Stationary Bootstrap to Determine the Optimal Portfolio Weights and obtain the value at risk

For the Bachelor of Science Honours Degree in Financial Mathematics and Industrial Statistics

By

SC/2021/12450 - K.S.H Kuruppu

Supervisor : Prof. Leslie Jayasekara
Department of Mathematics
University of Ruhuna
Matara

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Declaration

I declare that this dissertation titled ”Stationary Bootstrap to Determine the Optimal Portfolio Weights” and the work presented has not been previously submitted and accepted for any degree in the University of Ruhuna or any other institution in it are my own. I confirm that this dissertation was written by me under the supervision of Prof. Leslie Jayasekara, Department of Mathematics, Faculty of Science, University of Ruhuna.

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K.S.H Kuruppu
B.Sc. Honours in Financial Mathematics and Industrial Statistics
Department of Mathematics
Faculty of Science
University of Ruhuna
Matara
..../..../ 2025

....

Prof. Leslie Jayasekara
Department of Mathematics
Faculty of Science
University of Ruhuna
Matara
..../..../2025

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Abstract

The objective of this study is to investigate the best portfolio with optimal weights applied by each firm in the Colombo Stock Exchange market (CSE) that maximum return while keeping the minimum risk using the Stationary Bootstrapping method. This method is a statistical resampling technique to estimate Value at Risk (VaR) using monthly stock price data from January 2016 to June 2020 for four selected companies: LOLC, Abans, United Motors, and Sampath Bank sourced from the Kaggle website. The results suggest that this method provides a more reliable framework for optimal portfolio allocation under uncertain market conditions. The findings provide actionable insights for investors seeking to make informed decisions in the CSE market.

Keywords: Portfolio weights, Stationary Bootstrap, VaR, United Motors, LOLC, Sampath Bank, Abans

Chapter 1

Introduction

1.1 Background of the study

Colombo stock exchange stock investment is high risk because the financial uncertainty. Investors use various methods for that , asset diversification method is a good solution. Investors can select the several companies for invest according to the fundamental analysis after that need to identify the how to arrange the weights in portfolio. In this case study focus on to the how to obtain the optimal portfolio weights and calculate the future risk of this. By using that investors can get better solutions.

There are various methods for obtain the optimal portfolio weights, among them the stationary bootstrapping method is used for this study. It is more accuracy rathe than only use historical values. According to our stock market data set, it depends on time. For that reason, we can use the stationary bootstrapping method, which is one of the techniques of the time dependent method of bootstrapping. It generates the samples by resampling blocks of consecutive data points according to the internal structures of the data.

Use bootstrap sample, can obtain the var value and It is one of the popular methods for estimating Value at Risk (VaR). Value at Risk is defined as the maximum potential loss expected over a specific period at a certain confidence level. Investors can use this value identify the future risk of there portfolio and they can compare the portfolios for get better decisions. It is widely used because of its simplicity of calculation and interpretation.

1.2 Significance of the study

This study is notable for employing the stationary bootstrapping method, which offers a more precise approach to portfolio optimization by addressing market uncertainties and volatility. By determining optimal portfolio weights that maximize returns while minimizing potential losses, analysts can assist their organizations in making better-informed investment allocation decisions. Similarly, investors can gain valuable insights into how to distribute their capital across different companies to reduce expected losses.

1.3 Research Question

The study explores how stationary bootstrapping method can be used to determine optimal portfolio weights using four companies in the Colombo Exchange market. This approach helps investors and managers make better decisions by providing risk assessments and creating portfolios that are more resilient to market volatility.

What are the optimal portfolio weights to invest the LOLC and UM company and future risk of this portfolio ?

1.4 Research Objectives

The primary objective of this study is to apply the stationary bootstrapping method to determine the optimal portfolio weights that minimize risk and effectively maximize return.

This study aims to,

- Evaluating the optimal portfolio weights under uncertain and calculate the portfolio future risk.
- Create the model for any stock market portfolio and determine the optimal weight and risk then help to the investors decision.

Chapter 2

Literature Review

Modern Portfolio Theory also quantify the benefits of diversification and explore how risk-averse investors construct portfolios to optimize expected returns against market risks.

