$$p = \frac{\rho_0 \, c_0^2}{\gamma} \left[\left(\frac{\rho}{\rho_0} \right)^{\gamma} - 1 \right]$$

$$\boldsymbol{v} = \boldsymbol{v}_{irr} + \boldsymbol{v}_{sol}$$

$$\nabla^2\phi = \nabla \cdot \boldsymbol{v}$$

$$\boldsymbol{u}^* = \boldsymbol{u}^n + \Delta t \boldsymbol{RHS}^n$$

$$\nabla^2 p^{n+1} = \frac{\rho}{\Delta t} \nabla \cdot \boldsymbol{u}^*$$

$$\boldsymbol{u}^{n+1} = \boldsymbol{u}^* - \frac{\Delta t}{\rho} \nabla p^{n+1}$$

$$\boldsymbol{u}^* = \boldsymbol{u}^n - \frac{\Delta t}{\rho}\nabla p^n + \Delta t \boldsymbol{RHS}^n$$

$$\nabla^2\phi = \frac{\rho}{\Delta t}\nabla \cdot \boldsymbol{u}^*$$

$$\boldsymbol{u}^{n+1} = \boldsymbol{u}^* - \frac{\Delta t}{\rho} \nabla \phi$$

$$p^{n+1} = p^n + \phi$$

$$
\begin{array}{c|cccc}
0 & & & & \\
1/2 & 1/2 & & & \\
1/2 & 0 & 1/2 & & \\
1 & 0 & 0 & 1 & \\
\hline
 & 1/6 & 1/3 & 1/3 & 1/6
\end{array}
$$

$$\frac{\partial \boldsymbol{W}}{\partial t} + \nabla \cdot \boldsymbol{\Psi}\left(\boldsymbol{W}\right) = \boldsymbol{S}$$

$$\Omega = \bigcup_{i=1}^{N} \Omega_i$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{\Omega_i}\boldsymbol{W}\,\mathrm{d}\Omega-\int_{\partial\Omega_i}\boldsymbol{W}\left(\boldsymbol{u}_{\Omega_i}\cdot\boldsymbol{n}\right)\mathrm{d}S+\int_{\partial\Omega_i}\boldsymbol{\Psi}\left(\boldsymbol{W}\right)\cdot\boldsymbol{n}\,\mathrm{d}S=\int_{\partial\Omega_i}\boldsymbol{S}\,\mathrm{d}\Omega$$

$$\int_{\partial\Omega_i} \boldsymbol{\Psi}\left(\boldsymbol{W}\right) \cdot \boldsymbol{n} \mathrm{d}S = \sum_{j=1}^{M} \int_{A_{ij}} \boldsymbol{\Psi}\left(\boldsymbol{W}\right) \cdot \boldsymbol{n} \mathrm{d}S = \sum_{j=1}^{M} \boldsymbol{F}_{ij}\, A_{ij}$$

$$\overline{\boldsymbol{W}}_i = \frac{1}{\Omega_i} \int_{\Omega_i} \boldsymbol{W} \, \mathrm{d}\Omega \,; \qquad \overline{\boldsymbol{S}}_i = \frac{1}{\Omega_i} \int_{\Omega_i} \boldsymbol{S} \, \mathrm{d}\Omega \,;$$

$$\frac{\mathrm{d}\overline{\boldsymbol{W}}_i}{\mathrm{d}t} = -\sum_{j=1}^{M} \frac{A_{ij}}{\Omega_i} \boldsymbol{F}_{ij} + \overline{\boldsymbol{S}}_i$$

$$\frac{\partial y}{\partial t} + u\frac{\partial y}{\partial x} = 0$$

$$y_i^{n+1} - y_i^n = -u\frac{\Delta t}{2\Delta x}\left(y_{i+1}^n - y_{i-1}^n\right)$$

$$\alpha_i = \frac{d_i^L}{\mathrm{IS}_i + \varepsilon} \qquad \omega_i^L = \frac{\alpha_i}{\sum_{j=1}^{3} \alpha_j} \qquad i = 0,\ 1,\ 2$$

$$\left.\frac{\mathrm{d}\boldsymbol{u}_i}{\mathrm{d}t}\right|^n = (1 - H_i)\ \left.\frac{\mathrm{d}\boldsymbol{u}_i}{\mathrm{d}t}\right|^n_{fluid} + H_i\ \left.\frac{\mathrm{d}\boldsymbol{u}_i}{\mathrm{d}t}\right|^n_{solid}$$

