Welcome to the course

INTRODUCTION TO PORTFOLIO ANALYSIS IN R

Kris Boudt

Professor, Free University Brussels & Amsterdam

Is investing monkey-business?

[1] Source: Eric Isselee, Getty Images

Who am I?

- Professor of Finance

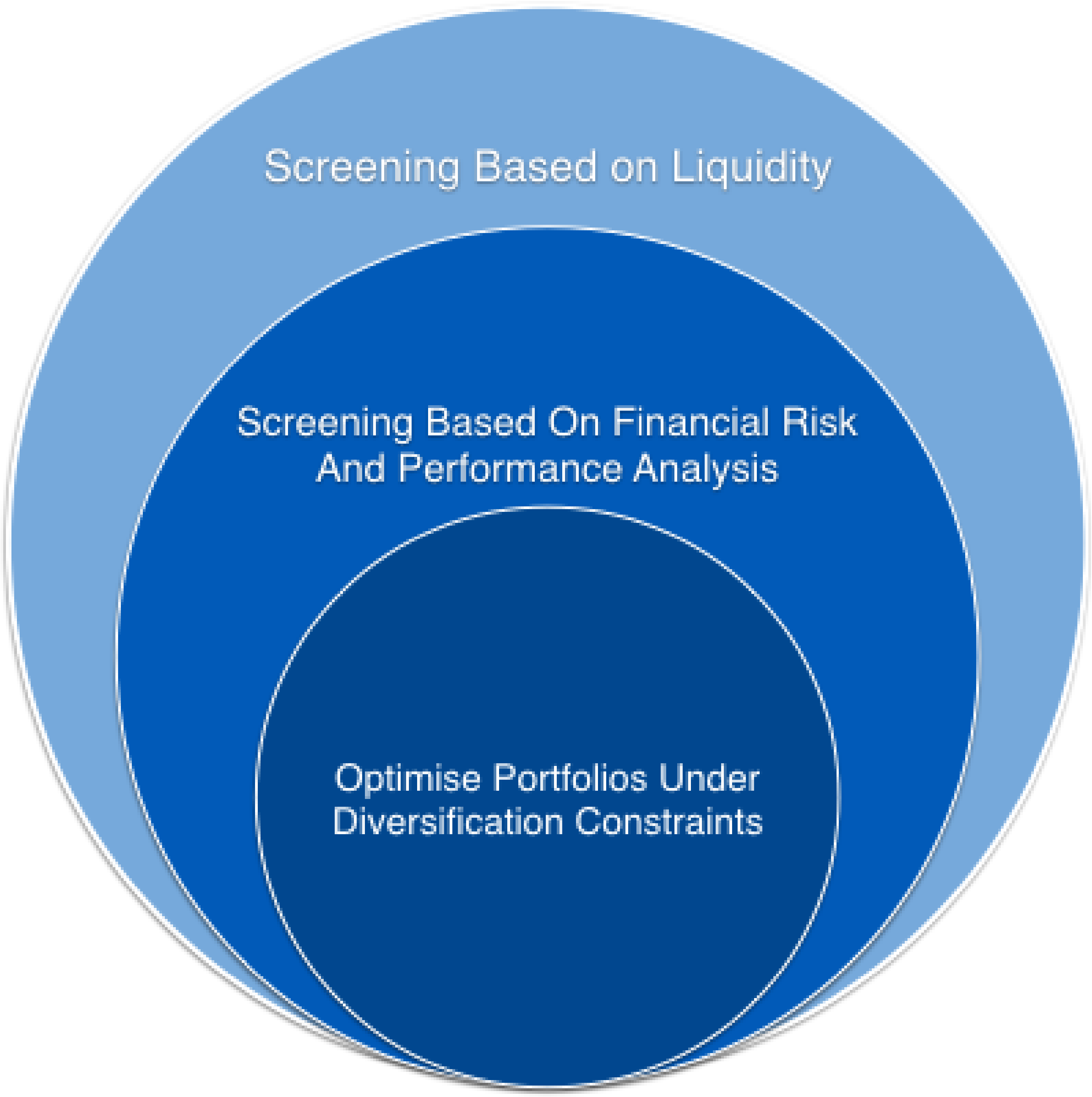
Vrije Universiteit Brussel

VRIJE UNIVERSITEIT AMSTERDAM

Who am I?

- Advisor to investment companies about risk optimized investment: Winning by losing less.

Diversify to avoid losses



Simple tricks

- To avoid large losses:
  - Carefully select diversified portfolios
  - Use backtesting and online performance monitoring

Simple tricks

- DataCamp

Course overview

Chapter 1: Portfolio Weights & Returns

Course overview

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Chapter 1: Portfolio Weights & Returns

Chapter 2: Portfolio Performance Evaluation

Course overview

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Chapter 2: Portfolio Performance Evaluation

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Chapter 1: Portfolio Weights & Returns

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Chapter 2: Portfolio Performance Evaluation

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Chapter 3: Drivers of Performance

Course overview

- Chapter 1: Portfolio Weights & Returns
- Chapter 2: Portfolio Performance Evaluation
- Chapter 3: Drivers of Performance

Course overview

Chapter 1: Portfolio Weights & Returns

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Chapter 2: Portfolio Performance Evaluation

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Chapter 3: Drivers of Performance

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Chapter 4: Portfolio Optimization

Let's practice!

INTRODUCTION TO PORTFOLIO ANALYSIS IN R

The portfolio weights

INTRODUCTION TO PORTFOLIO ANALYSIS IN R

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Investment decision choices

There are two similar companies: Do you invest in either of them based on a coin toss?

[1] Source: ICMA Photos, Flickr

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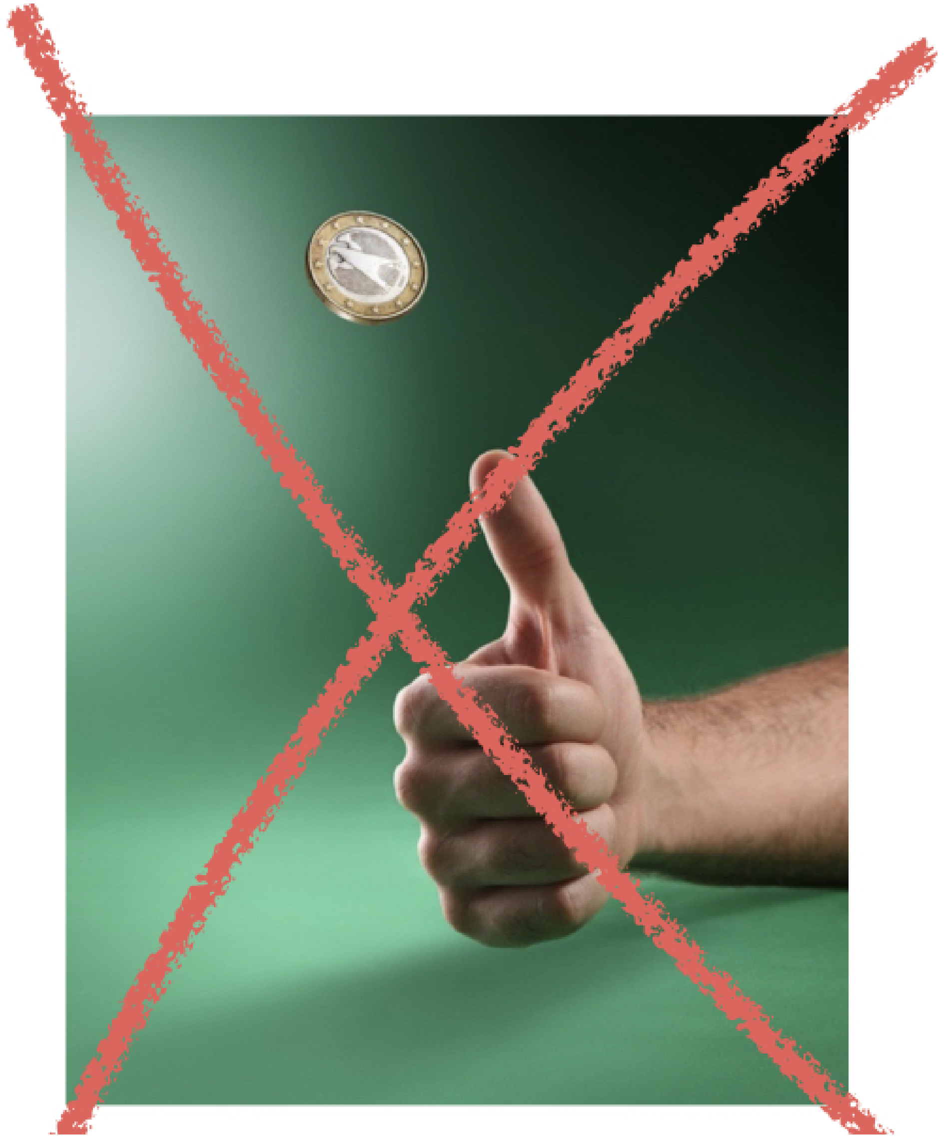

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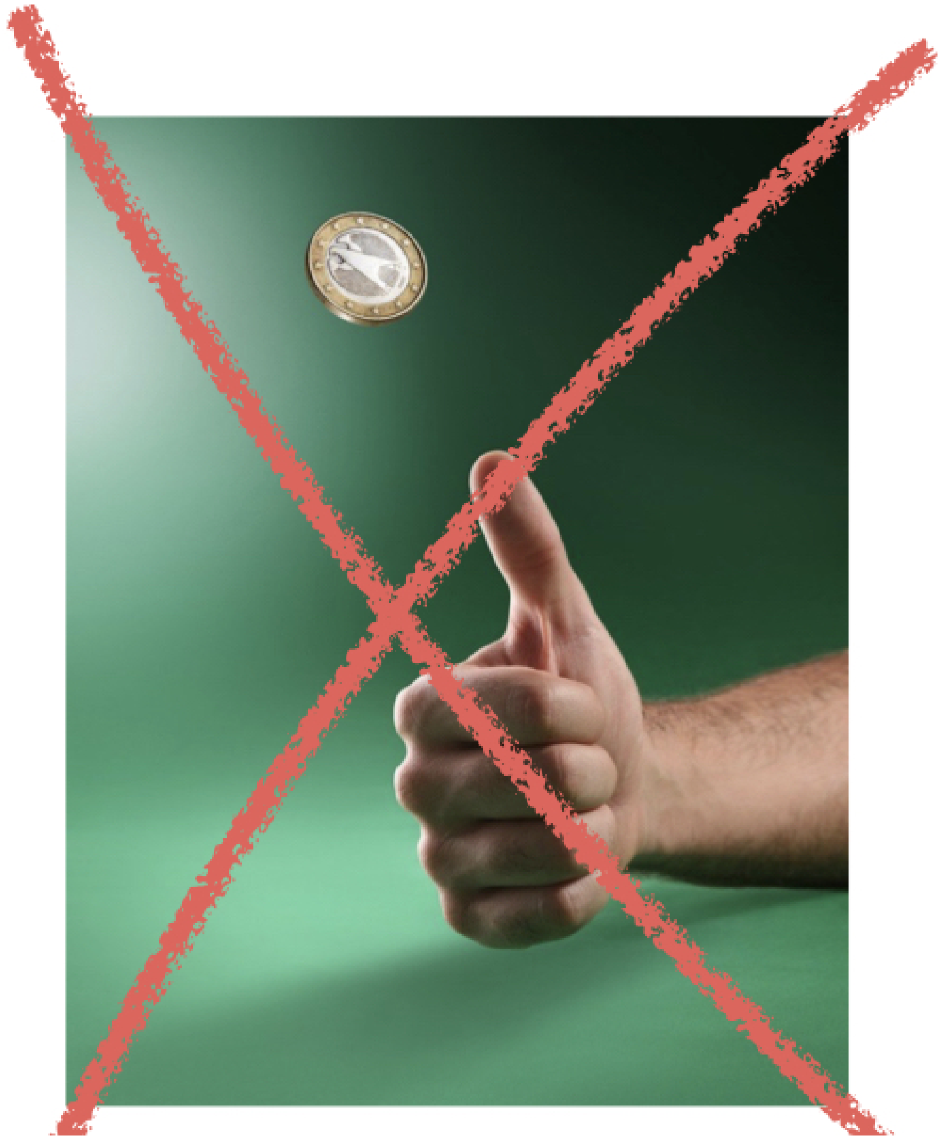

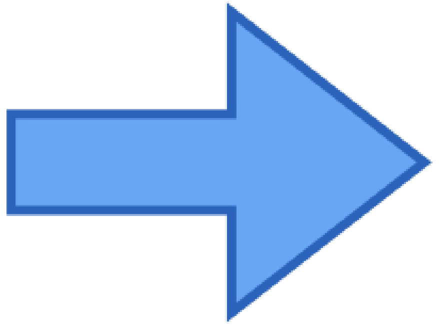

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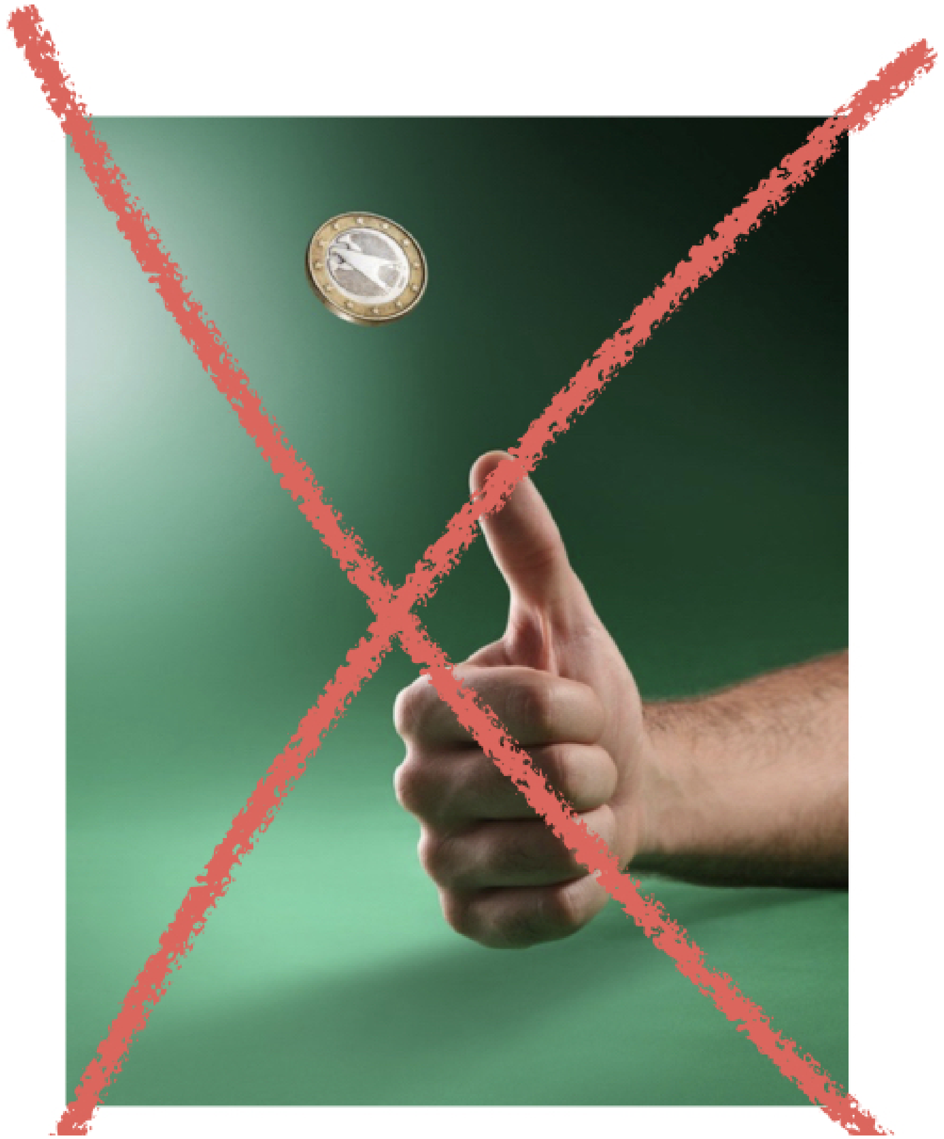

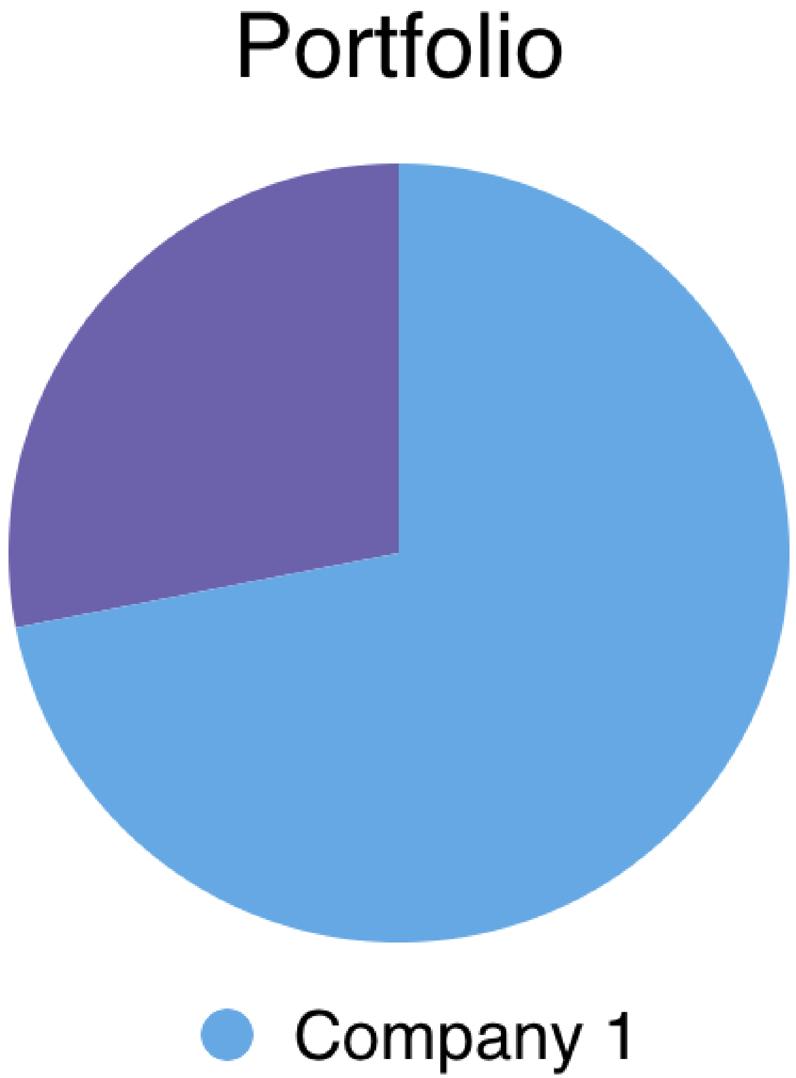

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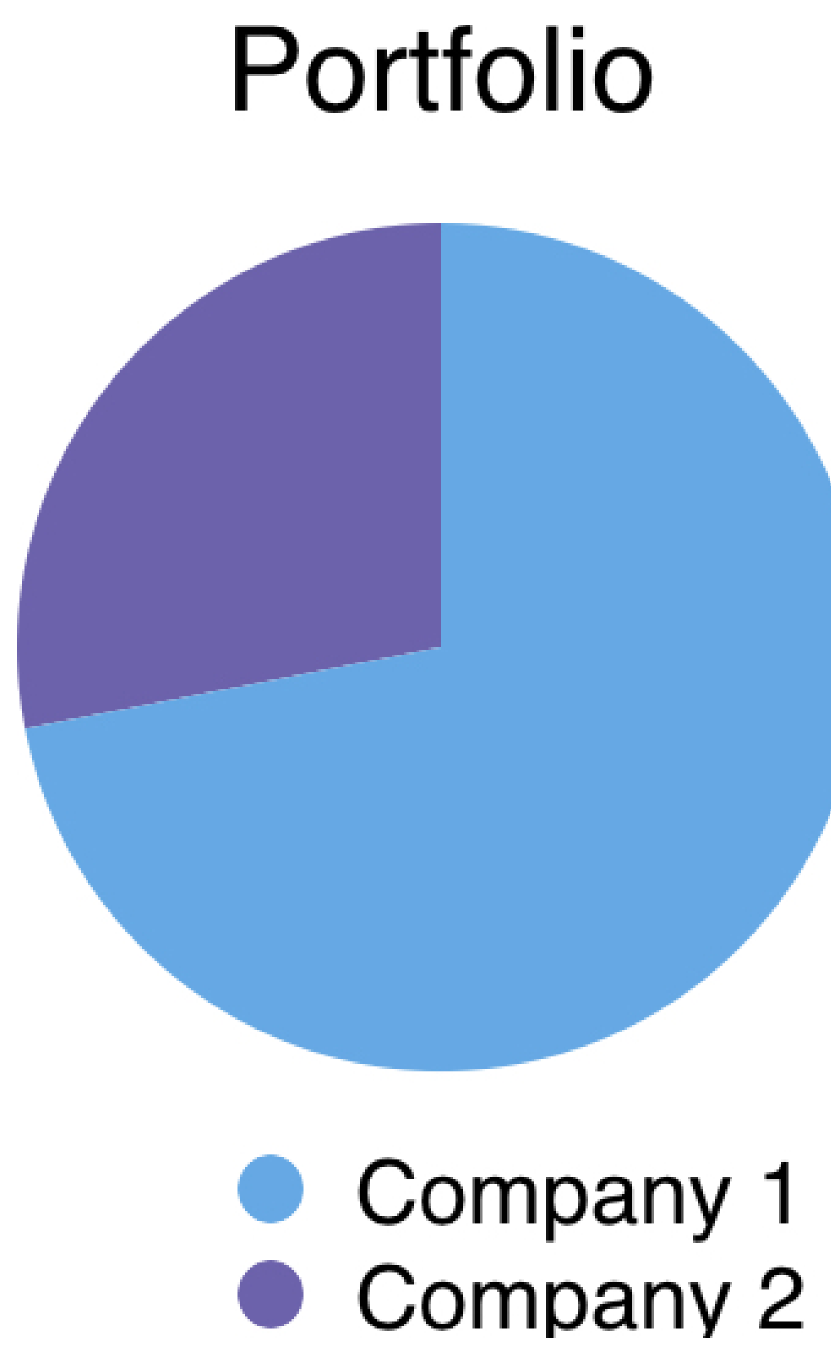

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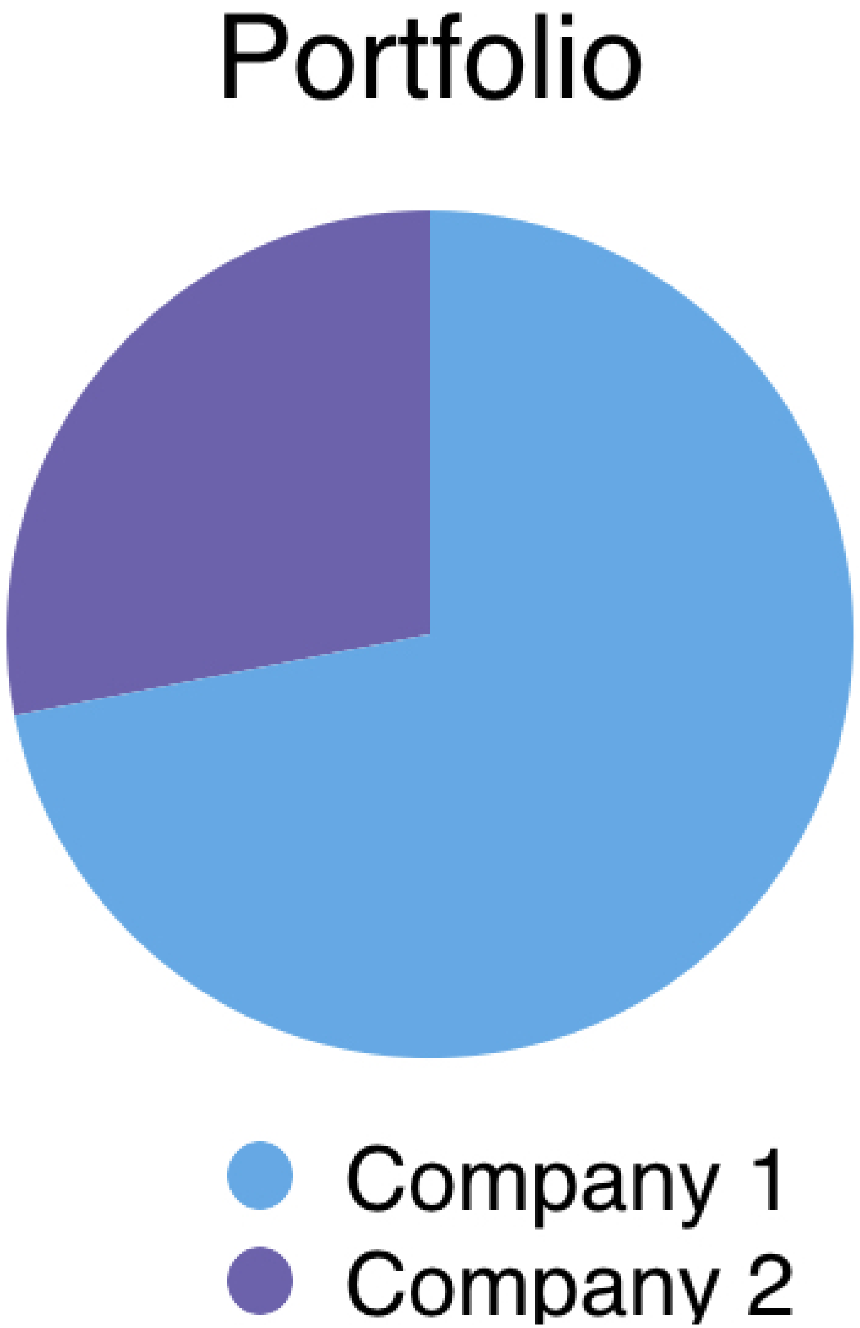

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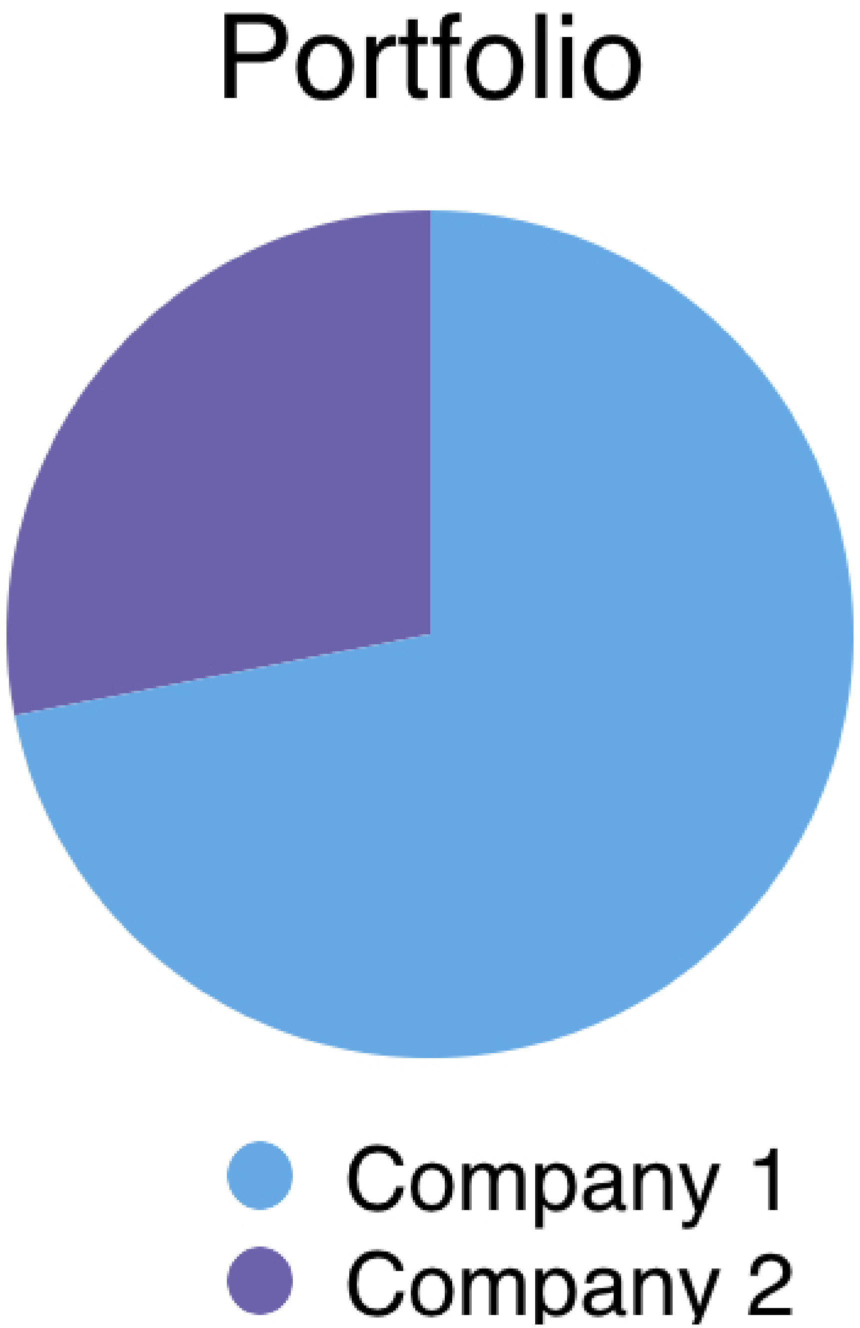

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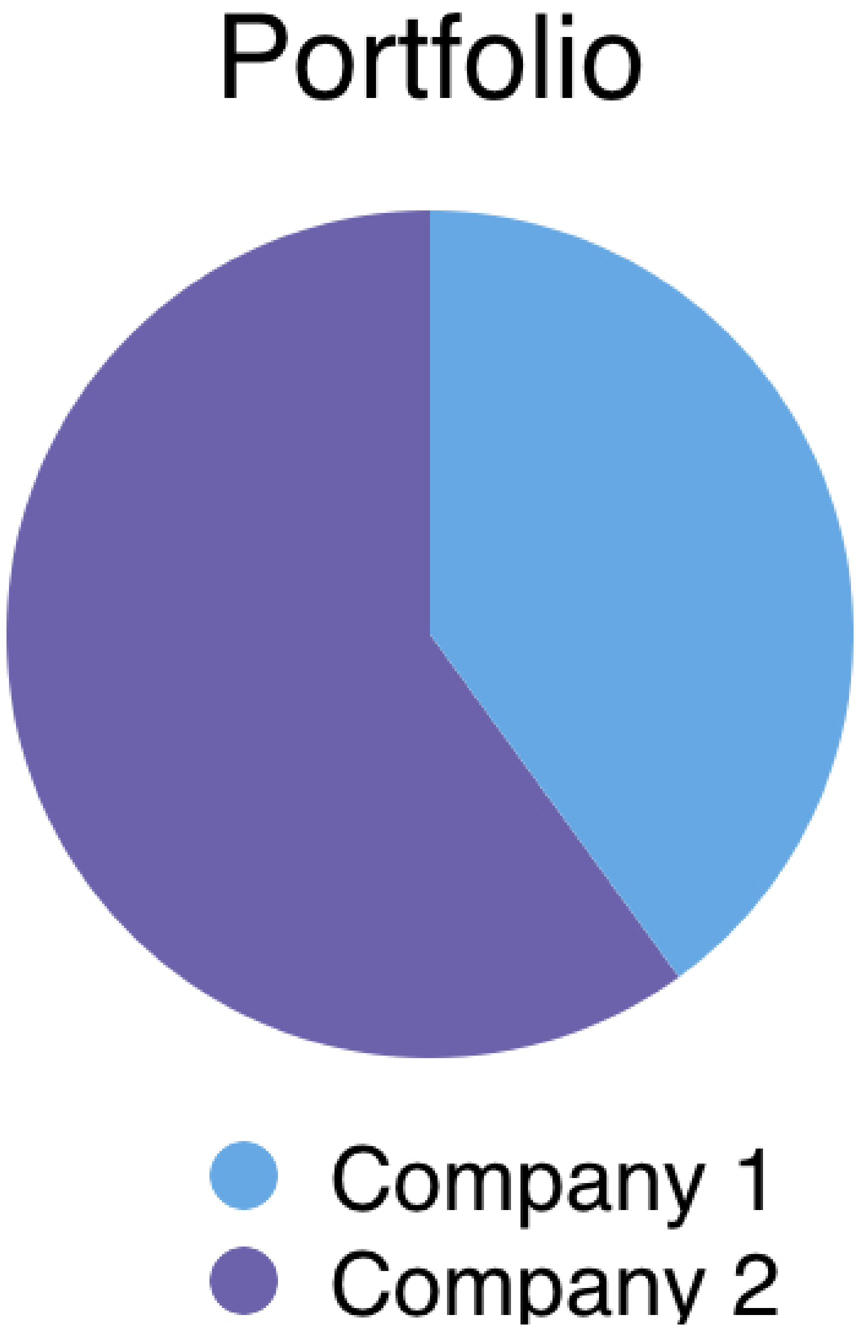

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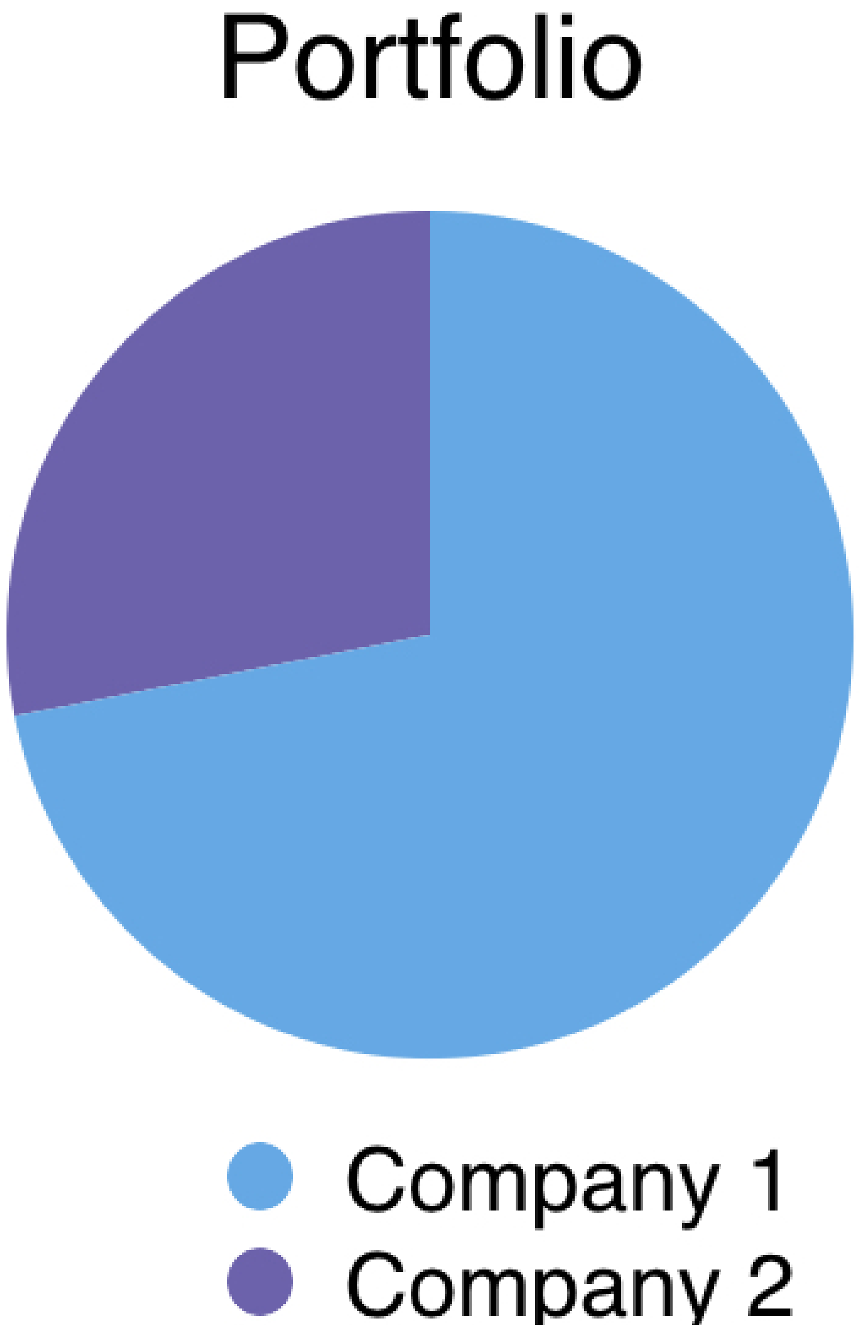

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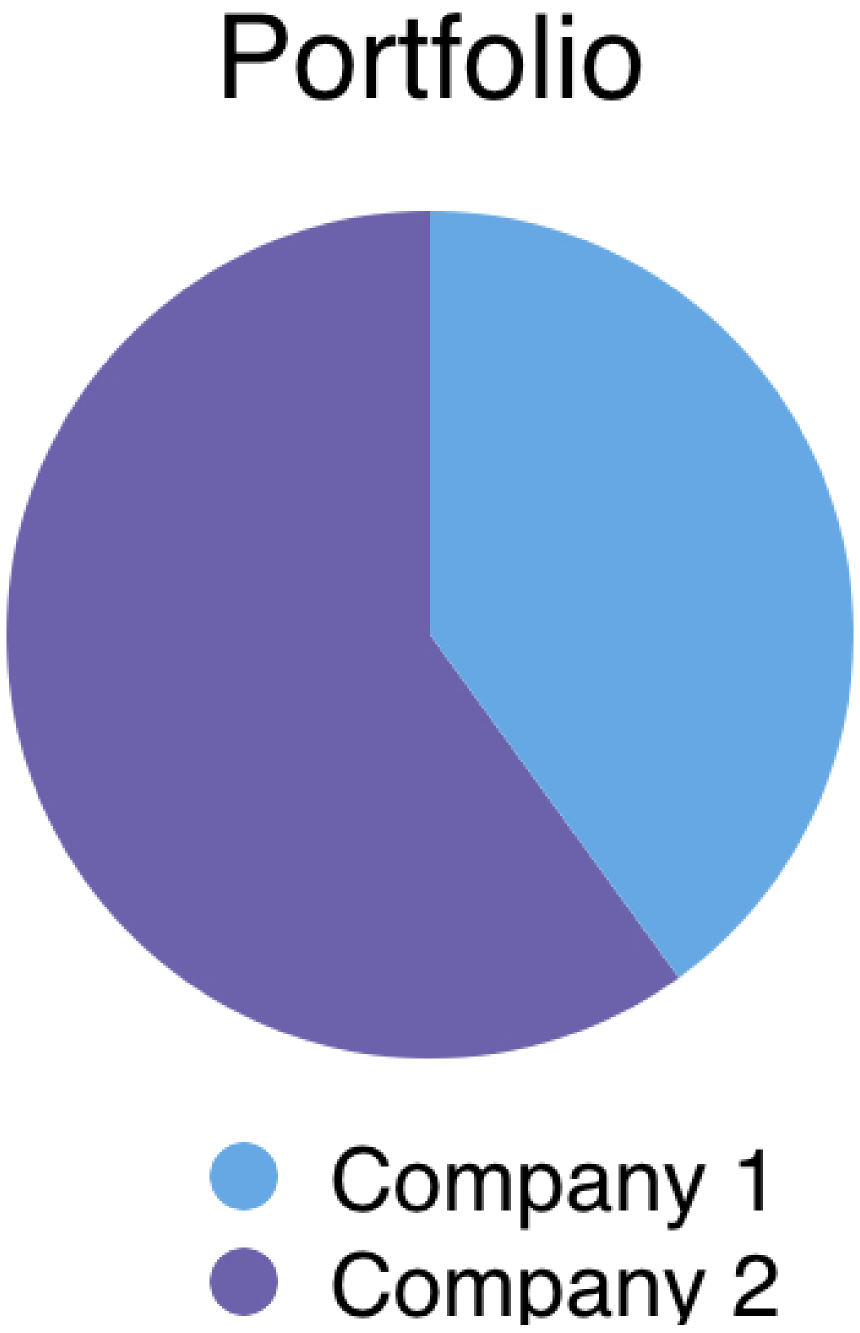

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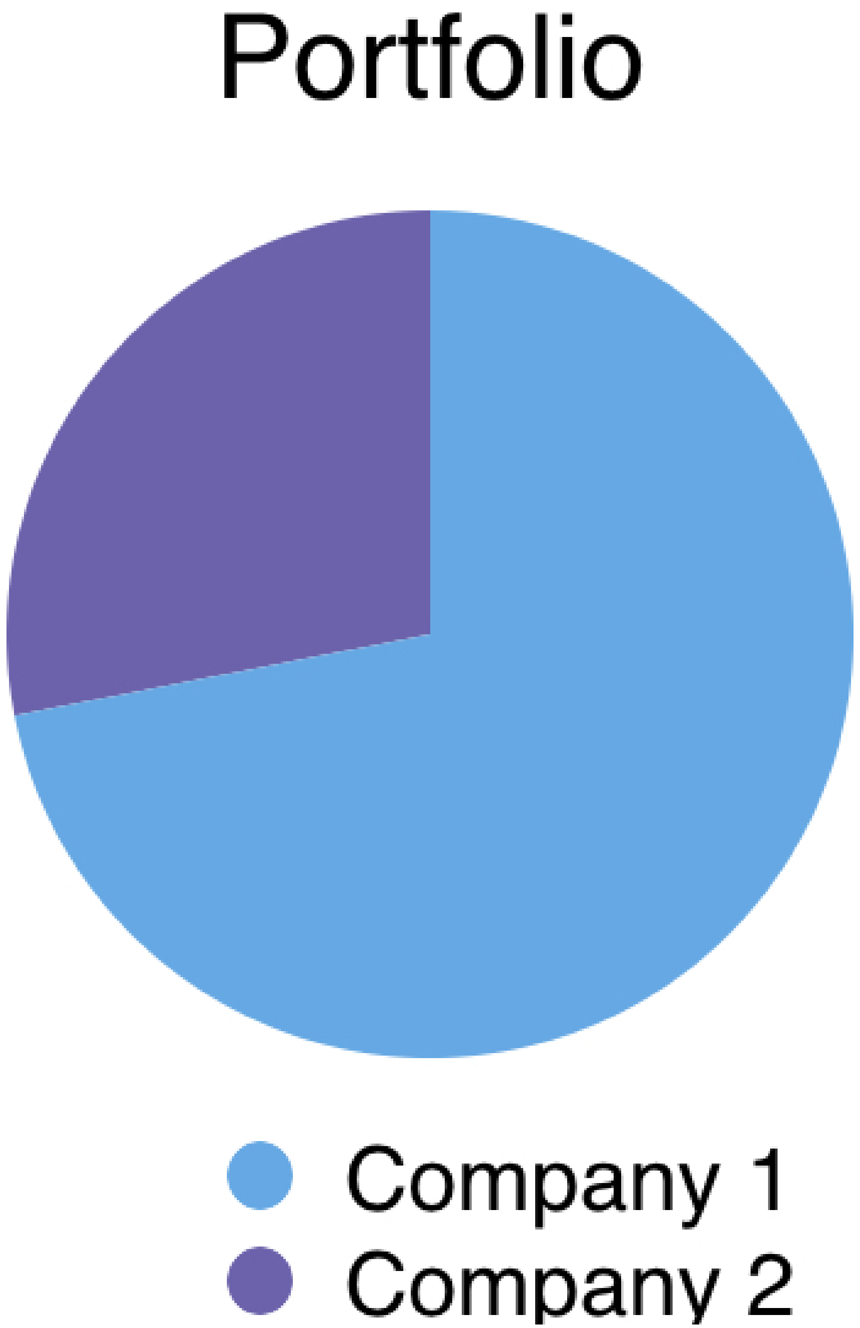

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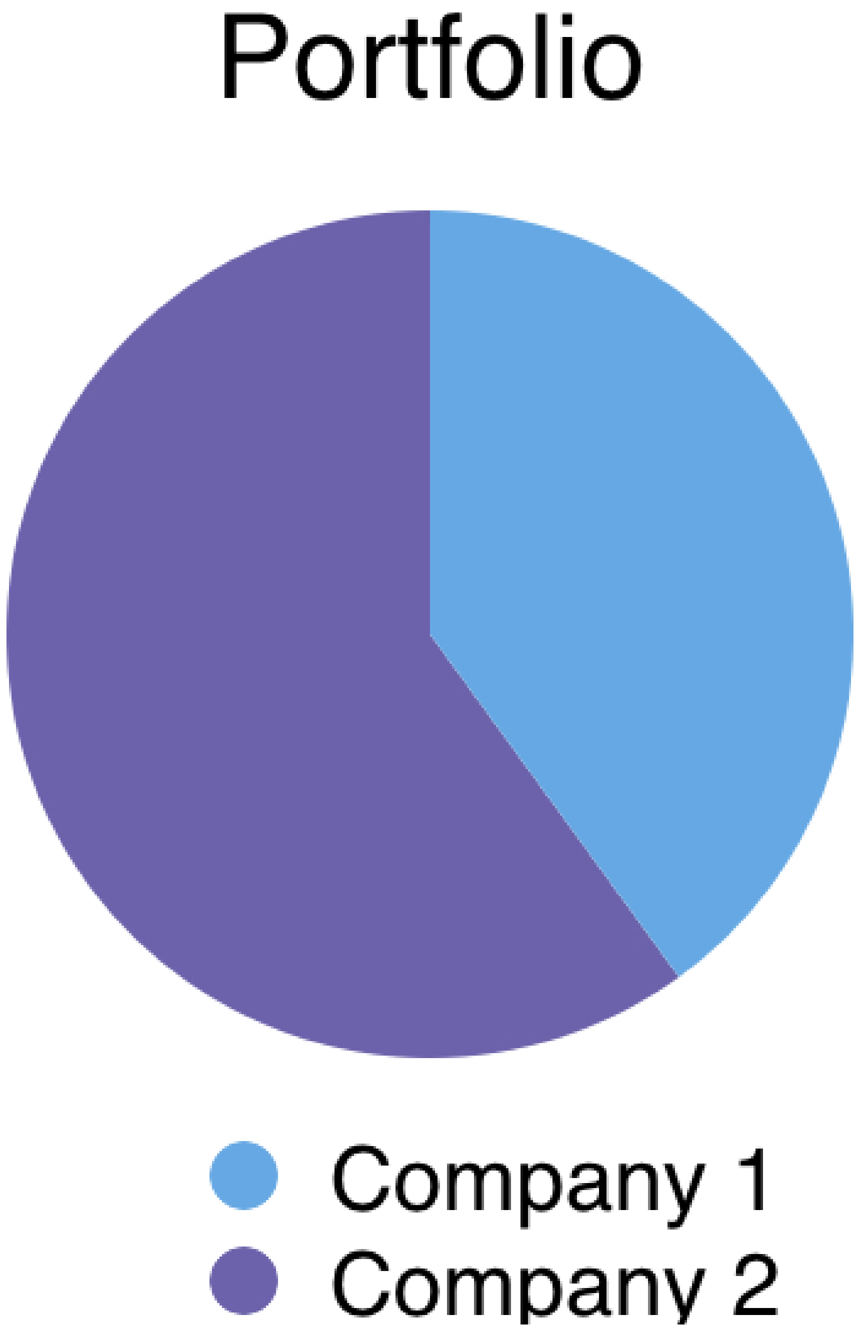
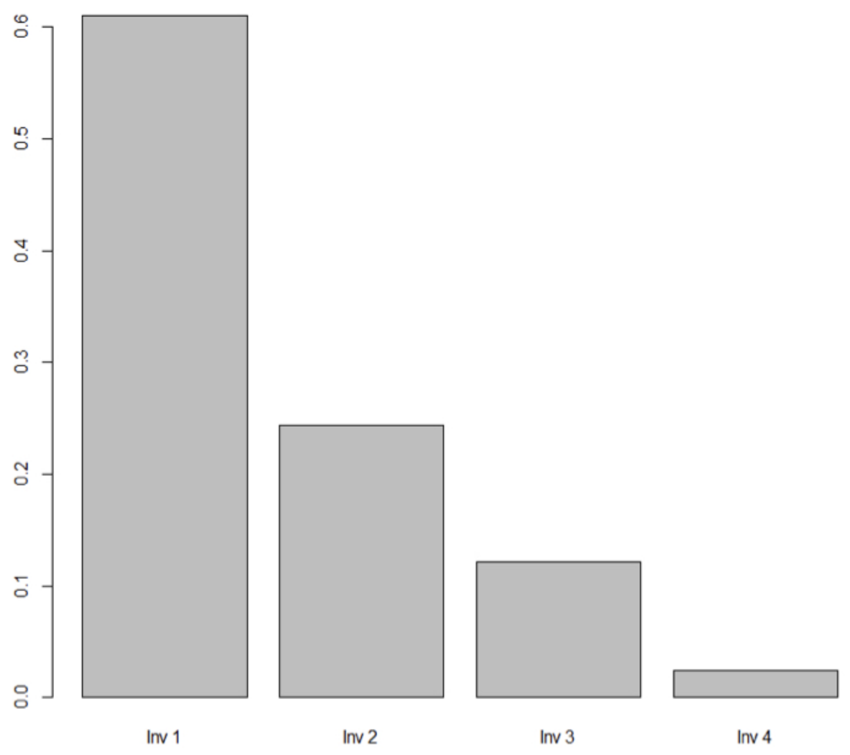


or ...