As the investors want maximum return in future when minimum risk then according to the historical data we can calculate , how to invest money from capital each assets. Colombo stock Exchange have only stocks as the assets for invest then can be made stock portfolio to invest. This literature review related to how to invest money for each stocks in stock market . There are two assets , Ajinamoto company and UMW company. Result indicates the efficient frontier for investment is started with 42.5% investment in Ajinomoto and 57.5% investment in UMW. The expected portfolio re turn using this investment combination is 0.14 percentages. This is basics part of the portfolio allocation according to the portfolio theory, we can develop that by using future data predictions. [1]

There are several methods for predict the future portfolio returns using non parametric and parametric methods , Bootstrap is the non parametric. This paper focuses on developing a stable investment portfolio optimization model that emphasizes diversification and minimizes abrupt allocation changes by monitoring impact factor dynamics. It explores a bootstrap-based approach, leveraging Michaud's (1999) re-sampled efficiency method, which reduces reliance on excessive constraints and uncertain information. The study compares block bootstrap optimization models with the traditional Markowitz model using frequently traded stocks on the BSE. Results indicate that the bootstrap approach achieves better out-of-sample performance, incorporates more stocks into the portfolio, and reduces estimation errors due to limited sample sizes. However, the traditional method provides optimal solutions under stricter constraints, highlighting differences in efficiency limits between the models. [4]

Value-at-risk (Var) is an estimate of the amount that can be lost from a financial position over a specific time interval. The time horizons commonly used to predict this interval range from one day to month or a year, taken at a predefine confidence interval. Thus, Var is a method used to quantify exposure to market risk through statistical techniques. Obtain the most appropriate non parametric and parametric method for obtain the optimal portfolio using value at risk values. [3]

We can apply this method for Colombo Stock Exchange and minimize the risk and maximize the returns of the investors. The common financial term use for that sharp ratio. Maximum sharp ratio of the portfolio is the optimal for invest. [2] Use average variance and Var method for that conclusion. We can create model for that then it can be applied for every assets and every market conditions.

Chapter 3

Material and Methods

3.1 Research Design

The study aims to investigate the optimal portfolio weights by estimating the Value at Risk using the Stationary Bootstrapping method. The steps are outlined below.

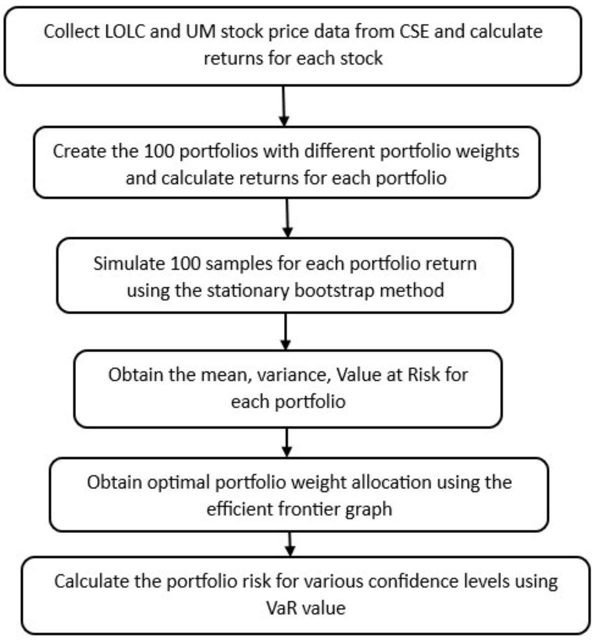

Figure 3.1: Procedure of study

3.2 Research Approach

This study employs a quantitative methodology centered on designing and testing a statistical bootstrapping algorithm to determine the optimal portfolio weights. By collecting numerical data and utilizing statistical techniques for analysis and interpretation, this approach enables the evaluation of various investment scenarios. It also facilitates identifying the most effective portfolio allocation strategies under varying conditions.

3.3 Methods

3.3.1 Stationary Bootstrapping Method

Several bootstrapping methods are used to generate data samples while maintaining various types of dependencies. Stationary bootstrap is a time-dependent bootstrapping approach. It involves resampling blocks of consecutive data points, according to the internal structures of the data. According to our stock market data set, stock prices change from time to time under different market conditions. Therefore, we can use the stationary bootstrapping method.

The stationary bootstrapping method is the most widely used resampling technique in statistics and finance. It can be used to generate the distribution of a sample without relying on assumptions about the historical data. It is a historical simulation technique used to estimate value at risk (VaR). It is also a nonparametric method that generates potential values for risk factors using historical data.

Mathematical formulation of the Bootstrapping Method

Resample B datasets $X_1^*, X_2^*, \ldots, X_B^*$ of size n with replacement from the original dataset X. For each bootstrap sample X_b^*, calculate the bootstrap statistic:

$$\theta_b^* = t(X_b^*), \quad b = 1, 2, \ldots, B.$$

The bias of the statistic θ can be estimated as:

$$\text{Bias}(\hat{\theta}) = \mathbb{E}[\theta^*] - \hat{\theta},$$

where :

$$\mathbb{E}[\theta^*] \approx \frac{1}{B}\sum_{b=1}^{B} \theta_b^*.$$

$$\bar{\theta}^* = \frac{1}{B}\sum_{b=1}^{B} \theta_b^*.$$

3.3.2 Value at Risk

Value at Risk (VaR) is a statistical technique used to measure the potential risk of loss of investments. VaR analysis takes into account variables such as market volatility, economic trends, and other key factors that can influence investment outcomes. Value-at-Risk is used to estimate the maximum loss that an investment portfolio or financial institution can face in a given time frame and within a certain level of confidence.

