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Master Thesis

Comparing Hierarchical and On-The-Fly Model Checking for Java Pointer Programs

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Aachen, den 19. Juli 2019, Sally Chau

Abstract

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Chapter 1

Introduction

Nowadays software systems are to be found in every corner of our everyday lives and their impact is rather increasing. Smart devices, automated processes, and digital support in economy and industry are indispensable. Therefore, their reliability is of outmost importance in order to allow for bug-free behavior. Think of onboard computers for vehicles, like cars and planes, that monitor the characteristics of the vehicle. The system helps to communicate information of the vehicle between the conductor, the vehicle's components, and possibly a third instance, such as a control station. An error within this software system might cause substantial damage. Imagine a faulty distance value that is displayed to a pilot of a plane. Therefore, it is important to ensure that a system is free of errors.

In order to ensure the absence of errors in a software system, software verification checks that a specified property is satisfied by the system under consideration. In contrast to testing, software verification does not only discover errors, but ensures their absence. Examples for properties checked during verification are reachability properties, i.e., a certain point in the program can be reached, memory leaks, or cycle detection.

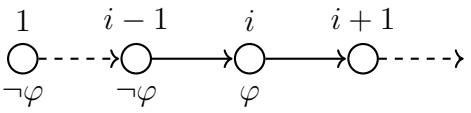
A formal verification technique that examines whether the system under consideration satisfies a specified property is *model checking*. The analysis yields three possible outcomes:

1. The program *satisfies* the property.

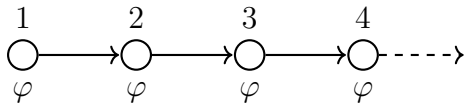
2. The program *violates* the property. The analysis returns a counterexample indicating at which state of the program the property is violated.

3. It is *unknown* whether the program satisfies the property. This answer is returned if the analysis terminates as computational resources are exceeded, e.g., the computation runs out of memory.

The property φ is eventually fulfilled.



The property φ is always fulfilled.



The property φ_2 holds until the property φ_1 holds.

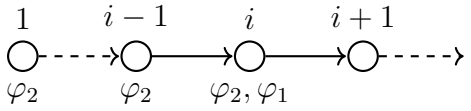


Figure 1.1: Examples for modalities of time in linear temporal logic.

In order to apply model checking for system verification, two ingredients are required: a suitable *model* of the system and a formal description of the *properties* that are checked.

According to the ATTESTOR framework by Heinen et al. [8], a model checking tool for JAVA pointer programs, we model the system (or program) under consideration as a graph, called *state space* or *transition system*, where vertices represent states of the system and edges between two states model a transition from one state to another. A transition can be interpreted as a change in a system's state. Each state models a point during program execution. It contains a representation of the program heap which is depicted by a graph. Pointer programs contain dynamic data structures that induce potentially infinite state spaces as objects are created at runtime. Also arbitrary sizes of input to a program have to be considered. In order to finitely represent possibly infinite states, Heinen et al. employ *hypergraphs* that summarize data structures by replacing subgraphs with placeholders (also called non-terminals) [8]. Placeholders represent several subgraphs and offer a compact representation of possibly infinite data structures. A hypergraph containing placeholders is called abstract while a hypergraph without any placeholders is called concrete. The process of replacing subgraphs by placeholders is called abstraction while the reverse is called concretisation. The rules that define how abstraction and concretisation are performed are summarized in graph grammars.

Based on the generated state space, model checking is performed for a specified property and returns whether the property is satisfied, violated or that the result is unknown. We focus on linear-time properties specified by formulae in *linear temporal logics* (LTL), a logic that formulates properties for paths with respect to modalities of time, e.g., a property is eventually fulfilled, a property is always satisfied, or a property holds until another one becomes true. In LTL, time is understood as a

linear sequence of moments where each moment is followed by a well-defined successor moment. Figure 1.1 illustrates the aforementioned examples. In order to verify LTL formulae for a state space, ATTESTOR implements the tableaux construction algorithm by Grumberg et al. [4] that constructs a proof structure based on a set of rules in order to infer whether a property is satisfied by a state [4]. The approach thus successively divides a verification goal into subgoals until a statement can be made about the result of the verification goal.

Often software programs contain calls to procedures that induce (procedure) state spaces. In ATTESTOR, effects of procedure calls are applied to the return state in the state space of the main method of an input program such that the resulting state space reflects the behavior of the main method. Hence, model checking is executed for the top level state space only while procedure state spaces are not directly included. The question arises whether solely verifying a program's top level state space is sufficient to conclude about the entire validity of the system with respect to a specified property.

```
1  public class SLList {
2    private SLList next;
3    public SLList(SLList list) {
4      this.next = list;
5    }
6  }
```

Listing 1.1: JAVA class definition for singly-linked lists.

Now, consider the JAVA class `SLList` specified in Listing 1.1 that defines a singly-linked list with a `next` variable that points to the next element in a list. A method `traverse` that traverses a singly-linked list is given in Listing 1.2. We added a procedure `swap` that takes a list as an input and dereferences the list's `next` pointer, but references the original value again at the end of the procedure. Thus, the original list is modified during the execution of the procedure `swap`, but not in the main procedure `traverse`. We demand that the method should satisfy the property that a list is not modified during the run of the program. We express this property by the LTL formula

$$\square\,\{\texttt{identicNeighbours}\}$$

which states that for every state all neighbors stay the same during the execution of the program.

```
1  public static void traverse(SLList head) {
2    SLList current = head;
3    while (current.next != null) {
```

```
4             current = current.next;
5             head = swap(head);
6           }
7         }
8
9         public static SLList swap(SLList head) {
10          SLList list = head.next;
11          head.next = null;
12          head.next = list;
13          return head;
14        }
```

Listing 1.2: JAVA code for traversing a singly-linked list that contains erroneous behaviour in a called procedure.

When performing top level model checking, as implemented in the ATTESTOR tool, for the given input program in Listing 1.2 and the above mentioned LTL formula, the result is that the property, that the list is never mutated, is satisfied, although the list is modified during the procedure `swap`. As this modification has no effect on the top level state space, the failure cannot be detected by the currently implemented top level model checking algorithm. Therefore, in this thesis, we analyze two algorithms that do not only check the top level state space but also verify procedure state spaces. We refer to these algorithms as *hierarchical model checking* algorithms as they directly perform verification on the procedure state spaces induced by possibly recursive procedure calls.

Hierarchical model checking includes three main challenges. One challenge is that procedure state spaces need to be computed and model checked. This induces an increase in time and space demands compared to top level model checking as several state spaces need to be considered. Moreover, reoccuring procedure calls need to be handled such that already generated state spaces or checked verification goals are not computed repeatedly. Another aspect to consider in hierarchical model checking is that the recursion depth of a procedure can be unbounded. Therefore, we have to account for possibly infinite recursion depth.

As a first algorithm for hierarchical model checking, we introduce an on-the-fly approach that combines an on-the-fly state space generation with the tableaux construction for model checking. This approach allows for efficient detection of errors in a program that does not require an entirely generated state space (including procedure state spaces). The second algorithm is based on the concept of recursive state machines (RSM) that capture the hierarchical structure of methods with (recursive) procedure calls in a compact manner [1]. After generating an RSM for an input program, the tableaux construction is performed for the RSM in order to verify given

LTL specifications.

A detailed description of the Attestor tool is given in the next section while the theoretical foundations of this thesis are described in Chapter 2 including the concepts of hypergraphs by Heinen et al. [8], recursive state machines by Alur et al. [1], and linear temporal logics. Chapter 3 presents the tableaux construction for model checking LTL formulae by Grumberg et al. [4]. Thereupon, Chapters 4 and 5 introduce the on-the-fly hierarchical and RSM-based model checking algorithms, respectively. The thesis concludes with experimental results on the effectiveness and efficiency of the introduced algorithms compared to the top level model checking procedure in Chapters 6 and 7.

1.1 Attestor

Attestor is a verification tool that checks specified properties for a Java program. The tool generates a state space for the program under consideration, where each state represents the current heap and transitions between states model changes in a state. Procedures of the input program are summarized by procedure contracts that specify the heap prior to and after their execution. Thus, procedure state spaces do not need to be computed repeatedly for the same heap. After the state space has been generated, model checking is performed by checking the resulting state space against LTL specifications. Possible outcomes for the model checking procedure are that the input program either fulfills the property, violates the property, or the result is unknown. In case of a property violation, a counterexample is provided.

The Attestor architecture divides up into input, back-end, front-end, and output as depicted in Figure 1.2. The tool offers several options to specify a verification task including a Java program to be analyzed and LTL formulae that describe the properties verified during model checking. Moreover, Attestor requires graph grammars that define data structures present in the input program. Next to the pre-defined grammars in the tool (singly-linked lists, doubly-linked lists, trees), the user has the option to define custom ones. Further options include the specification of initial heaps or properties for state space generation. Attestor's core is the back-end which contains the program analysis. The front-end communicates via the Attestor API with the back-end to visualize the output such as the generated state space.

The Attestor back-end constitutes the core process of the tool which is divided into six phases. The first three phases comprise the preprocessing of the verification task, followed by the state space generation phase and the model checking phase.

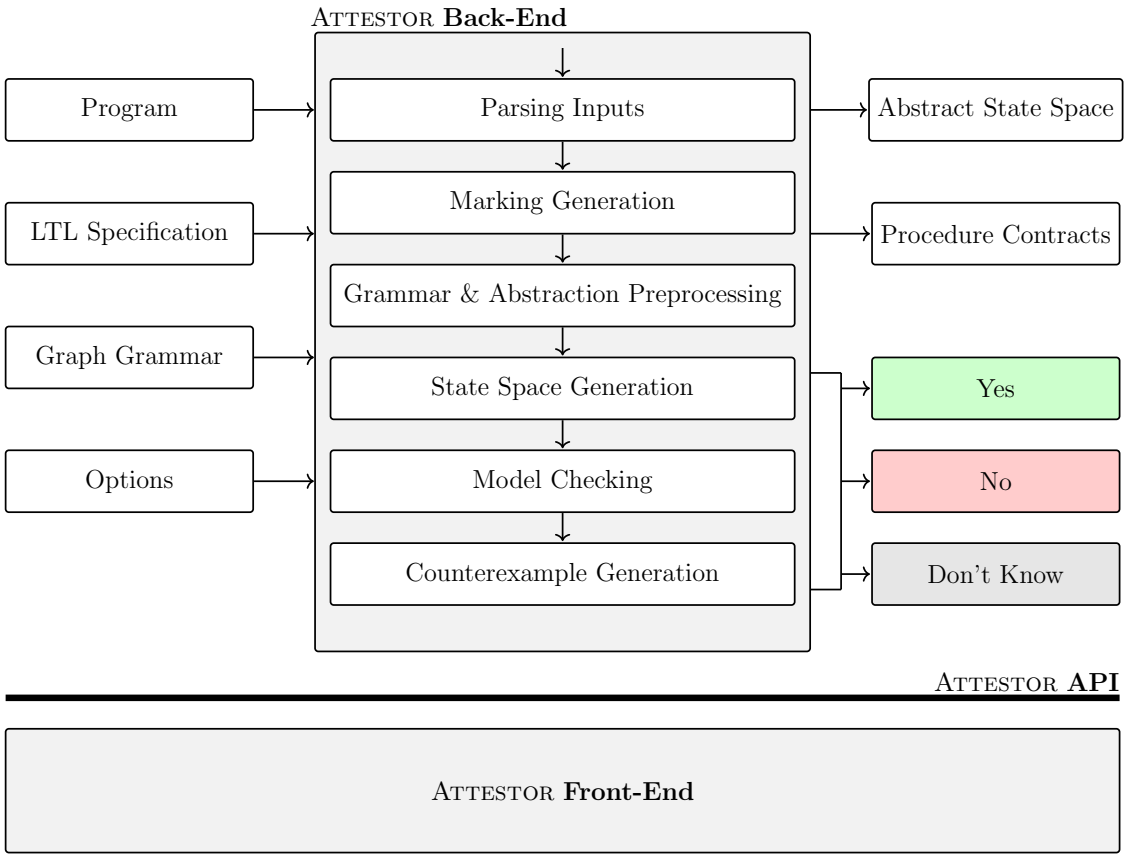


Figure 1.2: ATTESTOR architecture. [2]

Phase 1: Parsing Inputs In the first phase, the supplied input options passed to ATTESTOR, including the input program and optional parameters, are parsed.

Phase 2: Marking Generation After parsing the input, markings are added to the initial heap if required by the specified LTL properties [7]. The markings track object identities during program execution along sequences of states so that properties such as neighborhood preservation can be checked.

Phase 3: Grammar and Abstraction Preprocessing In this phase, state space generation is prepared by refining the input grammar such that properties can be decided more efficiently, e.g., by only considering hypergraphs that satisfy a specified property. Furthermore, abstraction preprocessing computes the transformers required for state space generation itself, e.g., garbage collection.

Phase 4: State Space Generation After the stages of preprocessing, the state space of the input program is generated by executing program statements on the initial heap such that new states are added. The procedure is illustrated in Figure 1.3. The abstract execution loop is executed until either there are no more states left to process, a fixpoint has been reached or computational resources are exceeded. The loop starts with picking a state, which has not been processed yet, in a DFS manner, i.e., unprocessed states are added to and removed from the end of a list of unprocessed states. The abstract semantics of the next statement are applied to the state. As hypergraphs potentially contain abstracted parts representing a number of concrete subgraphs, the heap potentially needs to be concretised so that the statement is executed on a concrete heap. Concretisation is achieved by applying grammar rules in a forward manner. Thereafter, the heap is cleared, e.g., dead variables are removed and the garbage collector performs its actions. In order to obtain a compact heap representation, heap abstraction is performed by applying grammar rules in a backward manner introducing placeholders for fragments of the graph. Finally, the resulting state is labeled with atomic propositions that are satisfied by the heap. Labels provide information about structural properties about the heap, e.g., the shape of the heap, reachability information or the state is a terminal state. The labeling is implemented by heap automata [2]. Before adding the resulting state to the state space, it is checked whether a more abstract state already covers the current one. If not, the state is added to the state space and the algorithm continues with the next state. Otherwise, ATTESTOR checks whether a fixpoint has been reached and terminates the procedure in this case. The generated state space represents the transformation of the initial heap during program execution for the main method of the input program.

