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WaveAR: A software tool for calculating parameters for water waves with incident and reflected components

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ABSTRACT

The ability to determine wave heights and phases along a spatial domain is vital to understanding a wide range of littoral processes. The software tool presented here employs established Stokes wave theory and sampling methods to calculate parameters for the incident and reflected components of a field of weakly nonlinear waves, monochromatic at first order in wave slope and propagating in one horizontal dimension. The software calculates wave parameters over an entire wave tank and accounts for reflection, weak nonlinearity, and a free second harmonic. Currently, no publicly available program has such functionality. The included MATLAB®-based open source code has also been compiled for Windows®, Mac® and Linux® operating systems. An additional companion program, *VirtualWave*, is included to generate virtual wave fields for *WaveAR*. Together, the programs serve as ideal analysis and teaching tools for laboratory water wave systems.

1. Introduction

Surface wave characteristics are of great importance to a wide range of littoral processes, such as wave attenuation and transformation, sediment transport and bed morphodynamics. In the laboratory, accurately resolving water wave characteristics is paramount for controlling experimental conditions. Despite the growing use of more complex wave spectra (due to advances in wavemaker controls), monochromatic or Stokes waves are still integral to many experiments, such as mine burial studies (Cataño-Lopera and García, 2005, 2007), wave friction factor estimates (Carter, 2002), and bedform evolution (Cataño-Lopera and García, 2006a, b; Hancock et al., 2008; Landry and Garcia, 2007; Landry et al., 2007). For such studies, researchers generally need to resolve and quantify the incident wave amplitude, reflection coefficient, and other key parameters.

Existing methods to calculate wave parameters include the two-wave gage method (Goda and Suzuki, 1976; Thornton and Calhoun, 1972), three-wave gage method (Isaacson, 1991; Mansard and Funke, 1980), and maximum/minimum wave height method. These methods estimate the incident wave amplitude and reflection coefficient based on wave measurements at each gage. In addition, since the three-gage method measures the phase lags between gages, it can also estimate the phase of the reflection coefficient. The maximum/minimum method, though simple, is regarded as time consuming and subject to human error (Nallayarasu et al., 1995). The two- and three-gage methods are error prone due to their sensitivity to frequency range, noise, nonlinear harmonics, and probe spacing. Most problematic, though, is the fact that these methods only employ measurements from up to three fixed positions. In a wave field with incident and reflected components, the wave height is modulated over each wavelength. Thus, the previous methods will ultimately fail to accurately resolve the incident and reflected components unless the gages are accurately positioned with prior knowledge of the wave field.

Beyond accurate first harmonic wave parameters, additional information is often required to understand and model the wave field. For example, complete knowledge of the wave field's spatial variation is crucial to understanding bedform ripple geometries and migration velocities (Landry, 2011; Landry et al., 2009). To properly model sandbar formation under Stokes waves, the first and second wave harmonics and the second order Stokes flow must be resolved, including the free and bound components of the second harmonic (Hancock et al., 2008; Landry et al., 2007). The presence of a pronounced, free second harmonic can significantly alter the surface wave envelope and impact bedform development (Hancock et al., 2008).

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The user-friendly wave analysis software tool presented herein, *WaveAR*, estimates wave parameters based on an arbitrary number of wave gage positions. In general, the more wave gage positions that are distributed along the wave field, the more accurately the wave parameters may be resolved. *WaveAR* was developed as part of a combined experimental and numerical effort to observe, measure, and predict the evolution of water waves over sandy bedforms evolving in response to the waves (Hancock, 2005; Hancock et al., 2008; Landry, 2004; Landry et al., 2007). In the regime of interest, and therefore the regime of validity of our software, water waves are adequately described by second order Stokes wave theory. *WaveAR* resolves the full Stokes wave field to second order in wave slope, including the amplitudes and phases of the first and second harmonic wave components. Parameters are estimated by a least squares fit to the measured wave elevations along the wave flume. The theory underlying *WaveAR* is valid for both low and high reflection coefficients and is a generalization of Rosengaus' (1987) reference measuring method valid for low reflection coefficients (Hancock, 2005; Hancock et al., 2008). *WaveAR* has been continuously improved and validated throughout its development against a host of experimental conditions (Hancock et al., 2008; Landry et al., 2007, 2009; Landry and Garcia, 2007).

