Computational Physics / PHYS-GA 2000 / Problem Set 3

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Problem 1

The matrix multiplication time vs. N is plotted together with the plot of N^3. As the graph below showed, the time rises with the matrix size, and it aligns with the plot of N^3 as predicted. The plot of using dot() method is also shown. Taken N = 1000 as the example, the time calculated through explicit function is 604.0759706497192 seconds, but the time used for dot() method is only 0.03136086463928223 seconds.

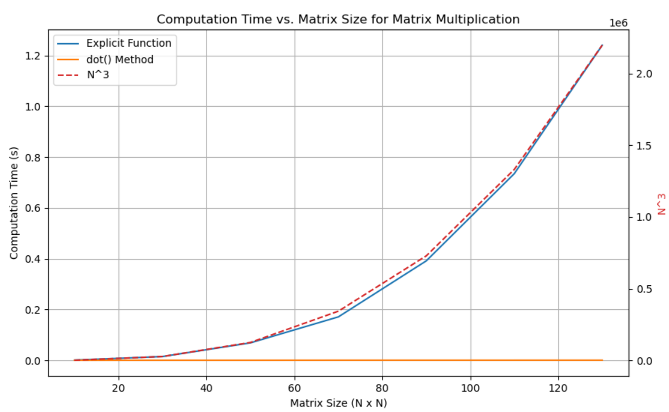

Figure 1: Computation time using different methods

Problem 2

Description: We started with 10000 atoms. Four lists are created to track the number of different isotopes at each time step. The simulation loops follow the decay routes with the listed probabilities and half-time.

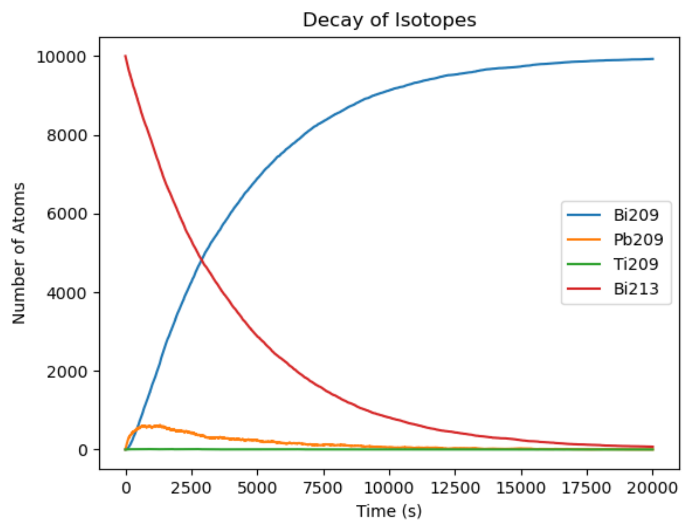

Figure 2: Radioactive decay chain, numbers of atoms for each element as function of time

Problem 3

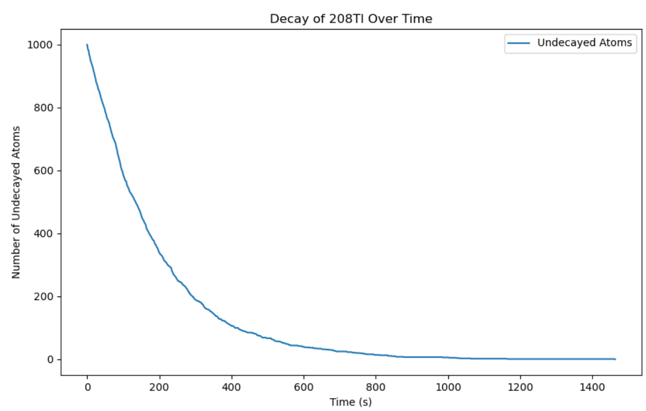

Figure 3: Radioactive decay simulation using faster nonuniform distribution method, numbers of atoms for each element as function of time

Description: In the code, besides the constants, random numbers are generated from the nonuniform distribution and decay times are calculated using the transformation method. The sort() function is used to sort the decay times in the increasing order and the time array was created. The undecayed atoms are counted at each time step and the numbers are plotted as a function of time.

