$$
\begin{cases}
\dfrac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0 \\
\dfrac{\partial \rho \boldsymbol{u}}{\partial t} + \nabla \cdot (\rho \boldsymbol{u} \otimes \boldsymbol{u}) = -\nabla p + \nabla \cdot \boldsymbol{\tau} + \rho \boldsymbol{f}
\end{cases}
$$

$$
\begin{cases} \dfrac{\partial \boldsymbol{u}}{\partial t} + \nabla \cdot (\boldsymbol{u} \otimes \boldsymbol{u}) = -\dfrac{\nabla p}{\rho} + \dfrac{1}{\rho} \nabla \cdot \boldsymbol{\tau} + \boldsymbol{f} \\ \nabla \cdot \boldsymbol{u} = 0 \end{cases}
$$

$$\begin{cases} \dfrac{\boldsymbol{u}^{n+1} - \boldsymbol{u}^n}{\Delta t} = -\dfrac{\nabla p^{n+1}}{\rho} + \boldsymbol{RHS}^n \\ \nabla \cdot \boldsymbol{u}^{n+1} = 0 \end{cases}$$

$$
\begin{cases}
\dfrac{\boldsymbol{u}^* - \boldsymbol{u}^n}{\Delta t} = \boldsymbol{RHS}^n \\
\dfrac{\boldsymbol{u}^{n+1} - \boldsymbol{u}^*}{\Delta t} = -\dfrac{\nabla p^{n+1}}{\rho} \\
\nabla \cdot \boldsymbol{u}^{n+1} = 0
\end{cases}
$$

$$
\begin{aligned}
\frac{\mathrm{d}\overline{\boldsymbol{W}}_{i,j,k}}{\mathrm{d}t} = & - \frac{\cancel{\Delta y \Delta z}}{\Delta x \cancel{\Delta y \Delta z}} \left(\boldsymbol{F}_{i+\frac{1}{2},j,k} - \boldsymbol{F}_{i-\frac{1}{2},j,k} \right) - \\
& - \frac{\cancel{\Delta z \Delta x}}{\Delta y \cancel{\Delta z \Delta x}} \left(\boldsymbol{G}_{i,j+\frac{1}{2},k} - \boldsymbol{G}_{i,j-\frac{1}{2},k} \right) - \\
& - \frac{\cancel{\Delta x \Delta y}}{\Delta z \cancel{\Delta x \Delta y}} \left(\boldsymbol{H}_{i,j,k+\frac{1}{2}} - \boldsymbol{H}_{i,j,k-\frac{1}{2}} \right) + \overline{\boldsymbol{S}}_{i,j,k-\frac{1}{2}}
\end{aligned}
$$

$$
\begin{aligned}
y_i^{n+1} - y_i^n &= -u\frac{\Delta t}{\Delta x}\left(y_{i+1}^n - y_i^n\right) && \text{if } u < 0 \\
y_i^{n+1} - y_i^n &= -u\frac{\Delta t}{\Delta x}\left(y_i^n - y_{i-1}^n\right) && \text{if } u > 0
\end{aligned}
$$

$$
\begin{cases}
\boldsymbol{F}_{i+\frac{1}{2},j,k} = \boldsymbol{F}^{L}_{i+\frac{1}{2},j,k} & \text{if } u_{i+\frac{1}{2},j,k} > 0 \\
\boldsymbol{F}_{i+\frac{1}{2},j,k} = \dfrac{1}{2}\left(\boldsymbol{F}^{L}_{i+\frac{1}{2},j,k} + \boldsymbol{F}^{R}_{i+\frac{1}{2},j,k}\right) & \text{if } u_{i+\frac{1}{2},j,k} = 0 \\
\boldsymbol{F}_{i+\frac{1}{2},j,k} = \boldsymbol{F}^{R}_{i+\frac{1}{2},j,k} & \text{if } u_{i+\frac{1}{2},j,k} < 0
\end{cases}
$$

$$\begin{cases} u_{i-\frac{1}{2}}^{R} = u_i - \dfrac{1}{2}\Theta\left(\kappa_i\right)\left(u_{i+1} - u_i\right) \\ u_{i+\frac{1}{2}}^{L} = u_i + \dfrac{1}{2}\Theta\left(\kappa_i\right)\left(u_{i+1} - u_i\right) \end{cases}$$

$$
\begin{cases}
u^L_{i+\frac{1}{2}} = \ \omega^L_0 \left(\dfrac{1}{3}u_i + \dfrac{5}{6}u_{i+1} - \dfrac{1}{6}u_{i+2}\right) \ + \omega^L_1 \left(-\dfrac{1}{6}u_{i-1} + \dfrac{5}{6}u_i + \dfrac{1}{3}u_{i+1}\right) + \\
\qquad\quad + \ \omega^L_2 \left(\dfrac{1}{3}u_{i-2} - \dfrac{7}{6}u_{i-1} + \dfrac{11}{6}u_i\right) \ , \\
u^R_{i-\frac{1}{2}} = \omega^R_0 \left(\dfrac{11}{6}u_i - \dfrac{7}{6}u_{i+1} + \dfrac{1}{3}u_{i+2}\right) + \ \omega^R_1 \left(\dfrac{1}{3}u_{i-1} + \dfrac{5}{6}u_i - \dfrac{1}{6}u_{i+1}\right) \ + \\
\qquad\quad + \omega^R_2 \left(-\dfrac{1}{6}u_{i-2} + \dfrac{5}{6}u_{i-1} + \dfrac{1}{3}u_i\right) \ .
\end{cases}
$$