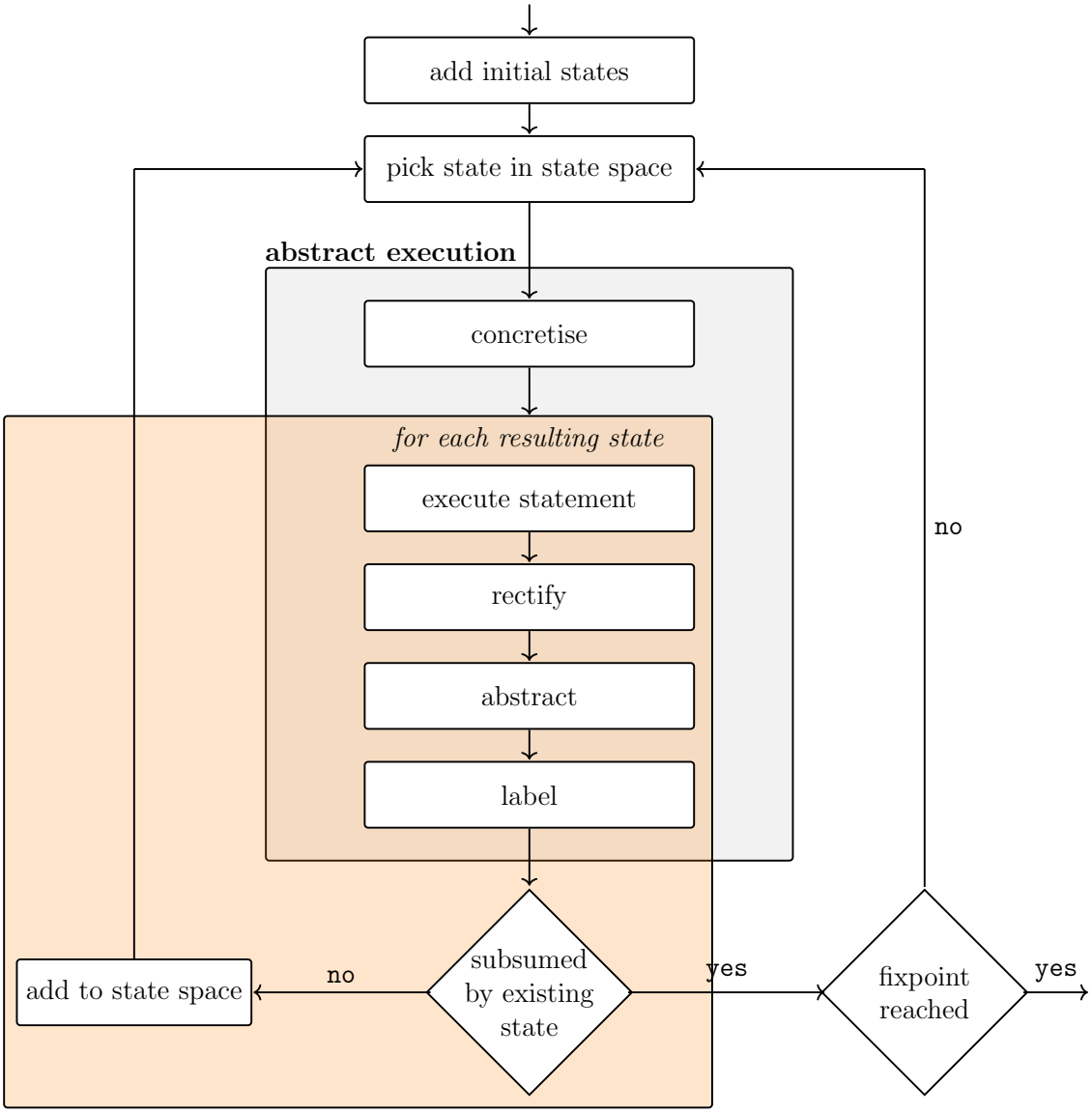


Figure 1.3: Phase 4: State space generation in Attestor [2].

Phase 5: Model Checking State space generation is followed by the model checking phase if LTL formulae have been specified. ATTESTOR currently implements the tableaux method described in Section 3.1 which verifies LTL formulae for the main state space. In case a formula is violated, a failure trace is returned that constitutes a counterexample generated in the next phase. If all properties are satisfied, ATTESTOR outputs accordingly. A drawback of the current model checking procedure is that procedural programs contain (recursive) methods and method calls with inherent state spaces that are not (directly) checked in the current implementation. Rather, procedure calls are woven into the main state space by considering their influence on the heap only after the execution. However, properties should also be checked for the procedure state spaces themselves as they might introduce violations not visible in the main state space. We approach this gap by considering hierarchical model checking in Chapters 4 and 5 that also verifies procedure state spaces for specified LTL properties.

Phase 6: Counterexample Generation In case an LTL formula is found to be violated, the model checking phase returns a failure trace. Together with the violated formula a counterexample is generated in this phase in order to provide an instance for debugging purposes.

1.2 Related Work

Several algorithms on model checking recursive software programs have been developed in the research of LTL model checking. In [5], Esparza et al. propose an automata-theoretic approach that reduces the model checking problem to the accepting runs problem of a Büchi pushdown system. A Büchi pushdown system is the product of a pushdown system that represents the underlying input program and a Büchi automaton that corresponds to the negation of the LTL formula to be verified. Thus, an empty set of configurations that have an accepting run in the Büchi pushdown system denotes that no run in the pushdown system satisfies the negated input formula, i.e., the formula is satisfied. The algorithm runs in $\mathcal{O}(|\mathcal{P}^3||\mathcal{B}^3|)$ time and $\mathcal{O}(|\mathcal{P}^2||\mathcal{B}^2|)$ space, where $|\mathcal{P}|$ and $|\mathcal{B}|$ denote the size of the pushdown system and the size of a Büchi automaton for the negation of the LTL formula, respectively. Hence, Esparza et al. show that the model checking problem for linear temporal logics can be solved in ploynomial time.

In [6], Esparza et al. further extend the automata-theoretic approach for boolean programs with possibly recursive procedures by representing the underlying input program as a symbolic pushdown system. A symbolic pushdown system is a compact representation of a pushdown system where a symbolic rule denotes a set of transition rules in the corresponding pushdown system. Thus, symbolic pushdown systems finitely capture possibly infinite states. Furthermore, they use BDDs to

encode the values of Boolean variables.

Alur et al. further address the problem of model checking recursive input programs that induce possibly infinite hierarchies in state spaces by introducing recursive state machines in [1]. Closely related to pushdown systems, recursive state machines model the control flow of input programs with recursive procedure calls. Every procedure is represented by a component within the state machine. A state in a recursive state machine is either a state in a component or a box that represents a procedure call. Alur et al. adapt the automata-theoretic approach for model checking recursive state machines.

We base our work on the previously introduced results and employ the notion of recursive state machines for model checking LTL formula. However, we will refrain from an automata-theoretic approach and focus more on an on-the-fly model checking algorithm as introduced in [4] that constructs a proof structure for the underlying state space in order to verify a linear time property. By combining the compact representation of recursive programs in recursive state machines and the efficient on-the-fly model checking algorithm, we introduce algorithms that do not require the construction of several automata, but rather operate on the underlying state space itself. We introduce two algorithms for hierarchical model checking. The first algorithm is an on-the-fly hierarchical model checking approach that works completely on-the-fly, i.e., state space generation is executed on demand if the tableaux construction requires so. This approach is promising in detecting property violations at an early stage avoiding the need to generate an entire state space. The second algorithm, we introduce in this thesis, executes the tableaux construction on recursive state machines. This approach offers an efficient way to verify a set of linear time properties for a (hierarchical) state space once computed. Finally, the thesis concludes with an evaluation of the introduced algorithms and compares them to ATTESTOR's top level model checking algorithm.

Chapter 2

Preliminaries

This chapter introduces the foundations of this thesis. Sections 2.1 and 2.2 describe how an input program is modeled by a state space including hypergraphs that represent heaps for each state according to Heinen et al. [8]. In order to describe hierarchical structures relevant to modeling programs with procedure calls, we focus on recursive state machines in Section 2.3 based on the work of Alur et al. [1]. The chapter concludes by presenting linear temporal logic that formalizes properties with temporal modalities in order to verify the input program under consideration.

2.1 Heap Representation

Our focus in model checking lies on pointer-manipulating programs. Therefore, we consider the heap of the program under consideration at every state. These are generated during ATTESTOR's state space generation phase (see Figure 1.2). The heap encompasses a set of locations with variables and pointers, i.e., references to locations. It is represented by a graph where vertices represent heap objects, directed edges depict selectors, and labeled edges depict the mapping of program variables to heap objects. Selectors are pointer variables that are connected to two vertices, i.e., a selector points from its source to its target node.

Consider the class definition of a singly-linked list (*SLL*) in JAVA code in Listing 1.1 in Chapter 1. It defines a singly-linked list with a selector `next` that points to an element's successor node. Figure 2.1 illustrates a heap for a singly-linked list. The list consists of five elements represented by circles. The `next`-selectors are represented by directed edges. Furthermore, the program variables `head` and `tail` are attached to the first and the last vertex of the list, respectively.

Pointer-manipulating operations are represented by graph transformations. For instance, executing the operation `tail := head.next` on the heap given in Figure 2.1 attaches the variable `tail` to the vertex pointed to by the `next` selector of the first

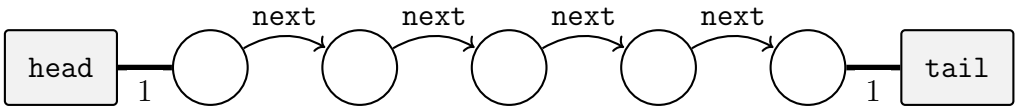


Figure 2.1: Heap for a singly-linked list.

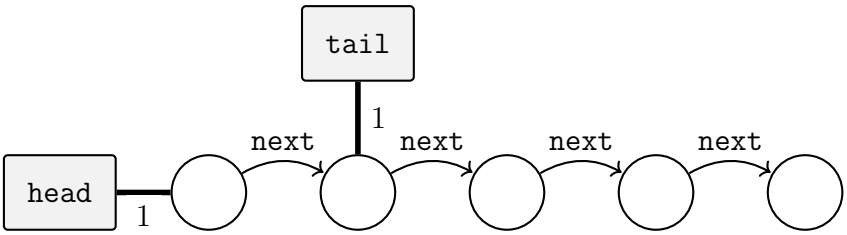


Figure 2.2: Modified heap of a singly-linked list after pointer-operation `tail := head.next`.

vertex. The resulting heap is shown in Figure 2.2.

The number of objects in a heap can become unboundedly large. Consider a program that contains a loop in which a new element is added to a list with every iteration. If the loop is visited infinitely often, the resulting heap size will increase with every iteration. In order to obtain a finite representation of a heap, parts of the graph are abstracted, i.e., a subgraph is replaced by a placeholder. An example for an abstracted graph is depicted in Figure 2.3. The graph shows a singly-linked list with a placeholder labeled *SLL* which represents a singly-linked list of arbitrary length. The example illustrates the concept of *hypergraphs*. Hypergraphs are graphs in which an edge is connected to an arbitrary number of nodes. These edges are called *hyperedges*. The number of vertices a hyperedge connects is captured in its *rank*. Hypergraphs represent heaps that are partially concrete and partially abstract. In our example from Figure 2.3, the hyperedge labeled *SLL* is an abstracted part of the hypergraph in the otherwise concrete graph. In order to express the abstracted parts of the heap, we require a *ranked alphabet* $\Sigma = \Sigma_N \uplus \Sigma_T$, where Σ_N denotes a finite set of *nonterminal symbols* and $\Sigma_T = Var \uplus Sel$ denotes the terminal symbols including the set Var of variables and the set Sel of selectors. Program variables are of rank one, while selectors are of rank two. Hypergraphs over the alphabet Σ_T describe *concrete* heaps that do not contain an abstract part such as the heap depicted in Figure 2.1.

Definition 2.1: Hypergraph [7]

Given a finite ranked alphabet $\Sigma = \Sigma_N \uplus \Sigma_T$ with associated ranking function

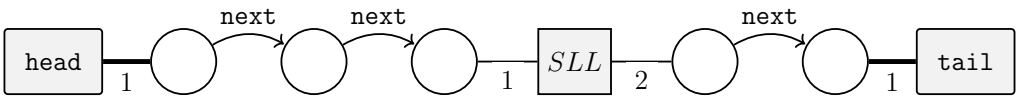


Figure 2.3: A singly-linked list with an abstracted subgraph represented as a hypergraph.

$\mathsf{rk} : \Sigma \to \mathbb{N}$. A (labeled) hypergraph over Σ is a tuple

$$H = (V, E, \mathsf{att}, \mathsf{lab}, \mathsf{ext})$$

where

- V is a finite set of vertices,
- E is a finite set of hyperedges,
- $\mathsf{att} : E \to V^*$ is the attachment function that maps each hyperedge to a sequence of incident vertices,
- $\mathsf{lab} : E \to \Sigma$ is the hyperedge-labeling function that maps a label to an edge, and
- $\mathsf{ext} \in V^*$ is the (possibly empty) sequence of pairwise distinct external vertices.

For every $e \in E$, we let $\mathsf{rk}(e) = |\mathsf{att}(e)|$ and we require $\mathsf{rk}(e) = \mathsf{rk}(\mathsf{lab}(e))$. The set of all hypergraphs over Σ is denoted by HG_Σ.

In order to obtain all possible heaps represented by an abstract subgraph, we require the concept of graph grammars. Graph grammars define a set of production rules that define how nonterminals can be replaced by hypergraphs. Nonterminals represent abstract parts of the graph such as SLL in Figure 2.3. Exhaustively applying production rules to a graph gradually replaces nonterminals by graphs so that concrete graphs without nonterminals can be reached eventually. Graph grammars are therefore comparable to string grammars that define rules to manipulate strings. As we consider hypergraphs in our analysis, we focus on hyperedge replacement grammars that are graph grammars working on hypergraphs, where nonterminals represent hyperedges.

Definition 2.2: Hyperedge Replacement Grammar [7]

A *hyperedge replacement grammar* G over a ranked alphabet Σ is a set of production rules of the form $X \to R$, where $X \in \Sigma_N$ is a nonterminal and $R \in HG_\Sigma$ is a rule graph, i.e., a hypergraph with $|ext_R| = \mathsf{rk}(X)$.

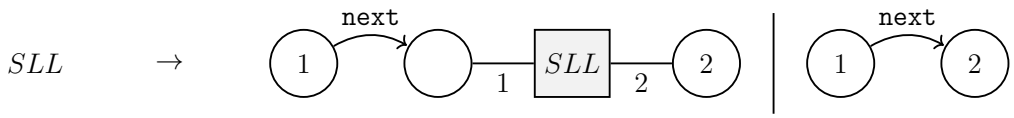


Figure 2.4: A hyperedge replacement grammar for singly-linked lists.

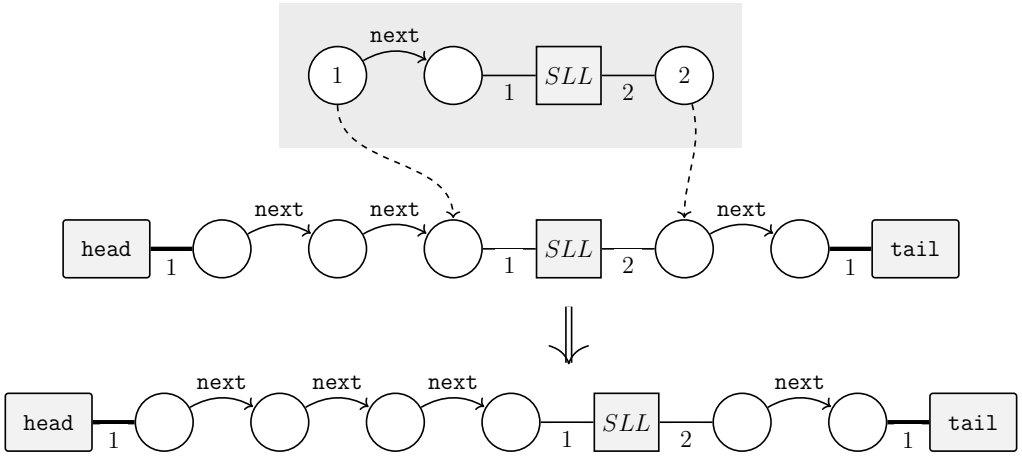


Figure 2.5: By applying the first production rule from Figure 2.4 a list element is added to the (abstract) hypergraph.

The language of a hyperedge replacement grammar contains all concrete hypergraphs that are obtained by repeatedly applying the production rules to a given hypergraph. In this context, a nonterminal in a hypergraph is replaced by a hypergraph according to the production rules. The replacement requires that the external nodes of the replacing hypergraph are mapped to the placeholders in the hypergraph. To illustrate an hyperedge replacement, consider the hyperedge replacement grammar given in Figure 2.4. The grammar describes the language of all singly-linked lists with at least two elements. The first production rule recursively adds an element to the existing list introducing a new nonterminal SLL in order to allow for adding more elements during another production. The second rule terminates the production by replacing the nonterminal SLL by a concrete graph. Let us consider the hypergraph from Figure 2.1. We have two options to apply the hyperedge replacement grammar for singly-linked lists to the hypergraph under consideration. Applying the first rule yields the (abstract) hypergraph depicted in Figure 2.5, while applying the second rule yields a concrete hypergraph as shown in Figure 2.6.