Asset weighting

Investment	Value Invested	Weight
1	V_1	$w_1 = \frac{V_1}{V_1 + \ldots + V_N}$
2	V_2	$w_2 = \frac{V_2}{V_1 + \ldots + V_N}$
$\vdots$	$\vdots$	$\vdots$
N	V_N	$w_N = \frac{V_N}{V_1 + \ldots + V_N}$

```r
values <- c(500000, 200000, 100000, 20000)
names(values) <- c("Inv 1", "Inv 2", "Inv 3", "Inv 4")
weights <- values/sum(values)
barplot(weights)
```



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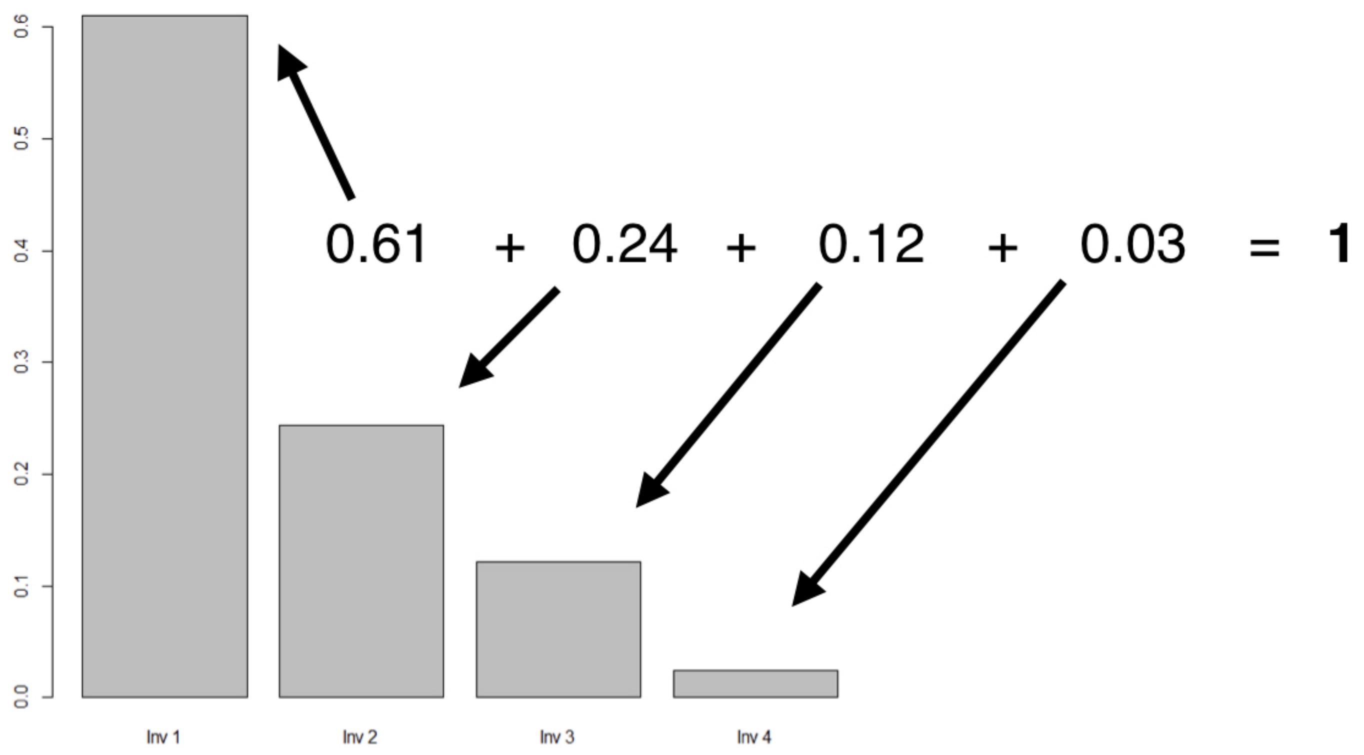
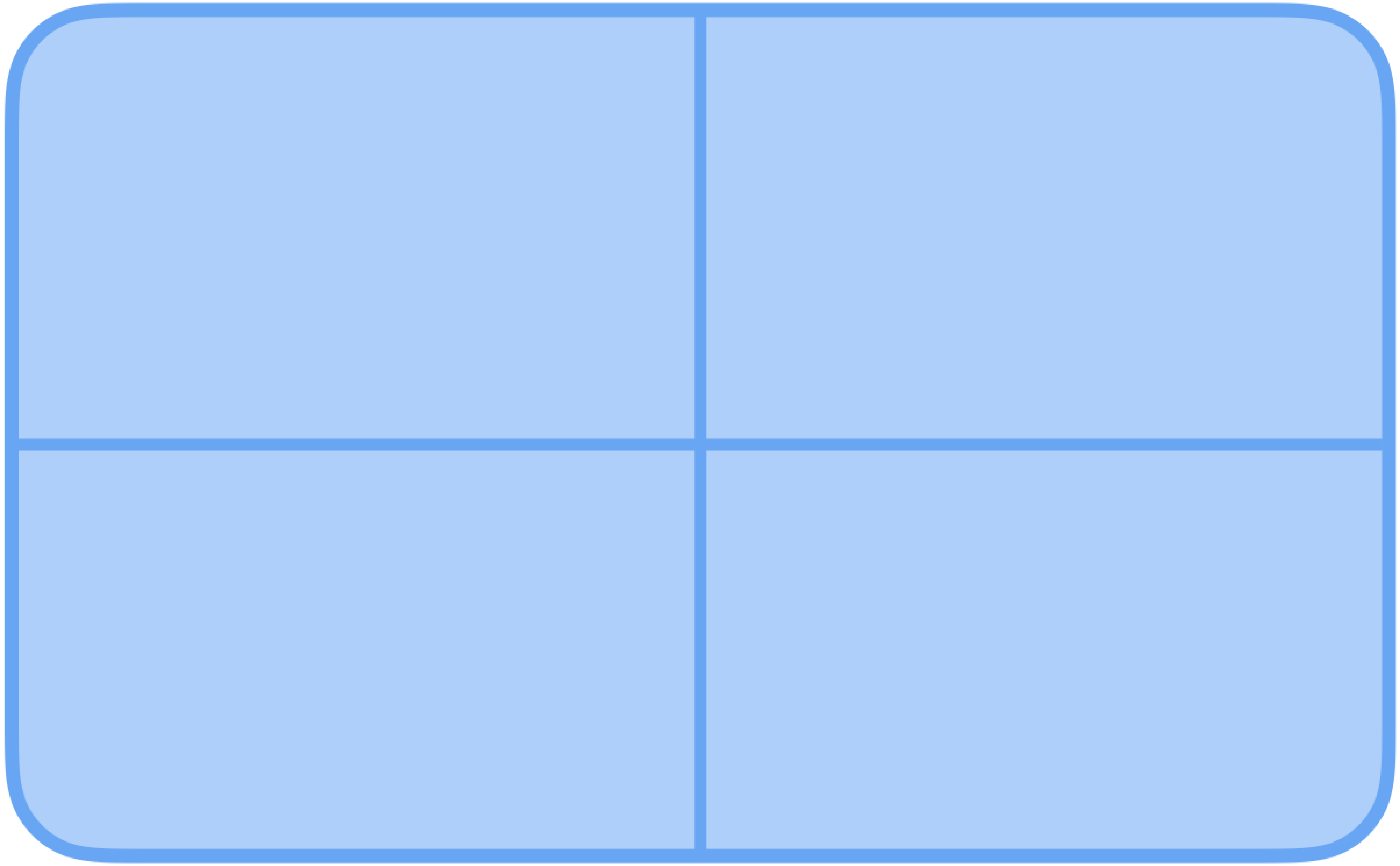


Allocation strategies



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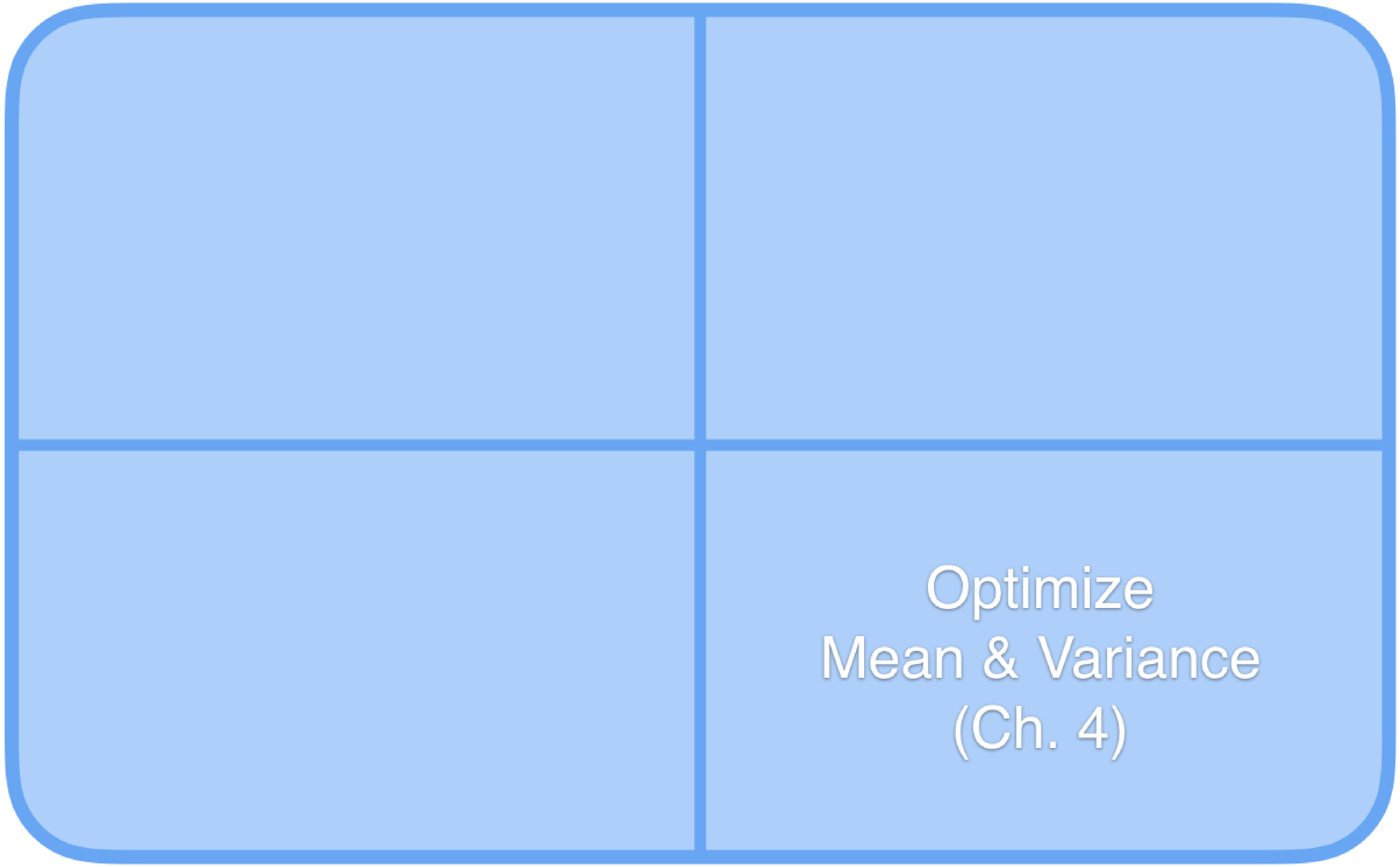

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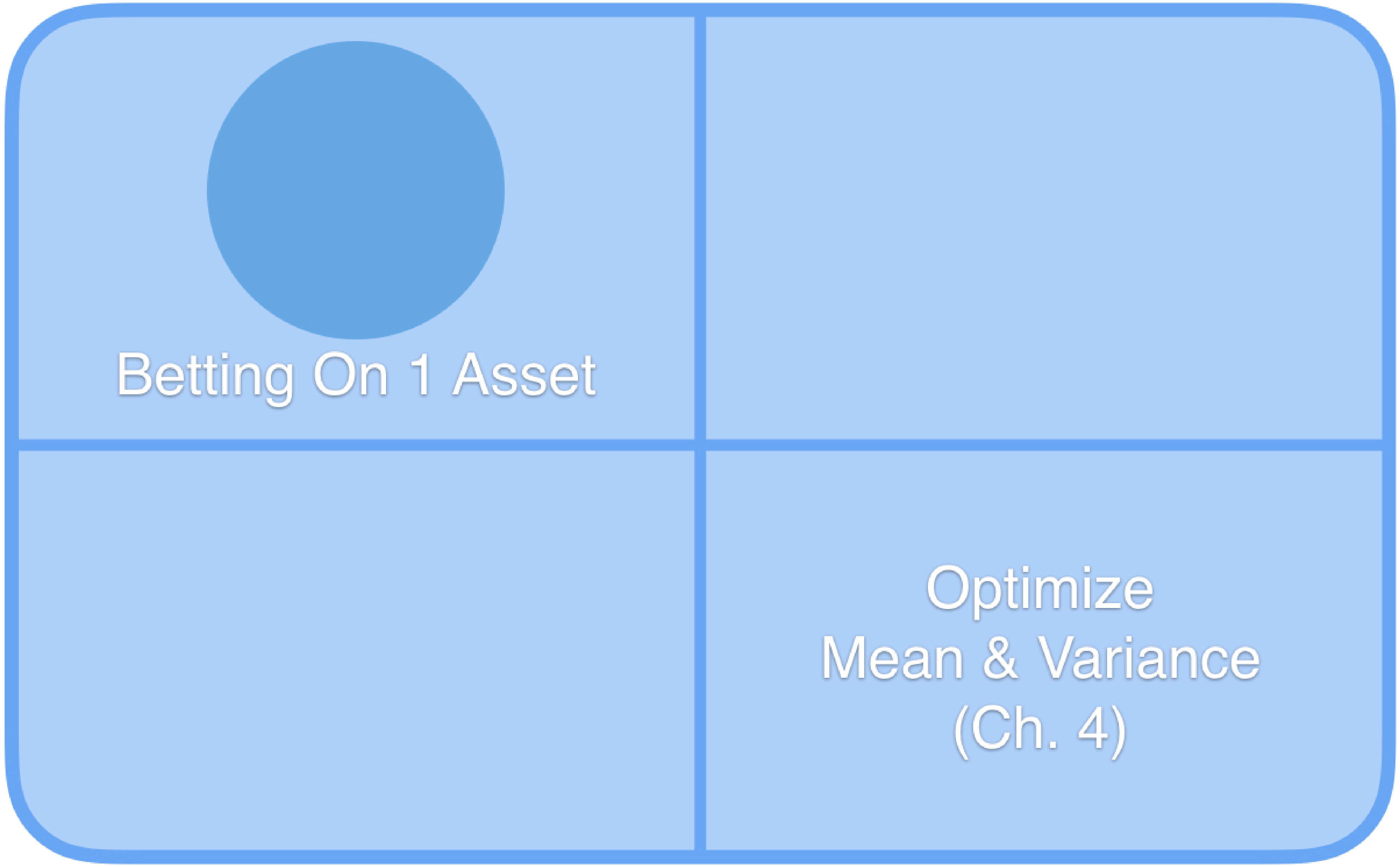

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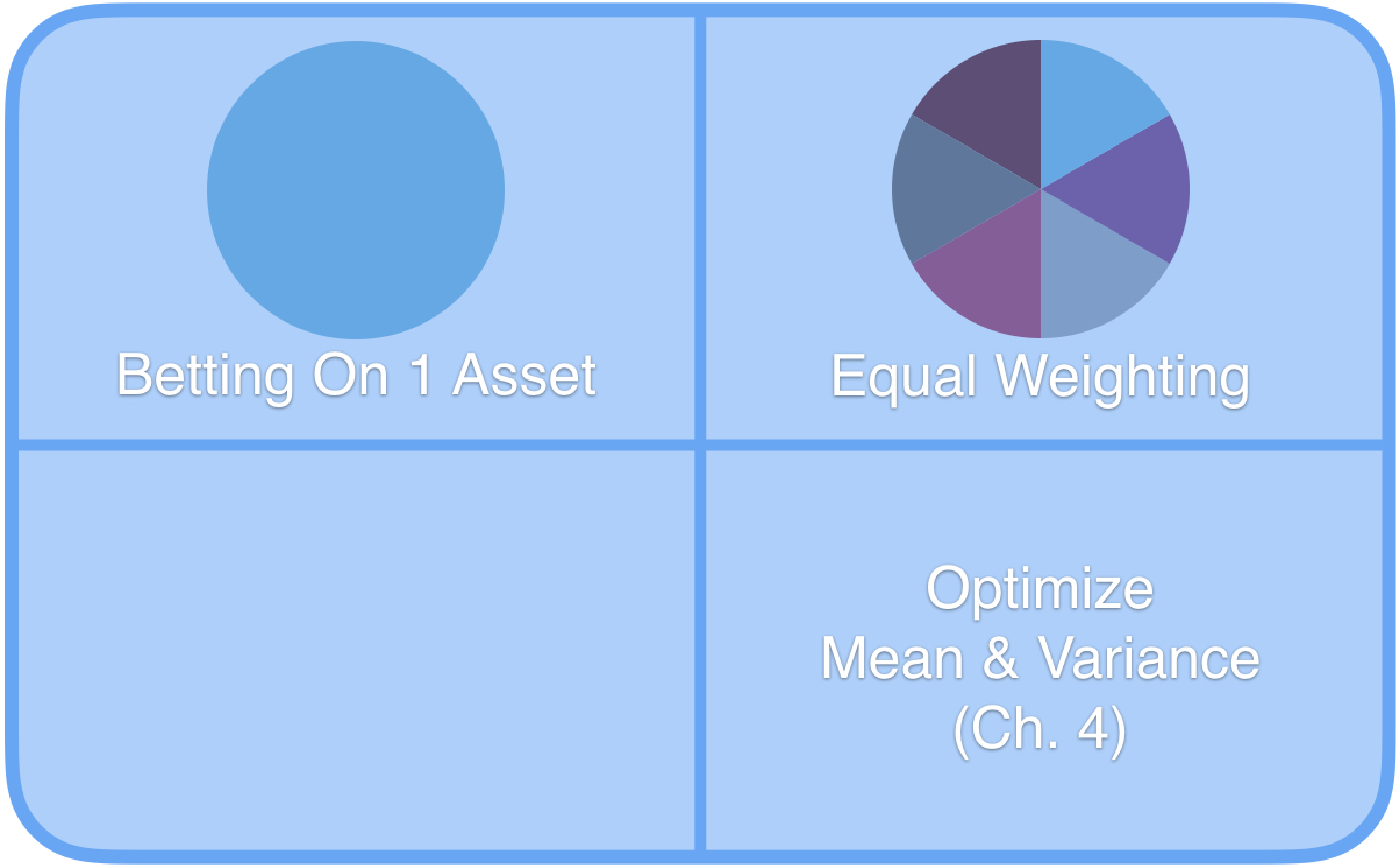

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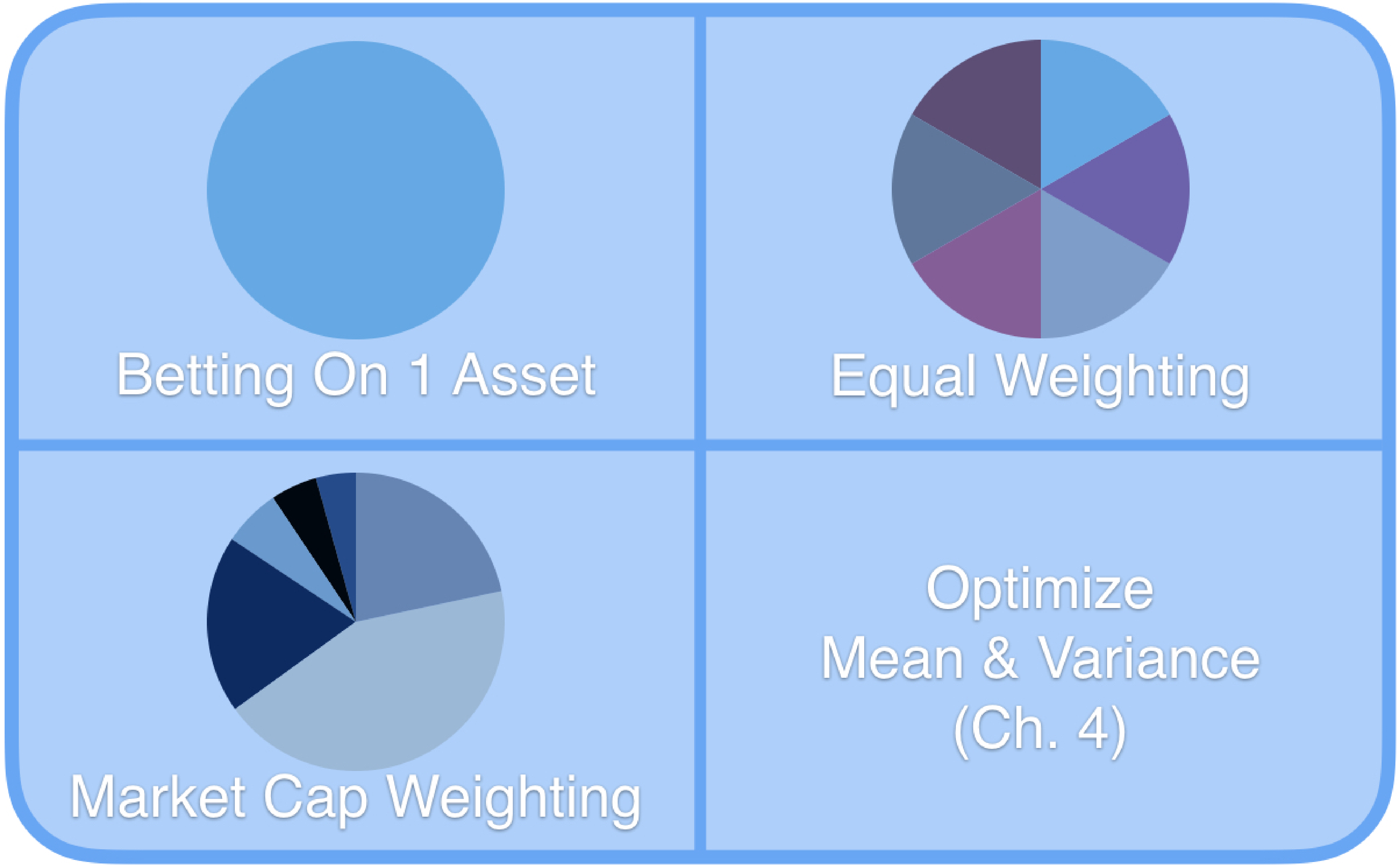

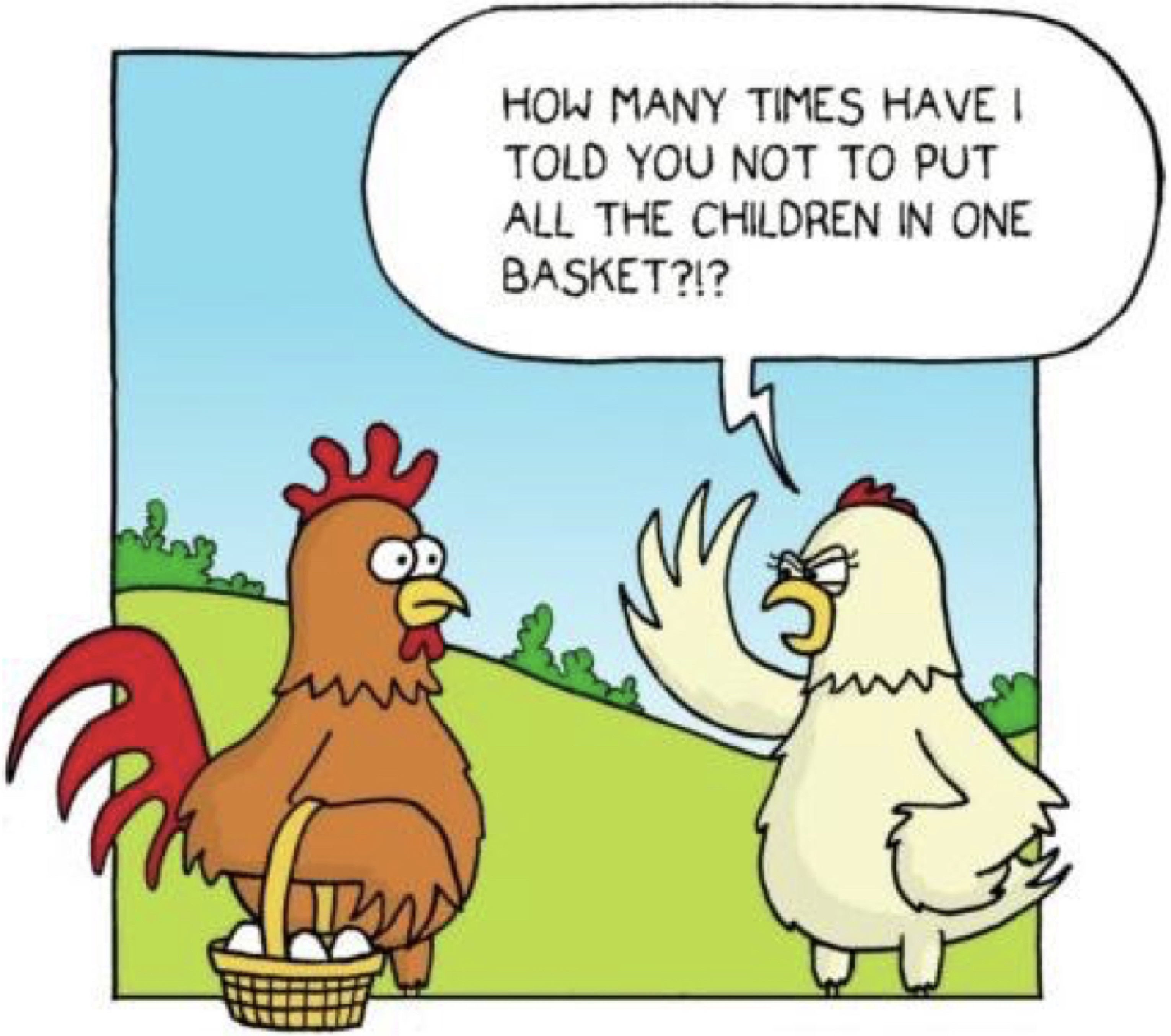

[1] Source: http://www.falibo.com/vocabulary/idiom-dont-put-all-your-eggs-in-one/

Let's practice!

INTRODUCTION TO PORTFOLIO ANALYSIS IN R

The portfolio return

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Portfolio returns: relative value

- Weights reveal active investment bets
- Returns are the relative changes in value:

$$\frac{final\,value\ -initial\,value}{initial\,value}$$

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Final Value	120

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Initial Value	100
Final Value	120

$$\frac{120 - 100}{100} = 20\%$$

Three steps

$Asset_1$	…	$Asset_N$
$InValue.Asset_1$	…	$InValue.Asset_N$
$FinValue.Asset_1$	…	$FinValue.Asset_N$

$InValue.Portfolio = InValue.Asset_1 + \ldots + InValue.Asset_N$
$FinValue.Portfolio = FinValue.Asset_1 + \ldots + FinValue.Asset_N$

Three steps

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$$Portfolio\ Return = \frac{FinValue.Portfolio - InValue.Portfolio}{InValue.Portfolio}$$

Example: two assets

$Asset_1$	$Asset_2$
$InValue.Asset_1 = \$200$	$InValue.Asset_2 = \$300$
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InValue.Portfolio = \$200 + \$300 = \$500
FinValue.Portfolio = \$180 + \$330 = \$510

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InValue.Portfolio = \$200 + \$300 = \$500
FinValue.Portfolio = \$180 + \$330 = \$510

$$Portfolio\ Return = \frac{FinValue.Portfolio - InValue.Portfolio}{InValue.Portfolio} = \frac{510-500}{500} = 2\%$$

Example: two assets

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Portfolio returns: weighted average return

$$Portfolio\ Return = w_1R_1 + w_2R_2 + ... + w_nR_n$$

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Where: $$w_i = \frac{InValue.Asset_i}{\sum_{j=1}^{N} InValue.Asset_j}$$

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Three steps

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Asset $_1$	Asset$_N$
$w_1 = \frac{InValue.Asset_1}{InValue.Portfolio}$	$w_n = \frac{InValue.Asset_n}{InValue.Portfolio}$
$R_1 = \frac{FinValue.\,Asset_1 - InValue.\,Asset_1}{InValue.\,Asset_1}$	$R_n = \frac{FinValue.\,Asset_n - InValue.\,Asset_n}{InValue.\,Asset_n}$

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⇩

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$FinValue.Asset_1$ = \$180	$FinValue.Asset_2$ = \$330

$Asset_1$	$Asset_2$
$w_1 = \frac{200}{500} = 40\%$	$w_2 = \frac{300}{500} = 60\%$
$R_1 = \frac{180 - 200}{200} = -10\%$	$R_2 = \frac{330 - 300}{300} = 10\%$

Example: two assets

Asset $_1$	Asset$_2$
InValue.Asset$_1$ = \$200	InValue.Asset$_2$ = \$300
FinValue.Asset$_1$ = \$180	FinValue.Asset$_2$ = \$330

$\Downarrow$

Asset $_1$	Asset$_2$
$w_1 = \frac{200}{500} = 40\%$	$w_2 = \frac{300}{500} = 60\%$
$R_1 = \frac{180 - 200}{200} = -10\%$	$R_2 = \frac{330 - 300}{300} = 10\%$

$\Downarrow$

$$Portfolio\ Return = 0.4^{*}(-10\%) + 0.6^{*}(10\%) = 2\%$$

Let's practice!

INTRODUCTION TO PORTFOLIO ANALYSIS IN R

PerformanceAnalytics

INTRODUCTION TO PORTFOLIO ANALYSIS IN R

Kris Boudt
Professor, Free University Brussels & Amsterdam

The practitioner's challenge

- In practice, time series of portfolio returns
- Longer history → more info on portfolio
- Good package = `PerformanceAnalytics`

The creators

- `PerformanceAnalytics` is the go-to package for portfolio return analysis in R

Peter Carl

Brian Peterson

[1] https://tradeblotter.files.wordpress.com/2012/02/bwauthorpcc.jpeg

Calculating returns

- `Return.calculate` : to compute the asset returns
- `Return.portfolio` : to compute the portfolio return
- `Return.calculate(prices)`
  - `xts` -object
- Dates structure: `YYYY-MM-DD`

Calculating returns

- `Return.calculate`

```
returns <- Return.calculate(prices)
```

```
returns <- returns[(-1),]
```

```
head(prices)
```

```
AAPL    MSFT
2006-01-03 9.829465  21.07395
2006-01-04 9.858394  21.17603
2006-01-05 9.780810  21.19173
...
```

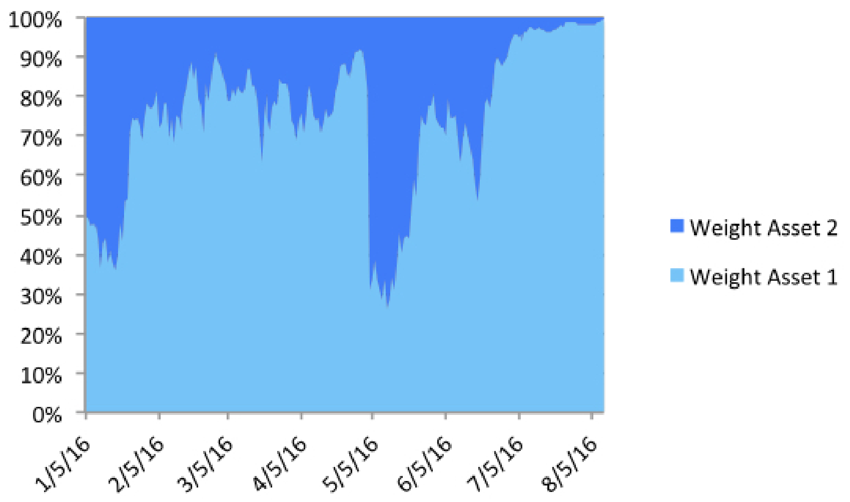
```
head(returns)
```

```
                  AAPL         MSFT
2006-01-03   NA           NA
2006-01-04   0.002943090  0.0048434670
2006-01-05  -0.007869842  0.0007415934
...
```

Dynamics of portfolio weights

Set Initial Weights & Do Not Intervene

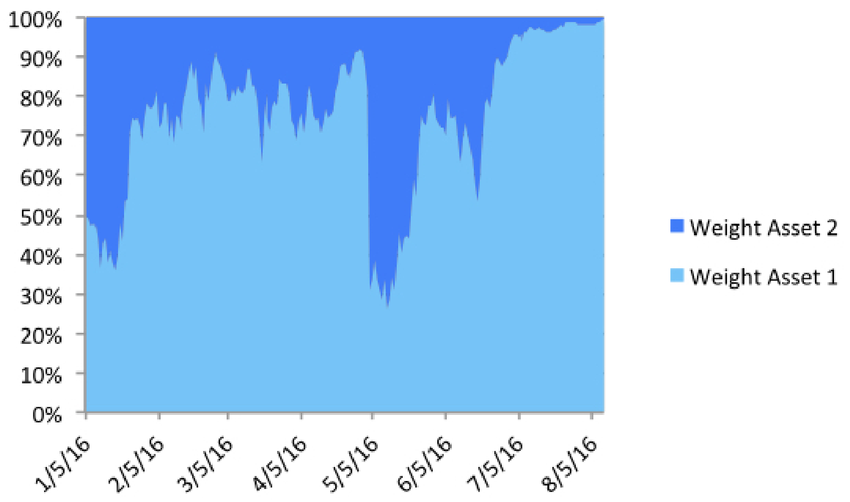
Example: Initial 50/50 weight



Dynamics of portfolio weights

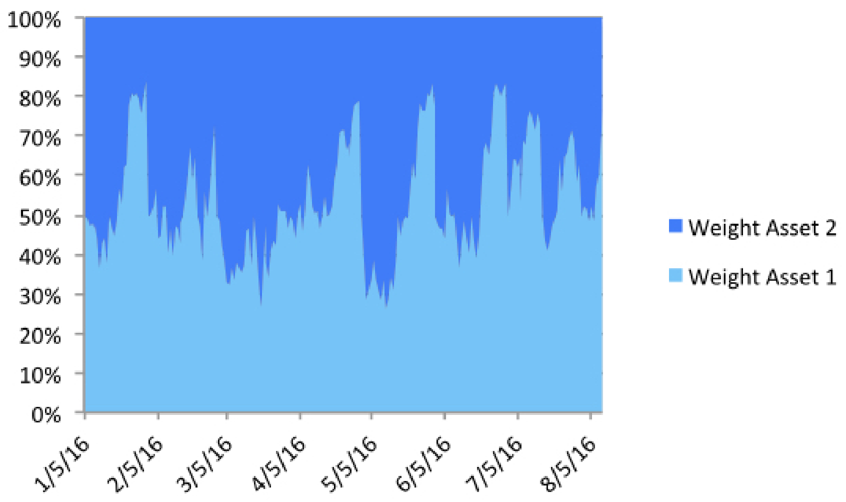
Set Initial Weights & Do Not Intervene

Example: Initial 50/50 weight



Actively Change Portfolio Weights

Example: 50/50 Weight With Rebalance



Portfolio returns

```
Return.portfolio <- function(R, weights = NULL,
    rebalance_on = c(NA, "years", "quarters",
                     "months", "weeks", "days"))
```

- Three arguments to be specified:
  - return data
  - weights
  - rebalancing

Let's practice!

INTRODUCTION TO PORTFOLIO ANALYSIS IN R

Dimensions of portfolio performance

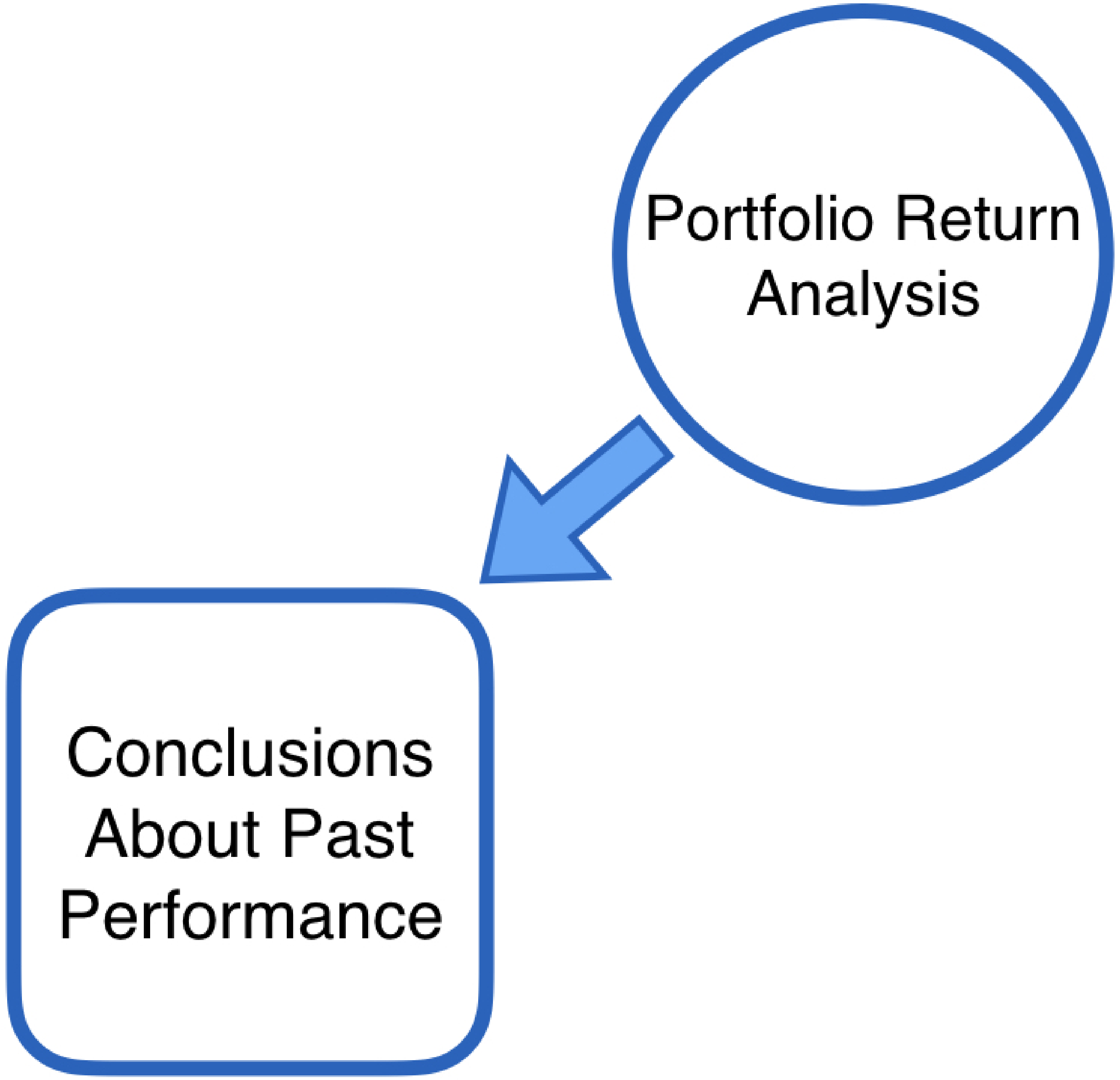
INTRODUCTION TO PORTFOLIO ANALYSIS IN R

Kris Boudt

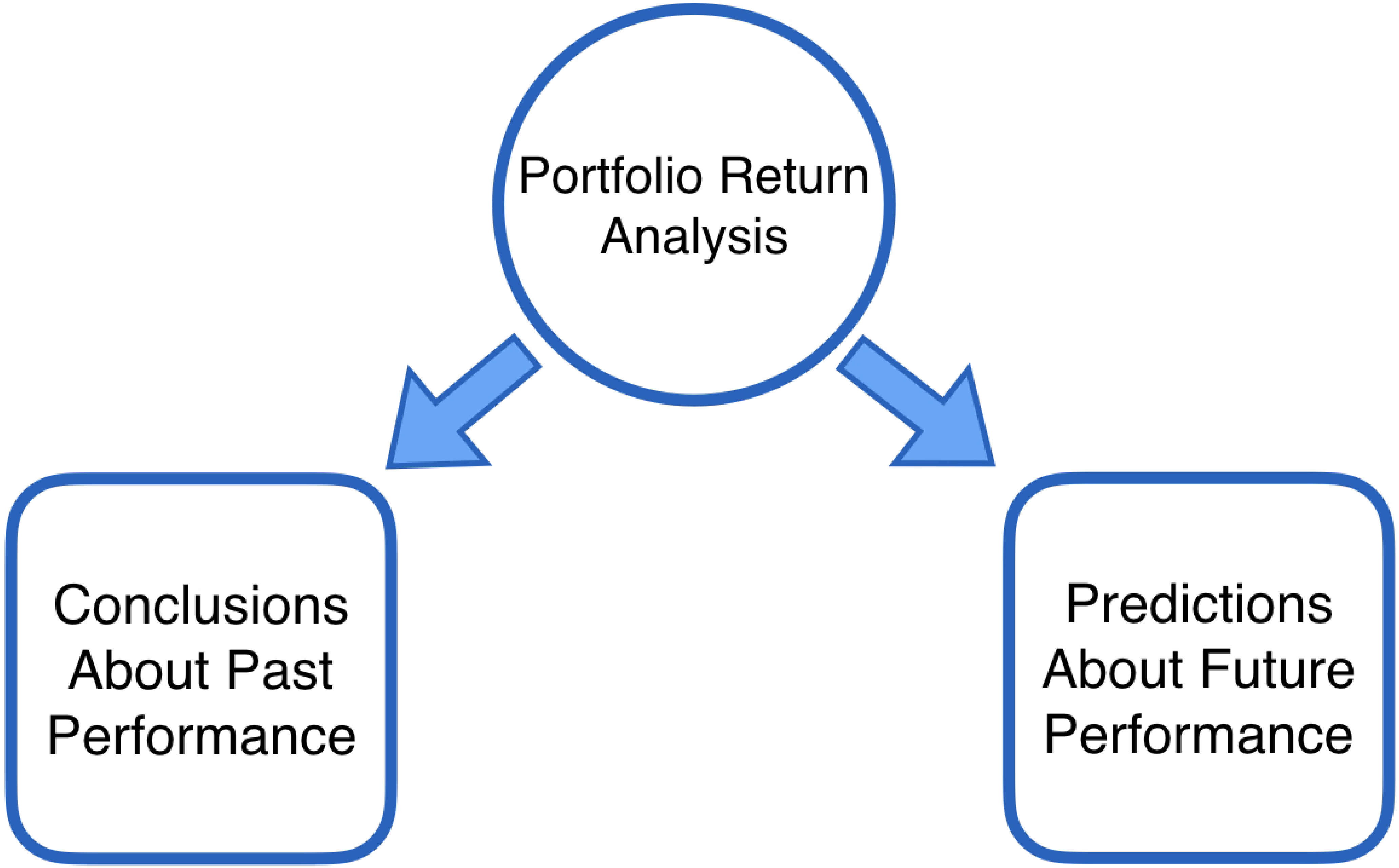
Professor, Free University Brussels & Amsterdam