$$
\begin{cases}
\mathrm{IS}_0 = \dfrac{13}{12}\left(u_{i+2} - 2u_{i+1} + u_i\right)^2 + \dfrac{1}{4}\left(u_{i+2} - 4u_{i+1} + 3u_i\right)^2 \\
\mathrm{IS}_1 = \dfrac{13}{12}\left(u_{i+1} - 2u_i + u_{i-1}\right)^2 + \dfrac{1}{4}\left(u_{i+1} - u_{i-1}\right)^2 \\
\mathrm{IS}_2 = \dfrac{13}{12}\left(u_i - 2u_{i-1} + u_{i-2}\right)^2 + \dfrac{1}{4}\left(3u_i - 4u_{i-1} + u_{i-2}\right)^2
\end{cases}
$$

$$\begin{cases} \rho\left(\phi\right) = \rho_1 + \left(\rho_2 - \rho_1\right) H\left(\phi\right) \\ \mu\left(\phi\right) = \mu_1 + \left(\mu_2 - \mu_1\right) H\left(\phi\right) \\ \boldsymbol{f}_{sf}\left(\phi\right) = \sigma\kappa\left(\phi\right) \nabla H \end{cases}$$

$$H(\phi) = \begin{cases} 0 & \phi < -\varepsilon \\ \dfrac{1}{2} + \dfrac{\phi}{2\varepsilon} + \dfrac{1}{2\pi}\sin\left(\dfrac{\pi\phi}{\varepsilon}\right) & -\varepsilon \leq \phi \leq \varepsilon \\ 1 & \phi > \varepsilon \end{cases}$$

$$
\begin{aligned}
\boldsymbol{D}_x &: \Omega_x \longrightarrow \Omega_c & \qquad \boldsymbol{G}_x &: \Omega_c \longrightarrow \Omega_x \\
\boldsymbol{D}_y &: \Omega_y \longrightarrow \Omega_c & \qquad \boldsymbol{G}_y &: \Omega_c \longrightarrow \Omega_y \\
\boldsymbol{D}_z &: \Omega_z \longrightarrow \Omega_c & \qquad \boldsymbol{G}_z &: \Omega_c \longrightarrow \Omega_z
\end{aligned}
$$

$$
\left\{
\begin{aligned}
\Delta t_h &= \mathrm{CFL}_h \frac{\Delta x}{\max_{\Omega}\left(|u_i|\right)} \\
\Delta t_v &= \mathrm{CFL}_v \frac{\Delta x^2}{\nu} \\
\Delta t &= \min\left(\Delta t_h,\ \Delta t_v\right)
\end{aligned}
\right.
$$

$$\begin{cases} u = & -\Omega\left(y - 0.5\right) \\ v = & \Omega\left(x - 0.5\right) \end{cases}$$

$$
\begin{cases}
u\left(x,y\right)= & U_0\sin\left(2\pi\dfrac{x}{L}\right)\cos\left(2\pi\dfrac{y}{L}\right)\\
v\left(x,y\right)= & -U_0\cos\left(2\pi\dfrac{x}{L}\right)\sin\left(2\pi\dfrac{y}{L}\right)\\
p\left(x,y\right)= & \dfrac{\rho U_0^2}{4}\left[\cos\left(4\pi\dfrac{x}{L}\right)+\cos\left(4\pi\dfrac{y}{L}\right)\right]
\end{cases}
$$

$$\begin{cases} u = \dfrac{-2\Delta t\pi\sin\left(2\pi x\right)\cos\left(2\pi y\right)}{\rho\left(x,\, y\right)} \\ v = \dfrac{-2\Delta t\pi\cos\left(2\pi x\right)\sin\left(2\pi y\right)}{\rho\left(x,\, y\right)} \end{cases}$$

$$k_n = \frac{n\pi}{L}$$

$$\omega_n^2 = \frac{g_0 k_n \left(\rho_2 - \rho_1\right)}{\rho_1 \coth\left(\omega_{2n+1} t\right) + \rho_2 \coth\left(k_{2n+1} x\right)}$$

$$y\left(x,\, t\right) = h_2 + \frac{a}{g_0}\left[x - \frac{L}{2} + \sum_{n=0}^{\infty} \frac{4}{L k_{2n+1}^2} \cos\left(\omega_{2n+1} t\right) \cos\left(k_{2n+1} x\right)\right]$$