$$p = \frac{\rho_0 c_0^2}{\gamma} \left[ \left( \frac{\rho}{\rho_0} \right)^{\gamma} - 1 \right]$$

$$oldsymbol{v} = oldsymbol{v}_{irr} + oldsymbol{v}_{sol}$$

$$abla^2\phi=
abla\cdotoldsymbol{v}$$

$$oldsymbol{u}^* = oldsymbol{u}^n + \Delta t oldsymbol{R} oldsymbol{H} oldsymbol{S}^n$$

$$abla^2 p^{n+1} = rac{
ho}{\Delta t} 
abla \cdot oldsymbol{u}^*$$

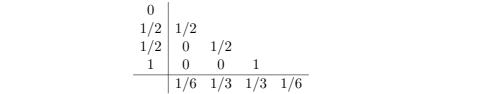
$$oldsymbol{u}^{n+1} = oldsymbol{u}^* - rac{\Delta t}{
ho} 
abla p^{n+1}$$

 $oldsymbol{u}^* = oldsymbol{u}^n - rac{\Delta t}{2} 
abla p^n + \Delta t oldsymbol{R} oldsymbol{H} oldsymbol{S}^n$ 

 $\nabla^2 \phi = \frac{\rho}{\Delta t} \nabla \cdot \boldsymbol{u}^*$ 

$$oldsymbol{u}^{n+1} = oldsymbol{u}^* - rac{\Delta t}{
ho} 
abla \phi$$

$$p^{n+1} = p^n + \phi$$



 $\frac{\partial \boldsymbol{W}}{\partial t} + \nabla \cdot \boldsymbol{\Psi} \left( \boldsymbol{W} \right) = \boldsymbol{S}$ 

$$\Omega = \bigcup_{i=1}^N \Omega_i$$

 $\frac{\mathrm{d}}{\mathrm{d}t} \int_{\Omega_{i}} \mathbf{W} \mathrm{d}\Omega - \int_{\partial\Omega_{i}} \mathbf{W} \left( \mathbf{u}_{\Omega_{i}} \cdot \mathbf{n} \right) \mathrm{d}S + \int_{\partial\Omega_{i}} \mathbf{\Psi} \left( \mathbf{W} \right) \cdot \mathbf{n} \mathrm{d}S = \int_{\partial\Omega_{i}} \mathbf{S} \mathrm{d}\Omega$ 

$$\int_{\partial\Omega_{i}}\mathbf{\Psi}\left(\mathbf{W}\right)\cdot\mathbf{n}\mathrm{d}S=\sum_{j=1}^{M}\int_{A_{ij}}\mathbf{\Psi}\left(\mathbf{W}\right)\cdot\mathbf{n}\mathrm{d}S=\sum_{j=1}^{M}\mathbf{F}_{ij}\,A_{ij}$$

 $\overline{\boldsymbol{W}}_i = \frac{1}{\Omega_i} \int_{\Omega_i} \boldsymbol{W} d\Omega;$ 

 $\overline{m{S}}_i = rac{1}{\Omega_i} \int_{\Omega_i} m{S} \mathrm{d}\Omega \, ;$ 

 $\frac{\mathrm{d}\overline{\boldsymbol{W}}_{i}}{\mathrm{d}t} = -\sum_{j=1}^{M} \frac{A_{ij}}{\Omega_{i}} \boldsymbol{F}_{ij} + \overline{\boldsymbol{S}}_{i}$ 

 $\frac{\partial y}{\partial t} + u \frac{\partial y}{\partial x} = 0$ 

$$y_i^{n+1} - y_i^n = -u \frac{\Delta t}{2\Delta x} (y_{i+1}^n - y_{i-1}^n)$$

$$\alpha_i = \frac{d_i^L}{\mathrm{IS}_i + \varepsilon}$$
  $\omega_i^L = \frac{\alpha_i}{\sum_{j=1}^3 \alpha_j}$   $i = 0, 1, 2$ 

 $\frac{\mathrm{d}\boldsymbol{u}_i}{\mathrm{d}t}\bigg|^n = (1 - H_i) \left. \frac{\mathrm{d}\boldsymbol{u}_i}{\mathrm{d}t} \right|_{fluid}^n + H_i \left. \frac{\mathrm{d}\boldsymbol{u}_i}{\mathrm{d}t} \right|_{solid}^n$ 