Interpretation of portfolio returns

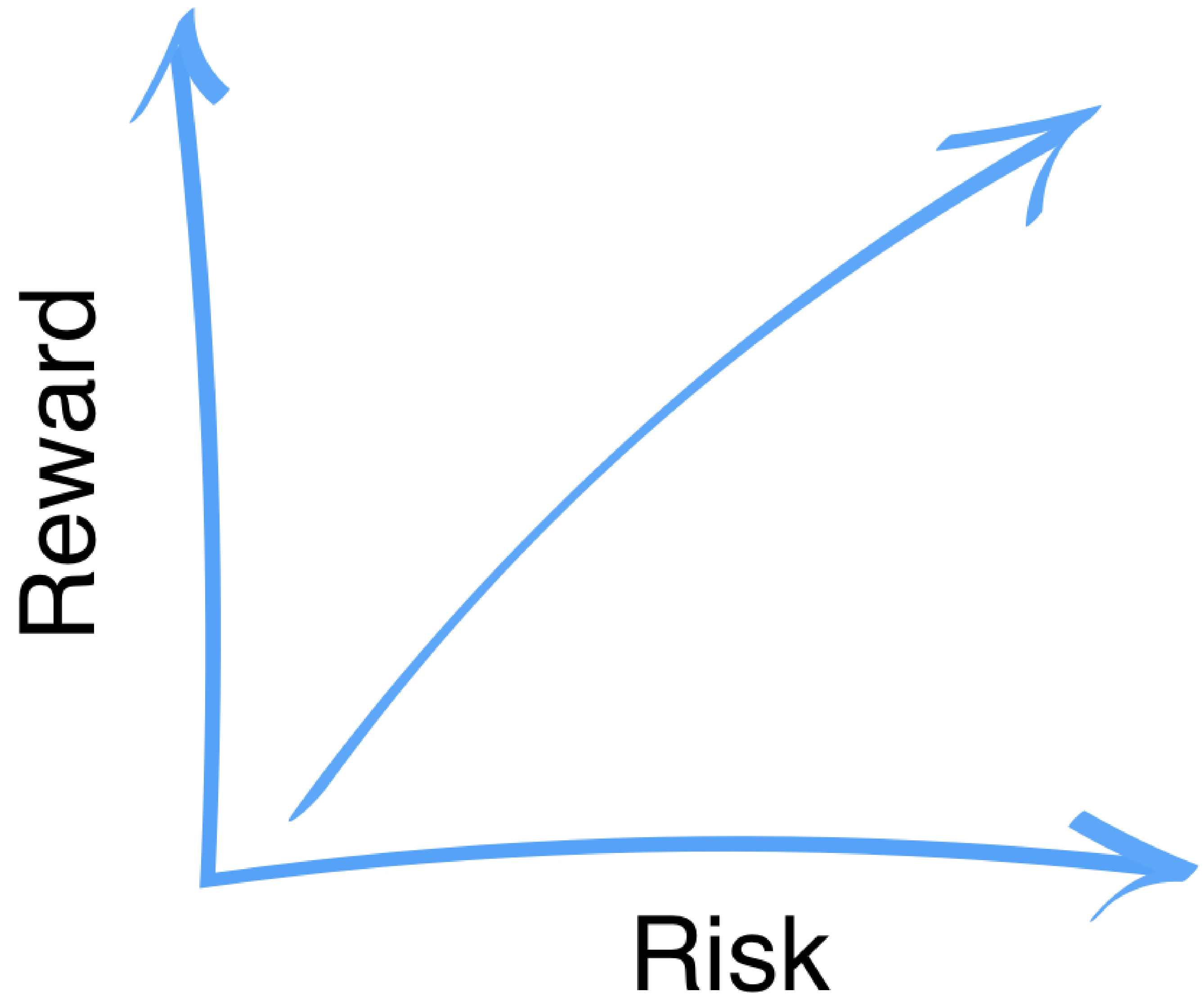
Interpretation of portfolio returns



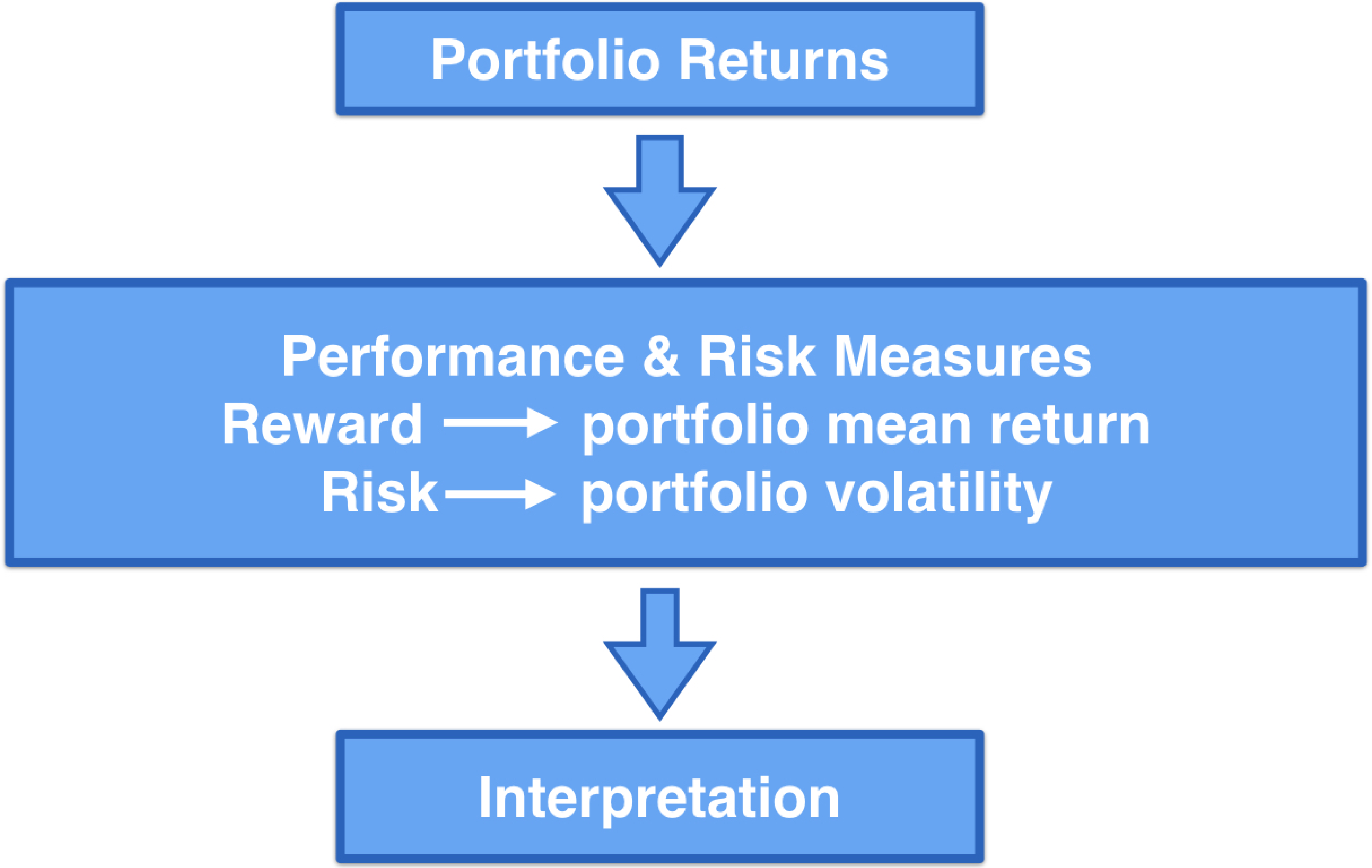
Interpretation of portfolio returns



Risk vs. reward



Need for performance measure



Arithmetic mean return

- Assume a sample of T portfolio return observations:
  - $R_1, R_2, ..., R_T$
- Reward measurement: Arithmetic mean return is given:
  - $\hat{\mu} = \dfrac{R_1, R_2, ..., R_T}{T}$
- It shows how large the portfolio return is on average

Risk: portfolio volatility

- De-meaned return
  - $R_i - \hat{\mu}$
- Variance of the portfolio
  - $\hat{\sigma}^2 = \dfrac{(R_1 - \hat{\mu})^2 + (R_2 - \hat{\mu})^2 + ... + (R_T - \hat{\mu})^2}{T_1}$
- Portfolio volatility:
  - $\hat{\sigma} = \sqrt{\hat{\sigma}^2}$

No linear compensation in return

- Mismatch between average return and effective return

final value=
initial value * (1 +0.5)*(1-0.5)= 0.75 * initial value

No linear compensation in return

- Mismatch between average return and effective return

final value=
initial value * (1 +0.5)*(1-0.5)= 0.75 * initial value

No linear compensation in return

- Mismatch between average return and effective return

final value=
initial value * (1 +0.5)*(1-0.5)= 0.75 * initial value

↓

Average Return = (0.5 - 0.5) / 2 = 0

Geometric mean return

- Formula for Geometric Mean for a sample of T portfolio return observations $R_1, R_2, ..., R_T$:

Geometric mean = $[(1+R_1)\cdot(1+R_2)\cdot ...(1+R_T)]^{1/T}-1$

- Example: +50% & -50% return
  - Geometric mean = $[(1+0.50)\cdot(1-0.50)]^{1/2}-1$
  - = $0.75^{1/2}-1$
  - = -13.4%


Google
amazon.com
ebay
priceline.com
E*TRADE
NETFLIX
ALEXION
EXPRESS SCRIPTS
Ford
LEGG MASON
RALPH LAUREN
WHOLE FOODS
comcast
NVIDIA
CITRIX
Ameriprise
JANUS CAPITAL
FIFTH THIRD BANK
SUNTRUST
McAfee
Cognizant
IPG
TEXTRON
LEXMARK
TERADYNE
TSYS
QUALCOMM
WYNDHAM
POLO RALPH LAUREN
Expedia
CBRE
metroPCS
EA
xerox
Lincoln
Morgan Stanley
C.H. ROBINSON
ALLERGAN
E·T·N
STATE STREET
NOVELLUS
Robert Half
Cigna
PROLOGIS
Pfizer
DaVita
KeyCorp
ups
vf
COVENTRY
EXXON Mobil
Microsoft
Walmart
Johnson & Johnson
GE
WELLS FARGO
Chevron
JPMorganChase
P&G
IBM
verizon
Bank of America
ORACLE
at&t
Coca-Cola
MERCK
citi
Walt Disney
VISA
Sara Lee
The Washington Post
CME Group
Lorillard
BLACKROCK
Prudential
Public Storage
LOCKHEED MARTIN
Avalon
SIMON
Entergy
COACH
Kimberly-Clark
MetLife
CenturyLink
3M
Adobe
AA
BOEING
Capital One
CISCO
COSTCO WHOLESALE
DELTA
FOSSIL
GM
Kraft
MICHAEL KORS
NIKE
ROSS
SEAGATE
SOUTHWEST AIRLINES
Symantec.
Time Warner Cable
AMERICAN EXPRESS
CAT
Baxter
BB&T
CSX

Let's practice!

INTRODUCTION TO PORTFOLIO ANALYSIS IN R

The (annualized) Sharpe ratio

INTRODUCTION TO PORTFOLIO ANALYSIS IN R

Kris Boudt
Professor, Free University Brussels & Amsterdam

Benchmarking performance

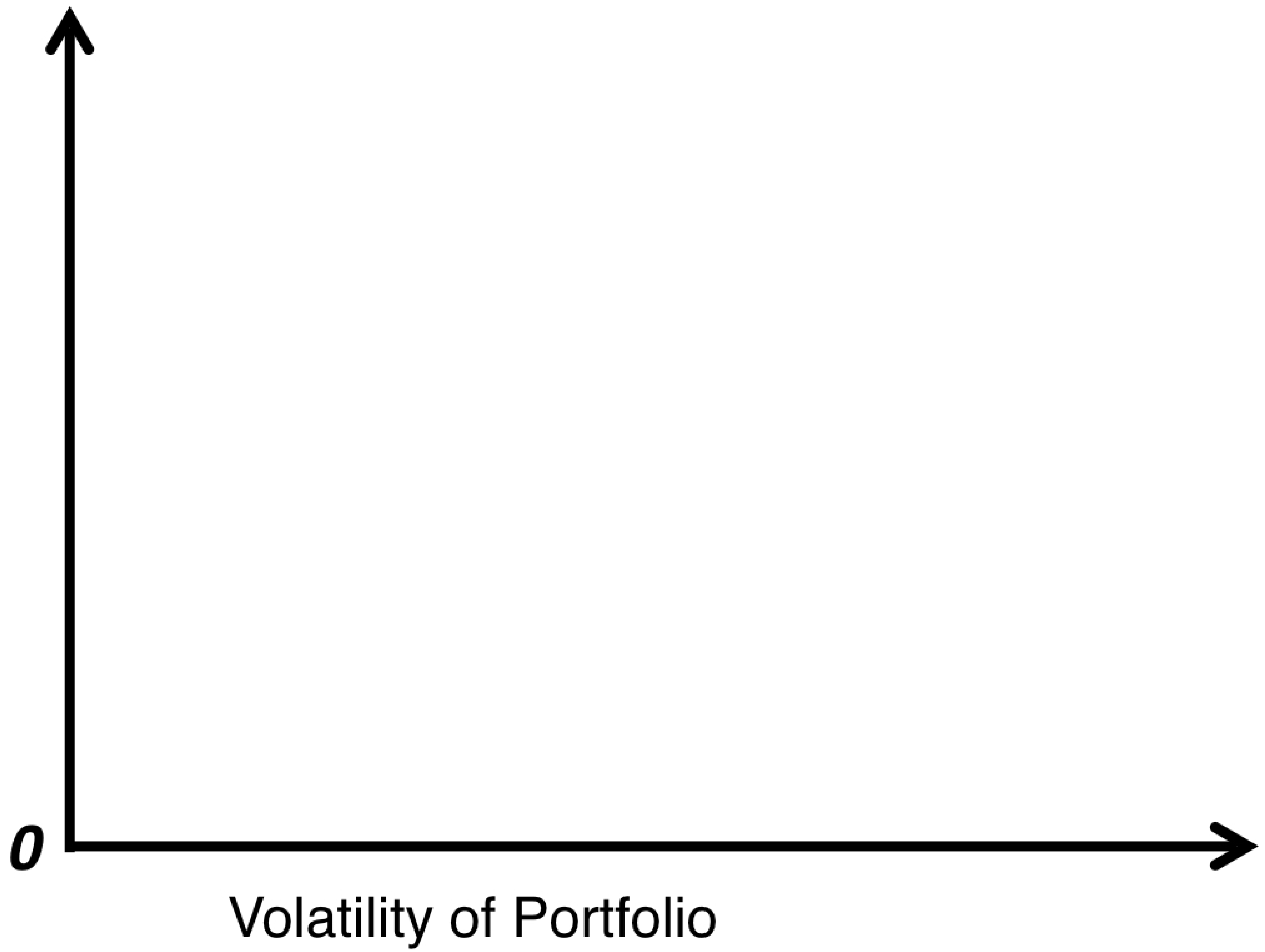
Risky Portfolio E.g: portfolio invested in stocks, bonds, real estate, and commodities
Reward: measured by mean portfolio return
Risk: measured by volatility of the portfolio returns

Benchmarking performance

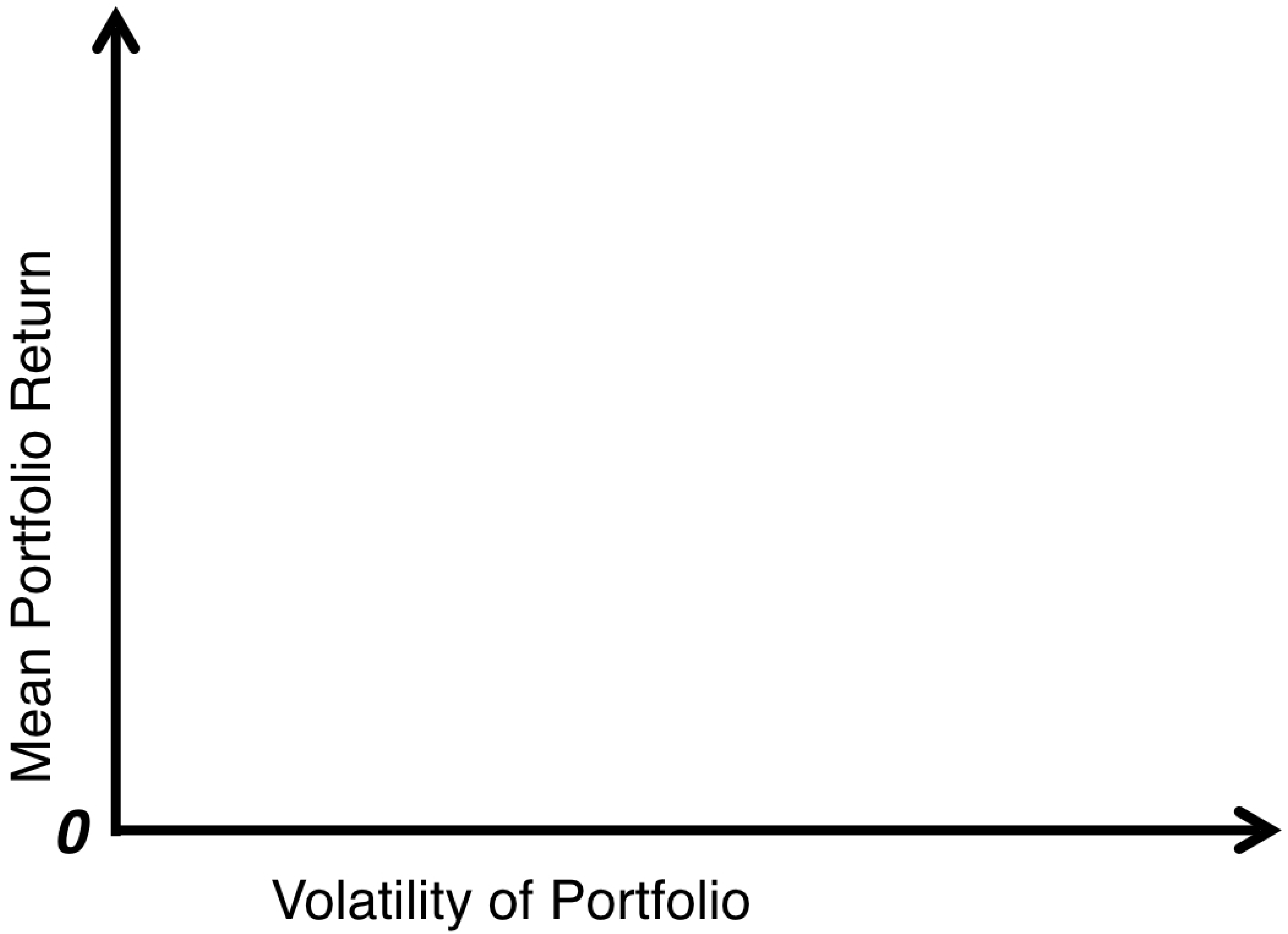
Risky Portfolio E.g: portfolio invested in stocks, bonds, real estate, and commodities
Reward: measured by mean portfolio return
Risk: measured by volatility of the portfolio returns

Risk Free Asset E.g: US Treasury Bill
Reward: measured by risk free rate
Risk: No risk, volatility = 0 return = risk free rate

