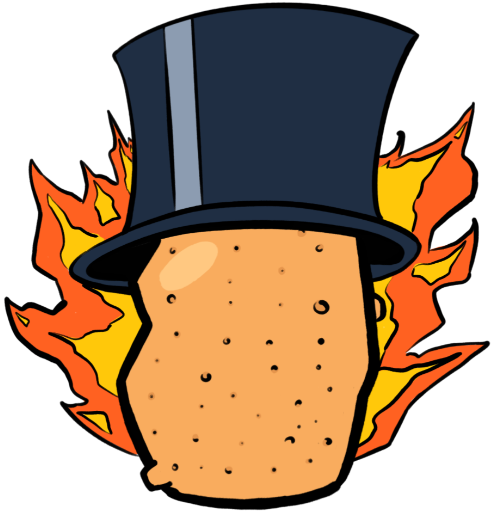
Hot Potato Theatre

A tool for building, analysing and playing
Simple Hot Potato Games

Presented by: **Salvatore Salerno**



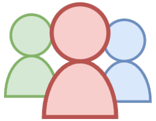
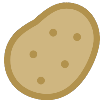
Academic Year 2024/2025

Outline

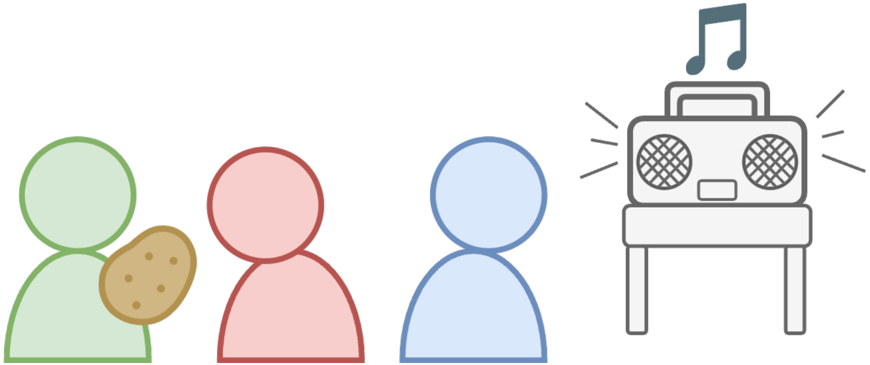
1. Introduction to Simple Hot Potato Games
2. Developing Hot Potato Theatre
3. Modelling players
4. Finding a longest chain of play
5. Live Demo
6. Conclusions

The classic Hot Potato Game

Recall the rules of the classic children's game of the Hot Potato where there are:

- A group of people 
- An object referred to as the *hot potato* 

At the start the potato is considered **good** and players are incentivised to pass it between each other.



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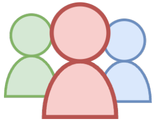
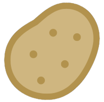
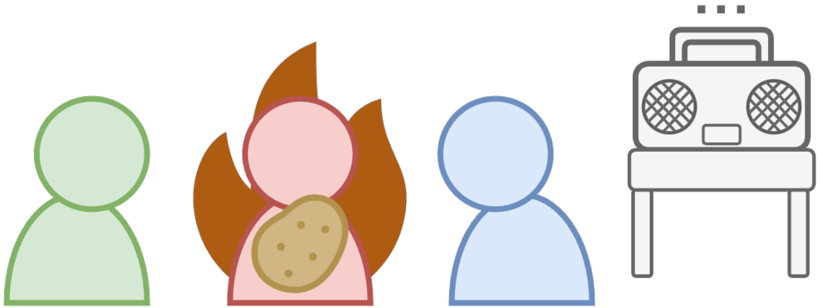
- A group of people 
- An object referred to as the *hot potato* 

After a certain event occurs, the hot potato becomes **bad**, causing the last player holding it to lose, being eliminated from the game.



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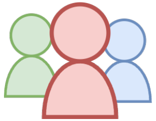
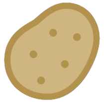
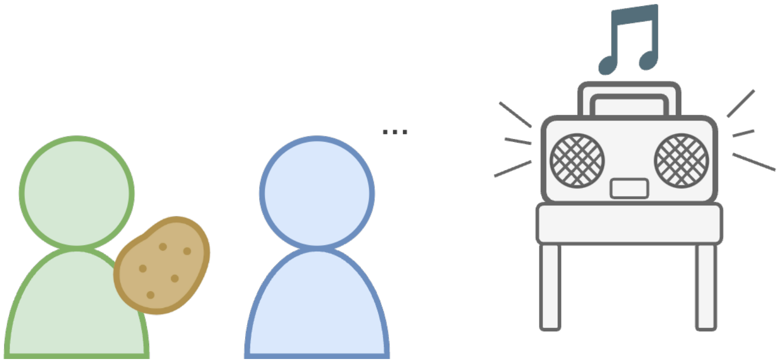
- A group of people 
- An object referred to as the *hot potato* 

The game continues until there is just one player left.