In what follows, we give a brief theoretical background that includes the mathematical representation of the various wave harmonics and describes how *WaveAR* resolves the bound and free second wave harmonics. We then take the reader on a tour of the *WaveAR* tool and work through two demonstrative examples, including free second harmonic cancellation.

2. Brief theoretical background

A description of Stokes wave theory is found in most standard wave mechanics texts (Dean and Dalrymple, 1991; Mei, 1989). A piston wavemaker with a monochromatic sinusoidal motion will generate a water wave at the fundamental frequency, a bound second harmonic wave associated with Stokes theory (Madsen, 1971) and a free second harmonic. The bound harmonic is an artifact of nonlinearities at the free surface, while the free second harmonic results from the mismatch between the motion of the vertical wall of the wavemaker piston and the fluid particle orbital displacement. The wavelength mismatch between the bound and free second harmonics modulates the total amplitude of the second harmonic along the wave tank. While the derivation details are left to previous work (Hancock et al., 2008), the formulas used by *WaveAR* to resolve the various wave parameters are listed below. The free surface elevation is described mathematically by

$$\eta(t) = A\,\mathrm{Re}(e^{i(kx-\omega t)} + R\,e^{i(\theta-kx-\omega t)}) + \mathrm{Re}((\eta_{2b}+\eta_{2f})e^{-2i\omega t}) \tag{1}$$

where the complex amplitudes of the bound and free second harmonic components are, respectively,

$$\eta_{2b} = \frac{A^2 k(1+2\cosh^2(kh))\cosh(kh)}{4\sinh^3(kh)}(e^{2ikx} + R^2\,e^{2i(\theta-kx)}) \tag{2}$$

$$\eta_{2f} = A_2\,e^{i\alpha_2}(e^{ik_2 x} + R_2\,e^{i(\theta_2-k_2 x)}) \tag{3}$$

The total amplitude of the second harmonic is

$$\eta_2 = |\eta_{2b}+\eta_{2f}| \tag{4}$$

where vertical bars denote the modulus of a complex number. All parameters in Eqs. (1)–(3) are defined in Table 1. The wavenumbers are related to the angular frequency ω and water depth h by

Table 1
Wave parameters used in *WaveAR* and *VirtualWave*.

Variable	Description
First harmonic	
A	Incident amplitude
R	Reflection coefficient
θ	Relative phase of reflection coefficient
k	Wavenumber
T	Wave period
ω	Angular frequency
Free second harmonic	
A_2	Incident amplitude
k_2	Wavenumber
R_2	Reflection coefficient
θ_2	Relative phase of reflection coefficient R_2
α_2	Relative phase of incident amplitude A_2
Other parameters	
x	Longitudinal distance from the wavemaker
t	Time
h	Still water depth
L	Wave amplitude attenuation coefficient

the dispersion relations

$$\omega^2 = gk\tanh(kh), \quad (2\omega)^2 = gk_2\tanh(k_2 h) \tag{5}$$

The wavelengths are given by $\lambda = 2\pi/k$ and $\lambda_2 = 2\pi/k_2$. In general, $\lambda_2 \neq 2\lambda$, as noted above, leading to second harmonic modulation over many wavelengths. The envelope of the first harmonic is

$$\eta_1 = A\sqrt{1+R^2+2R\cos(2kx-\theta)} \tag{6}$$

and the rms amplitude of the Stokes wave is

$$\eta_{\mathrm{rms}} = \sqrt{\frac{\eta_1^2+\eta_2^2}{2}} \tag{7}$$

WaveAR performs Fourier decompositions of the wave elevation time series measured at fixed positions along the wave tank. The resulting wave harmonic amplitudes are compared to Eqs. (4) and (6) and the parameters in Table 1 are found by least squares fitting. For validation and instruction, the companion tool *VirtualWave* performs the reciprocal calculation: given the wave parameters in Table 1, *VirtualWave* uses Eqs. (1)–(7) to produce wave elevation time series, and harmonic and rms amplitudes. For purely incident waves ($R=0$), the parameter L specifies the wave damping and A is replaced by $A\,e^{-Lkx}$ in both *WaveAR* and *VirtualWave*. Lastly, Eqs. (1)–(6) are technically valid only on a flat bottom; bedforms such as sandbars can alter the spatial variation of A, R, and θ along the tank (Hancock et al., 2008). In fact, the reflection of incident waves from bedforms can lead to an apparent damping of the incident wave (Hancock et al., 2008).