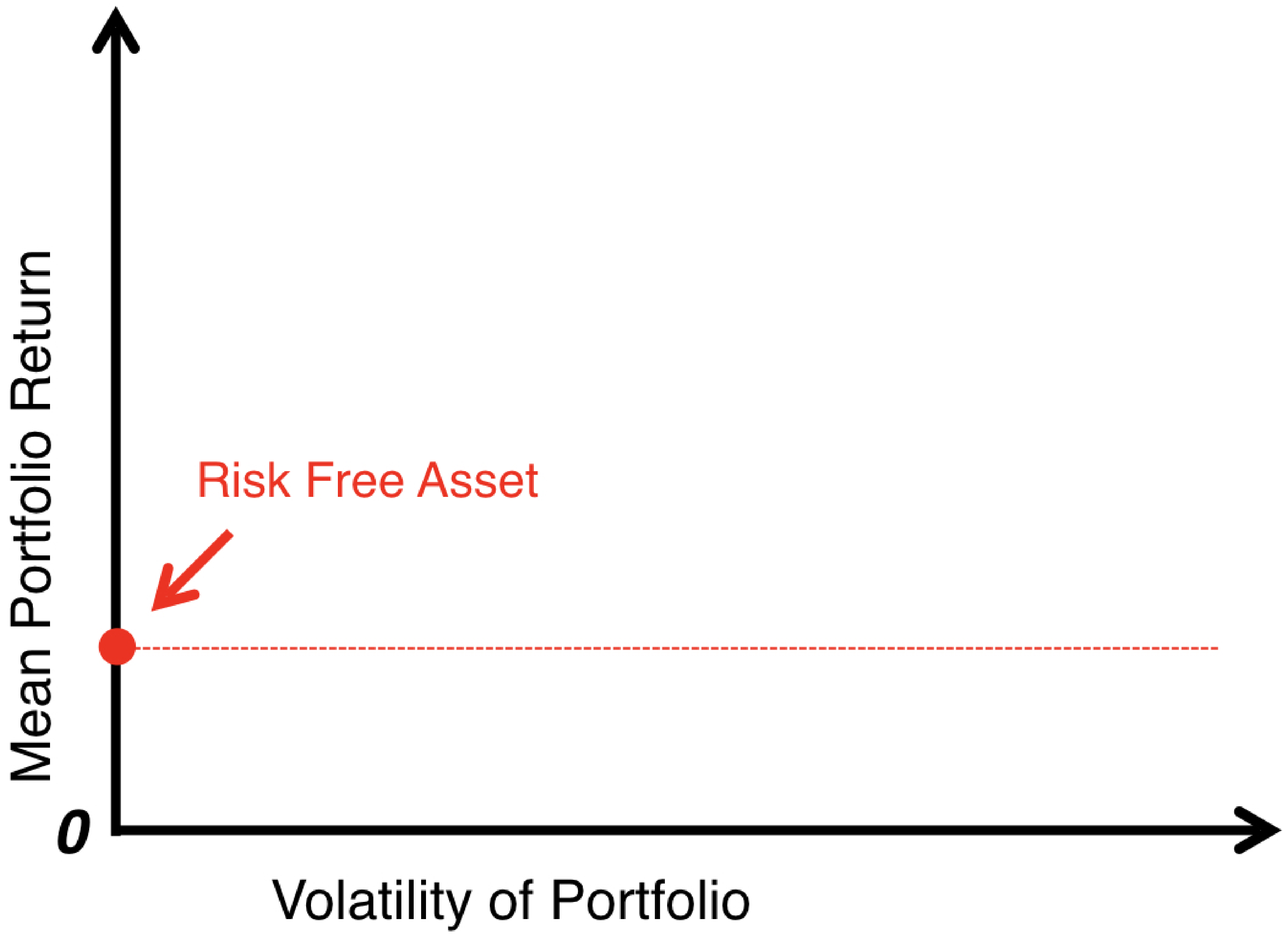
Risk-return trade-off



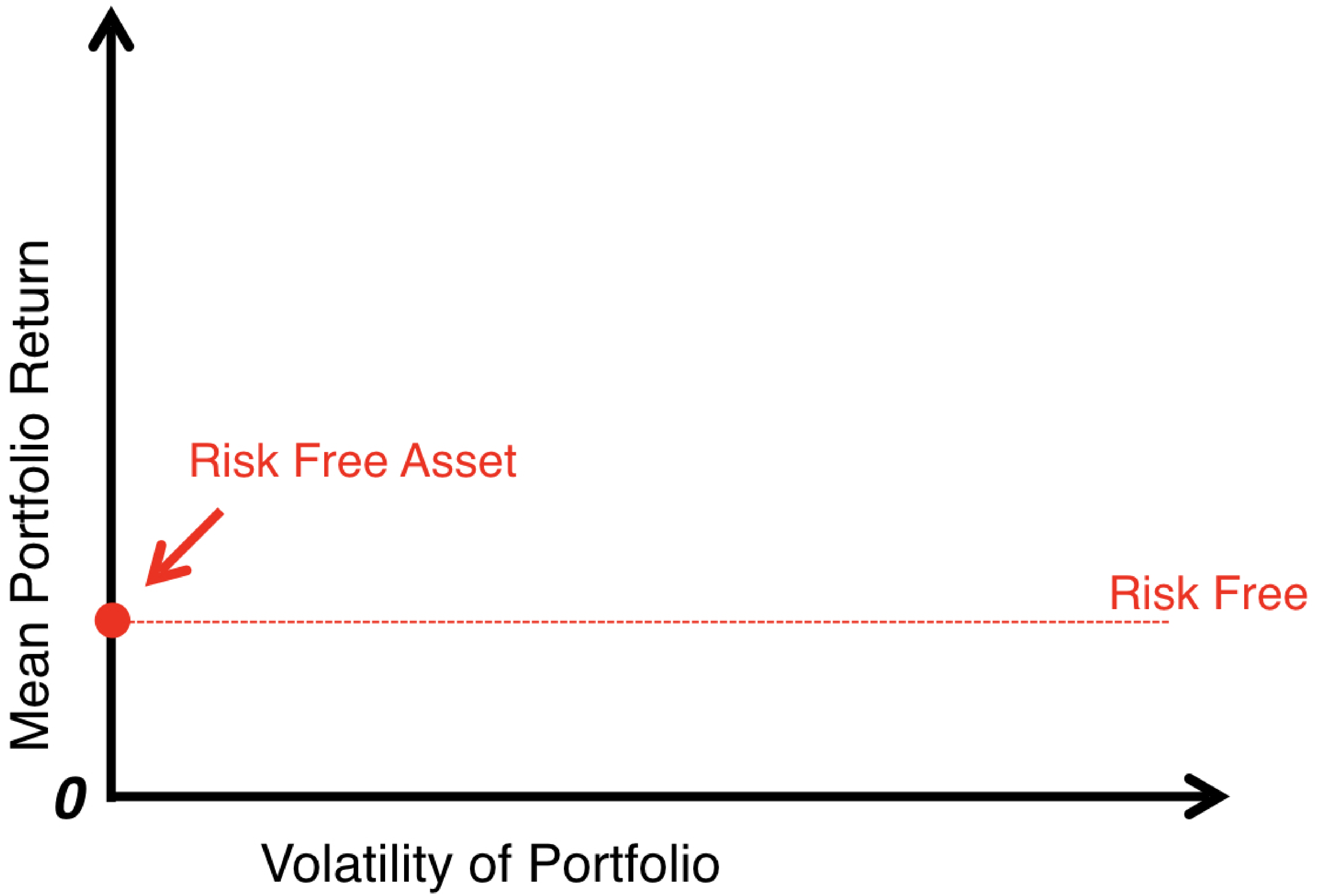
Risk-return trade-off



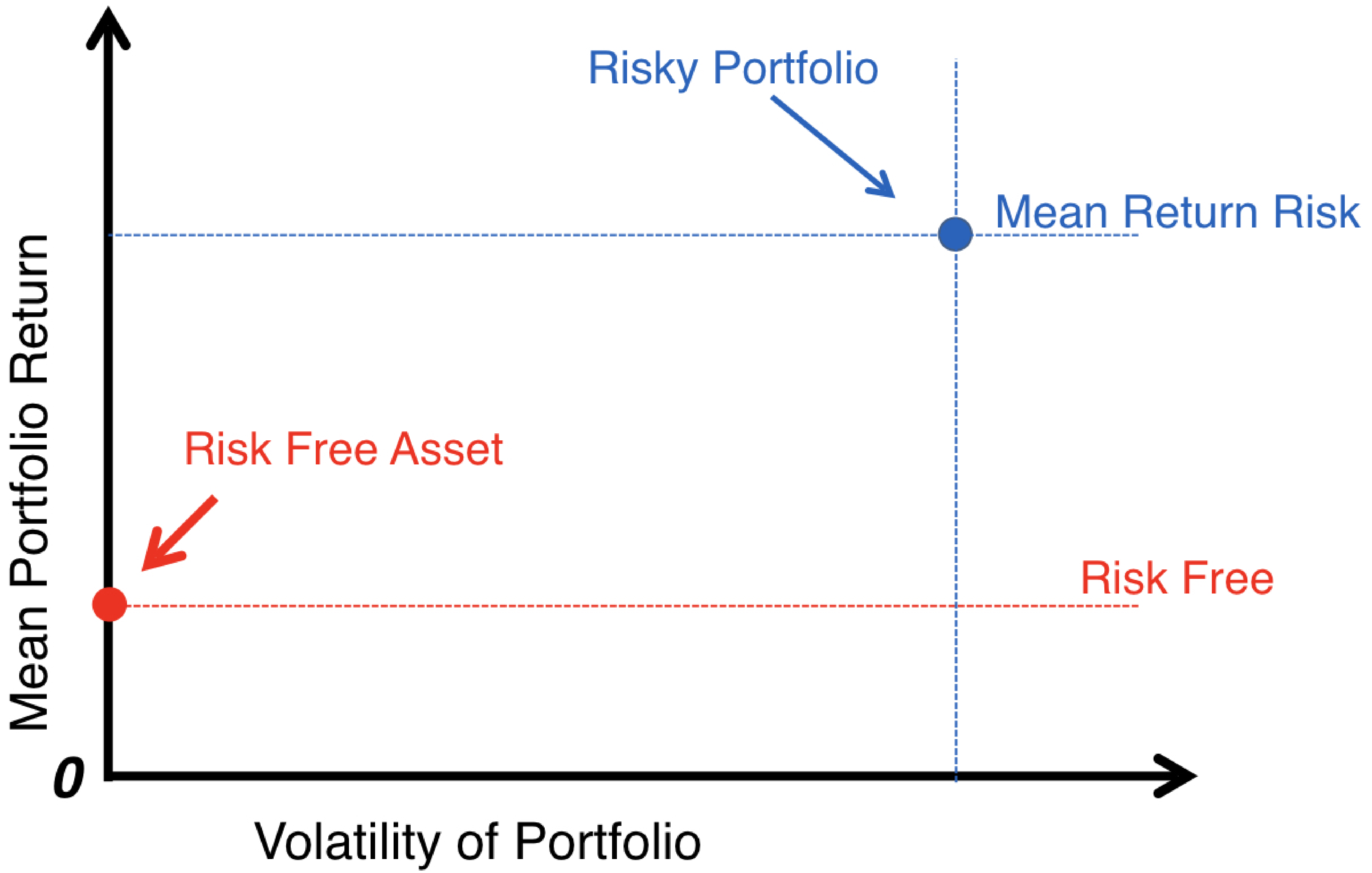
Risk-return trade-off



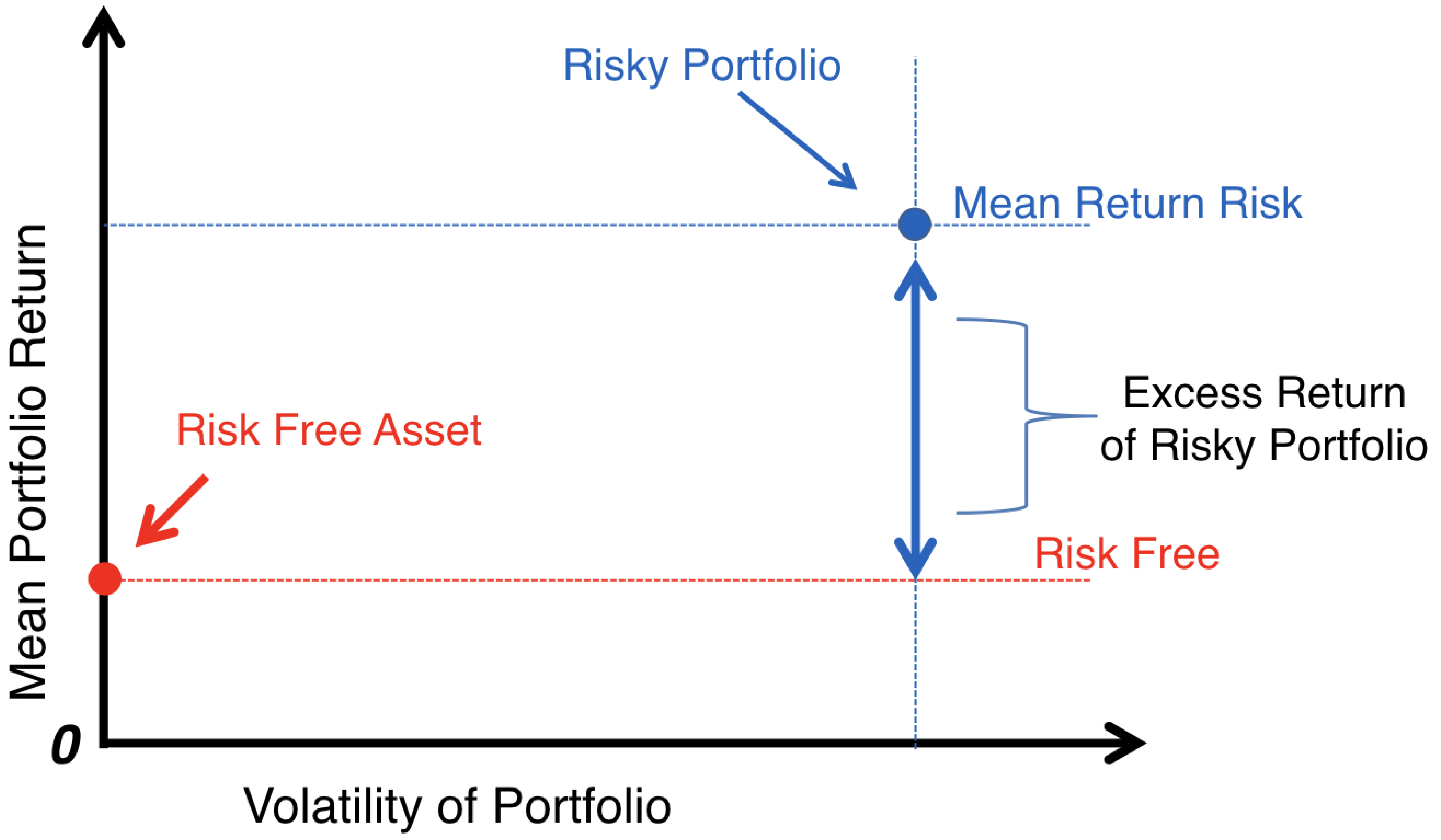
Risk-return trade-off



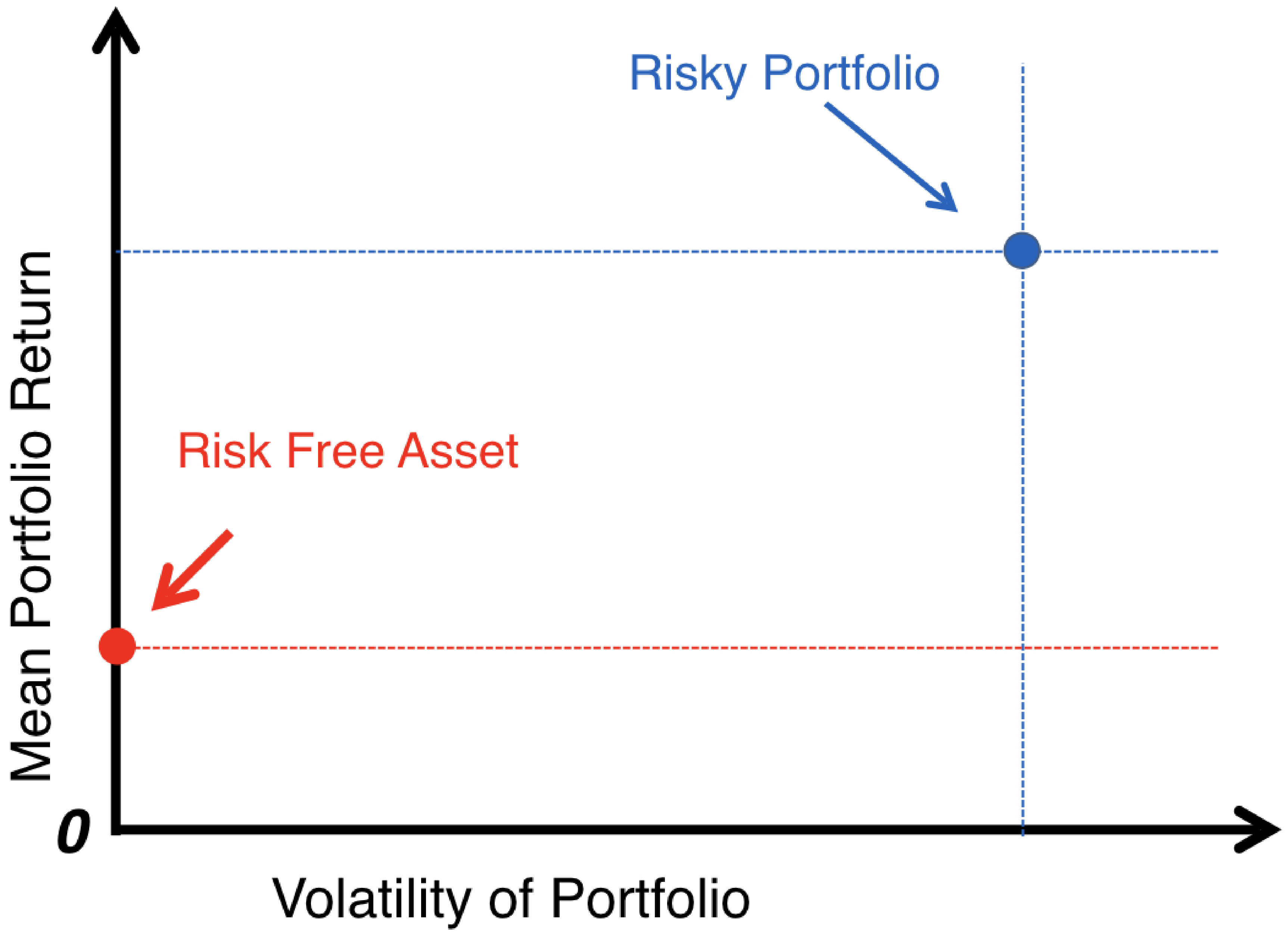
Risk-return trade-off



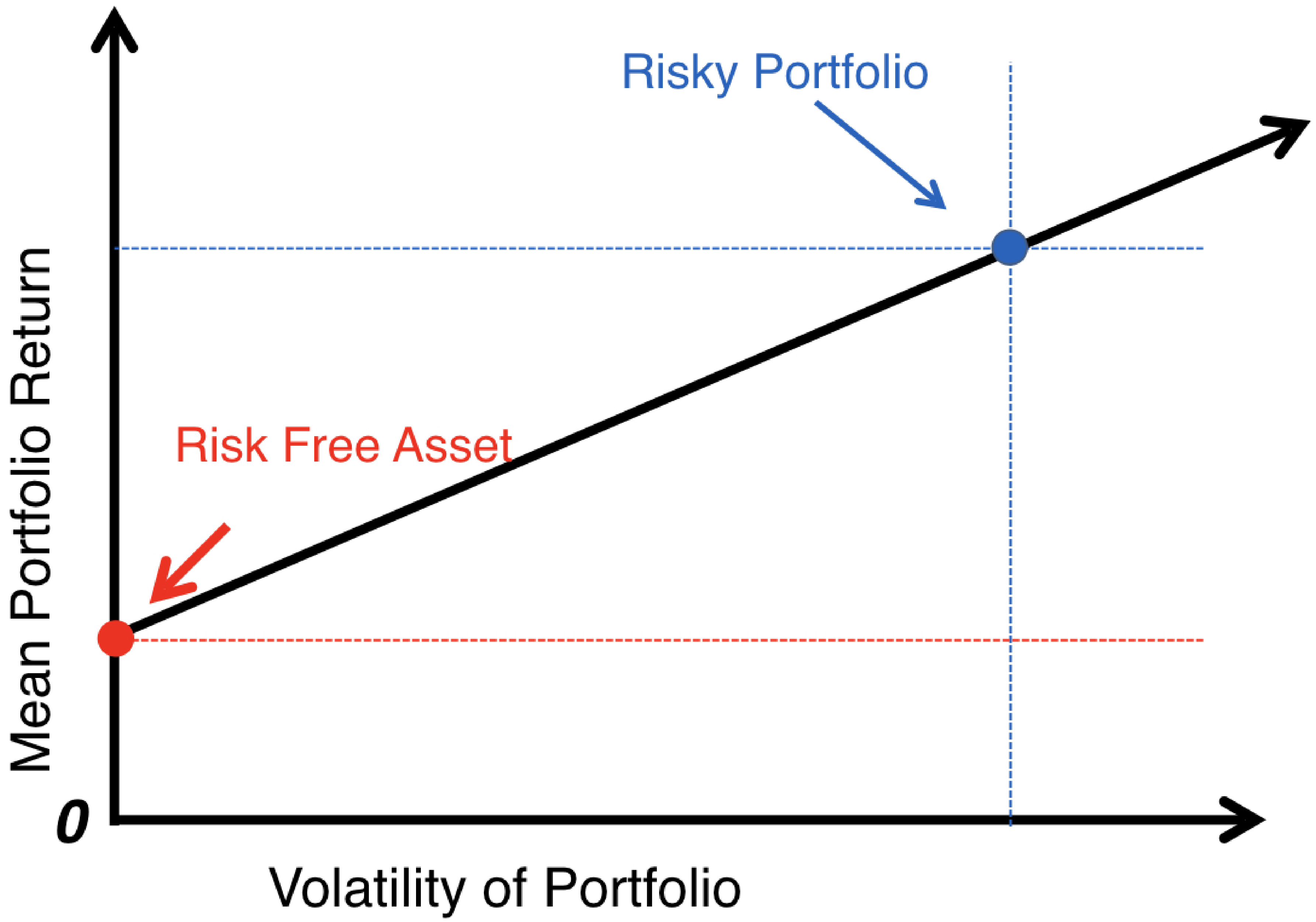
Risk-return trade-off



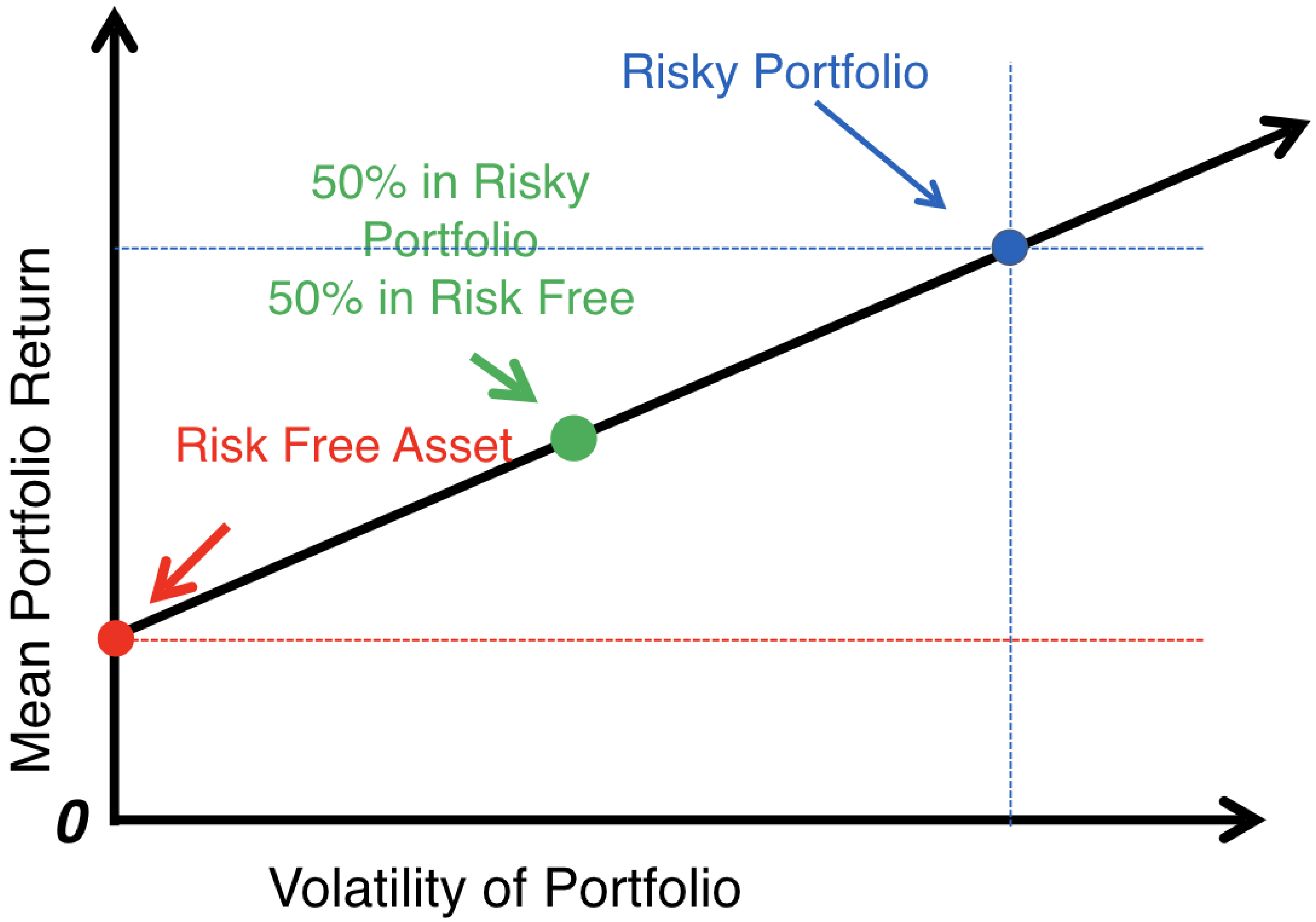
Capital allocation line



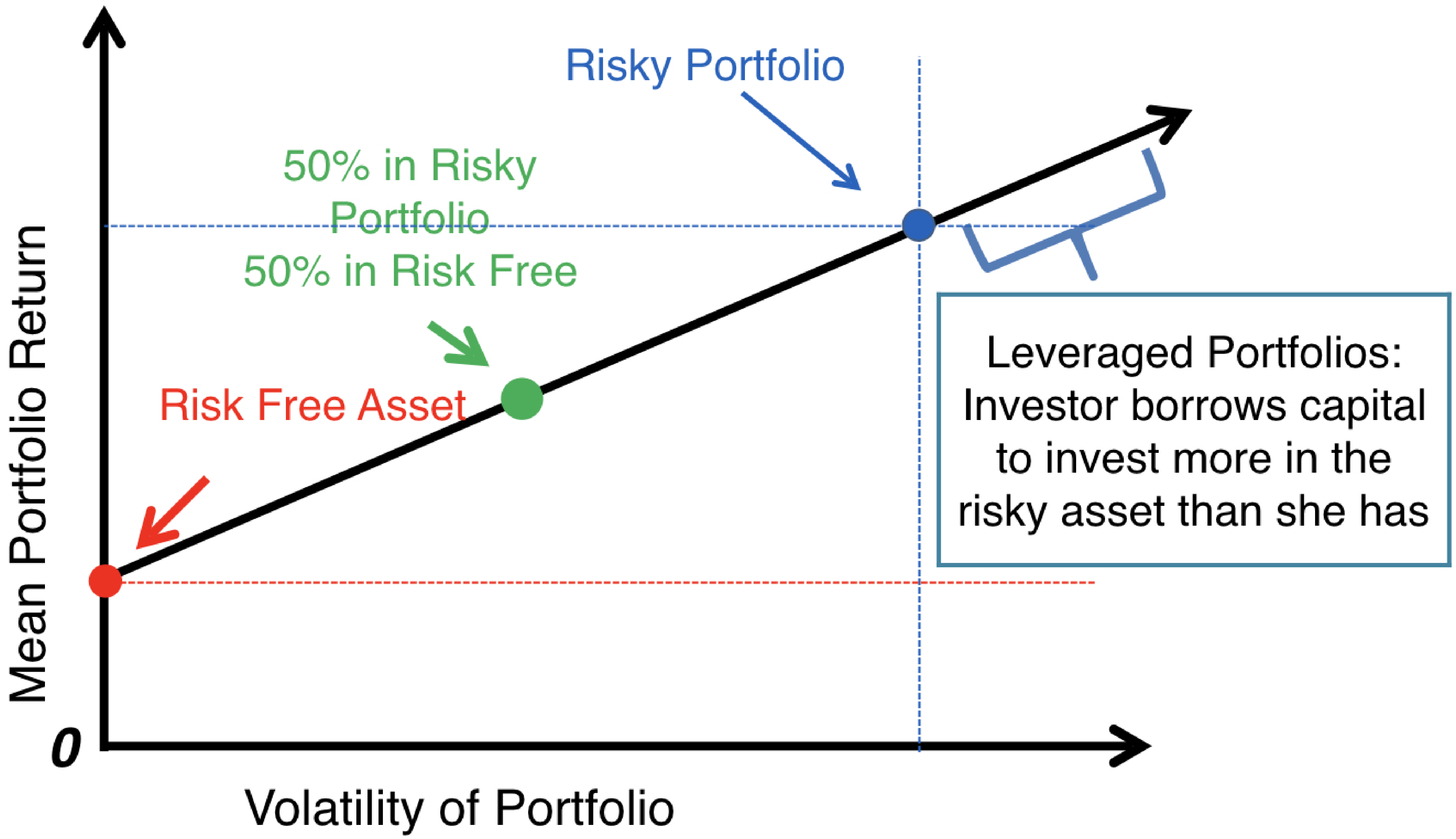
Capital allocation line



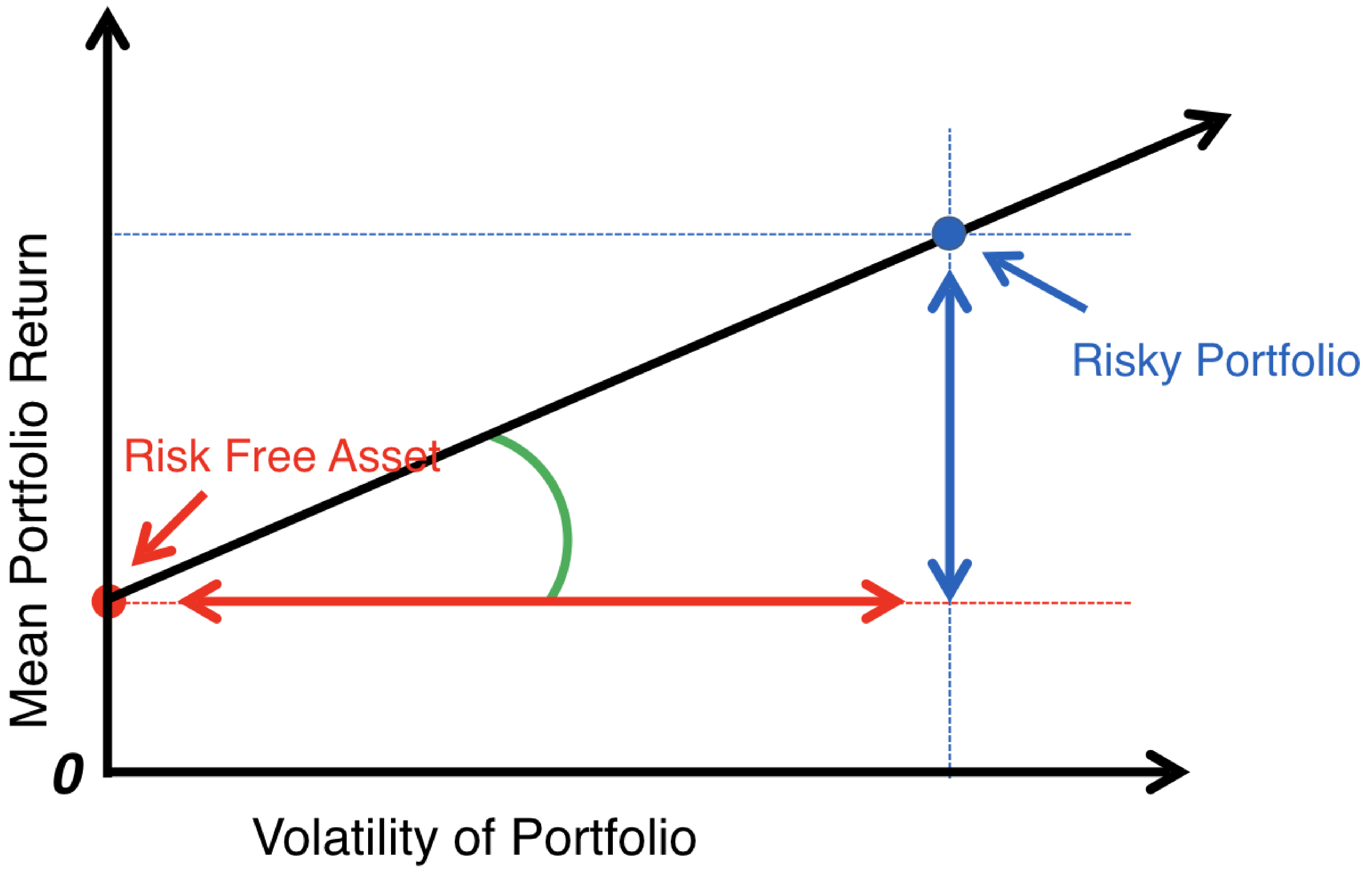
Capital allocation line



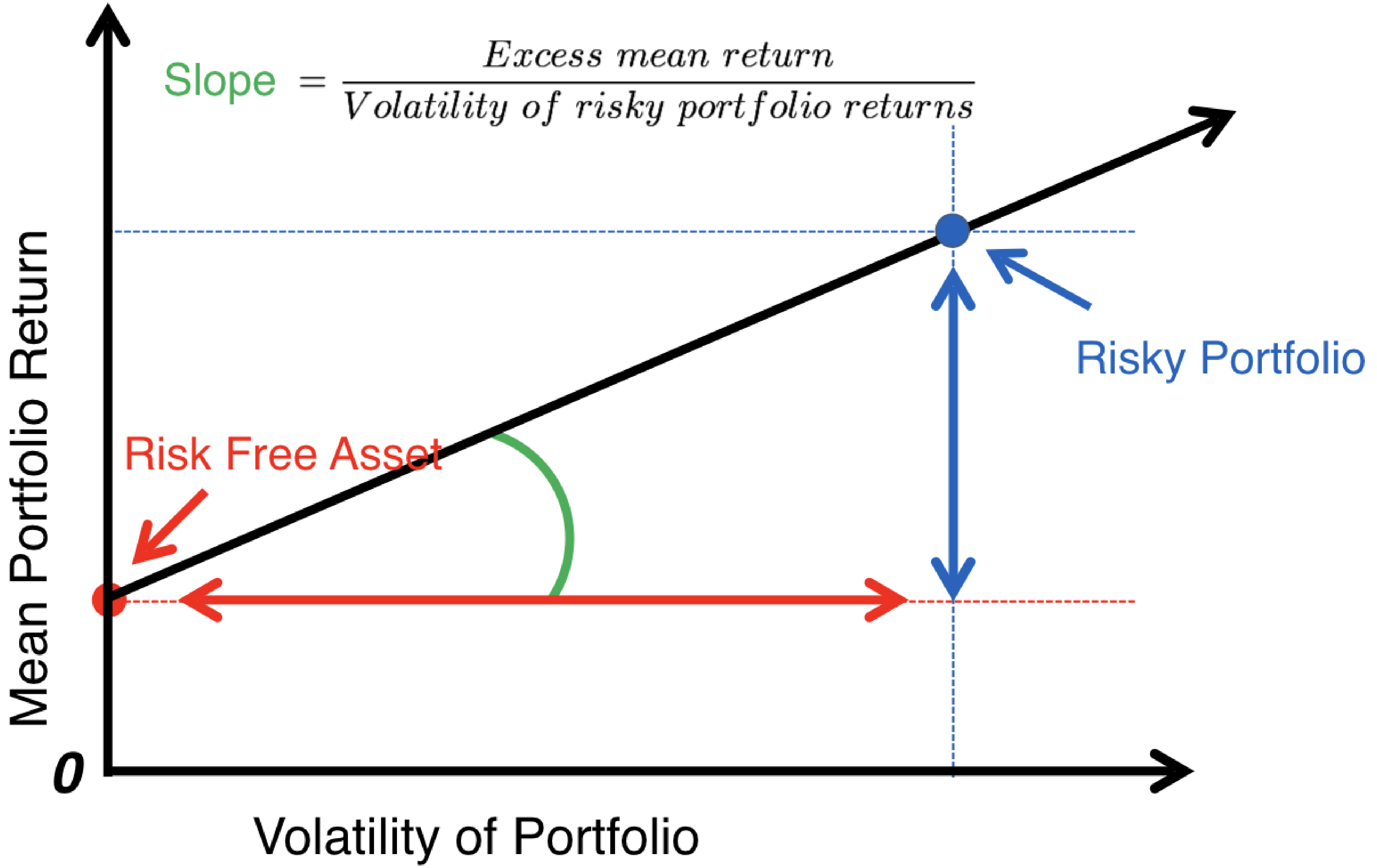
Capital allocation line



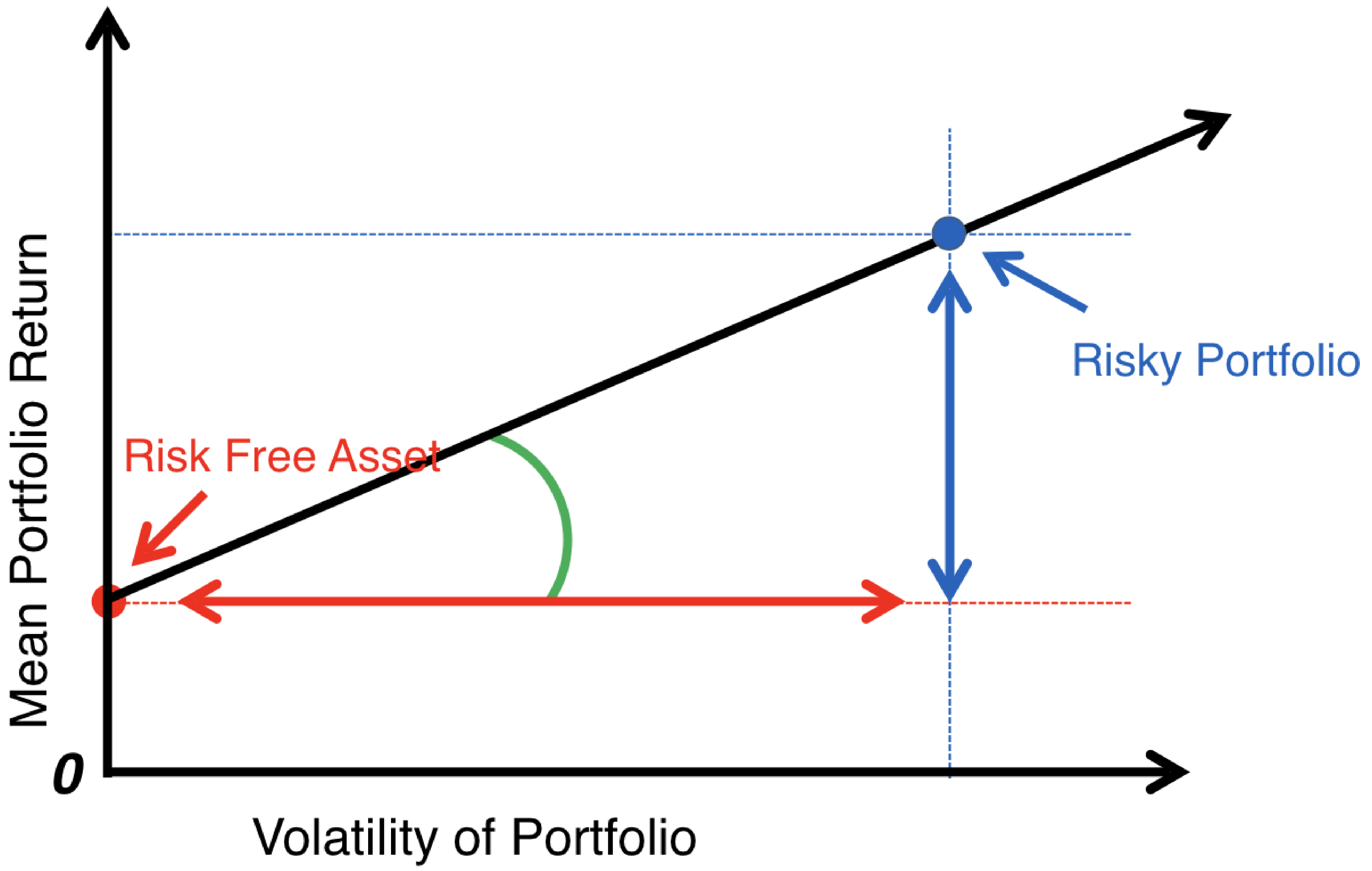
The Sharpe ratio



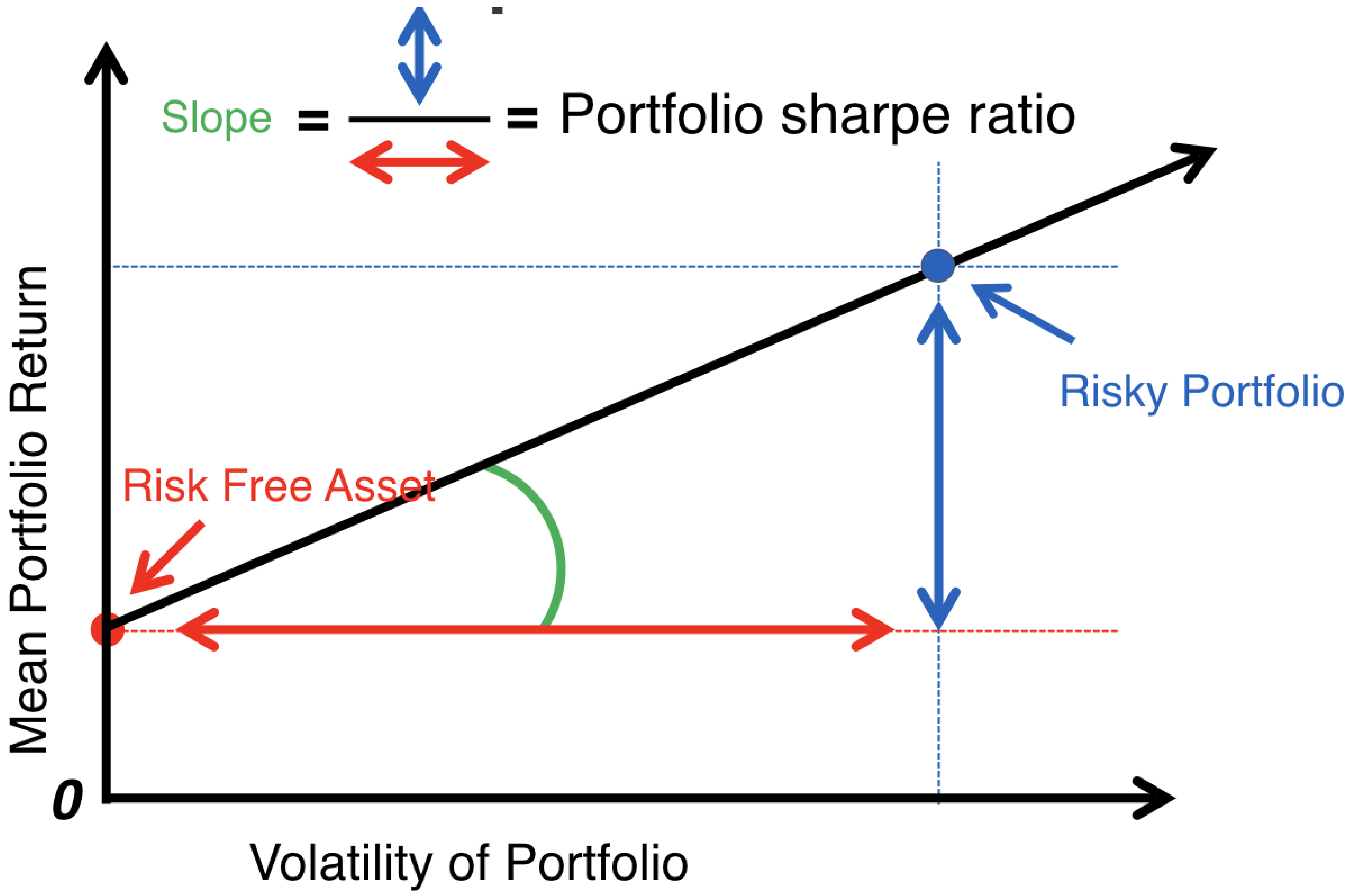
The Sharpe ratio



The Sharpe ratio



The Sharpe ratio



Performance statistics in action

```
library(PerformanceAnalytics)
sample_returns <- c(-0.02, 0.00, 0.00, 0.06, 0.02, 0.03, -0.01, 0.04)
```

returns	-0.02, 0 , 0 , 0.06, 0.02, 0.03, -0.01, 0.04
arithmetic mean	
geometric mean	
volatility	
sharpe ratio	

Performance statistics in action

```
library(PerformanceAnalytics)
sample_returns <- c(-0.02, 0.00, 0.00, 0.06, 0.02, 0.03, -0.01, 0.04)
mean(sample_returns)
```

returns	-0.02, 0 , 0 , 0.06, 0.02, 0.03, -0.01, 0.04
arithmetic mean	0.015
geometric mean	
volatility	
sharpe ratio	

Performance statistics in action

```
library(PerformanceAnalytics)
sample_returns <- c(-0.02, 0.00, 0.00, 0.06, 0.02, 0.03, -0.01, 0.04)
mean.geometric(sample_returns)
```

returns	-0.02, 0 , 0 , 0.06, 0.02, 0.03, -0.01, 0.04
arithmetic mean	0.015
geometric mean	0.01468148
volatility	
sharpe ratio	

Performance statistics in action

```
library(PerformanceAnalytics)
sample_returns <- c(-0.02, 0.00, 0.00, 0.06, 0.02, 0.03, -0.01, 0.04)
StdDev(sample_returns)
```

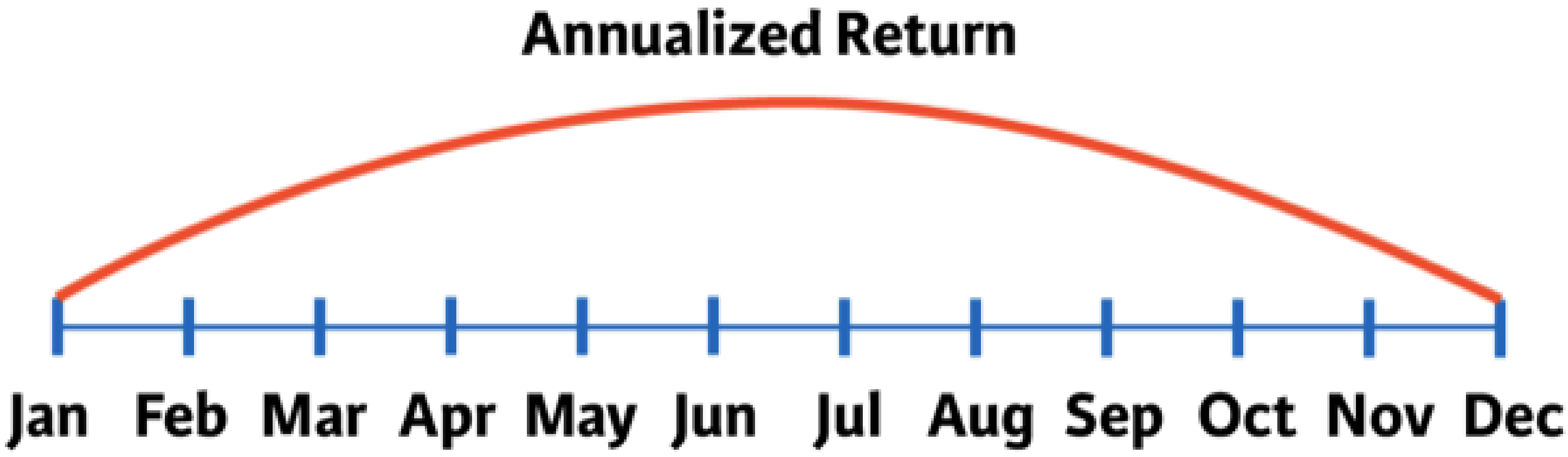
returns	-0.02, 0 , 0 , 0.06, 0.02, 0.03, -0.01, 0.04
arithmetic mean	0.015
geometric mean	0.01468148
volatility	0.02725541
sharpe ratio	

Performance statistics in action

```
library(PerformanceAnalytics)
sample_returns <- c(-0.02, 0.00, 0.00, 0.06, 0.02, 0.03, -0.01, 0.04)
(mean(sample_returns)-0.004)/StdDev(sample_returns)
```

returns	-0.02, 0 , 0 , 0.06, 0.02, 0.03, -0.01, 0.04
arithmetic mean	0.015
geometric mean	0.01468148
volatility	0.02725541
sharpe ratio	0.4035897

Annualize monthly performance



- Arithmetic mean: monthly mean * 12
- Geometric mean, when R_i are monthly returns:
  - $[(1+R_1)\cdot(1+R_2)\cdot\ldots\cdot(1+R_T)]^{12/T}-1$
- Volatility: monthly volatility * sqrt(12)

Performance statistics in action

```
library(PerformanceAnalytics)
sample_returns <- c( -0.02, 0.00, 0.00, 0.06, 0.02, 0.03, -0.01, 0.04)
```

	monthly	FACTOR	annualized
arithmetic mean	0.015		
geometric mean	0.01468148		
volatility	0.02725541		
sharpe ratio	0.4035897		

Performance statistics in action

```
library(PerformanceAnalytics)
sample_returns <- c( -0.02, 0.00, 0.00, 0.06, 0.02, 0.03, -0.01, 0.04)
Return.annualized(sample_returns, scale = 12, geometric = FALSE)
```

	monthly	FACTOR	annualized
arithmetic mean	0.015	12	0.18
geometric mean	0.01468148		
volatility	0.02725541		
sharpe ratio	0.4035897		

Performance statistics in action

```
library(PerformanceAnalytics)
sample_returns <- c( -0.02, 0.00, 0.00, 0.06, 0.02, 0.03, -0.01, 0.04)
Return.annualized(sample_returns, scale = 12, geometric = TRUE)
```

	monthly	FACTOR	annualized
arithmetic mean	0.015	12	0.18
geometric mean	0.01468148	$[0.98 * \ldots * 1.04]^{\frac{12}{8}} - 1$	0.1911235
volatility	0.02725541		
sharpe ratio	0.4035897		

Performance statistics in action

```
library(PerformanceAnalytics)
sample_returns <- c( -0.02, 0.00, 0.00, 0.06, 0.02, 0.03, -0.01, 0.04)
Std.Dev.annualized(sample_returns, scale = 12)
```

	monthly	FACTOR	annualized
arithmetic mean	0.015	12	0.18
geometric mean	0.01468148	$[0.98 * ... * 1.04]^{\frac{12}{8}} - 1$	0.1911235
volatility	0.02725541	sqrt(12)	0.0944155
sharpe ratio	0.4035897		

Performance statistics in action

```
library(PerformanceAnalytics)
sample_returns <- c( -0.02, 0.00, 0.00, 0.06, 0.02, 0.03, -0.01, 0.04)
Return.annualized(sample_returns, scale = 12)/
    Std.Dev.annualized(sample_returns, scale = 12)
```

	monthly	FACTOR	annualized
arithmetic mean	0.015	12	0.18
geometric mean	0.01468148	$[0.98 * \ldots * 1.04]^{\frac{12}{8}} - 1$	0.1911235
volatility	0.02725541	sqrt(12)	0.0944155
sharpe ratio	0.4035897	sqrt(12)	1.398076

Let's practice!