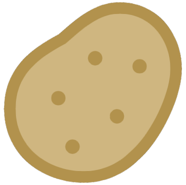


The Simple Hot Potato Game

Inspired by it, *Carter T. Butts* and *David C. Rode* wrote the paper *Rational and Empirical Play in the Simple Hot Potato Game,* designing a new class of mathematical games.

The **hot potato** is an exchangeable object whose nature changes over time. Formally:

- l : **lifetime**, that is the number of turns in which it remains beneficial.
- P : **gain**, the positive payoff for players who took it but are not the last to hold it.
- C : **loss**, the negative payoff of the last player holding the potato.

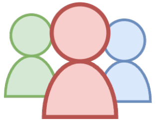
$$X = (l, P, C) =$$ 

The Simple Hot Potato Game

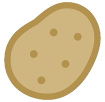
Inspired by it, *Carter T. Butts* and *David C. Rode* wrote the paper *Rational and Empirical Play in the Simple Hot Potato Game,* designing a new class of mathematical games.

The **Simple Hot Potato Game** (SHPG for short) is defined by:

- $N = \{x_1; \dots; x_n\}$: the set of players, all having initial payoff set to 0.



- $X = (l, P, C)$: the hot potato object



SHPG Example

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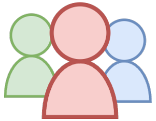
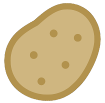
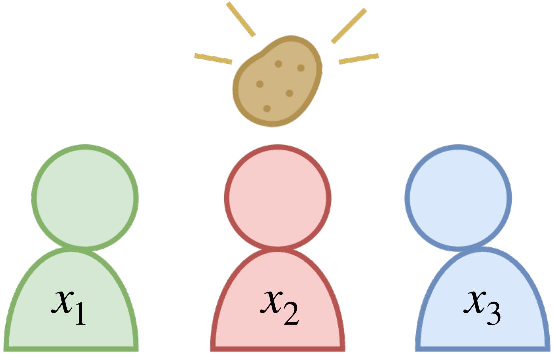
- $N = \{x_1; x_2; x_3\}$: the set of players, all having initial payoff set to 0.



- $X = (l = 3, P = 5, C = 10)$: the hot potato object



1) At the start the game chooses a random player to offer the good.



$$turn = 1$$
$$lifetime\ left = 2$$

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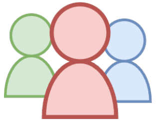
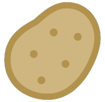
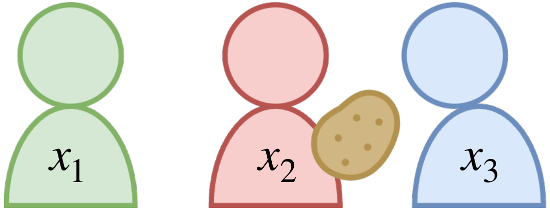
- $N = \{x_1; x_2; x_3\}$: the set of players, all having initial payoff set to 0. 
- $X = (l = 3, P = 5, C = 10)$: the hot potato object 

2) The current holder attempts to pass it to another player who never took the good prior.



$$turn = 2$$
$$lifetime\ left = 1$$

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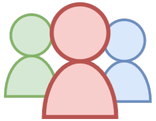

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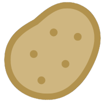
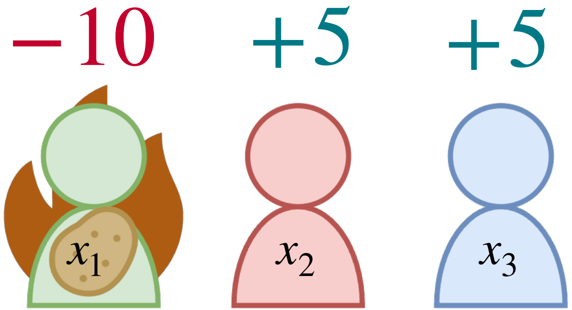


3) If no player is willing to accept the hot potato, or if its lifetime expires, the game ends.



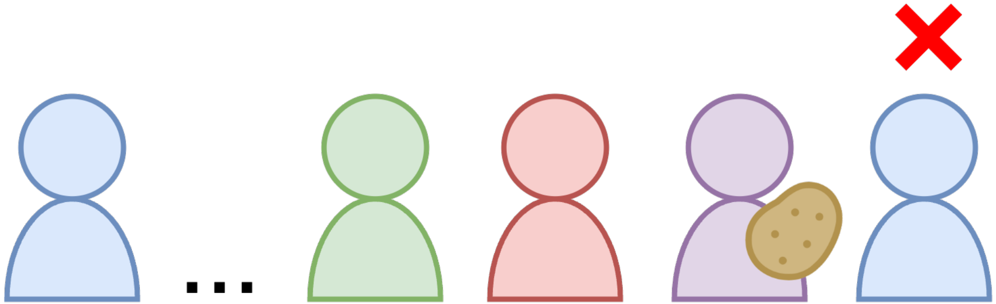
$$turn = 3$$
$$lifetime\ left = 0$$

Theorem - Optimal Play for Rational Actors

Carter T. Butts and *David C. Rode* proved for the SHPG that, assuming all players are **rational actors** with **complete knowledge** of the game state then:

It is a unique *sub-game perfect Nash equilibrium* for **no** player in the SHPG to accept the Hot Potato good X

The proof works by **backward induction**:



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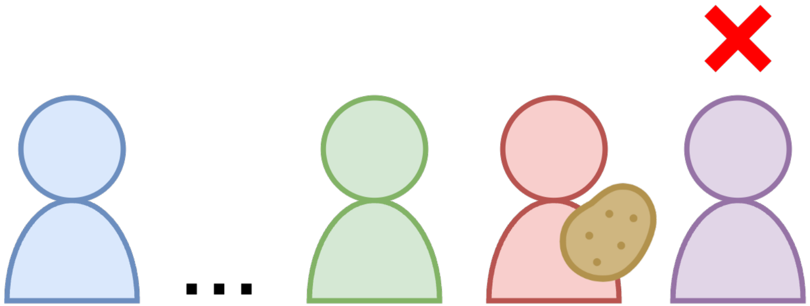

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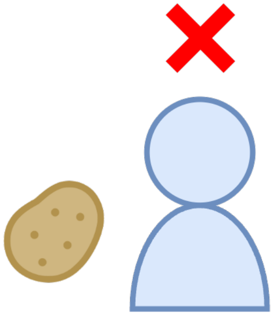

Behavioural Models

What the theorem proves does not accurately reflect r**eal human behaviour.**

For this reason, the paper introduces several behavioural models of players whose knowledge is limited and/or whose motivations are not purely egoistical:

- **Assumption of irrational actors:** the mere possibility of irrational play could alter the strategic reasoning of players *(Aumann 1995; Kreps et al. 1982; Andreoni and Miller 1993).*

- ***Effect of bounded rationality:*** players have limited computational capacity to evaluate all outcomes or incomplete knowledge of the game's current state *(Jehiel 2004, 2001; Camerer 2003; de Groot 1965).*

Behavioural Models

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For this reason, the paper introduces several behavioural models of players whose knowledge is limited and/or whose motivations are not purely egoistical:

- **Effects of altruism:** actual human behaviour frequently includes some degree of altruism, even when individuals believe they are acting in their own self-interest *(Andreoni and Miller 2002).*
- **Cooperation and trust:** human players have been noticed to form informal coalitions in order to reduce the individual risk of incurring a loss, honouring the commitments they make *(Orbell et al. 1988; Burt and Knez 1995).*

Outline

Hot Potato Theatre

This seminar presents **Hot Potato Theatre**, a program written in *Kotlin.*

The project is publicly available on GitHub in the following repository for those interested in trying it out in the following link:

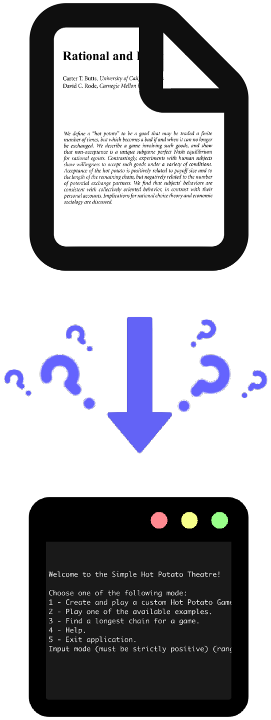
https://github.com/Sallo97/Hot-Potato-Theatre

Hot Potato Theatre

This seminar presents **Hot Potato Theatre**, a program written in *Kotlin.*

The program offers:

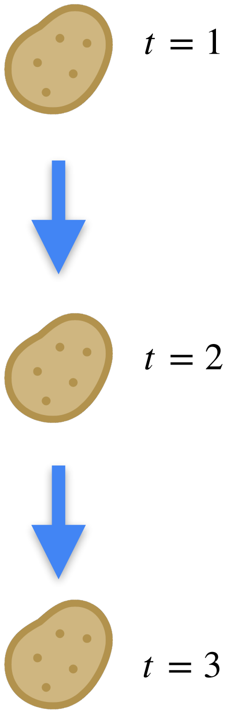
- A **concrete implementation** of the SHPG, being able to construct, analyse and play game.
- An extension of the game, proposing the **Mutable Potato**, that is a new kind of exchangeable object.
- An **algorithm for finding a possible longest chain** (if any exists) for any SHPG.



Implementing the Hot Potato object

In Hot Potato Theatre there are implemented two types of potatoes:

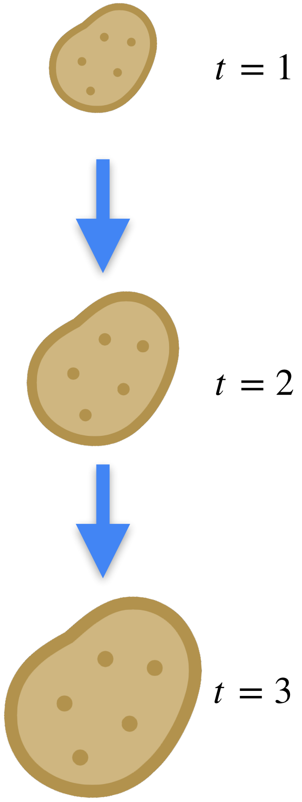
- **Fixed Potato**: this is the standard hot potato we have seen previously, being composed of:
  - **lifetime** (l): represented as an *absolute integer*.
  - **gain** (P): represented as an *absolute double*.
  - **loss** (C): represented as an *absolute double*
  - **current holder** (H): the player holding the potato at the moment.