 $\frac{\mathrm{d} \boldsymbol{u}_i}{\mathrm{d} t}\Big|^n$ 

fluid

 $pprox m{RHS}_i^n$ 

 $\frac{\mathrm{d}\boldsymbol{u}_i}{|}^n$ 

solid

 $\overline{\mathrm{d}t}$ 

 $pprox oldsymbol{a}_i^n$ 

$$\left. rac{\mathrm{d} oldsymbol{u}_i}{\mathrm{d} t} 
ight|^n pprox (1-H_i) oldsymbol{R} oldsymbol{H} oldsymbol{S}_i^n + H_i oldsymbol{a}_i^n = oldsymbol{R} oldsymbol{H} oldsymbol{S}_i^n + oldsymbol{S}_i^n$$

$$oldsymbol{S}_{i}^{n} = H_{i}\left(rac{oldsymbol{u}_{i}^{n+1}ig|_{solid} - oldsymbol{u}_{i}^{n}}{\Delta t} - rac{oldsymbol{u}_{i}^{n+1}ig|_{fluid} - oldsymbol{u}_{i}^{n}}{\Delta t}
ight) = H_{i}rac{oldsymbol{u}_{i}^{n+1}ig|_{solid} - oldsymbol{u}_{i}^{n+1}}{\Delta t}$$

$$H\left(\phi\right) = \frac{1}{2}\left[1 + \tanh\left(\alpha \frac{\phi}{\Delta x}\right)\right]$$

$$\partial\Omega=\left\{ \left.oldsymbol{x}
ight|\phi\left(oldsymbol{x},\,t
ight)=0
ight\}$$

$$m{n} = rac{
abla \phi}{|
abla \phi|} \hspace{1cm} \kappa = -
abla \cdot \left(rac{
abla \phi}{|
abla \phi|}
ight)$$



 $\alpha = \frac{1}{\Omega_i} \int_{\Omega_i} \mathbf{H} (\phi) \, d\Omega \qquad \text{where} \qquad \mathbf{H} (\phi) = \left\{ \begin{array}{ll} 1 & \text{if} \quad \phi > 0 \\ 0 & \text{if} \quad \phi \leq 0 \end{array} \right.$ 

$$\left. rac{\mathrm{d} oldsymbol{x}}{\mathrm{d} t} \right|_{oldsymbol{X},\,t} = oldsymbol{u}\left( oldsymbol{x}\left( oldsymbol{X},\,t 
ight),\,t 
ight)$$

$$rac{\mathrm{D}\phi}{\mathrm{D}t} = rac{\partial\phi}{\partial t} + oldsymbol{u}\cdot
abla\phi = 0$$

$$\frac{\partial d}{\partial \tau} + \operatorname{sign}(\phi)(|\nabla d| - 1) = 0$$

$$[m{ au}m{n}-pm{n}]=-\sigma\kappam{n}$$

$$rac{\partial \phi}{\partial t} + 
abla \cdot (\boldsymbol{u}\phi) = 0$$

$$\rho(\phi) = \rho_1 + \frac{\rho_2 - \rho_1}{2} \left[ 1 + \tanh\left(\alpha \frac{\phi}{\Delta x}\right) \right]$$

 $\nabla \cdot \left(\frac{1}{\rho} \nabla p\right) = \frac{1}{\Delta t} \nabla \cdot \boldsymbol{u}^*$ 

 $\left. \frac{\partial}{\partial x} \left( \frac{1}{\rho} \frac{\partial p}{\partial x} \right) \right|_{i} \approx \frac{1}{\Delta x} \left[ \frac{1}{\rho_{i+\frac{1}{2}}} \frac{p_{i+1} - p_{i}}{\Delta x} - \frac{1}{\rho_{i-\frac{1}{2}}} \frac{p_{i} - p_{i-1}}{\Delta x} \right]$ 

$$rac{\phi^{n+1}-\phi^n}{\Delta t} = -
abla \cdot (oldsymbol{u}^n\phi^n)$$

$$rac{oldsymbol{u}^* - oldsymbol{u}^n}{\Delta t} = -
abla \cdot (oldsymbol{u}^n \otimes oldsymbol{u}^n) + oldsymbol{g}$$