INTRODUCTION TO PORTFOLIO ANALYSIS IN R

Time-variation in portfolio performance

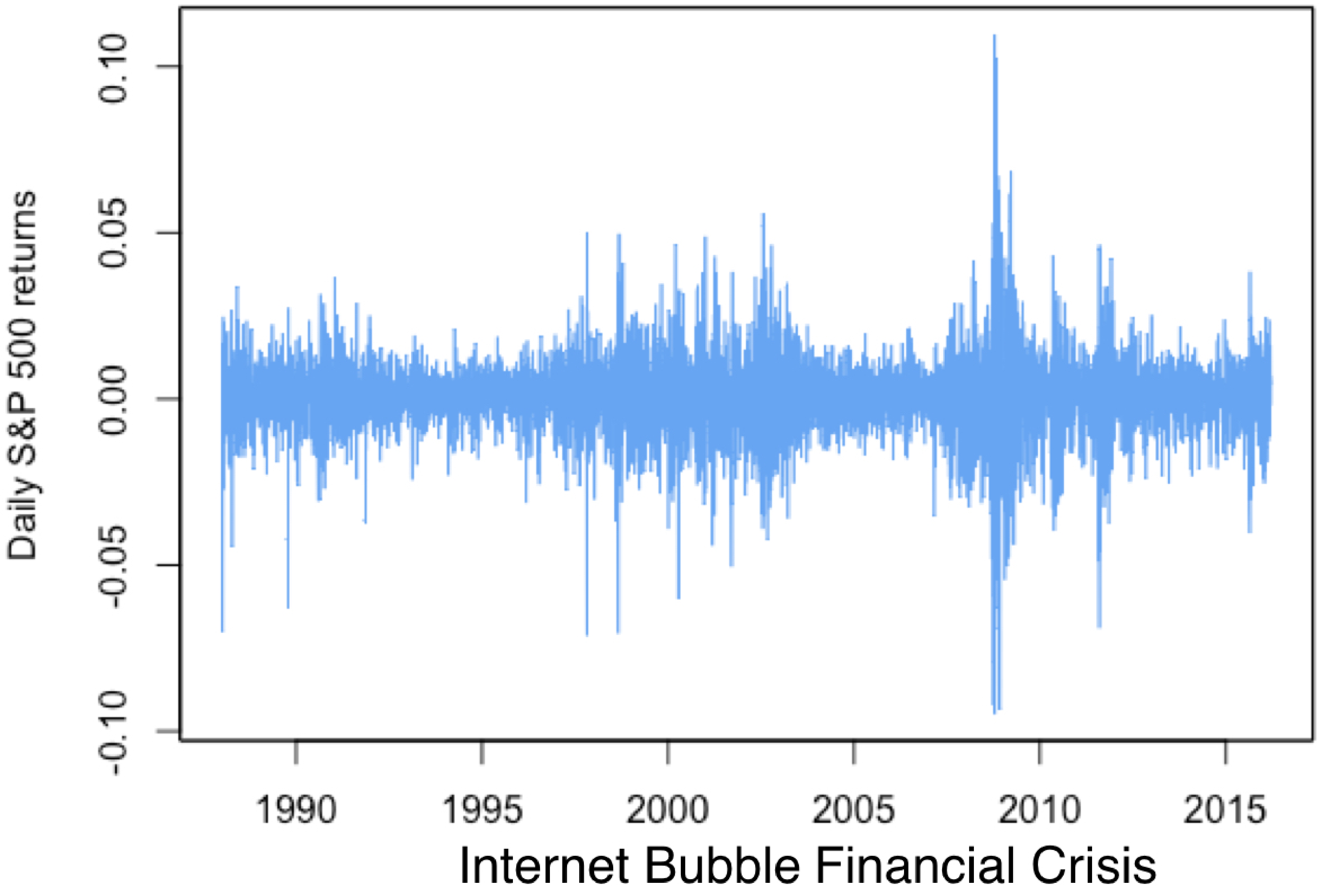
INTRODUCTION TO PORTFOLIO ANALYSIS IN R

Kris Boudt
Professor, Free University Brussels & Amsterdam

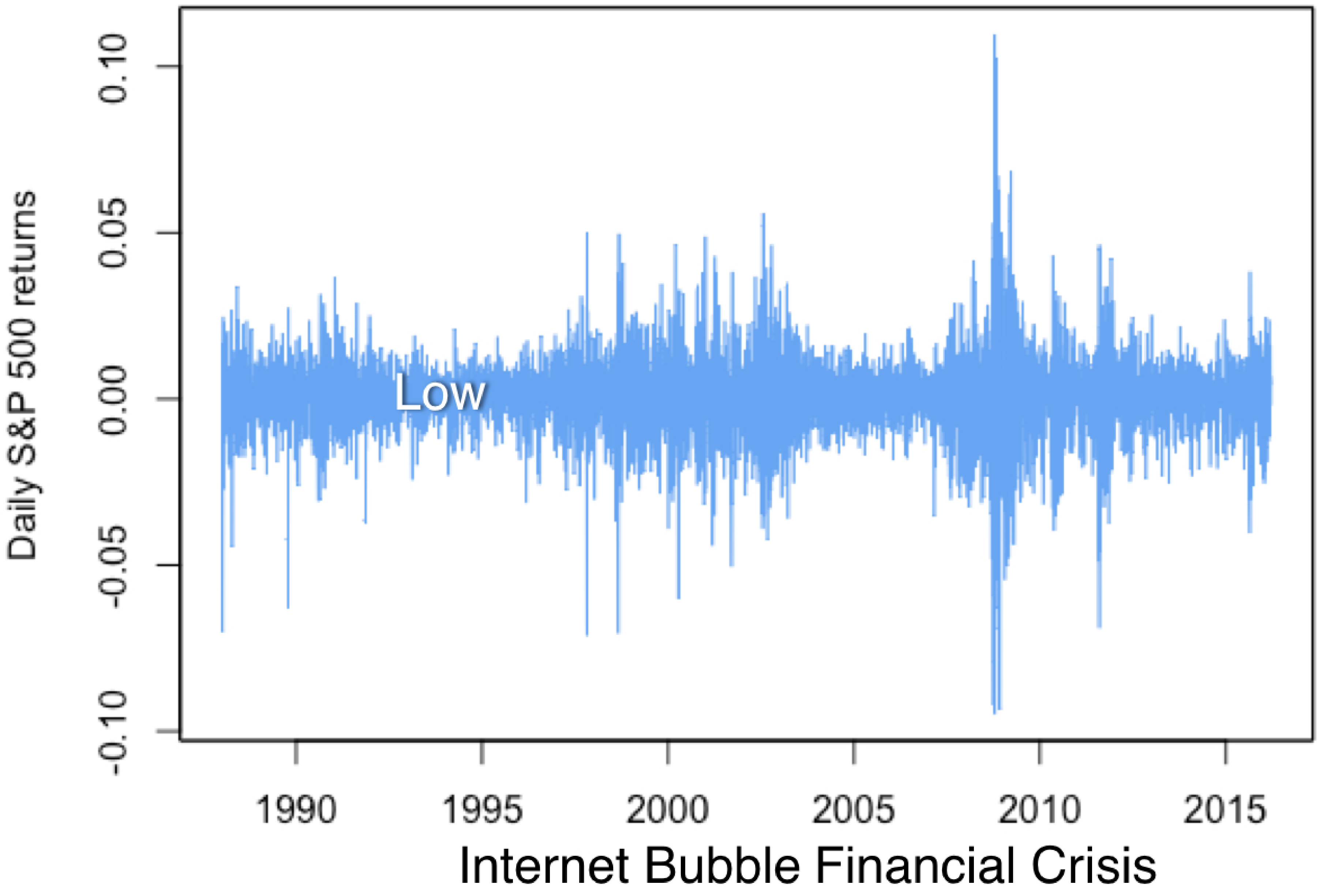
Bulls & bears

- Business cycle, news, and swings in the market psychology affect the market

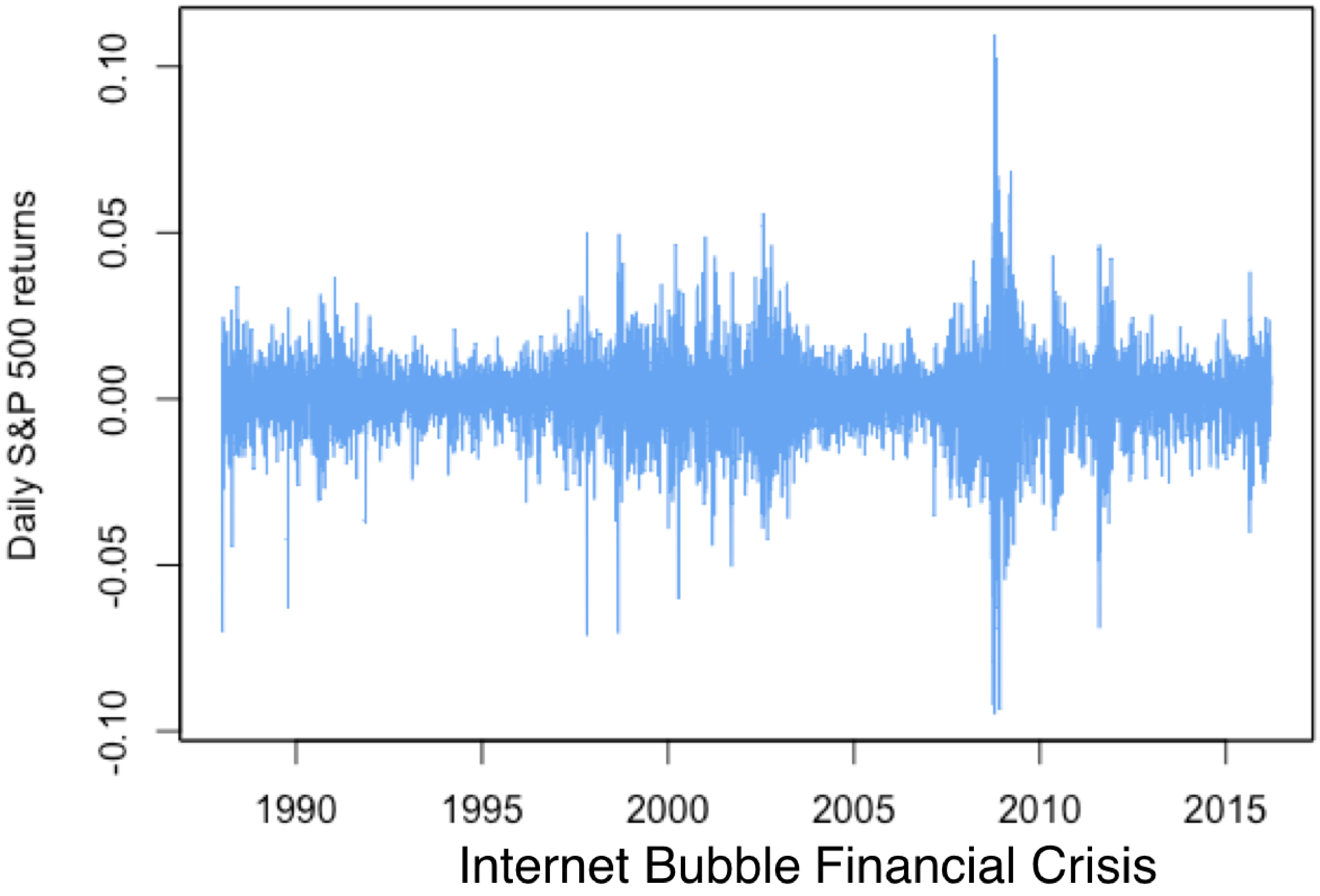
Clusters of high & low volatility



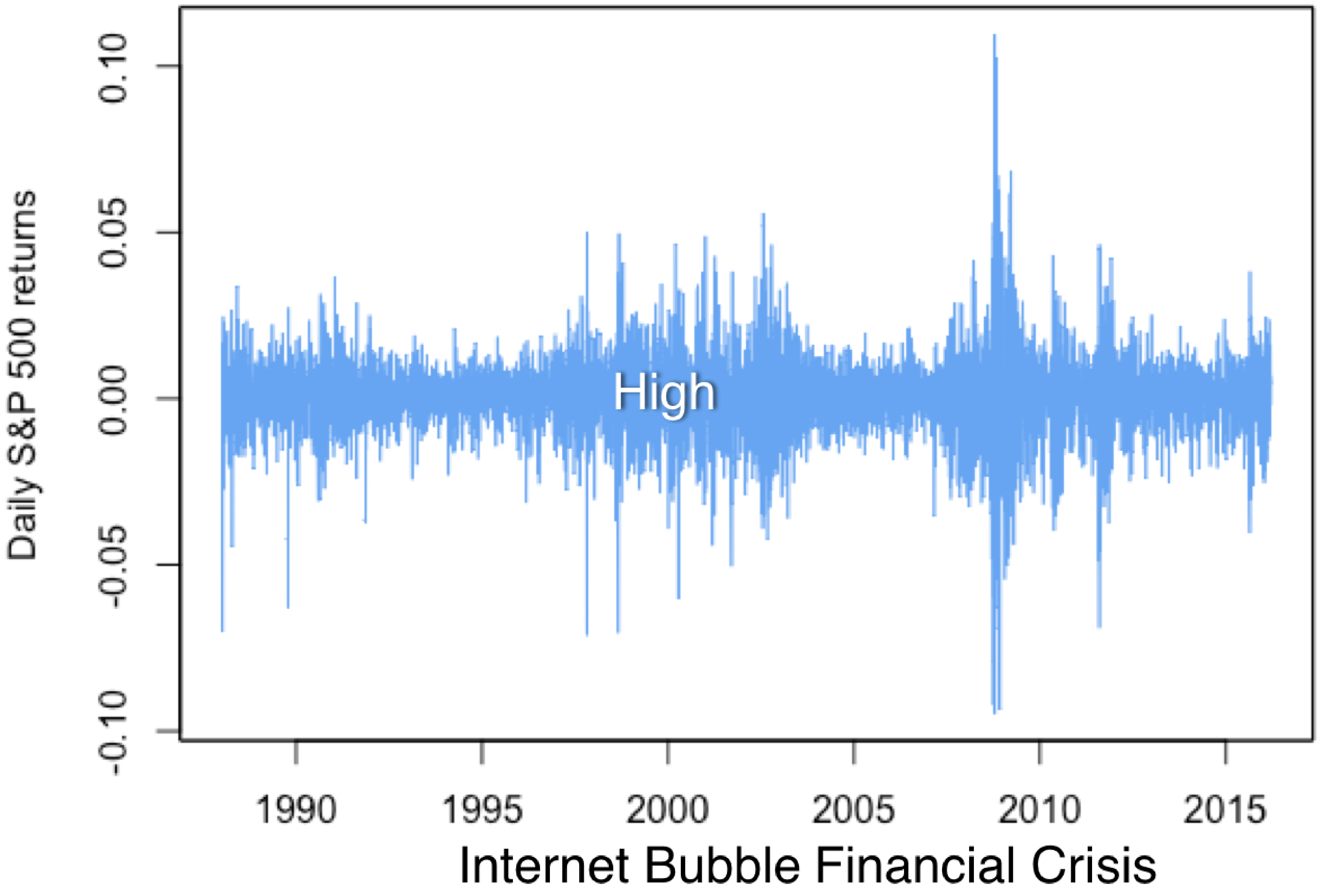
Performance statistics in action



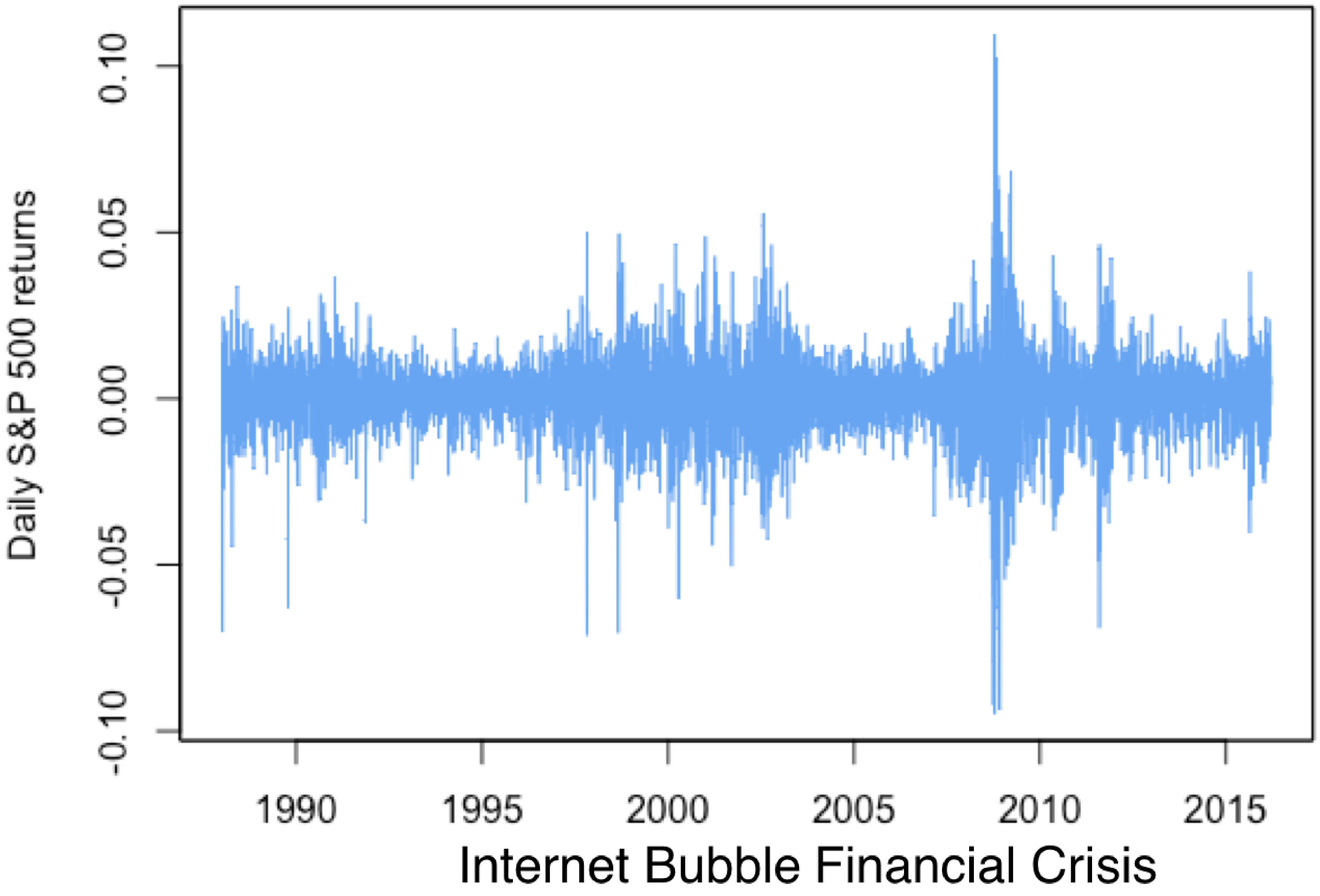
Performance statistics in action



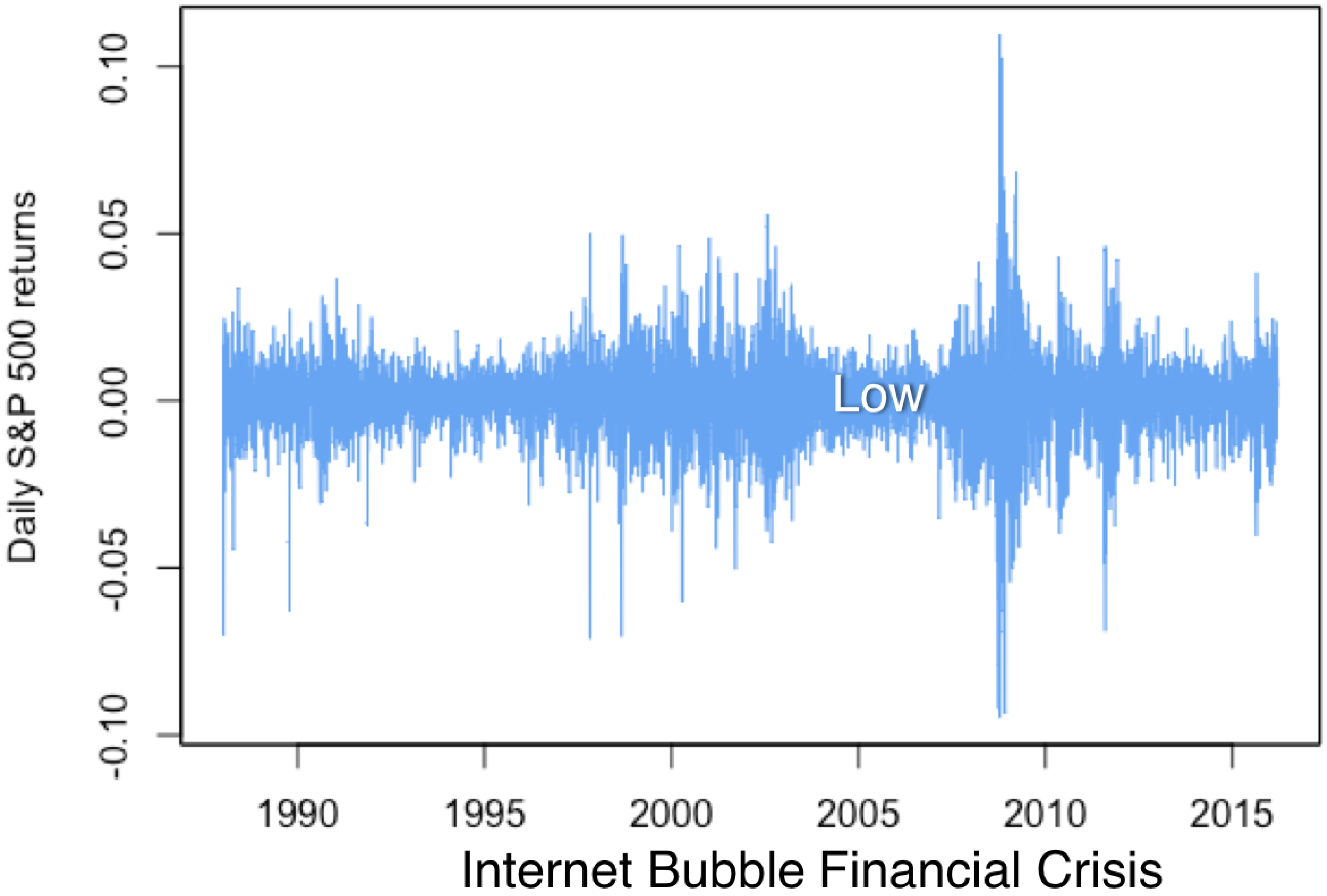
Performance statistics in action



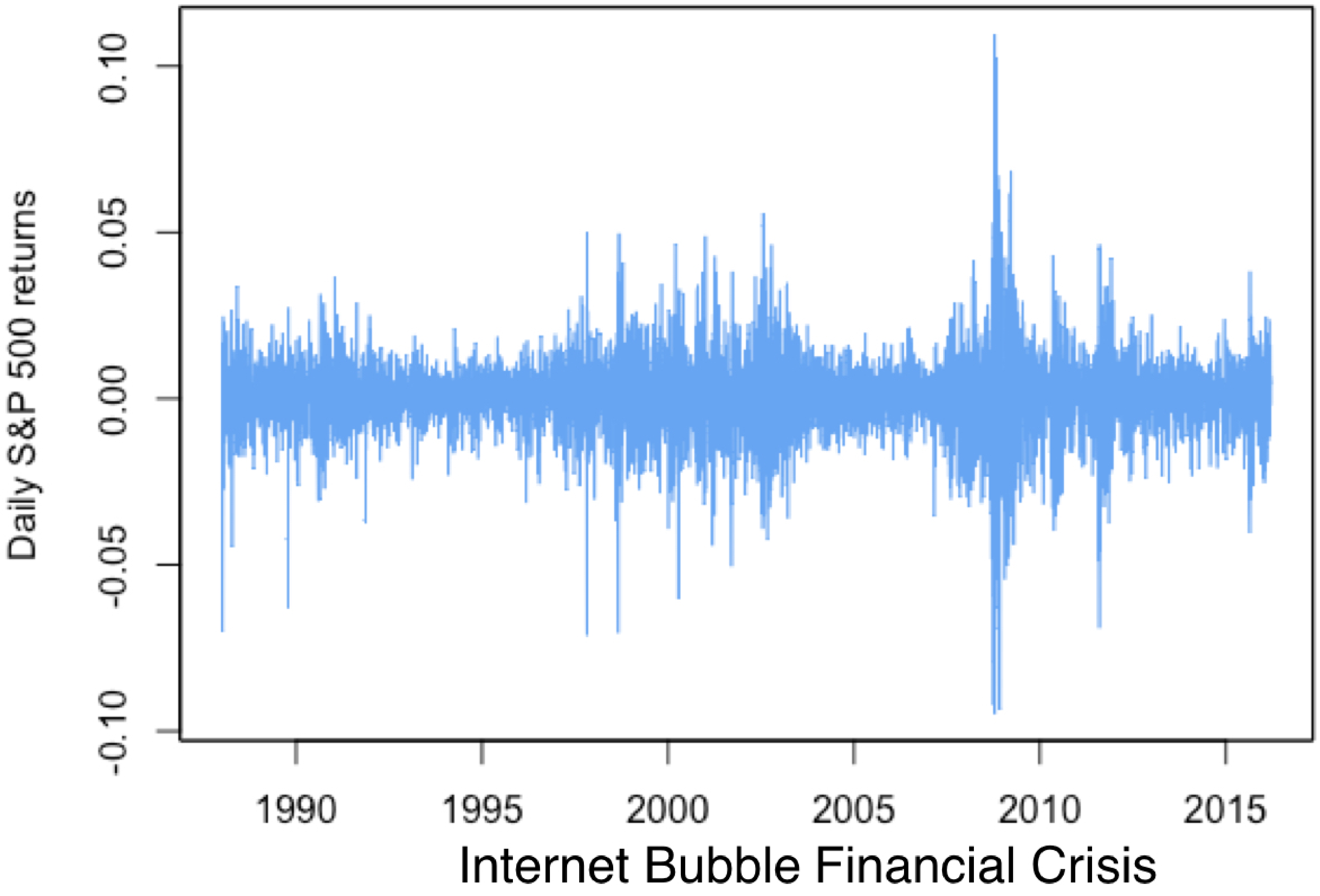
Performance statistics in action



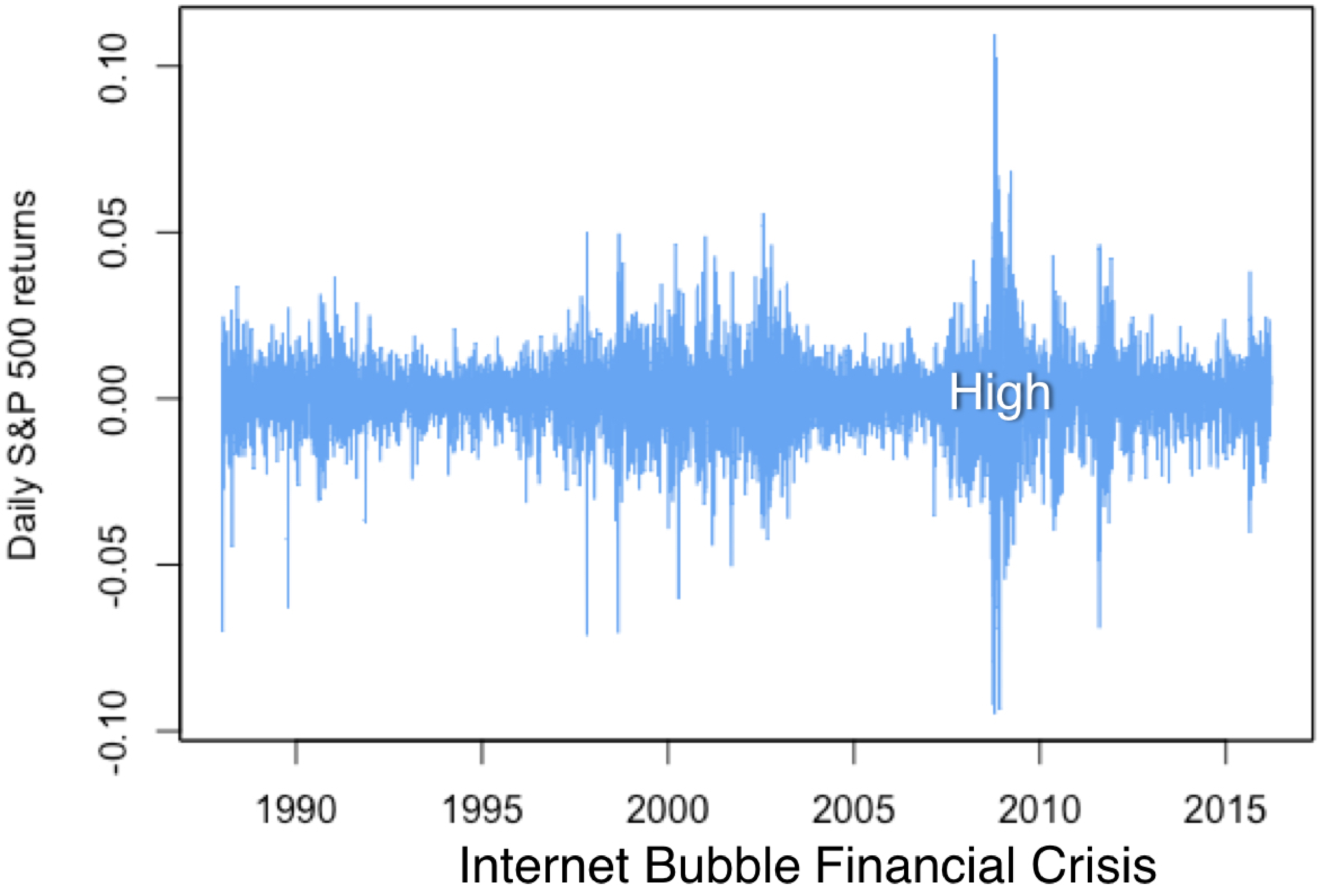
Performance statistics in action



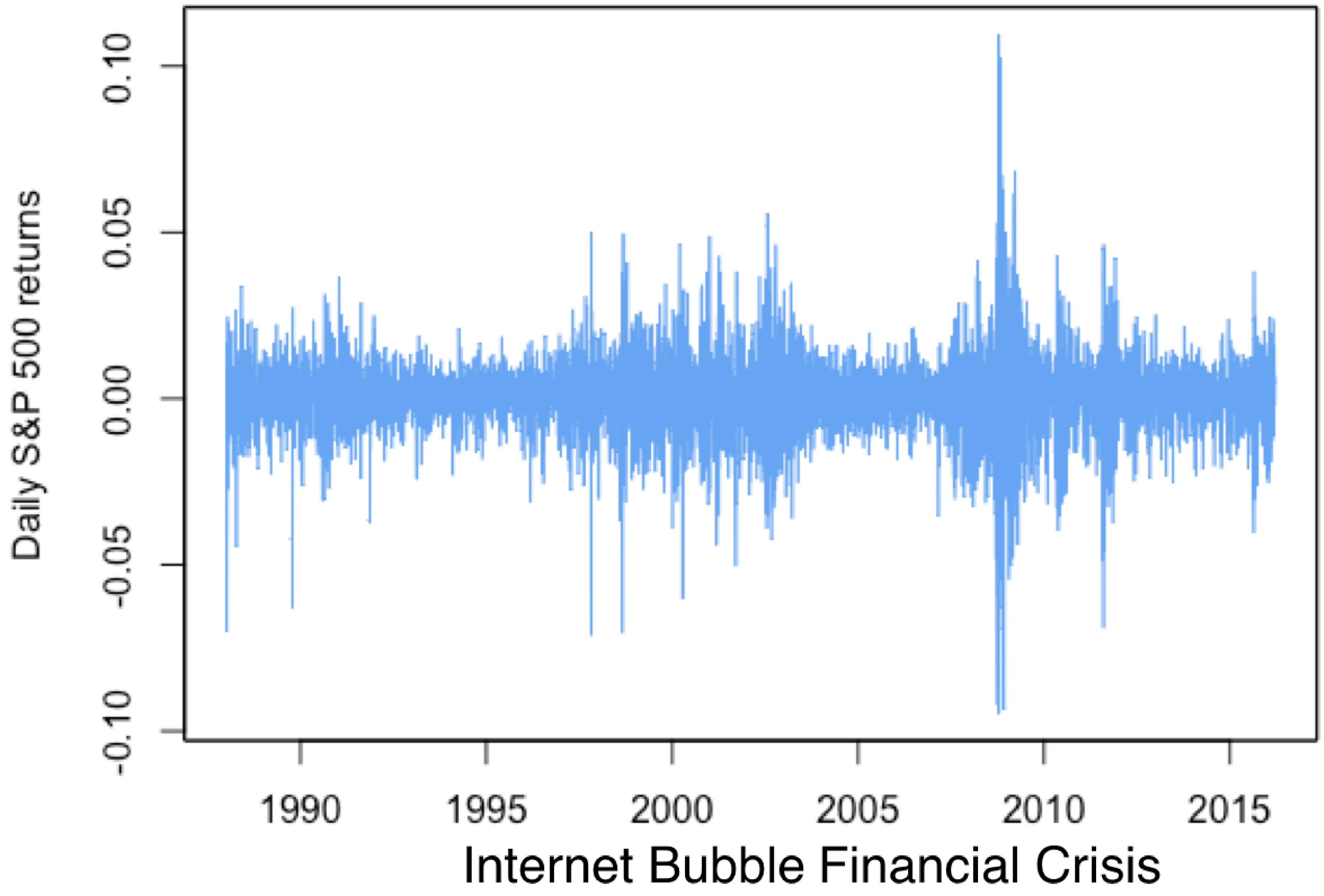
Performance statistics in action



Performance statistics in action



Performance statistics in action



Rolling estimation samples

- Rolling samples of K observations:
  - Discard the most distant and include the most recent

R_{t-k+1}	R_{t-k+2}	R_{t-k+3}	...	R_t	R_{t+1}	R_{t+2}	R_{t+3}

Rolling estimation samples

- Rolling samples of K observations:
  - Discard the most distant and include the most recent

R_{t-k+1}	R_{t-k+2}	R_{t-k+3}	...	R_t	R_{t+1}	R_{t+2}	R_{t+3}

Rolling estimation samples

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R_{t-k+1}	R_{t-k+2}	R_{t-k+3}	...	R_t	R_{t+1}	R_{t+2}	R_{t+3}

Rolling estimation samples

- Rolling samples of K observations:
  - Discard the most distant and include the most recent

R_{t-k+1}	R_{t-k+2}	R_{t-k+3}	...	R_t	R_{t+1}	R_{t+2}	R_{t+3}

Rolling estimation samples

- Rolling samples of K observations:
  - Discard the most distant and include the most recent

R_{t-k+1}	R_{t-k+2}	R_{t-k+3}	…	R_t	R_{t+1}	R_{t+2}	R_{t+3}

Rolling estimation samples

- Rolling samples of K observations:
  - Discard the most distant and include the most recent

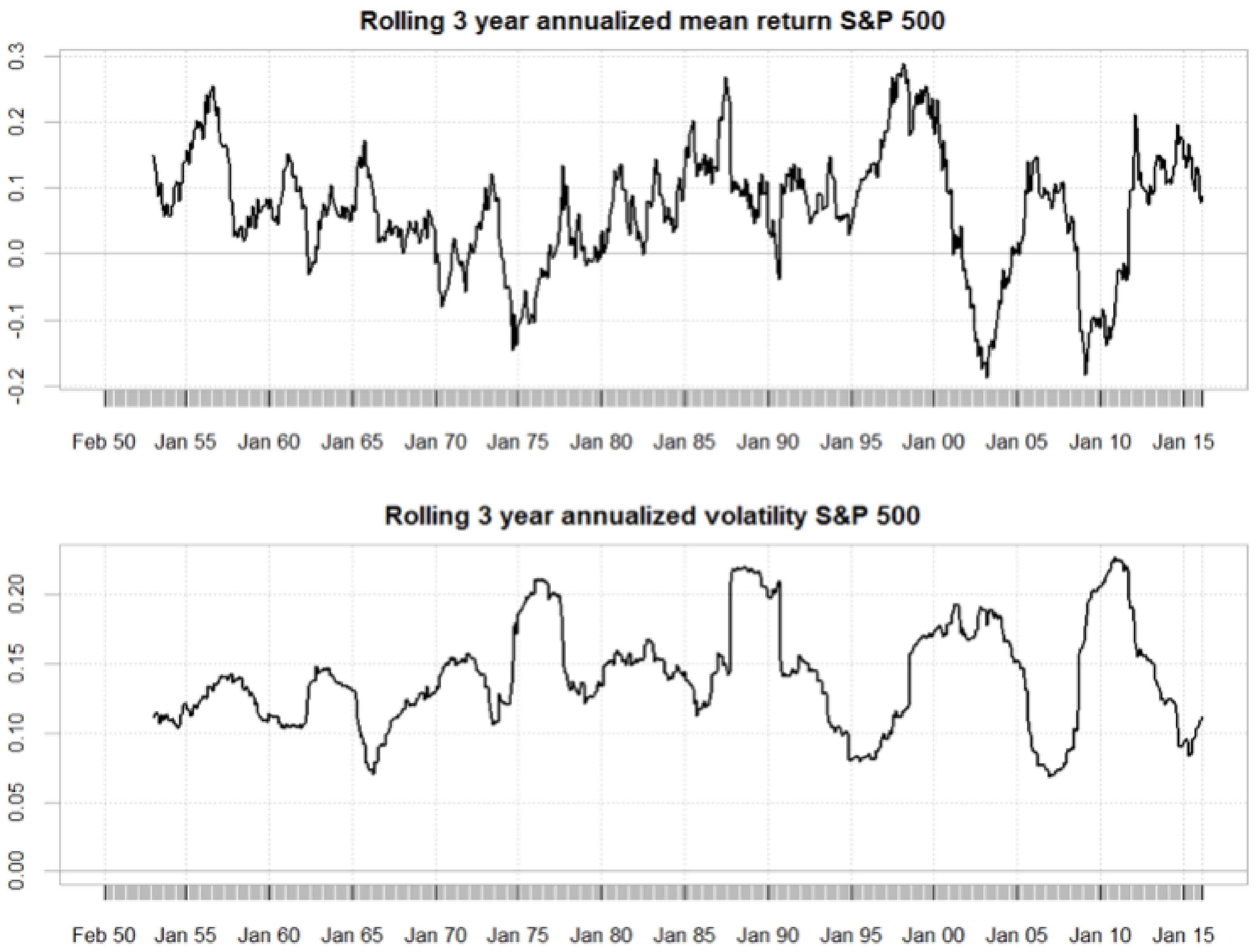
R_{t-k+1}	R_{t-k+2}	R_{t-k+3}	...	R_t	R_{t+1}	R_{t+2}	R_{t+3}

Rolling estimation samples

- Rolling samples of K observations:
  - Discard the most distant and include the most recent

R_{t-k+1}	R_{t-k+2}	R_{t-k+3}	...	R_t	R_{t+1}	R_{t+2}	R_{t+3}

Rolling performance calculation



Choosing window length

- Need to balance noise (long samples) with recency (shorter samples)
- Longer sub-periods smooth highs and lows
- Shorter sub-periods provide more information on recent observations

Let's practice!

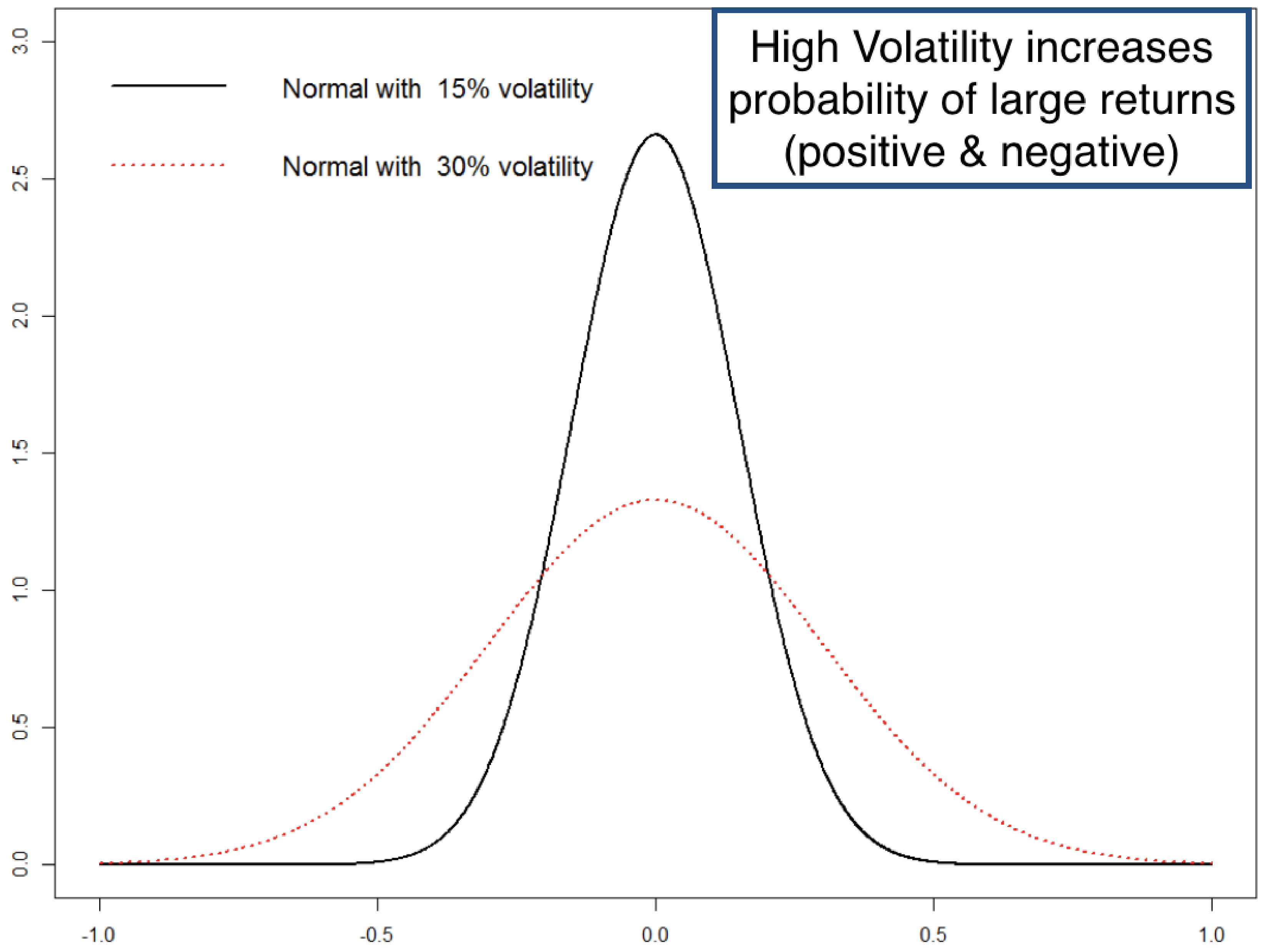
INTRODUCTION TO PORTFOLIO ANALYSIS IN R

Non-normality of the return distribution

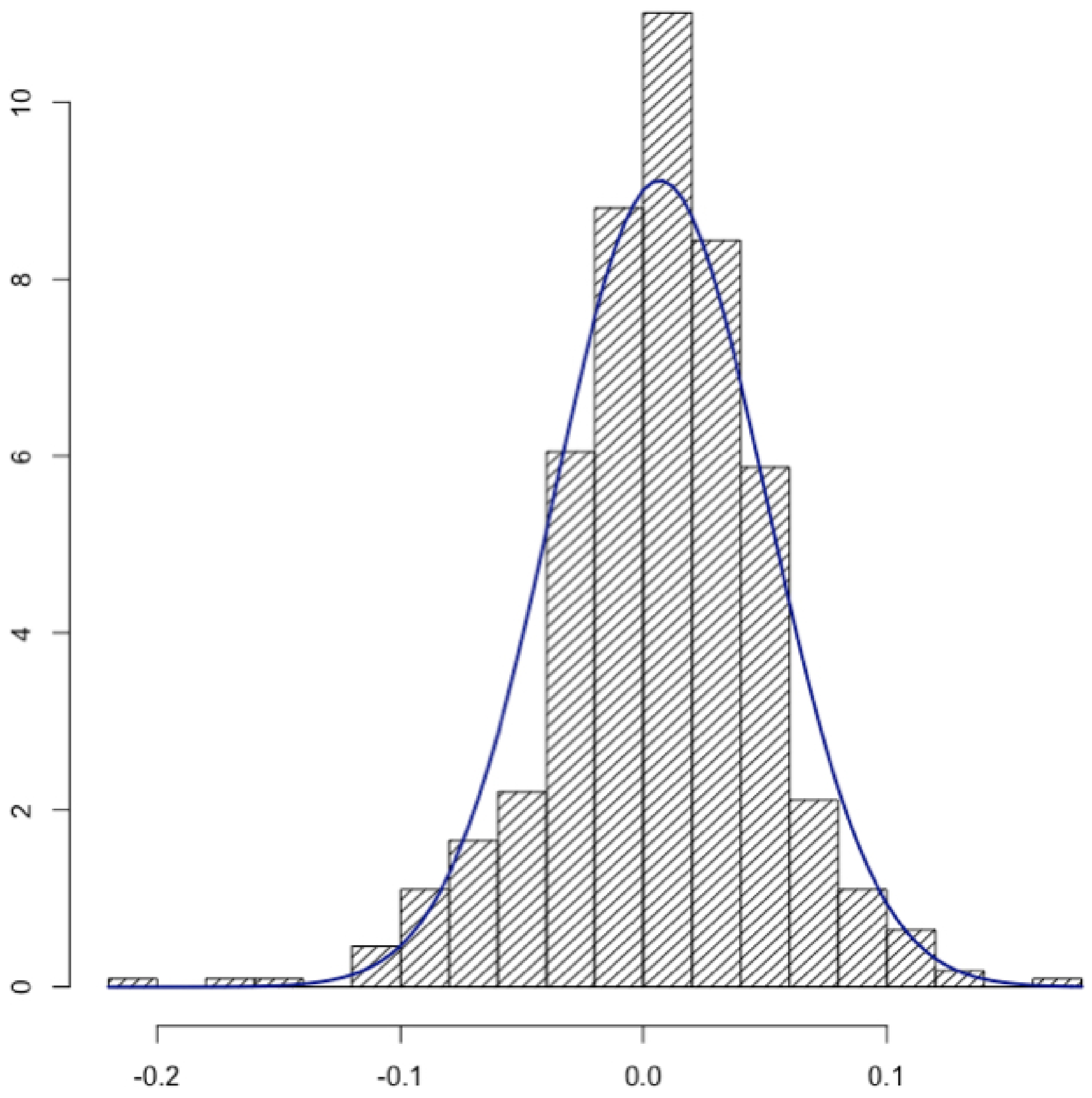
INTRODUCTION TO PORTFOLIO ANALYSIS IN R

Kris Boudt
Professor, Free University Brussels & Amsterdam

Volatility describes "normal" risk



Non-normality of return



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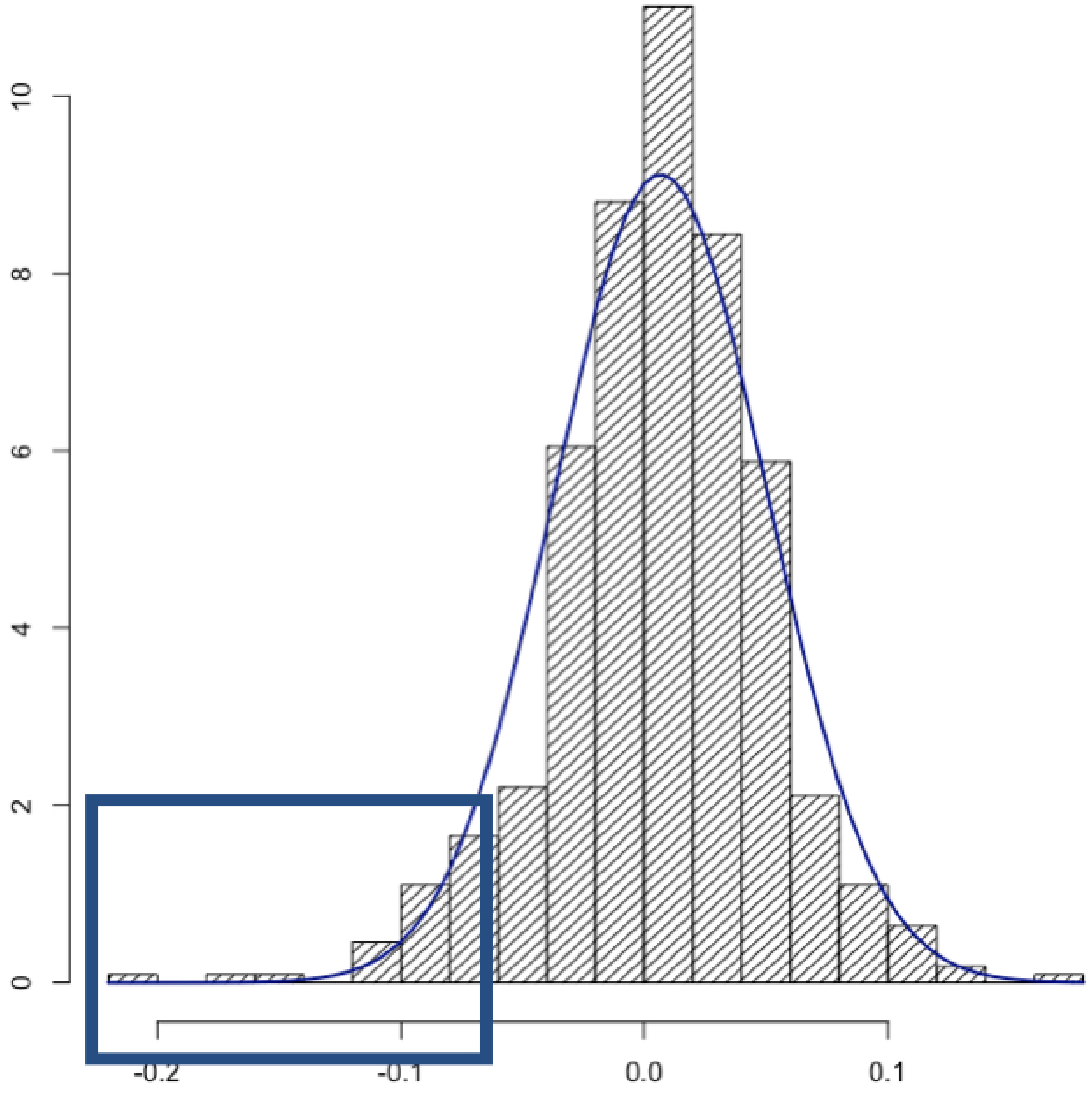

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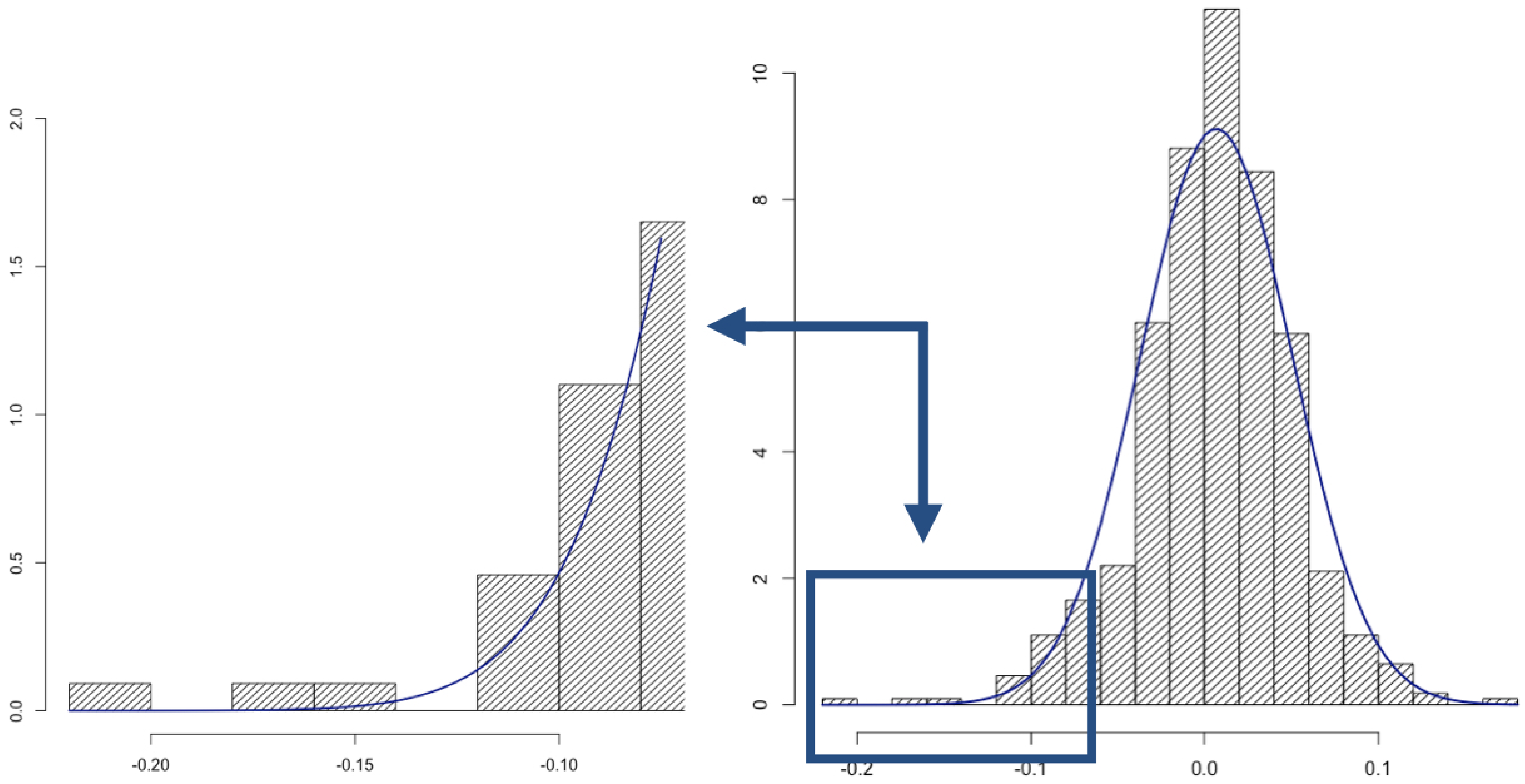


Portfolio return semi-deviation

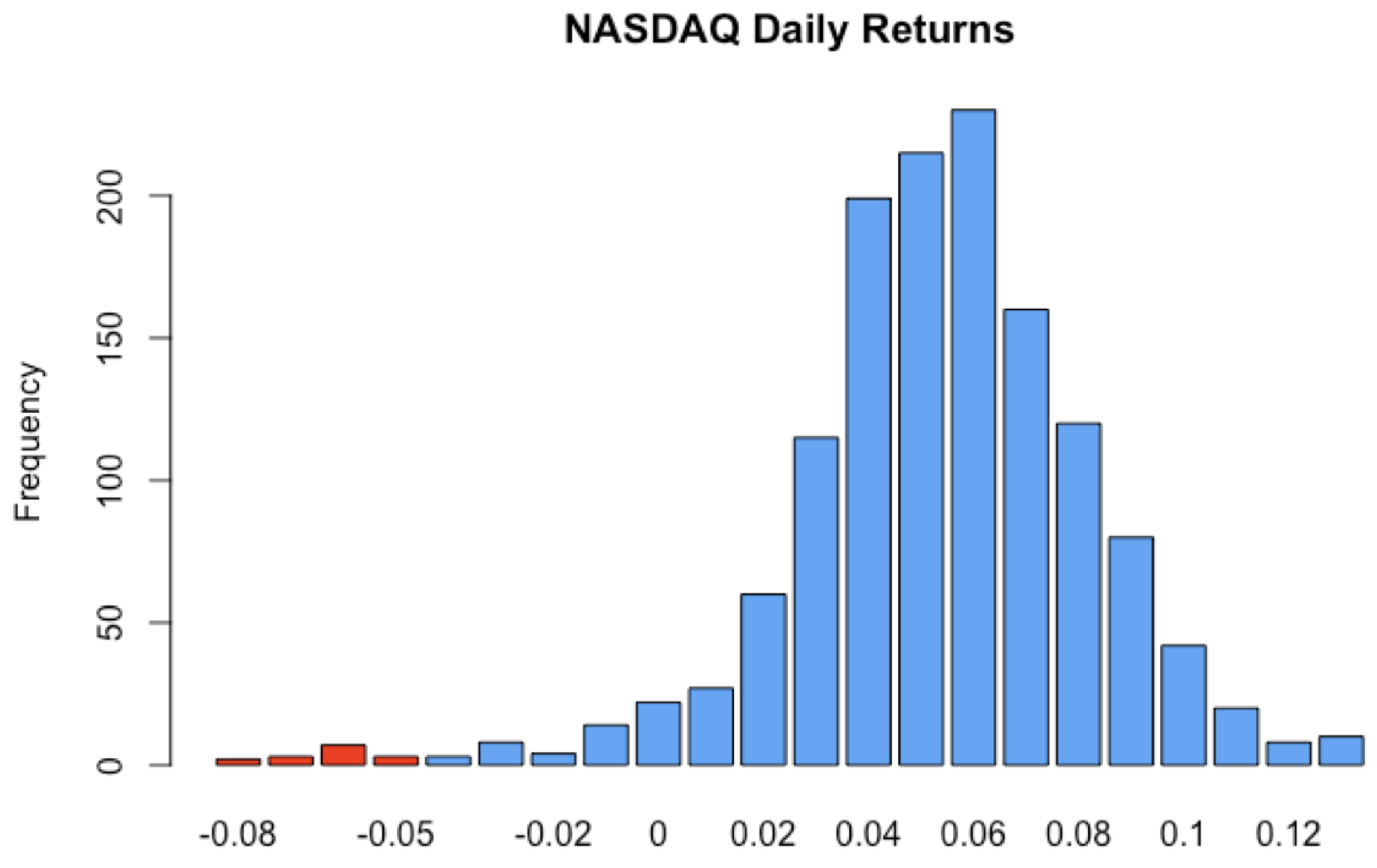
- Standard Deviation of portfolio returns:
  - Take the full sample of returns

$$SD = \sqrt{\frac{(R_1 - \mu)^2 + (R_2 - \mu)^2 + ... + (R_T - \mu)^2}{T - 1}}$$

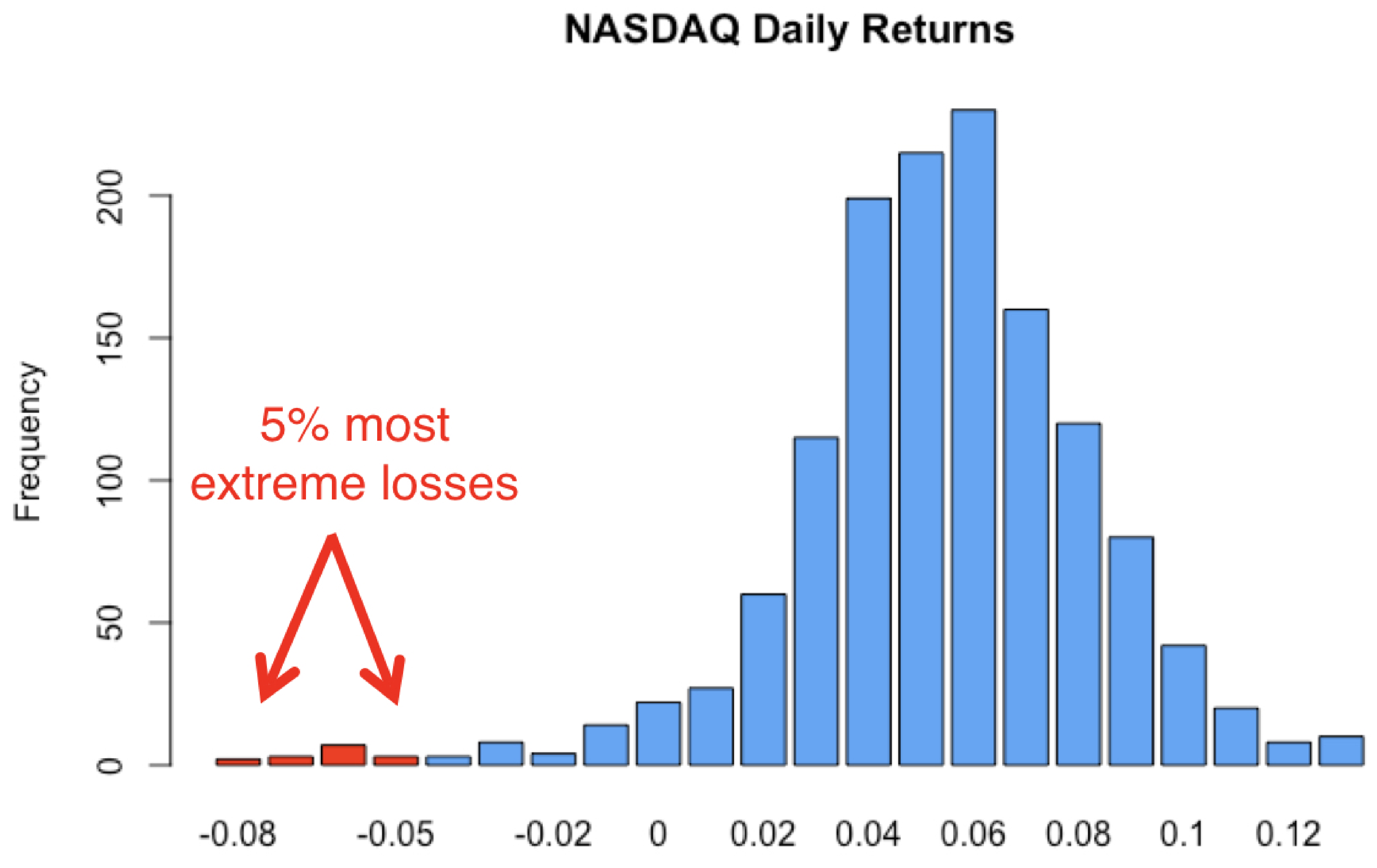
- Semi-Deviation of portfolio returns:
  - Take the subset of returns **below the mean**

$$SemiDev = \sqrt{\frac{(Z_1 - \mu)^2 + (Z_2 - \mu)^2 + ... + (Z_n - \mu)^2}{n}}$$

Value-at-risk & expected shortfall



Value-at-risk & expected shortfall



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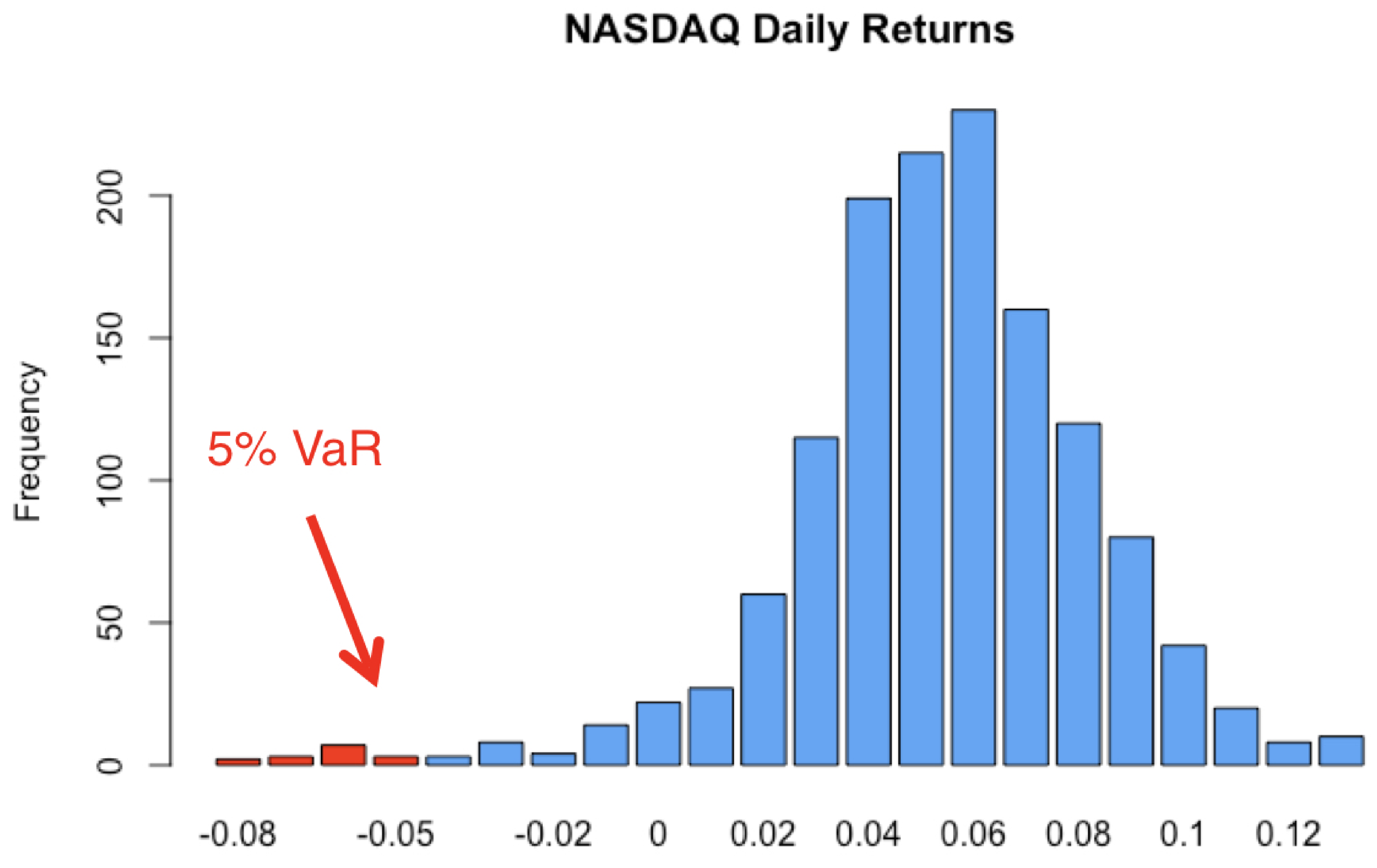

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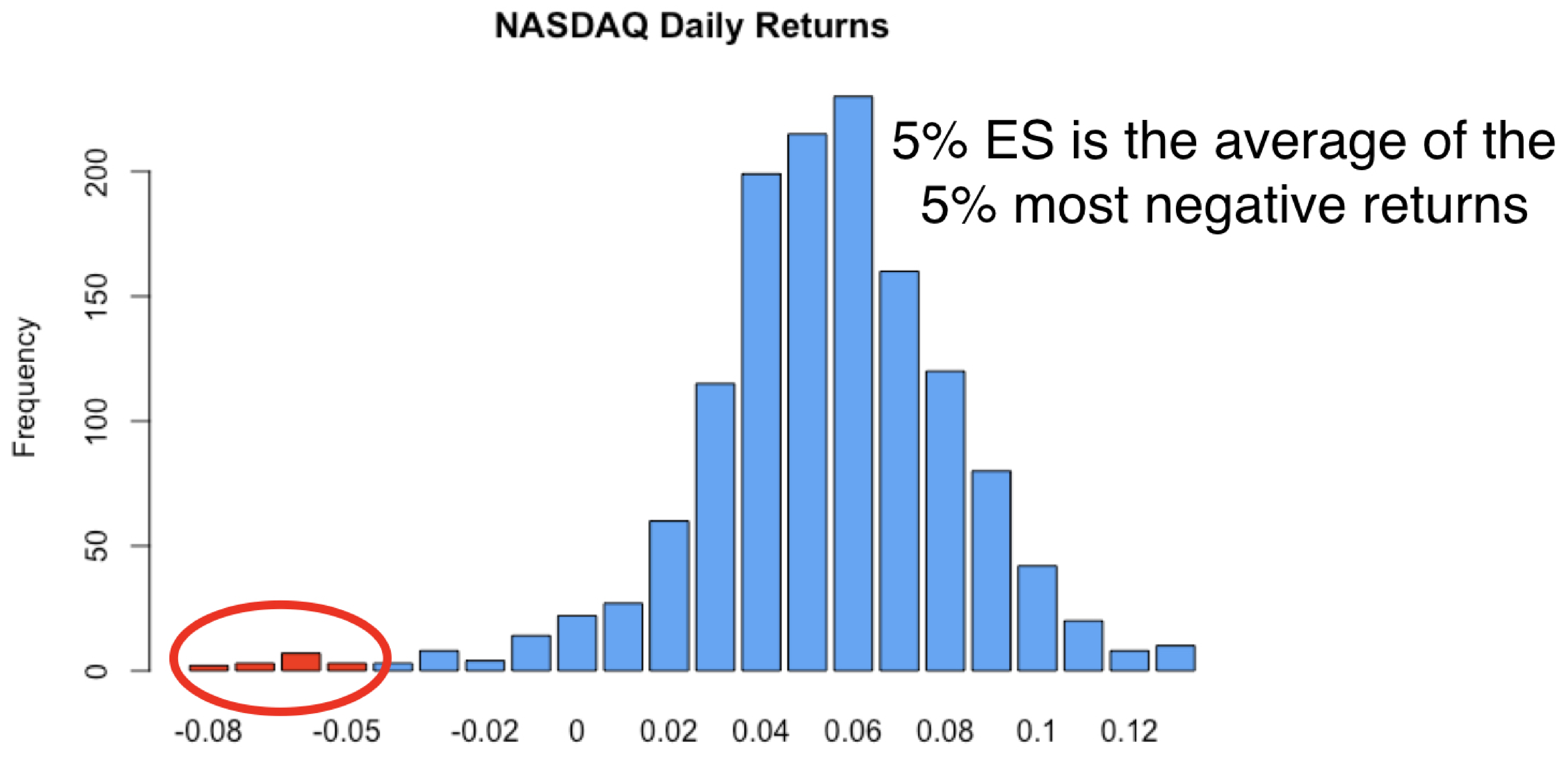
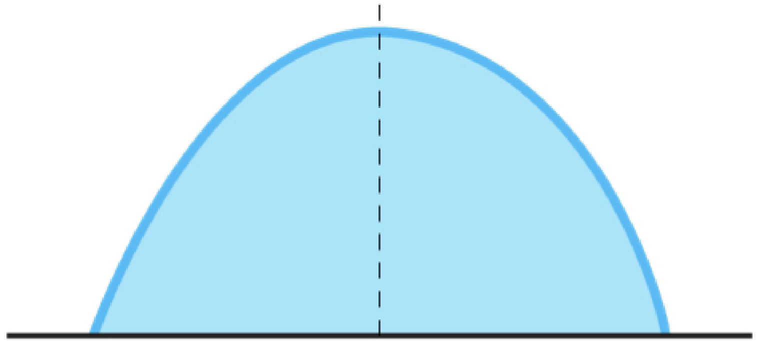