VaR is determined by three variables such as duration, confidence level and amount of potential loss. It gives the probability of losing more than a given amount in a given portfolio.

Figure 3.2: VaR value at 95% confidence level

Mathematical formulation of the VaR

$$\mathrm{VaR}_\alpha = \mu + \sigma \cdot Z_\alpha$$

Where:

- VaR_α : Value at Risk at confidence level α
- μ: Mean (expected return) of the portfolio
- σ: Standard deviation (volatility) of the portfolio returns
- Z_α: Z score associated with confidence level α.

Chapter 4

Data

4.1 Dataset

Daily stock price data from the Colombo Stock Exchange (CSE) for four selected companies from January 2016 to June 2020 were obtained from the Kaggle website.

4.2 Data Dictionary

Company Name	**Symbol**	**Sector**	**Data Type**
LOLC Holdings PLC	LOLC	Financials (Diversified Financial Services)	Float
Abans Finance PLC	AFSL	Financials (Finance and Leasing)	Float
United Motors Lanka PLC	UML	Consumer Discretionary (Automobile)	Float
Sampath Bank PLC	SAMP	Financials (Banking)	Float

Table 4.1: Selected Companies from Different Sectors in Sri Lanka

4.3 Preparation for analysis

Daily stock prices were converted to monthly stock prices by taking the average of stock prices. Stock prices were missing for some dates in the data set. Therefore, this process can help reduce errors in the analysis.

Before conducting the analysis, it is necessary to calculate the stock price returns. We calculate the historical returns of the stock prices in our data set by determining the percentage change in the stock price for each period. The return formula is given by;

$$\text{Return} = \frac{\text{Current stock price} - \text{Previous stock price}}{\text{Previous stock price}}$$

Calculating the mean and standard deviation based on historical stock prices

Among the selected companies, we should first identify the best ones to invest in, minimizing losses. To achieve this, historical stock returns from each company were used. The historical mean can be calculated by averaging the stock returns of four companies: Abans, Sampath Bank (SB) , United Morters (UM), and LOLC. Similarly, the standard deviation can be determined for each company's stock returns. The mean values can be plotted against the standard deviation values for the four companies.

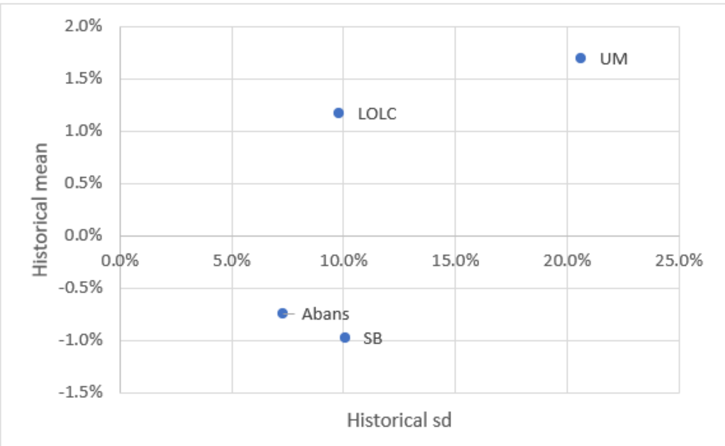

Figure 4.1: Mean and sd related to historical stock prices

Based on this figure, Sampath Bank was excluded from the analysis due to its losses. Therefore, only LOLC, Abans and United Motors were considered to construct the optimal portfolio. Here, four distinct portfolio scenarios can be created using these three companies.

The portfolio models are ;

Model 01 : Abans, LOLC, United Motors
Model 02 : Abans, LOLC
Model 03 : Abans, United Motors
Model 04 : LOLC, United Motors

From this point on, the study will focus on the model developed using **LOLC and United Motors**. In this manner, models can also be created for other portfolio scenarios. Among these, the portfolio scenario with the lowest VaR value at the specified confidence level is considered the best optimal portfolio.

Mathematical formulation of the portfolio return

$$R_p = w_{\text{LOLC}} R_{\text{LOLC}} + w_{\text{United Motors}} R_{\text{United Motors}}$$

Where:

- R_p: Portfolio return
- w_{LOLC}: Weight of LOLC stock in the portfolio (the proportion of the total portfolio invested in LOLC).
- $w_{\text{United Motors}}$: Weight of United Motors stock in the portfolio (the proportion of the total portfolio invested in United Motors).
- R_{LOLC}: Stock return of LOLC
- $R_{\text{United Motors}}$: Stock return of United Motors

Chapter 5

Results

5.1 Exploratory data Analysis

Exploratory Data Analysis is the initial process of investigating, analyzing, and summarizing a data set to understand its main characteristics.