$$\rho\left(\phi^{n+1}\right) = \rho_1 + \frac{\rho_2 - \rho_1}{2} \left[ 1 + \tanh\left(\alpha \frac{\phi^{n+1}}{\Delta x}\right) \right]$$

$$abla \cdot \left( rac{1}{
ho \left( \phi^{n+1} 
ight)} 
abla p^{n+1} 
ight) = rac{1}{\Delta t} 
abla \cdot oldsymbol{u}^*$$

$$oldsymbol{u}^{n+1} = oldsymbol{u}^* - rac{\Delta t}{
ho\left(\phi^{n+1}
ight)}
abla p^{n+1}$$

## $D_x; \qquad D_y; \qquad D_z$

$$m{D}_x = rac{\Delta y \Delta x}{\Delta x} \left[ egin{array}{cccccc} -1 & 1 & 0 & 0 & 0 & 0 \ 0 & -1 & 1 & 0 & 0 & 0 \ 0 & 0 & 0 & -1 & 1 & 0 \ 0 & 0 & 0 & 0 & -1 & 1 \end{array} 
ight]$$

$$oldsymbol{G}_x = ext{diag}\left\{rac{1}{V_i}
ight\}oldsymbol{D}_x^T; \qquad oldsymbol{G}_y = ext{diag}\left\{rac{1}{V_i}
ight\}oldsymbol{D}_y^T; \qquad oldsymbol{G} = ext{diag}\left\{rac{1}{V_i}
ight\}_z oldsymbol{D}_z^T$$

$$\boldsymbol{L} = \boldsymbol{D}_x \boldsymbol{G}_x + \boldsymbol{D}_y \boldsymbol{G}_y + \boldsymbol{D}_z \boldsymbol{G}_z$$

$$oldsymbol{b} = rac{
ho}{\Delta t} \left( oldsymbol{D}_x oldsymbol{u} + oldsymbol{D}_y oldsymbol{v} + oldsymbol{D}_z oldsymbol{w} 
ight)$$

$$oldsymbol{p} = oldsymbol{L}^{-1} oldsymbol{b}$$

 $oldsymbol{u} = oldsymbol{u} - rac{\Delta t}{a} oldsymbol{G}_x oldsymbol{p}\,; \qquad oldsymbol{v} = oldsymbol{v} - rac{\Delta t}{a} oldsymbol{G}_z oldsymbol{p}\,$ 

$$\operatorname{diag}\left\{\frac{1}{\rho}\right\}_{x}:\,\Omega_{x}\,\longrightarrow\,\Omega_{x}\,;\qquad\operatorname{diag}\left\{\frac{1}{\rho}\right\}_{y}:\,\Omega_{y}\,\longrightarrow\,\Omega_{y}\,;\qquad\operatorname{diag}\left\{\frac{1}{\rho}\right\}_{z}:\,\Omega_{z}\,\longrightarrow\,\Omega_{z}$$

$$oldsymbol{L} = oldsymbol{D}_x ext{diag} \left\{rac{1}{
ho}
ight\}_x oldsymbol{G}_x + oldsymbol{D}_y ext{diag} \left\{rac{1}{
ho}
ight\}_y oldsymbol{G}_y + oldsymbol{D}_z ext{diag} \left\{rac{1}{
ho}
ight\}_z oldsymbol{G}_z$$

$$oldsymbol{u} = oldsymbol{u} - \Delta t \operatorname{diag} \left\{ rac{1}{
ho} 
ight\}_x oldsymbol{G}_x oldsymbol{p}; \quad oldsymbol{v} = oldsymbol{v} - \Delta t \operatorname{diag} \left\{ rac{1}{
ho} 
ight\}_y oldsymbol{G}_y oldsymbol{p}; \quad oldsymbol{w} = oldsymbol{w} - \Delta t \operatorname{diag} \left\{ rac{1}{
ho} 
ight\}_z oldsymbol{G}_z oldsymbol{p}$$

$$\Delta t = \min \left( \Delta t_h, \, \Delta t_v, \, \Delta t_{max} \right)$$

$$p = \cos(2\pi x)\cos(2\pi y)$$