\omega_0 n)$? Explain your reasoning.

   Solution:

   For the case $x_2(n) = \cos 2\omega_0 n$ is easy. You'll get $y(n) = 2[1, -\frac{7}{8}, \frac{6}{8}, -\frac{5}{8}, \frac{4}{8}, -\frac{3}{8}, \frac{2}{8}, -\frac{1}{8}]$ For the case $_{1/2}(n) = \cos(\frac{\omega_0}{2}n)$ is not simple to find the values without explicitly computing them. But is definetivly easy to see that since the frequency is halved, the period is double hence, the negative "spike" will move from a lag of 2 samples to lag of 4 samples. Similarly, the positive "spike" in the original autocorrelation will move from a lag of 4 samples to a lag of 8 samples.

   (d) Compute the circular autocorrelation of $x(n)$ (without zero-padding).

   Solution:

   Given that

   $$\hat{r}_x(l) = (x \star x)_l = \frac{1}{N}\sum_{n=0}^{N-1} \overline{x(n)}x(n+l)$$

   is easy to verify that $\hat{r}_x(l) = [0.5, 0, -0.5, 0, 0.5, 0, -0.5, 0]$

Lab Assignments

Follow the same file naming convention of the previous lab.

1. (30 pts) [Windows] Matlab has functions for creating windows, including the followings:

   - `boxcar`
   - `bartlett`
   - `hann (hanning)`

   (a) Write a script to plot these windows (of length 128) both in the time domain and their magnitude spectra using your `plotspec` function. For magnitude spectra, zero-pad your windows with a zpf of 8.

   (b) Describe the characteristics of each window spectrum (main lobe width, relative height of sidelobes, etc.) and what consequence it might have on a sinusoid you are analyzing.

   Solution:

```
clear all
close all

N  = 128;
w_hann  = window(@hann,N)';
w_bartlett = window(@bartlett,N)';
w_boxcar = ones(1,N);
plot(1:N, [w_boxcar;w_hann;w_bartlett]); axis([1 N 0 1]);
legend('Boxcar','Hann','Bartlett');
xlabel('time(samples)')
ylabel('amplitude')

zpf=8;
w_hann_zp = z_pad(w_hann,zpf);
w_bartlett_zp = z_pad(w_bartlett,zpf);
w_boxcar_zp = z_pad(w_boxcar,zpf);

plotspec(w_hann_zp)
title('Hann Spectrum')
plotspec(w_bartlett_zp)
title('Bartlett Spectrum')
plotspec(w_boxcar_zp)
title('Boxcar Spectrum')
```

2. (20 pts) [Windowed and zero-padded sinusoid] Apply a window of your choice to a 1 second sinusoid at 16.0625 Hz, using a sampling rate of 128 Hz.

(a) Plot your windowed sinusoid in the time domain with the title indicating the window type.

(b) Zero-pad your windowed signal with a zpf of 8, and plot the magnitude spectra of the windowed, zero-padded signal with the title indicating the window type.

Solution:

```
clear all
close all

% a)
%--------------------------------------------
dur=1; %(sec)
fs=128; %(Hz/sec)
A=1;
wo=2*pi*16.0625; %(rad)

dT=1/fs;
t=(0:dT:dur);
SINUSOID=A*sin(wo*t);
N=length(SINUSOID);

w_chebwin = window(@chebwin,N)';
SINUSOID_wc = w_chebwin .* SINUSOID;

plot(t,SINUSOID_wc)
grid
title('Sinusoid windowed with Chebwin')

% b)
%--------------------------------------------
zpf=8;
SINUSOID_wc_zp=z_pad(SINUSOID_wc,zpf);

plotspec (SINUSOID_wc_zp,fs)
title('Spectrum sinusoid widowed Chebwin')
```

3. (50 points) You are a recording engineer/audio detective. You wanted to test the sound of three different microphones on one of the three violinists in a string quartet. You asked your assistant (Jorge) to record a little with each of the three microphones placed next to a respective player. While Jorge was preparing the recordings for you, he stupidly mixed up the ordering of the files. As a result, you don't know which

recording corresponds to which microphone. As a matter of life or death, you need to solve this mystery!

To help recover the unknown ordering, you were able to recover a few seconds of each performer playing the same music individually at another time. Use this evidence (`cleanSignals`) along with the unordered (`mixedSignals`) to solve the mystery. See the following Matlab starter code and data file:

```
Lab7_3.m
data.mat
```

Solution:

```
 % Courtesy of Oriol Nieto
%% Clear all the data
clear all; close all; clc;

% Load the data
load data

% Plot
plot(violinist(1,:), violinist(2,:), 'x', 'MarkerSize', 14)
grid on; hold on;; axis([-.5 1.5 -.5 1.5])
text(violinist(1,1), violinist(2,1), '  v  1');
text(violinist(1,2), violinist(2,2), '  v  2');
text(violinist(1,3), violinist(2,3), '  v  3');

plot(mic(1,:), mic(2,:), 'ro', 'MarkerSize', 14)
legend('violinist', 'Mic')
text(mic(1,1), mic(2,1), '  mic  1');
text(mic(1,2), mic(2,2), '  mic  2');
text(mic(1,3), mic(2,3), '  mic  3');
title('Recording Configuration')
xlabel('X dimension ')
ylabel('Y dimension ')

%% Your Code here

% Compute the Correlation between each of the signals:
csignals = cleanSignals;
mixed1 = fftfilt(flipud(csignals),[mixedSignals(:,1); zeros(length(csignals),1)]);
mixed2 = fftfilt(flipud(csignals),[mixedSignals(:,2); zeros(length(csignals),1)]);
mixed3 = fftfilt(flipud(csignals),[mixedSignals(:,3); zeros(length(csignals),1)]);
```

```
%Now find the index where the correlation is max
[max1, ind1] = max(mixed1);
[max2, ind2] = max(mixed2);
[max2, ind3] = max(mixed3);

%Show them in a matrix (rows are Mixed Signals and columns are the
%violins):
ind1, ind2, ind3

% We see the distances here. By watching at the graph, we can
% see that the mixed signal 1 corresponds to the mic 2, since there
% are two equal distances, and the mic 2 is the only one that has
% two violins at the same distance.
% Then for the mixed signal 2, we see that the closest violin is
% violin 1. So it has to be the microphone 1.
% Finally, the mixed signal 3 corresponds to the mic 3, since the
% violin 3 is the closests from it (as we see in the matrix)
%
% So:
% Mixed Signal 1 -> Mic 2
% Mixed Signal 2 -> Mic 1
% Mixed Signal 3 -> Mic 3
```