

Search for the rare fully leptonic decay

$$B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu \text{ at LHCb}$$

Slavomira Stefkova

High Energy Physics

Blackett Laboratory

Imperial College London

A thesis submitted to Imperial College London
for the degree of Doctor of Philosophy

Abstract

This thesis reports the branching fraction measurement of the rare Cabibbo-suppressed decay $\Lambda_b^0 \rightarrow p\pi^-\mu^+\mu^-$. The decay is observed for the first time with a 5.5σ deviation from the background-only hypothesis. This is the first observation of a $b \rightarrow d$ quark transition in the baryon sector. The dataset used for the measurement corresponds to 3 fb^{-1} of pp collisions collected at the LHCb experiment at CERN. The branching fraction is measured using $\Lambda_b^0 \rightarrow J/\psi (\rightarrow \mu^+\mu^-)p\pi^-$ as a normalisation channel and is measured as

$$\mathcal{B}(\Lambda_b^0 \rightarrow p\pi^-\mu^+\mu^-) = (6.9 \pm 1.9 \pm 1.1_{-1.0}^{+1.3}) \times 10^{-8},$$

where the first error is the statistical uncertainty, the second is the systematic uncertainty and the third is the uncertainty on $\mathcal{B}(\Lambda_b^0 \rightarrow J/\psi p\pi^-)$. The measurement of $\mathcal{B}(\Lambda_b^0 \rightarrow p\pi^-\mu^+\mu^-)$ can be combined with the branching fraction measurement for $\Lambda_b^0 \rightarrow pK^-\mu^+\mu^-$ to give constraints on the ratio of CKM matrix elements $|\frac{V_{td}}{V_{ts}}|$. Such a determination of $|\frac{V_{td}}{V_{ts}}|$ requires a theory prediction for the ratio of the relevant form factors.

This thesis also reports the ratio of tracking efficiencies, ϵ_{rel} , between data and simulation for $K_s^0 \rightarrow \pi^+\pi^-$ decays occurring within the LHCb detector acceptance. As K_s^0 particles are long-lived, their associated tracking efficiencies are less precisely determined compared to those of shorter-lived particles. The average value of ϵ_{rel} for $K_s^0 \rightarrow \pi^+\pi^-$ decays, where the K_s^0 has a flight distance of $\gtrsim 1 \text{ m}$, is found to be

$$\epsilon_{\text{rel}} = 0.70 \pm 0.02.$$

To perform this calibration measurement a novel technique was developed which has the potential to be used in measuring the value of ϵ_{rel} for other decays involving

long-lived particles.

Contents

Declaration of originality	6
Copyright declaration	7
Acknowledgements	8
List of Figures	10
List of Tables	15
List of abbreviations and definitions	16
1 Introduction	20
2 The LHCb detector	21
2.1 LHCb Layout	22
2.2 VERtex LOCator	26
2.3 Tracking System	28
2.3.1 Tracking Algorithms	29
2.4 Ring Imaging Detectors	31
2.5 RICH Reconstruction and Performance	33
2.5.1 RICH performance	33
2.6 Calorimetry	35
2.7 Muon Stations	36

2.7.1	Muon Identification	37
2.7.2	Muon Performance	39
2.8	Trigger	40
2.9	Simulation	44
2.9.1	Differences in Simulation And Data	45
3	Handling of trimuon correlations at LHCb	47
3.1	Muon PID variables	47
3.1.1	Binary Muon PID variables	47
3.1.2	Muon PID variables based on sharing hits	48
3.1.3	Muon PID variables based on regression techniques	51
3.2	Clones	52
3.3	Probability of $K/\pi \rightarrow \mu$ misidentification at LHCb	55
3.3.1	Specific control sample for $K/\pi \rightarrow \mu$ misid rates	56
3.3.2	Selection for $B^0 \rightarrow J/\psi(\rightarrow \mu^+ \mu^-)K^*$	56
3.3.3	Fitting Strategy for $B^0 \rightarrow J/\psi(\rightarrow \mu^+ \mu^-)K^*$ decay	56
4	Discovering (Setting Limit for) $B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu$ at LHCb	59
4.1	Topology of at LHCb $B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu$ at LHCb	59
4.2	Sources of Backgrounds	62
4.3	Analysis strategy	63
4.4	Samples	63
4.4.1	Data Samples	63
4.4.2	Simulation Samples	64
4.5	Preselection for $B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu$	66
4.6	Trigger Selection	67
4.7	q^2 selection	69
4.8	Further Selection	70
	Bibliography	71

Appendices	76
A Boosted Decision Trees	76
B The <i>sPlot</i> technique	79
C Fitting functions	81
C.1 Crystal Ball function	81
C.2 Double-sided Ipatia function	82
C.3 Rookeys function from R00FIT package	82

Declaration of originality

The work presented in this thesis is the result of collaborative work between members of the LHCb collaboration and myself. All the analysis work (chapters ??–??) presented in this thesis was performed by myself, with the exception of producing the simulation and data samples used in the analysis in ??. All work and plots presented in this thesis that were not the product of my own work are appropriately referenced.

This thesis has not been submitted for any other qualification.

Eluned Smith

September 2016

Copyright Declaration

The copyright of this thesis rests with the author and is made available under a Creative Commons Attribution Non-Commercial No Derivatives licence. Researchers are free to copy, distribute or transmit the thesis on the condition that they attribute it, that they do not use it for commercial purposes and that they do not alter, transform or build upon it. For any reuse or redistribution, researchers must make clear to others the license terms of this work.

Acknowledgements

Firstly, I would like to thank my supervisor Ulrik Egede. I would like to thank you for all the support, time and article corrections that you have invested in me. You have given me the opportunity to grow as physicist as well as you taught me to appreciate solving problems differently. You have given me the freedom to follow many different activities such as conferences, hardware, as well as detector maintenance all leading to meeting many great people that I am happy to call my friends.

To you Patrick Owen, I owe a big thank you for showing me all the support throughout my entire analysis. Without your great insight and motivation that you have kept even when you changed institute, it would have been completely different journey.

I would also like to express special thanks to Mike McCainn, whose patience knows no limits, but apparently LHC's is 15 minutes. I learnt how handle many power-cycling buttons as well as croquet. Here I would also like to extend my gratitude to many of the RICH collaborators such as Antonis, Silvia, Roberta whom I have met while working with the upgrade.

And ofcourse all the Imperial crew: Dave, Andrei, Mitesh, Paula, Eluned, Will, Fede, Sophie, Matt, Felix and Malte, who all have their part in this work.

None of this would be possible without the STFC, the financial body on behalf of British government, who financially supported my PhD and the LHCb collaboration as a whole.

Lastly but most importantly, I would like to thank my family. I have never heard from you that something cannot be done. You have taught me to be curious, to be independent, to be strong. We have sacrificed the most precious commodity, time spent

together, in order for me to follow my interests. And below is just one example it was all worth it.

List of Figures

2.1	Accelerator complex at CERN. The image is taken from [1].	22
2.2	Angular production and acceptance of LHCb's $b\bar{b}$ pair (in red) as well as General Purpose Detector (in yellow). LHCb covers region with highest production cross-section at 8 TeV. These plots were produced using PYTHIA8 [2] simulation. This plot was taken from [3] (left). Probability of interaction per bunch crossing as a function of instantaneous luminosity. This figure was obtained from [4] (right).	23
2.3	The schematic slice of LHCb detector in y, z plane where z is defined to be the direction parallel to beamline, and x, y define the plane perpendicular to the beamline. θ , the opening angle in $y-z$ plane with $\theta = 0$ along z -axis. The figure was taken from [7].	24
2.4	Integrated luminosity collected in different years of data-taking. This plot is taken from [8] (left). Development of the instantaneous luminosity for ATLAS, CMS and LHCb during LHC fill 2651. After ramping to the desired value of $4 \times 10^{32} cm^{-2}s^{-1}$ for LHCb, the luminosity is kept stable in a range of 5% for about 15 hours by adjusting the transversal beam overlap. The difference in luminosity towards the end of the fill between ATLAS, CMS and LHCb is due to the difference in the final focusing at the collision points, commonly referred to as the beta function, β^* . This plot was obtained from [9] (right).	26

2.5	Schematic plot of VELO detector configuration along the beam pipe showing the layout as well as positions while in stable beams (discs have slight overlap) and injection. Figure taken from [10].	27
2.6	Two key variables which quantify performance of the VELO detector. IP resolution which is worse for low momentum tracks (left) and PV resolution dependent on the number of tracks forming the primary vertex N (right). Figures taken from [11].	28
2.7	Visualisation of use of different technology with silicon technology in violet and straw-tube technology in cyan. The Figure was obtained in [12](left). Track types visualisation depending on which track stations provided hits. For the study of $B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu$ decays only, Long track , are considered as muons will travel to the end of the detector leaving the hits all along. Figure is taken from [13] (right).	29
2.8	Momentum resolution of long tracks measured using "tag and probe" method at LHCb. The decay channel $J/\psi^+ \rightarrow \mu^+ \mu^-$ is analysed.	31
2.9	Separation power for different species of particles in momentum-Cerenkov angle plane (left). Schematic diagram of RICH1 layout (right). Both figures are taken from [10].	32
2.10	Invariant mass distribution of D^0 data sample (in black) overlaid with fit to both background and signal (in blue) (left). An example of kaon ID (red) and MisID (black) efficiency as a function of momentum under two PID hypotheses, $DLL_K > 0$ (empty) and $DLL_K > 5$ (filled) (right). Both figures are taken from [16].	34
2.11	$\pi^+ \pi^-$ invariant mass distributions obtained using kinematic constraints only (left) and also using PID constraints (right) in order to isolate $B^0 \rightarrow \pi^+ \pi^-$ peak. This figure is taken from [17].	35
2.12	Granularity of ECAL (left) and HCAL (right) detectors. The figure was taken from [10].	36

2.13	(a) Layout of the muon detector x-z plane and (b) x-y plane. This figure is taken from [18].	38
2.14	(a) Probability of correctly identifying muons as a function of momentum p in the bins of p_T for $J/\psi \rightarrow \mu^+ \mu^-$ with IsMuon constraint. (c) Probability of incorrectly identifying pion (b) proton and (d) kaon as muon with IsMuon. This figure is taken from [19].	40
2.15	TOS efficiency as a function of p_T for muon-based decisions (left). TOS efficiency for different decays using L0 hadron trigger lines. This figure is taken from [18].	41
2.16	HLT1 efficiencies of the corresponding triggers using the same proxy as in Figure 2.15. This figure is taken from [20].	42
2.17	Trigger scheme differences between Run I and Run II. Figures obtained from [21]	44
2.18	IP resolution in x-direction comparing the data and simulation output for 2012 data-taking period (left). IP resolution in x-direction comparing the data and simulation output for 2011 data-taking period as a function of angle, ϕ (right). This figures are taken from [11].	46
3.1	Schematic view of the muon station slicing into x-y pads. This is left quadrant of M1 station, showing decreasing granularity of the muon stations away from the beam. This figure has been taken from [18]. M1R1 is the innermost region and M1R4 is the outermost region of the M1 station.	49
3.2	(a) The nShared variable in simulation, (b) data for $B^+ \rightarrow (J/\psi \rightarrow \mu^+ \mu^-) K^+$ in different stripping versions corresponding to 2012 (<i>Stripping 21</i>), 2011 (<i>Stripping 21r1</i>), 2016 (<i>Stripping 26</i>) data-taking. There is shift of distribution in <i>Stripping 26</i> towards less isolated tracks.	50

3.3	ID and misID probabilities from standard calibration datasets from 2012 (<i>Stripping 21</i>) 2016 (<i>Stripping 26</i>), binned using default 2-dimensional binning scheme in momentum p and pseudorapidity η . In this plot, ID and misID rate in central bin of η and first and second bin in p are compared. This demonstrates that for same ID efficiency, the misID rate is significantly higher in 2016.	51
3.4	(a) Visible and (b) corrected mass of B , shows a clear peak coming from clones in 2012 same sign data sample.	52
3.5	(a) Clone and (b) no clones flight distance properties. It can be seen that <i>clone</i> tracks have their decay vertex placed at the end of the detector, whereas regular good tracks will decay within VELO	53
3.6	The occupancy difference in the OT detector between (a) clones and (b) real tracks in the OT at the distance 9450 mm along the LHCb . <i>Clones</i> are concentrated along the inner edge of the OT	54
3.7	(a) Clone and (b) no clone distribution for $\cos(\theta_B)$	54
3.8	(a) Clone and (b) no clone distribution for sum of all muon n_{Shared} . Since in this case the clones are of each other, for clone there is clear peak at three.	55
4.1	Schematic view of $B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu$ decay. At pp interaction point, or PV , $b\bar{b}$ pair hadronizes into B^\pm . B^\pm flies some distance before decaying into three muons and neutrino. All charged tracks (in filled-blue) seen can be combined into four-vector representing the visible part of the decay (semi filled-blue). Information about invisible neutrino (semi filled-red) are deduced from the conservation of momentum with respect to the direction of the flight of B^\pm . Neglecting momentum component parallel to the direction of flight for neutrino, transverse component of momentum is given.	60

4.2	Distributions for signal MC in using Pythia 6.4 [23] conditions. (a) $K^*(\mu^+, \nu_\mu)$ (b) $q(\mu^+, \mu^-)$ distributions under different K^* mass hypotheses. The most flat distribution in $K^*(\mu^+, \nu_\mu)$ is plotted in yellow.	65
4.3	(a) Signal simulation sample distribution in <i>minq</i> and <i>maxq</i> variables. Values below 980 MeV/c ² (red line) are accepted. (b) Combinatorial data sample after <i>stripping</i> selection with no other cuts shows clearly the J/ψ (green) and $\Psi(2S)$ (blue) resonances which are vetoed and the measurement region (red).	70
A.1	An example decision tree. The S and B stand for ‘Signal-like’ and ‘Background-like’. The β_i variables refer to the cut values chosen by the machine learning algorithm after the tree has been trained on signal and background samples. The blue ovals represent final nodes called leafs, which each leaf having an associated purity, i.e. the fraction of the weight of a leaf due to signal events.	77

List of Tables

2.1	Running conditions of LHC and LHCb in different years of data-taking. The statistics of LHCb's instantaneous luminosity is extracted using run database.	25
2.2	Momentum-dependent definition IsMuon variable.	39
3.1	Offline selection for $B^0 \rightarrow J/\psi(\rightarrow \mu^+\mu^-)K^*$ decay.	57
4.1	Summary of signal simulation samples used in this analysis with different decay models. In all cases the daughters of B^\pm are required to be within LHCb acceptance. All of this samples are mixture under magnetic polarity up and magnetic polarity down conditions.	64
4.2	Selection of events based on muon and the B^+ candidate requirements. <i>Stripping selection</i> for the signal decay $B^+ \rightarrow \mu^+\mu^-\mu^+\nu_\mu$ is the same for both Run1 and 2016 data.	67
4.3	Trigger selection applied on both signal and normalisation samples . .	68
4.4	Veto for J/ψ and $\Psi(2S)$ contributions.	69
4.5	Offline selection performed after stripping. Differences can be seen between Run I and Run II datasets	70

List of abbreviations and definitions

$IP\chi^2$ The $IP\chi^2$ is the difference in the χ^2 of the fit to the primary vertex, when the track whose $IP\chi^2$ is being measured is added and then removed.

ALICE A Large Ion Collider Experiment.

ATLAS A Toroidal LHC ApparatuS.

BDT Boosted Decision Tree, a BDT employs multivariate analysis techniques to combine a set of weakly discriminating variables into a single discriminating variable.

CB Crystal Ball function.

CMS Compact Muon Solenoid.

$\text{Cos}(\theta_B)$ The cosine of the angle between the momentum vector of the B^+ meson and the direction of the flight of the B^+ meson from its primary vertex to its secondary vertex .

ECAL Electromagnetic calorimeter.

FD Flight Distance, how far a particle flies before decaying.

$FD\chi^2$ The $FD\chi^2$ is defined as the increase in χ^2 when the primary and secondary vertex are fitted separately as compared to single vertex fit.

FOI Field of Interest.

HCAL hadronic calorimeter.

HLT High Level Trigger. The HLT is the software trigger which is applied after the **L0** trigger.

HLT1 First stage of high level trigger.

HLT2 Second stage of high level trigger.

HPD Photomultiplier tubes that collect Cerenkov light.

ID Probability of correctly identifying particle, given PID requirement.

IP Impact Parameter. The IP is defined as the distance between a track and the **PV** at the track's closest point of approach.

IT Inner trackers, the inner section of the T stations.

L0 Level-0 trigger. The L0 is the first trigger to be applied and uses hardware to make decisions on events.

LHCb The Large Hadron Collider beauty experiment.

long track Long track is track category which classifies tracks that have hits in the VELO and the T stations. Hits in the TT stations are optional.

LS1 Long Shutdown 1.

M1-5 The five muon stations.

MC Monte Carlo Simulation.

Min $IP\chi^2$ The minimum impact parameter χ^2 is the minimal difference in fit χ^2 (quality of the fit) to the primary vertex between fit with this track added and removed.

misID Probability of incorrectly identifying particle given PID requirement.

MWPCs multi-wire proportional chambers.

OT Outer trackers, the outer section of the T stations.

P_{ghost} Ghost Probability is probability of misreconstruction of the track, where for each track 0 is most signal-like and 1 is most ghost-like. A charged particle is not considered to be a ghost if 70% of the hits match between the reconstructed and simulated true tracks. Similarly, neutral particles are ghosts if simulated particle contributes less than 50% of the reconstructed cluster energy from calorimeter.

PID Particle IDentification.

PRS pre-shower.

PV Primary Vertex, the pp interaction vertex.

QED Quantum Electrodynamics.

RICH Ring Imaging Cherenkov detectors, provide particle identification by using Cherenkov radiation.

SM Standard Model.

SPD Scintillator Pad Detectors.

SV Secondary Vertex.

T1, T2 and T3 Trackers downstream of the magnet composed of silicon micro-strips strips in the inner section and straw tubes in the outer section..

TIS Events which are Triggered Independent of Signal.

TISTOS Events which require both the presence of signal and the rest of the event to fire the trigger.

TOS Events which are Triggered On Signal.

Track χ^2/ndof The track χ^2 per degree of freedom is the minimal difference in fit χ^2 (quality of the fit) to the primary vertex between fit with this track added and removed.

TT The tracking station upstream of the magnet composed of silicon micro-strips..

VELO VERtEX LOcator. Subdetector of LHCb, placed around the pp interaction point, used to realise the precise measurements of vertices and tracks.

Vertex χ^2/ndof The vertex χ^2 per degree of freedom in a vertex fit.

Chapter 1

Introduction

The Standard Model ([SM](#)) is an effective theory which describes fundamental particles and their interactions to an impressive precision.

bla

Chapter 2

The LHCb detector

In this section, overview of accelerator complex at CERN as well as physics motivation behind [LHCb](#) detector and its details will be described.