5.1.1 Checking the normality of portfolio weight returns

First,we checked whether the portfolio weights of the returns followed a normal distribution. Based on the results, either a parametric or non-parametric method was selected for the study.

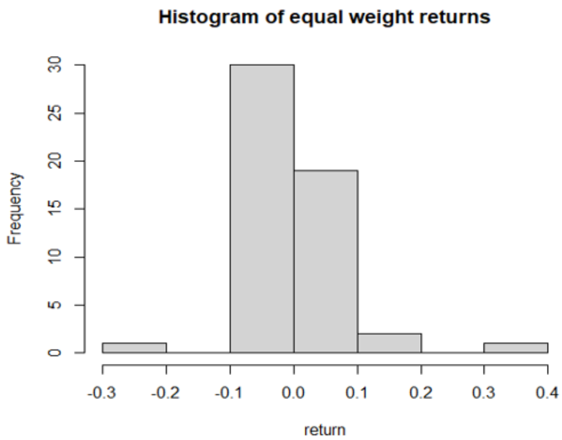

Figure 5.1: Histogram of equal weight returns

This histogram displays the historical portfolio return of equal weight allocation (LOLC – 50% and UM – 50 %). It also shows that stock returns do not follow a normal distribution. According to the skewness and kurtosis test, the values were 0.4779 and 5.9501. Both of these tests confirm that equal weight returns are not normal.

Therefore, a non-parametric approach is required to determine optimal portfolio values. And also, in this case study the aim is to obtain the optimal portfolio allocation for these stocks, then this process is not practical because it involves 100 number of portfolios, requiring the normality of each to be checked individually. Since the Monte Carlo simulation relies on normality assumptions, it is not suitable for this analysis. Therefore, the bootstrapping method was applied in our case study to ensure more accurate results.

5.2 Quantitative analysis

Quantitative analysis involves the computation of returns, value at risk (VaR), and portfolio optimization using historical stock price data to assess and manage risk.

5.2.1 Portfolio weight allocation

This table shows the head of portfolio weight allocations, we generate 100 number of portfolios according to the uniform distribution, and all portfolio weights are greater than 10%.

Portfolio	LOLC	UM
1	0.4380039	0.5619961
2	0.4015565	0.5984435
3	0.2365534	0.7634466
4	0.5776600	0.4223400
5	0.8106526	0.1893474
6	0.5243222	0.4756778

5.2.2 Equal weight allocation for stock returns

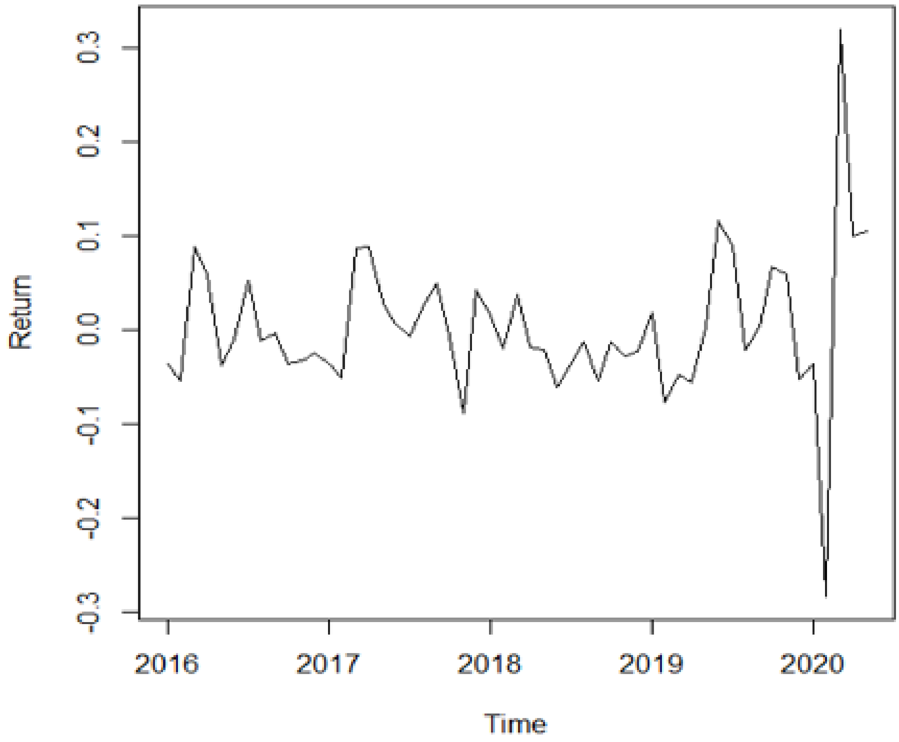

Figure 5.2: Time series plot of equal weight return

This figure shows the time series plot of the historical returns of the equal weight allocation portfolio. After 2019 there was a high fluctuation in historical returns. Thus, it can be observed that the equal weight allocation of returns has shown a distinct time dependency since 2019.