3. Software overview

The software package includes two programs: the main program *WaveAR* and a simple companion program, *VirtualWave*. *WaveAR* is short for "wave amplitude and reflection" and is pronounced "Wave · A · R". The software provides an interactive graphical user interface (GUI) for wave parameters to be fit to measured data. *VirtualWave* generates virtual wave elevations at specified numerical probe locations and is intended to be used as a validation and instruction tool to complement *WaveAR*. The open source MATLAB® files for *WaveAR* and *VirtualWave* are available as Electronic Supplementary Information (ESI) included with this paper. The latest version of the software including

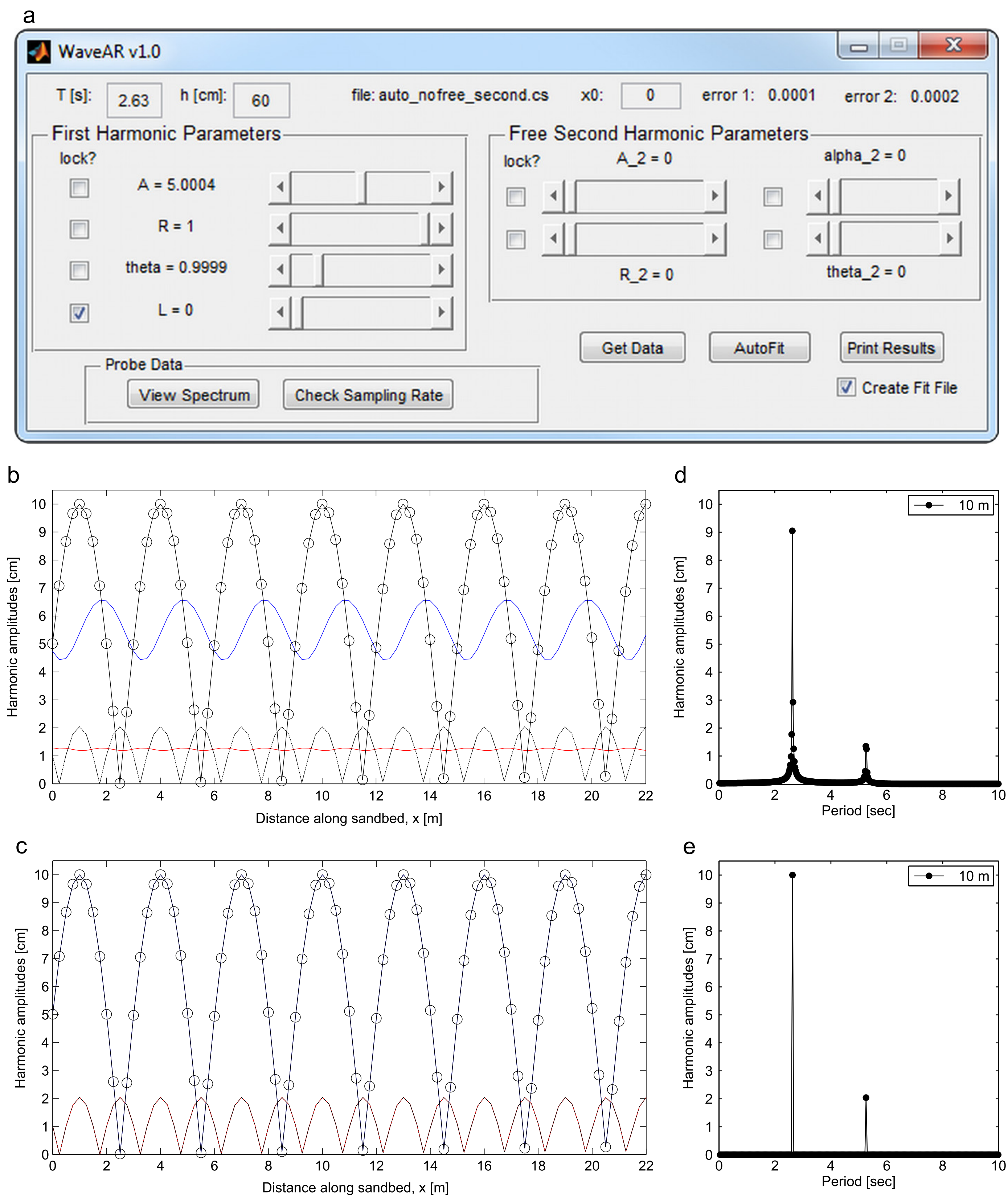

Fig. 1. Interface and output of *WaveAR* program: (a) main GUI for the *WaveAR* program allows the user to interactively correctly fit the measured wave data. (b) Illustration of dynamic data fitting with *WaveAR*. Measured first and second harmonics denoted by black line with circles and black line, respectively. Theoretical fits for first and second harmonics, not optimized here for visualization, are denoted by blue and red lines, resp. (c) Lines same as in (b) with fitting parameters optimized. (d) Measured wave amplitude spectrum at the sampling location 10 m along the sand bed. Spectral leakage about each of the dominant harmonics indicates improper data sampling. (e) Same as (d) with correct sampling frequency which avoids spectral leakage. *WaveAR* can help determine the proper sampling rate for wave conditions (see Eq. (9) and ESI, User's Manual). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

standalone executable versions for Windows®, Mac®, and Linux® may be obtained from 〈http://vtchl.illinois.edu/software/〉 or from the authors.

3.1. Main program: WaveAR

WaveAR's GUI incorporates dynamic controls to facilitate user interaction (Fig. 1a). First, the user loads a comma separated value (CSV) data file consisting of the experimental parameters and wave elevation time series at fixed positions along the wave tank (see ESI, User's Manual). Once loaded, *WaveAR* decomposes the input into wave harmonics and plots these along the tank (Fig. 1b and c). Theoretical fit lines are plotted for comparison. By adjusting the salient wave parameters via slider controls, the numerical fit is updated and replotted in real-time.

WaveAR can perform automatic least squares fitting and also allows the user to manually adjust the wave parameters. The automatic fitting is accomplished with MATLAB®'s lsqcurvefit routine. In all cases, *WaveAR* reports the relative error (upper right, Fig. 1a) between the measured and fitted amplitudes of the given harmonic (first or second), defined as