CERN built one of the most exciting laboratories to study elementary particle interactions. The complex set of particle accelerators and detectors is shown in [Figure 2.1](#). The process of accelerating protons starts with the source of protons. Protons are obtained from hydrogen gas bottle by applying and an electric field separates hydrogen into positively and negatively charged constituents. The first proton accelerator in the chain, Linac 2, accelerates the protons to the energy of 50 MeV. Is is a tank composed of several chambers where the resonant cavity is tuned to a specific frequency which creates potential differences in the cavities making accelerate the protons. These are then injected into the Proton Synchrotron Booster (PSB). Here the protons are accelerated to 1.4 GeV. The next line is the Proton Synchrotron (PS) reaching energy of 25 GeV. Before either entering the LHC or North Area (mainly used as testing facility for experiment upgrades) Super Proton Synchrotron (SPS) is the last stop. Here proton acceleration to 450 GeV is achieved.

The Large Hadron Collider (LHC) is a complex machine which accelerates beams of protons in opposite directions in

~

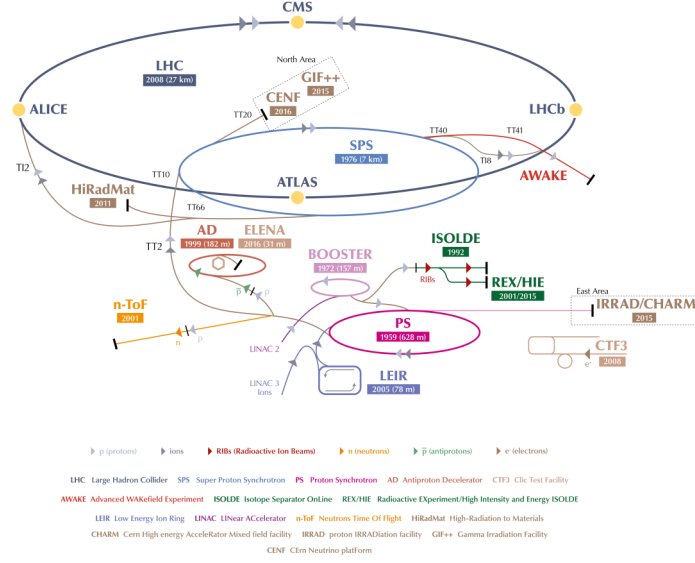


Figure 2.1: Accelerator complex at CERN. The image is taken from [1].

27km circular tunnel. It is located 50-157 m below ground on the border of Switzerland and France. Once the desired energy is achieved proton-proton, pp , or ion, collisions happen at four distinct points, where different detectors with different physics focus are located. These are **ATLAS**, **CMS**, **ALICE** and **LHCb**. Study of $B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu$ was performed using data obtained at **LHCb**.

2.1 LHCb Layout

LHCb differs from the other general purpose detectors on the LHC ring as its studies properties of heavy particles containing b or c quarks. This can be attributed to the geometrical acceptance and unique vertex resolution as well as excellent **PID**.

Contrary to the two general purpose detectors where the collisions are occurring in the centre of the detector, **LHCb**'s collision point is located at one end of the detector, hence its description as a forward single-arm spectrometer. This means that information about products outside of its scope are not known, meaning that there is no overall

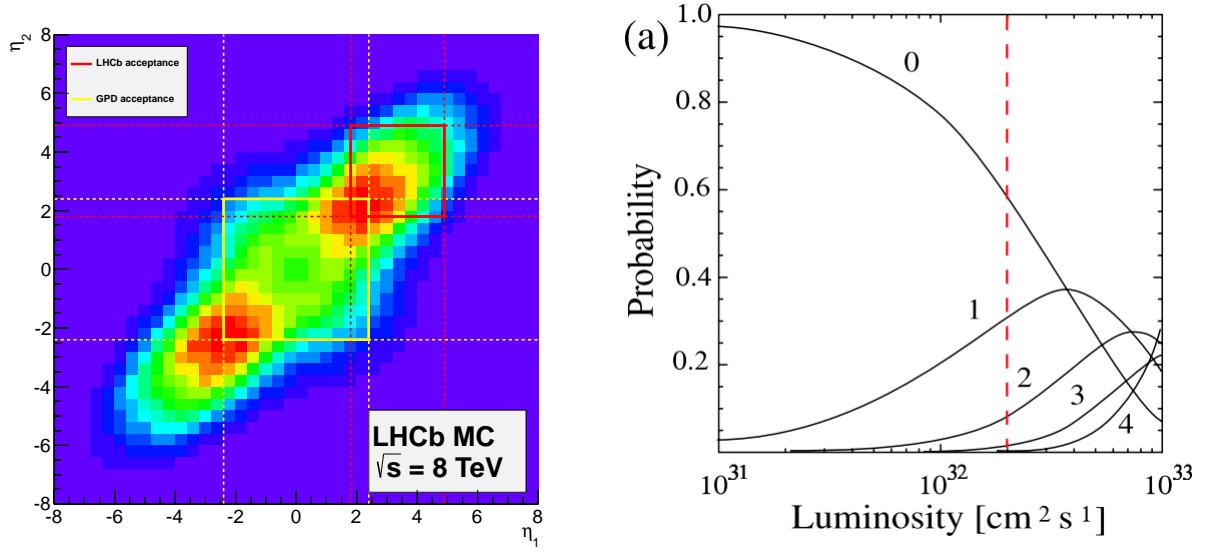


Figure 2.2: Angular production and acceptance of LHCb's $b\bar{b}$ pair (in red) as well as General Purpose Detector (in yellow). LHCb covers region with highest production cross-section at 8 TeV. These plots were produced using PYTHIA8 [2] simulation. This plot was taken from [3] (left). Probability of interaction per bunch crossing as a function of instantaneous luminosity. This figure was obtained from [4] (right).

constraint on collision information, unlike in other flavour experiments. This is compensated by production mechanism of $b\bar{b}$ and $c\bar{c}$ in pp interactions, which occurs via gluon-gluon fusion. In this process, each gluon will carry part of proton's momentum. If the two gluons from two protons carry significantly different momentum, the $b\bar{b}$ system will be boosted with respect to the pp rest frame, either in forward or backward cone closely to the beamline, as can be seen in Figure 2.2.

The angular coverage of LHCb is formally defined using pseudorapidity η ,

$$\eta = -\ln\left(\tan\frac{\theta}{2}\right) \quad (2.1)$$

where θ is defined in Figure 2.3. LHCb detector, hence, covers the region $2 < \eta < 5$. The production cross-section of the fundamental process of $pp \rightarrow b\bar{b}X$ was measured in this region yielding, $\sigma(pp \rightarrow b\bar{b}X) = 75.3 \pm 5.4 \pm 13.0 \text{ } \mu\text{bb}$ at 7 TeV [5] and $144 \pm 1 \pm 21 \text{ } \mu\text{b}$ at 13

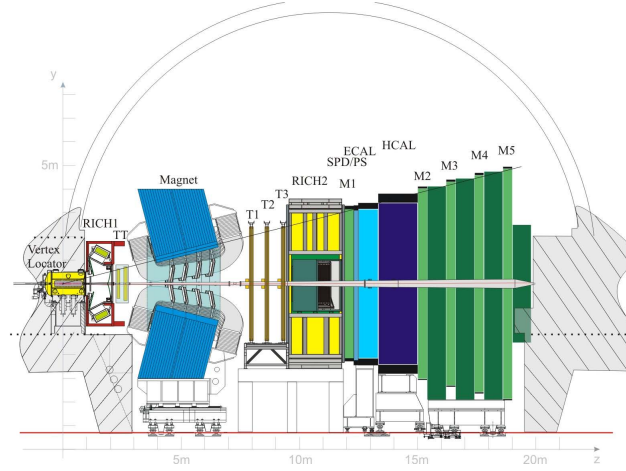


Figure 2.3: The schematic slice of LHCb detector in y, z plane where z is defined to be the direction parallel to beamline, and x, y define the plane perpendicular to the beamline. θ , the opening angle in $y-z$ plane with $\theta = 0$ along z – axis. The figure was taken from [7].

TeV [6], which shows that the production cross-sections scales roughly linearly with the centre-of-mass energy. Assuming design conditions of LHCb, seen in Table 2.1, 2 fb^{-1} of data would correspond to 10^{12} of $b\bar{b}$ pairs being produced.

Despite such impressive statistics of $b\bar{b}$ pairs available to LHCb, the bottleneck arises in much more copious inelastic background. It mostly originates from soft QCD processes which are related to the amount of pile-up, the visible number of pp interaction in the visible events. By looking at the probability of number of pp interaction per bunch crossing as a function of luminosity, shown in Figure 2.2, it can be noted that the maximum probability for only one pp interaction (and hence minimizing the background) is found to be at $\sim 2 \times 10^{32} \text{ cm}^{-2} \text{ s}^{-1}$, hence the LHCb design luminosity. In addition, to keep the occupancy of the detector reasonable, global event cuts, GECs, are put in the place, where only events with 600 (in 7,8 TeV) and 450 (in 13 TeV) tracks and less are allowed to be processed.

As LHCb requires much lower luminosity compared to other LHC detectors, it is achieved by LHCb-specific control of luminosity known as *luminosity levelling*. This

procedure achieves stable instantaneous luminosity by controlling that the two beams do not collide straight head-on at collision point, but are moved with respect to each other. It limits the effects of luminosity decay, which can lead to trigger alterations during specific data taking run, resulting in systematic uncertainties.

So far, the detector has been running since 2010, with collected integrated luminosity shown in [Figure 2.4](#). As compared to [ATLAS](#) and [CMS](#) the integrated luminosity is much lower, due to allowed pile-up conditions. In 2017, there were two pp collision energies at which the data was taken: at $\sqrt{s} = 13$ and 5 TeV. Run I data-taking (2010-2012) was paused by Long Shutdown 1 ([LS1](#)) and followed with Run II data-taking (2015-2018). The summary of LHCb running conditions is provided in [Table 2.1](#), showing the evolution of the instantaneous luminosity as well as the frequency of collisions compared to the design proposal.

year	\sqrt{s} [TeV]	Instantaneous Luminosity \mathcal{L} [$\times 10^{32} cm^{-2} s^{-1}$]
Design	Up to 14	2
2011	7	$\sim 3.0-3.5$
2012	8	~ 4.0
2015	13	$\sim 0.5-4.5$
2016	13	~ 4.0
2017	13	$\sim 4.0-6.0$

Table 2.1: Running conditions of LHC and [LHCb](#) in different years of data-taking. The statistics of LHCb’s instantaneous luminosity is extracted using run database.

In the following sections, brief discussion of different subdetectors is presented. Both hardware and software overview will be presented with particular emphasis given to Muon Station and Simulation of LHCb.

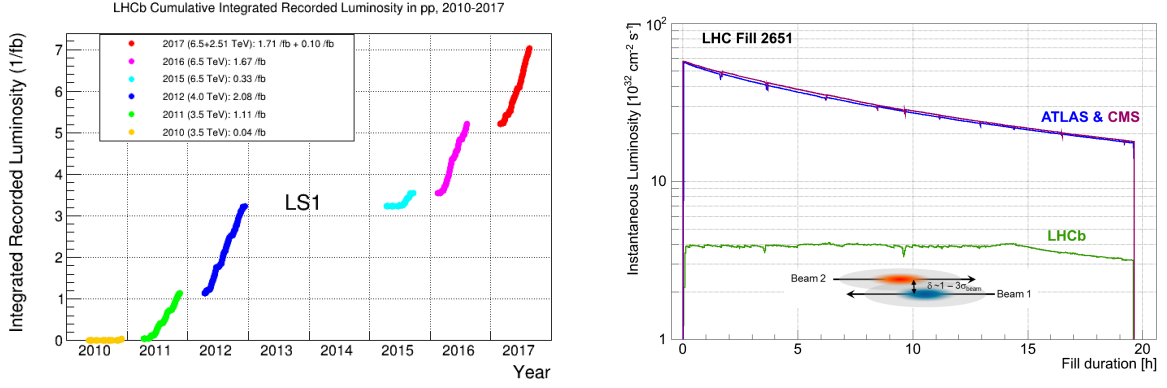


Figure 2.4: Integrated luminosity collected in different years of data-taking. This plot is taken from [8] (left). Development of the instantaneous luminosity for ATLAS, CMS and LHCb during LHC fill 2651. After ramping to the desired value of $4 \times 10^{32} \text{ cm}^{-2} \text{ s}^{-1}$ for LHCb, the luminosity is kept stable in a range of 5% for about 15 hours by adjusting the transversal beam overlap. The difference in luminosity towards the end of the fill between ATLAS, CMS and LHCb is due to the difference in the final focusing at the collision points, commonly referred to as the beta function, β^* . This plot was obtained from [9] (right).

2.2 VErteX LOcator

The closest detector around the collision point is VErteX LOcator (VELO). This silicon-strip based detector, that extends 1 m along the beam axis, is primarily used for to distinguish events from prompt background. The typical differing property of a B hadron decay includes large impact parameter (IP), the minimal distance between the track and primary vertex, in addition to significantly higher transverse momentum p_T . Main uses of this subdetector include finding:

- primary vertices positions
- secondary vertices of short-lived particles (heavy quark hadrons)
- tracks that did NOT originate from primary vertex

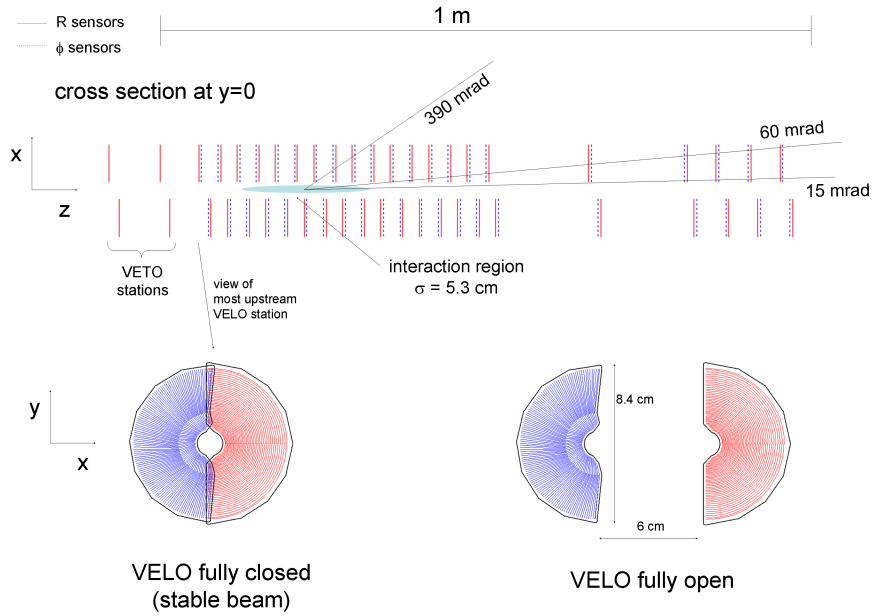


Figure 2.5: Schematic plot of **VELO** detector configuration along the beam pipe showing the layout as well as positions while in stable beams (discs have slight overlap) and injection. Figure taken from [10].

The detector consists of two sets of 21 silicon modules positioned around the beam pipe, where each module has 2 types of half-moon-shaped discs as seen in Figure 2.5. In the first disc type the strips are arranged to provide radial information (R), whereas the second type provides azimuthal (ϕ) information. As pp interaction point brings high radiation dose for this detector, the first sensitive strip starts only at a distance of 8 mm once stable beams are declared. Throughout the beam injection, when the beam radius may be larger, the two sets are moved away 3 cm perpendicularly from the interaction point. For the (R) sensor, the individual module's strip pitch, distance between two strips, varies from $38 \mu\text{m}$ to $102 \mu\text{m}$ away from the beam pipe, so that the hit occupancy is roughly even. Each **VELO** half is kept within an aluminium welded box causing material overlap once stable beams are declared. These boxes then create their own vacuum which is different to the nominal LHC vacuum in order to protect

the detector from any electromagnetic interference with the beam.

This setup brings outstanding hit resolution (4-40 μm), which in turn allows for very high impact parameter resolution and very good primary vertex resolution, as seen in Figure 2.6. This is indispensable not only in order to perform the precise measurements of B and D lifetimes, but also to resolve oscillations caused by $B_s^0 - \bar{B}_s^0$ mixing occurring at 3 trillion Hz rate.

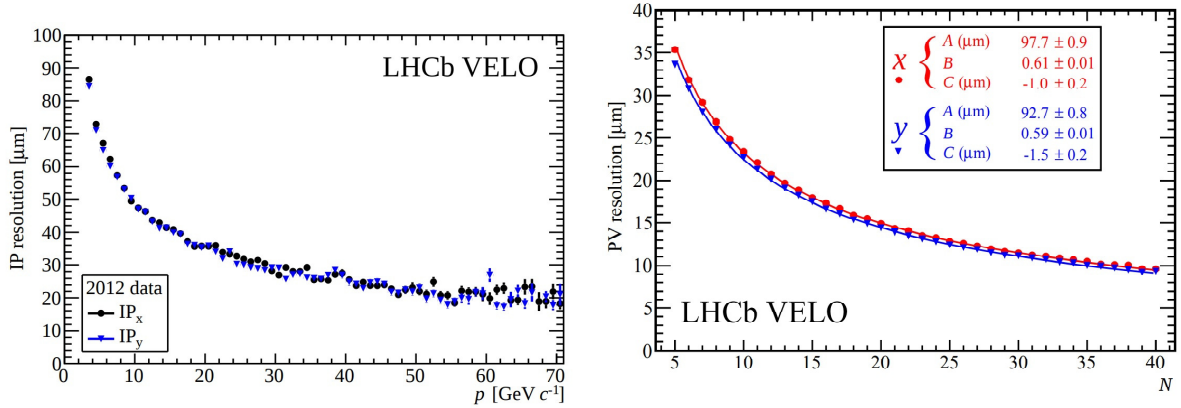


Figure 2.6: Two key variables which quantify performance of the **VELO** detector. **IP** resolution which is worse for low momentum tracks (left) and **PV** resolution dependent on the number of tracks forming the primary vertex N (right). Figures taken from [11].

2.3 Tracking System

In addition to tracking information provided by **VELO**, the trajectories of charged particles are monitored by series of tracking subdetectors. The main task of these tracking subdetectors is to provide efficient reconstruction and precise measurement of particle's momentum. There are four tracking stations apart from **VELO**: Tracker Turicensis (**TT**), positioned upstream from magnet, and **T1**, **T2** and **T3** tracking stations on the other side from the magnet. The 10 m dipole magnet with ≈ 4 Tm integrated field provides enough strength to bend charged particles with p of 200 GeV/c^2 .

Two different detection technologies are used in these trackers reflecting the nature

of track occupancy as function of distance from beam pipe. The tracker's part close to the beam pipe, **TT** station together with central region of T1-T3, also known as Inner Tracker (**IT**), expects higher occupancy and makes use of the silicon microstrip detection mechanism. The outer part of **T1**, **T2** and **T3** stations, also known as Outer Tracker **OT**, is made of a straw-tube detectors. It measures the hit position by measuring the drift-time of ionized electrons. This two technologies are seen in [Figure 2.7](#).

