Music 421
Spring 2010-2011
Homework #3
More Windows, Window Design, FIR Filter Design
Due in one week (April 21 and 22, by 5pm)

Theory Problems

1. (5 pts) Show that the $\cos^p(t)$ window has p leading zeros in the series expansion about its right endpoint.

2. (5 pts) Sketch the window w, corresponding to the window transform

$$W(\omega) \triangleq M\text{asinc}_M(\omega) \triangleq \frac{\sin(M\omega/2)}{\sin(\omega/2)}$$

where M is an even integer. [Hint: Note that W is not 2π-periodic, but it is $4\pi-$ *periodic*. The purest way to deal with this is to define the IDTFT from -2π to 2π. Another approach is to use this function to define W on $[-\pi, \pi]$ and define it to be zero outside that interval (the "properly bandlimited" assumption).]

3. (10 pts) Suppose we want to window a measured system's impulse response $h(n)$ in order to shorten it and fit a one-pole filter to it. The window used is a 15-point zero-phase (centered) Poisson window with decay time-constant $\tau = 1$ second (assuming $f_s = 1$), where all *negative* time (anti-causal) samples are set to zero (since only measurement noise occurs for negative-time samples). Recall that the Poisson window is a rectangularly windowed sampled exponential. To confirm that you got the description right, the window should have a support of only 8 samples (at most 8 nonzero samples).

   (a) (2 pts) Write down the window function $w_0(n)$ in time domain. (It can be written in terms of the rectangular window $w_R(n)$ and step function $u(n)$.)

   (b) (3 pts) Write down the transfer function $W_0(z)$ of the window function in the z-domain. (It can be written in terms of $W_R(z)$.)

   (c) Suppose you now apply the window w_0 to recorded impulse-response data $h_a(n)$ that has been converted to analytic form (*i.e.*, its negative frequencies have been filtered out):

$$h_w(n) = w_0(n)h_a(n)$$

   And suppose you fit a one-pole filter to $h_w(n)$ and obtain the filter

$$V_0(z) = \frac{1}{1 - e^{-1+j\pi/2}z^{-1}}.$$

Specify the filter $V(z)$ obtained by removing the exponential decay of the analysis window. That is, find the filter that would have been obtain is the analysis window were rectangular over the full duration of the impulse response. [Hint: this is accomplished by moving the pole along a radial line in the z plane.]

i. (2 pts) Write down the new filter transfer function $V(z)$.
   This is the analytic form of the filter (positive frequencies only). The corred-ponding real filter can be obtained by assuming Hermitian symmetry.

ii. (3 pts) Express $V(z)$ in terms of V_0 and the pole of $V_0(z)$ *only*.

4. (5 pts) **System Identification Using the Poisson Window**
   Suppose now that we are attempting to measure and model a *loudspeaker* by measuring its impulse of a speaker $h(t)$.

   You want to model the speaker impulse response using a one-pole filter:

$$y(n) = b_0 x(n) + a_0 y(n-1)$$

   where, b_0, and a_0 are your design parameters you will be fitting.

   You record the speaker impulse response with $fs = 50,000$, but it rings for over 2 minutes. You have limited memory to load the file for your model fitting and can only use 2 seconds of data.

   (a) (3 pts) Explain how the Poisson window can effectively suppress the ringing of the speaker response and allow you to fit your one-pole model on shorter windowed recording with a post-processing step to correct the fitted pole so it's unknowing of the windowing.

   (b) (2 pts) You want to only estimate the model using the 2 seconds of data with a Poisson window T60 time of the same length (2 seconds). What is the α parameter of the Poisson window to acheive this specification? The Poisson window is defined as:
   $w_P(n) = w_R(n) e^{-\alpha \frac{|n|}{\frac{M-1}{2}}}$.

Lab Problems

1. (5 pts) Find out where the Chebyshev window "breaks down" in Matlab. Let the length be fixed at $M = 31$, and try various ripple specifications until there is an obvious error in the window obtained. Describe the source of the failure when the ripple specification is (a) too large and (b) too low. The following Matlab code can be used as a starting point:

```
N = 8192;
M = 31;
w = chebwin(M,rip);
W = fft(w,N);
% normalize and clip the window transform in dB:
Wdb = 20*log10(max(abs(W)/max(abs(W)),...
                    10^(-rip*1.5/20)));
f = [0:N-1]/N - 0.5;
plot(f,fftshift(Wdb)); grid;
axis([-0.5 0.5 -rip*1.5 0]);
hold on;
plot([f(1),f(N)],[-rip,-rip],'--k');
text(f(1)+0.02,-rip+rip*1.5/20,...
      sprintf('rip = %0.1f dB',rip));
xlabel('Normalized Frequency (cycles/sample)');
ylabel('Magnitude (dB)');
title(sprintf('Length %d Chebyshev window',M));
```

2. (10 pts) For $\omega_c T = \pi/2$, design a length $M = 100$ real FIR lowpass filter using the window method. Plot the amplitude response (dB gain versus frequency) for the following windows:

   - Hamming
   - Hann
   - Blackman
   - Kaiser with $\beta = 10$

   Explain why the stopband rejection is so different from that in the previous problem.

3. Design a length $M = 51$ Chebyshev window with 40 dB of sidelobe attenuation using

   - `chebwin`
   - `firpm` (formerly called `remez`)
   - `linprog`

   in matlab. Normalize each window such that the main lobe of the window transform has peak magnitude 0 dB. For `firpm` and `linprog`, set the normalized transition bandwidth to 0.068 (where 0 is dc and 1 is half the sampling rate, is as commonly used in matlab). Using `cputime`, measure the average compute-time for ten iterations of each of the three methods above.

   (a) (5 pts) Report the three average compute times. Divide the two longer compute-times by the smallest compute time and report those two speed ratios.

   (b) (5 pts) Plot an overlay of the three windows in the time domain. Repeat with the chebwin case subtracted out.

(c) (5 pts) Plot an overlay of the window magnitude spectra in the frequency domain. Use at least a factor of five zero-padding.

(d) (2 pts) Which method is the most accurate and why do you think that is? Describe any numerical considerations you can see.

(e) **Optional:** For each method, find the order (to within 10%) above which numerical problems become significant. [It is probably easiest to inspect the magnitude spectra to detect numerical troubles.] [To obtain an accuracy of 10%, one can increase the order by 10% each trial, or double it each trial, followed by 10% increments over the last interval, etc.]