$$\text{error} = \sqrt{\frac{\sum_j (a_{j,\text{fit}} - a_{j,\text{measured}})^2}{\sum_j a_{j,\text{measured}}^2}} \tag{8}$$

where a is the amplitude of the given harmonic and j ranges over the wave gage x positions. The final fitting results may be exported to a standard text file for further use (Fig. 1c and Table S1).

WaveAR can also report the single-sided Fast Fourier Transform (FFT) spectrum at a specified sampling location (ESI, User's Manual). The user may examine the spectrum to check for problems in the Fourier decomposition, for example spectral leakage and aliasing due to sampling and windowing errors (Fig. 1d and e). For the Stokes waves in the regime of interest, these errors are prevented provided that the wave frequency is known a priori and that the sampling frequency f_s is chosen as (Bendat and Piersol, 2000)

$$f_s = 2^{n-m} f \tag{9}$$

where f is the wave frequency (=1/period), 2^n is the number of (equi-spaced) sampling points in time, 2^m is the number of sampled wave periods, and n and m are user-specified positive integers (Table S2). The optimal sampling frequency must be equal to or above the Nyquist frequency, which is twice the frequency of the harmonic to be resolved. For example, to accurately sample the first and second harmonics, we must have $f_s \geq 4f = 4/T$. In addition, the optimal sampling frequency must be below the maximum sampling frequency of the device. These criteria restrict the optimal values for n and m. *WaveAR* can calculate the optimal sampling frequency given user input (ESI, User's Manual).

3.2. Companion program: VirtualWave

In addition to analyzing laboratory measurements, *WaveAR* can process virtual data sets, especially useful for educational settings without access to a wave facility. The companion software *VirtualWave* is an interactive GUI tool for creating artificial wave data sets (Fig. 18 in ESI, User's Manual). The user specifies the first and free second harmonic wave characteristics (i.e., amplitude, reflection, period, phase, etc.) and whether to include the bound second harmonic. Additional parameters, such as spatial and temporal ranges, enable full control of the visualized simulation domain and duration. Wave gage positions and the temporal sampling frequency are specified separately, allowing the user to explore effects such as sparse gage placement and spectral leakage. During data generation, the program saves the sampled data to CSV files that can be directly processed by *WaveAR*.