2.3.1 Tracking Algorithms

Different particles will leave different footprint in the detector. Charged particles will form tracks. Depending on presence of hits in individual subdetectors, they are grouped into several categories, visualized in [Figure 2.7](#).

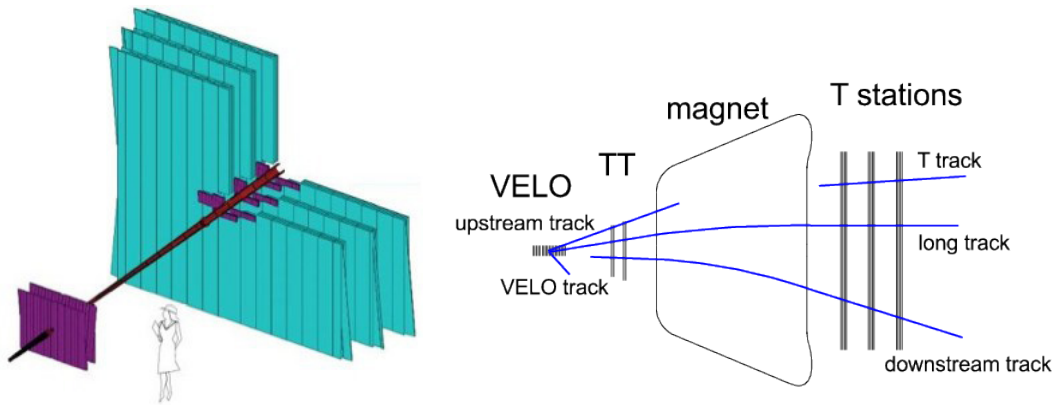


Figure 2.7: Visualisation of use of different technology with silicon technology in violet and straw-tube technology in cyan. The Figure was obtained in [\[12\]](#)(left). Track types visualisation depending on which track stations provided hits. For the study of $B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu$ decays only, **Long track**, are considered as muons will travel to the end of the detector leaving the hits all along. Figure is taken from [\[13\]](#) (right).

Most of the physics analyses use long tracks, tracks leaving hits in **VELO** and **T1**, **T2** and **T3**, as they give most precise momenta estimates. VELO tracks leave hits only in R and Φ sensor, but not in any other tracking stations, meaning that they must have left **LHCb** acceptance or they come from particles produced backwards and hence are

useful for PV reconstruction. Upstream tracks are formed by tracks leaving hits in **VELO** and **TT** only. These are usually low momentum particles, which are bent out **LHCb** acceptance while traversing the magnet. Long-lived particles such as Λ or K_s^0 will only decay outside of the **VELO** acceptance and hence will produce no hits until **TT** and **T1, T2 and T3** forming downstream tracks. T-track is track type that only have hits in **T1, T2 and T3**. Again this could be due to presence of long-lived particles or due to secondary interactions in the detector.

In general, the track reconstruction software starts with *pattern recognition*, where several hits in one part of a tracking subdetector are identified and form *track seeds*, which are then extrapolated and combined with hits in other tracking subdetector provided this subdetector sits in low magnetic field. The long track candidates are formed and fitted with a Kalman filter [14], where because of the material present in the detector, corrections for energy losses as well as multiple scattering are incorporated.

Sometimes *pattern recognition* may combine random hits into a track, *ghost track*, or several tracks could be made out of same hits, *clone track*. Presence of these tracks are heavily suppressed with different techniques - such as establishing ghost probability (P_{ghost})- variable based on the output of neural network combining track χ^2 and missing hits in the subdetectors.

Mass uncertainty is one of the crucial parameters to minimize as it provides opportunity for high precision measurements by better separations from backgrounds. It strongly correlates with momentum resolution that is obtained using tracking. Resulting relative momentum uncertainty (0.5-1.1%) on long tracks using $J/\psi \rightarrow \mu^+ \mu^-$ using *tag and probe* can be seen in [Figure 2.8](#). It varies logarithmically with increasing momentum.

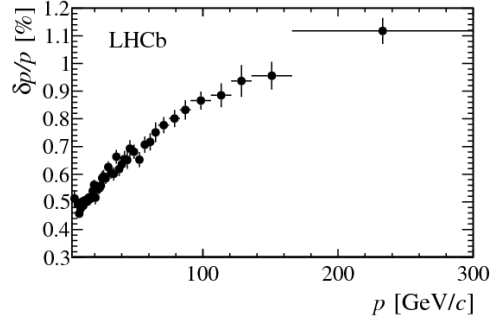


Figure 2.8: Momentum resolution of long tracks measured using "tag and probe" method at LHCb. The decay channel $J/\psi^+ \rightarrow \mu^+ \mu^-$ is analysed.

2.4 Ring Imaging Detectors

Particle identification, [PID](#), at [LHCb](#) relies heavily on two dedicated Ring Imaging subdetectors. These detectors take advantage of the phenomena, emission of Cerenkov light, which happens when a charged particle travels through a medium at a speed faster than the phase velocity of light in that medium. This cone of light is emitted at an angle θ with respect to the charged particle's trajectory. Using the knowledge of refractive index of the medium, n , and momentum p that is measured using tracking, mass m of the particle can be obtained through:

$$\cos \theta_c = \frac{\sqrt{m^2 + p^2}}{pn}. \quad (2.2)$$

As the momentum and mass are intrinsic properties of passing particle, the momentum identification range is limited by the choice of medium, also known as radiator. For very low-momentum particle, as $\cos \theta_c \rightarrow 1$, the particle is not producing any Cerenkov light cone. At the very high momentum, as $\cos \theta_c \rightarrow 0$, there is saturation point as all species of particles will emit the light at the same Cherenkov angle, hence all the

discriminating power will be lost.

Low momentum (2-60 GeV) particles are identified in the upstream RICH1 detector and high momentum particles (15-100) GeV are analyzed downstream in RICH2. RICH1 covers ± 25 -300 mrad in x-z plane, ± 250 mrad in the x-y plane, using either gaseous Aerogel ($n = 1.03$) and C_4F_{10} ($n = 1.0014$) as radiators. RICH2 has more limited acceptance of ± 15 -120 mrad in x-y plane and ± 100 mrad in x-z plane and uses CF_4 as radiator, with lower $n = 1.0005$. The discrimination power between different particles can be seen [Figure 2.9](#).

Both RICH1 and RICH2 use set of spherical primary mirrors to guide the photons onto the flat secondary mirrors which are then further focused into Cerenkov rings onto the surface of Hybrid Photon Multipliers, HPD. The schematic view of a particle passing through RICH1 can be also seen in [Figure 2.9](#).

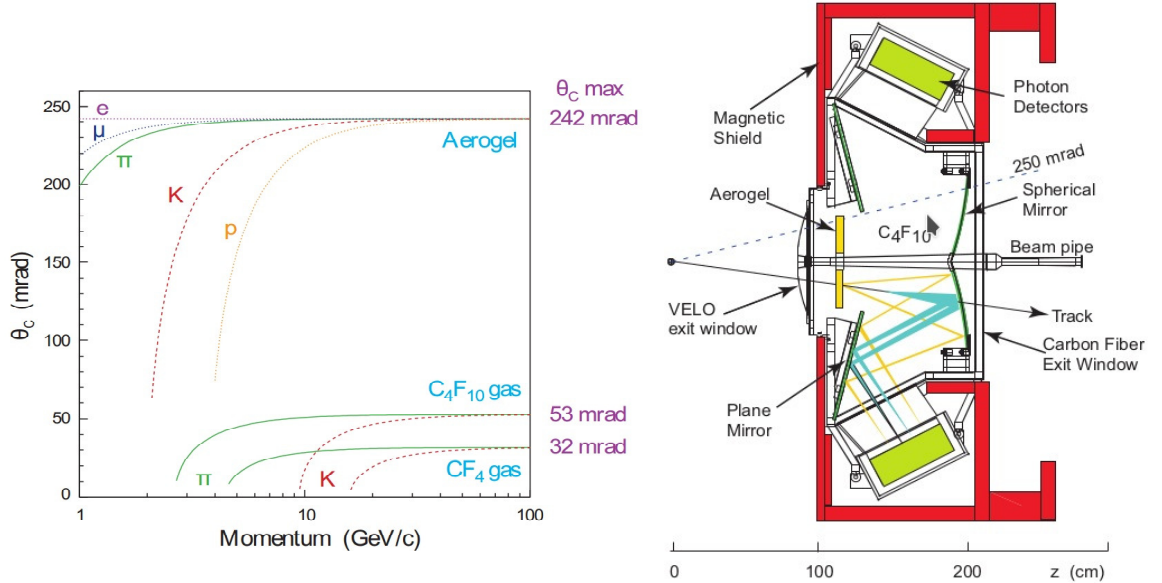


Figure 2.9: Separation power for different species of particles in momentum-Cerenkov angle plane (left). Schematic diagram of RICH1 layout (right). Both figures are taken from [\[10\]](#).

2.5 RICH Reconstruction and Performance

In order to establish species of particles for each track, the Cerenkov angle is combined with the track momentum measured by tracking. In practice, however, as RICH detectors operate in high track density environment, many Cerenkov rings will be overlapping and hence a complex pattern recognition algorithm is deployed [15].

For each event, the RICH computes full event likelihood that is consistent with assigning pion mass hypothesis for all tracks given the observed hit distribution read out by HPD. The algorithm then iterates through all other possible particle species, (e, μ, π, K , proton, deuteron), assigning new full event likelihood for a given track, having all other hypotheses fixed. The mass hypothesis with the highest full event likelihood is assigned to the track and this process is repeated for all the tracks in the event, until no improvement is found.

Results of this algorithm provide likelihood variables, DLL_x , that quantify the strength of the chosen species hypothesis against pion hypothesis,

$$DLL_x = \log(\mathcal{L})_x - \log(\mathcal{L})_\pi \quad x \in e, \mu, K, \text{proton}, \text{deuteron}. \quad (2.3)$$

By calculating $DLL_{x_1} - DLL_{x_2}$, one can obtain discriminative strength between any two species.

2.5.1 RICH performance

In order to measure the performance of the PID computed by RICH, populous calibration samples with very little background contamination are required. In order not to bias results, these samples have no PID constraints themselves and are reconstructed solely using kinematic information. For studies of pion/kaon efficiencies, $D^{*+} \rightarrow D^0(K^-\pi^+)\pi^+$ background-subtracted samples are used, whereby the daughter tracks of D^0 become proxies for evaluation. The probability of correctly identifying kaon given certain constraint on DLL_K , identification efficiency (ID), and probability of mistakenly swapping pion identification, MisID efficiency, are summarized in

Figure 2.10. Identification probabilities of $\approx 85\%$ with misidentification rate of $\approx 3\%$ provide invaluable discriminating separation between kaon and pion.

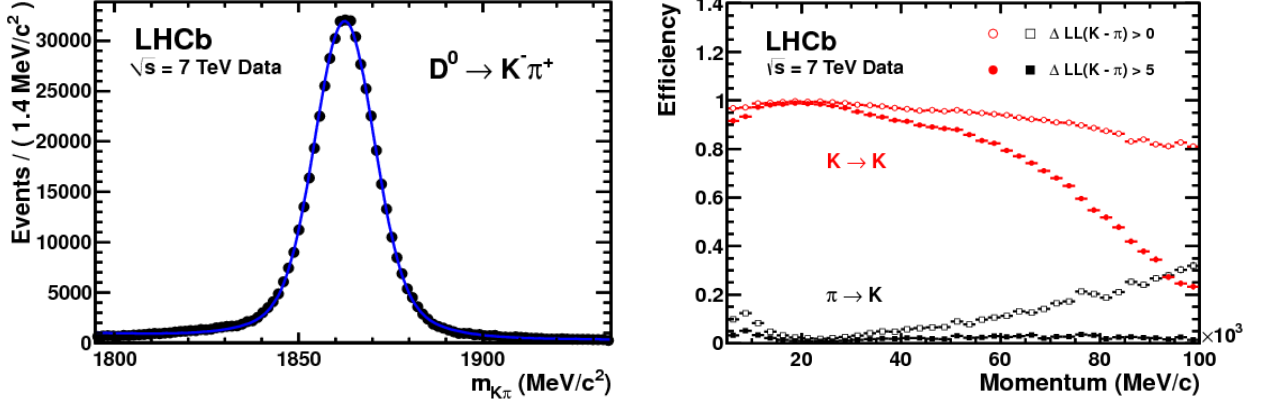


Figure 2.10: Invariant mass distribution of D^0 data sample (in black) overlaid with fit to both background and signal (in blue) (left). An example of kaon ID (red) and MisID (black) efficiency as a function of momentum under two PID hypotheses, $DLL_K > 0$ (empty) and $DLL_K > 5$ (filled) (right). Both figures are taken from [16].

In search for B^0 and B_s^0 decaying to $h^+ h^-$, where $h \in K, \pi$, $\pi^+ \pi^-$ invariant mass spectra with and without PID DLL_x requirements can be seen in Figure 2.11. These plots clearly demonstrate increase in sensitivity searching for $B^0 \rightarrow \pi^+ \pi^-$ signal amongst other components.

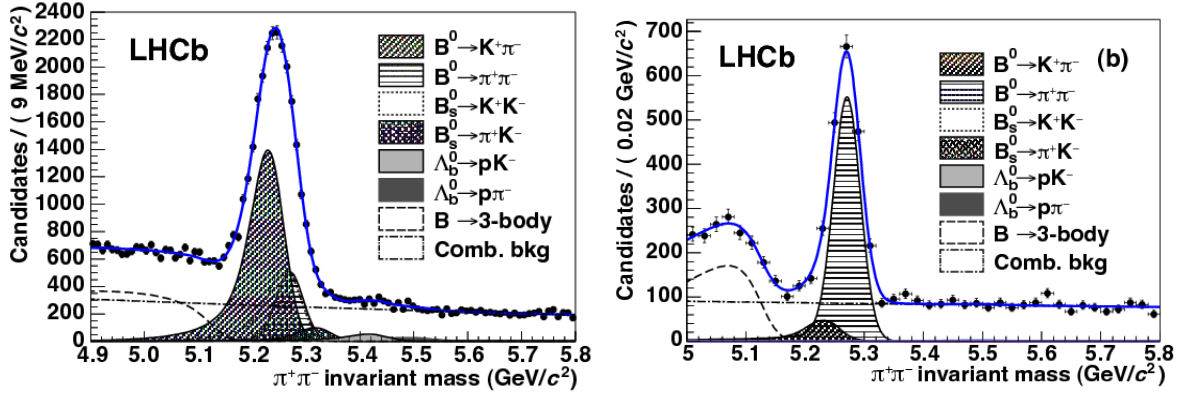


Figure 2.11: $\pi^+\pi^-$ invariant mass distributions obtained using kinematic constraints only (left) and also using PID constraints (right) in order to isolate $B^0 \rightarrow \pi^+\pi^-$ peak. This figure is taken from [17].

2.6 Calorimetry

As many other particle physics detectors, **LHCb** is equipped with series of subdetectors providing separation and **PID** tool for electrons, pions and photons. This separation is achieved because different particles interact at different distances, producing differently shaped showers of light. This part of detector is not only integral to the way **LHCb** trigger system works but it also provides precise measurement of energies of these objects. All the subcomponents discussed here operate on the same principle. The light from the scintillating material is guided to photomultiplier tubes by wavelength shifting fibres.

Electrons, pions and photons firstly encounter two planes of scintillating tiles: Scintillating Pad Detector (**SPD**), Preshower Detector (**PRS**) intersected by a wall of lead. The **SPD** senses the passage of charged particles whereas neutral particles will not be affected, distinguishing electrons from photons. The wall of lead initiates the electromagnetic shower, where photons are converted into electron-positron pairs, depositing sizable energy in the **PRS** allowing electron/pion separation.

The following Electromagnetic Calorimeter (**ECAL**) is based on sampling shashlik-

type technology, where scintillating tiles are alternated by lead plates measuring the energy deposit of electromagnetic showers. As the best energy resolution requires full energy deposit of energetic photons along the **ECAL**, the thickness is equivalent to 25 radiation lengths. The resulting resolution of **ECAL** is $\frac{\sigma_E}{E} = \frac{10\%}{\sqrt{E}} \oplus 1\%$, where E is in GeV.

On the other hand, **HCAL** sandwiches iron instead of lead as the absorber with thickness of 5.6 interaction length only, achieving resolution of $\frac{\sigma_E}{E} = \frac{70\%}{\sqrt{E}} \oplus 10\%$ in beam tests. This poorer resolution however fulfils the requirements necessary for the main purpose of this detector, hadron trigger. Away from beampipe the granularity of cells is coarser to mirror the track occupancy as seen in Figure 2.12.

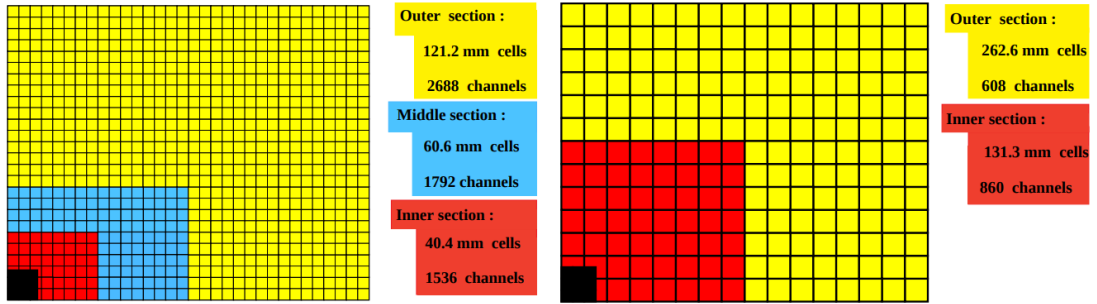


Figure 2.12: Granularity of **ECAL** (left) and **HCAL** (right) detectors. The figure was taken from [10].

2.7 Muon Stations

Muons are considered to be of fundamental importance to many flagship analyses by **LHCb**, such as the search for the rare $B_s^0 \rightarrow \mu^+ \mu^-$ decay. Analysis of $B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu$ of course relies heavily on good performance of this part of detector. Muon stations are positioned at the end of the detector, taking advantage of the low probability of interaction of muon previously in the detector.

LHCb's five rectangular muon stations **M1-5** are positioned before and after calorime-

try system, with first station M1 upstream of the SPD, and four stations (M2-M5) downstream of HCAL as shown in Figure 2.13. The M1 station consists of 12 sets of three gas electron multiplier foils (triple-GEMs) in the region closest to the beam pipe, resisting the highest dose of radiation due to the highest particle flux. Its main use lies in improving the p_T resolution by $\approx 10\%$. M2-M5 station each consist of 276 multi-wire proportional chambers (MWPCs) filled with Ar – CO₂ – CF₄ gas mixture. They are interlayed with 0.8m iron walls, to provide stopping target to all particles, other than muons with momentum higher than 6 GeV/c. In order to ease the accessibility, like in VELO, all the stations are split into two independent mechanical sides, also known A and C side.

Each station is then further segmented into four increasingly larger regions away from the beam, R1 to R4. All the regions were constructed to cover the same acceptance, keeping the track occupancy constant across the station. The granularity of the readout is higher in the horizontal plane to take advantage of magnet's horizontal bending plane.