Figures 2.5 and 2.6 display the forward application of production rules on a hypergraph also called *concretisation* as an abstract fragment is replaced by a (more) concrete subgraph. A concretisation step can yield more than one concrete hypergraph if several production rules are applicable. Therefore, concretisation needs to

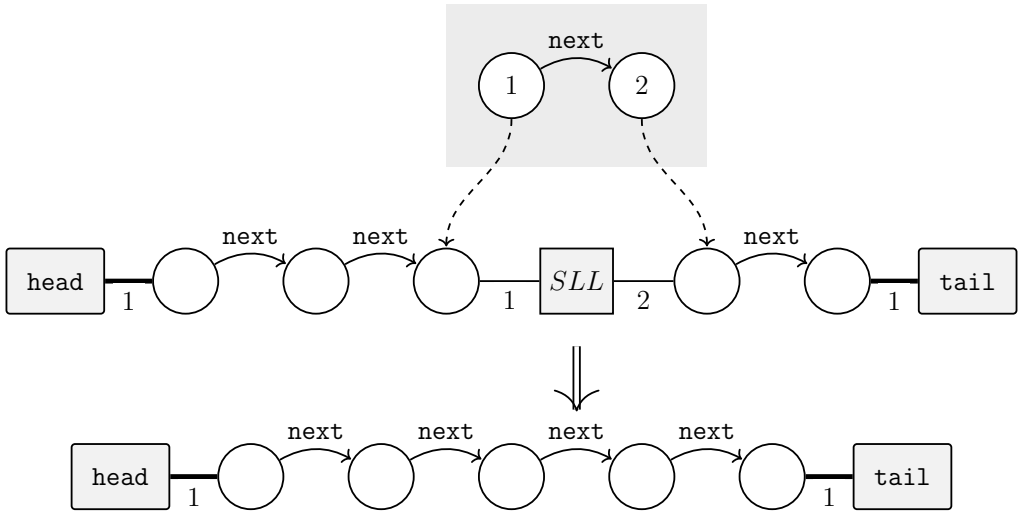


Figure 2.6: By applying the second production rule from Figure 2.4 a concrete hypergraph is obtained.

consider all possible hypergraphs. In fact, the application of the production rules of the hyperedge replacement grammar for singly-linked lists in Figures 2.5 and 2.6 is an example for a case where more than one rule is applicable.

In contrast to concretisation, *abstraction* describes the backward application of production rules such that a subgraph is replaced by a nonterminal. Abstraction of subgraphs yield an over-approximation of the current set of concrete hypergraphs, since information on what exactly has been abstracted is lost during abstraction. Abstraction allows us to represent possibly unboundedly large graphs in a finite manner.

2.2 Transition Systems

Hypergraphs model the heap of a program at a certain moment., i.e., a single state during program execution. In order to describe the overall behavior of a program, we use the notion of a *transition system*. A transition system is a graph where nodes represent *states* reachable by a program and edges indicate *transitions* between states. A transition reflects how a state of the system changes, e.g., due to a statement execution. Therefore, the state space resulting from Attestor's state space generation phase (see Figure 1.2) is a transition system, where the states compose of hypergraphs.

Definition 2.3: Transition System [3]

A *transition system* is a tuple

$$T = (S, I, \delta, AP, L)$$

where

- S is a set of states,
- $I \subseteq S$ is a set of initial states,
- $\delta \subseteq S \times S$ is a transition relation,
- AP is a set of atomic propositions, and
- $L : S \to 2^{AP}$ is a labeling function.

T is called *finite* if S, Act, and AP are finite.

We consider transition systems where a state is a hypergraph that encompasses the current heap of the program under consideration. The set AP of atomic propositions consists of the properties under consideration. The labeling function L maps a state s to a set $L(s) \in 2^{AP}$ of atomic propositions satisfied by s. For example, the atomic proposition "$i < 0$" holds for a state where the variable i is set to the value -2. Based on the set $L(s)$, we can specify that s satisfies a propositional logic formula ϕ if the evaluation induced by $L(s)$ fulfills the formula ϕ. Therefore,

$$s \models \phi \quad \text{iff} \quad L(s) \models \phi.$$

The transition relation δ formally describes how the transition system T evolves starting in a state $s \in S$. Thus, the transition $(s, s') \in \delta$ defines that state s evolves to state s'. If a state has more than one outgoing transition, the next transition is chosen nondeterministically. A state without any outgoing transitions is called a *terminal state*.

Definition 2.4: Terminal State [3]

A state $s \in S$ in a transition system $T = (S, I, \delta, AP, L)$ is called *terminal* if and only if

$$\bigcup \{s' \in S \mid (s, s') \in \delta\} = \emptyset.$$

Figure 2.7 and 2.8 depict transition systems for the methods `traverse` and `swap` given in Listing 1.2, respectively. Let us exemplify the transition system $T =$

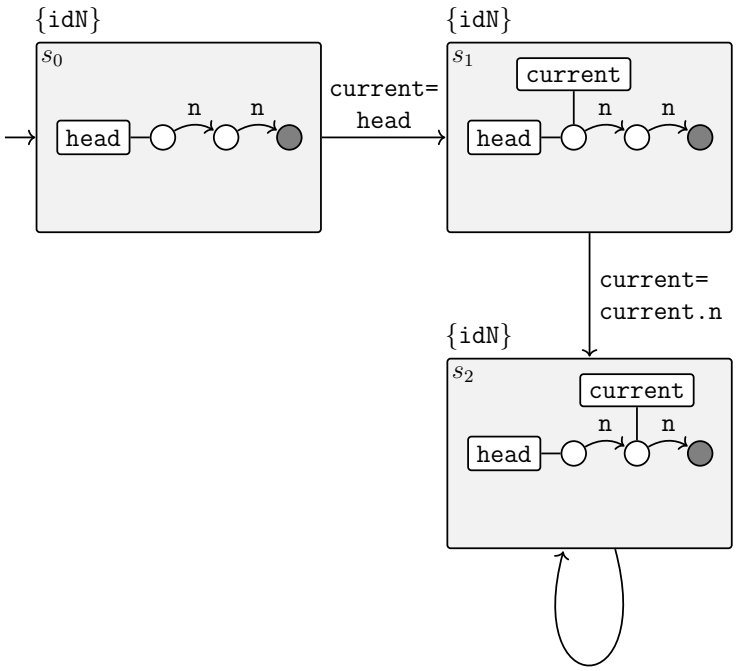


Figure 2.7: State space for method `SLList.traverse`.

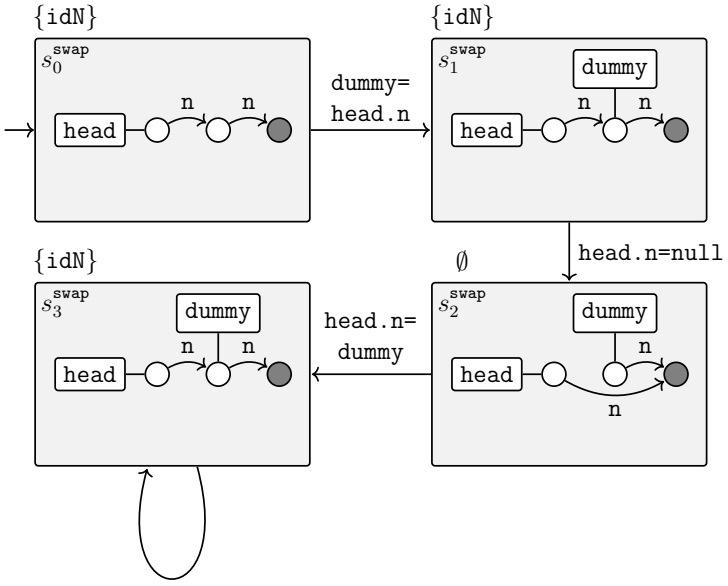


Figure 2.8: State space for method `SLList.swap`.

(S, I, δ, AP, L) for the method `traverse`. Every state encompasses a hypergraph that represents the heap at the current position in the analyzed program. The initial state $s_0 \in I$ is marked with a short incoming arrow. It consists of a singly-linked list with two elements. The variable `head` is attached to the first node of the list. s_0 is succeeded by state s_1 which contains a further variable `current` that points to the same node as the variable `head`. The succeeding state s_2 of s_1 contains a self-loop, i.e., s_2 is its own successor. It also marks the final state of the transition system. The transition relation δ is represented by arrows connecting two states. For example, $(s_0, s_1) \in \delta$ and $(s_1, s_2) \in \delta$. The self-loop at state s_2 depicts the transition $(s_2, s_2) \in \delta$. The transition system does not have any terminal states as all states have outgoing transitions. The states in T are labeled with atomic propositions that they satisfy, i.e., all states satisfy the proposition `identicNeighbours`, abbreviated as `idN`, that indicates that the order of the elements in the list is not modified. Therefore, $L(s_0) = L(s_1) = L(s_2) = \{\texttt{identicNeighbours}\}$. From the stated propositions, the set AP of atomic propositions is implicitly given as $AP = \{\texttt{identicNeighbours}\}$.

A sequence of transitions starting in an initial state $s_0 \in I$ and either ending in a terminal state $s \in S$ or infinitely prolonging, is called a *path* of transition system T.

Definition 2.5: Paths [3]

A *finite path* π of a transition system $T = (S, I, \delta, AP, L)$ is a finite sequence $s_0 s_1 \dots s_n$ such that

- $s_0 \in I$ is an initial state,
- $s_i \in \bigcup \{s \in S \mid (s_{i-1}, s) \in \delta\}$ for all $0 < i \leq n$, where $n \geq 0$, and
- s_n is a terminal state.

An *infinite path* π is an infinite sequence $s_0 s_1 s_2 \dots$ such that

- $s_0 \in I$ is an initial state and
- $s_i \in \bigcup \{s \in S \mid (s_{i-1}, s) \in \delta\}$ for all $i > 0$.

$\mathsf{Paths}(T)$ denotes the set of all paths in T.

For a path π, $\pi[i]$ denotes the ith state of π, while $\pi[i..]$ denotes the ith suffix of π. Paths display the order of states that are traversed throughout a sequence of transitions. However, the related sets of atomic propositions of the traversed states, which are relevant for model checking, are not observable in the path itself. Therefore, we consider the notion of *traces* which are sequences of sets of atomic propositions that are satisfied along a path π.

Definition 2.6: Trace [3]

Let $T = (S, I, \delta, AP, L)$ be a transition system without terminal states. The *trace* of the finite path $\pi = s_0 s_1 \dots s_n$ is defined as

$$\mathsf{trace}(\pi) = L(s_0)L(s_1)\dots L(s_n).$$

The *trace* of the infinite path $\pi = s_0 s_1 \dots$ is defined as

$$\mathsf{trace}(\pi) = L(s_0)L(s_1)\dots.$$

$\mathsf{Traces}(s)$ denotes the set of traces of paths starting in state s and $\mathsf{Traces}(T)$ denotes the set of traces of the initial states of a transition system T.

The condition that a transition system does not have any terminal states is not a restriction since, for every transition system, it is possible to construct an equivalent one without terminal states. This is achieved by adding a new state s_{stop} with a self-loop to the transition system to which all terminal states have a transition. Thus, the resulting system does not contain any terminal states. In the following we assume that a transition system does not have any terminal states.

Consider the sample transition system T in Figure 2.7. A path π in T is the finite sequence $s_0 s_1 s_2$. The corresponding trace of π is given by

$$L(s_0)L(s_1)L(s_2) = \{\texttt{idN}\}\{\texttt{idN}\}\{\texttt{idN}\}.$$

An infinite path π' in T is the sequence $s_0 s_1 s_2 s_2 \dots$ that infinitely loops around the state s_2. π' implies the trace

$$L(s_0)L(s_1)L(s_2)L(s_2)\cdots = \{\texttt{idN}\}\{\texttt{idN}\}\{\texttt{idN}\}\{\texttt{idN}\}\dots.$$

2.3 Recursive State Machines

Often computer programs do not only consist of a sequence of commands, but also contain (recursive) calls to methods. The execution of procedural programs contains call- and return-statements to different sections of the input program. In order to capture the hierarchical (or recursive) structure, we introduce the notion of *recursive state machines*, as defined in [1], that encapsulate each method in its own *component*. Each component consists of a set of *nodes* that are the states of the model and *boxes* that are each mapped to a component in the recursive state machine. A box can be understood as an interface with entry and exit nodes that models the transition into another method environment, e.g. entering a box resembles a method

invocation while exiting a box represents the return from a method execution. Edges between states and boxes identify transitions. Thus, a component state machine is a transition system for a single procedure while a recursive state machine composes a transition system in which states can invoke procedures and hence enter different transition systems. We assume that the set AP of atomic propositions is implicitly defined by the context of the program under consideration.

Definition 2.7: Recursive State Machine [1]

A *recursive state machine* (RSM) $\mathcal{A}$ over a finite alphabet Σ is given by a tuple $(A_1, ..., A_k)$, where each *component state machine* (CSM)

$$A_i = (S_i, En_i, Ex_i, \delta_i, L_i),$$

$1 \leq i \leq k$, consists of

- a set $S_i = N_i \cup B_i$ of states composing of a set N_i of *nodes* and a (disjoint) set B_i of *boxes*,
- a set of *entry nodes* $En_i \subseteq N_i$,
- a set of *exit nodes* $Ex_i \subseteq N_i$,
- a *transition relation* δ_i, where transitions are of the form (u, σ, v), where
  - the source u is either a node of N_i or a pair (b, x), where b is a box in B_i and x is an exit node in Ex_j for $j = L_i(b)$,
  - the label σ is in Σ, and
  - the destination v is either a node in N_i or a pair (b, e), where b is a box in B_i and e is an entry node in En_j for $j = L_i(b)$, and
- a *labeling* $L_i : B_i \mapsto \{1, ..., k\}$ that assigns to every box an index $j \in \{1, ..., k\}$ referring to one of the component state machines $A_1, ..., A_k$.

A sample RSM with three components A_1, A_2, A_3 is depicted in Figure 2.9. The nodes drawn at the border of the components represent the entry and exit nodes, respectively. The arrows depict the transitions between the states of a component as well as between states and boxes. Each box is mapped to a component, e.g. box b_1 in component A_1 is mapped to component A_2. Box c_2 in component A_2 is mapped to component A_3. Thus, entering box b_1 or c_2 changes the current component under control from component A_1 to A_2 or from component A_2 to A_3, respectively. This can be understood as an invocation of program methods where entry nodes represent input arguments to the called method and exit nodes model return values.

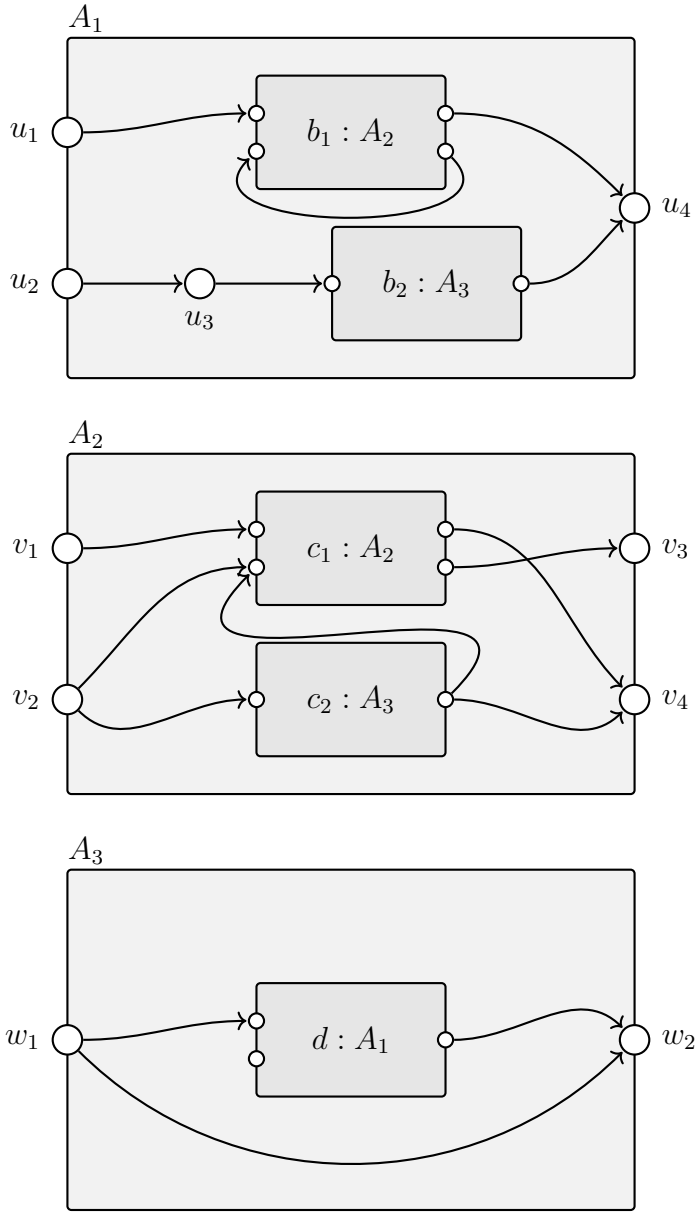


Figure 2.9: A sample recursive state machine. Adapted from [1].