Implementing the Hot Potato object

In Hot Potato Theatre there are implemented two types of potatoes:

- **Mutable Potato:** a potato whose gain an loss value is **updated over time** by some:
  - **gain factor** (f_P): represented as an *absolute double*.
  - **loss factor** (f_C): represented as an *absolute double*.
  - **gain at turn t:** $P_t = P \times f_P^t$
  - **loss at turn t:** $C_t = C \times f_C^t$



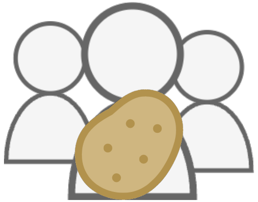
Implementing the SHPG

At its core the Simple Hot Potato Game is a class requesting by its constructor:

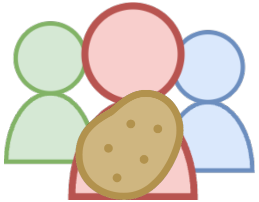
- a set of players $N = \{x_1; x_2; x_3\}$
- an hot potato $X = (l, P, C)$

In practice there is a third parameter, **type**, which identifies the game as either:

Homogeneous



Mixed



Implementing the Player

The Player class is an abstract entity, providing a blueprint which its sub-classes, the concrete players implementation, must follow. A player is recognised by:

- **payoff** (p): a property initially set to 0.
- **decideAcceptance**: it implements the decision-making logic for accepting and rejecting the hot potato.
    - It requires as argument the **current game state**, for which depending on the type of player will use distinct information to reason its strategy.
    - The function returns a boolean, stating if it accepted the potato or not.

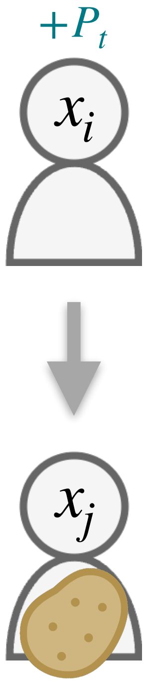
Executing a SHPG

The game begins by invoking the public method **run** :

1. The system checks the game status and attempts to find a player willing get the potato.
   For proposing the good to a player its **decideAcceptance** method is called.

2. If the response was positive, the game's state is updated: player is added to chain, the turn is increased, coalition and potato are updated.

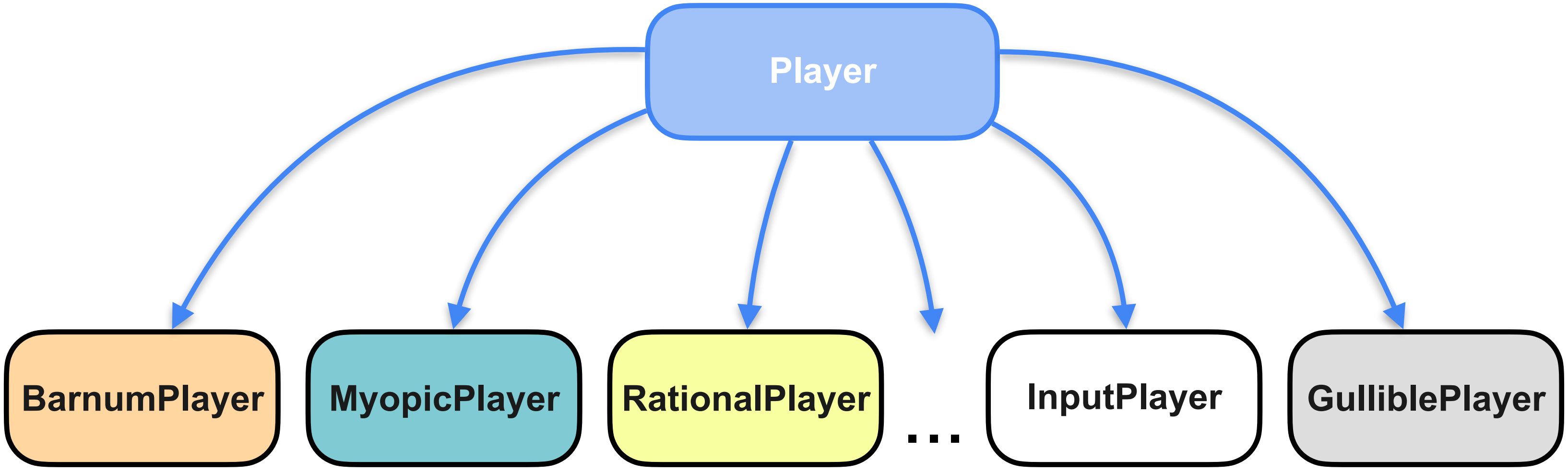
3. If the hot potato exceeded its lifetime or the system was not able to find a willing player, the game ends. The chain of willing players and the eventual coalition formed is returned.