Both GEM and MWPCs operate on a same principle. In each station, position in the $x - y$ plane is determined by ionizing electrons that come from muons passing through the detector, which are then attracted either to the closest anode mesh or wire mesh. The trigger is fired if the corresponding rectangular region in each station registered positive binary decision. This means the efficiency of each station must be $\geq 99\%$ to give overall 95% trigger efficiency. Geometrical layout covers $\approx 20\%$ muons originating in semileptonic b decays.

2.7.1 Muon Identification

Apart from triggering events with high enough p_T muons, muon stations provide necessary PID information for muon analyses. Offline variables mostly used for muon ID by analysts are

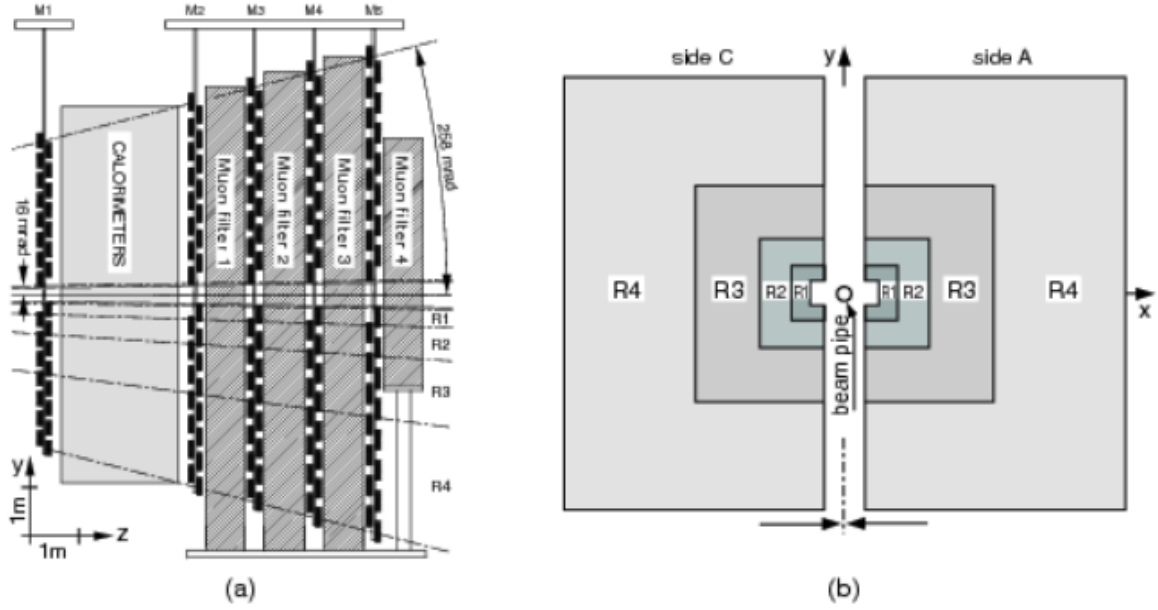


Figure 2.13: (a) Layout of the muon detector x-z plane and (b) x-y plane. This figure is taken from [18].

- **IsMuon**: Boolean decision of muon candidates with momentum-dependent categorisation. Long tracks with $p > 3 \text{ GeV}/c$ are extrapolated to muon stations yielding $x - y$ coordinates in $M2 - M5$, considering only tracks within acceptance. For each station, search for the hit information within elliptical area defined by momentum, field of interest (FOI), is performed. The hit requirements are summarized in Table 2.2.
- **muDLL**: Difference in log likelihoods computed using muon and non-muon hypothesis. These hypotheses are based on the proximity/distance D^2 of the track extrapolation into the muon stations and corresponding closest sensed hits in those stations. Muon-like particle will tend to have sharper distribution in D^2 as compared to other species. Protons were chosen for the calibration purposes giving broader distribution originating from punch-through protons, protons having coincident hit position to true muon, and random hits.

- **DLLmu**: For each track global likelihood is produced, by combination of muon and non-muon likelihood from **muDLL**, with the **RICH** different mass hypothesis likelihoods, and calorimetry likelihood exploiting the energy deposits information. Like in **RICH** likelihoods, the default hypothesis corresponds to separation between muon and pion hypothesis.

Particle Momentum p	Hits in Muon Stations
$3 \text{ GeV}/c < p < 6 \text{ GeV}/c$	M1 & M2
$6 \text{ GeV}/c < p < 10 \text{ GeV}/c$	M1 & M2 & (M3 M4)
$10 \text{ GeV}/c < p$	M1, M2, M3 and M4

Table 2.2: Momentum-dependent definition IsMuon variable.

2.7.2 Muon Performance

As in hadron performance measurements, muon ID is determined using high statistics decay channel $J/\psi \rightarrow \mu^+ \mu^-$ using tag-and-probe method. Mis-ID rates are computed using the same decay channels as previously. The summary of IsMuon **ID** and misid rates are presented in [Figure 2.14](#). Very high ID rate (above 90%) for relatively low misid probability (below 10%) performs the least well in low p_T bins as here the muons end up outside of the acceptance. The misid rates for kaon and pions are significantly higher in low momenta region as the dominant process causing this are prompt muons from decay-in-flight.

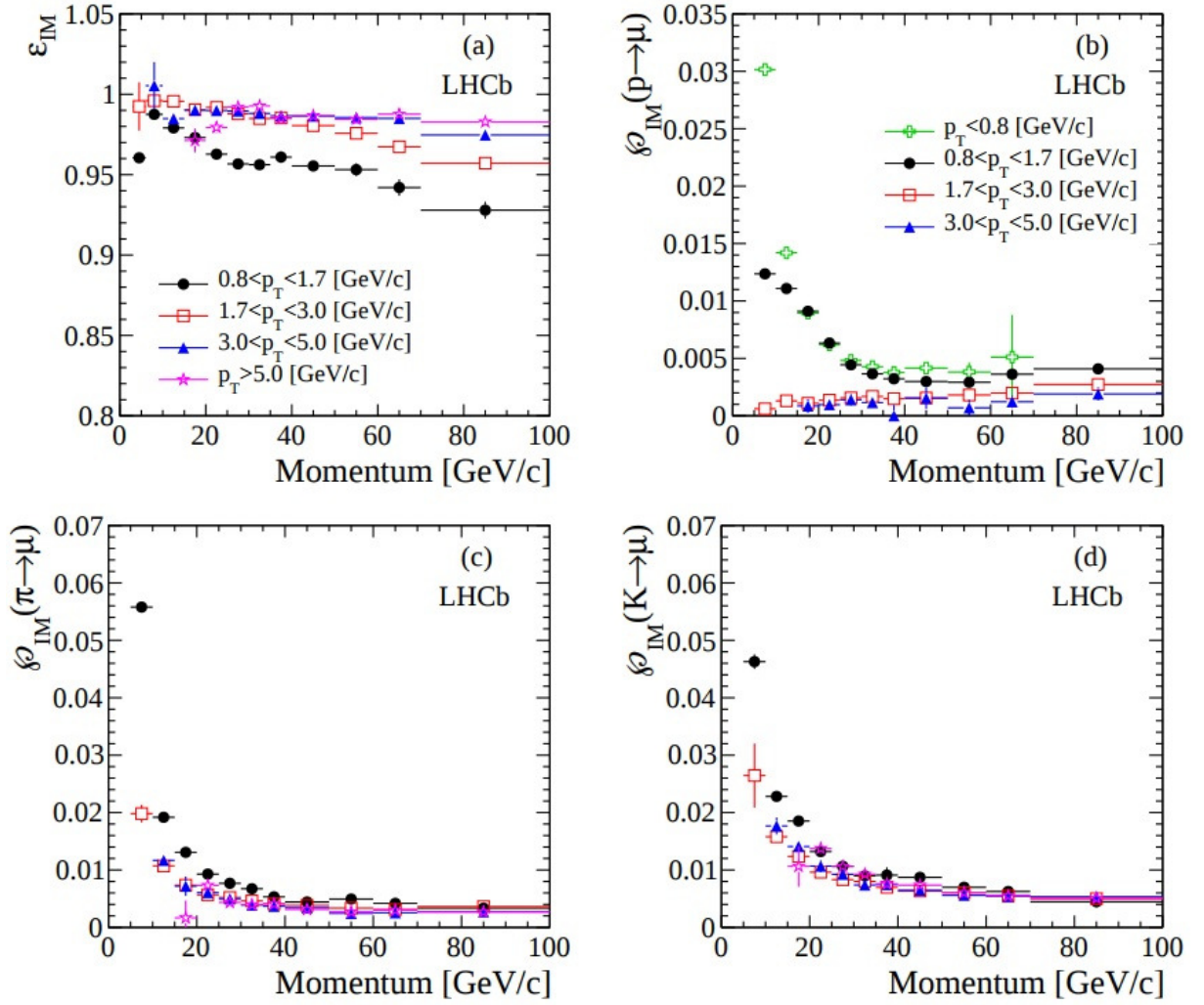


Figure 2.14: (a) Probability of correctly identifying muons as a function of momentum p in the bins of p_T for $J/\psi \rightarrow \mu^+ \mu^-$ with IsMuon constraint. (c) Probability of incorrectly identifying pion (b) proton and (d) kaon as muon with IsMuon. This figure is taken from [19].

2.8 Trigger

Nowadays, big-data physics experiments have to make decisions on what kind of data they want to keep. The choice of interesting events is performed by series of decisions, which is cumulatively known as trigger. LHCb trigger system was build

around constraints posed by the run conditions, read-out capabilities and available disk space. In Run I and Run II LHCb has at its disposal the multistage trigger consisting of hardware-based level 0 trigger (**L0**) and software-based high level trigger (**HLT**).

L0 reduces the rate of data from 40 MHz to 1 MHz by employing five trigger decisions, also known as lines. First three lines make decision using calorimeter information about the transverse energy, E_T , whether it is photon, electron or hadron causing the shower energy deposit. Two other lines are reading out information from the muon system by looking for transverse momentum, p_T , of muon and dimuon (two muon tracks) objects. Efficiencies of L0 muon triggers are evaluated using $B^+ \rightarrow (J/\psi \rightarrow \mu^+ \mu^-) K^+$ decays. Hadron trigger efficiency in different decay channels can be seen in Figure 2.15.

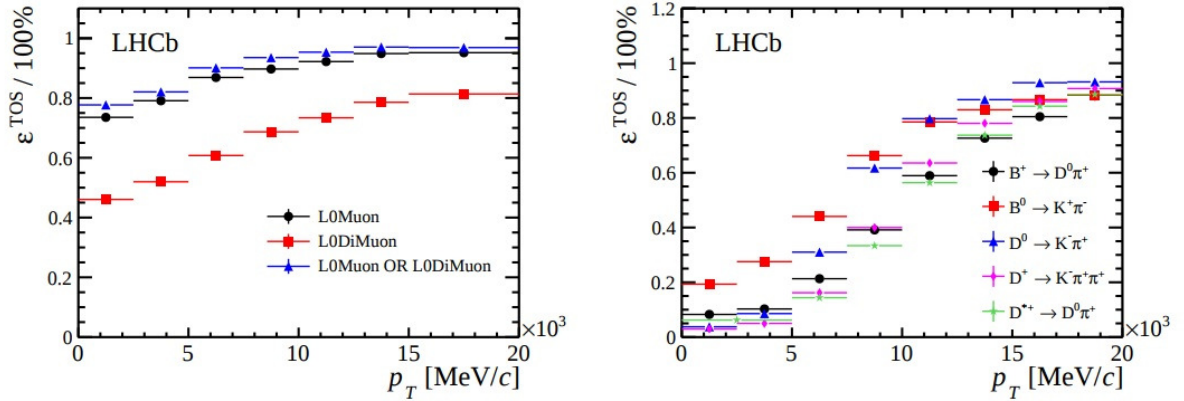


Figure 2.15: TOS efficiency as a function of p_T for muon-based decisions (left). TOS efficiency for different decays using L0 hadron trigger lines. This figure is taken from [18].

Software-based **HLT** then further reduces the rate from 1 MHz down to 40 – 80 kHz which can be safely stored to disk. The first stage of the **HLT**, (**HLT1**), performs limited track reconstruction and hence makes decision based on the presence of charged tracks in the event. **HLT1** uses **VELO** hits to reconstruct **PVs** and **VELO** tracks by using 3D pattern recognition. As LHCb's primary mission is to study decays of hadrons containing b and c quark, **HLT1** will make decision based on the track segments being

displaced (having high IP) with respect to the PV . For events selected by the $L0Mu\text{on}$, an attempt is made to match the $VELO$ tracks to hits observed in the vertical plane in the muon chambers due to magnet bending plane. By computing the track χ^2 , the potential muon track candidates are selected. Finally, the $VELO$ tracks and muon tracks are extrapolated into the OT or IT trackers, allowing for so called *forward tracking*, whereby p and p_T requirements are imposed to reduce processing time. Each track is then fitted with fast Kalman filter providing the χ^2 of the fit. The corresponding performance of $HLT1$ trigger lines are shown in Figure 2.16.

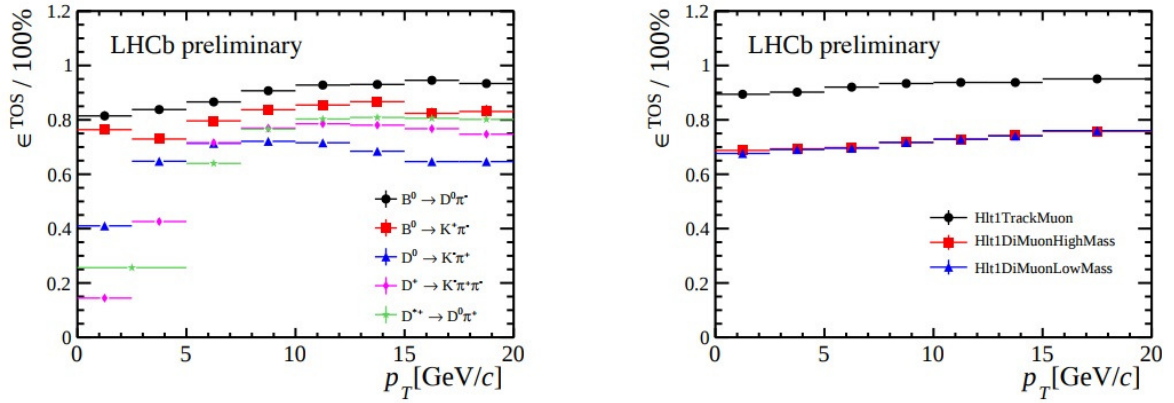


Figure 2.16: $HLT1$ efficiencies of the corresponding triggers using the same proxy as in Figure 2.15. This figure is taken from [20].

The second stage $HLT2$ reduces the rate to 5 kHz that can be safely written to disk. $HLT2$ consists of a series of decisions based on a full reconstruction of either groups of decays or specific decay modes. *Topological triggers* exploit the vertex and track information (topology) of b -hadron decays. By employing multivariate techniques 2,3 or 4-body decays away from PV are reconstructed. To account for decays where final state particle is not fully reconstructed, corrected mass serves as an input variable in the the BDT . Dedicated lines are also written to reconstruct muon and dimuon channels allowing for both prompt J/ψ and $B \rightarrow J/\psi X$ studies. Finally *Exclusive triggers* concentrating on selecting events with $c\bar{c}$ do selection very similar to the offline selection

but without **PID** cuts and with *prescales*, only allowed in a certain fraction of events, is applied.

In the end, selected events have their trigger decisions categorized by different type. An event where signal candidate caused the trigger to fire is known to be Trigger on Signal (**TOS**). An event where it is non-signal like particle causing the trigger decision to occur, Trigger Independent of Signal (**TIS**) is used. Finally, if only by combination of signal particle together with other particle's properties in the event produce affirmative decision, then these events are categorized as **TIS** & **TOS** = **TISTOS**.

Between the Run I and Run II period there has been a change in how the software trigger operates, which can be seen in [Figure 2.17](#). As more timing budget was introduced for both **HLT1** and **HLT2**, **LHCb** took advantage in upgrading the trigger system. By introducing update of calibration and alignment constants of the relevant subdetectors before the data is sent to permanent disk, *online reconstruction*, defined as being produced at trigger farm, is the same as the *offline reconstruction*, defined as reconstruction made when data reached the permanent disk. Hence, there is enhancement of available information, such as the **PID** in the **HLT**, which can be then used at the trigger level. (Mention Turbo?).

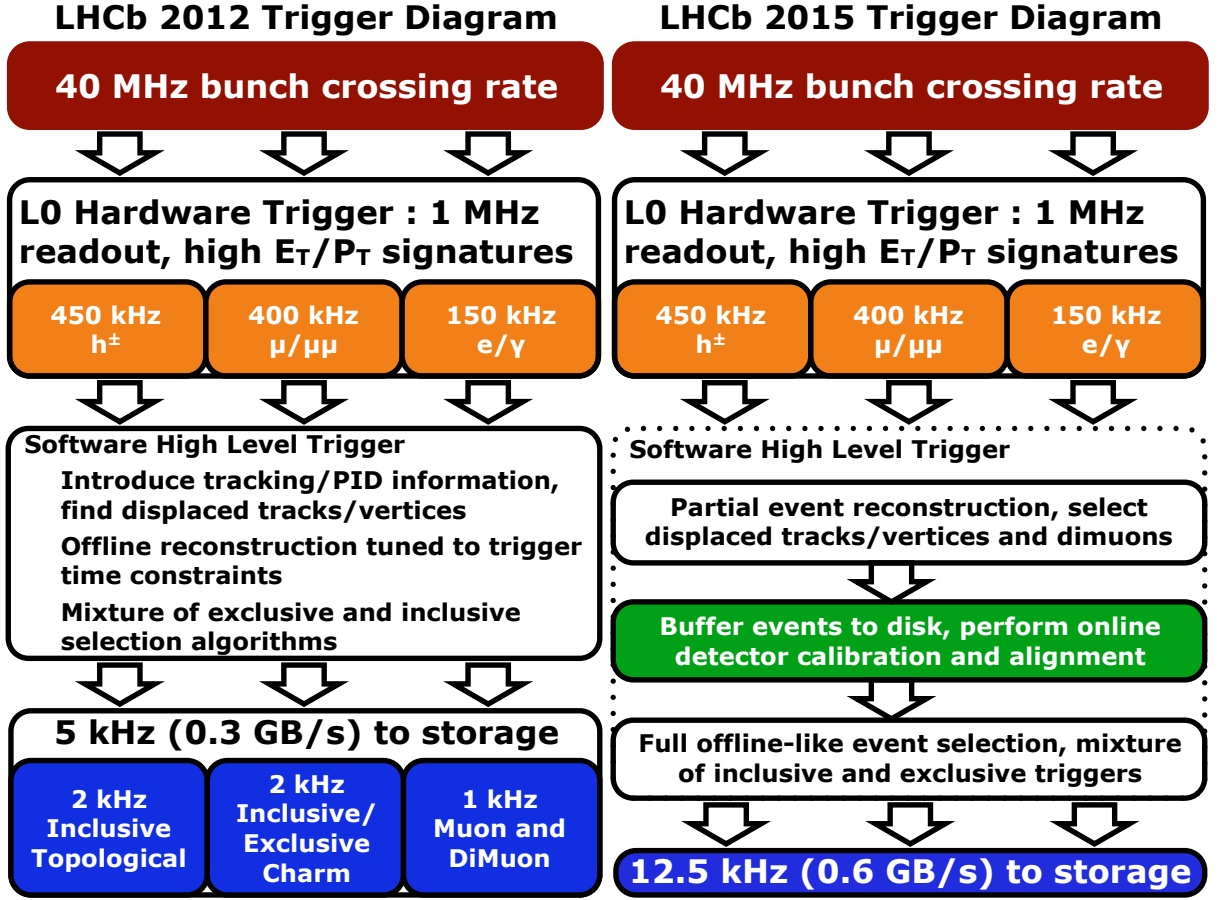


Figure 2.17: Trigger scheme differences between Run I and Run II. Figures obtained from [21]

2.9 Simulation

In order to optimise the event selections, extract efficiencies and model the backgrounds, a full Monte Carlo Simulation **MC** starting from simulation the pp collision to detector readout of decay of interest is produced. The pp collisions within **LHCb** configuration [22] are simulated with Pythia 6.4 [23] and Pythia 8.1 [2]. **LHCb** specific settings are mostly related to running conditions: luminosity, number of collisions per bunch crossing as well as contamination from other bunches, *spill-over*.