Problem 4

The Central Limit Theorem (CLT) states that the distribution of the sample mean will tend to be normally distributed, regardless of the original distribution, as the sample size increases. We define a random variable y as the average of N independent exponentially distributed random variables x_i:

$$y = \frac{1}{N} \sum_{i=1}^{N} x_i$$

The mean and variance of the exponential distribution are given by:

$$\mu = E[x_i] = 1, \quad \sigma^2 = \mathrm{Var}(x_i) = 1$$

For the average y, the expected mean and variance can be calculated as:

$$E[y] = \frac{1}{N} \sum_{i=1}^{N} E[x_i] = 1$$

$$\mathrm{Var}(y) = \frac{1}{N^2} \sum_{i=1}^{N} \mathrm{Var}(x_i) = \frac{1}{N}$$

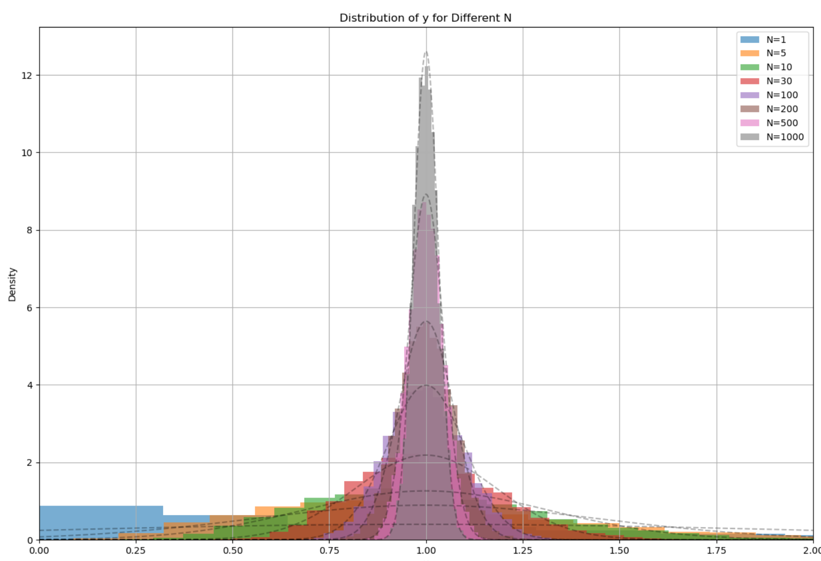

Figure 4: Distribution of y for Different N Values

In the code samples of y for various values of N (specifically $N = 1, 5, 10, 30, 100, 200, 500, 1000$) are generated. As N increases, the distribution of y approaches a normal distribution, as shown in Figure. Additionally, we calculate the mean, variance, skewness, and kurtosis of y for a range of N values. For estimating at which N the skewness and kurtosis have reached about 1% of their value for $N = 1$, we draw an expectation line of 1%) on each graph and make the estimation but it was of hard to tell. So I asked the program to print out the first number to reach below 1%). After run the program for a few times, I found that for skewness is around 500, and for kurtosis is around 30.

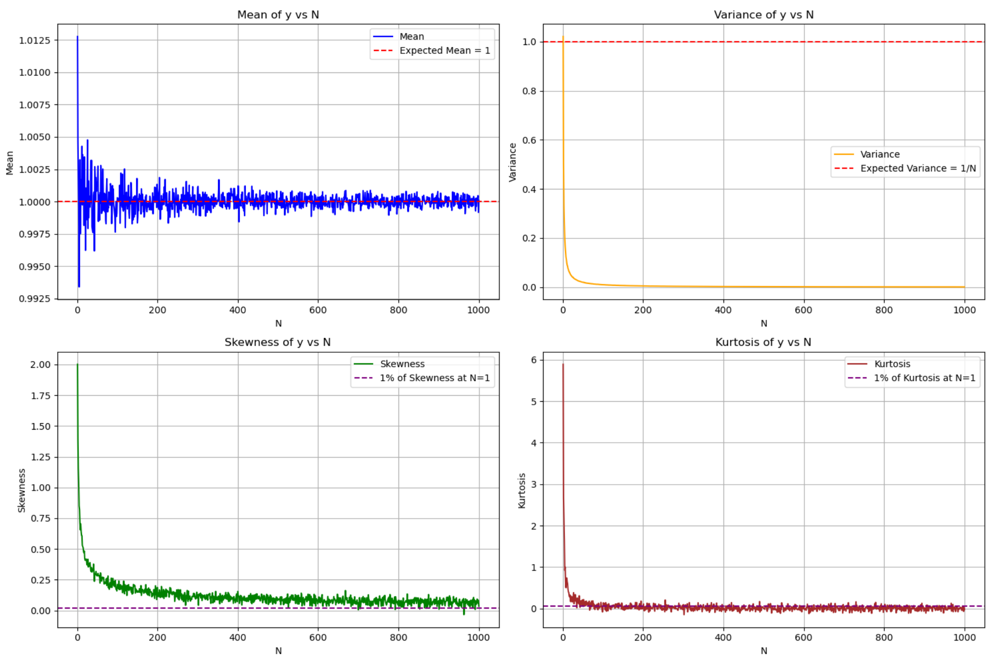

Figure 5: Statistical Properties of y as Functions of N