$$\left.\frac{\mathrm{d}\boldsymbol{u}_i}{\mathrm{d}t}\right|_{fluid}^{n} \approx \boldsymbol{RHS}_i^n \qquad \left.\frac{\mathrm{d}\boldsymbol{u}_i}{\mathrm{d}t}\right|_{solid}^{n} \approx \boldsymbol{a}_i^n$$

$$\left.\frac{\mathrm{d}\boldsymbol{u}_i}{\mathrm{d}t}\right|^n \approx (1 - H_i)\,\boldsymbol{RHS}_i^n + H_i \boldsymbol{a}_i^n = \boldsymbol{RHS}_i^n + \boldsymbol{S}_i^n$$

$$\boldsymbol{S}_i^n = H_i \left(\frac{\boldsymbol{u}_i^{n+1}\big|_{solid} - \boldsymbol{u}_i^n}{\Delta t} - \frac{\boldsymbol{u}_i^{n+1}\big|_{fluid} - \boldsymbol{u}_i^n}{\Delta t} \right) = H_i \frac{\boldsymbol{u}_i^{n+1}\big|_{solid} - \boldsymbol{u}_i^{n+1}\big|_{fluid}}{\Delta t}$$

$$H\left(\phi\right)=\frac{1}{2}\left[1+\tanh\left(\alpha\frac{\phi}{\Delta x}\right)\right]$$

$$\partial\Omega = \{\, \boldsymbol{x} |\, \phi\,(\boldsymbol{x},\, t) = 0 \}$$

$$\boldsymbol{n} = \frac{\nabla \phi}{|\nabla \phi|} \qquad\qquad \kappa = -\nabla \cdot \left(\frac{\nabla \phi}{|\nabla \phi|} \right)$$

$$|\nabla \phi| = 1$$

$$\alpha = \frac{1}{\Omega_i} \int_{\Omega_i} \mathrm{H}\left(\phi\right) \mathrm{d}\Omega \qquad \text{where} \qquad \mathrm{H}\left(\phi\right) = \begin{cases} 1 & \text{if} \quad \phi > 0 \\ 0 & \text{if} \quad \phi \leq 0 \end{cases}$$

$$\left.\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t}\right|_{\boldsymbol{X},\,t} = \boldsymbol{u}\left(\boldsymbol{x}\left(\boldsymbol{X},\,t\right),\,t\right)$$

$$\frac{\mathrm{D}\phi}{\mathrm{D}t} = \frac{\partial \phi}{\partial t} + \boldsymbol{u} \cdot \nabla \phi = 0$$

$$\frac{\partial d}{\partial \tau} + \operatorname{sign}(\phi)\left(|\nabla d| - 1\right) = 0$$

$$[\boldsymbol{\tau}\boldsymbol{n} - p\boldsymbol{n}] = -\sigma\kappa\boldsymbol{n}$$

$$\frac{\partial \phi}{\partial t} + \nabla \cdot (\boldsymbol{u}\phi) = 0$$

$$\rho\left(\phi\right) = \rho_1 + \frac{\rho_2 - \rho_1}{2}\left[1 + \tanh\left(\alpha\frac{\phi}{\Delta x}\right)\right]$$

$$\nabla \cdot \left(\frac{1}{\rho} \nabla p \right) = \frac{1}{\Delta t} \nabla \cdot \boldsymbol{u}^*$$

$$\frac{\partial}{\partial x}\left(\frac{1}{\rho}\frac{\partial p}{\partial x}\right)\bigg|_i \approx \frac{1}{\Delta x}\left[\frac{1}{\rho_{i+\frac{1}{2}}}\frac{p_{i+1}-p_i}{\Delta x} - \frac{1}{\rho_{i-\frac{1}{2}}}\frac{p_i - p_{i-1}}{\Delta x}\right]$$

$$\frac{\phi^{n+1} - \phi^n}{\Delta t} = -\nabla \cdot (\boldsymbol{u}^n \phi^n)$$

$$\frac{\boldsymbol{u}^* - \boldsymbol{u}^n}{\Delta t} = -\nabla \cdot (\boldsymbol{u}^n \otimes \boldsymbol{u}^n) + \boldsymbol{g}$$