Outline

Modelling players

As described previously, the hierarchy of player's class is structured in the following way:



The difference lies on their **decideAcceptance** method.

Rational, Gullible and Input players

These players are included for convenience, as they are useful for debugging purposes and for analysing the behaviour of other players.

- **RationalPlayer**: follow the theorem of *Optimal Play for Rational Actors*, they will never accept the hot potato.
- **GulliblePlayer**: always accept the hot potato, regardless of the game's current state.
- **InputPlayer:** are controlled by the user from standard input

Assumption of irrational actors - Barnum players

BarnumPlayers is a class of rational actors who believe in the possibility that irrational players exist among the active population. This belief is modelled by the parameter:

$$p_b = \mathscr{P}(\exists x \in N_{active}/\{player\} \, . \, x \; is \; rational)$$

Then the decision-logic is:

- $min(l - t, |N| - t) > 0$ that is the are still turns left.
- $\mathscr{P}(\forall x \in N_{active} \, . \, x \; is \; rational) = p_B^{n-t-1}$
- $\mathscr{P}(\forall x \in N_{active} \, . \, x \; is \; not \; rational) = 1 - p_B^{n-t-1}$

Assumption of irrational actors - Barnum players

Then the decision-logic is:

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- $\mathscr{P}(\forall x \in N_{active} \,.\, x \text{ is not rational}) = p_B^{n-t-1}$

$$\left|\frac{P_t}{C_t}\right| \geq \frac{\mathscr{P}(\forall x \in N_{active} \,.\, x \ is \ rational)}{\mathscr{P}(\forall x \in N_{active} \,.\, x \ is \ not \ rational)}$$

Note how the condition does not take into account the behaviour of other Barnum actors.

Effect of bounded rationality - Myopic players

MyopicPlayer is a class of actors who perceives the game as effectively infinite below a certain threshold τ. However, once the remaining chain falls below τ , the player act rationally and refuses the potato.

Then the decision-logic is:

$$min(l - t, |N| - t) > \tau$$

Effect of bounded rationality - Stochastic players

StochasticPlayer do not interpret the game as being influenced by their own choices or those of others. Instead, they treat it as a stochastic process. We model them by the parameter:

$$p = \mathscr{P}(\text{another player at the next turn will reject the hot potato})$$

Then the decision-logic is:

- $\mathscr{P}(\text{the game ends at the next turn}) = p^{n-t-1}$
- $\mathscr{P}(\text{the game continues at the next turn}) = 1 - p^{n-t-1}$

$$\mathscr{P}(\text{game continues}) \times P_t \geq \left| \mathscr{P}(\text{game ends}) \times C_t \right|$$

Effects of altruism - Direct altruist players

DirectAltruistPlayer describes actors concerned for the well-being of another player. It is willing to accept the hot potato if doing so could help this beneficiary.

The paper does not define an exact acceptance function, rather outlines some key property of the decision-logic:

- the higher the potential gain, the more incentive has to accept.
- the potential cost should be:
    - discounted by the player’s willing to help its beneficiary.
    - increased by the player’s perceived responsibility, that is if he/she is aware that other players could help the same individual.

Effects of altruism - Direct altruist players

To model the acceptance method, two hyper-parameters are defined:

- $\alpha \in [0,1]$: the **altruistic factor**, how much the player is willing to help.

- $\beta \in [0,1]$: the **alter belief**, how much the current player believe another actor is willing to help the same individual.

Then the decision-logic is:
- otherHelpers = $n - t - 2$
- responsibility = if (otherHelpers == 0) then 1 else $\beta^{otherHelpers}$

$$\boxed{\frac{P_t}{\alpha} \geq |C_t \times (\alpha - (1 - responsability))|}$$

Effects of altruism - Benthamite players

BenthamitePlayer describes *global altruist* actors, aiming to maximise the game duration.

Like other altruistic agents, their sense of responsibility depends on the current game state and their expectations about how other players will behave.

To model this behaviour, a hyper-parameters has been introduced:

- $\alpha \in [0,1]$: the weight assigned for accepting the hot potato.

Effects of altruism - Benthamite players

To model this behaviour, a hyper-parameters has been introduced:

- $\alpha \in [0,1]$: the weight assigned for accepting the hot potato.

Then the decision-logic is:

- $\beta = 1 - \alpha$
- $r = min(l - t, |N| - t)$
- $g = \alpha \times (P_t \times r - C_t)$ (potential of the maximum chain)
- $w =$ *if* $(n - t == 0)$ *then* 0 *else* $\beta * \dfrac{C_t}{n - t}$ (risk of taking the loss)

Effects of altruism - Benthamite players

Then the decision-logic is:

- $r = min(l - t, |N| - t)$
- $g = \alpha \times (P_t \times r - C_t)$ (potential of the maximum chain)
- $w =$ *if* $(n - t - 1 == 0)$ *then* 0 *else* $\beta * \dfrac{C_t}{n - t}$ (risk of taking the loss)

$$\boxed{g \geq w}$$

Cooperation and trust - Coalitional players

CoalitionalPlayers aim to form alliances with other players of the same type in order to share risks and rewards.