In the pp collision, the b and c production mechanisms are simulated and then the following $b\bar{b}$ or $c\bar{c}$ pair is hadronized into hadrons of interest. In this thesis and the

analysis presented, B^+ is the hadron of interest. Hadrons are then further decayed using EVTGEN [24] into the chosen decay products. In this stage, different physics models or inputs from theory can be configured. At the same time some initial CPU-friendly selection is established, usually requiring the hadrons to be contained within the forward detector's acceptance. In order to account for the effects of QED radiative corrections, PHOTOS [25] algorithm can be used. All of this combined establishes *generator-level simulation* of LHCb.

In the next phase, *detector simulation*, the interactions of the all the particles with the detector, transport, as well as detector's response are simulated using the C++ GEANT4 toolkit [26], [27]. LHCb's interface to GEANT4 is detailed in [28].

2.9.1 Differences in Simulation And Data

Despite the complexity and best intention of the LHCb simulation, there are several shortcomings that require correction treatment. The most affected variables necessary for physics analyses that one needs to consider are IP resolution, track reconstruction efficiencies, PID variables and track occupancy.

The IP resolution shows better trend in the simulation then in the data due to the mismodelling of material description of VELO simulation. As shown in Figure 2.18 IP resolution does greatly differ depending the variation of material density of VELO. Around $\phi = \pm\pi/2$, where the two VELO parts overlap, the material difference causes the discrepancy. It can be corrected either by reweighting to data or by smearing the resolution wit Gaussian distribution.

Track reconstruction efficiency is also not reproduced very well in certain kinematical bins, again due to modelling of scattering interactions.

The most critical problem that needs to be addressed in the presented analysis are the inaccuracies of PID variables, which are mismodelled in the simulation. The origin of this problem arises as a consequence of much lower estimate of low momentum tracks in the detector making the photoelectron background underestimated. This

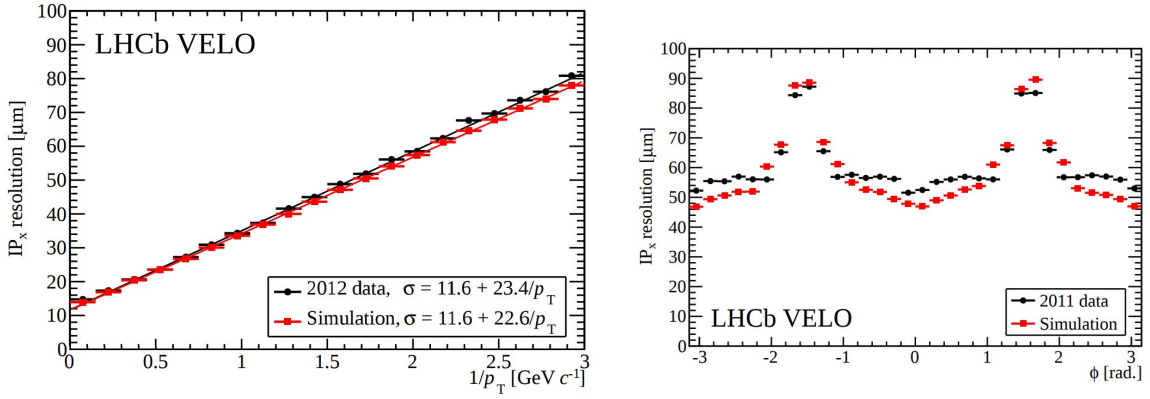


Figure 2.18: IP resolution in x-direction comparing the data and simulation output for 2012 data-taking period (left). IP resolution in x-direction comparing the data and simulation output for 2011 data-taking period as a function of angle, ϕ (right). These figures are taken from [11].

results in better performance of separation power in simulation and is corrected using real data calibration.

Chapter 3

Handling of trimuon correlations at LHCb

Having three muons passing through the detector may result in few issues. Two collimated muons may be passing through the same parts of the detector if they bear the same charge, which causes problems in resolving them. From tracking point of view, ghosts and clones are hence much more likely to occur. In LHCb, plethora of muon [PID](#) variables can be used to suppress these spurious tracks, however, one must be careful once estimating the relevant [PID](#) efficiencies as most these efficiencies will be dependant on number of muons in the detector.

3.1 Muon PID variables

There is a further set of muon variables apart from the ones mentioned in [subsection 2.7.1](#) that are available for [PID](#) of muon. In this section there is summary of the variables used in analysis of $B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu$.

3.1.1 Binary Muon PID variables

Similar to `isMuon` shown in [Table 2.2](#), there are two more binary variables, `isMuonTight` and `isMuonLoose`, that can help with classification of muons.

In each station (M1-M4) and in each dimension, $d = (x \text{ or } y)$, FOI is defined as a function of momentum p ,

$$FOI_d = \rho_{d,1} + \rho_{d,2} \cdot \exp\left(\frac{\rho_{d,3} \cdot p}{\text{GeV}/c}\right). \quad (3.1)$$

Muon passing through the detector will leave hit in pad_d of each muon station and have h_d coordinate. Extrapolations from tracking system are made into the stations are denoted E_d . The hits are considered to be within the FOI for given d if $\|h_d - E_d\| < FOI_d \cdot pad_d$.

Because the detector information is read out in x y direction separately, the pad slicing according to read-out is known as *physical* slicing of pads. However, as seen in Figure 3.1, the overlapping *physical* pads are grouped into *logical* pads. Uncrossed hits are registered within *physical* pads only and crossed hits are given by *logical* pads. `isMuon` only requires positive decision from uncrossed hits, whereas `isMuonTight` requires crossed hits.

3.1.2 Muon PID variables based on sharing hits

The `nShared` variable for muons gives number of tracks with shared hits in the muon stations. For each hit within FOI of the extrapolated track, the `nShared` algorithm will check whether any other track has been established using the given hit. In this case, the muon track which has bigger distance between the extrapolation and the hit is increased by 1 and the other tracks becomes the hit owner track. Hence this integer PID variable, `nShared`, can help greatly reduce *ghost* tracks and *clones* if `nShared`==0.

To make sure that muons for $B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu$ are not coming from these spurious tracks, `nShared`==0 for all three muons, i.e, not to share hits in muon stations with any other downstream or long tracks. In analysing data, there were features in the software which changed between Run I and Run II.

The first feature that is different comes from the calculation of the distance between the extrapolation and the hit in `nShared` algorithm. In *Stripping 21* used for 2012 and *21r1* used for 2011 data, it was discovered that the distance between extrapolated

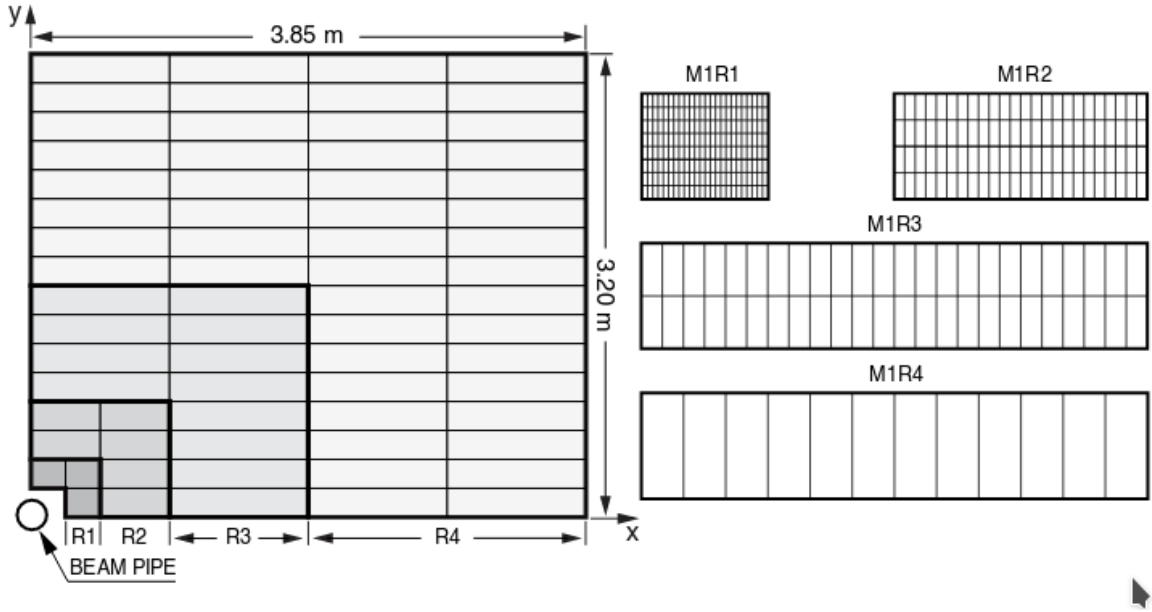


Figure 3.1: Schematic view of the muon station slicing into x-y pads. This is left quadrant of M1 station, showing decreasing granularity of the muon stations away from the beam. This figure has been taken from [18]. M1R1 is the innermost region and M1R4 is the outermost region of the M1 station.

track and hit was wrongly calculated, and hence this was corrected before *Stripping 23*, used for 2015 data. Additionally, information from M1 station was used to calculate distances, even though M1 information is not usually used for muon ID algorithm. For analysts, this feature was present accross all reconstruction software and hence it is consistent within stripping version, meaning that simulation and data is affected in the same way.

In *Stripping 23*, the Muon ID alghorithm was rewritten to adapt for parallelisation that needs to be done in order to meet criteria for upgrade of **LHCb**. There were two mistakes introduced prior to 2015 datataking. Firstly, an array was defined with 4-elements [0,3] to store information about x and y coordinates of the hits. However, an iteration occured by filling [1,4] array (M2-M5 station) resulting in 5-element array where 0-th element was not filled randomnly filled. However it turns out that it is

well-behaved and has no impact on physics.

Futher in the process, however, this information is used to calculate the sum and average of distances per station between the hits and extrapolations. This algorithm again iterates over $[0, 3]$ arrays, meaning that no information is used from M5 muon station. This obviously has effect, but again it is consistent accross the reconstruction version.

The interplay between all these features for $B^+ \rightarrow (J/\psi \rightarrow \mu^+ \mu^-) K^+$ can be seen in Figure 3.2.

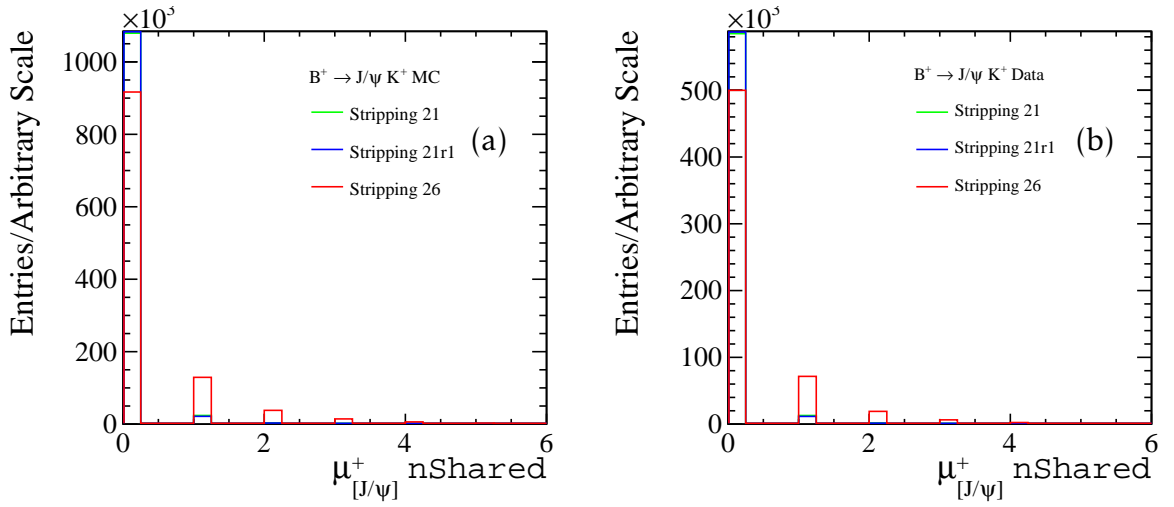


Figure 3.2: (a) The n_{Shared} variable in simulation, (b) data for $B^+ \rightarrow (J/\psi \rightarrow \mu^+ \mu^-) K^+$ in different stripping versions corresponding to 2012 (*Stripping 21*), 2011 (*Stripping 21r1*), 2016 (*Stripping 26*) data-taking. There is shift of distribution in *Stripping 26* towards less isolated tracks.

This will have inevitably and impact on the PID probabilities. Using the same calibration channels as in subsection 2.7.2, misID and ID rates can be seen in Figure 3.3. As the tracks tend to be less isolated in *Stripping 26*, typical of non-signal like events, the misID rate is expected to be higher for the same working point (ID efficiency).

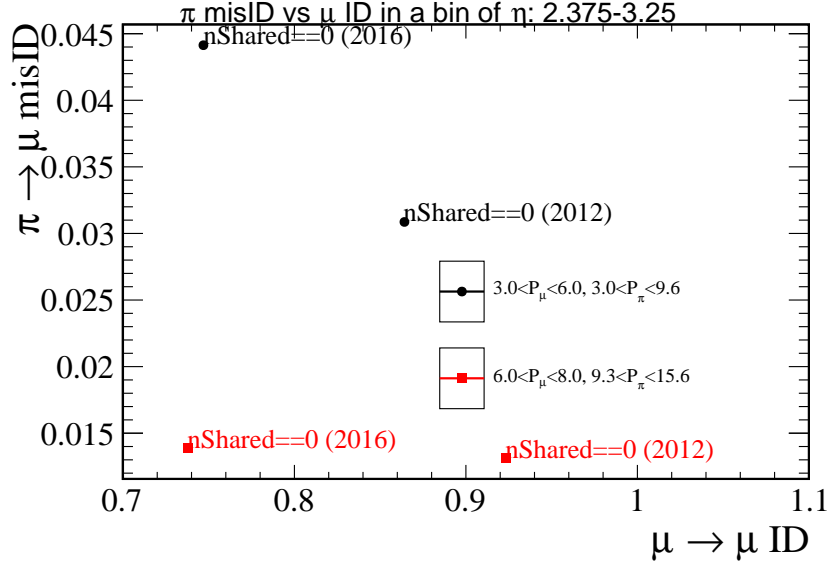


Figure 3.3: ID and misID probabilities from standard calibration datasets from 2012 (*Stripping 21*) 2016 (*Stripping 26*), binned using default 2-dimensional binning scheme in momentum p and pseudorapidity η . In this plot, ID and misID rate in central bin of η and first and second bin in p are compared. This demonstrates that for same ID efficiency, the misID rate is significantly higher in 2016.

3.1.3 Muon PID variables based on regression techniques

Similar to the DLL_{mu} variable in [subsection 2.7.1](#), which combines all the information from the detector into a global likelihood, it is possible to feed all the different variables to a neural network, which can then produce an output corresponding to the probability of a particle to be of a certain species. $Probnn_x$ where, x is the species of interest, is calculated and can be used also for muonID. Compared to DLL_x variables, $Probnn_x$ variables tend to have smaller correlation with the kinematics of the particle, and hence are more useful with decays where particles are soft, such as $B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu$. As with any machine learning algorithms, the selection of the training sample is important as well as input variables. In I there were two tunings introduced V2 and V3, with more variables in V2. Depending on the species of particles, V2 or V3 performed better. In the analysis of $B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu$ $Probnn_{x_V2}$ is used.

3.2 Clones

When analysing decays with two muons of the same charge, LHCb magnet bends these two muons in two separate planes. With two muons of the same sign, it is more likely that their two tracks will be collimated. This poses more difficult task for tracking as it distinguishes these two tracks less well. It is even possible that these two same sign muon tracks are not genuine tracks, but rather subtracks or a copy of another track, *clone tracks*. Two tracks are clones if they share at least 70% of the hits in the VELO and at least 70% of the hits in the other T-stations. Ofcourse, once it is established that two tracks share this percentage of hits, it has to be established which track is the clone track. This decision is based on the total number of hits and the track χ^2/ndof comparison of the two tracks.

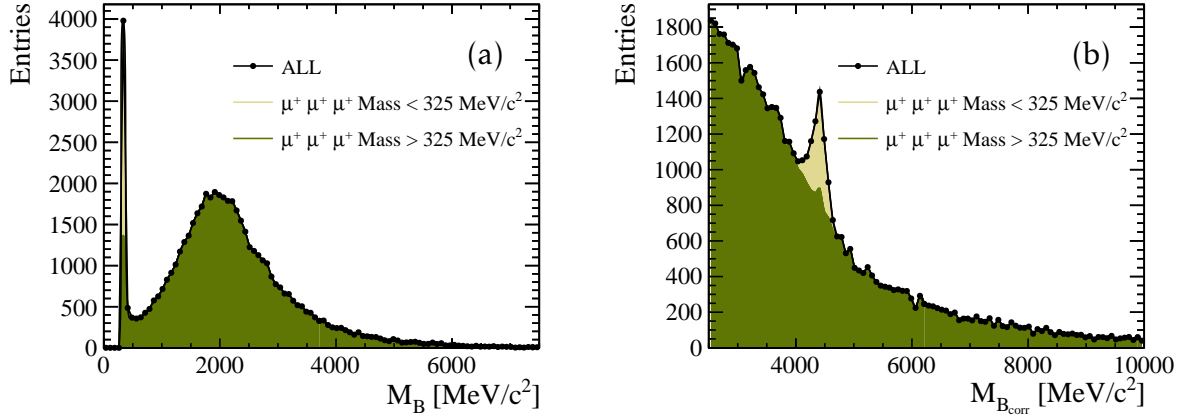


Figure 3.4: (a) Visible and (b) corrected mass of B , shows a clear peak coming from clones in 2012 same sign data sample.