In order to define the execution of an RSM $\mathcal{A} = (A_1, ..., A_k)$, we first describe the global relation between its component state machines $A_i, 1 \leq i \leq k$. A *global state* of an RSM is a sequence of boxes ending in a node of a component.

Definition 2.8: Global State [1]

A *global state* of an RSM $\mathcal{A} = (A_1, ..., A_k)$ is a tuple $(b_1, ..., b_r, u)$, where $b_1 \in B_1, ..., b_r \in B_r$ are boxes and u is a node. The set S of global states of $\mathcal{A}$ is B^*N, where $B = \bigcup_i B_i$ and $N = \bigcup_i N_i$. A state $(b_1, ..., b_r, u)$ with $b_i \in B_{j_i}$ for $1 \leq i \leq r$ and $u \in N_j$ is *well-formed* if $L_{j_i}(b_i) = j_{i+1}$ for $1 \leq i < r$ and $L_{j_r}(b_r) = j$.

A well-formed state $(b_1, ..., b_r, u)$ of an RSM $\mathcal{A} = (A_1, ..., A_k)$ corresponds to a path through the components A_j of $\mathcal{A}$, where we enter component A_j via box b_r of component A_{j_r}.

Consider the sample RSM given in Figure 2.9. A global state in the sample RSM is given by

$$(b_1, c_1, c_1, c_1, c_2, d, b_2, d, u_3),$$

where the sequence of boxes reflects which components are visited before reaching the current state u_3. The state is well-formed as for every box $b \in \{b_1, b_2, c_1, c_2, d\}$ the labeling function L_i coincides with the component referred to by the next box in the sequence, e.g., $L_1(b_1) = A_2$, which is exactly the component in which the following box c_1 is defined.

In order to move between global states of an RSM $\mathcal{A}$, we require the notion of a *global transition relation* δ which enables us to not only transition between states within a CSM A_j as defined by its transition relation δ_j, but also between pairs of CSMs.

Definition 2.9: Global Transition Relation [1]

Let $s = (b_1, ..., b_r, u) \in S$ be a state with $u \in N_j$ and $b_r \in B_m$ for an RSM $\mathcal{A} = (A_1, ..., A_k)$. The *global transition relation* δ for $\mathcal{A}$ defines $(s, \sigma, s') \in \delta$ if and only if one of the following holds:

1. $(u, \sigma, u') \in \delta_j$ for a node u' of A_j and $s' = (b_1, ..., b_r, u')$.
2. $(u, \sigma, (b', e)) \in \delta_j$ for a box b' of A_j and $s' = (b_1, ..., b_r, b', e)$.
3. u is an exit-node of A_j, $((b_r, u), \sigma, u') \in \delta_m$ for a node u' of A_m, and $s' = (b_1, ..., b_{r-1}, u')$.

4. u is an exit-node of A_j, $((b_r, u), \sigma, (b', e)) \in \delta_m$ for a box b' of A_m, and $s' = (b_1, ..., b_{r-1}, b', e)$.

Definition 2.9 specifies the possible kinds of transitions between global states $s, s' \in S$ of an RSM $\mathcal{A}$. Again, consider the RSM given in Figure 2.9. For each case depicted in Definition 2.9, we illustrate the global transition relation:

Case 1 describes the scenario where the source and the destination nodes are both within the same component A_j. For instance, the component A_1 defines $(u_2, \sigma, u_3) \in \delta_1$, thus, in terms of the global transition relation δ, a valid global transition is $((b_2, d, u_2), \sigma, (b_2, d, u_3)) \in \delta$.

Case 2 depicts that a new component is entered via box b' of A_j. Thus, the current node of the destination state s' is the entry-node e. An example for this case is given by regarding the global state (b_1, c_1, c_2, d, u_3) which is located in component A_1. The local transition relation δ_1 contains the transition $(u_3, \sigma, (b_2, v_1))$. Therefore, globally $((b_1, c_1, c_2, d, u_3), \sigma, (b_1, c_1, c_2, d, b_2, v_1)) \in \delta$ which corresponds to entering component A_2 via box b_2 and transitioning to state $(b_1, c_1, c_2, d, b_2, v_1)$.

Case 3 and 4 are both exiting component A_j via the exit-node u. While case 3 returns to component A_m, from where we entered A_j before, case 4 directly enters a new component via box b' of component A_m. An example for case 3 is given by the transition $((b_1, c_1, c_2, d, u_4), \sigma, (b_1, c_1, c_2, w_2)) \in \delta$, where we return from component A_1 via box d to component A_3. If we continue the return action for state (b_1, c_1, c_2, w_2), we get $((b_1, c_1, c_2, w_2), \sigma, (b_1, c_1, c_1, v_2)) \in \delta$ as a sample transition for case 4. The transition describes that we exit component A_3 entered via box c_2 and directly enter component A_2 via box c_1 as $((c_2, w_2), \sigma, (c_1, v_2))$ is a valid transition according to δ_2.

After defining the terms of global states and the global transition relation for an RSM $\mathcal{A}$, we can summarize these components together with the finite alphabet Σ within the concept of a *labeled transition system* (LTS) $T_{\mathcal{A}}$ induced by $\mathcal{A}$. The LTS encodes the execution of $\mathcal{A}$.

Definition 2.10: Labeled Transition System induced by an RSM [1]

The *labeled transition system* (LTS) $T_{\mathcal{A}} = (S, \Sigma, \delta)$ induced by an RSM $\mathcal{A} = (A_1, ..., A_k)$ consists of

- the set of global states S,
- the finite alphabet Σ, and

- the global transition relation δ.

The LTS of an RSM is basically the flattening of the hierarchical structure induced by the components and boxes of an RSM. Therefore, an LTS corresponds to our initial definition of a transition system, where the set I of initial states, the set AP of atomic propositions, and the labeling function L are implicitly specified by the underlying RSM (cf. Definition 2.3). The notion of paths and traces of transition systems also carry over to LTS. Thus, traces of an RSM are the traces of the corresponding LTS.

We specified the relevant framework for model checking pointer-manipulating programs. We model the program execution by recursive state machines capturing the hierarchical nature of method calls, while hypergraphs offer a finite representation of heaps that constitute the states of the model under consideration. Another ingredient to model checking is the formal definition of the properties the model is to be validated for. Here, we focus on *linear temporal logic* described in the following section.

2.4 Linear Temporal Logic

First proposed by Pnueli in 1977, *Linear Temporal Logic* (LTL) is a logic suited to describe *linear-time properties*. Linear-time properties specify requirements on paths (or rather their traces) and can be understood as a set of (infinite) words over a set AP of atomic propositions.

Definition 2.11: Linear-Time Property [3]

A *linear-time property* over the set of atomic propositions AP is a subset of the set $(2^{AP})^\omega$, i.e., all infinite words defined over the alphabet 2^{AP}.

The satisfaction relation $\models$ for linear-time properties defines that a transition system T satisfies a linear-time property $P \subseteq (2^{AP})^\omega$ if and only if all traces of T are included in the set P, i.e., every trace of T is a word in the language induced by P.

Definition 2.12: Satisfaction Relation for Linear-Time Properties [3]

Let P be a linear-time property over AP and $T = (S, I, \delta, AP, L)$ a transition system. Then, T *satisfies* P, denoted $T \models P$, if and only if $\mathsf{Traces}(T) \subseteq P$. A state $s \in S$ satisfies P, denoted $s \models P$, if and only if $\mathsf{Traces}(s) \subseteq P$.

Linear-time properties can be specified by LTL formulae that encode temporal specifications for paths.

LTL formulae are composed of three components: the Boolean operators *negation* ($\neg$) and *conjunction* ($\wedge$), the temporal operators *next* ($\bigcirc$) and *until* ($\mathbf{U}$), and a set of atomic propositions AP. Atomic propositions are state labels of a transition system, which express properties that hold for a single state, e.g., "$i = 1$". Formally, the syntax of LTL formulae is defined as follows:

Definition 2.13: Syntax of LTL [3]

Given a set AP of atomic propositions with $a \in AP$, *LTL formulae* are given by the context-free grammar below:

$$\varphi := \texttt{true} \mid a \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \bigcirc\varphi \mid \varphi_1 \mathbf{U} \varphi_2.$$

Further temporal operators that are commonly used, but are not included in the definition of LTL formulae, are the temporal modalities *eventually* ($\Diamond$), *globally* ($\Box$), and *release* ($\mathbf{R}$). They are derived by the operators given in Definition 2.13 by

$$\begin{aligned} \Diamond\varphi &:= \texttt{true}\mathbf{U}\varphi \\ \Box\varphi &:= \neg\Diamond\neg\varphi \\ \varphi_1 \mathbf{R} \varphi_2 &:= \neg(\neg\varphi_1 \mathbf{U} \neg\varphi_2). \end{aligned}$$

The following definition captures the relation between linear-time properties and LTL formulae as the latter can be interpreted as words over the alphabet 2^{AP}.

Definition 2.14: Semantics of LTL (Interpretation over Words) [3]

Let φ be an LTL formula over AP. The linear-time property induced by φ is

$$\mathsf{Words}(\varphi) = \{\sigma \in (2^{AP})^\omega \mid \sigma \models \varphi\}$$

where the satisfaction relation $\models \subseteq (2^{AP})^\omega \times LTL$ is the smallest relation with the following properties:

$$\begin{array}{lll} \sigma \models \texttt{true} & & \\ \sigma \models a & \text{iff} & a \in A_0, \text{ where } \sigma = A_0 A_1 A_2 \ldots \\ \sigma \models \varphi_1 \wedge \varphi_2 & \text{iff} & \sigma \models \varphi_1 \text{ and } \sigma \models \varphi_2 \\ \sigma \models \neg\varphi & \text{iff} & \sigma \not\models \varphi \\ \sigma \models \bigcirc\varphi & \text{iff} & \sigma[1\ldots] = A_1 A_2 A_3 \ldots \models \varphi \\ \sigma \models \varphi_1 \mathbf{U} \varphi_2 & \text{iff} & \exists j \geq 0. \sigma[j\ldots] \models \varphi_2 \text{ and } \sigma[i\ldots] \models \varphi_1, \forall 0 \leq i < j. \end{array}$$

We constitute an intuitive understanding of temporal operators by visualizing their semantics in Figure 2.10.

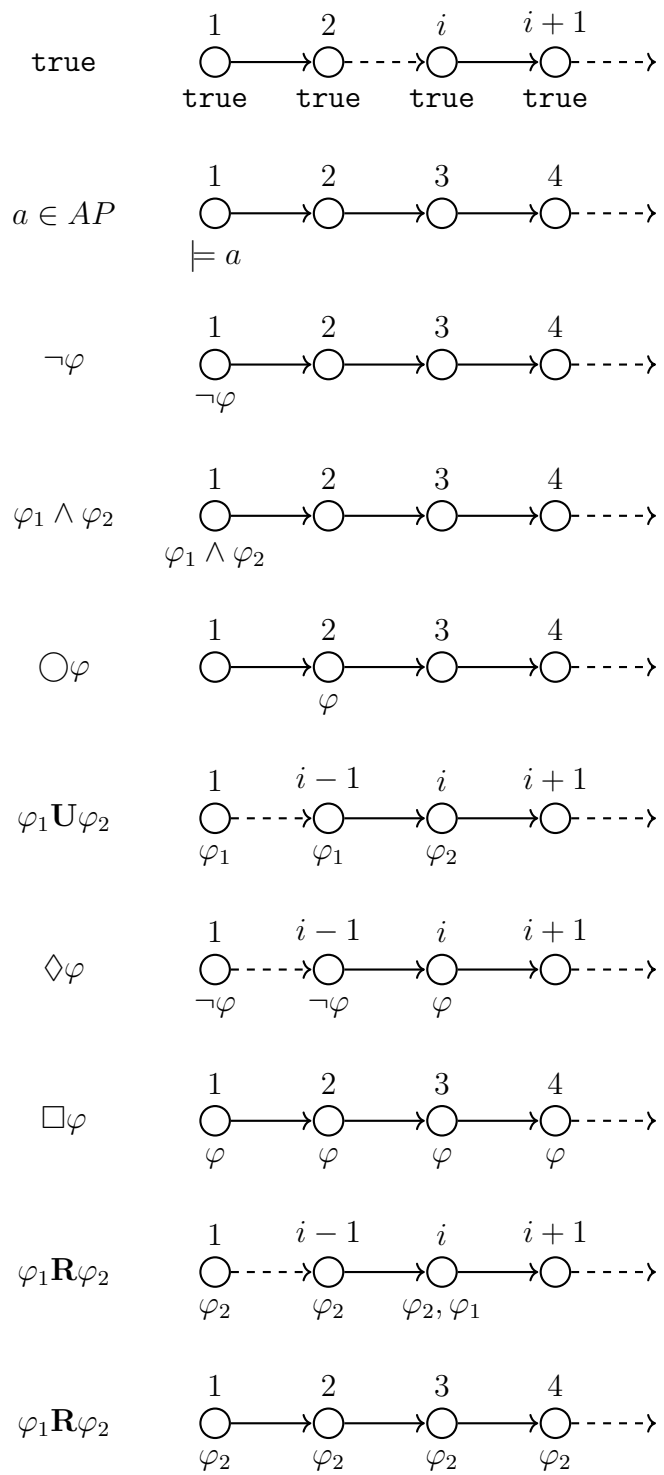


Figure 2.10: Intuitive semantics of temporal operators for a path π.

The interpretation of LTL formulae over words can be used to describe the semantics of LTL formulae over paths and states of a transition system T.

Definition 2.15: Semantics of LTL over Paths and States [3]

Let $T = (S, I, \delta, AP, L)$ be a transition system without terminal states, and let φ be an LTL-formula over AP. For an infinite path π of T, the satisfaction relation is defined by

$$\pi \models \varphi \quad \text{iff} \quad \mathsf{trace}(\pi) \models \varphi.$$

For a state $s \in S$, the satisfaction relation $\models$ is defined by

$$s \models \varphi \quad \text{iff} \quad (\forall \pi \in \mathsf{Paths}(T).\pi \models \varphi).$$

T satisfies φ, denoted $T \models \varphi$, if $\mathsf{Traces}(T) \subseteq \mathsf{Words}(\varphi)$.

From Definition 2.15, it follows that

$$T \models \varphi \quad \text{iff} \quad \forall s_0 \in I : s_0 \models \varphi.$$

Based on the satisfaction relation of LTL formulae over paths and states, we can specify the semantics of LTL for a transition system T.