For simplicity, it is assumed that only one coalition can be formed during the execution of a game.

To handle this type of players a **Coalition** has been designed, with the aim to manage a coalition. It contains the following properties:

- The current payoff promised to each player within the coalition .
- The set of current coalition members.
- The optimal coalition size in case of an *Homogeneous* game.

Cooperation and trust - Coalitional players

In **Homogeneous games** the system precomputes the **optimal coalition size**:

- **Fixed potato**: since the gain and loss are fixed then
  - either this optimal size always includes all players
  - no coalition is profitable since the loss exceed the total gain

- **Mutable potato**: decreasing gains or increasing losses over time may make it preferable to end earlier.

During gameplay a player will ask the coalition if it can enter:

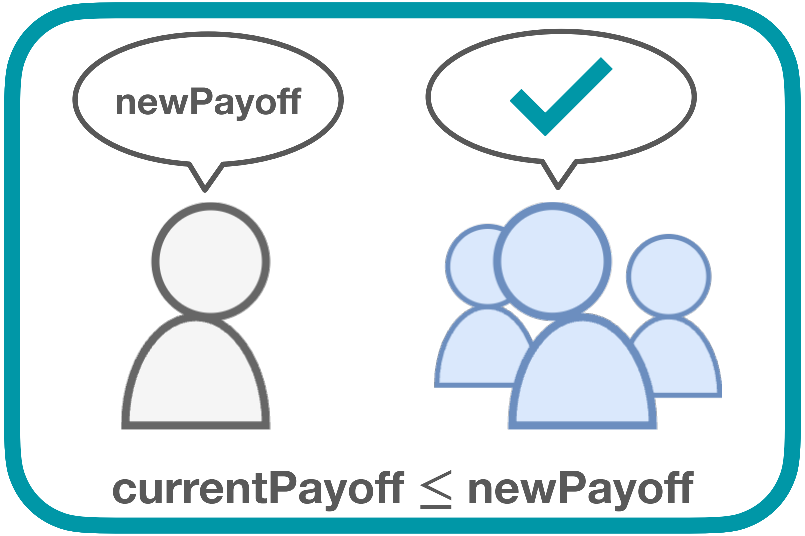
If coalition.size $<$ optimalSize ✓

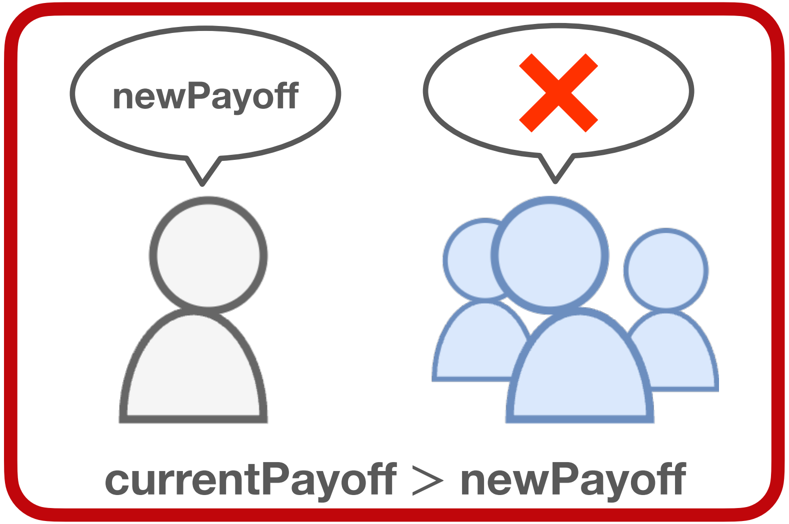
If coalition.size $\geq$ optimalSize ✗

Cooperation and trust - Coalitional players

In **Mixed games** with **Mutable** potato, the decision is done dynamically.

If a coalition already exists a player proposes to it to be a member and their decision depends on their current payoff:





Cooperation and trust - Coalitional players

In **Mixed games** with **Mutable** potato, the decision is done dynamically.

If a coalition does not exists, then the current player needs to decide if its convenient to start one. This is determined by an heuristic on:

- c **:** the number of coalitional members still in the game.
- $m = \arg\min_{1 \le i \le c} \left(\frac{\left(\sum_{j=1}^{i} P_j\right) - L}{i} \right) > 0$ (minimum number of members required to get a positive payoff)
- $p = \left(\frac{\left(\sum_{j=1}^{m} P_j\right) - L}{m} \right) > 0$ (minimum estimated payoff)

Cooperation and trust - Coalitional players

- $\alpha = \dfrac{\binom{(n-t)-m}{t-m}}{\binom{n-t}{t}} = \mathscr{P}(\text{at least } m \text{ coalitional players will be proposed the potato})$
- $\beta = \dfrac{\binom{(n-t)-m}{t}}{\binom{n-t}{t}} = \mathscr{P}(\text{no coalitional player will be proposed the potato})$

$$\boxed{p \times \alpha \geq C \times \beta}$$

If it is convinced, then the player will ask the game to initiate the coalition.