In search for $B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu$ two muons have the same charge, and hence are affected by the *clones*, which needs to be understood. In this case, sample with three muons of same sign was obtained, where the effect is even more prominent and can create potentially *fake peaks* in visible mass spectrum. *Clones* peak at well defined visible mass

$$M_B = \sqrt{(3 \times M_\mu)^2} \approx 318 \text{ MeV}/c^2 \quad (3.2)$$

Once translated into corrected mass, these *fake peaks* are smeared and look like genuine resonances with resolution as seen in Figure 3.4.

The shape of genuine resonance comes as a result of LHCb many factors: vertexing, tracking and trigger selection. As there are three parallel tracks, the vertex of the system is not well defined. However, the vertex fitting of the PV and SV is functional and $\text{vertex } \chi^2/\text{ndof}$ is good as these tracks are subtracks of each other. *Clones* can be, however, differentiated by the position of the decay vertex of *B*, Figure 3.5 as well as the occupancy in the tracking, OT as seen in Figure 3.6.

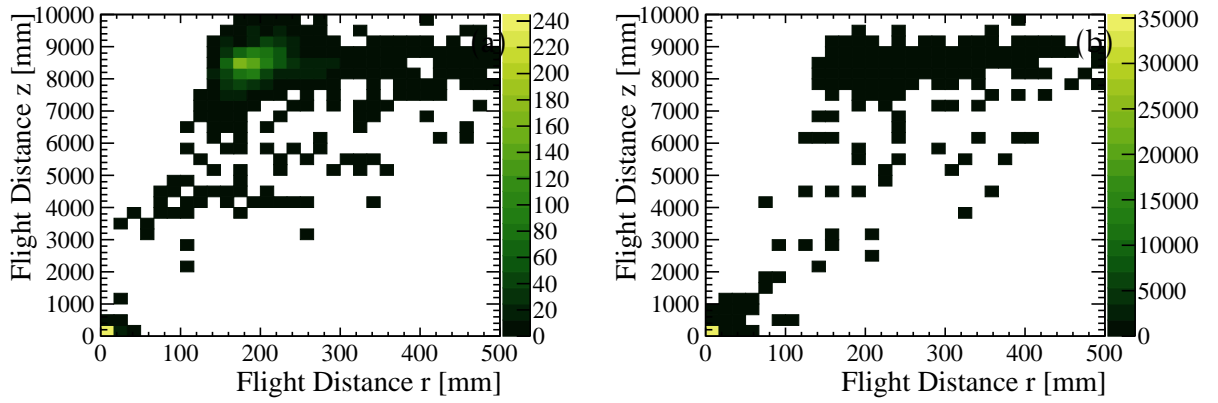


Figure 3.5: (a) Clone and (b) no clones flight distance properties. It can be seen that *clone* tracks have their decay vertex placed at the end of the detector, whereas regular good tracks will decay within VELO.

With this typical path for the clones there is a fixed angle of the clones through the detector (the angle between muon momentum the z-axis), which is calculated using information from OT in a following way:

$$\arctan(\theta) = \arctan\left(\frac{\text{FD radius}}{\text{FD distance along } z}\right) = \arctan\left(\frac{200 \text{ mm (Figure 3.6)}}{8500 \text{ mm}}\right) = 0.023 \text{ rad.} \quad (3.3)$$

With the L0Muon p_T threshold of 1.76 GeV/c for 2012 [20], the typical momentum from about 75 to 120 GeV/c is yielded because

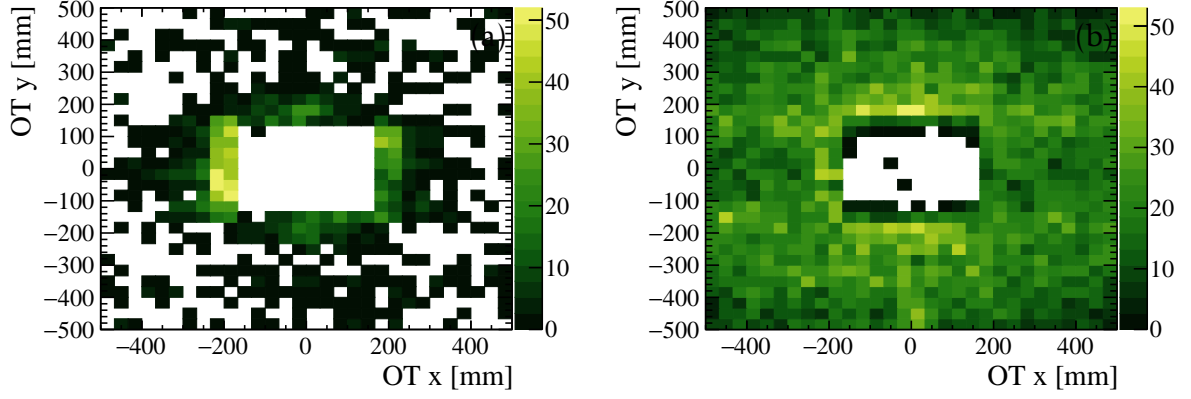


Figure 3.6: The occupancy difference in the OT detector between (a) clones and (b) real tracks in the OT at the distance 9450 mm along the LHCb. Clones are concentrated along the inner edge of the OT.

$$p = 1.76 \text{ GeV}/c / \sin \left(\arctan(\theta) = \arctan \left(\frac{200 \text{ mm}}{8500 \text{ mm}} \right) \right). \quad (3.4)$$

The angle between B flight and trimuon momentum vector, $\cos(\theta_B)$, will also be fixed and have typical value of 0.7 mrad as seen in Figure 3.7.

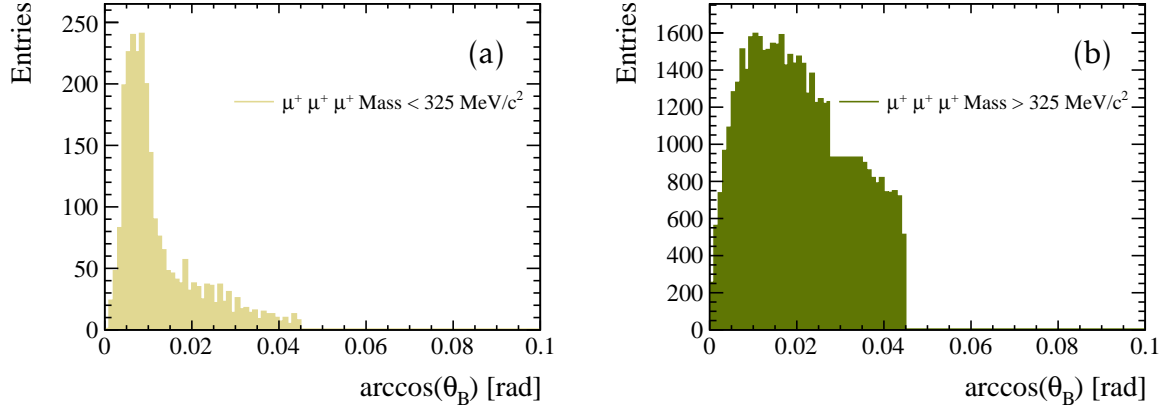


Figure 3.7: (a) Clone and (b) no clone distribution for $\cos(\theta_B)$.

Hence, missing p_T in the direction of the flight can be calculated using $\cos(\theta_B)$ and typical p ,

$$100 \text{ GeV}/c \times \sin(0.0007) = 0.7 \text{ GeV}/c. \quad (3.5)$$

Hence corrected mass $M_{corr} = \sqrt{M^2 + |p_T^2|} + |p_T| = 4.2 \text{ GeV}/c^2$, using missing p_T from Equation 3.5 and visible mass of clones from Equation 3.2.

In order to suppress these tracks in analysing $B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu$, where two muons have the same sign, any distinguishing features mentioned could be used. But the most powerful PID-wise is requiring $n_{\text{Shared}}=0$, as this requirement removes all of the clones, as seen in Figure 3.8.

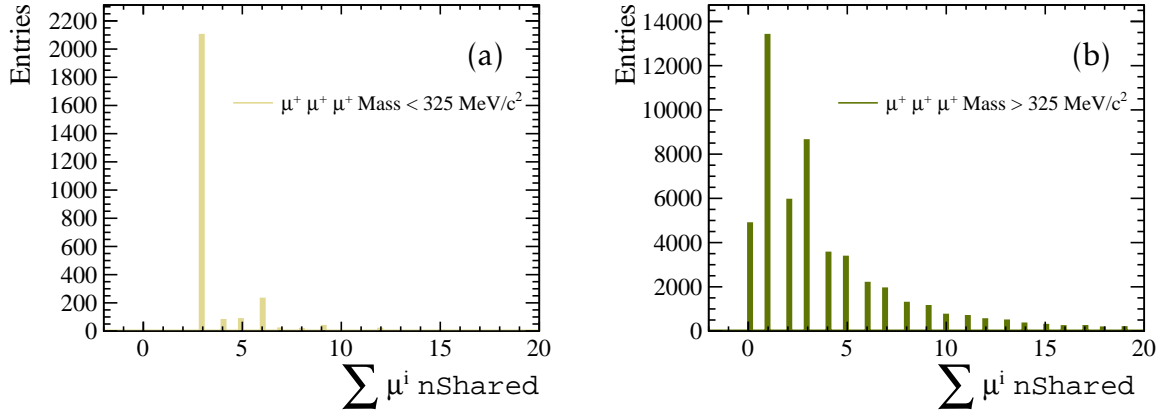


Figure 3.8: (a) Clone and (b) no clone distribution for sum of all muon n_{Shared} . Since in this case the clones are of each other, for clone there is clear peak at three.

3.3 Probability of $K/\pi \rightarrow \mu$ misidentification at LHCb

Usually, in order to estimate background coming from misidentification of species into muons in the detector, data samples with *non-muon* (identified not to be muons) tracks are obtained. They are assigned parametrically probabilities of misid rates from some specific control samples, which were discussed in subsection 2.5.1. However, three muon signature will induce problems for PID variables that are correlated with the number of muons in the detector and other control samples need to be used.

3.3.1 Specific control sample for $K/\pi \rightarrow \mu$ misID rates

At LHCb, PIDCalib package can usually provide misID and ID rates obtained from standard K/π control samples $D^{*+}(\rightarrow D^0(\rightarrow \underline{K^+\pi^-})\pi^+)$. These statistically populated background-free *sWeighted* samples, for which it is possible to extract misID and ID rates as a function of kinematics given certain PID criteria, however do not have other muons in the final state.

More specifically, the topology of the mis-ID background component, which is two real muon track with an additional *fake* muon track is very different to PIDCalib sample $D^0 \rightarrow K^+\pi^-$. This influences rest of misID rates as some of the PID variables are strongly correlated with number of muons in the decay, due to the fact that the mis-identified particle can share hits with other muons in the rest of the decay. This should be reflected mostly in high momenta region, where the three particles tend to be collimated and share hits most often.

For this reason, an alternative sample (should also be statistically populated background-free), $B^0 \rightarrow J/\psi(\rightarrow \mu^+\mu^-)K^*$, which mimics the two real muon plus fake muon correctly, will be used to obtain misID rates.

3.3.2 Selection for $B^0 \rightarrow J/\psi(\rightarrow \mu^+\mu^-)K^*$

Data samples for each year of data taking were obtained from *stripping* dedicated to look for this type of decay, but with both kaon and pion having no particle identification applied, which ensures that PID performance of different variables can be studied. Some initial selection was applied together with more stringent $B^+ \rightarrow \mu^+\mu^-\mu^+\nu$ selection including trigger (but on J/ψ candidate rather than B). Finally to remove most of the backgrounds that still pollute the signal samples following selection summarized in Table 3.1 are used.

3.3.3 Fitting Strategy for $B^0 \rightarrow J/\psi(\rightarrow \mu^+\mu^-)K^*$ decay

After all the selection, there will be still some background that needs to be modelled.

Idea	Cut
ID K^*	$ m(K\pi) - m_{PDG}(K_0) < 100 \text{ MeV}/c^2$
Compatible with <i>PIDCalib</i>	for K, π , $p_T > 250 \text{ MeV}/c$
Compatible with <i>PIDCalib</i>	for μ , $p_T > 800 \text{ MeV}/c$
Muon swap veto	$ m((h \rightarrow \mu)\mu) - m_{PDG}(J/\psi) > 60 \text{ MeV}/c^2$
Veto $B^+ \rightarrow K^+ \mu^+ \mu^-$	$\max(m(K^+ \mu^+ \mu^-), m((\pi^+ \rightarrow K)\mu^+ \mu^-)) < 5100 \text{ MeV}/c^2$
Veto $B_s^0 \rightarrow \phi \mu^+ \mu^-$	$m(K(\pi \rightarrow K)) > 1040 \text{ MeV}/c^2$
ID muons	$\text{mu1_ProbNNmu} > 0.5$ and $\text{mu2_ProbNNmu} > 0.5$
For kaon misID rates:	
ID pion	$\text{DLLK} < 0$ $\text{DLLp} < 0$ and $\text{IsMuon} == 0$
For pion misID rates:	
ID kaon	$\text{DLLK} > 0$ and $\text{DLLK} - \text{DLLp} > 0$ and $\text{IsMuon} == 0$

 Table 3.1: Offline selection for $B^0 \rightarrow J/\psi(\rightarrow \mu^+ \mu^-)K^*$ decay.

The signal component, $B^0 \rightarrow J/\psi K^*$, is obtained by fixing the shape from simulation apart from mean μ and width σ . It is fitted with double-sided Hypatia function [29] (more in section C.2).

Background that peaks in the upper mass sideband, coming from heavier $B_s^0, B_s^0 \rightarrow \pi^+ K^-(J/\psi \rightarrow \mu^+ \mu^-)$ is also modelled using simulation, using the same function as a signal but with offset of μ by PDG difference between B_s^0 and B^0 .

It is also possible that kaons and pions are swapped between themselves. Background coming from $K \leftrightarrow \pi$ swaps is modelled from simulation where mass hypotheses were swapped. Its distribution is fitted with double sided Crystal Ball function [30] (more in section C.1).

Possibility of misidentified background comes from $\Lambda_b \rightarrow K^- p \mu^+ \mu^-$ where proton is misidentified as pion. This background is modelled from simulation and fitted with RooKeys (more in section C.3).

And finally combinatorial component is modelled by the exponential function. In order to obtain these K/π misid rates fit to constrained mass between 5150 - 5450 MeV/c^2 was performed, where all of the yields for different components are left floating.

Chapter 4

Discovering (Setting Limit for)

$B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu$ at LHCb

LHCb's flagship analyses contain several muons in the final state coming from differently flavoured B mesons. Despite being in this category, search for $B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu$ is limited by the rareness of its occurrence as well as different backgrounds that can mimic its signature in the detector. Moreover, presence of invisible neutrino does induce uncertainties into reconstruction. This [chapter 4](#) will concentrate on characterisation of backgrounds as well as selection that is performed in order to reduce the backgrounds.

4.1 Topology of $B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu$ at LHCb

Upon hadronisation of $b\bar{b}$ pair B^\pm particle will travel less than a millimetre in the laboratory frame of reference before it decays into its decay products. This allows reconstruction of a primary vertex [PV](#) and its decay vertex, *secondary vertex* [SV](#). By joining these vertices, direction as well as length of the B^\pm existence, also known as flight distance ([FD](#)), can be established. In order to infer information about kinematic properties of B^\pm , the decay products are studied. All three muons are used to reconstruct the visible four-momentum. By conservation of momentum with respects to the direction of the flight of B^\pm , neutrino is assigned all missing momentum transverse to

the direction of the flight of B^\pm . The schematic diagram can be seen in Figure 4.1.

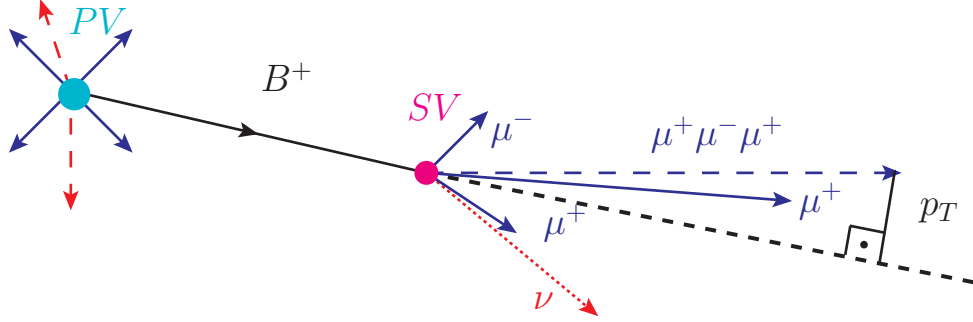


Figure 4.1: Schematic view of $B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu$ decay. At pp interaction point, or PV , $b\bar{b}$ pair hadronizes into B^\pm . B^\pm flies some distance before decaying into three muons and neutrino. All charged tracks (in filled-blue) seen can be combined into four-vector representing the visible part of the decay (semi filled-blue). Information about invisible neutrino (semi filled-red) are deduced from the conservation of momentum with respect to the direction of the flight of B^\pm . Neglecting momentum component parallel to the direction of flight for neutrino, transverse component of momentum is given.

Altogether it allows for reconstruction of a quantity, *corrected mass*, that plays similar role to invariant mass in fully reconstructed decays. Invariant mass is usually used in LHCb for fitting distribution from which physics results are extracted as it distinguishes well signal from background and there is minimal modelling problem.

Corrected mass is defined as

$$M_{corr} = \sqrt{M^2 + |p_T^2| + |p_T|}, \quad (4.1)$$

where the M^2 is the invariant visible mass squared and p_T^2 is the missing momentum squared transverse to the direction of B^+ flight. Usually the corrected mass of B^\pm will be denoted as $M_{B_{corr}}$.