Definition 2.16: Semantics of LTL [3]

Given an LTL formula φ, a concrete transition system T, and a path $\pi \in \mathsf{Paths}(T)$, the model relation $\models$ for LTL formulae is defined by

$$\begin{aligned}
&\pi \models \texttt{true} \\
&\pi \models a && \Leftrightarrow \pi[1] \models a \\
&\pi \models \neg\varphi && \Leftrightarrow \text{ not } \pi[1] \models \varphi \\
&\pi \models \varphi_1 \wedge \varphi_2 && \Leftrightarrow (\pi \models \varphi_1) \text{ and } (\pi \models \varphi_2) \\
&\pi \models \bigcirc\varphi && \Leftrightarrow \pi[2...] \models \varphi \\
&\pi \models \varphi_1 \mathbf{U} \varphi_2 && \Leftrightarrow \exists i \geq 1.(\pi[i...] \models \varphi_2 \wedge (\forall 1 \leq k < i.\pi[k...] \models \varphi_1)).
\end{aligned}$$

Given a state $s \in S$, $s \models \varphi$ if for all $\pi \in \mathsf{Paths}(T)$ it holds that $\pi \models \varphi$. For a transition system T, $T \models \varphi$ if for all $\pi \in \mathsf{Paths}(T)$ it holds that $\pi \models \varphi$.

For the operators *eventually* ($\Diamond$), *globally* ($\Box$), and *release* ($\mathbf{R}$), the semantics are defined similarly:

$$\begin{aligned}
\pi \models \Diamond\varphi \quad & \Leftrightarrow \exists i \geq 1.\pi[i...] \models \varphi \\
\pi \models \Box\varphi \quad & \Leftrightarrow \forall i \geq 1.\pi[i...] \models \varphi \\
\pi \models \varphi_1 \mathbf{R} \varphi_2 \quad & \Leftrightarrow \forall i \geq 1.\pi[i...] \models \varphi_2 \text{ or} \\
& \exists i \geq 1.(\pi[i...] \models \varphi_1 \wedge (\forall 1 \leq k < i.\pi[k...] \models \varphi_2)).
\end{aligned}$$

The following LTL formulae are examples for specifying properties for model checking pointer-manipulating programs. The formula

$$\bigcirc\{\texttt{SLList}\}$$

employs the $\bigcirc$ operator and the atomic proposition `SLList`, describing that the heap is a singly-linked list, to express that the shape of the heap of the next state is a singly-linked list. Another example is the formula

$$\Box\{\texttt{SLList}\}$$

which requires the heap of every state to be a singly-linked list. Thus, any state not satisfying the atomic proposition `SLList` falsifies the formula $\Box\{\texttt{SLList}\}$. The formula

$$\Box\Diamond\{\texttt{terminated}\} \rightarrow \Box\Diamond\{\texttt{SLList}\}$$

includes another atomic proposition, `terminated`, that describes that a state is a terminating state. Thus, the above formula states that the heap is a singly-linked list upon termination of the analyzed program.

Two LTL formulae are semantically equivalent if they evaluate to the same results under all interpretations. For every LTL formula, there exists an equivalent formula in *positive normal form* (PNF), where negations are only allowed on the level of literals [3].

Definition 2.17: Positive Normal Form [3]

Given a set AP of atomic propositions with $a \in AP$, LTL formulae in *positive normal form* (PNF) are defined by

$$\varphi := \texttt{true} \mid \texttt{false} \mid a \mid \neg a \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2 \mid \bigcirc\varphi \mid \varphi_1 \mathbf{U} \varphi_2 \mid \varphi_1 \mathbf{R} \varphi_2.$$

The existence of an equivalent PNF formula for every LTL formula is due to the following equivalences that allow to push negations inside [7]:

$$\neg\neg\varphi = \varphi$$
$$\neg\texttt{false} = \texttt{true}$$
$$\neg(\varphi_1 \wedge \varphi_2) = \neg\varphi_1 \vee \neg\varphi_2$$
$$\neg \bigcirc \varphi = \bigcirc\neg\varphi$$
$$\neg(\varphi_1 \mathbf{U} \varphi_2) = \neg\varphi_1 \mathbf{R} \neg\varphi_2$$

As an example, consider the LTL formula

$$\Box\Diamond\{\texttt{terminated}\} \rightarrow \Box\Diamond\{\texttt{SLList}\}$$

where `terminated` and `SLList` are atomic propositions as described above. An equivalent formula in PNF is achieved by the following equivalences:

$$\begin{aligned}
& \Box\Diamond\{\texttt{terminated}\} \rightarrow \Box\Diamond\{\texttt{SLList}\} \\
\equiv\ & \neg(\Box\Diamond\{\texttt{terminated}\}) \vee (\Box\Diamond\{\texttt{SLList}\}) && \text{(definition of } \rightarrow\text{)} \\
\equiv\ & \neg(\Box(\texttt{true}\ \mathbf{U}\ \{\texttt{terminated}\})) \\
& \vee (\Box(\texttt{true}\ \mathbf{U}\ \{\texttt{SLList}\})) && \text{(definition of } \Diamond\text{)} \\
\equiv\ & \neg(\neg\Diamond\neg(\texttt{true}\ \mathbf{U}\ \{\texttt{terminated}\})) \\
& \vee (\neg\Diamond\neg(\texttt{true}\ \mathbf{U}\ \{\texttt{SLList}\})) && \text{(definition of } \Box\text{)} \\
\equiv\ & \neg(\neg(\texttt{true}\ \mathbf{U}\ \neg(\texttt{true}\ \mathbf{U}\ \{\texttt{terminated}\}))) \\
& \vee (\neg(\texttt{true}\ \mathbf{U}\ \neg(\texttt{true}\ \mathbf{U}\ \{\texttt{SLList}\}))) && \text{(definition of } \Diamond\text{)} \\
\equiv\ & \neg(\neg\texttt{true}\ \mathbf{R}\ (\texttt{true}\ \mathbf{U}\ \{\texttt{terminated}\})) \\
& \vee (\neg\texttt{true}\ \mathbf{R}\ (\texttt{true}\ \mathbf{U}\ \{\texttt{SLList}\})) && \text{(duality of } \mathbf{U} \text{ and } \mathbf{R}\text{)} \\
\equiv\ & \texttt{true}\ \mathbf{U}\ (\neg\texttt{true}\ \mathbf{R}\ \neg\{\texttt{terminated}\}) \\
& \vee (\neg\texttt{true}\ \mathbf{R}\ (\texttt{true}\ \mathbf{U}\ \{\texttt{SLList}\})). && \text{(duality of } \mathbf{U} \text{ and } \mathbf{R}\text{)}
\end{aligned}$$

The following theorem states the existence of an equivalent formula in PNF for every LTL formula.

Theorem 2.18: Existence of Equivalent PNF Formulae [3]

For each LTL formula there exists an equivalent LTL formula in weak-until PNF.

Chapter 3

Hierarchical Model Checking

One of the main challenges in model checking programs with method calls is that we do not only need to consider the state space of the main method, but also the state spaces induced by method executions, denoted as *procedure state spaces*. Here, we face three main difficulties:

- How can we finitely represent the state space of a program with recursive method calls avoiding repetitions in the state space?
- How can we efficiently model check procedure state spaces, at best avoiding checking a procedure state space multiple times?
- How do we deal with unbounded recursion depth?

The underlying state space of procedural programs is a hierarchical one where each procedure contributes an own state space that is connected to nodes of other state spaces reflecting method invocation. In a flat setting, where hierarchy is not actively considered, this corresponds to an edge connecting the calling state with the "entry" state of the procedure state space. For the flat setting, we introduce the tableaux construction to model checking LTL properties for pointer-manipulating programs with method calls by Grumberg et al. [4]. Based on this algorithm, we introduce our modified approaches to hierarchical model checking in Chapters 4 and 5.

3.1 Tableaux Construction

This section presents the on-the-fly approach by Grumberg et al. [4] that constructs a *proof structure* based on a set of *tableaux rules* in order to show whether an LTL formula φ in PNF is satisfied by a transition system T. We assume all LTL formula to be in PNF which is not a restriction due to Theorem 2.18.

$$(R^{\models}) \quad \frac{s \vdash \Phi \cup \{a\}}{\texttt{true}} \text{ if } s \models a$$

$$(R^{\not\models}) \quad \frac{s \vdash \Phi \cup \{a\}}{s \vdash \Phi} \text{ if } s \not\models a$$

$$(R^{\vee}) \quad \frac{s \vdash \Phi \cup \{\varphi_1 \vee \varphi_2\}}{s \vdash \Phi \cup \{\varphi_1\} \cup \{\varphi_2\}}$$

$$(R^{\wedge}) \quad \frac{s \vdash \Phi \cup \{\varphi_1 \wedge \varphi_2\}}{s \vdash \Phi \cup \{\varphi_1\} \qquad s \vdash \Phi \cup \{\varphi_2\}}$$

$$(R^{\mathbf{U}}) \quad \frac{s \vdash \Phi \cup \{\varphi_1 \mathbf{U} \varphi_2\}}{s \vdash \Phi \cup \{\varphi_2, \varphi_1\} \qquad s \vdash \Phi \cup \{\varphi_2, \bigcirc(\varphi_1 \mathbf{U} \varphi_2)\}}$$

$$(R^{\mathbf{R}}) \quad \frac{s \vdash \Phi \cup \{\varphi_1 \mathbf{R} \varphi_2\}}{s \vdash \Phi \cup \{\varphi_2\} \qquad s \vdash \Phi \cup \{\varphi_1, \bigcirc(\varphi_1 \mathbf{R} \varphi_2)\}}$$

$$(R^{\bigcirc}) \quad \frac{s \vdash \{\bigcirc\varphi_1, \ldots, \bigcirc\varphi_n\}}{s_1 \vdash \{\varphi_1, \ldots, \varphi_n\} \qquad \ldots \qquad s_m \vdash \{\varphi_1, \ldots, \varphi_n\}}$$

Figure 3.1: Tableaux rules for LTL model checking.

A proof structure is a directed graph (V, E), where the set of vertices V are composed of *assertions* and the set E of edges contains the edge (λ_1, λ_2) between two assertions λ_1 and λ_2 if the underlying tableaux contains the inference rule $\frac{\lambda_1}{\lambda_2}$.

Definition 3.1: Proof Structure [4]

A *proof structure* for $\lambda \in \Lambda$ is a tuple (V, E) with $V \subseteq (\Lambda \cup \texttt{true})$ and $E \subseteq V \times V$, such that for any λ' it holds that λ' is reachable from λ and that the successors of λ' are the ones that result from applying some of the rules, i.e.

$$(\lambda_1, \lambda_2) \in E \quad \text{iff} \quad \frac{\lambda_1}{\lambda_2 \quad s \ldots}.$$

The underlying tableaux rules for the proof structure are specified in Figure 3.1. They model the semantics of LTL. The rules for the operators **U** and **R** follow from the expansion law for LTL formulae [3]. Accordingly,

$$\varphi_1 \mathbf{U} \varphi_2 \equiv \varphi_2 \vee (\varphi_1 \wedge \bigcirc(\varphi_1 \mathbf{U} \varphi_2))$$

and

$$\varphi_1 \mathbf{R} \varphi_2 \equiv \varphi_2 \wedge (\varphi_1 \vee \bigcirc(\varphi_1 \mathbf{R} \varphi_2)).$$

A vertex in the set V of a proof structure (V, E) is an assertion of the form $s \vdash \Phi$, where s is a state in T and Φ is a set of LTL formulae. An assertion $s \vdash \Phi$ holds if at least one formula $\varphi \in \Phi$ is satisfied by the state s. Thus, an assertion can be interpreted as a verification goal that aims at proving that $s \models \bigvee_{\varphi \in \Phi} \varphi$. In order to do so, the assertion is broken down into subgoals according to the tableaux rules. By proving a sequence of subgoals the validity of the assertion λ can be concluded from the validity of the subgoals. Hence, the proof structure of an assertion λ contains all subgoals of λ.

The rules $(R^{\mathbf{U}})$ and $(R^{\mathbf{R}})$ can introduce cycles into a proof structure if φ_1 is fulfilled for every state in the underlying transition system for formulae of the form $\varphi_1 \mathbf{U} \varphi_2$ or $\varphi_1 \mathbf{R} \varphi_2$, while no state fulfills φ_2. Therefore, a cycle in a proof structure represents an *infinite path* in the underlying state space. In an infinite path, $\varphi_1 \mathbf{U} \varphi_2$ can never be fulfilled whereas $\varphi_1 \mathbf{R} \varphi_2$ is fulfilled according to the definition of $\mathbf{R}$. Consequently, a cycle in the proof structure originating from successively applying rule $(R^{\mathbf{U}})$ evaluates to a violated assertion, while a cycle arising from applying rule $(R^{\mathbf{R}})$ fulfills the subgoal. The other rules specified in the tableaux cannot introduce infinite paths as their application reduces the size of the formulae.

In the following, we describe when a proof structure (V, E) for an assertion λ can be concluded to be *successful* [4]:

- If $s \vdash \emptyset \in V$, then (V, E) is unsuccessful as an empty assertion can never be fulfilled.
- $\lambda \in V$ is a leaf of the proof structure if there is no $\lambda' \in V$ with $(\lambda, \lambda') \in E$. A leaf λ is called successful if $\lambda = \texttt{true}$.
- An infinite path $\lambda_1 \lambda_2 \ldots$ in (V, E) is called successful if and only if there exists a position $i \in \mathbb{N}$ with $\varphi_1 \mathbf{R} \varphi_2 \in \lambda_i$ and for all $j \geq i$ it holds that $\varphi \notin \lambda_j$.
- The proof structure (V, E) is called successful if every leaf as well as every of its infinite paths is successful.

Theorem 3.2 states that the tableaux construction is indeed a suitable procedure to model check a transition system T for an LTL formula φ as the success of the proof structure $s \vdash \{\varphi\}$ for a state s in T coincides with the validity of $T \models \varphi$.

Theorem 3.2: Correctness of the Tableaux Construction [4]

Given a concrete transition system $T = (S, I, \delta, AP, L)$ with $s \in S$ and an LTL formula φ. Let (V, E) be the proof structure for $s \vdash \{\varphi\}$. Then it holds that

$s \models \varphi$ if and only if (V, E) is successful.

In order to illustrate the tableaux construction algorithm, consider the method `reverse` for reversing a singly-linked list given in Listing 3.1.

```
1   public static SLList reverse(SLList head) {
2
3     SLList revList = null;
4     SLList current = head;
5
6     while (current != null) {
7       SLList next = current.next;
8       current.next = reversedList;
9       revList = current;
10      current = next;
11    }
12    return reversedList;
13  }
```

Listing 3.1: JAVA method for reversing a singly-linked list.

Figure 3.2 shows the state space of reversing a singly-linked list with two elements according to the method `reverse`. For this state space, we employ tableaux construction to check whether the formula $\varphi = \Box\Diamond\{\texttt{SLList}\}$ is satisfied. φ states that the heap will always be a singly-linked list. An equivalent formula to φ in PNF is

$$(\neg\texttt{true}\ \mathbf{R}\ (\texttt{true}\ \mathbf{U}\ \{\texttt{SLList}\})).$$

The proof structure is depicted in Figure 3.3. It starts with the initial formula φ in PNF and the initial state s_0 of the state space, i.e., $\lambda_0 = (s_0 \vdash \{\varphi\})$ is the first assertion of the proof structure. Applying the rule $(R^{\mathbf{R}})$ to λ_0, we split λ_0 into two assertions that are connected to the root assertion via directed edges. Thereupon, the rules $(R^{\mathbf{U}})$ and $(R^{\not\models})$ are applied to the resulting assertions $s_0 \vdash \{\texttt{true}\ \mathbf{U}\ \{\texttt{SLList}\}\}$ and $s_0 \vdash \{\neg\texttt{true}, \bigcirc\varphi\}$, respectively. While the first branch terminates in a leaf with the Boolean value `true`, the second branch expands the proof structure for successor states of s_0, i.e., s_1, by applying the rule $(R^{\bigcirc})$. Thus, φ is validated for state s_1. For the sake of readability, the rule applications for states $s_2, \ldots, s_9$ are omitted as they are analogous to the rule application for state s_0. All paths ending in the leaf `true` are successful. For state s_{10}, the proof structure induces two infinite paths: the infinite path π_0 looping around the sets of LTL formulae $\{\varphi\}, \{\neg\texttt{true}, \bigcirc\varphi\}$, and $\{\bigcirc\varphi\}$, and the infinite path π_2 looping around the sets $\{\texttt{true}\ \mathbf{U}\ \{\texttt{SLList}\}\}, \{\texttt{SLList}, \bigcirc(\texttt{true}\ \mathbf{U}\ \{\texttt{SLList}\})\}$, and $\bigcirc(\texttt{true}\ \mathbf{U}\ \{\texttt{SLList}\})$. The loop in π_2 is marked red. As the sets of formulae in the loop on path π_1 contain an $\mathbf{R}$-operator, π_1 is considered as successful. Opposed to this, the sets of formulae

in the loop on path π_2 do not contain an **R**-operator, but originate from applying the rule ($R^{\mathbf{U}}$). Thus, π_2 is not successful. Therefore, the proof structure for λ_0 is unsuccessful concluding that the state space, i.e., the method `reverse`, does not satisfy the LTL formula ¬`true` **R** (`true` **U** {`SLList`}).