5.2.3 Using Stationary Bootstrap Method

Based on the weight allocation described above, the returns are calculated for 100 portfolio scenarios. The stationary bootstrapping method is then applied to generate 100 samples for each portfolio weight. Calculate mean, variance and var value for each simulations then obtain the mean value of every one. This values are assign to the each portfolios.

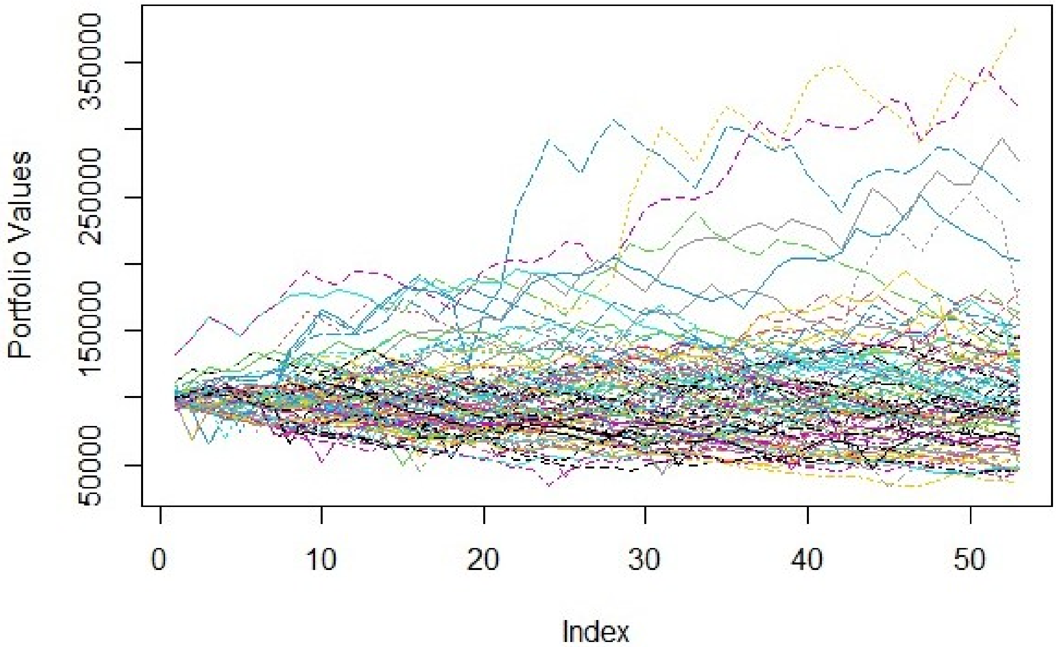

Figure 5.3: Bootstrap simulation of equal returns portfolio

This plot shows the results of bootstrap simulations for equal return portfolios. Obtain the compounded return value for each simulations and assuming current market portfolio value is 100,000/=. multiply by these two values and plot the outputs for each simulations. Each line represents a simulated path of portfolio returns over multiple iterations, illustrating the variability of each portfolio.

5.2.4 Efficient Frontier for portfolio investment

The Optimal (Efficient) Frontier is the set of portfolios that achieve the highest possible return for a given level of risk (or the lowest risk for a given return). It is the upper portion of the risk-return scatter plot of all feasible portfolios.

We calculated the variance (risk) and returns using the stationary bootstrap method and determined the portfolio weights that maximize returns while minimizing risk.

Sharp Ratio ;

$$\max\left(\frac{\text{return}}{\text{risk}}\right)$$

This formula shows that the optimal portfolio can be identified by selecting the highest sharpe ratio among all the sharpe ratios.

According to that we obtain the optimal portfolio weights are
LOLC : 61.71% and UM : 38.29% (Sharpe Ratio is 0.1718)