M_{corr} can be thought of as the minimal correction to the visible mass to account for the missing neutrino information. The resolution on the *corrected mass* hence becomes a critical quantity that needs to be understood. As the method of reconstruction of

corrected mass relies heavily on the knowledge of B^\pm flight direction, the resolution of **PV** position and **SV** vertex is crucial. Let $\vec{x}_{PV} = \{x_{PV}, y_{PV}, z_{PV}\}$, $\vec{x}_{SV} = \{x_{SV}, y_{SV}, z_{SV}\}$ be **PV** and **SV** vertex position and $\vec{p} = \{p_x, p_y, p_z\}$ be the visible trimuon momentum. Then the missing transverse momentum to the direction of the flight p_T (momentum of the neutrino) is

$$p_T^2 = |\vec{p} - (\vec{x}_{SV} - \vec{x}_{PV}) \frac{\vec{p} \cdot (\vec{x}_{SV} - \vec{x}_{PV})}{|\vec{x}_{SV} - \vec{x}_{PV}|^2}|^2 \quad (4.2)$$

In general in order to propagate error on $f(x, y, z)$, where x, y, z are independent variables, the variance of $f(x, y, z)$

$$\langle f^2 - \langle f \rangle^2 \rangle = \langle f(x + \delta x, y + \delta y, z + \delta z)^2 - f(\langle x \rangle, \langle y \rangle, \langle z \rangle)^2 \rangle \quad (4.3)$$

Using first order Taylor expansion of variance and rewriting into the matrix form:

$$\begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z} \end{bmatrix} \begin{bmatrix} \delta x^2 & \delta x \delta y & \delta x \delta z \\ \delta y \delta x & \delta y^2 & \delta y \delta z \\ \delta z \delta x & \delta z \delta y & \delta z^2 \end{bmatrix} \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{bmatrix} \quad (4.4)$$

So now assuming that $x = \vec{x}_{PV}$, $y = \vec{x}_{SV}$ and $z = \vec{p} = \{E, p_x, p_y, p_z\}$,

$$\nabla_{x_{PV}}^T \text{COV}_{x_{PV}} \nabla_{x_{PV}} + \nabla_{x_{SV}}^T \text{COV}_{x_{SV}} \nabla_{x_{SV}} + \nabla_p^T \text{COV}_p \nabla_p \quad (4.5)$$

where COV is the covariance matrix.

In conclusion in order to calculate error on *corrected mass*, $\delta_{corr m}$

$$\delta_{corr m} = \sqrt{\langle f^2 - \langle f \rangle^2 \rangle} = \sqrt{\nabla_{x_{PV}}^T \text{COV}_{x_{PV}} \nabla_{x_{PV}} + \nabla_{x_{SV}}^T \text{COV}_{x_{SV}} \nabla_{x_{SV}} + \nabla_p^T \text{COV}_p \nabla_p} \quad (4.6)$$

which can be calculated analytically (method used for all the plots) or using numerical approximation of first derivative of *finite differences*.

4.2 Sources of Backgrounds

The largest background that can be will be looking similar to signal comes from *cascade decays*, where semileptonic $b \rightarrow c \rightarrow s$ or ($\bar{b} \rightarrow \bar{c} \rightarrow \bar{s}$) transition occurs. A typical example of this background in hadronic terms is $B^+ \rightarrow (\bar{D}^0 \rightarrow (K^+ \rightarrow \mu^- \nu) \mu^+ \nu)$, where K^+ is misidentified as muon. Because K^+ is misidentified as muon, this type of background is denoted as misID background.

In fact, any other particle species that is misidentified, belongs to the misID background category. If the sign of the misidentified particle agrees with the sign of the mother B^\pm , it belongs to the same sign misID background (*SS misID*) background. In the event where opposite sign particle to the mother B^\pm is misidentified, this background will be referred to as (*OS misID*) background. However, *OS misID* background is expected to have smaller rate as the misidentified particle would have to proceed via decays with additional particles or if coming as product from other b hadronization.

The presence of other B -hadron from $b\bar{b}$ pair does create its own decay chain and hence it is possible to combine one of its muon tracks with two muons from the "signal" B . This is denoted as combinatorial background.

Then presence of neutrino in a final state allows for certain uncertainty regarding the information of the fourth decay product. If some of the tracks of the decays are not reconstructed, either because they are neutral, or either they are charged but they are soft, it means that the missing information may be attributed to the neutrino. *Missing tracks* will hence create partially reconstructed background. Some of the most dangerous are $B^+ \rightarrow D \mu^+ \nu$ type partially reconstructed backgrounds where $B^+ \rightarrow (D^0 \rightarrow K^- \pi^+ \mu^+ \mu^-) \mu \nu$, where $\mathcal{B}(D^0 \rightarrow K \pi^+ \mu^+ \mu^-) \approx 4.17 \times 10^{-6}$ and $B^+ \rightarrow D^0 \mu \nu \approx 10\%$. This predicts $\mathcal{B}(B^+ \rightarrow K^+ \pi^- \mu^+ \mu^-) = 1 \times 10^{-7}$.

4.3 Analysis strategy

The analysis of the $B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu_\mu$ decay is divided into several different parts; signal selection, optimisation, normalisation, fitting and limit setting. Throughout this document, charge conjugates of the decays are assumed unless stated otherwise. Results presented are based on the analysis of the full 3 fb^{-1} Run 1 dataset as well $\approx 1.7 \text{ fb}^{-1}$ Run 2 data (not using 2015 dataset due to very low sensitivity (high muon trigger thresholds)). Additionally the search will be conducted in a particular $\min q^2 = \min(q^2(\mu_1^+, \mu^-), q^2(\mu^-, \mu_2^+))$ region.

To perform the search for $B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu_\mu$, a specific preselection was applied to form potential signal candidates. To reconstruct the mass of the B^+ with missing information about the neutrino, a corrected mass variable $M_{B_{corr}} = \sqrt{M_{3\mu}^2 + |p_\perp^2|} + |p_\perp|$, where $M_{3\mu}^2$ is the invariant visible mass squared and p_\perp^2 is the missing momentum squared transverse to the direction of flight of B^+ , is introduced. A simulation sample that mimics the decay of the $B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu_\mu$ passing through preselection was used to develop further discriminating selection. To get the selection efficiency for different types of backgrounds, different proxy samples are used. For more details about samples used see Section ??.

Combinatorial background, which arises as random combinations of tracks passing the preselection, is taken from the upper corrected $\mu^+ \mu^- \mu^+$ mass side band, $M_{B_{corr}} > 5.5 \text{ GeV}$, where very few signal candidates are expected.

4.4 Samples

4.4.1 Data Samples

Results presented in this thesis are based on the analysis of the full 3 fb^{-1} Run I dataset at $\sqrt{s} = 7, 8 \text{ TeV}$ as well $\approx 1.7 \text{ fb}^{-1}$ Run II data at $\sqrt{s} = 13 \text{ TeV}$.

4.4.2 Simulation Samples

For signal simulation, three different decay models were exploited and are summarized in Table 4.1.

Channel	Year	Pythia	EVTGEN	Size	Stage
Simulation used for fitting mass shapes					
$B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu$	2012	Pythia 6.4 [23]	PHSP	0.5M	<i>generator-level+detector</i>
$B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu$	2012	Pythia 8.1 [2]	PHSP	0.5M	<i>generator-level+detector</i>
$B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu$	2012	Pythia 6.4 [23]	MINE	0.5M	<i>generator-level+detector</i>
$B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu$	2012	Pythia 8.1 [2]	MINE	0.5M	<i>generator-level+detector</i>
$B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu$	2016	Pythia 8.1 [2]	MINE	1.0M	<i>generator-level+detector</i>
Simulation used for evaluating <i>generator-level</i> efficiencies					
$B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu$	2012	Pythia 6.4 [23]	PHSP	25000	<i>generator-level</i>
$B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu$	2012	Pythia 6.4 [23]	MINE	25000	<i>generator-level</i>
$B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu$	2012	Pythia 8.1 [2]	MINE	25000	<i>generator-level</i>
Simulation used for ratification of minq^2 selection					
$B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu$	2012	Pythia 6.4 [23]	NIKI	25000	<i>generator-level</i>

Table 4.1: Summary of signal simulation samples used in this analysis with different decay models. In all cases the daughters of B^\pm are required to be within LHCb acceptance. All of this samples are mixture under magnetic polarity up and magnetic polarity down conditions.

Full phase space model, *PHSP*, only takes into account the kinematic constraints of the decay without taking into account any input from theoretical considerations as the matrix element is constant and hence angular momentum is disregarded.

In order to produce simulation with decay model which is more representative of the spin structure involved, following strategy is adapted. In this picture, the decay proceeds as follows: B^\pm decays into W^\pm and a pair of opposite sign muons and then

W^+ is decayed to $\mu^+ \nu$. *BTOSLLBALL* model [31], traditionally used for $B \rightarrow (K, K^*) l^+ l^-$ decay, with the form factor calculations can be used to simulate $B^\pm \rightarrow W^\pm l^+ l^-$ decay. After that, W^+ is decayed to $\mu^+ \nu$ using *PHSP*. For semileptonic $b \rightarrow sl^+ l^-$ transitions, there is a characteristic photon pole for low $q(\mu^+, \mu^-)$, invariant mass of the opposite muon pair, and flat distribution for $K^*(\mu^+, \nu_\mu)$, invariant mass of the muon and neutrino pair. In order to achieve this, a new pseudo-particle is introduced to *EVTGEN* with specific properties, $K^*(\mu^+, \nu_\mu)$, and the best output can be seen to be for a particle $K^*(\mu^+, \nu_\mu)$ with mass to be set to $0.1 \text{ GeV}/c^2$, and width, corresponding to $\tau = 1.3 \times 10^{-17}$ nanoseconds as can be seen in Figure 4.2. This procedure was also applied for the charge conjugate case. This model is denoted as *MINE* and is used as default in mass fits and efficiency calculations.

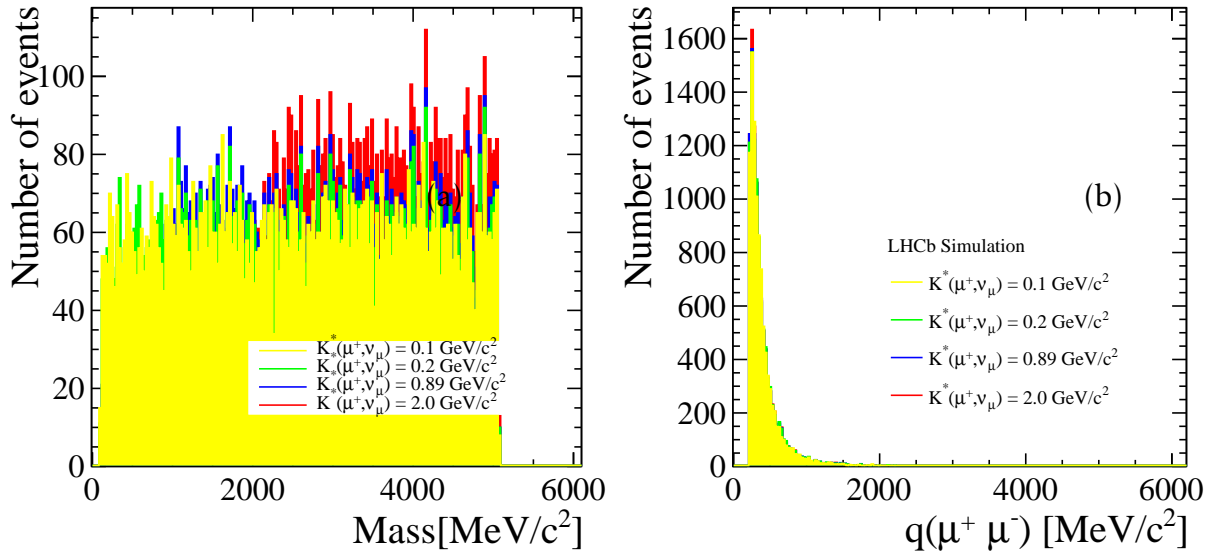


Figure 4.2: Distributions for signal MC in using Pythia 6.4 [23] conditions. (a) $K^*(\mu^+, \nu_\mu)$ (b) $q(\mu^+, \mu^-)$ distributions under different K^* mass hypotheses. The most flat distribution in $K^*(\mu^+, \nu_\mu)$ is plotted in yellow.

Finally, exclusively for this decay, a new decay model *B2MuMuMuNu* was added to *EVTGEN*, based on work performed by theorist Nikola Nikitin (write more once theory chapter is done and refer to it.). This model denoted as *NIKI*, is used mainly for

validation purposes.

4.5 Preselection for $B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu$

Set of initial identification for signal $B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu$ summarized in Table 4.2, also known as *stripping* selection was developed in order to improve signal to background ratio.

Firstly, all three muon tracks are required to have a significant IP with respect to the primary vertex. Minimum Impact Parameter χ^2 , ($\text{min IP } \chi^2$), gives the minimum significance of a particles's trajectory to the primary vertex. Hence by requiring $\text{min IP } \chi^2 > 9$ for muons is consistent with the hypothesis that the muon is 3σ away from the primary vertex and hence can be well differentiated. In addition, the change in the χ^2 if PV and SV vertices are fitted separately as opposed to common vertex fit, $\text{FD } \chi^2$, suppresses prompt backgrounds.

Each muon track is required to have good track χ^2 per number of degrees of freedom (ndof), ($\text{track } \chi^2/\text{ndof}$), of the fit as well as low P_{ghost} . This removes spurious tracks as well as tracks with low quality.

Each muon candidate is also identified with initial basic PID variables. Firstly muons are chosen due to their signature in the muons stations with the binary `i sMuon` decision. Secondly, muons candidates are chosen such that it is more likely that the candidate is muon than pion or kaon using global DLLmu variables defined in subsection 2.7.1. This reduces the background from misidentified muons.

In order to only select events which are compatible with the three muons originating from the same point in the space, ($\text{vertex } \chi^2/\text{ndof}$), the χ^2 of the trimuon vertex per degree of freedom fit required to be small. This decreases the contamination from *cascade decays* where the particle with the c quark content from $b \rightarrow \bar{c} \rightarrow s$, such as D , would have non-negligible lifetime leading to higher $\text{vertex } \chi^2/\text{ndof}$.

Requiring that B^+ direction points in the same direction as the line from PV to SV, ($\cos(\theta_B)$ - which measures the angle between these two vectors), is close to unity

Candidate	Stripping Selection	
muon	$\min \text{IP} \chi^2 > 9$	} track
muon	$p_T > 0$	
muon	$\text{track } \chi^2/\text{ndof} < 3$	
muon	$DLL_\mu > 0$	} PID
muon	$DLL_\mu - DLL_K > 0$	
muon	<code>isMuon==true</code>	
combination	$\cos(\theta_B) > 0.999$	
combination	$p_T > 2000 \text{ MeV}$	
combination	$\text{FD } \chi^2 > 50$	
combination	$\text{vertex } \chi^2/\text{ndof} < 4$	
combination	$0 \text{ MeV}/c^2 < M_B < 7500 \text{ MeV}/c^2$	
combination	$2500 \text{ MeV}/c^2 < M_{B_{\text{corr}}} < 10000 \text{ MeV}/c^2$	

Table 4.2: Selection of events based on muon and the B^+ candidate requirements. *Stripping selection* for the signal decay $B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu_\mu$ is the same for both Run1 and 2016 data.

translates into a well reconstructed event, which minimizes combinatorial background, where random track makes this pointing worse. Putting bounds on mass window, whether it is visible or corrected mass, also suppresses combinatorial events. This is because of on average higher momentum of combinatorial muon leading to higher mass.

4.6 Trigger Selection

In order to obtain triggered data, $B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu_\mu$ candidates are required to pass certain set of trigger decisions at L0, HLT1 and HLT2 levels summarized in Table 4.3. It can be noted that the decision is applied at the mother B^\pm level. In particular, Bplus_L0MuonDecision_T0S decision, means that one of the muons from B^\pm in an

event has triggered and made positive decision.

Trigger Selection	
Bplus_L0MuonDecision_T0S	
Bplus_Hlt1TrackMuonDecision_T0S	
Bplus_Hlt2TopoMu2BodyBBDTDecision_T0S	} OR
Bplus_Hlt2TopoMu3BodyBBDTDecision_T0S	
Bplus_Hlt2DiMuonDetachedDecision_T0S	
Bplus_Hlt2DiMuonDetachedHeavyDecision_T0S	

Table 4.3: Trigger selection applied on both signal and normalisation samples

As discussed in [section 2.8](#) L0MuonDecision decides on whether an event is accepted depending on the p_T of muon and the number of hits in the SPD. Run I can be split into 2011 and 2012 conditions where, in 2011 the most used threshold for positive decision is 1.48 GeV/c [32] and 1.76 GeV/c [20]. Run I SPD rate only accepts events below 600. In Run II, the trigger thresholds varied more but the most representative acceptance for muon p_T was above 1.85 GeV/c with SPD multiplicity below 450.

Hlt1TrackMuonDecision accepts events where mother particle has certain lifetime, such as B, D by requiring certain $IP\chi^2$ of the track with respect to all of its PVs. There has to be at least one muon (isMuon==true) in its final state with certain kinematic thresholds on p and p_T , for example in 2011, the identified muons that triggered positive decision had to have p above 8 GeV/c [32].

At HLT2 level, the candidates are required to pass through at least one of the four decisions. Hlt2TopoMu[2,3]BodyBBDTDecision belong to the *topological triggers* category with extra requirement of muon being identified by isMuon decision. Hlt2DiMuonDetachedDecision and Hlt2DiMuonDetachedHeavyDecision reconstruct decays with two muons in a final state, dimuon. The two lines differ in selection that is optimized either for heavy or light dimuon pair. For example,

Hlt2DiMuonDetachedDecision accepts events dimuon p_T above 1.5 GeV/ c and with mass above 1 GeV/ c^2 , whereas Hlt2DiMuonDetachedHeavyDecision accepts dimuon pairs with any p_T but above 2.95 GeV/ c^2 in mass. The reason why these lines are called detached are because individual muons are required to have high $IP\chi^2$.

4.7 q^2 selection

In $B^+ \rightarrow \mu^+ \mu^- \mu^+ \nu$, two pairs of opposite sign muons can be formed, namely $q^2(\mu_1, \mu_2)$ and $q^2(\mu_2, \mu_3)$ where $\mu_1 = \mu^+, \mu_2 = \mu^-, \mu_3 = \mu^+$. From the two invariant mass squared pairs one can define, $minq^2 = \min[q^2(\mu_1, \mu_2), q^2(\mu_2, \mu_3)]$ and $maxq^2 = \max[q^2(\mu_1, \mu_2), q^2(\mu_2, \mu_3)]$. This measurement is made in region where $\sqrt{minq^2} = minq < 980$ MeV/ c^2 because of two main reasons: most of the contributions to the amplitude of the decay is below this value and combinatorial background is greatly reduced if $minq^2 < 1$ (GeV/ c^2)², see Figure 4.3.

In order to remove backgrounds that proceed via resonant J/ψ and $\Psi(2S)$ contributions, vetoes in invariant mass are placed in the corresponding regions, see Table 4.4 for more details.

Veto	$minq$ [MeV/ c^2]	$maxq$ [MeV/ c^2]
J/ψ	!(2946.0 < $minq$ < 3176.0)	!(2946.0 < $maxq$ < 3176.0)
$\Psi(2S)$!(3586.0 < $minq$ < 3766.0)	!(3586.0 < $maxq$ < 3766.0)

Table 4.4: Vetoes for J/ψ and $\Psi(2S)$ contributions.