The tableaux construction can be combined with an on-the-fly state space generation such that not all states need to be computed if not required during the run of the tableaux construction. Consider the LTL formula ◯{`SLList`} which describes that the heap of the next states of the system under consideration is a singly-linked list. In order to check this formula only the successors of the current state are required. For the state space given in Figure 3.2, starting in state s_0, the only successor state is s_1. Thus, checking the heap of s_1 is sufficient to decide whether ◯{`SLList`} is satisfied. An on-the-fly approach thus circumvents the generation of the complete state space.

Considering our example introduced in Chapter 1 in Listing 1.2, we construct the proof structure for the method `traverse` based on the state space given in Figure 2.7 for the property □{`identicNeighbours`} according to ATTESTOR's top level model checking algorithm. Figure 3.4 shows the resulting proof structure, which is indeed successful as no violating path has been encountered. However, including the state space of the method `swap` from Figure 2.8 into the proof structure construction a violating point is identified. Figure 3.5 shows the according proof structure. The violated assertion is marked as red. Therefore, top level model checking is not sufficient in order to detect property violations within procedure state spaces. We introduce two algorithms that aim at model checking procedure state spaces efficiently.

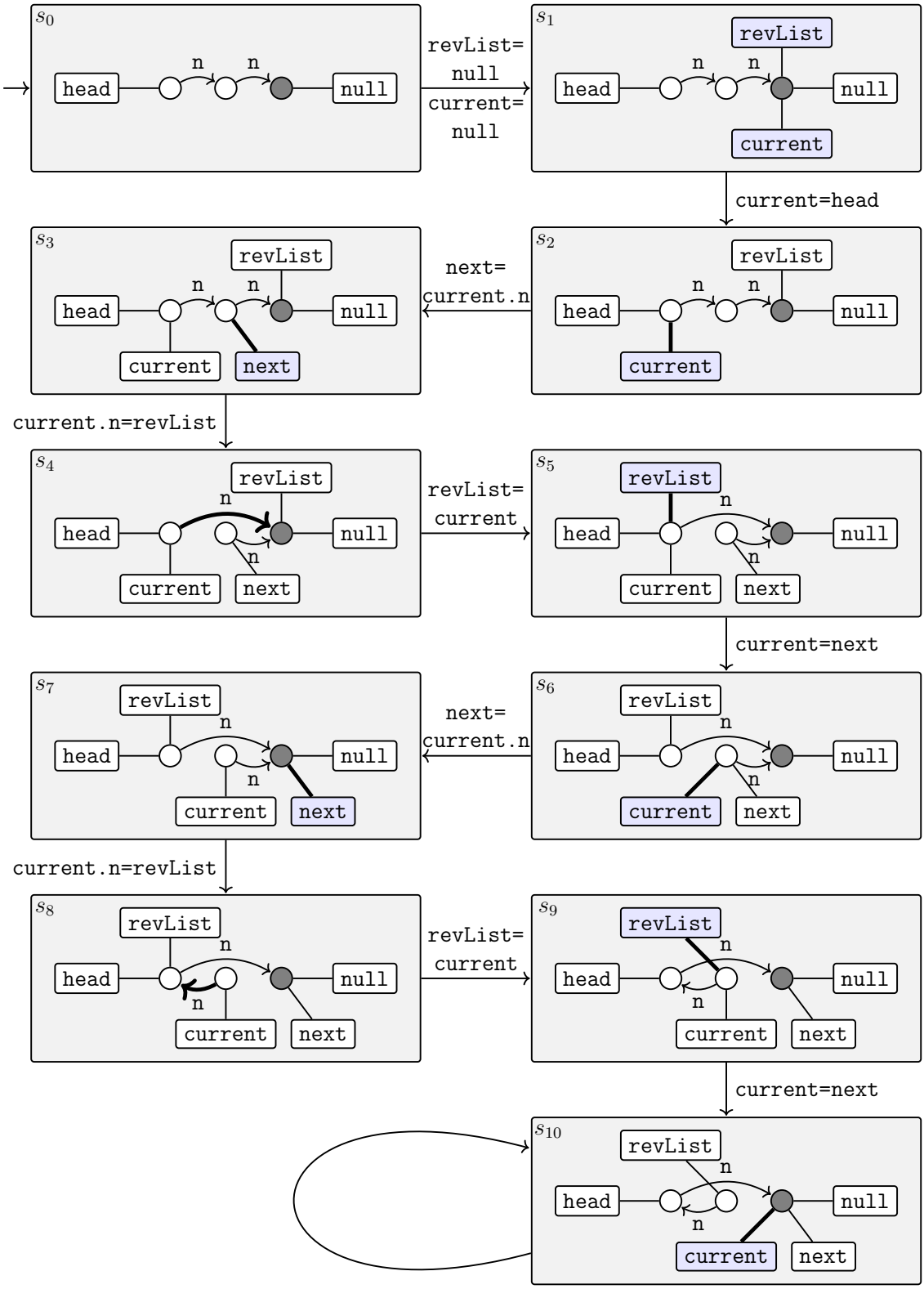


Figure 3.2: State space for reversing a singly-linked list. Adapted from [7].

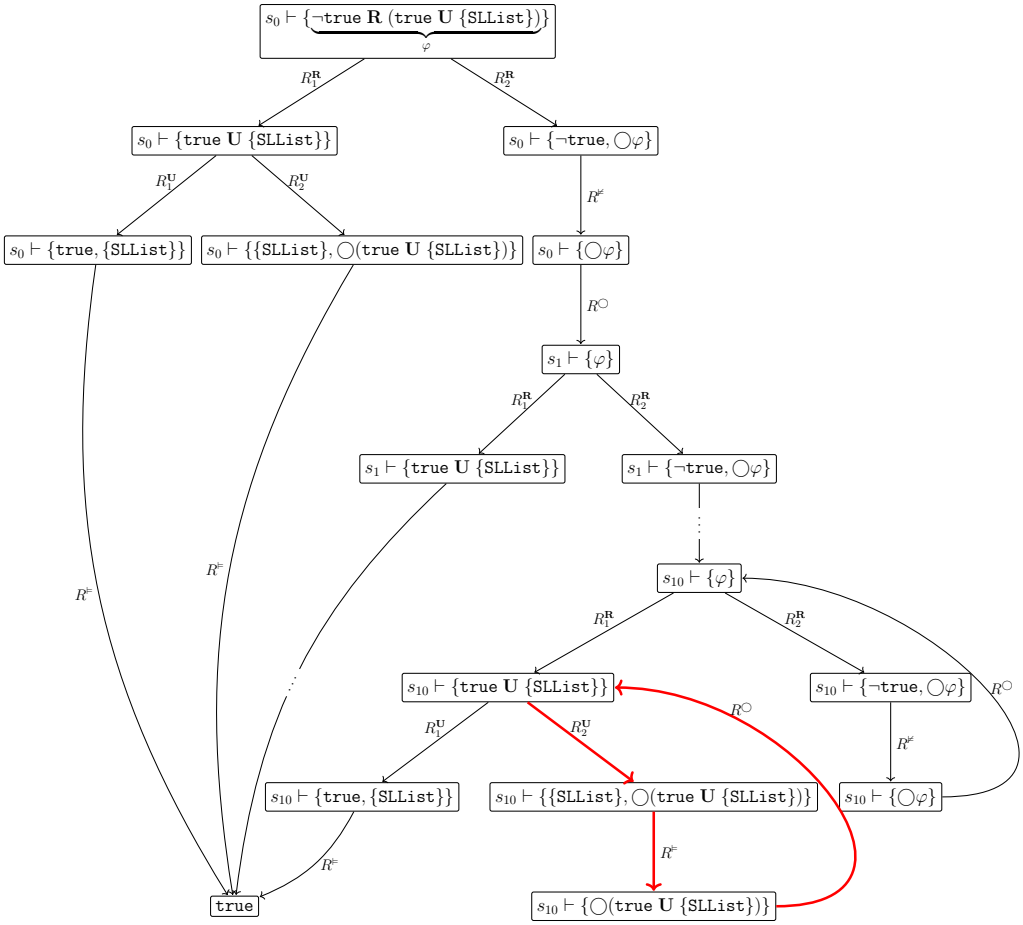


Figure 3.3: Proof structure for model checking method `reverse` for formula $\varphi = (\neg\texttt{true}\ \mathbf{R}\ (\texttt{true}\ \mathbf{U}\ \{\texttt{SLList}\}))$.

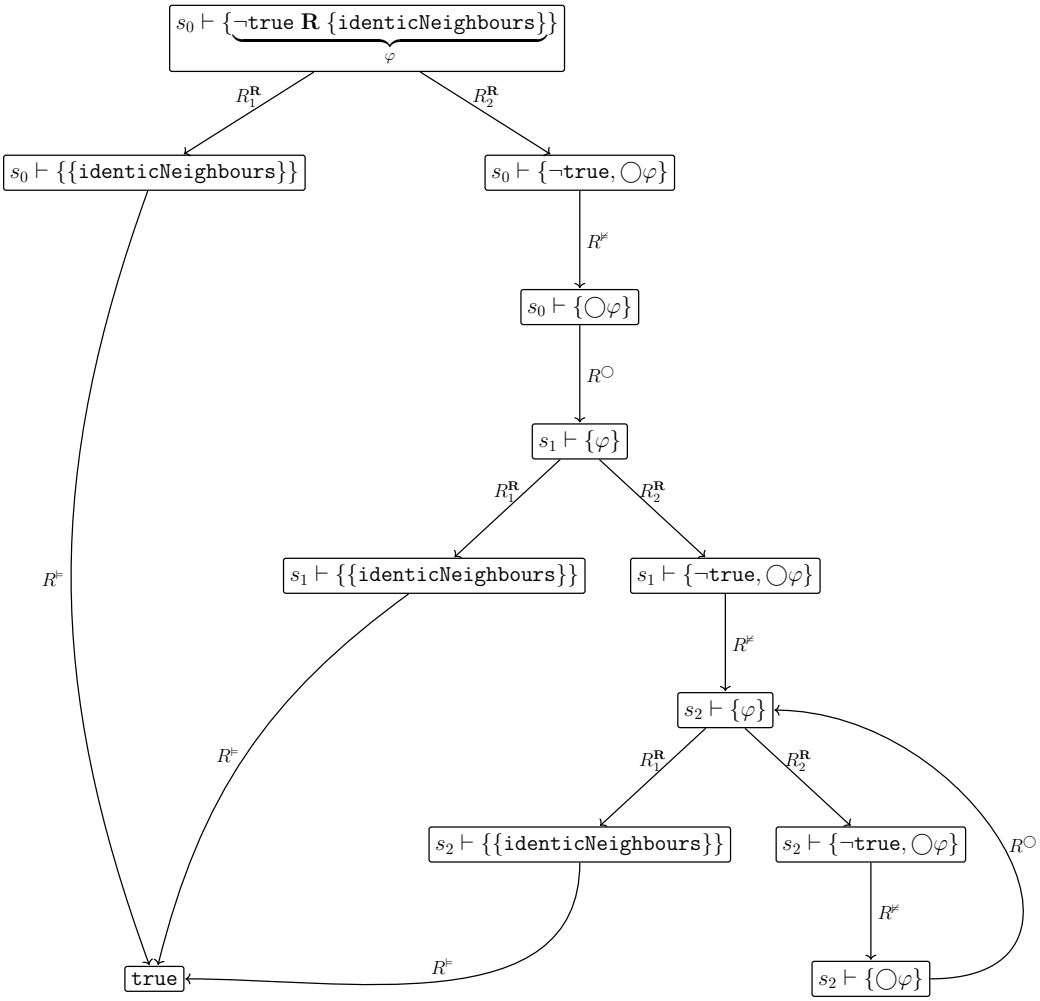


Figure 3.4: Proof structure for model checking method `SLList.traverse` for formula $\varphi = \Box\{\texttt{identicNeighbours}\}$.

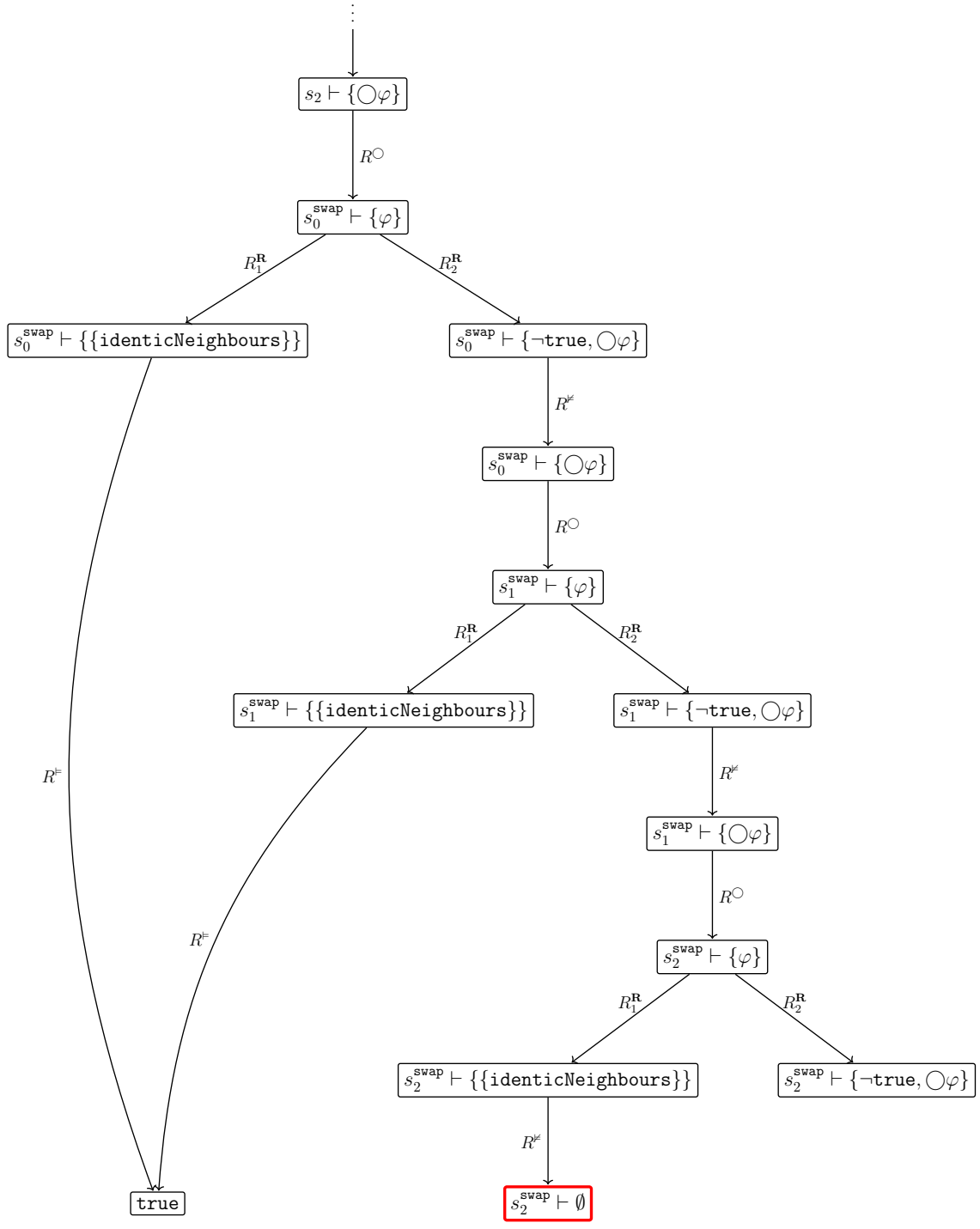


Figure 3.5: Proof structure for model checking method `SLList.swap` for formula $\varphi = \Box\{\texttt{identicNeighbours}\}$.