Considering the mutability in player's decision

In case of a **Mutable** potato, players consider how the gain and loss change over time:

- **If loss is increasing or gain is decreasing**, they are more incentivised to accept the good immediately.
- **If gain is increasing or loss is decreasing**, the player may prefer to deny the good for the time being

In the program, Player offers the method **mutablePotatoAcceptance**, which taken the current acceptance weight of the player and the current loss weight, checks:

$$\text{acceptanceWeight} \times (1/\text{gainFactor}) \geq \text{lossWeight} \times \text{lossFactor}$$

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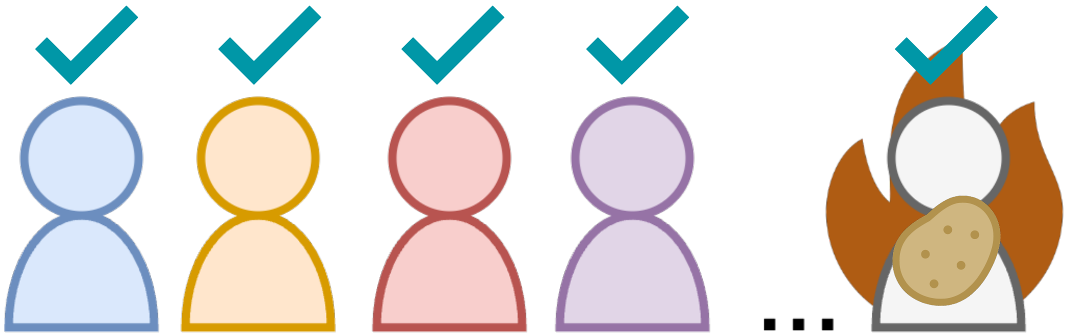
Longest chain of play

A **longest chain** for a SHPG is defined as a sequence of players in the population such that, if the game proposes the hot potato to them in the exact order of the chain, it will lead to the maximum duration of said game.

Hot Potato Theatre offers an **algorithm** for finding (if any exists) a longest chain for all SHPG.

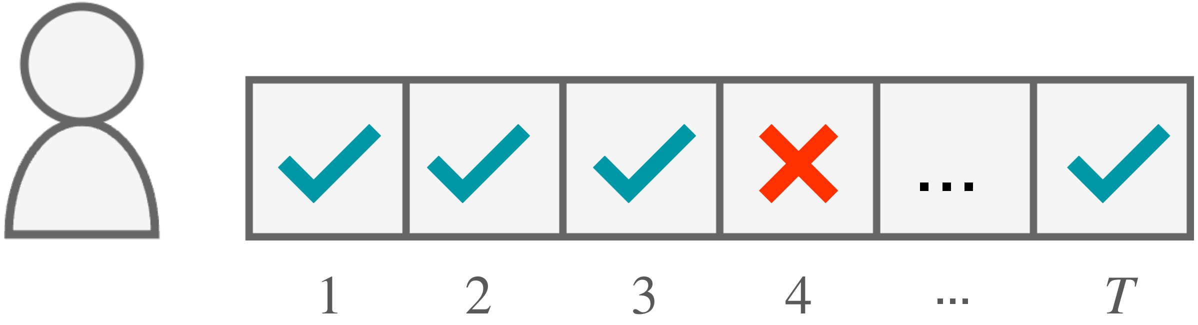
$$S = \{s_1; \ldots; s_n\} =$$



Acceptance Array

For each player x_i in the population $\mathcal{N}$ we define its **acceptance array** b_i s.t.:

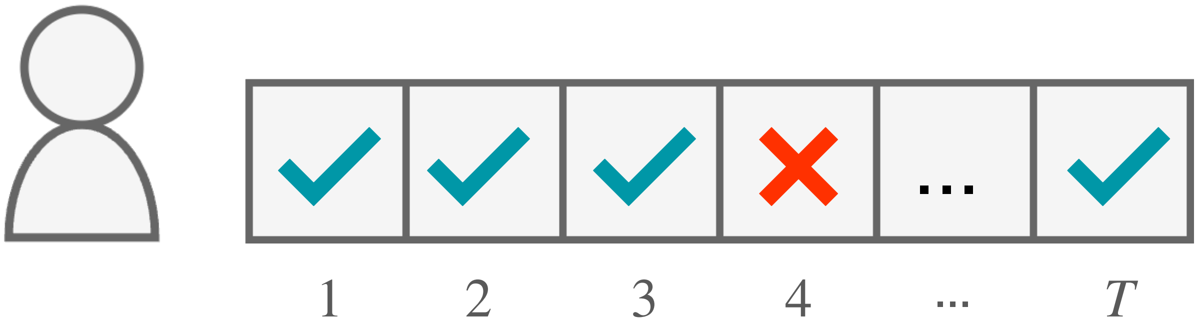
- $T = min(l, |N|)$: that is the maximum number of turns possible
- For each turn $j \in [1, T]$. $b_i[j]$ is a is **true** if player x_i would accept the potato at turn j, **false** otherwise



Constructing Acceptance Array

For **non-coalitional** players, we simulate **dummy game states** turn by turn and observe their behaviour.