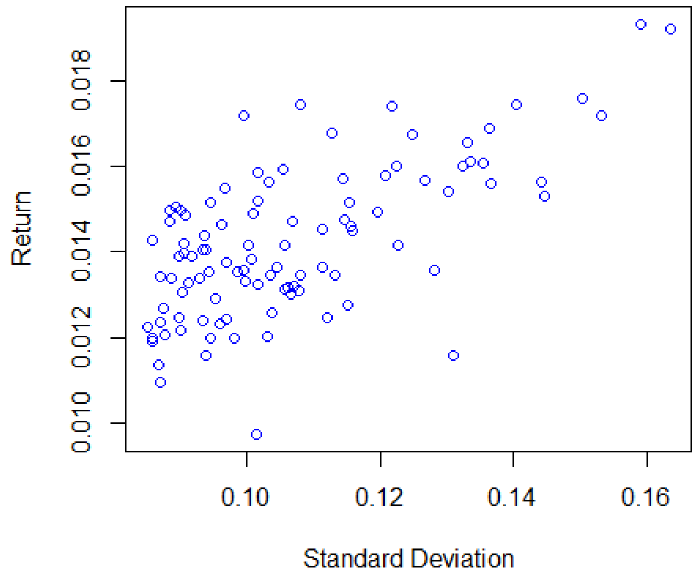

Figure 5.4: Efficient frontier of investment portfolio

This plot shows the return versus the standard deviation of each portfolio, and we can identify the portfolio with the highest Sharpe ratio.

5.2.5 Optimal weights portfolio VaR values for various confidence level

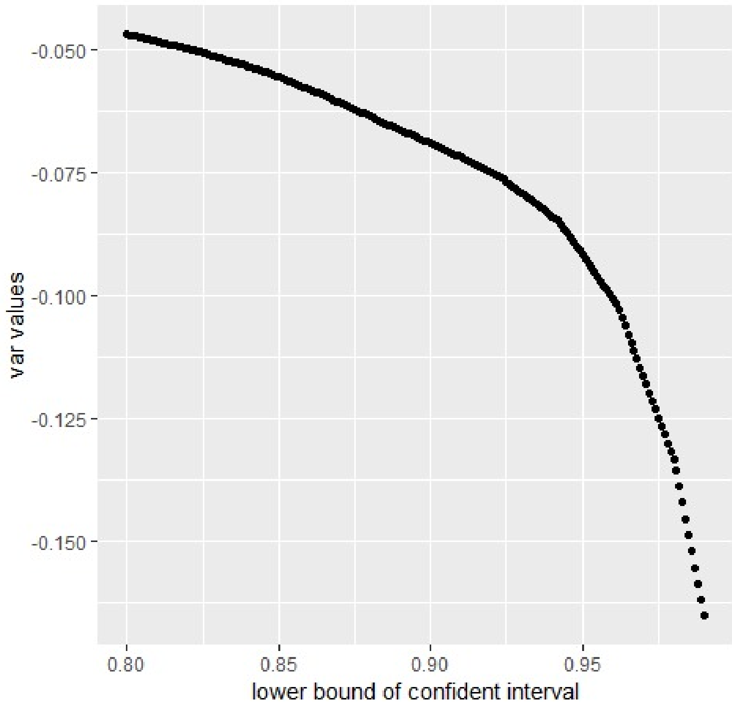

Figure 5.5: Optimal portfolio VaR values

Specially focus on optimal weight allocation portfolio and this plot shows the var value of each confident interval. When using a high confidence interval, the var value is increasing (the maximum loss of the portfolio is increasing).

We need to find the var value for each confidence level to determine the optimal portfolio weights. The plot above illustrates the VaR values for the optimal portfolio. From these, select the confidence level with the highest VaR value. This confidence level represents the maximum loss that the investor can tolerate.

5.2.6 VaR values of optimal portfolio weights with various confidence levels

Confidence Level	VaR value
99%	-0.1913 (19.13%)
98%	-0.1443 (14.43%)
95%	-0.0793 (7.93%)
90%	-0.0580 (5.8%)
85%	-0.0483 (4.83%)
80%	-0.0414 (4.14%)

Table 5.1: VaR values of optimal portfolios

The above table shows the optimal allocation of portfolio weights for various lower bounds of confidence levels. When we consider the 99% confidence level, the maximum loss is 19.13% from the investment value. This method can determine the var values for the optimal portfolio weights under various confidence levels. The chosen confidence interval depends on the preferences of the investor or entity.

5.2.7 Risk Tolerance

It is difficult to determine the exact confidence level for an optimal portfolio for investors. It depends on the preferences of the investor or the entity. We mainly focus on how comfortable the investor feels about taking a risk. The optimal confidence level for a portfolio is influenced by the investor's risk tolerance, as it reflects their willingness to accept losses in pursuit of returns.

A more risk-averse investor might prefer a lower confidence level, such as 80%, to ensure that losses are minimized, even if it means accepting lower returns. Investments can be made with an expected loss not exceeding 4.14% of the investment value.

In addition, a risk seeker investor may be comfortable with a higher confidence level, such as 99% , to take on more risk for higher returns. This means that the maximum loss could be 19.13% of the investment value. Ultimately, the choice of confidence level is a key factor in portfolio optimization, aligning the portfolio risk with the investor's individual goals and preferences.

In this case study, we can obtain the optimal portfolio according to their risk tolerance of the investors that depends on the investor's financial stability.