3.2 Model Checking Contracts

The tableaux construction by Grumberg et al. is an efficient approach at verifying LTL formulae for software programs [4]. A solution to model checking hierarchical systems is to flatten the underlying state space such that the hierarchy is removed from the model. Then, the state space can be verified by performing tableaux construction on the flattened system. However, this solution yields a number of problems. These include repetitive verification of states in case procedures are called multiple times. Also the size of the state space resulting from flattening a hierarchical model can become unboundedly large due to possibly unbounded recursion depth. Therefore, we propose the concept of *model checking contracts* that allow for model checking hierarchical systems without flattening the underlying state space. Model checking contracts can be included in hierarchical model checking algorithms in order to avoid re-computation of previously encountered verification tasks for a state space.

As the hierarchical structure of the system under consideration is kept, it follows that a hierarchy of model checking procedures is constructed where each level represents the verification of a procedure state space. Once state space generation and model checking is completed for a method execution, the successor states and model checking results are returned to the above lying level. Figure 3.6 visualizes the flow of model checking information throughout a hierarchy of method calls $m_i(), 1 \leq i \leq 3$. Every executed method i contributes a state space that is checked during generation. The state labels $\Phi^{(k)}$ denote the set of LTL formulae to be validated.

Important for model checking procedure state spaces is the labeling of the contained states with atomic propositions. An atomic proposition $a \in AP$ is added to a state s if $s \models a$. In ATTESTOR, only the states of the top-level state space are labeled, as procedure state spaces were not considered in the tableaux construction. However, in order to model check procedure state spaces, the corresponding states need to be labeled as well. When a method is invoked at a state s, the heap of s is adapted to the scope of the invoked method such that local variables are excluded. Thus, we obtain a *scoped heap*. This might produce faulty results when labeling states within procedure state spaces, as parts of the heap are disguised. For example, consider the property of the heap being a singly-linked list. Assume a heap, where this is in fact true. Now, if we enter a method `foo()` that does nothing, the heap will be scoped to an empty heap. Therefore, the property, that the heap is a singly-linked list, is violated. Thus, the state will not be labeled with the corresponding atomic proposition, resulting in a negative outcome of the model checking procedure despite the fact that the heap is still a singly-linked list outside of the current scope. In order to account for procedure state labeling, we introduce the notion of a *scope hierarchy* which tracks which parts of the heap have been excluded for state space generation.

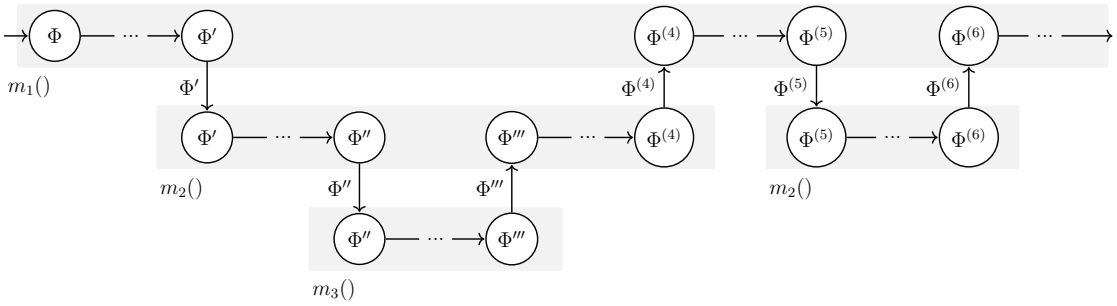


Figure 3.6: Conceptual flow of model checking information in a state space with method invocations.

As there might be a hierarchy of method calls, the heap under consideration might have been scoped multiple times. Thus, the collection of scopes is summarized in a scope hierarchy. Now, before computing the labels for a state under consideration, the scope hierarchy is applied to the state in reversed order, such that the original heap is restored. Based on the resulting state, the corresponding atomic propositions are determined.

In order to avoid repetitive verification of procedure state spaces for a same set of LTL formulae, model checking contracts store verification results for procedure state spaces with respect to a verified set of LTL formulae. Hence, model checking results that have been computed beforehand can be reused if the same verification task is encountered at a later stage. Model checking results include information on whether a formula is satisfied as well as the updated set of formulae for which the successor states need to be checked for. The model checking results are stored in contracts of the form

$$(HC_{in}, \Phi) \mapsto (\Phi', \delta_{0/1}, \pi),$$

where HC_{in} denotes the input heap for which the set Φ of formulae is to be validated. The tuple (HC_{in}, Φ) is mapped to a tuple of model checking results containing the resulting set Φ' of formulae to be checked for possible successor states, a Boolean value $\delta_{0/1}$ that indicates whether model checking was successful, and a failure trace π in case any formulae is found to be violated. Model checking contracts are stored in the context of *procedure contracts* which unambiguously define the executed method. Procedure contracts capture the overall effect of a procedure by defining pairs of pre- and post-conditions. Pre- and post-conditions specify the heap prior to and after the execution of a procedure, respectively. Details on procedure contracts are described in [10]. Thus, model checking contracts can be used to avoid repetitive model checking of a procedure state space for a set of input formulae.

Chapter 4

On-The-Fly Hierarchical Model Checking

The tableaux construction by Grumberg et al. [4] is an on-the-fly approach to model checking transition systems against an LTL formula φ. In this chapter, we describe how we adapt the algorithm to account for model checking of hierarchical structures including procedure state spaces such that not only model checking is executed on-the-fly but also state space generation is performed in an on-the-fly manner. We further present the implementation of the algorithm within the ATTESTOR framework and conclusively discuss our proceedings.

4.1 Algorithm

Given an input program and an LTL formula φ, our goal is to verify whether the program under consideration satisfies the temporal property specified by φ. Currently, ATTESTOR solves this problem by first generating a state space for the input program and model checking the resulting state space in a second step. Therefore, the tool implements the tableaux construction by Grumberg et al. (see Figure 1.2). As the state space generation phase only returns the state space of the main method of the input program, procedure state spaces induced by procedure executions are not directly verified during model checking. However, as seen in the example given in Listing 1.2 procedure state spaces might contain erroneous behavior that are not detectable from top level analysis. Hence, we present an on-the-fly hierarchical model checking algorithm that includes procedure state spaces in model checking. As a set of state spaces is included in the process of hierarchical model checking, we aim at providing an approach that does not require to generate all state spaces entirely. This is especially beneficial if verification can be terminated without checking every single state in a state space in case a verification decision can already be made after the analysis of a subset of states. Therefore, we extend the tableaux construction approach by the following functionalities:

- Model checking procedure state spaces, and
- On-the-fly state space generation, therefore interweaving state space generation and model checking.

The on-the-fly hierarchical model checking algorithm is an automated model checking procedure that does not only verify an LTL formula on-the-fly according to the tableaux construction, but also generates the required states of the program under consideration entirely on-the-fly. We illustrate the procedure of the algorithm in Figure 4.1.

The two core parts of the on-the-fly algorithm are the on-the-fly state space generation and the on-the-fly model checking algorithm. The on-the-fly state space generation is based on the ATTESTOR state space generation described in Section 1.1 which provides the frame for the on-the-fly procedures as depicted in Figure 4.1. ATTESTOR's procedure in generating a state space from an input program as shown in Figure 1.3 are extended by further processes (marked yellow) that enable the conjunction of both on-the-fly state space generation and on-the-fly model checking. After initializing the state space and the proof structure for an input program, the proof structure is constructed based on the tableaux rules defined in Section 3.1. The construction starts with a single provided state until a $\bigcirc$-formula is encountered. $\bigcirc$-formulae define properties for successor states of the current state. Hence, the tableaux rule $(R^{\bigcirc})$ specifies a state switch such that successor states need to be generated. State generation involves the execution of the program statement at the current state, i.e. abstract execution. At this point, we extend ATTESTOR's abstract execution by an intermediate step that model checks the just generated state. Therefore, we differentiate between two types of statements: ones that invoke a method execution, and ones that are executed in place. Statements that invoke a method execution trigger the model checking of a procedure state space induced by the called procedure. That is, the on-the-fly hierarchical model checking procedure is executed on the calling state of the method and the current set Φ of LTL formulae. The set Φ is a set of formulae resulting from successively applying tableaux rules to the initial LTL formula φ. Model checking results for a procedure state space and a set of LTL formulae are stored in model checking contracts.

After abstract execution, it is checked whether model checking of the resulting successor states is successful. In case the verification returns `true`, new assertions induced by the successor states and a set Φ' of resulting LTL formulae are added to the proof structure and the procedure is continued. Otherwise, a violating path has been found and the proof structure is declared to be unsuccessful. Hence, the proof structures of the above lying state spaces can be aborted as a violating path has been found such that it can be concluded that the set of formulae Φ is not satisfied for the complete program state space. Thus, an early termination of the

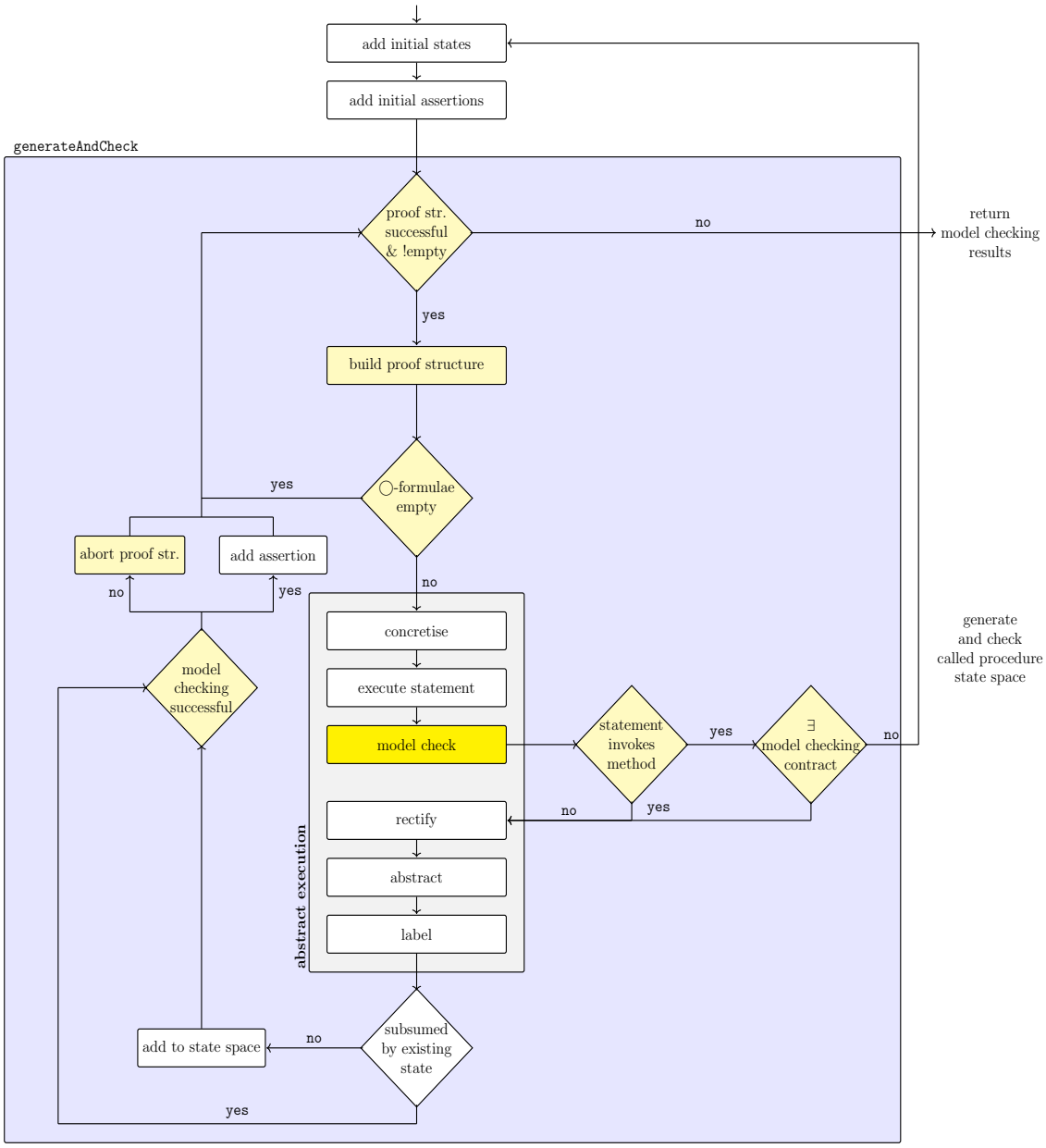


Figure 4.1: On-the-fly hierarchical model checking by a hierarchical tableaux construction.

model checking procedure is possible that does not require the computation of an entire state space.

The correctness of the on-the-fly tableaux construction for hierarchical state spaces follows from the correctness of the tableaux construction [4] and the correctness of the state space generation algorithm defined in [2].

4.2 Implementation

We implement the on-the-fly hierarchical model checking algorithm within the ATTESTOR framework written in JAVA. The code is available at `https://github.com/SallyChau/master_thesis/tree/master/attestor`. As both state space generation and model checking are performed, the algorithm encompasses ATTESTOR's state space generation and model checking phases. Therefore, the on-the-fly hierarchical model checking algorithm constitutes a separate phase in ATTESTOR refered to as the *on-the-fly model checking phase*. It is hence an alternative to the execution of ATTESTOR's verification phases.

The implementation of the on-the-fly model checking phase requires two main adaptations in the ATTESTOR framework. First, we need to adapt the tableaux construction algorithm such that it does not work on a complete state space, but rather successively demands for new states as soon as they are required for building the proof structure. Second, instead of generating an entire state space during a separated phase, we require a possibility to query for a list of successor states for a given state s. Figure 4.2 sketches the architecture of our implementation.

We first describe the implementation of the on-the-fly model checking procedure. In ATTESTOR, the `ProofStructure` class implements the tableaux construction in order to verify an LTL formula for a given input state space. By calling the method `ProofStructure.build` the proof structure is constructed. It requires a completely generated state space to operate on. As we aim at an approach that only computes states when required for expanding a proof structure, we modify the current implementation by introducing the class `OnTheFlyProofStructure`. Here, the tableaux construction is realized in a way such that it is capable of constructing the proof structure for an on-the-fly constructed state space. Instead of requiring a completely generated state space, the on-the-fly version solely requires initial assertions. These are expanded according to the tableaux rules until an assertion which only contains $\bigcirc$-formula is encountered. Since the underlying state space is not present, the proof structure construction cannot be continued. Thus, we require the generation of the successors of the state associated with the current assertion. At this point, the control is handed to the `OnTheFlyStateSpaceGenerator`, where successor states are generated for a given state. This step is indicated by a directed arrow labeled

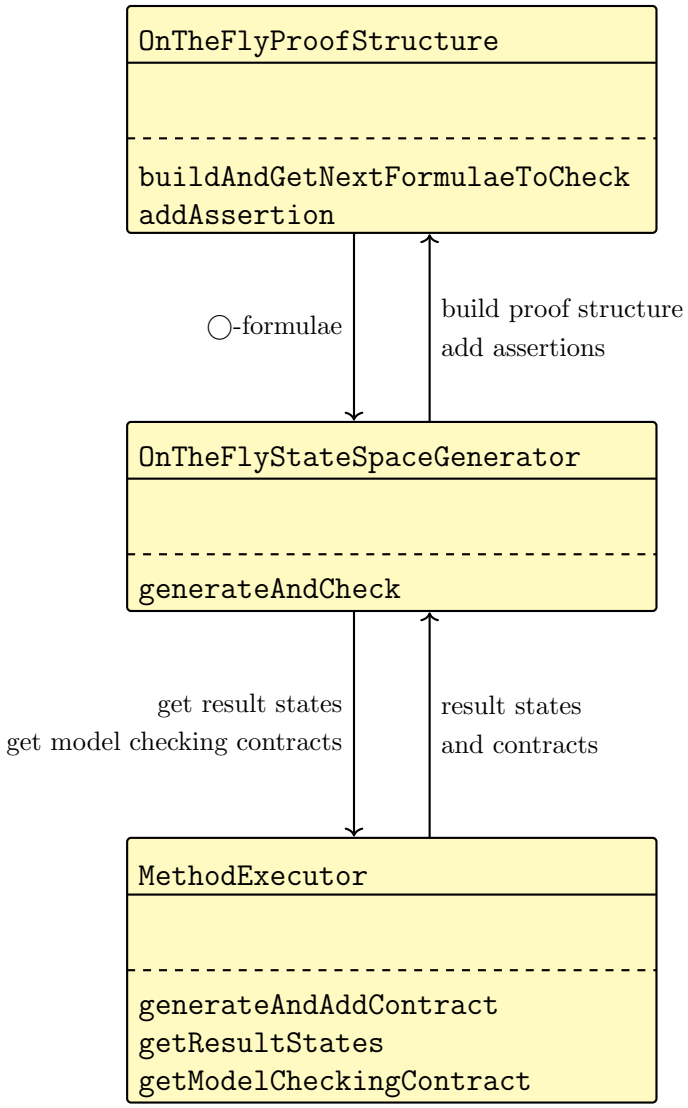


Figure 4.2: Conceptual UML-diagram for on-the-fly hierarchical model-checking.