Shape of the distribution

- Is it symmetric?
  - Check the skewness
- Are the tails fatter than those of the normal distribution?
  - Check the excess kurtosis

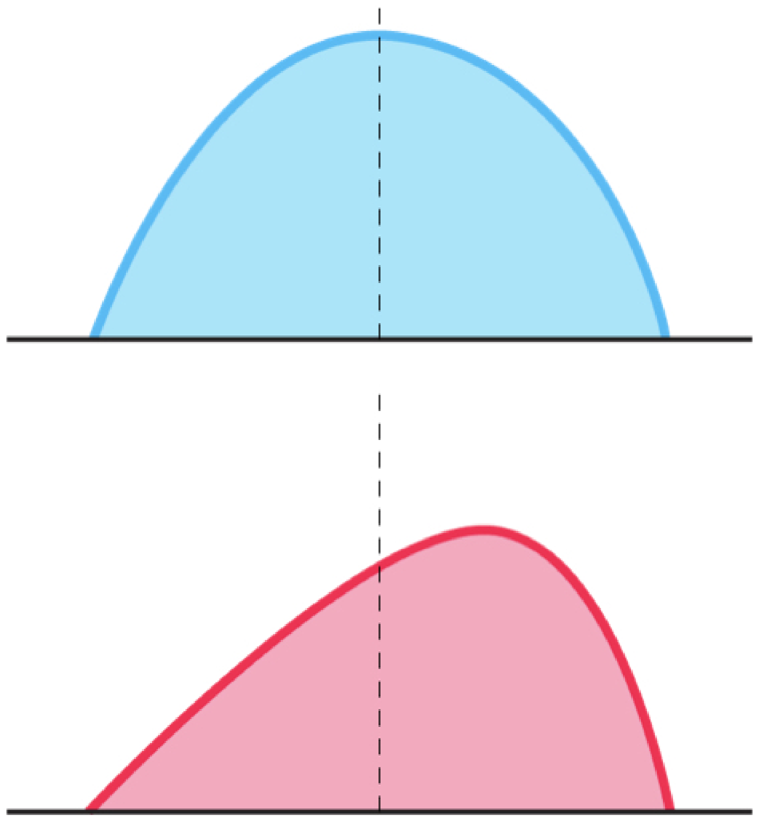
Skewness

- **Zero Skewness**
  - Distribution is symmetric



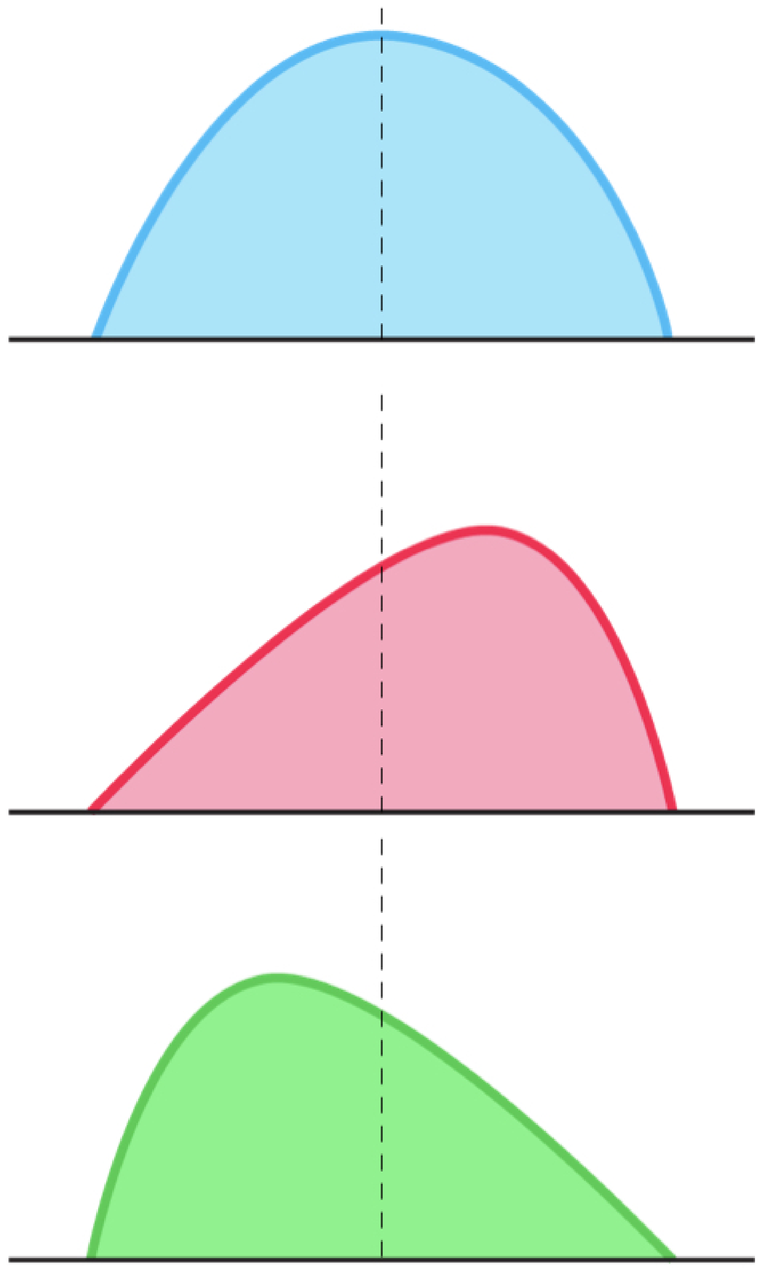
Skewness

- **Zero Skewness**
  - Distribution is symmetric
- **Negative Skewness**
  - Large negative returns occur more often than large positive returns



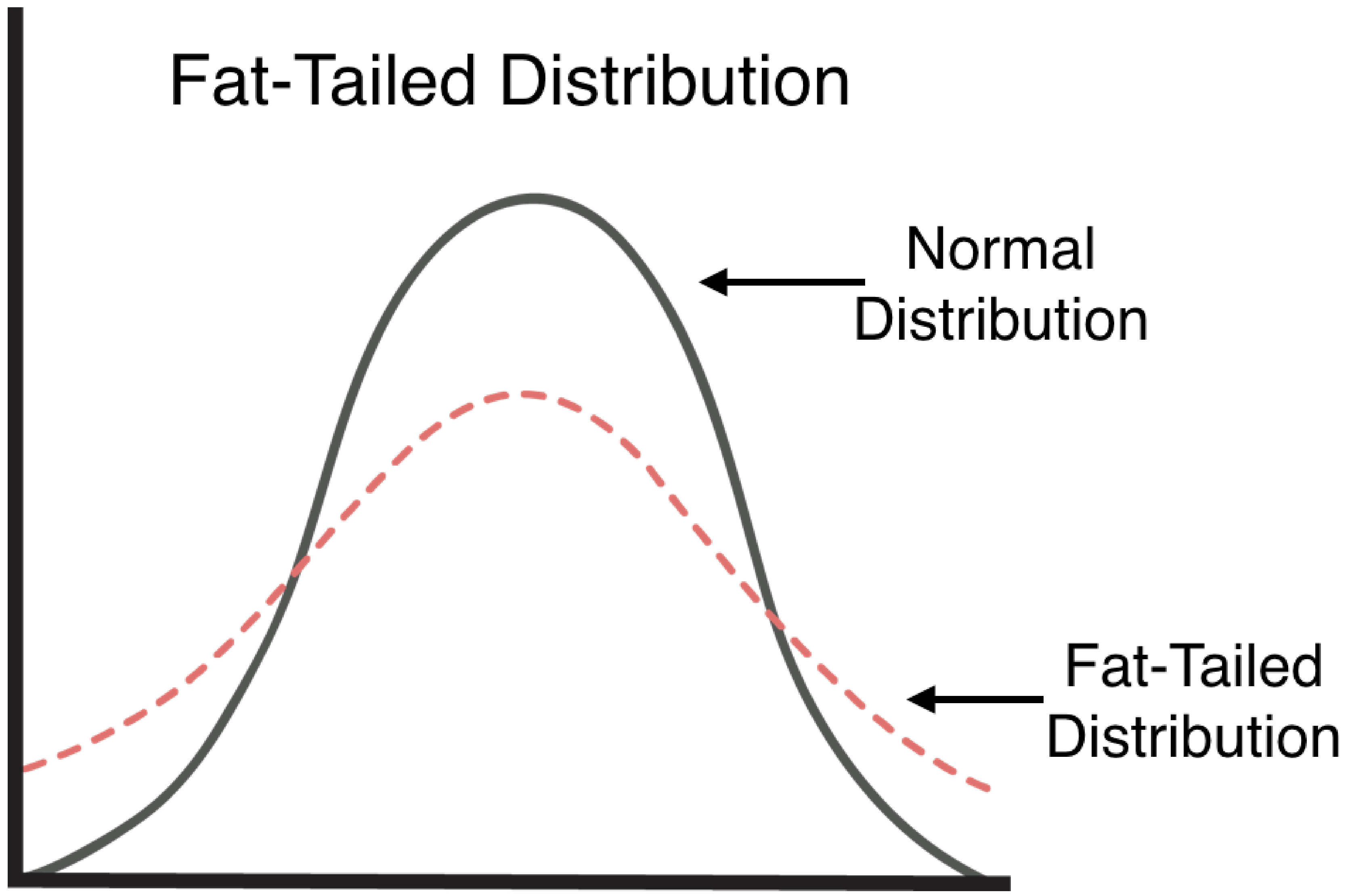
Skewness

- **Zero Skewness**
  - Distribution is symmetric
- **Negative Skewness**
  - Large negative returns occur more often than large positive returns
- **Positive Skewness**
  - Large positive returns occur more often than large negative returns



Kurtosis

- The distribution is fat-tailed when the excess kurtosis > 0



Let's practice!

INTRODUCTION TO PORTFOLIO ANALYSIS IN R

Drivers in the case of two assets

INTRODUCTION TO PORTFOLIO ANALYSIS IN R

Kris Boudt

Professor, Free University Brussels & Amsterdam

Future returns are random in nature

Optimizing Portfolio requires expectations:

- about average portfolio return (mean)
- about how far off it may be (variance)

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Why?

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Future returns are random in nature

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Why?

$\Downarrow$

Portfolio Return Is A Random Variable

Past performance to predictions

	Mean Portfolio Return
Computed on a sample of T Historical Returns	$\hat{\mu} = \frac{R_1 + R_2 + \ldots + R_T}{T}$
When the return is a random variable	$\mu = E[R]$

Past performance to predictions

	Mean Portfolio Return
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When the return is a random variable	$\mu = E[R]$

	Portfolio Return Variance
Computed on a sample of T Historical Returns	$\hat{\sigma}^2 = \frac{(R_1 - \hat{\mu})^2 + (R_2 - \hat{\mu})^2 + \ldots + (R_T - \hat{\mu})^2}{T - 1}$
When the return is a random variable	$\sigma^2 = E[(R - \mu)^2]$

Past performance to predictions

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Drivers of mean & variance

- Assume two assets:

Asset 1	Asset 2
Weight: w_1	Weight: w_2
Return: R_1	Return: R_2

- Portfolio Return, $P = w1 \cdot R1 + w2 \cdot R2$
- Thus: $E[P] = w_1 \cdot E[R_1] + w_2 \cdot E[R_2]$

Portfolio return variance

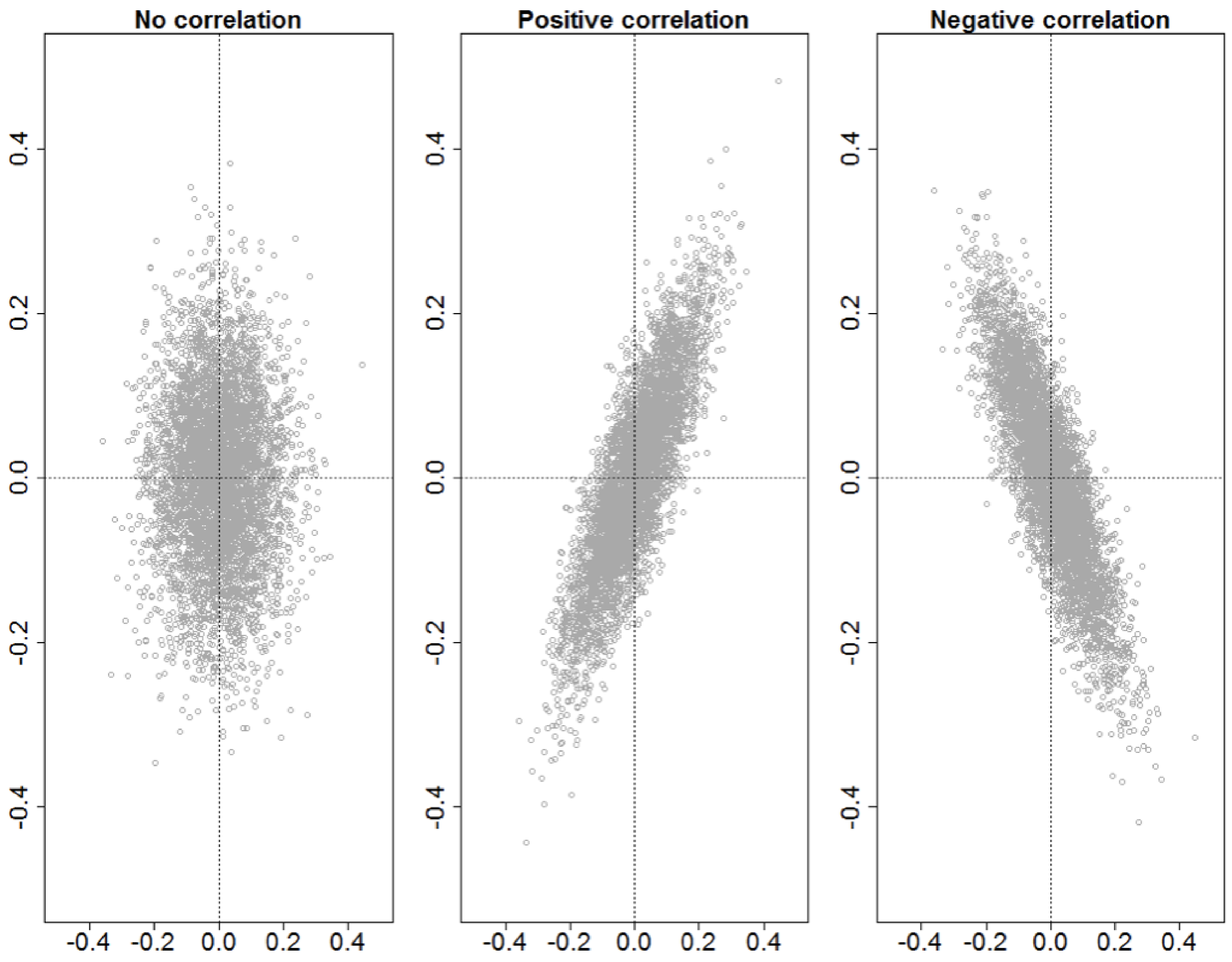
Again, for a portfolio with 2 assets

- $var(P) = w_1^2 \cdot var(R_1) + w_2^2 \cdot var(R_2) + 2 \cdot w_1 \cdot w_2 \cdot cov(R_1, R_2)$

Covariance between return 1 and 2

- $Cov(R_1, R_2)$
  - $= E[(R_1 - E[R_1])(R_2 - E(R_2))]$
  - $= StdDev(R_1) \cdot StdDev(R_2) \cdot corr(R_1, R_2)$

Correlations



Take away formulas

- **E[Portfolio Return] =** $E[P] = w_1 \cdot E[R_1] + w_2 \cdot E[R_2]$
- **var(Portfolio Return) =** $var(P)$ = $w_1^2 \cdot var(R_1) + w_2^2 \cdot var(R_2) + 2 \cdot w_1 \cdot w_2 \cdot cov(R_1, R_2)$

Let's practice!

INTRODUCTION TO PORTFOLIO ANALYSIS IN R

Using matrix notation

INTRODUCTION TO PORTFOLIO ANALYSIS IN R

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Professor, Free University Brussels & Amsterdam

Variables at stake for n assets

w: the N x 1 column-matrix of portfolio weights:

$$w = \begin{bmatrix} w_1 \\ w_2 \\ \dots \\ w_N \end{bmatrix}$$

μ: the N x 1 column-matrix of expected returns:

$$\mu = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \dots \\ \mu_N \end{bmatrix}$$

R: the N x 1 column-matrix of asset returns:

$$R = \begin{bmatrix} R_1 \\ R_2 \\ \dots \\ R_N \end{bmatrix}$$

Σ: The N x N covariance matrix of the N asset returns:

$$w = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \dots & \sigma_{1N} \\ \sigma_{21} & \sigma_2^2 & \dots & \sigma_{2N} \\ \dots & \dots & \dots & \dots \\ \sigma_{N1} & \sigma_{N2} & \dots & \sigma_N^2 \end{bmatrix}$$

Generalizing from 2 to n assets

Portfolio Return

$\boxed{w_1 * R_1 + w_2 * R_2}$ $\Rightarrow$ $\boxed{w_1 * R_1 + \ldots + w_N * R_N}$

Generalizing from 2 to n assets

Portfolio Return

$$\boxed{w_1 * R_1 + w_2 * R_2} \Rightarrow \boxed{w_1 * R_1 + \ldots + w_N * R_N}$$

Portfolio Expected Return

$$\boxed{w_1 * \mu_1 + w_2 * \mu_2} \Rightarrow \boxed{w_1 * \mu_1 + \ldots + w_N * \mu_N}$$

Generalizing from 2 to n assets

Portfolio Return

$$\boxed{w_1 * R_1 + w_2 * R_2} \Rightarrow \boxed{w_1 * R_1 + \ldots + w_N * R_N}$$

Portfolio Expected Return

$$\boxed{w_1 * \mu_1 + w_2 * \mu_2} \Rightarrow \boxed{w_1 * \mu_1 + \ldots + w_N * \mu_N}$$

Portfolio Variance

$$\boxed{\begin{array}{c} w_1^2 * var(R_1) + w_2^2 * var(R_2) \\ +2 * w_1 * w_2 * cov(R_1, R_2) \end{array}} \Rightarrow \boxed{\begin{array}{l} w_1^2 + var(R_1) + \ldots + w_N^2 * var(R_N) \\ +2 * w_1 * w_2 * cov(R_1, R_2) + \ldots \\ +2 * w_{N-1} * w_N * cov(R_{N-1}, R_N) \end{array}}$$

Matrices simplify the notation

- Avoid large number of terms by using matrix notation
- We have 4 matrices:
  - weights (w), returns (R), expected returns (μ), and covariance matrix (Σ)

$$w = \begin{bmatrix} w_1 \\ w_2 \\ \ldots \\ w_N \end{bmatrix}$$

$$w' = \left[\ w_1 \ w_2 \ \ldots \ w_N \ \right]$$

Simplifying the notation

Portfolio Return

$$\boxed{w_1 * R_1 + \ldots + w_N * R_N} \Rightarrow \boxed{w' R}$$

Simplifying the notation

Portfolio Return

$$\boxed{w_1 * R_1 + \ldots + w_N * R_N} \Rightarrow \boxed{w' R}$$

Portfolio Expected Return

$$\boxed{w_1 * \mu_1 + \ldots + w_N * \mu_N} \Rightarrow \boxed{w' \mu}$$

Simplifying the notation

Portfolio Return

$$\boxed{w_1 * R_1 + \ldots + w_N * R_N} \Rightarrow \boxed{w'R}$$

Portfolio Expected Return

$$\boxed{w_1 * \mu_1 + \ldots + w_N * \mu_N} \Rightarrow \boxed{w'\mu}$$

Portfolio Variance

$$\boxed{\begin{array}{l} w_1^2 + var(R_1) + \ldots + w_N^2 * var(R_N) \\ +2 * w_1 * w_2 * cov(R_1, R_2) + \ldots \\ +2 * w_{N-1} * w_N * cov(R_{N-1}, R_N) \end{array}} \Rightarrow \boxed{w'\Sigma w}$$

Let's practice!

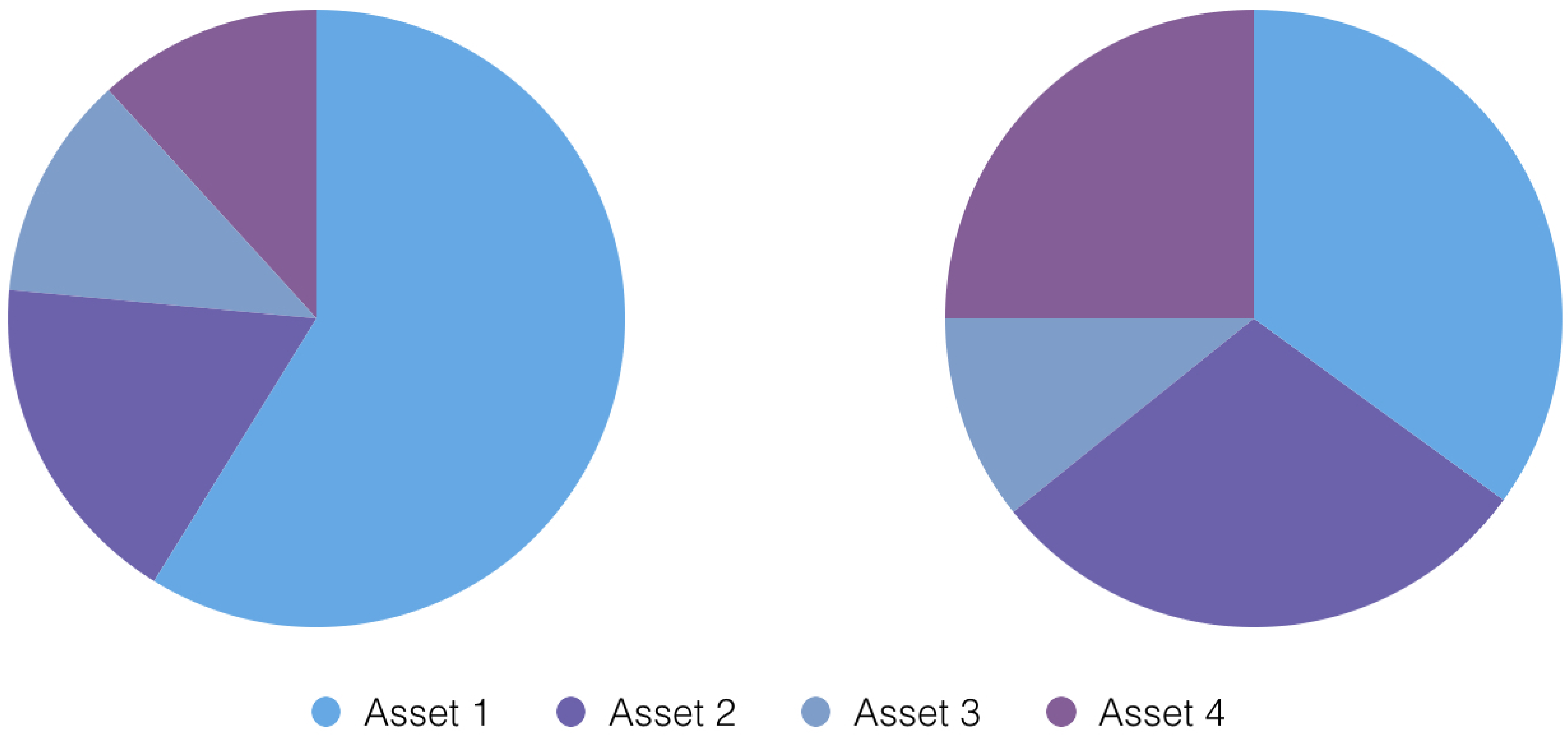
INTRODUCTION TO PORTFOLIO ANALYSIS IN R

Portfolio risk budget

INTRODUCTION TO PORTFOLIO ANALYSIS IN R

Kris Boudt
Professor, Free University Brussels & Amsterdam

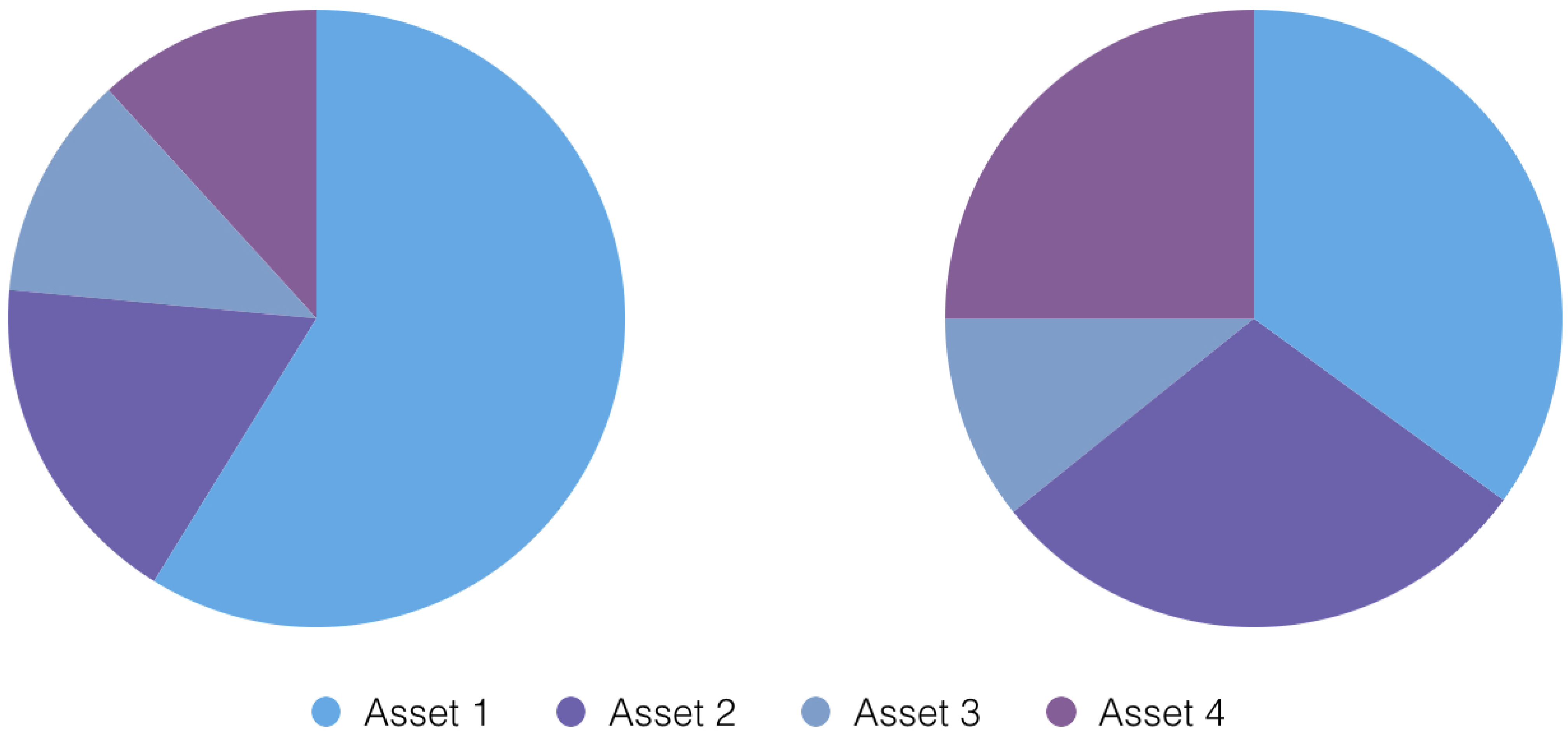
Who did it?



Who did it?

Capital Allocation Budget

Portfolio Volatility Risk



Portfolio volatility in risk contribution

$$\text{Portfolio Volatility} = \sum_{i=1}^{N} RC_i$$

where: $RC_i = \frac{w_i(\Sigma w)_i}{\sqrt{w'\Sigma w}}$

- Risk contribution of asset i depends on
  1. the complete matrix of weights w
  2. the full covariance matrix Σ

Percent risk contribution

$$\%RC_i = \frac{RC_i}{\text{Portfolio volatility}}$$

- where $\sum_{i=1}^{N} \%RC_i = 1$

Relatively less risky assets: $\%RC_i > w_i$

Relatively more risky assets: $\%RC_i < w_i$

Let's practice!

INTRODUCTION TO PORTFOLIO ANALYSIS IN R

Modern portfolio theory of Harry Markowitz

INTRODUCTION TO PORTFOLIO ANALYSIS IN R

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Professor, Free University Brussels & Amsterdam

Portfolio weights are optimal

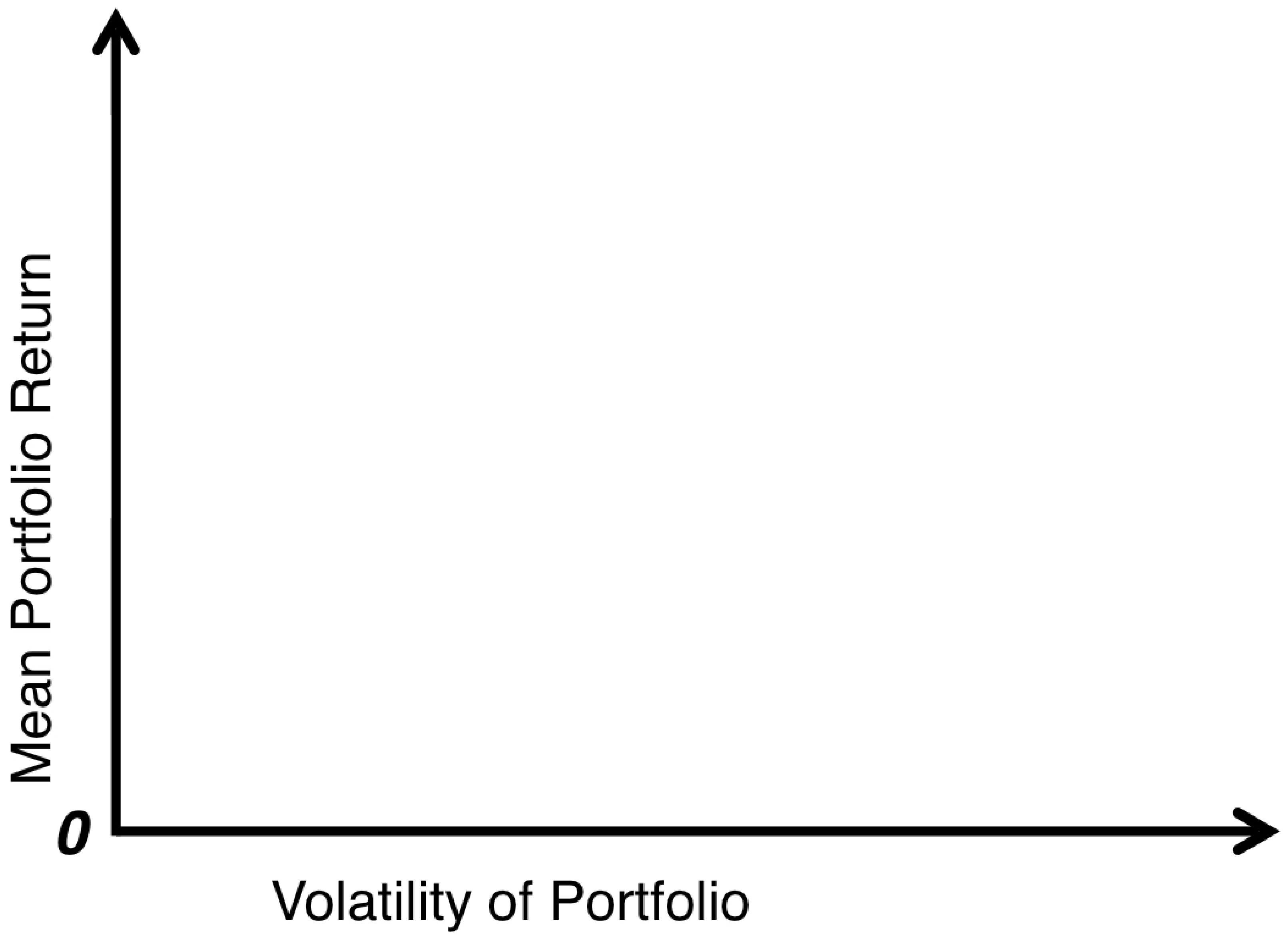
...when they optimize an objective function while satisfying the constraints.

Possible Objectives	**Possible Constraints**
Maximize expected return	Only positive weights
Minimize the variance	Weights sum to 1 (all capital needs to be invested)
Maximize the Sharpe ratio	Portfolio expected return equals a target value

Harry Markowitz

- Nobel Prize Winner
- Recommends finding optimal portfolios by
  - *Objective*: Minimize portfolio variance
  - *Constraints*:
    - Full investment
    - Expected return should be equal to a pre-specified target return

The H. Markowitz approach



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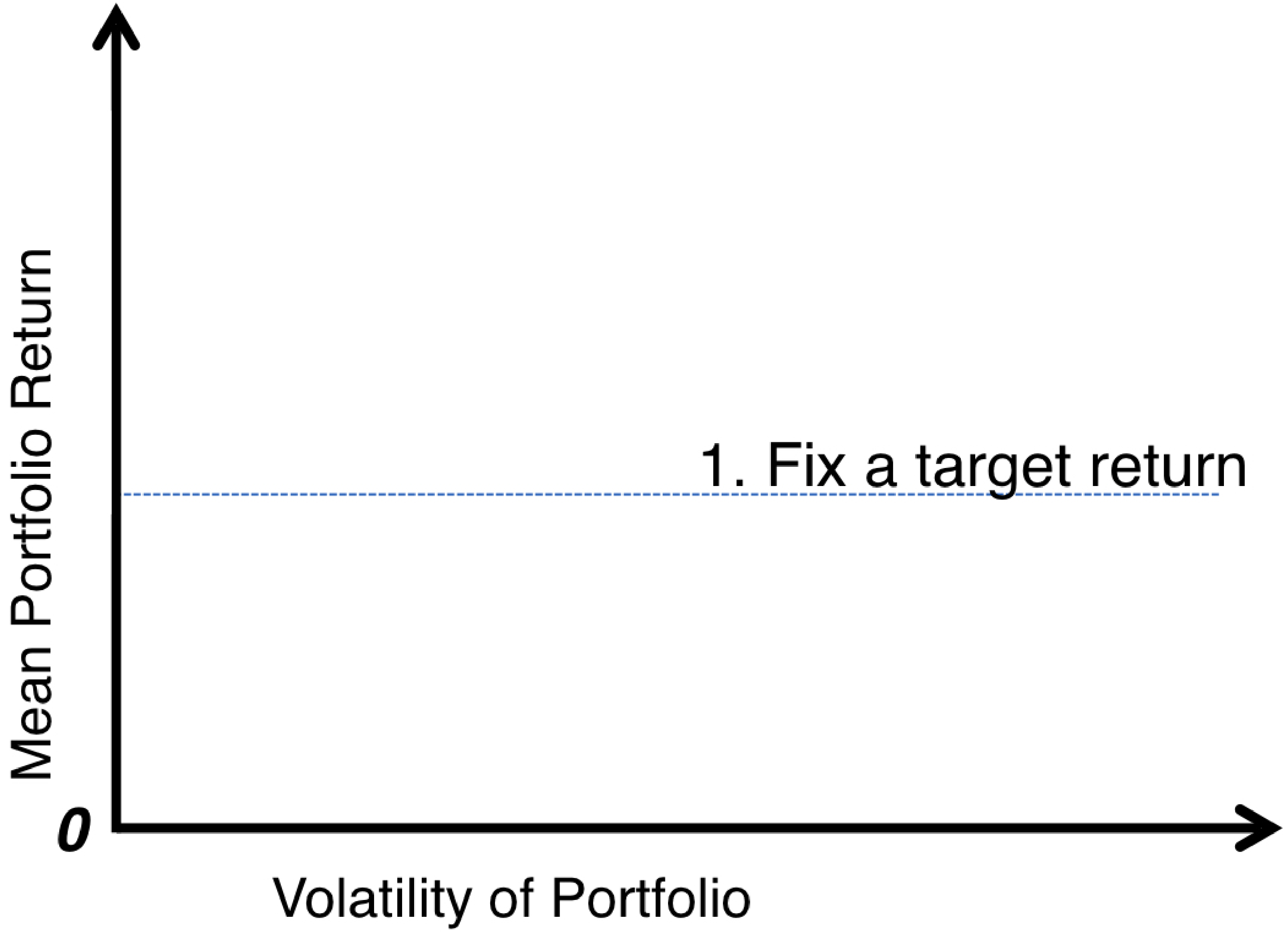

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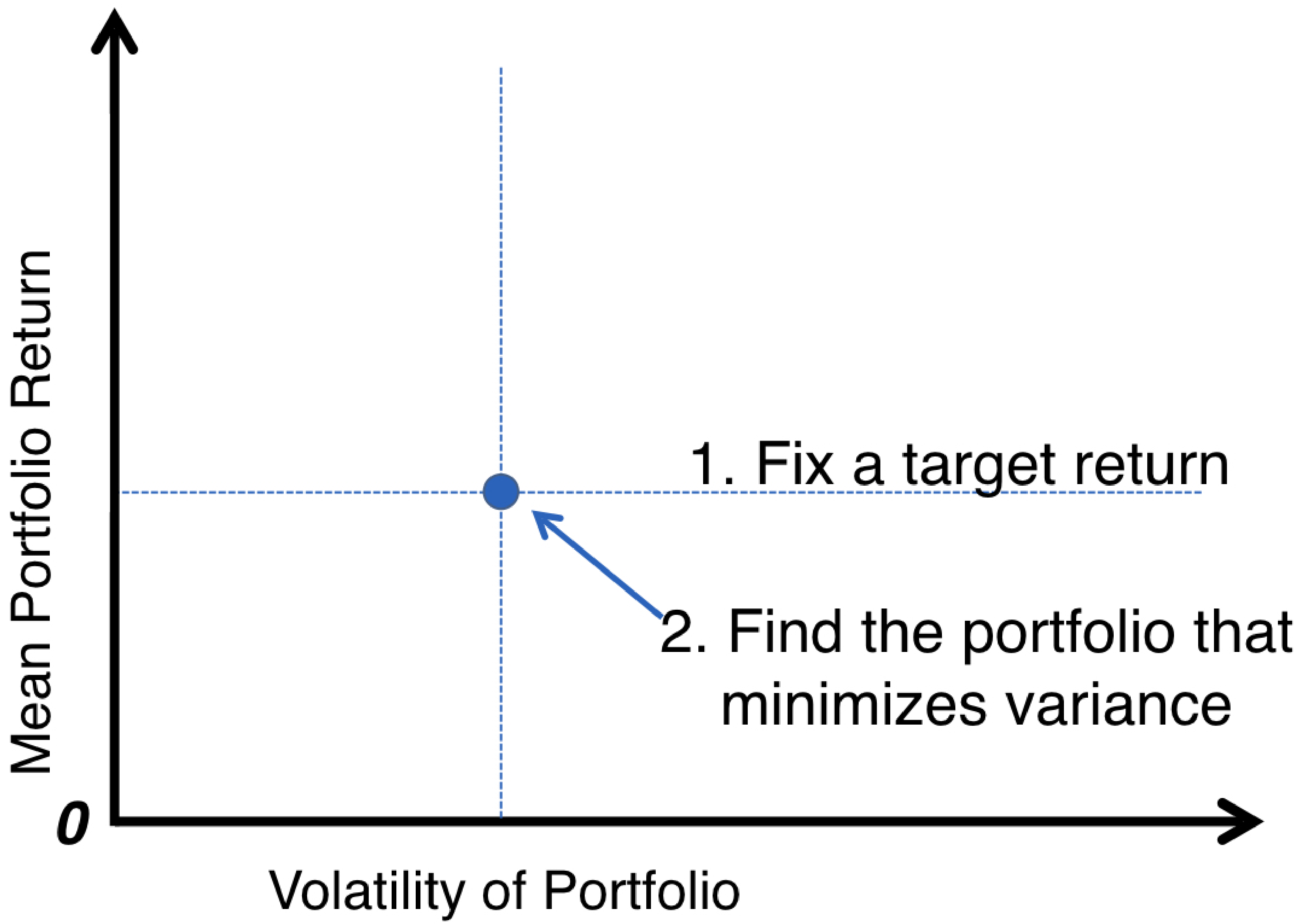

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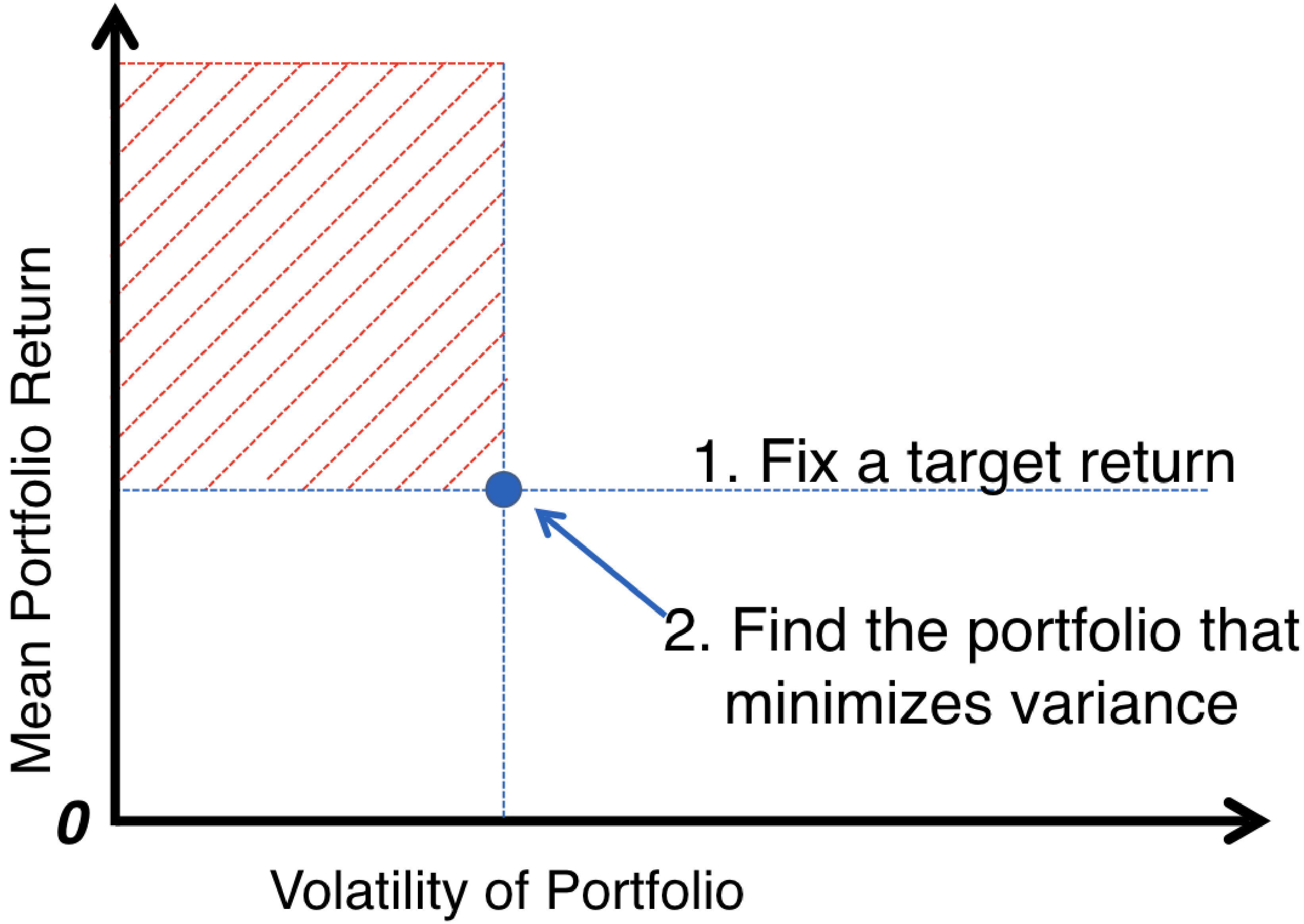

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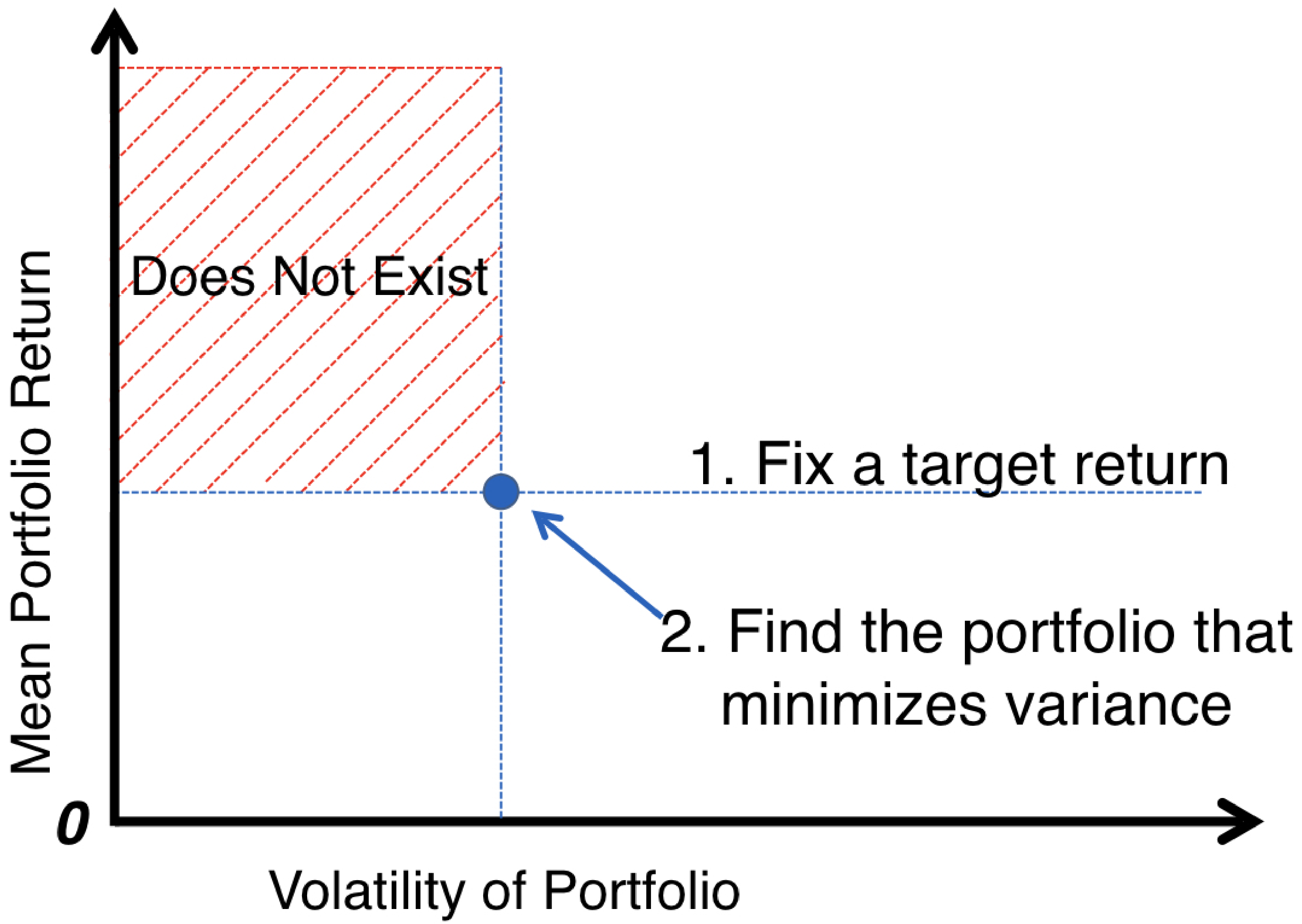

Let's practice!