This is feasible because their decisions depend solely on the current turn, the number of players in the chain, and their hyperparameters, **not on the actual game history or the specific players who took the potato prior.**



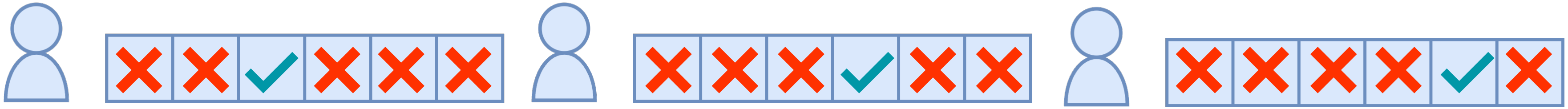
Constructing Acceptance Array

For **coalitional** players their acceptance arrays are constructed **collectively:**

1. We simulate turn-by-turn behaviour until the player decides to initiate a coalition.

2. Their acceptance array is finalised; the player is removed from the set, and a new coalition player is inserted **without resetting the current turn counter**.

3. The same procedure is repeated until all turns have been visited.



Algorithm for finding the longest chain

The algorithm works as follows:

1. We construct the acceptance array for all players in the population.

2. For each turn $i \in [1,T]$, we consider all players whose i-th entry in their acceptance array is set to **true**.

3. Among these candidates, we **greedily** select the player with the fewest total true entries, as postponing their selection may eliminate their only opportunity to participate.

If, at any turn, no such player is available, the algorithm terminates immediately, reporting the current computed chain.

Otherwise, the completed chain S is returned as a valid solution.

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Outline

Future Improvements

- Support the handling of multiple coalitions.
- Introduce another extension of the potato, whose lifetime is not fixed.
- Offering the possibility to export the result of an executed SHPG. This could be useful for applying analysis over the computed results.
- Specifying a specific benefactor for Direct Altruist players.
- Developing an algorithm for finding in a game the optimal hyper-parameter(s) of a player.
- Implement a GUI providing a better user experience.

References

- Paul Slovic Amos Tversky and Daniel Kahneman. The causes of preference reversal. American Economic Review 81:204-17, 1990.

- John Andreoni and John H. Miller. Rational cooperation in the finitely repeated prisoners-dilemma: Experimental evidence. The Economic Journal 103:570-85, 1993.

- John Andreoni and John H. Miller. Giving according to garp: An experimental test of the consistency of preferences for altruism. Econometrica 70:737-53, 2002.

- Robert J. Aumann. Backward induction and common knowledge of rationality. Games and Economic Behavior 8:6-19, 1995.

- Ronald S. Burt and Marc Knez. Kinds of third-party effects on trust. Rationality and Society 7:255-92, 1995.

References

- Carter T. Butts and David C. Rode. Rational and empirical play in the simple hot potato game. Social Forces Vol. 85 No. 4, 2007.
- Colin F. Camerer. Behavioral Game Theory. Experiments in Strategic Interaction. Princeton University Press, 2003.
- John Roberts David M Kreps Paul Milgrom and Robert Wilson. Rational cooperation in the finitely repeated prisoner's dilemma. Journal of Economic Theory 27:245-52, 1982.
- Robyn M. Dawes. Behavioral decision making and judgment. T. Daniel, S. T. Fiske and G. Lindzey, Eds., The Handbook of Social Psychology, Vol. 1-2, 4th Edition, 1996.
- Adriaan de Groot. Thought and Choice in Chess. Ishi Press, 1965.

References

- Ernst Fehr and Klaus Schmidt. A theory of fairness, competition, and cooperation. Quarterly Journal of Economics 114:817, 1999.
- Linda Thompson George Loewenstein and Max Bazerman. Social utility and decision making in interpersonal contexts. Journal of Personality and Social Psychology 57:426-41, 1989.
- David Grether and Charles R. Plott. Economic theory of choice and the preference reversal phenomenon. American Economic Review 69:623-38, 1979.
- Guillermina Jasso. A new representation of the just term in distributive-justice theory: Its properties and operation in theoretical derivation and empirical estimation. The Journal of Mathematical Sociology 12:251-74, 1986.

References

- Guillermina Jasso. How much injustice is there in the world? two new justice indexes. American Sociological Review 64:133, 1999.

- Philippe Jehiel. Limited foresight may force cooperation. Review of Economic Studies 68:369-91, 2001

- Philippe Jehiel. Bounded rationality and imperfect learning: Game theory vs ai. Transfer of Knowledge in Economic Decision Making, 2004.

- Alphons J. C. van de Kragt John M, Orbell and Robyn M. Dawes. Explaining discussion-induced cooperation. Journal of Personality and Social Psychology 54:811, 1988.

Thank you for your attention!

Recap

- It has been developed Hot Potato Theatre, a tool capable of defining, playing and analysing Simple Hot Potato Games (SHPG for short).
- It has been introduced a new kind of hot potato, the **Mutable potato**, that is an exchangeable good whose **gain** and **loss** values **change over time**.
- All the empirical players described have been implemented.
- An algorithm has been designed for finding a possible longest chain (if any exists) for any SHPG.