5.2.8 Selecting optimal portfolio weights

Among the confidence levels considered, the maximum VaR value is at the 99% confidence level. Finally, we can conclude that the portfolio maximizes return while maintaining minimum risk and has a value of 19.13% of its value at the 99% confidence level.

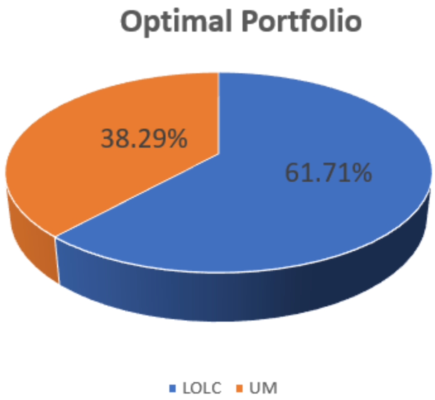

Figure 5.6: Optimal portfolio allocation

Allocate capital based on optimal weights:

- Invest 61.71% of the total capital in LOLC
- Invest 38.29% of the total capital in UM

This result suggests a higher weight in LOLC, possibly indicating lower risk or more stable returns compared to UM.

Chapter 6

Discussion and Conclusions

6.1 Discussion

Investors want to identify the optimal portfolio weights for their investment decisions. There are several methods use to obtain the optimal portfolios in past research papers. Global minimum variance portfolio , efficient frontier graph , sharp ratio are example for that in this case study use efficient frontier graph and sharp ratio methods. Most of studies use only the historical data for calculate that but in this we use stationary bootstrap method for calculate that. Then we can obtain the more accuracy value.

In practical situation according to the fundamental analysis investors can identify the several companies to invest. They need the compare the portfolio using financial term. Value at risk is the best solution for that in this case study we obtain optimal portfolio var value using stationary bootstrap simulations. We use historical simulation method for that but most of researchers calculate var value using different ways then compare the accuracy of methods. variance - covariance method and Tukey's g and h Family of distribution are the other methods. Tukey's and h family of distribution is non parametric method and its more accuracy most of research papers.

In this case study we use stationary bootstrap method for calculate more accuracy value for historical mean and variance and portfolio var value and we use sharp ratio and value at risk value obtain more useful output for support the investors decision making process.

6.2 Conclusion

Finally, according to the objectives obtain the optimal portfolio for invest and calculate the value at risk of this portfolio for difference confidence intervals. We use LOLC and UM company for obtain the results, according to that optimal portfolio weights are 61.71 % for LOLC and 38.29 % for UM and also value at risk of this portfolio is 19.13 % in 99% confidence interval. Its means maximum lost of this portfolio is 19.13 % from the portfolio value.

In this case study main object is create the model for calculate optimal portfolio and var value for any given stock portfolio. According to that this model can use for calculate the other portfolio optimal weights and var value. In this study we mention the four portfolios according to that investor can calculate the value at risk in 99% confidence interval of every portfolio then can identify the minimum var value portfolio among them. Not only that this obtain the optimal portfolio weights allocation for every portfolio then investors can get better decision using that.

Appendix

Excel file and R codes

Code of the obtain optimal portfolio and calculte the var value

```r
set.seed(1)
library(ggplot2)
library(boot)

# Parameters
simulate_portfolios_num <- 100
block_size <- sqrt(length(fin_data_set$LOLC))
n_bootstrap <- 100
data_set <- fin_data_set

## Generate Random Portfolio Weights ##
generate_random_weights <- function() {
  repeat {
    # Generate random weights between 0.1 and 1
    w <- runif(ncol(data_set), min = 0.1, max = 1)
    w <- w / sum(w)  # Normalize to sum to 1

    if (all(w > 0.1)) {  # Ensure all weights > 0.1
      return(w)
    }
  }
}

simulate_portfolios <- function(n) {
```

```r
  portfolio_weights <- matrix(0, nrow = n, ncol = ncol(data_set))
  for (i in 1:n) {
    portfolio_weights[i, ] <- generate_random_weights()
  }
  return(portfolio_weights)
}

simulated_weights <- simulate_portfolios(simulate_portfolios_num)
print(head(simulated_weights))  # Display a few sets of weights

## Stationary Block Bootstrap ##
stationary_block_bootstrap <- function(data, block_size, n_bootstrap) {
  n <- length(data)
  simulated_returns <- matrix(NA, nrow = n, ncol = n_bootstrap)

  for (b in 1:n_bootstrap) {
    indices <- integer(0)
    while (length(indices) < n) {
      block_start <- sample(1:n, 1)
      block <- seq(block_start, block_start + block_size - 1) %% n
      block[block == 0] <- n
      indices <- c(indices, block)
    }
    indices <- indices[1:n]  # Trim indices to match data length
    simulated_returns[, b] <- data[indices]
  }

  return(simulated_returns)
}

## Calculate Portfolio Returns ##
simulate_return_mat <- function(simulate_portfolios_num) {
  portfolio_return_matrix <- matrix(0, nrow = simulate_portfolios_num,
  ncol = ncol(data_set) +4 )

  for (i in 1:simulate_portfolios_num) {
    portfolio_stock_return <- numeric(nrow(data_set))