$$\rho\left(\phi^{n+1}\right) = \rho_1 + \frac{\rho_2 - \rho_1}{2}\left[1 + \tanh\left(\alpha\frac{\phi^{n+1}}{\Delta x}\right)\right]$$

$$\nabla \cdot \left(\frac{1}{\rho\left(\phi^{n+1}\right)} \nabla p^{n+1} \right) = \frac{1}{\Delta t} \nabla \cdot \boldsymbol{u}^{*}$$

$$\boldsymbol{u}^{n+1} = \boldsymbol{u}^{*} - \frac{\Delta t}{\rho\left(\phi^{n+1}\right)} \nabla p^{n+1}$$

$$\boldsymbol{D}_x\,; \qquad \boldsymbol{D}_y\,; \qquad \boldsymbol{D}_z$$

$$\boldsymbol{D}_x = \frac{\Delta y \cancel{\Delta x}}{\cancel{\Delta x}} \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

$$\boldsymbol{G}_x = \mathrm{diag}\left\{\frac{1}{V_i}\right\}\boldsymbol{D}_x^T\,; \qquad \boldsymbol{G}_y = \mathrm{diag}\left\{\frac{1}{V_i}\right\}\boldsymbol{D}_y^T\,; \qquad \boldsymbol{G} = \mathrm{diag}\left\{\frac{1}{V_i}\right\}_z \boldsymbol{D}_z^T$$

$$\boldsymbol{L} = \boldsymbol{D}_x \boldsymbol{G}_x + \boldsymbol{D}_y \boldsymbol{G}_y + \boldsymbol{D}_z \boldsymbol{G}_z$$

$$\boldsymbol{b} = \frac{\rho}{\Delta t} \left(\boldsymbol{D}_x \boldsymbol{u} + \boldsymbol{D}_y \boldsymbol{v} + \boldsymbol{D}_z \boldsymbol{w} \right)$$

$$\boldsymbol{p} = \boldsymbol{L}^{-1}\boldsymbol{b}$$

$$\boldsymbol{u} = \boldsymbol{u} - \frac{\Delta t}{\rho}\boldsymbol{G}_x\boldsymbol{p}\,; \qquad \boldsymbol{v} = \boldsymbol{v} - \frac{\Delta t}{\rho}\boldsymbol{G}_y\boldsymbol{p}\,; \qquad \boldsymbol{w} = \boldsymbol{w} - \frac{\Delta t}{\rho}\boldsymbol{G}_z\boldsymbol{p}$$

$$\operatorname{diag}\left\{\frac{1}{\rho}\right\}_x : \Omega_x \longrightarrow \Omega_x ; \qquad \operatorname{diag}\left\{\frac{1}{\rho}\right\}_y : \Omega_y \longrightarrow \Omega_y ; \qquad \operatorname{diag}\left\{\frac{1}{\rho}\right\}_z : \Omega_z \longrightarrow \Omega_z$$

$$\boldsymbol{L} = \boldsymbol{D}_x \mathrm{diag}\left\{\frac{1}{\rho}\right\}_x \boldsymbol{G}_x + \boldsymbol{D}_y \mathrm{diag}\left\{\frac{1}{\rho}\right\}_y \boldsymbol{G}_y + \boldsymbol{D}_z \mathrm{diag}\left\{\frac{1}{\rho}\right\}_z \boldsymbol{G}_z$$

$$\boldsymbol{u} = \boldsymbol{u} - \Delta t \mathrm{diag}\left\{\frac{1}{\rho}\right\}_x \boldsymbol{G}_x \boldsymbol{p}\,; \quad \boldsymbol{v} = \boldsymbol{v} - \Delta t \mathrm{diag}\left\{\frac{1}{\rho}\right\}_y \boldsymbol{G}_y \boldsymbol{p}\,; \quad \boldsymbol{w} = \boldsymbol{w} - \Delta t \mathrm{diag}\left\{\frac{1}{\rho}\right\}_z \boldsymbol{G}_z \boldsymbol{p}$$

$$\Delta t = \min\left(\Delta t_h,\, \Delta t_v,\, \Delta t_{max}\right)$$

$$p = \cos(2\pi x)\cos(2\pi y)$$