4. Applications

4.1. Resolution of wave parameters

WaveAR was created in response to a crucial need to rapidly resolve wave field measurements along a wave tank during bedform evolution studies (Hancock et al., 2008; Landry et al., 2007, 2009; Landry and Garcia, 2007). During each experiment, complete sets of wave measurements were recorded along the tank at regular time intervals (Fig. 3). For a particular experiment, the corresponding wave parameter sets resolved by *WaveAR* are reported in Table S3.

A typical experimental run from our past work provides a good example of the *WaveAR* usage to resolve wave parameters (Hancock et al., 2008; Landry, 2004; Landry et al., 2007). In these studies, three wave gages were spaced at 0.5-m intervals on a movable carriage that could translate along the entire 16.5-m test section. The gage positions coincided with a line along the center of the channel parallel to the direction of wave propagation (x-direction). The wave measurement and parameter calculation proceeded as follows:

1. Once the wave field had stabilized (after roughly 5–10 min from the onset of the wave generation), the wave gage carriage was positioned over an antinode or quasi-antinode, and wave measurements were taken at a sufficient sampling rate to resolve the first harmonic wave period T.
2. The wave measurement files were imported into *WaveAR* and the spectral data checked to ensure that no spectral leakage or aliasing occurred.
3. Wave measurements using the checked sampling rate were conducted at various locations along the tank. For experiments in which the (first harmonic) wavelength was approximately 6 m, wave gage measurements spaced 0.5 m along the tank provided adequate resolution and accuracy.

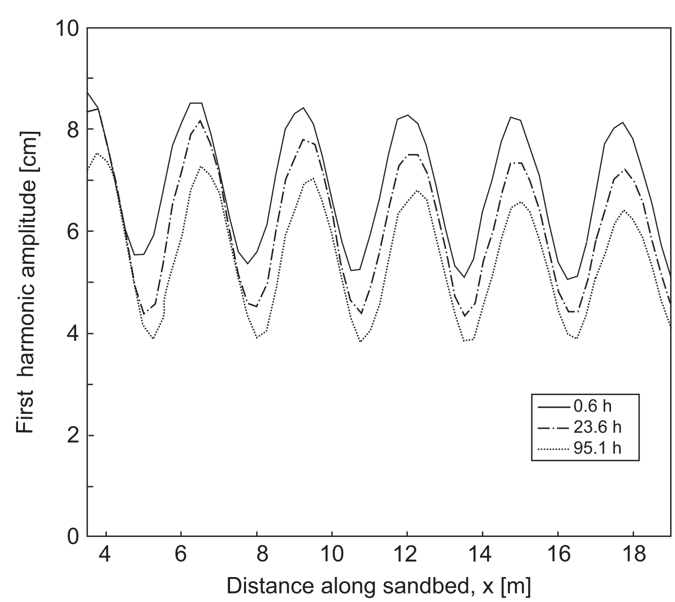

Fig. 2. Typical wave measurements. First harmonic amplitudes at specified times throughout the experiment illustrate the wave evolution during sandbar formation. Data from Hancock et al. (2008) and Landry (2004).