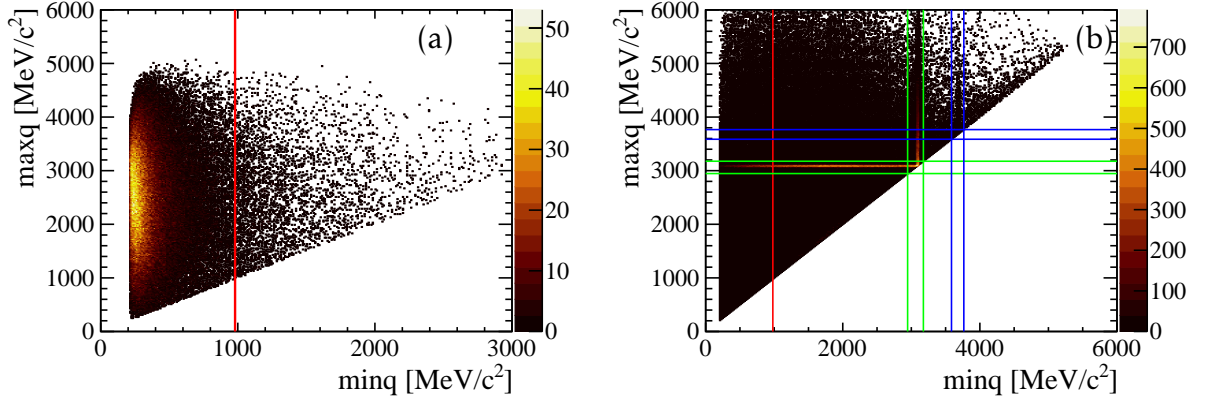


Figure 4.3: (a) Signal simulation sample distribution in $minq$ and $maxq$ variables. Values below $980 \text{ MeV}/c^2$ (red line) are accepted. (b) Combinatorial data sample after *stripping* selection with no other cuts shows clearly the J/ψ (green) and $\Psi(2S)$ (blue) resonances which are vetoed and the measurement region (red).

4.8 Further Selection

Further selection was performed as seen in [Table 4.5](#). It further helps to suppress backgrounds and but requires different treatment in Run I and Run II due to the different definitions of variables.

Idea	Object	Selection Run1	Selection 2016
Clean	Muon	-	IsMuonTight==1.0
Clone and ghost	Muon	Nshared==0	Nshared<2
Fit Region	B	$4000 \text{ MeV} < M_{B_{corr}} < 7000 \text{ MeV}$	$4000 \text{ MeV} < M_{B_{corr}} < 7000 \text{ MeV}$
Bkg Removal	event	Combinatorial BDT selection	Combinatorial BDT selection
Bkg Removal	event	Misid BDT selection	Misid BDT selection
Optimize FOM	Muon	$\text{PID}_{\mu} > 0.35$	$\text{PID}_{\mu} > 0.35$

Table 4.5: Offline selection performed after stripping. Differences can be seen between Run I and Run II datasets

Bibliography

- [1] *Image of the cern accelerator complex from* <https://cds.cern.ch/record/2225847>, Accessed in 2017.
- [2] T. Sjostrand, S. Mrenna, and P. Z. Skands, *A Brief Introduction to PYTHIA 8.1*, **Comput. Phys. Commun.** **178** (2008) 852, [arXiv:0710.3820](#).
- [3] LHCb collaboration, *Image of the acceptance of using simulation taken from* https://lhcb.web.cern.ch/lhcb/speakersbureau/html/bb_productionangles.html, Accessed in 2017.
- [4] G. Raven, *LHCb: Status and prospects*, **Nucl. Phys. Proc. Suppl.** **163** (2007) 153, [153(2007)].
- [5] LHCb collaboration, R. Aaij *et al.*, *Measurement of $\sigma(pp \rightarrow b\bar{b}X)$ at $\sqrt{s} = 7$ TeV in the forward region*, **Phys. Lett.** **B694** (2010) 209, [arXiv:1009.2731](#).
- [6] LHCb collaboration, R. Aaij *et al.*, *Measurement of the b -quark production cross-section in 7 and 13 TeV pp collisions*, **Phys. Rev. Lett.** **118** (2017) 052002, Erratum **ibid.** **119** (2017) 169901, [arXiv:1612.05140](#).
- [7] *Image of the lhcb detector from* <http://cds.cern.ch/record/1087860>, Accessed in 2017.
- [8] *Image of the luminosity overview from* <https://lbggroups.cern.ch/online/operationsplots>, Accessed in 2017.

- [9] LHCb collaboration, R. Aaij *et al.*, *LHCb detector performance*, *Int. J. Mod. Phys. A* **30** (2015) 1530022, [arXiv:1412.6352](#).
- [10] The LHCb collaboration, A. A. Alves, Jr. *et al.*, *The LHCb Detector at the LHC*, *JINST* **3** (2008) S08005.
- [11] R. Aaij *et al.*, *Performance of the LHCb Vertex Locator*, *JINST* **9** (2014) 09007, [arXiv:1405.7808](#).
- [12] *Image of the Tracker System taken from the LHCb public web page*, Accessed in 2017.
- [13] LHCb collaboration, R. Aaij *et al.*, *Measurement of the track reconstruction efficiency at LHCb*, *JINST* **10** (2015) P02007, [arXiv:1408.1251](#).
- [14] R. Hierk, M. Merk, M. Needham, and R. Van der Eijk, *Performance of the LHCb 00 track fitting software*, Tech. Rep. LHCb-2000-086, CERN, Geneva, Aug, 2000.
- [15] LHCb, R. Forty, *RICH pattern recognition for LHCb*, *Nucl. Instrum. Meth. A* **433** (1999) 257.
- [16] M. Adinolfi *et al.*, *Performance of the LHCb RICH detector at the LHC*, *Eur. Phys. J. C* **73** (2013) 2431, [arXiv:1211.6759](#).
- [17] LHCb collaboration, R. Aaij *et al.*, *Measurement of b -hadron branching fractions for two-body decays into charmless charged hadrons*, *JHEP* **10** (2012) 037, [arXiv:1206.2794](#).
- [18] A. A. Alves Jr. *et al.*, *Performance of the LHCb muon system*, *JINST* **8** (2013) P02022, [arXiv:1211.1346](#).
- [19] F. Archilli *et al.*, *Performance of the muon identification at LHCb*, *JINST* **8** (2013) P10020, [arXiv:1306.0249](#).
- [20] LHCb HLT project, J. Albrecht, V. V. Gligorov, G. Raven, and S. Tolk, *Performance of the LHCb High Level Trigger in 2012*, *J. Phys. Conf. Ser.* **513** (2014) 012001, [arXiv:1310.8544](#).

- [21] *Images of different trigger schemes taken from the LHCb speaker's bureau web page*, Accessed in 2018.
- [22] LHCb, I. Belyaev *et al.*, *Handling of the generation of primary events in Gauss, the LHCb simulation framework*, *J. Phys. Conf. Ser.* **331** (2011) 032047.
- [23] T. Sjostrand, S. Mrenna, and P. Z. Skands, *PYTHIA 6.4 Physics and Manual*, *JHEP* **05** (2006) 026, [arXiv:hep-ph/0603175](#).
- [24] D. J. Lange, *The EvtGen particle decay simulation package*, *Nucl. Instrum. Meth.* **A462** (2001) 152.
- [25] P. Golonka and Z. Was, *PHOTOS Monte Carlo: A Precision tool for QED corrections in Z and W decays*, *Eur. Phys. J.* **C45** (2006) 97, [arXiv:hep-ph/0506026](#).
- [26] Geant4 collaboration, J. Allison *et al.*, *Geant4 developments and applications*, *IEEE Trans. Nucl. Sci.* **53** (2006) 270.
- [27] Geant4 collaboration, S. Agostinelli *et al.*, *Geant4: a simulation toolkit*, *Nucl. Instrum. Meth.* **A506** (2003) 250.
- [28] LHCb, M. Clemencic *et al.*, *The LHCb simulation application, Gauss: Design, evolution and experience*, *J. Phys. Conf. Ser.* **331** (2011) 032023.
- [29] D. Martnnez Santos and F. Dupertuis, *Mass distributions marginalized over per-event errors*, *Nucl. Instrum. Meth.* **A764** (2014) 150, [arXiv:1312.5000](#).
- [30] T. Skwarnicki, *A study of the radiative CASCADE transitions between the Upsilon-Prime and Upsilon resonances*, PhD thesis, Cracow, INP, 1986.
- [31] A. Ali, P. Ball, L. T. Handoko, and G. Hiller, *A Comparative study of the decays $B \rightarrow (K, K^*)\ell^+\ell^-$ in standard model and supersymmetric theories*, *Phys. Rev.* **D61** (2000) 074024, [arXiv:hep-ph/9910221](#).

- [32] R. Aaij *et al.*, *The LHCb Trigger and its Performance in 2011*, [JINST 8 \(2013\) P04022](#),
[arXiv:1211.3055](#).

Appendices

Appendix A

Boosted Decision Trees

Many rare decay analyses make extensive use of BDTs and they are important in the $\Lambda_b^0 \rightarrow p\pi^-\mu^+\mu^-$ analysis. Firstly, the concept of a decision tree is introduced followed by a brief explanation of boosted decision trees.

A decision tree, in the context of data mining, is a supervised machine learning method which allows for the prediction of the value of a target variable based on several input variables. In particle physics, the purpose of the decision tree is to classify an event as being either signal or background, based on the event's input variables. The input variables, $\{x_i\}$, are various physics parameters. Each cut point in the tree is referred to as a node and the final nodes are referred to as leaves. A very simple example is shown in [Figure A.1](#). The purity, P , of a leaf refers to the fraction of the weight of a leaf due to signal events, e.g. if a leaf had 20 signal events and 15 background events it would have a purity of 0.75. If a leaf has a purity larger than 0.5 it is deemed to correspond to signal and if lower, to background.

A decision tree is constructed by a process called training. For this, samples of known signal and background events are used. These samples could be either simulation or data. For each x_i the best dividing point is decided, that is, the cut that gives the best separation between signal and background. This optimum point is decided by using the Gini index defined as

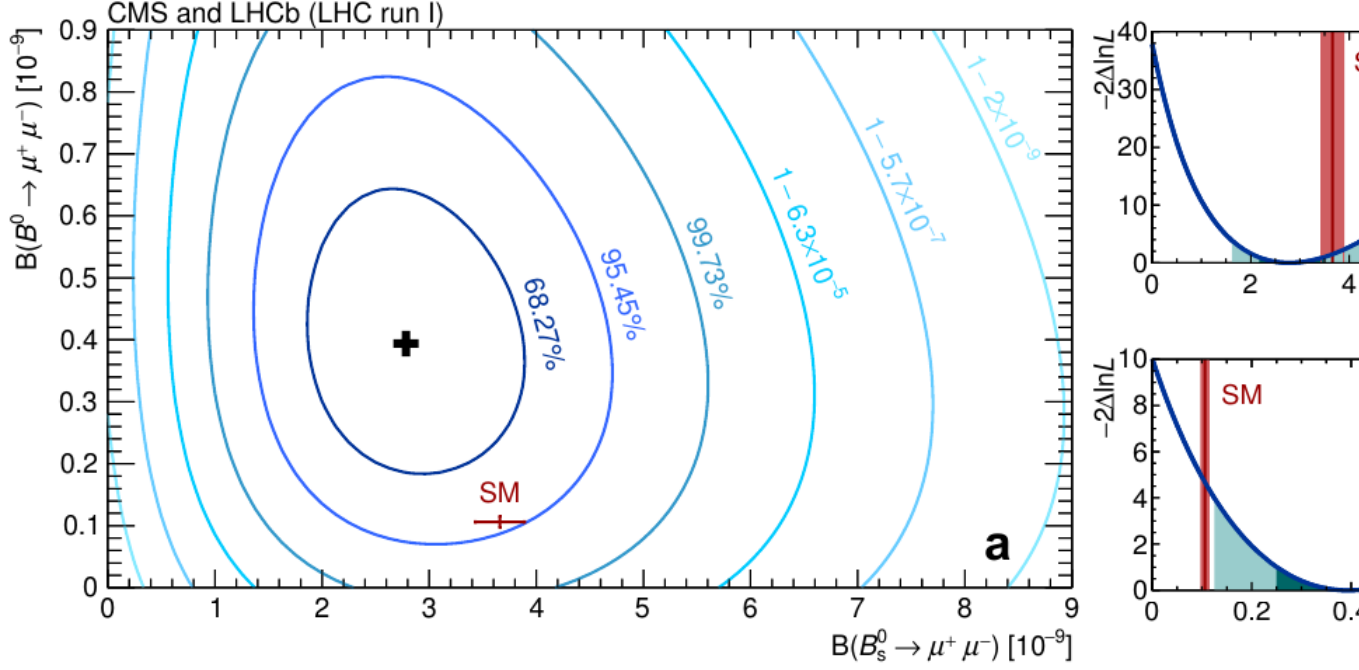


Figure A.1: An example decision tree. The S and B stand for ‘Signal-like’ and ‘Background-like’. The β_i variables refer to the cut values chosen by the machine learning algorithm after the tree has been trained on signal and background samples. The blue ovals represent final nodes called leafs, which each leaf having an associated purity, i.e. the fraction of the weight of a leaf due to signal events.

$$Gini = \sum_{i=1}^n W_i P(1 - P), \quad (\text{A.1})$$

where W_i is weight of the i^{th} event, which would generally be unity for the case of a non-boosted decision tree. The cutting point is then found by maximising the separation, Δ , between the Gini index of the parent node and the combined Gini index of the child nodes, as given in [Equation A.2](#)

$$\Delta = Gini_{parent} - Gini_{child_1} - Gini_{child_2}. \quad (\text{A.2})$$

The depth of a tree (the maximum number of cuts or nodes) is normally a number

specified before the training begins.

Boosting a decision tree involves training many trees ($\mathcal{O} \sim 1000$) and giving misclassified events a higher weight. A misclassified event is defined as a known signal event being placed on a background leaf and vice versa. By giving the events which are difficult to classify more weight, the next tree to be trained will effectively have to work harder in order to classify events correctly.

The total score on an event is deduced by following an event through from tree to tree and, for the algorithms used in this thesis, is simply given by the weighted sum of the scores over the individual trees.

Data sets are split into two (or more) sub samples, where one half is used for training the tree and the other is used for testing the tree, and the distributions of the event scores (the BDT output) for training and testing samples are compared for signal and background. Cases where the training sample performs better than the testing sample are referred to as over-trained trees, which is often due to the BDT becoming sensitive to the statistical fluctuations of the training sample.

The distribution of events scores for a given dataset can then be cut on in order to increase the fraction of signal events.

Appendix B

The *sPlot* technique

The *sPlot* technique is used extensively throughout this thesis. It is used in cases when there is a merged dataset which consists of data from different sources of data species, namely background and signal. These datasets are assumed to have two different sets of variables associated with the events they contain. Discriminating variables are those whose distributions are known for background and signal. Control variables are those whose distributions are unknown, or are assumed to be unknown.

The *sPlot* technique allows the distribution of the control variables for each data species to be deduced by using the species discriminating variable. This method relies on the assumption that there is no correlation between the discriminating variable and the control variable. The discriminating variable used in this thesis is always the mass distribution. The full mathematical description of the *sPlot* technique can be found in Ref [2], the key points are outlined here.

An unbinned extended maximum likelihood analysis of a data sample of several species is considered. The log-likelihood is expressed as

$$\mathcal{L} = \sum_{e=1}^N \left\{ \ln \sum_{i=1}^{N_s} N_i f_i(y_e) \right\} - \sum_{i=1}^{N_s} N_i, \quad (\text{B.1})$$

where N is the total number of events considered, N_s is the number of species of event (i.e. two - background and signal), N_i is the average number of expected events for

the i^{th} species, y represents the set of discriminating variables, $f_i(y_e)$ is the value of the Probability Density Function (PDF) of y for event e for the i^{th} species and the control variable, x , does not appear in the expression of \mathcal{L} by definition.

For the simple (and not particularly practical) case of the control variable x being a function of y , i.e. completely correlated, one could naively assume that the probability of a given event of the discriminating variable y being of the species n would be given by

$$\mathcal{P}_n(y_e) = \frac{N_n f_n(y_e)}{\sum_{k=1}^{N_s} N_k f_k(y_e)}. \quad (\text{B.2})$$

The distribution for a control variable x for the n^{th} species, $M_n(x)$, can be deduced by histogramming in x and applying $\mathcal{P}_n(y_e)$ as a weight to event e . In this scenario the probability, $\mathcal{P}_n(y_e)$, would run from 0 to 1.

In the case considered in this thesis, where x is entirely uncorrelated with y , it can be shown that $\mathcal{P}_n(y_e)$ can be written as

$$\mathcal{P}_n(y_e) = \frac{\sum_{j=1}^{N_s} V_{nj} f_j(y_e)}{\sum_{k=1}^{N_s} N_k f_k(y_e)}, \quad (\text{B.3})$$

where V_{nj} is the covariance matrix between the species n and the j^{th} species. The inverse of this covariance matrix is given by the second derivative of $-\mathcal{L}$ in [Equation B.1](#).

The quantity in [Equation B.3](#) is donated as the sWeight. In this thesis the species, n , in [Equation B.3](#) is always the signal. Because of the presence of the covariant derivative the sWeight of an event can be both positive and negative. The more negative an event is, the more likely it is to be background and vice versa for positive sWeights. The signal distribution for the control variable x , $M_s(x)$, can again be deduced by histogramming events in x , applying the sWeight to each event.

Appendix C

Fitting functions

C.1 Crystal Ball function

Crystal Ball (CB) function [30] is usually used for fitting of signal mass peaks in the invariant mass distributions. The CB function consists of Gaussian function (which usually describes mass peak) with a power-law tail below a certain threshold. Its PDF is defined as

$$f(x; \alpha, n, \bar{\mu}, \sigma) = N \cdot \begin{cases} e^{-\frac{(x-\bar{\mu})^2}{2\sigma^2}}, & \text{if } \frac{(x-\bar{\mu})}{\sigma} > \alpha \\ A \cdot \left(B - \frac{(x-\bar{\mu})}{\sigma}\right)^{-n}, & \text{otherwise} \end{cases} \quad (\text{C.1})$$

where A, B and N are all constants that depend on $\alpha, n, \bar{\mu}, \sigma$ ensuring correct normalisation and continuity of the first derivative. Thus, if α is positive, the tail, $A \cdot \left(B - \frac{(x-\bar{\mu})}{\sigma}\right)^{-n}$, will start below the mean, usually arising from the photon-radiating decay products (left tail) and vice versa for the case where α is negative, arising from non-Gaussian resolution effects (right tail).

If one has to deal with different per-event uncertainties on the mass, one way is to model this by a sum of two Crystal Ball functions, where then each uncertainty on the event, would correspond to sum of two delta functions. Hence, double-sided Crystall Ball is defined as a linear combination of $f(x; \alpha, n, \bar{\mu}, \sigma)$:

$$g(x; \alpha, n, \bar{\mu}, \sigma, f_{cb}) = f_{cb} \cdot f(x; \alpha, n, \bar{\mu}, \sigma) + (1 - f_{cb}) \cdot f(x; \alpha, n, \bar{\mu}, \sigma). \quad (\text{C.2})$$

C.2 Double-sided Ipatia function

Generalisation of (double-sided) Crystal Ball function where per-event uncertainty is taken into account, known as (double-sided) Ipatia function, [29].

C.3 Rookeys function from R00FIT package

A non-parametric function that is composed of superposition of Gaussians with equal surface, but with different widths σ , which are established by data at a given point.