INTRODUCTION TO PORTFOLIO ANALYSIS IN R

The efficient frontier

INTRODUCTION TO PORTFOLIO ANALYSIS IN R

Kris Boudt
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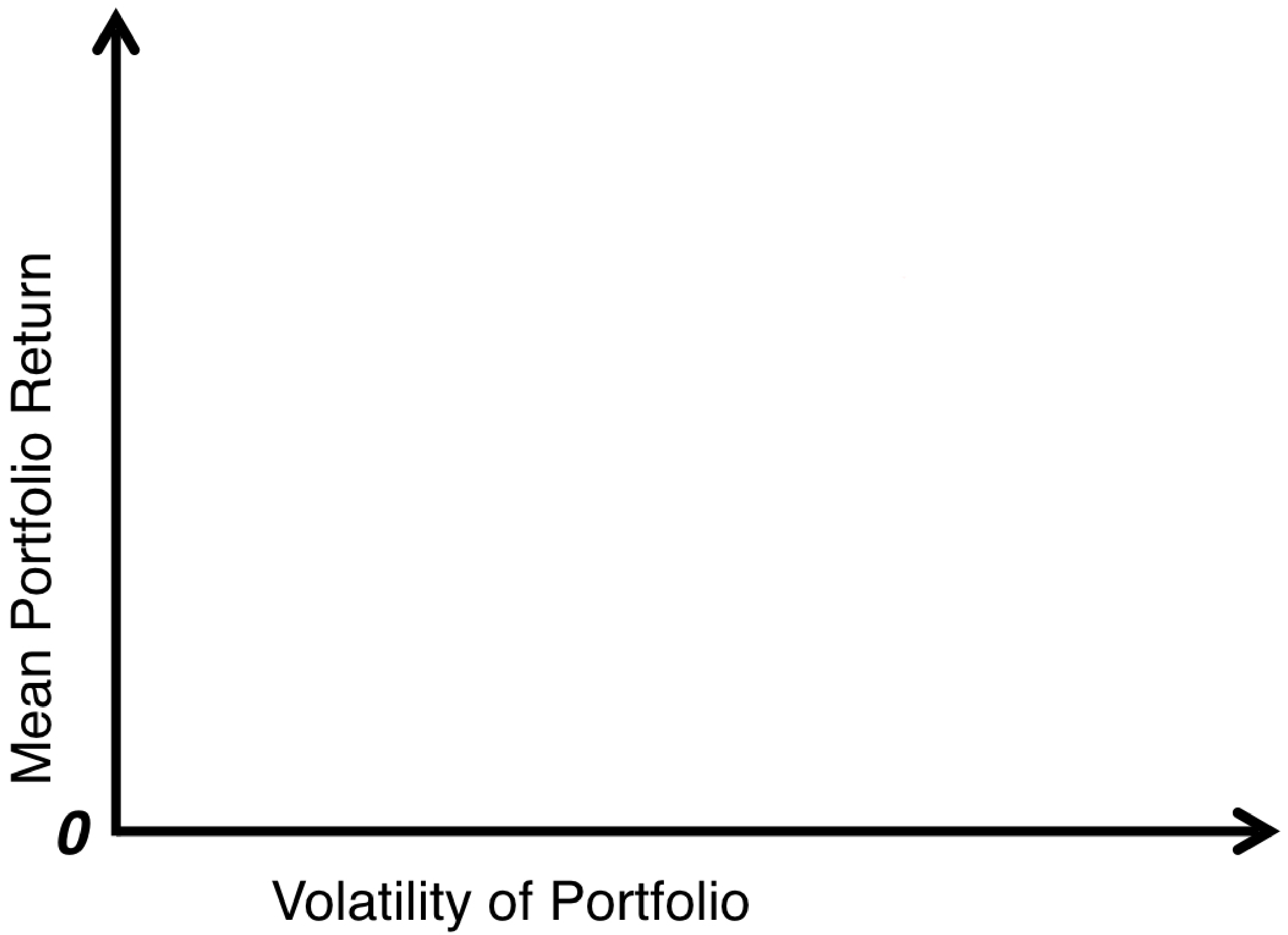
Changing target return



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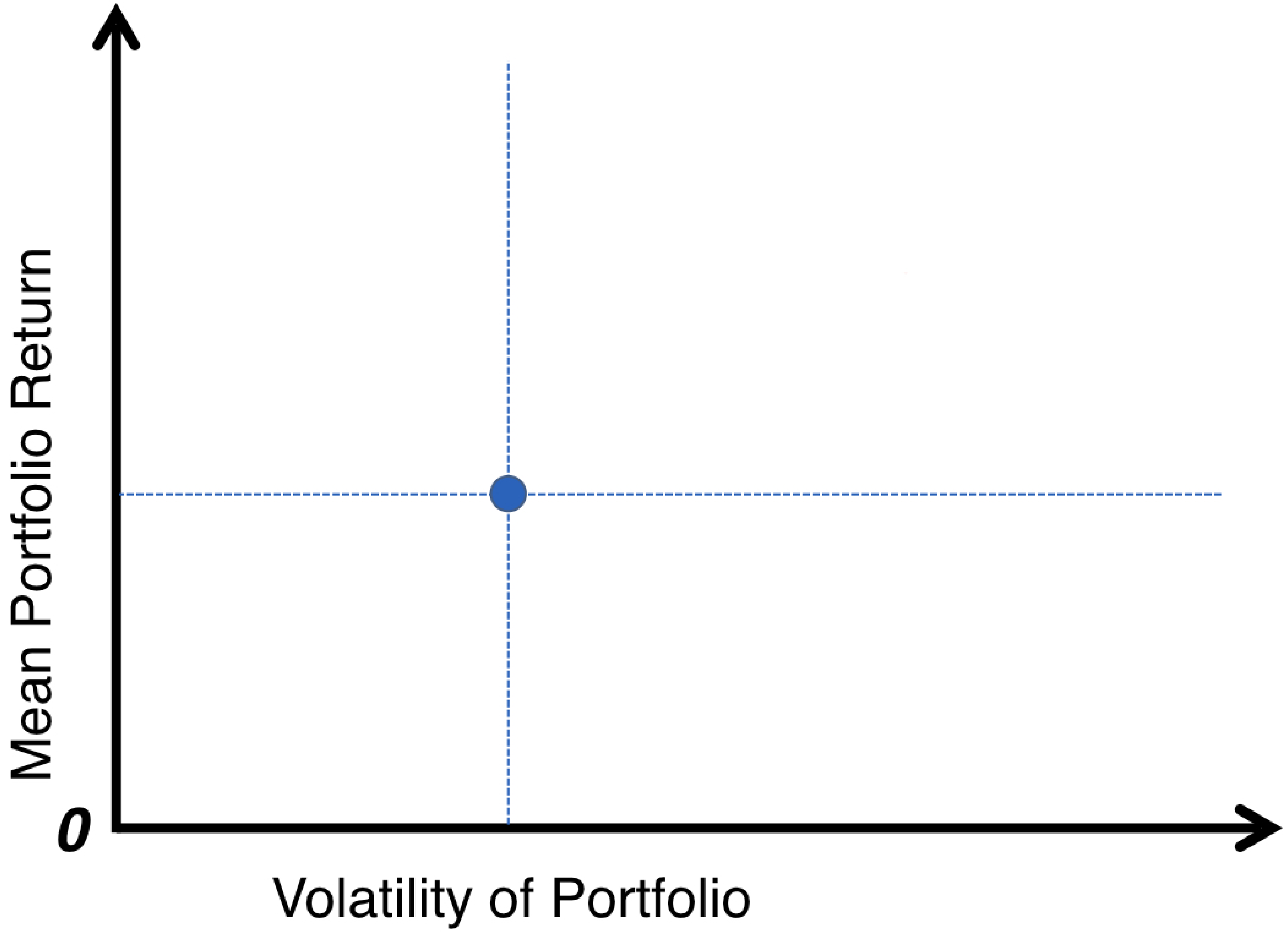

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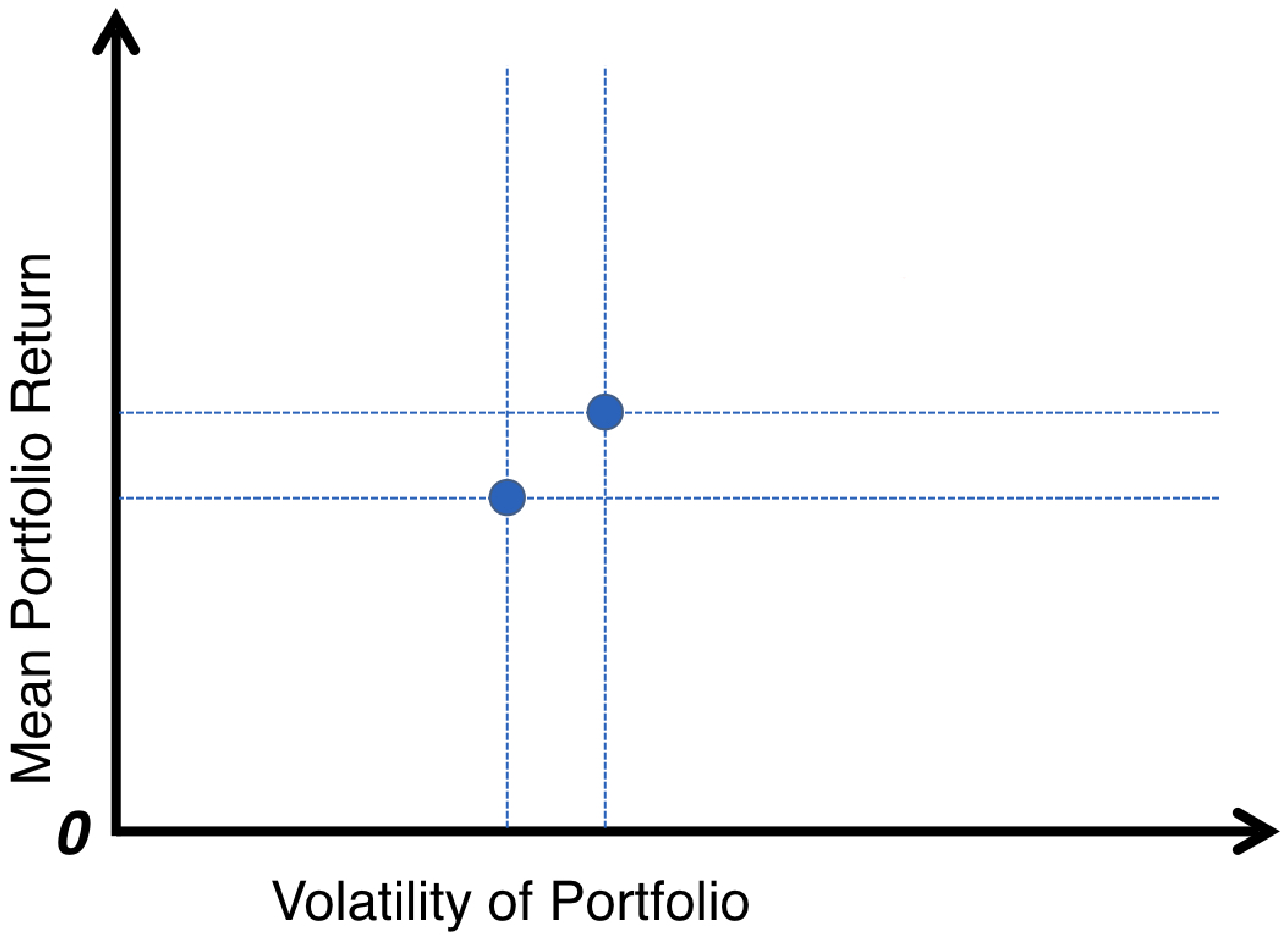

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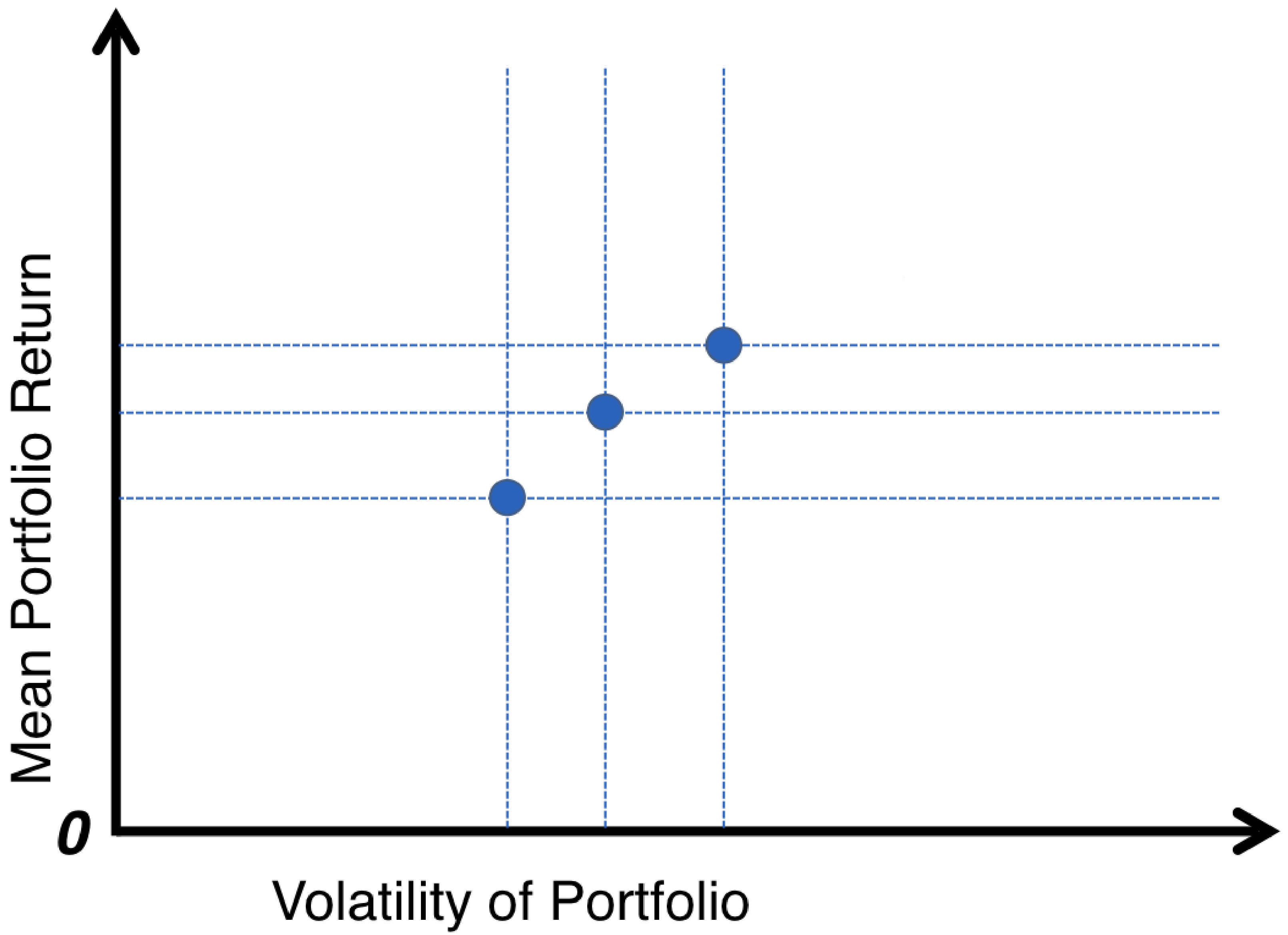

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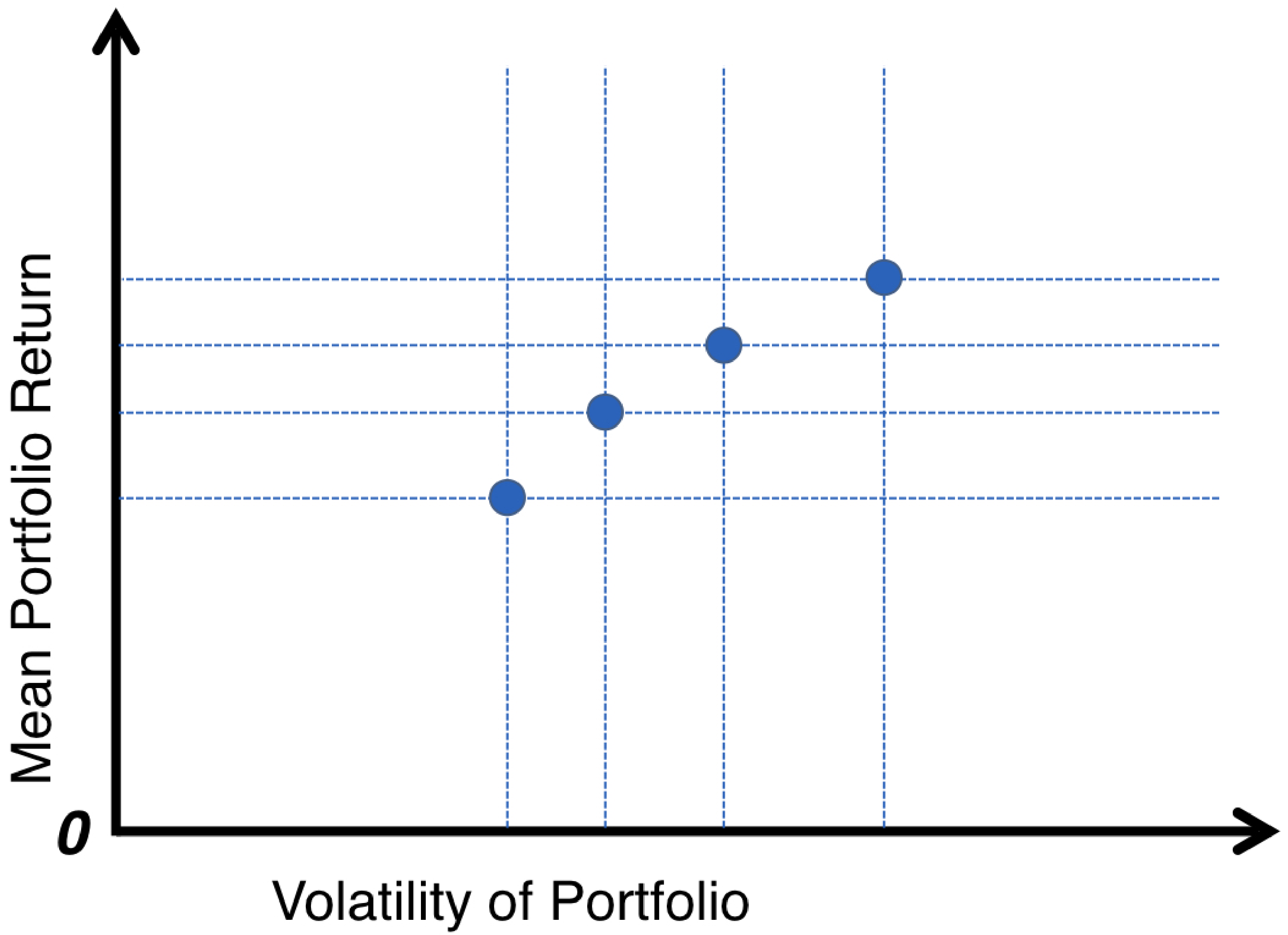

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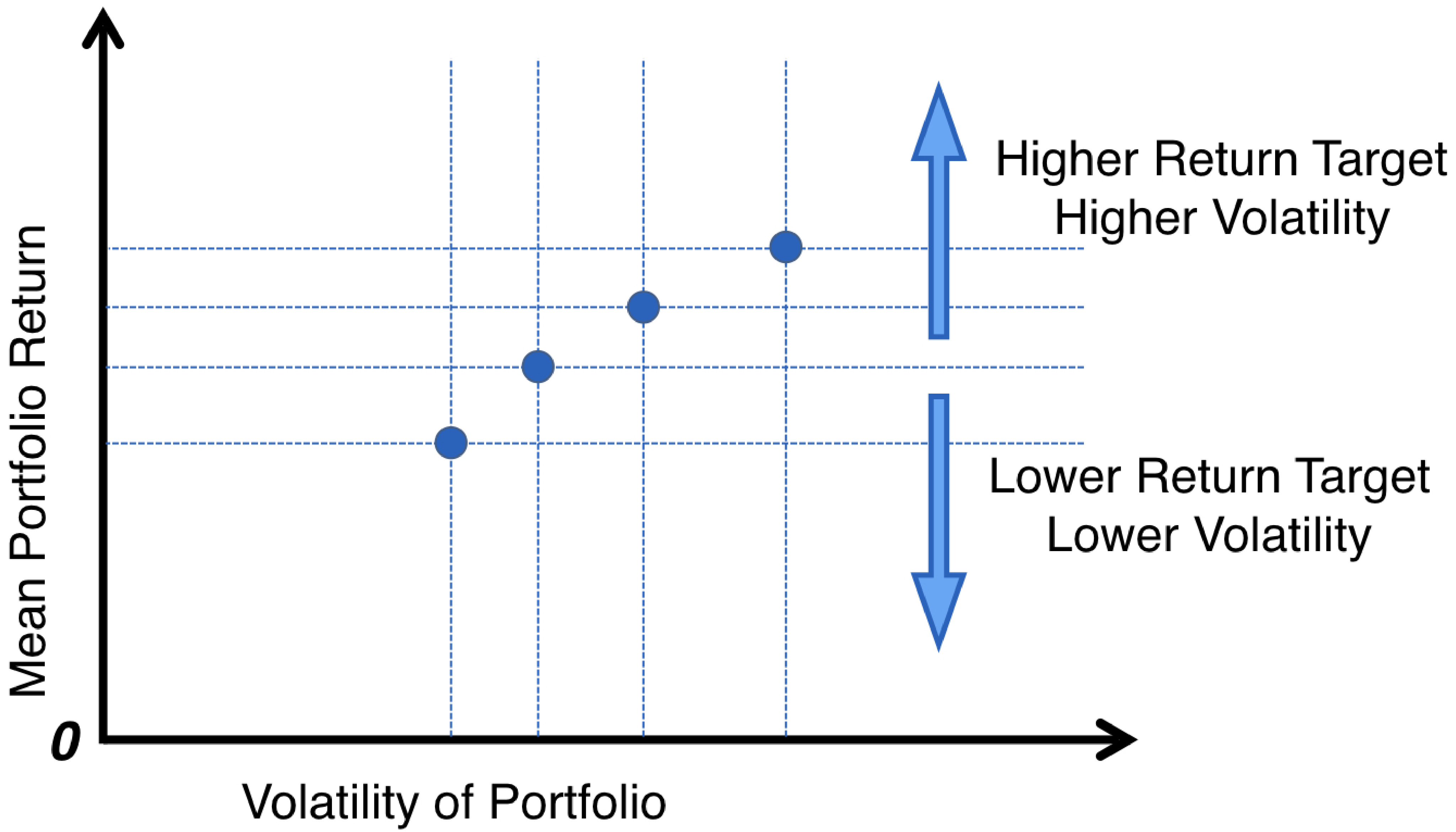

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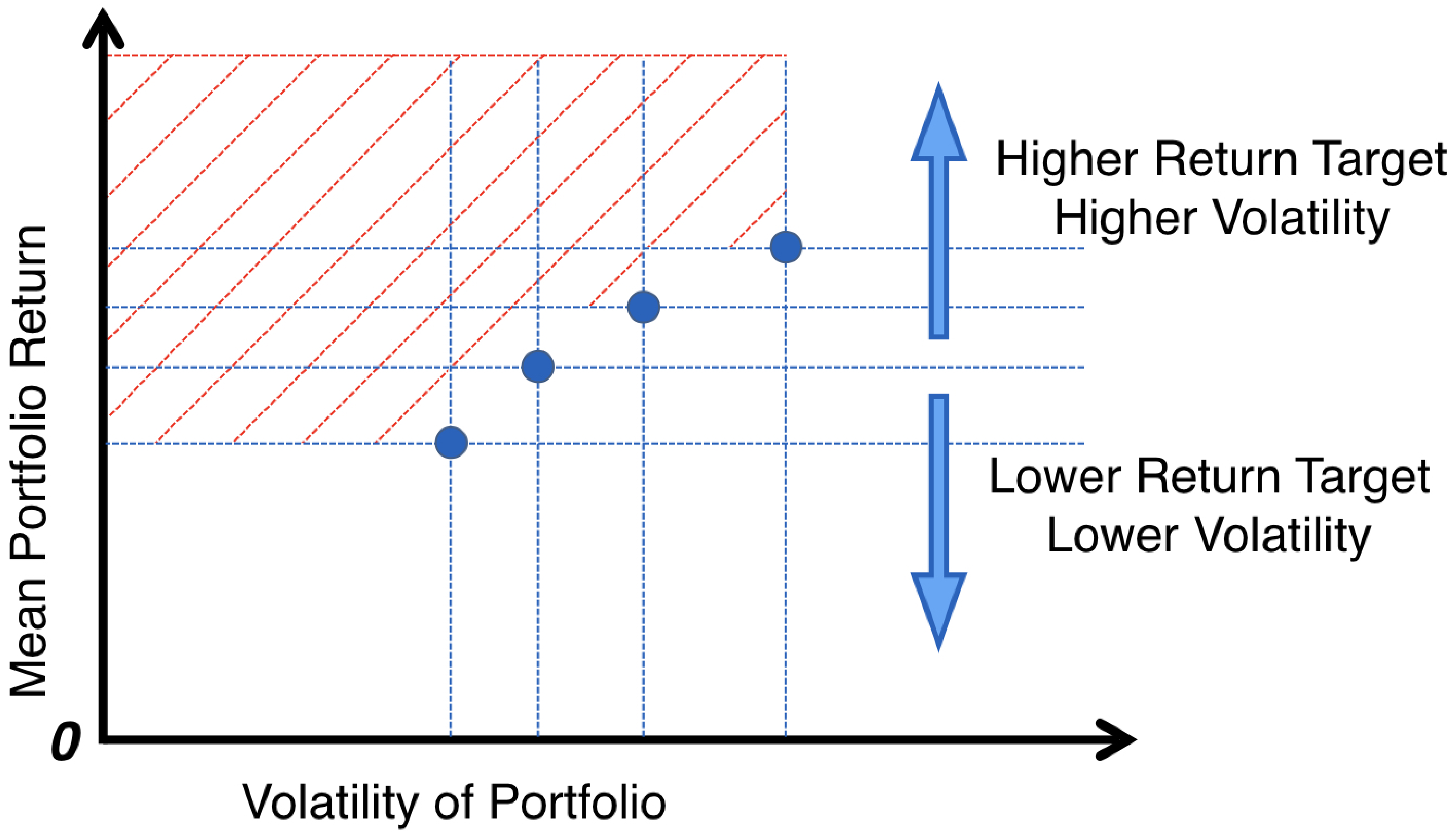
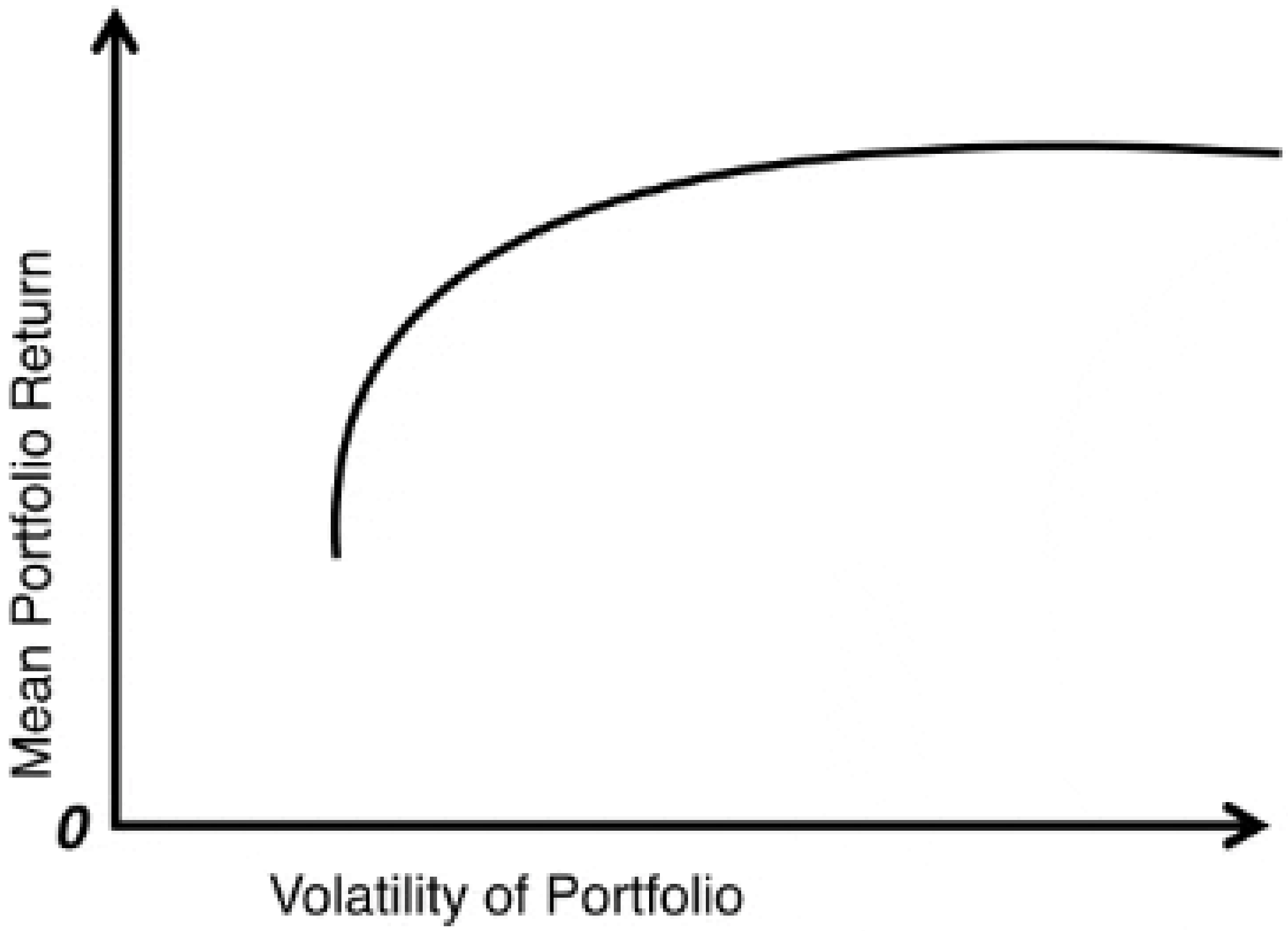
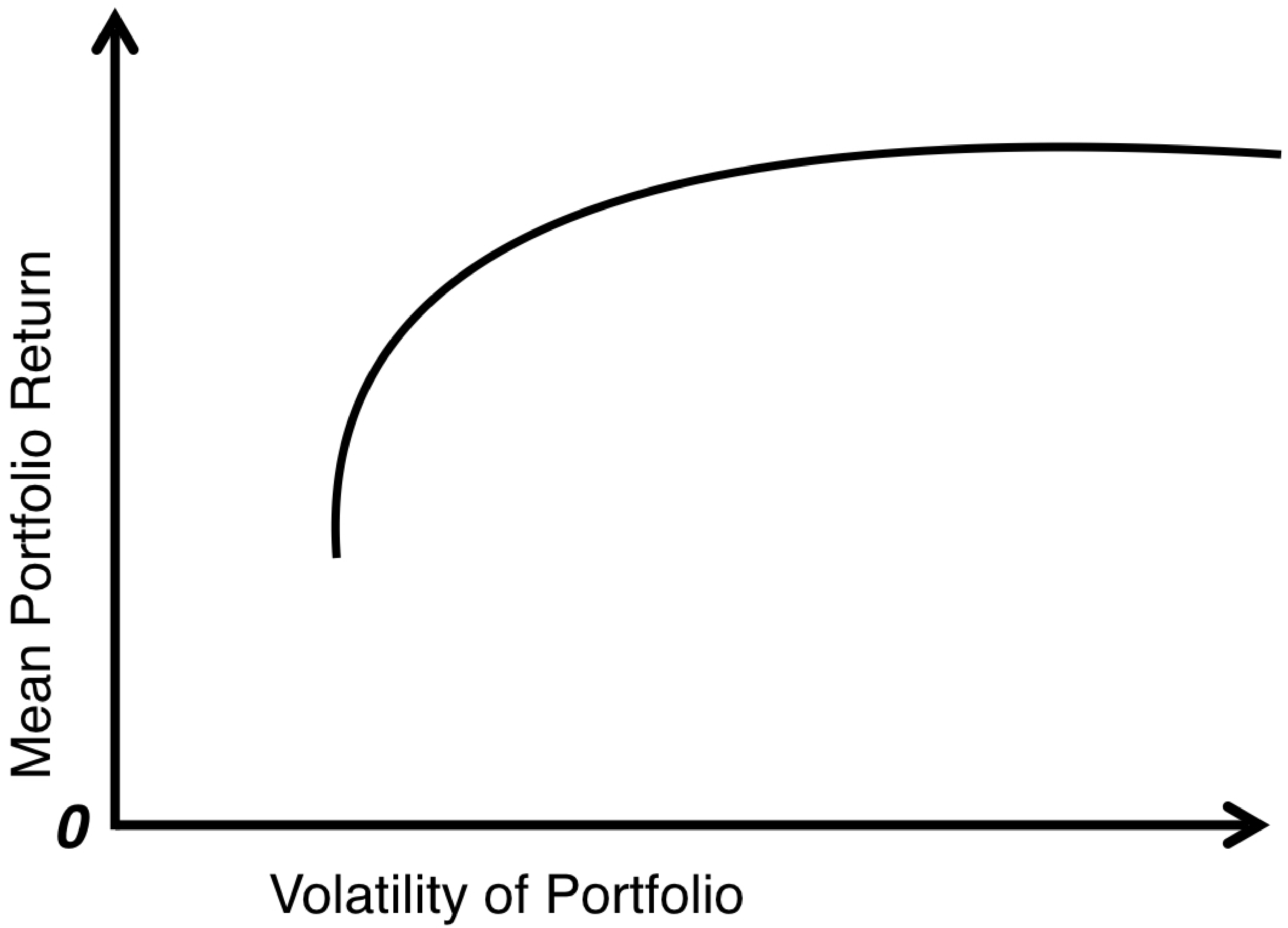


The efficient frontier



Minimum variance portfolio



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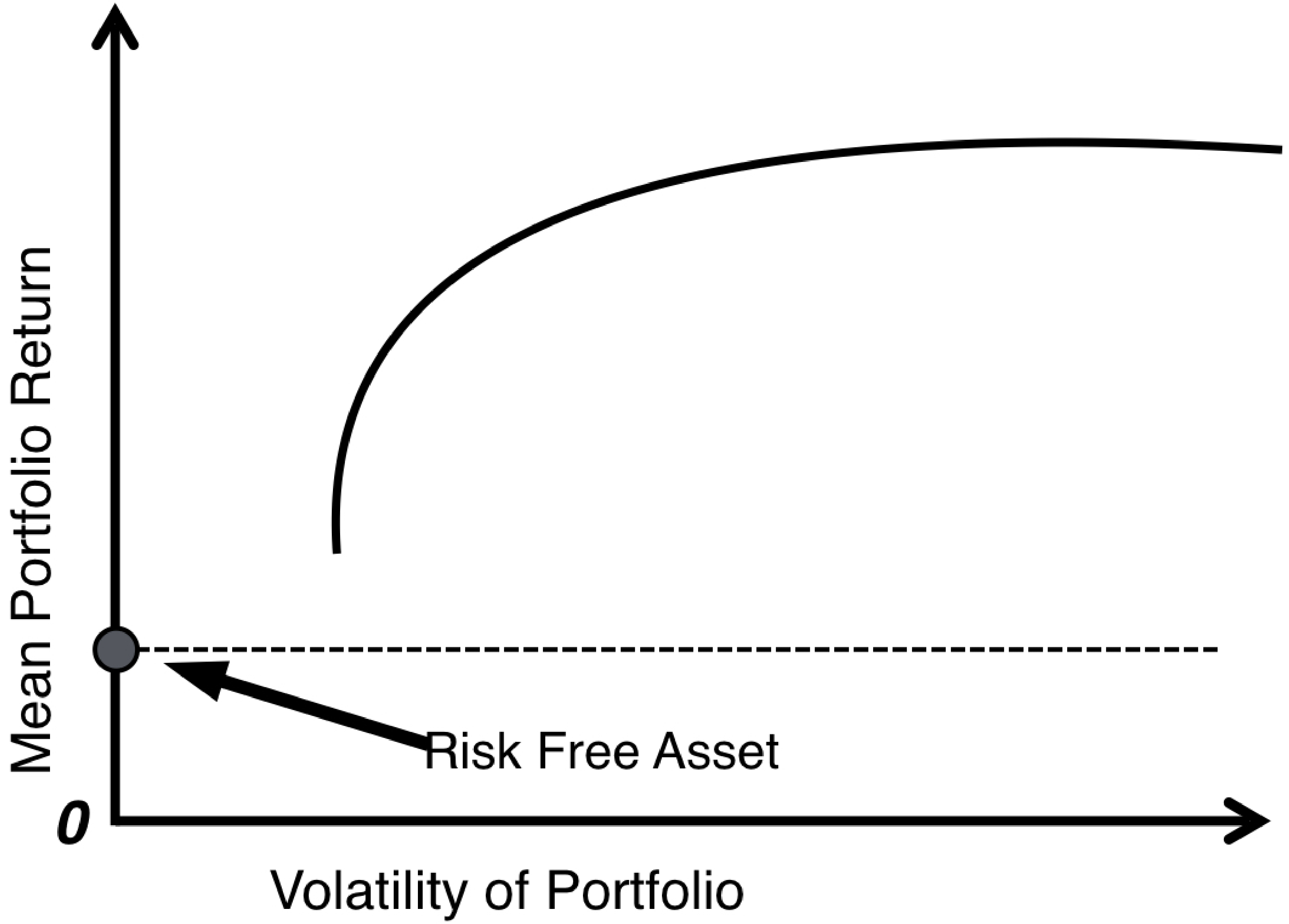

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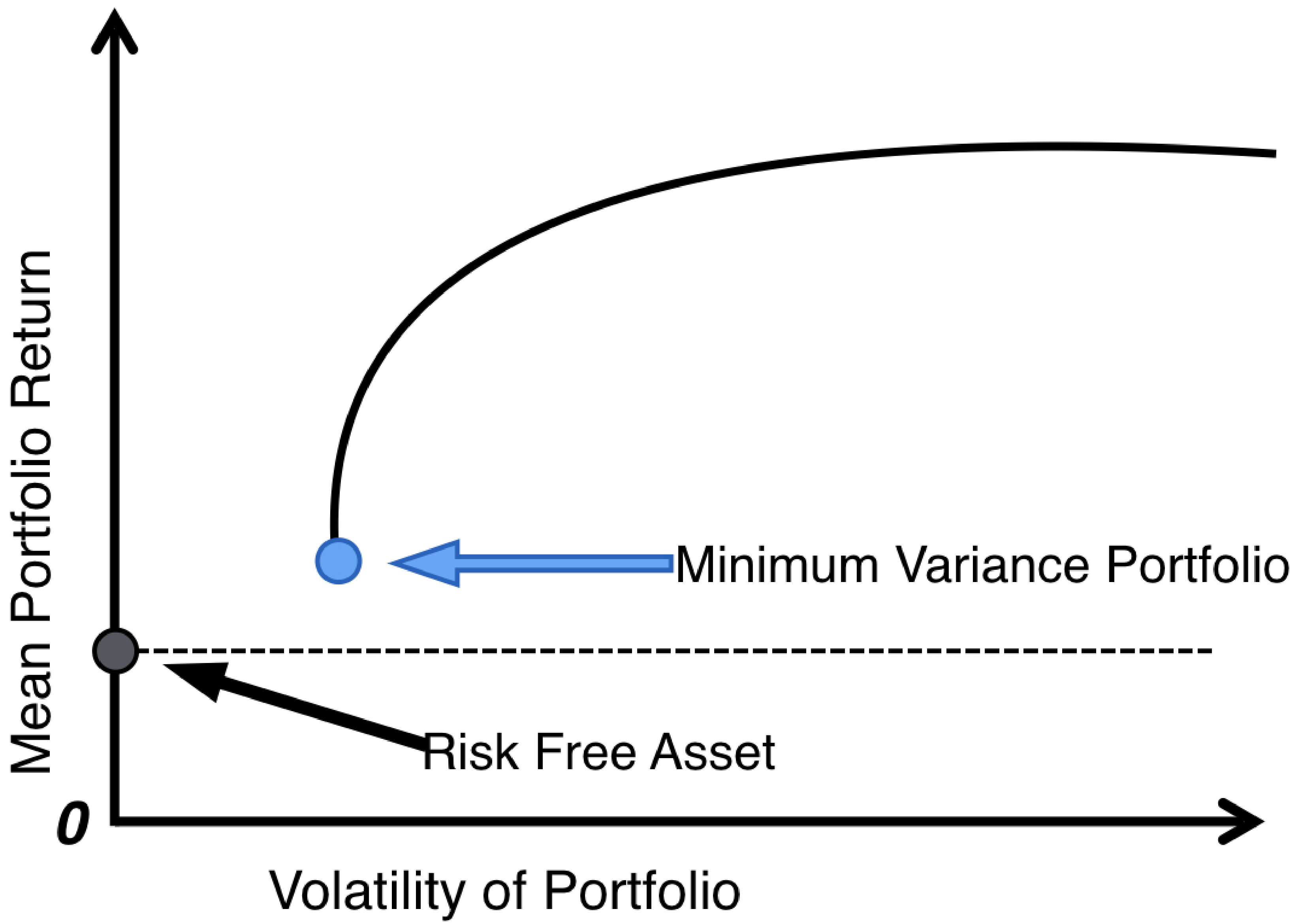
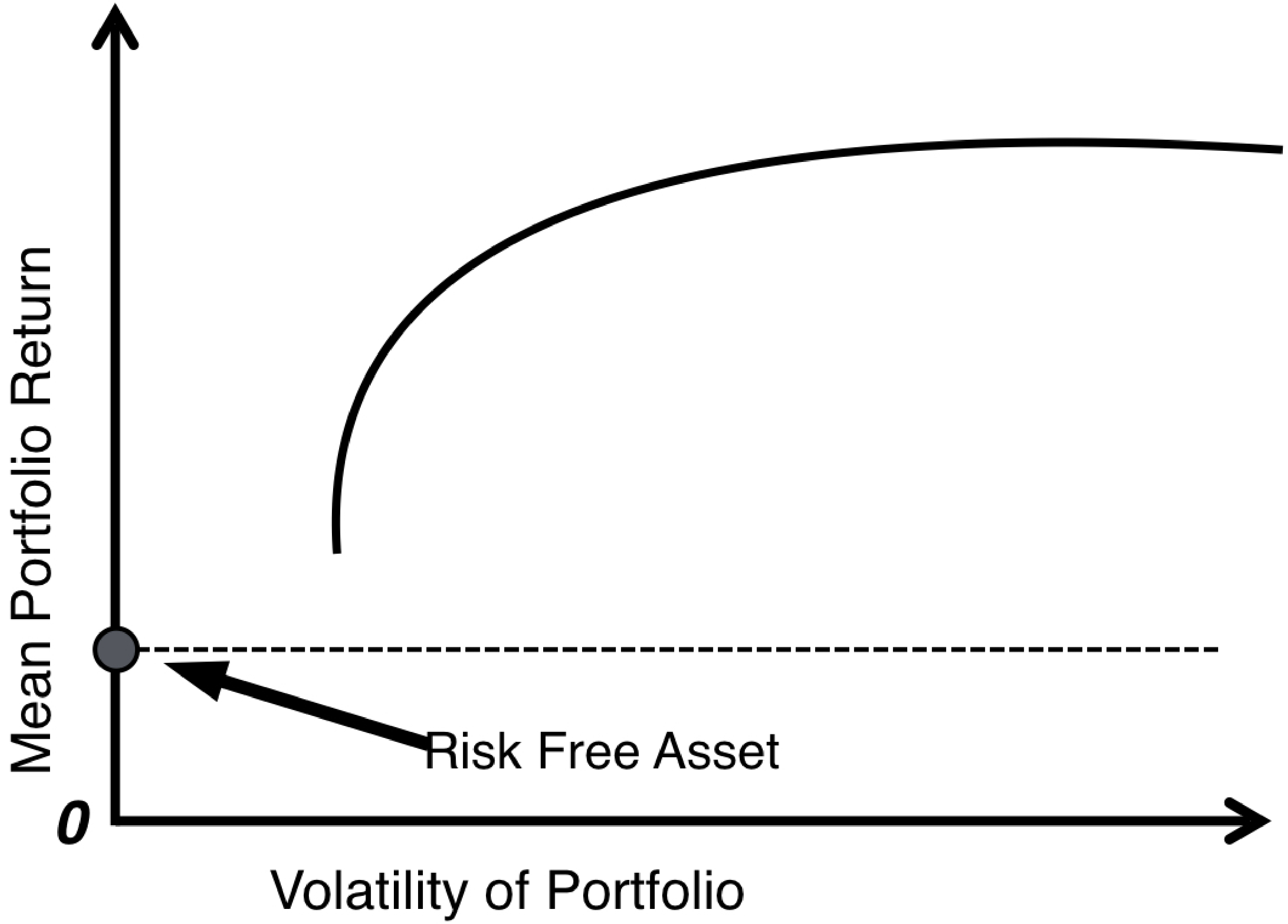