```

```
    # Calculate portfolio returns for the given weights
    for (j in 1:nrow(data_set)) {
      portfolio_stock_return[j] <- sum(simulated_weights[i, ] * data_set[j, ])
    }

    portfolio_return_historical <- portfolio_stock_return

    # Calculate historical mean and standard deviation
    historical_return_mean <- mean(portfolio_return_historical)
    historical_return_sd <- sd(portfolio_return_historical)

    # Use block bootstrap to simulate future returns
    simulated_returns <- stationary_block_bootstrap(portfolio_return_historical, block_size,
    n_bootstrap)

    # Calculate Value-at-Risk (VaR) for each bootstrap sample
    calculate_var <- apply(simulated_returns, 2, quantile, probs = 0.01)
    calculate_var_mean <- mean(calculate_var)

    mean <- mean(apply(simulated_returns,2,mean))
    sd <- mean(apply(simulated_returns,2,sd))
    mean_sd <- mean/sd

    # Store results
    portfolio_return_matrix[i, ] <- c(simulated_weights[i, ], calculate_var_mean,mean,
    sd, mean_sd)
  }

  return(portfolio_return_matrix)
}

# Generate the simulated return matrix
simulate_return_matrix <- simulate_return_mat(simulate_portfolios_num)
print(head(simulate_return_matrix))
var <- simulate_return_matrix[,3]
simulation_return_matrix[,5]

## Find Optimal Portfolio ##
```

```
plot(simulate_return_matrix[,5],simulate_return_matrix[,4], xlab = "Standard Deviation" ,
ylab = "Return",main = "Portfolio Risk and Return PLot",col=("blue"))
var <- simulate_return_matrix[,3]
mean_sd <- simulate_return_matrix[,6]
index_optimal <- which.max(mean_sd)
index <- which.max(var)
max(var)
optimal_weights <- simulate_return_matrix[index_optimal, ]
print(optimal_weights)
```

Code of the optimal portfolio var value changing according to the confidence interval

```
library(tseries)
library(e1071)
library(ggplot2)

attach(Returns)
data <- Returns$Return


Returns_ts <- ts(Returns, start = c(2016, 1), frequency = 12)
plot.ts(Returns_ts)
hist(Returns$Return,xlab = "return",main = "Histogram of equal weight returns")
#skewness(Returns$Return)
#kurtosis(Returns$Return)

block_size <- sqrt(length(Returns$Return))
n_bootstrap <- 100

historical_return_mean <- mean(Returns$Return)
historical_return_sd   <- sd(Returns$Return)


#generate random return using mean and sd

#use block stationary boostrap for equal weights
  stationary_block_bootstrap <- function(data, block_size, n_bootstrap) {
    n <- length(data)
```

```
    simulated_returns <- matrix(NA, nrow = n, ncol = n_bootstrap)

    for (b in 1:n_bootstrap) {
      indices <- integer(0)
      while (length(indices) < n) {
        block_start <- sample(1:n, 1)
        block <- seq(block_start, block_start + block_size - 1) %% n
        block[block == 0] <- n
        indices <- c(indices, block)
      }
      indices <- indices[1:n]  # Trim indices to match data length
      simulated_returns[, b] <- data[indices]
    }

    return(simulated_returns)
  }

simulated_returns <- stationary_block_bootstrap(data, block_size, n_bootstrap)

final_data_frame <- as.data.frame(simulated_returns)


#simulate_var_values <- matrix(NA,)
var_output <- matrix(NA,nrow = length(probabilities),ncol = 2)

probabilities <- 1- seq(0.8, 0.99, by = 0.001)
print(probabilities)

for(i in seq_along(probabilities) ){
prob <- probabilities[i]
calculate_var <- apply(final_data_frame, 2, quantile, probs = prob)
calculate_var_mean <- mean(calculate_var)
simulate_var_values[i] <- calculate_var_mean

#print(head(simulate_var_values))

var_output[i, ] <- c(1-prob , simulate_var_values[i])
}
```

```
print(var_output)
colnames(var_output) <- c("probabilities","var")
var_confident <- as.data.frame(var_output)
#plot(var_output[,1],-1*var_output[,2])
ggplot(var_confident, aes(x = var_confident$probabilities,y = var_confident$var))
+xlab("lower bound of confident interval")+
ylab("var values")+geom_point()
```

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