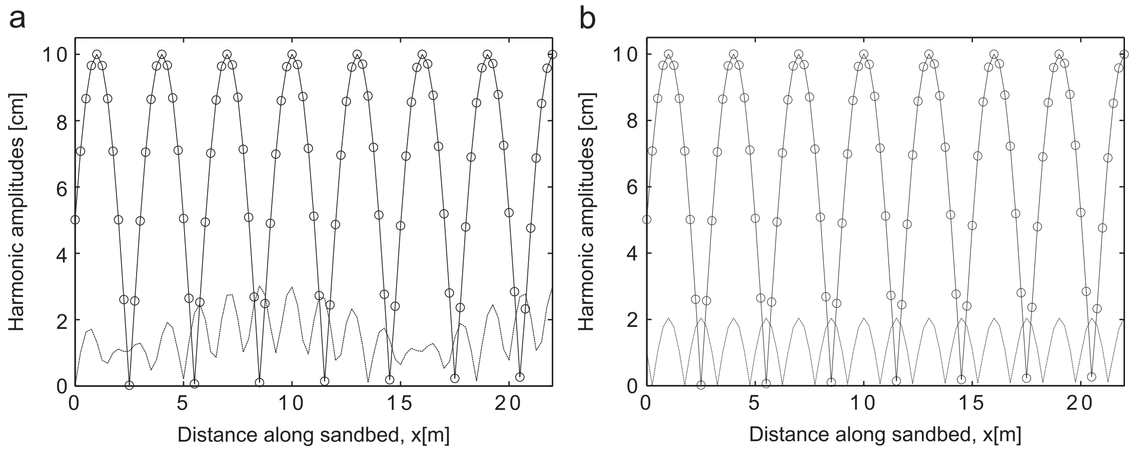

Fig. 3. Free second harmonic detection and cancellation: (a) first and second harmonic amplitudes indicated by lines with circles and without, resp. Free second harmonic present in the second harmonic amplitudes indicated by its slow modulation. (b) Same as (a) but with free second harmonic wave eliminated by correcting the second harmonic of the wavemaker motion.

4. Wave measurements from all wave gage positions were imported into *WaveAR* and the parameters fit by the procedure outlined above, producing a comparison plot of fitted wave profiles (Fig. 2). Parameter results were saved to a file.
5. The procedure was repeated for each successive set of wave measurements along the tank.

The overall time required for wave measurement and analysis is dominated by the wave measurements. Though other wave gage methods are simpler, these are either not applicable or not sufficiently accurate for wave fields with reflection. The time for wave measurement may be reduced by using more gages and employing advanced statistical methods to estimate parameters from sparse data. Alternatively, a single high resolution wave measurement and analysis procedure could first be performed as outlined above to find the wave nodes and antinodes. Then, two or three wave gages could be accurately placed at fixed positions to record the wave field at intermediate time points based on the simpler wave gage methods outlined above. For cases involving coupled bedform and wave dynamics, the wave field would evolve more rapidly and more frequent high-resolution sets of wave measurements would be required. In an experiment on wave generated sandbar formation, recording wave measurements along the tank at 3–12 h intervals was sufficient to capture the sandbar and surface wave interaction, depending on the stage of the coupled evolution (Hancock et al., 2008; Landry, 2004; Landry et al., 2007).

4.2. Free second harmonic cancellation

The presence of a pronounced, free second harmonic can significantly alter the surface wave envelope and impact bedform development. A well-known theory for canceling the free second harmonic (Madsen, 1971) is limited to a flat bottom with constant depth and zero beach reflection. This theory is therefore not applicable to wave tanks with bottom features such as ramps leading up to a sandy test bed, walls, or artificial beaches. In these more complex cases, an iterative procedure (Hancock et al., 2008) must be used to make the necessary second-order correction to the wavemaker motion and cancel the free second harmonic wave component. During each iteration of this cancellation procedure, the phase and amplitude of the first and second harmonics must be determined from the wave elevations along the wave tank. *WaveAR* is ideal for this purpose. The *WaveAR* outputs before and after the cancellation procedure are shown in Fig. 3a,b. The long scale of modulation present in the second harmonic amplitude (Fig. 3a) is absent in Fig. 3b, indicating the successful cancelation of the free second harmonic.

5. Conclusion

WaveAR is a user-friendly software tool for interactively processing Stokes wave elevation data in laboratory wave systems. The interactive and optimal estimates of wave parameters such as incident and reflected amplitudes and phases provide a unified framework for comparing multiple wave data sets. Although *WaveAR*'s automated fitting is limited to Stokes waves, including those with a free second harmonic component, the program may still be used to view and quantify the first and second harmonic components of more complex wave fields such as cnodial waves, JONSWAP spectra, etc. The companion *VirtualWave* generates synthetic Stokes wave data for use with *WaveAR*. For each tool, the open source MATLAB® code as well as the compiled versions for Windows®, Mac®, and Linux® are provided with this manuscript. Together, *WaveAR* and *VirtualWave* serve as excellent research and teaching tools for analyzing and exploring laboratory water waves and data sampling.

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Appendix. Supplementary material

Supplementary data associated with this article can be found in the online version at http://dx.doi.org/10.1016/j.cageo.2012.04.001.

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