Maximum Sharpe ratio portfolio



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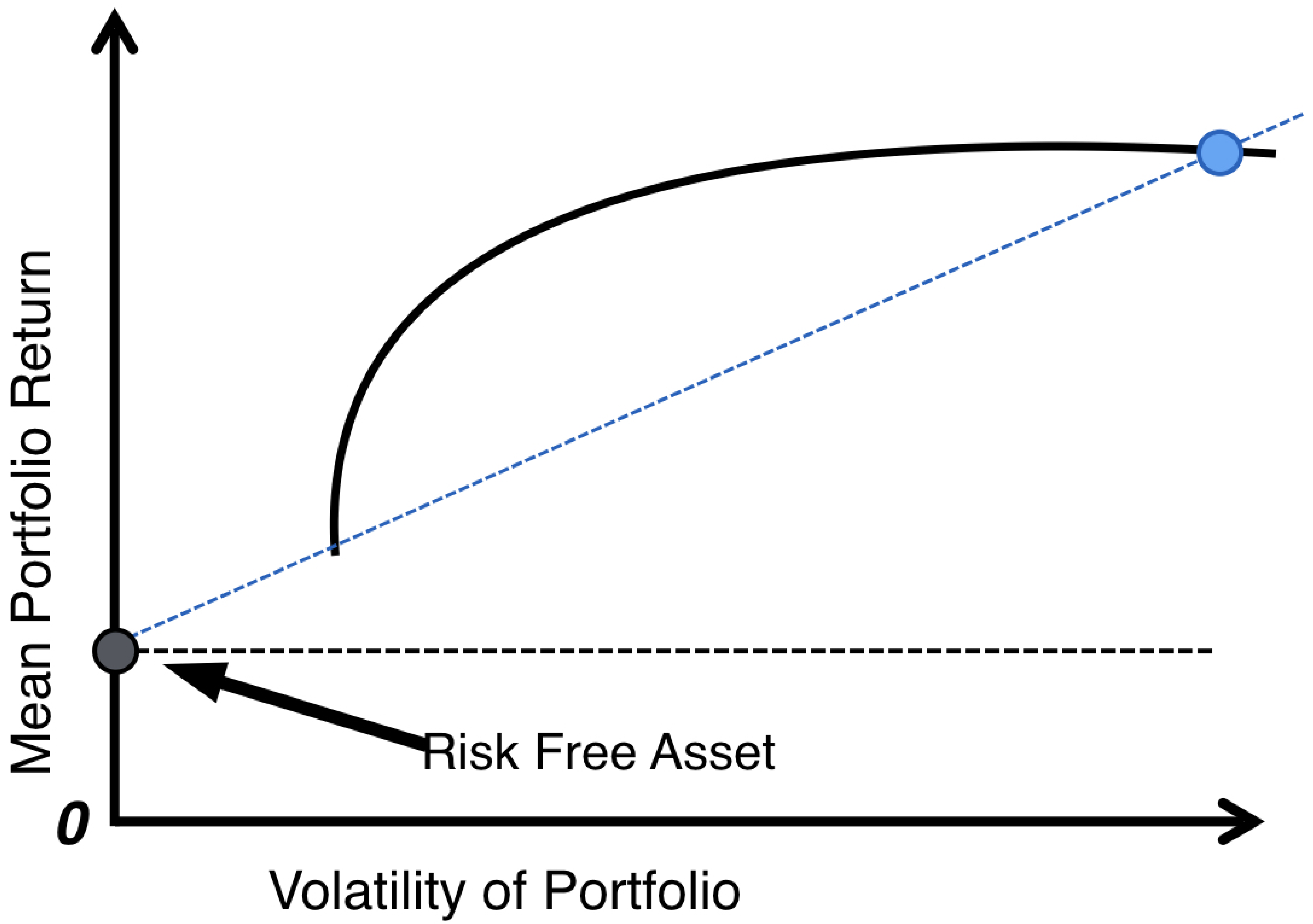

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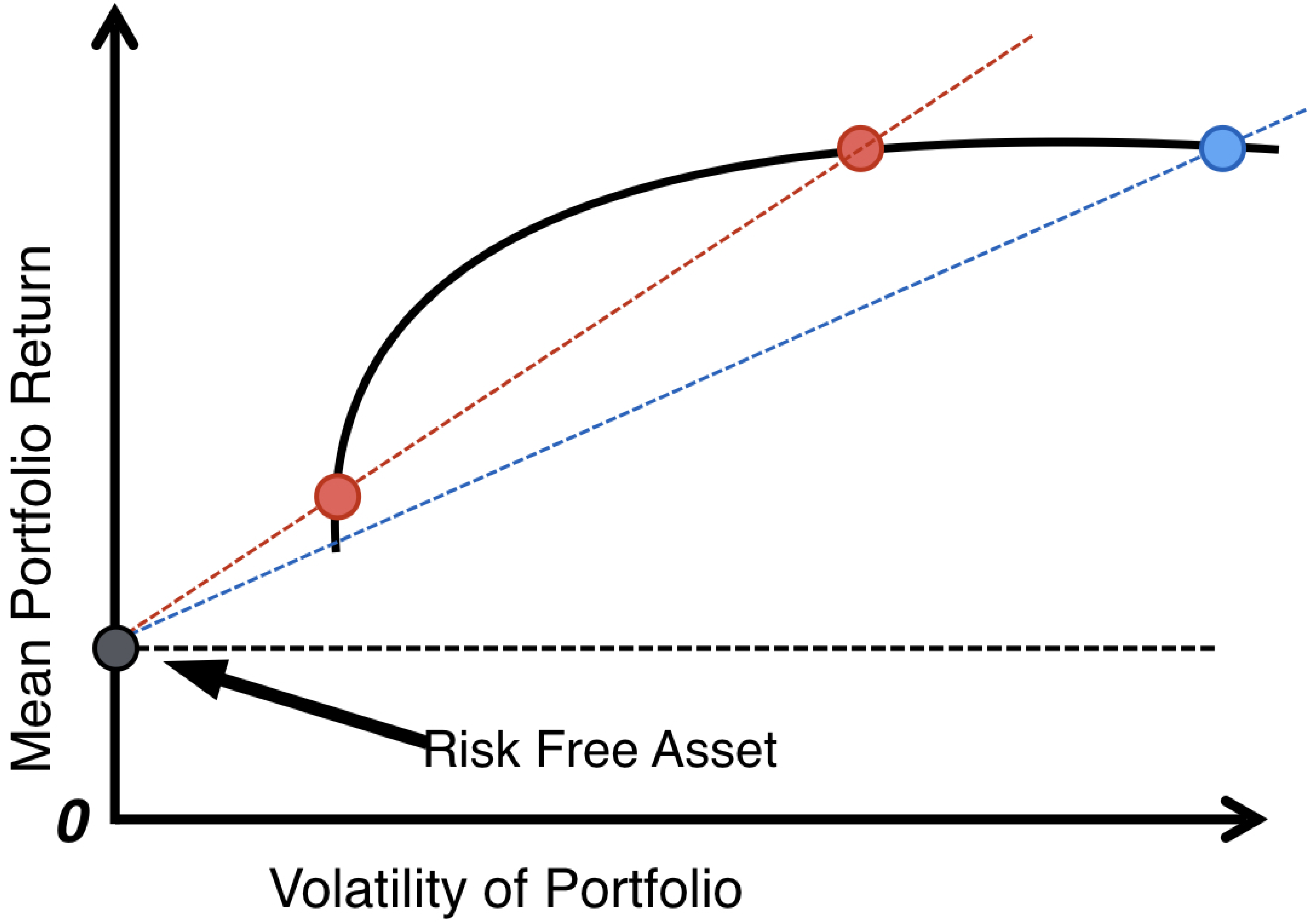

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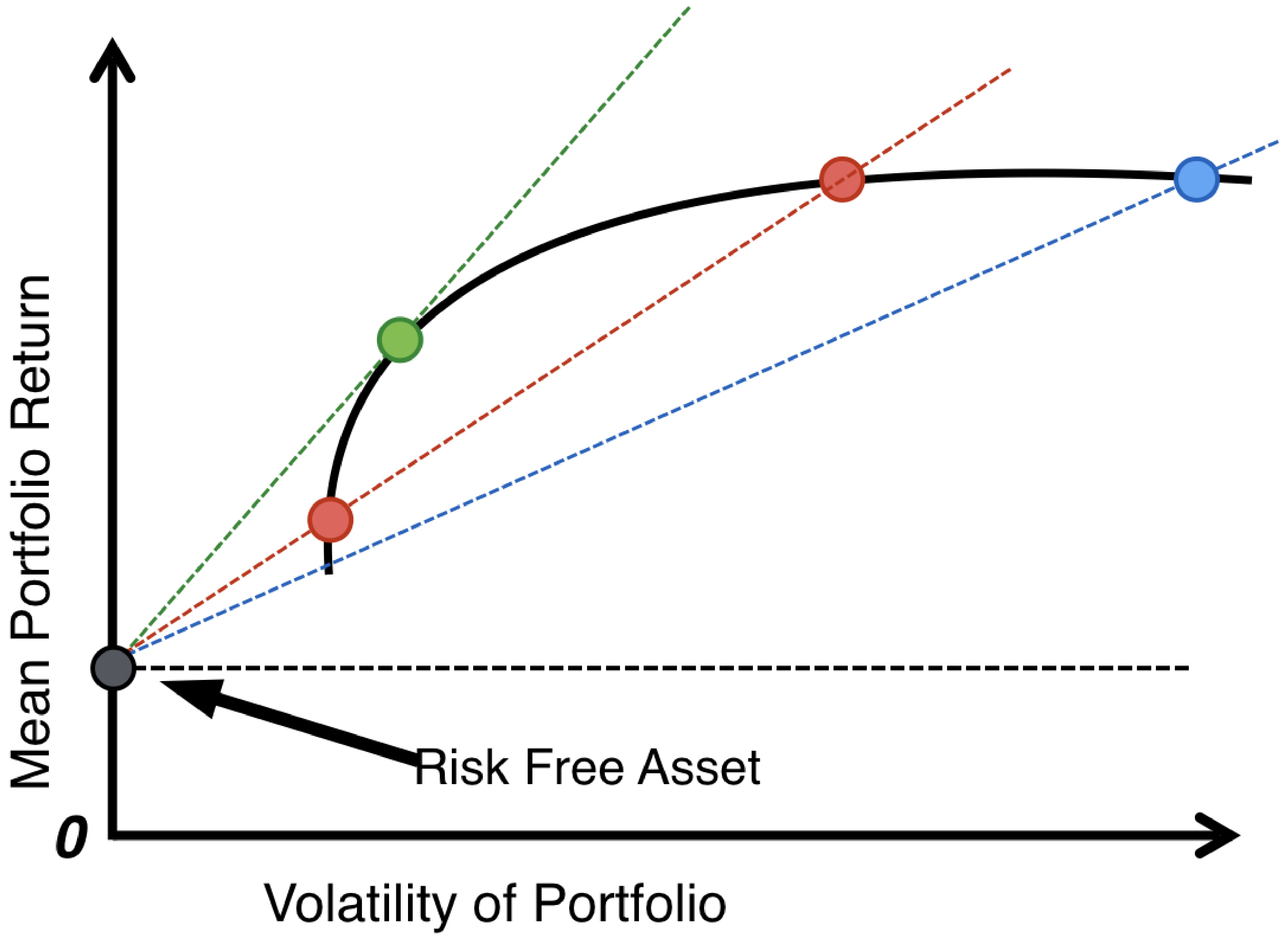

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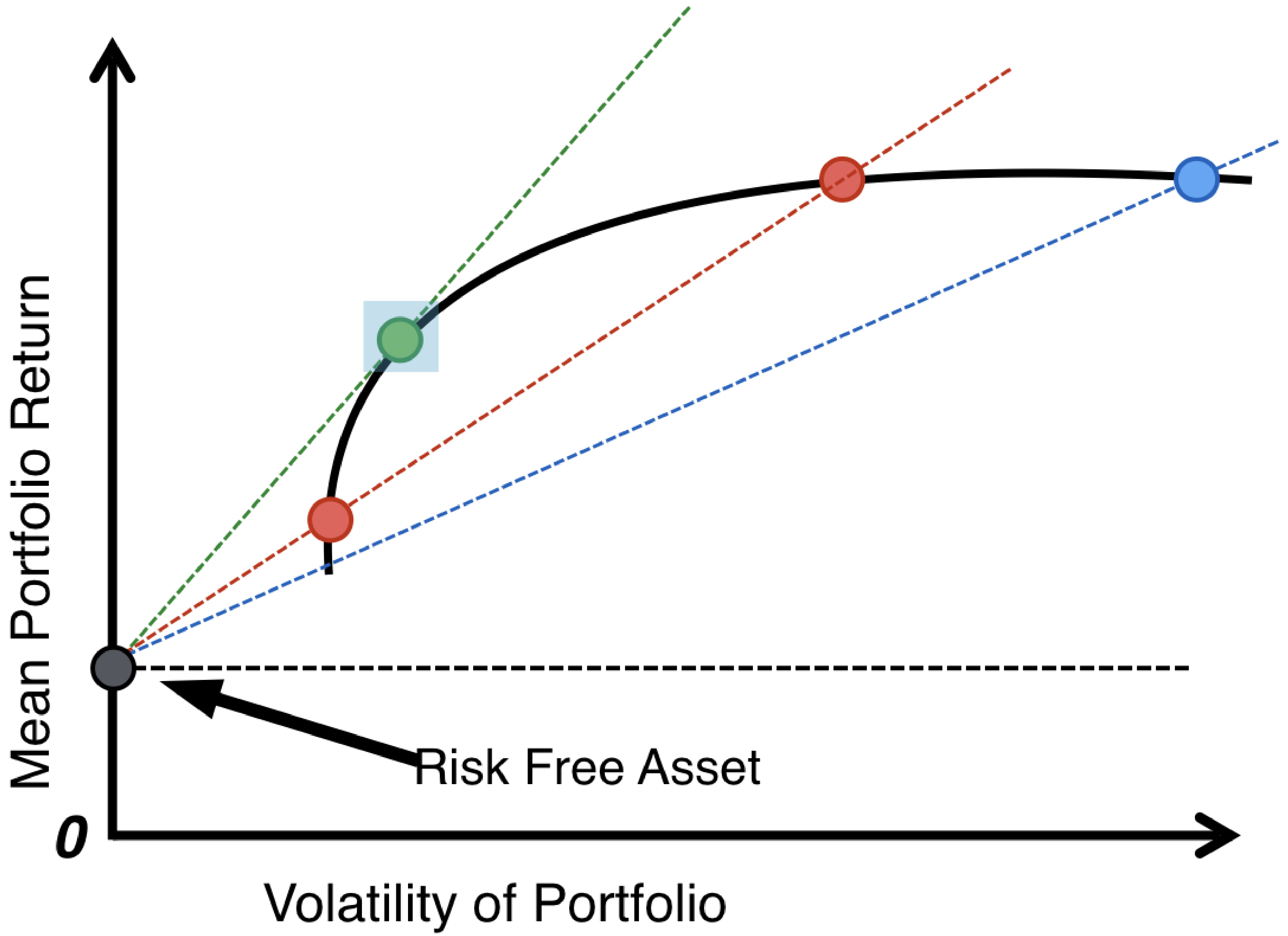
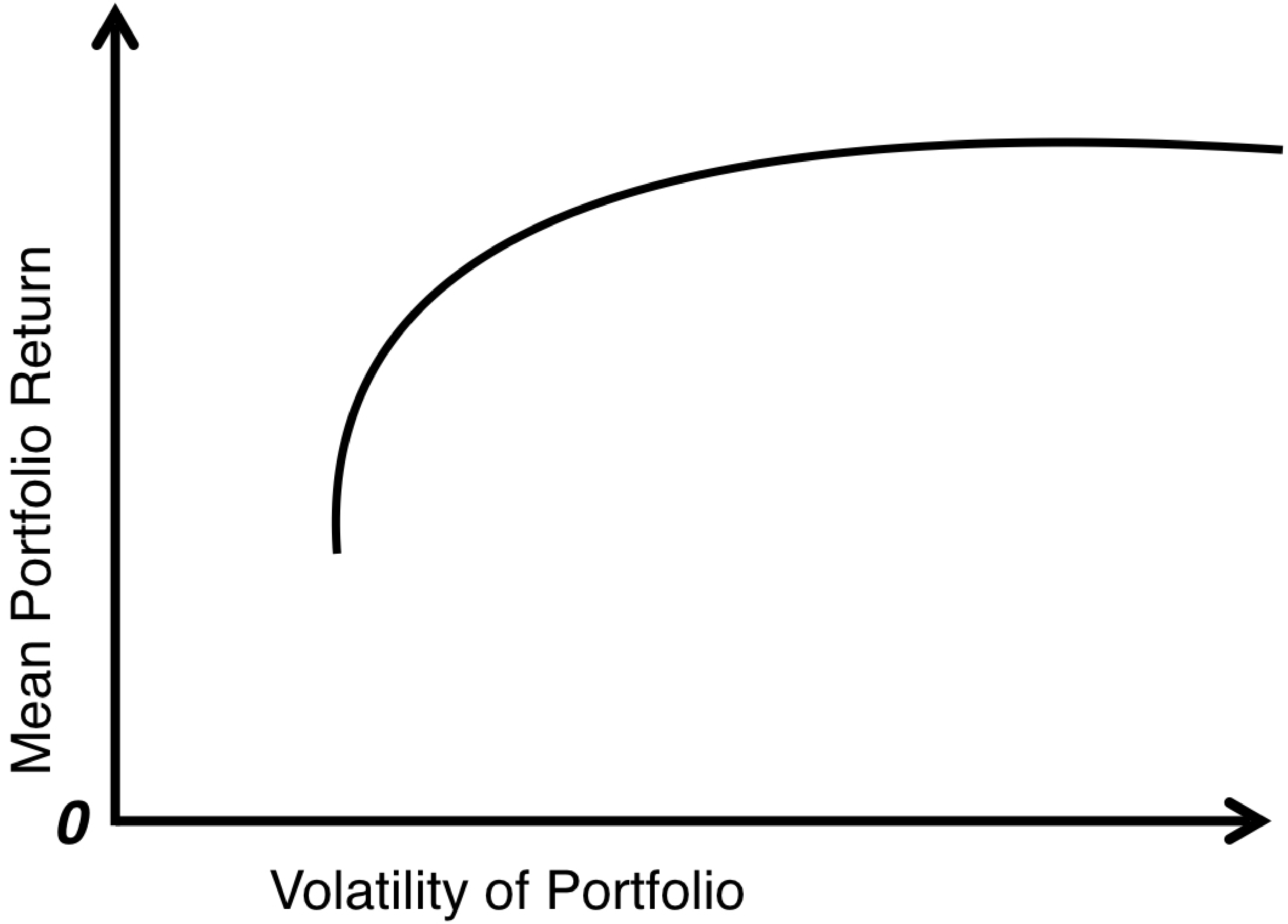


Time for practice



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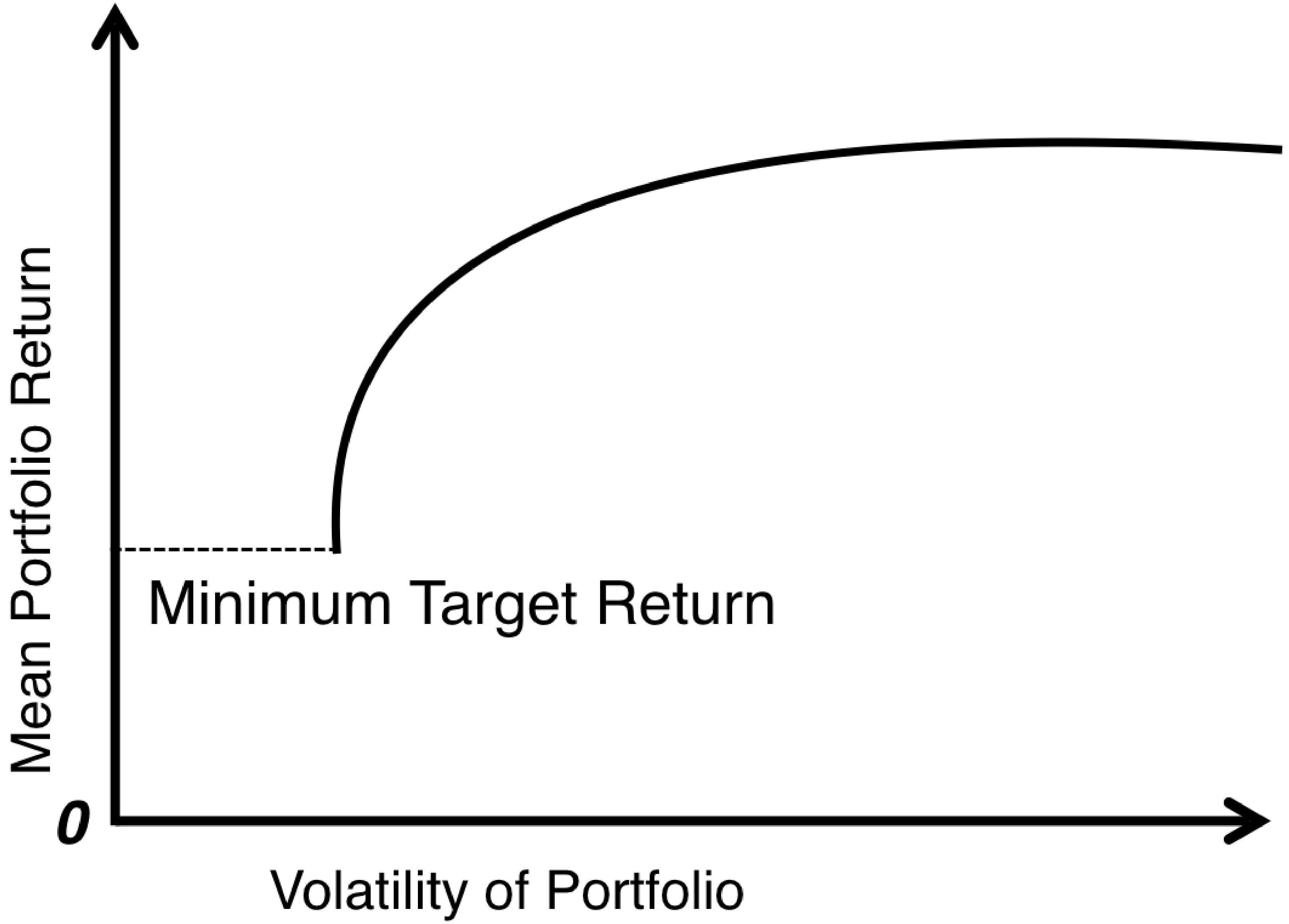

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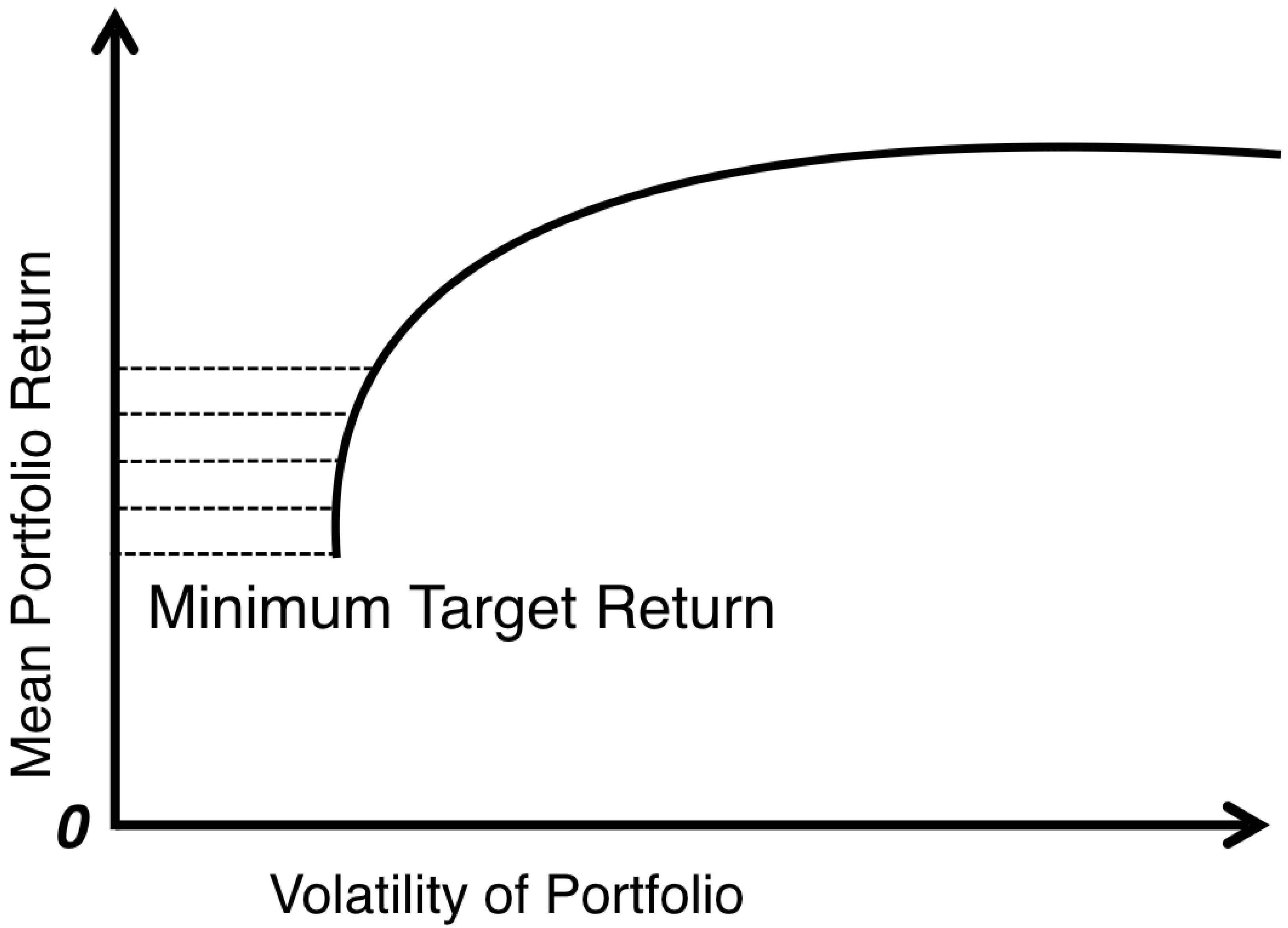

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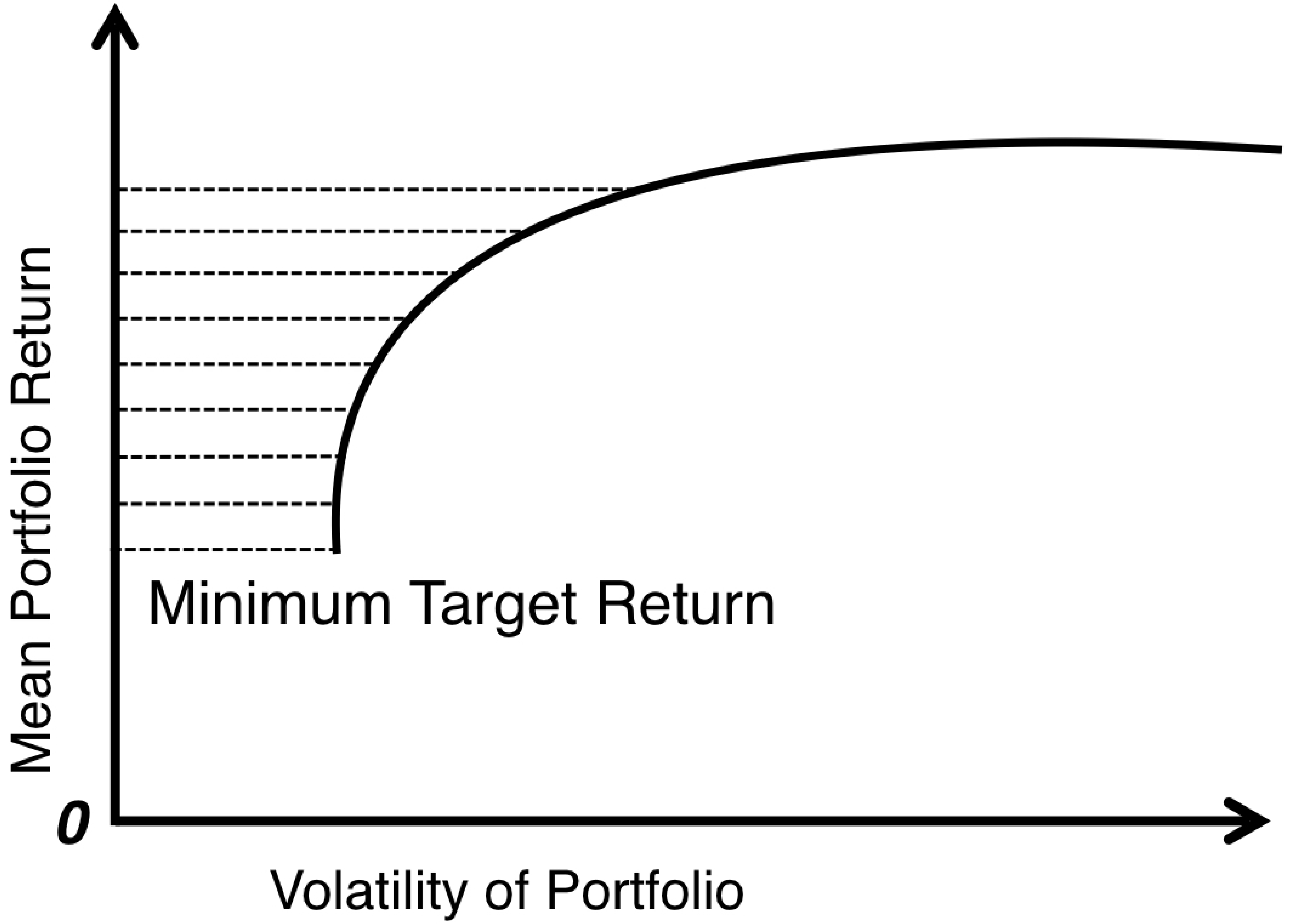

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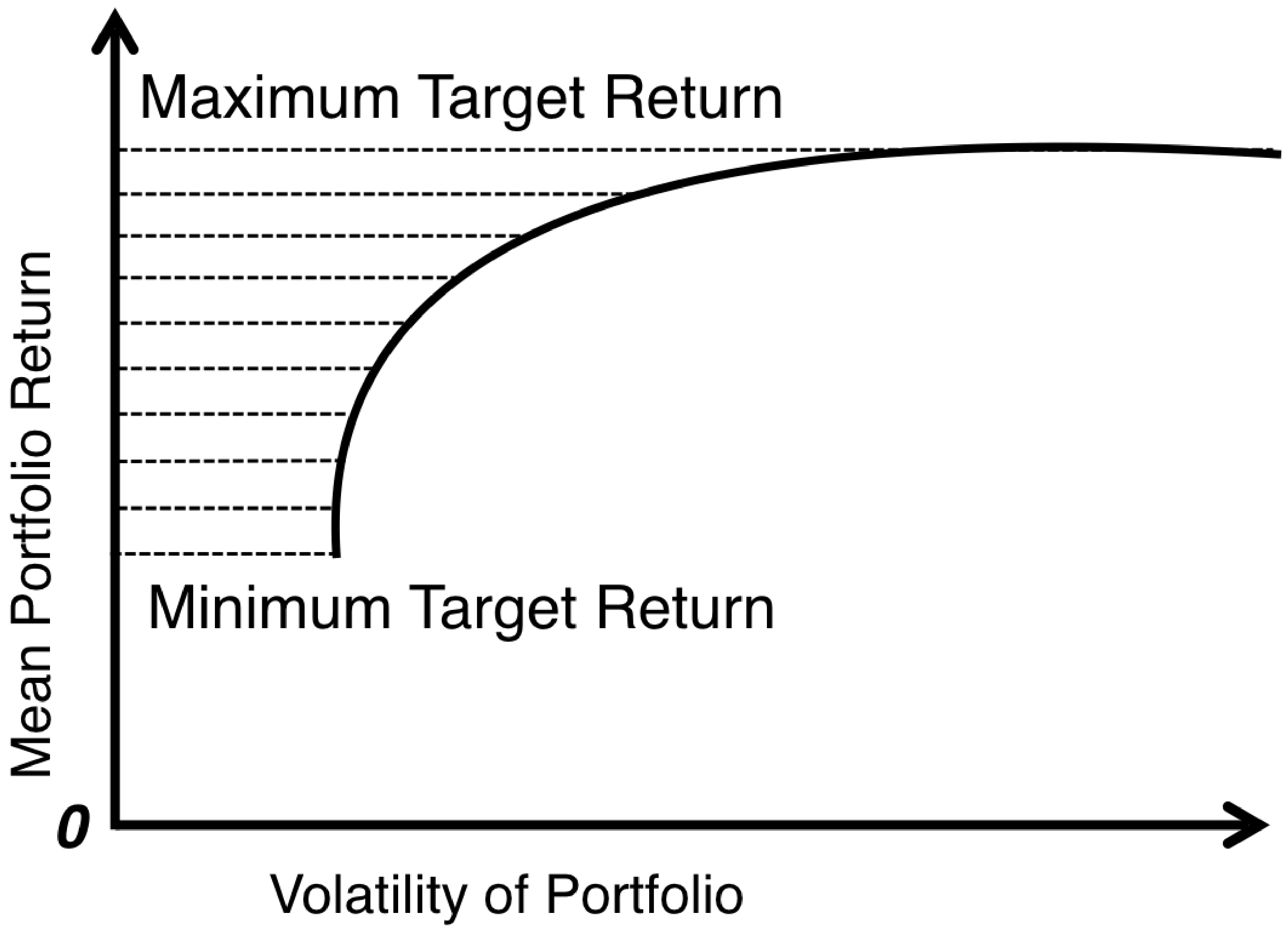

Let's practice!

In-sample vs. out-of-sample evaluation

INTRODUCTION TO PORTFOLIO ANALYSIS IN R

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Professor, Free University Brussels & Amsterdam

datacamp

Bad news: estimation error

- Limitation to data-driven portfolio allocation:

Use in Practice
Estimated mean $\hat{\mu}$
Estimated variance $\hat{\sigma}^2$

Bad news: estimation error

- Limitation to data-driven portfolio allocation:

Use in Practice
Estimated mean $\hat{\mu}$
Estimated variance $\hat{\sigma}^2$

Use In Theory
True (unknown) mean μ
True (unknown) variance σ^2

Bad news: estimation error

- Limitation to data-driven portfolio allocation:

Use in Practice	Use In Theory
Estimated mean $\hat{\mu}$	True (unknown) mean μ
Estimated variance $\hat{\sigma}^2$	True (unknown) variance σ^2

Optimized weights based on estimated mean & variance: $\hat{w}$	True optimal portfolio: w

Good news: opportunities

- Do not ignore estimation error
- Use split-sample analysis to do a realistic evaluation of portfolio performance

R_1	R_2	…	R_K	R_{K+1}	R_{K+2}	…	R_T

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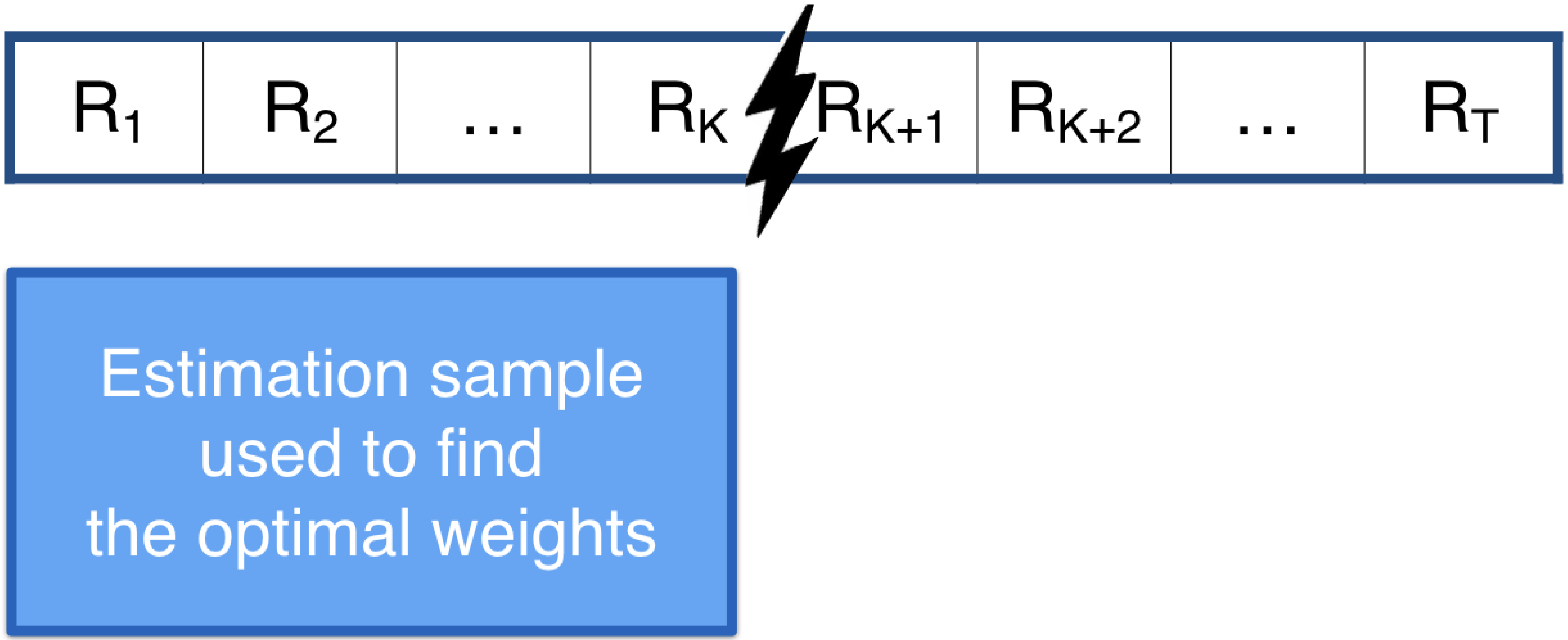

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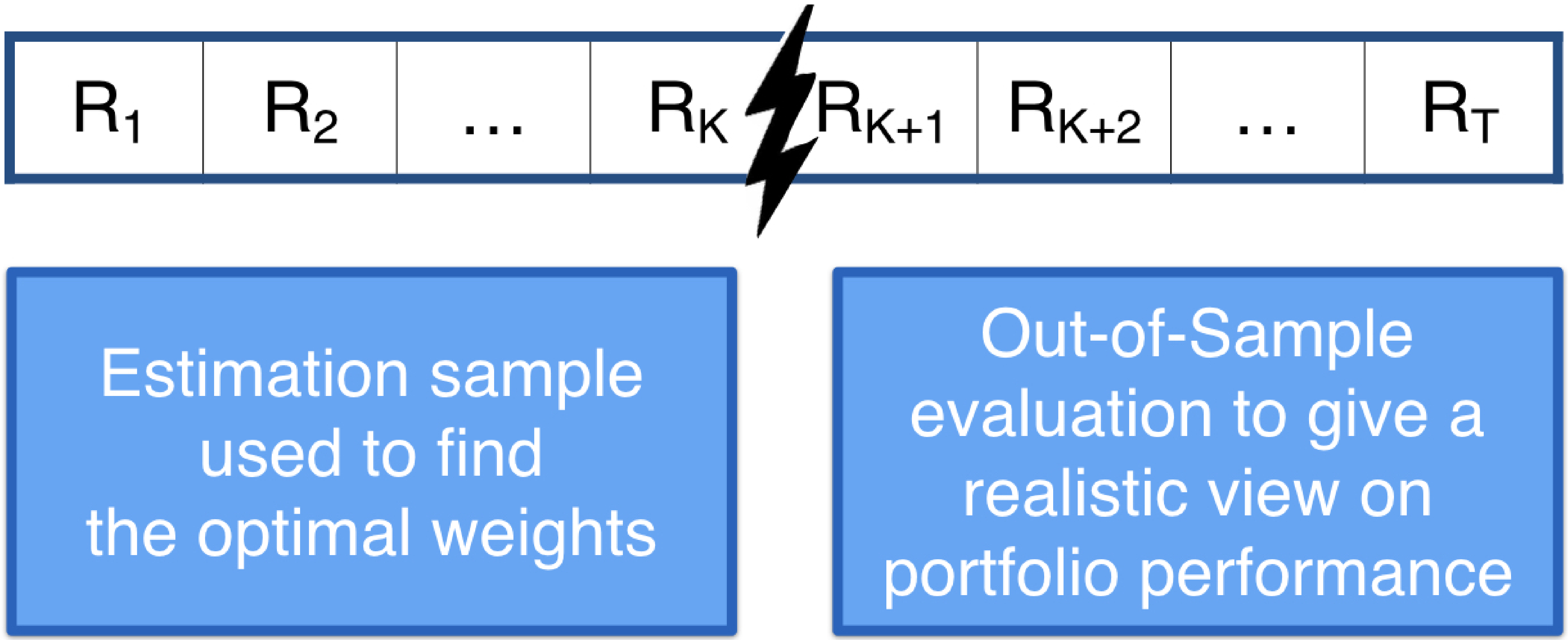
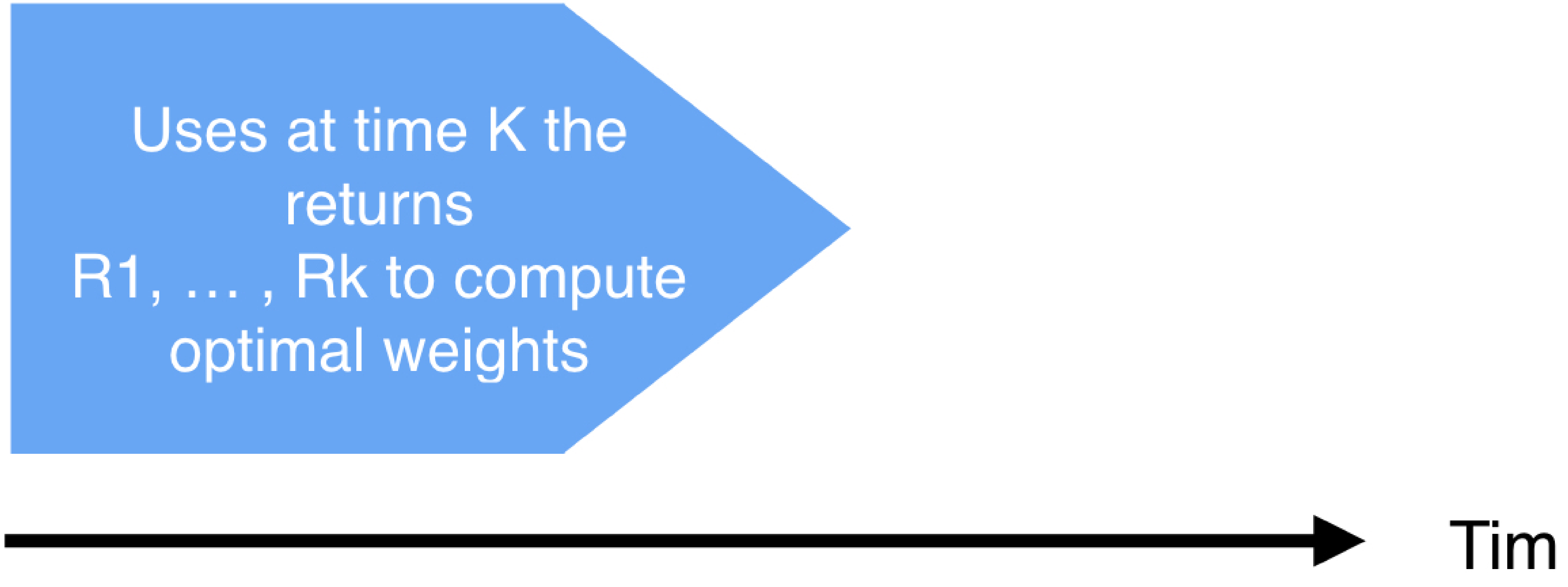


No look-ahead bias in optimized weights

- Split-sample design matches with the investor who:



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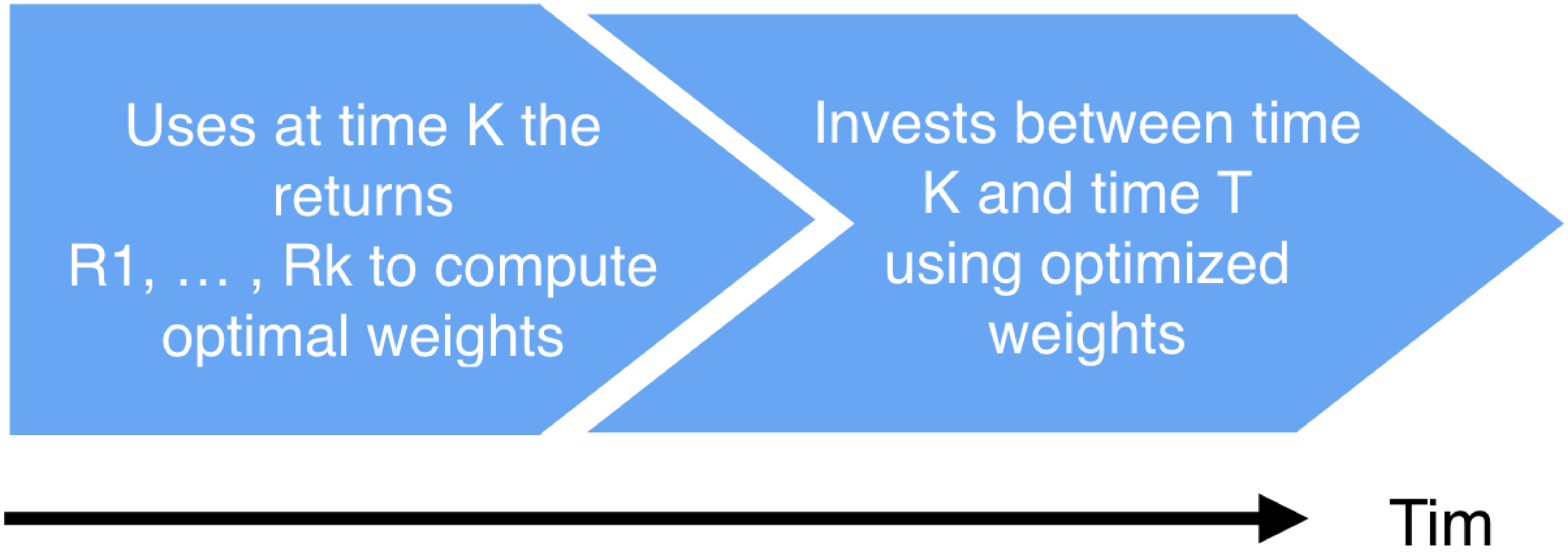

- Function `window()` to do split-sample analysis in R

Let's practice!

INTRODUCTION TO PORTFOLIO ANALYSIS IN R