"◯-formulae" in Figure 4.2 connecting the classes `OnTheFlyProofStructure` and `OnTheFlyStateSpaceGenerator`.

The class `OnTheFlyStateSpaceGenerator` realizes the computation of successor states for a given state s. It extends ATTESTOR's `StateSpaceGenerator` class, where an entirely generated state space is returned for an input program. In order to compute successor states for a state s, the program statement at state s is executed on the current heap. If the statement invokes a procedure execution, the verification of the called procedure is triggered. ATTESTOR's `MethodExecutor` class handles procedure calls by initiating (on-the-fly) state space generation for the called procedure. As procedure invocation induces a procedure state space, model checking is also performed for these such that the `MethodExecutor` class does not only return a list of successor states of the calling state s, but also information on model checking results. These are communicated to the proof structure by adding new assertions and reentering the model checking loop. This process is depicted by the directed arrow pointing from the `OnTheFlyStateSpaceGenerator` class to the `OnTheFlyProofStructure` class in Figure 4.1. The communication between the classes depicted in the diagram in Figure 4.1 continues until one of the following cases occur: a verification decision is made, i.e., the proof structure returns that the property under consideration is satisfied or violated, all states have been verified, or computational resources, i.e., memory, have been exceeded.

We execute the implementation of the on-the-fly hierarchical model checking for the example from Listing 1.2, where the `swap` method introduces errors in the execution of the main method `traverse`. The example shows that top level model checking is not sufficient to detect errors in procedure state spaces. In contrast to this, executing the on-the-fly hierarchical model checking algorithm for the erroneous `traverse` method effectively returns that the property, that a list is never mutated, is violated. The returned failure trace of state IDs

$$[[\underbrace{1, 14, 15, 44, 56, 67, 68}_{\texttt{traverse}}], [\underbrace{0, 1, 3, 5}_{\texttt{swap}}]]$$

indicates that the source of the failure lies within a procedure call, as the failure trace contains two distinct traces, i.e., the first list of states describes the trace in the top level state space of the main method `traverse`, the second list describes the trace in the state space of the procedure `swap`.

4.3 Discussion

When model checking procedural programs with possibly recursive procedure calls, procedure state spaces need to be taken into account. ATTESTOR's current model

checking approach only verifies the top-level state space and thus disregards possible erroneous behavior within method executions. The on-the-fly approach on hierarchical model checking presented in this chapter solves this gap by verifying procedure state spaces during their generation. Thus, the algorithm successfully interweaves state space generation and model checking. Violations can also be tracked on procedure state space level and hence offer more precise debugging instances. In order to avoid repetitive model checking of model checking instances, the algorithm applies model checking contracts for procedure state spaces in order to reuse model checking results that have been computed beforehand. Furthermore, the on-the-fly state space construction allows for early termination of the model checking procedure such that time and memory can be saved as not all states need to be determined.

However, in case of model checking a procedure state space for multiple distinct LTL formulae, the procedure state space needs to be computed afresh since they are not stored. This is due to the fact that the on-the-fly approach does not guarantee to return complete state spaces that can be reused. This might cause an overhead of state space generations.

Thus, the on-the-fly approach is especially suitable for cases in which erroneous behavior is expected within method executions in order to quickly find a counterexample, whereas positive validation of a property might cause an overhead of computations.

Chapter 6 presents benchmarks on the on-the-fly model checking algorithm described in this chapter.

Chapter 5

RSM-based Hierarchical Model Checking

Recursive state machines model the control flow of sequential imperative programs containing (recursive) procedure calls. Within the concept of RSMs, two kinds of states are differentiated: states that constitute a statement that is executed in place, and states that invoke method executions. This differentiation is helpful in modeling the hierarchical structure of procedural programs. Alur et al. present an automata-based approach for model checking recursive state machines [1]. The algorithm constructs a product automaton of the RSM under consideration and Büchi automaton representing the negation of an LTL formula to be verified. The product automaton is then checked for language emptiness. If the accepted language of the product automaton is empty, it can be concluded that the intersection of the languages accepted by the RSM and the Büchi automaton is empty. Therefore, the property specified by the LTL formula is satisfied. As the automata-based approach requires the construction of three automata (RSM, Büchi, and product automaton) for verifying an LTL formula, we propose an algorithm that only operates on the RSM itself. Therefore, we employ the tableaux construction by Grumberg et al. for model checking RSMs. In an RSM, component state machines store the pre-generated procedure state spaces of procedures. In order to account for verification of procedure state spaces, the tableaux construction approach is executed for a state space of a component if it the represented procedure is called. Results are stored as model checking contracts within a component state machine such that they can be reused if required. In the sequel of this chapter, we introduce the RSM-based algorithm in detail and describe its implementation within the Attestor framework. Finally, we evaluate the approach in terms of time and space efficiency.

5.1 Algorithm

Given a transition system T and an LTL formula φ, the goal of hierarchical model checking is to verify whether $T \models \varphi$. The RSM-based hierarchical model checking algorithm solves the verification problem in a two-step fashion: First, a recursive state machine is constructed for an input program. Second, model checking is executed on the generated RSM.

We illustrate the process of the RSM-based hierarchical model checking algorithm in Figure 5.1. In contrast to the on-the-fly approach described in the previous chapter, this approach separates state space generation and model checking into two phases. The first phase constructs an RSM for an input program. We assume that the (procedure) state spaces are generated in a previous step, e.g., in ATTESTOR's state space generation phase, and are therefore given. The construction of the RSM starts with generating a component state machine for every method of the input program. Procedure state spaces are mapped to the corresponding component based on the called method. Calling dependencies between states of a procedure state space and called procedures are stored in a set of boxes that indicate which method is invoked by which state within a component. In this context, one component might contain a set of state spaces as a procedure might have been called by different calling states.

The second phase executes LTL model checking for the generated RSM and a set Φ of LTL formulae. Model checking an RSM boils down to model checking its components. Thus, starting with the component A_1 corresponding to the main method of the program, a tableaux is constructed for the state space of A_1. If during the construction, a state s is encountered that invokes a method j, the according box is entered and model checking is continued for component A_j for the set Φ' of formulae. By this means, a hierarchy of procedure model checking processes is created, as presented in Figure 3.6. The model checking results for a component A_j are returned to the calling state, and included in the proof structure of the calling component. Similar to the on-the-fly hierarchical model checking approach, repetitive model checking of the same set of formulae for a given procedure state space is avoided by storing model checking results as model checking contracts. Thus, if a previously solved verification task appears, the proof structure does not need to be repeatedly computed, but stored results are applied. If a violating state has been encountered, the proof structure construction is aborted for all components, otherwise the procedure is continued until all relevant proof structures have been constructed such that the proof structure of the main state space, thus of the component A_1, comes to a conclusion.

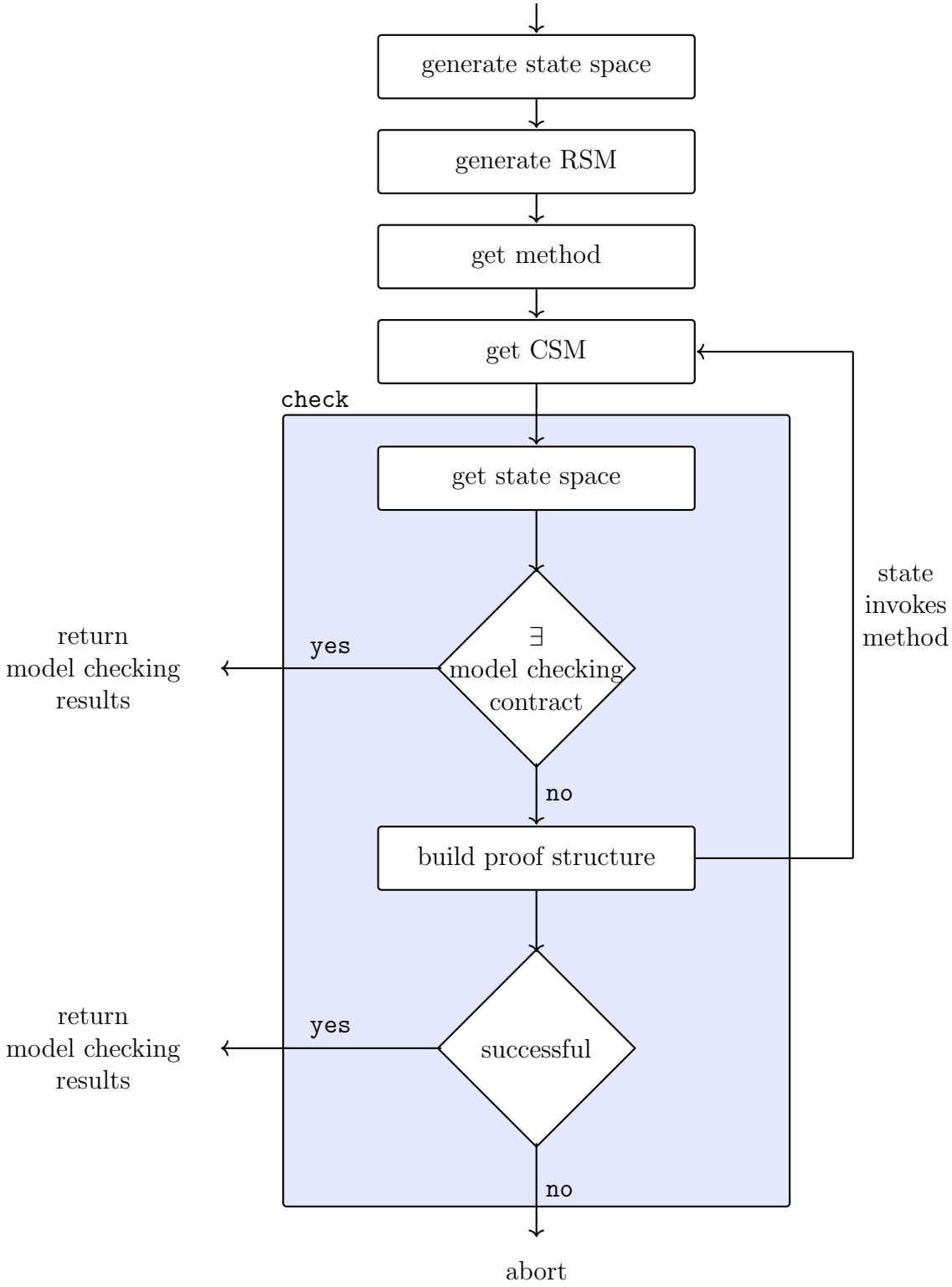


Figure 5.1: RSM-based hierarchical model checking using the tableaux construction.

5.2 Implementation

We implemented the RSM-based hierarchical model checking algorithm within the ATTESTOR framework. The implementation is available at `https://github.com/SallyChau/master_thesis/tree/master/attestor`. The approach is an alternative to the tool's currently implemented model checking phase that uses the tableaux construction to verify the top-level state space of an input program for an LTL formula φ. In order to realize the RSM-based algorithm within ATTESTOR, we require the following adaptations:

- Retrieve procedure state spaces from state space generation phase,
- Implement classes for recursive state machines and component state machines, and
- Enrich RSM and CSM with proof structures to account for model checking procedure state spaces.

Figure 5.2 sketches the implementation of the required adaptations.

We retrieve procedure state spaces and the corresponding calling states from ATTESTOR's state space generation phase. Given a set of procedure state spaces, the class `RecursiveStateMachine` builds a corresponding RSM by creating an object of the class `ComponentStateMachine` for each method of an input program. Since distinct calls to a method originating from distinct states might result in distinct state spaces, a `ComponentStateMachine` internally maps the calling state to the resulting state space. The class `RecursiveStateMachine` manages the calling information of a state s by creating boxes in the state's `ComponentStateMachine` that contains the information which `ComponentStateMachine` is called by s. Moreover, a `ComponentStateMachine` stores the model checking results for executed tableaux constructions in order to avoid repetitive computation of proof structures, similar to the implementation in Chapter 4. However, since the context of the method is unambiguously determined be the `ComponentStateMachine`, we do not require the context of procedure contracts as it is the case for on-the-fly model checking.

In order to account for model checking procedure state spaces, we require the class `ComponentStateMachine` to execute tableaux construction for the according state space. Therefore, we implement the class `HierarchicalProofStructure` which adapts ATTESTOR's proof structure to work on state spaces stored in a `ComponentStateMachine`. If during proof structure construction a state is encountered that invokes a method execution, the corresponding `ComponentStateMachine` representing the called method starts model checking for the according procedure state space. After termination of the sub-model checking procedure, we return to

the previous proof structure applying the recent model checking results. If the result is a final one, i.e., the proof structure is unsuccessful or the proof structure is ultimately successful (there are no more formulae to check), the result is reported to the dependent proof structures, such that model checking is terminated. Finally, the model checking result is returned including a failure trace in case of property violation.

In ATTESTOR, the RSM-based hierarchical model checking algorithm is executed by setting the option

```
--hierarchical-model-checking
```

in the settings of the verification task. The RSM-based approach then replaces ATTESTOR's top level model checking. Executing the RSM-based hierarchical algorithm on the example instance given in Listing 1.2, returns that the property, that the list is never mutated, is violated by the method `traverse`. In this context, the algorithm returns the failure trace of state IDs

$$[[\underbrace{7, 103, 104, 105, 106, 107, 108}_{\texttt{traverse}}], [\underbrace{0, 1, 2, 3}_{\texttt{swap}}]],$$

that indicates that the source of the violating path lies within the procedure `swap`. This result is similar to the one of the on-the-fly hierarchical model checking algorithm except for the state IDs as both algorithms employ differing state space generation algorithms.

5.3 Discussion

In this chapter, we presented an algorithm for model checking procedure state spaces represented as an recursive state machine. The hierarchical structure of the state space is captured by the RSM as every method is represented by a component. The component stores corresponding procedure state spaces, boxes that represent method invocations, and model checking results. Model checking of a procedure state space is controlled by the corresponding component, such that verification results are communicated to the calling state.

RSM-based hierarchical model checking has the advantage that procedure state spaces are stored within the component state machine of a method. Thus, procedure state spaces only need to be computed once during the state space generation phase and are reused in future model checking tasks. Similarly, model checking of a formula φ can be skipped if the results are known due to previous model checking instances as results are stored in contracts.

However, the advantage of reusing previously computed results induces the require-

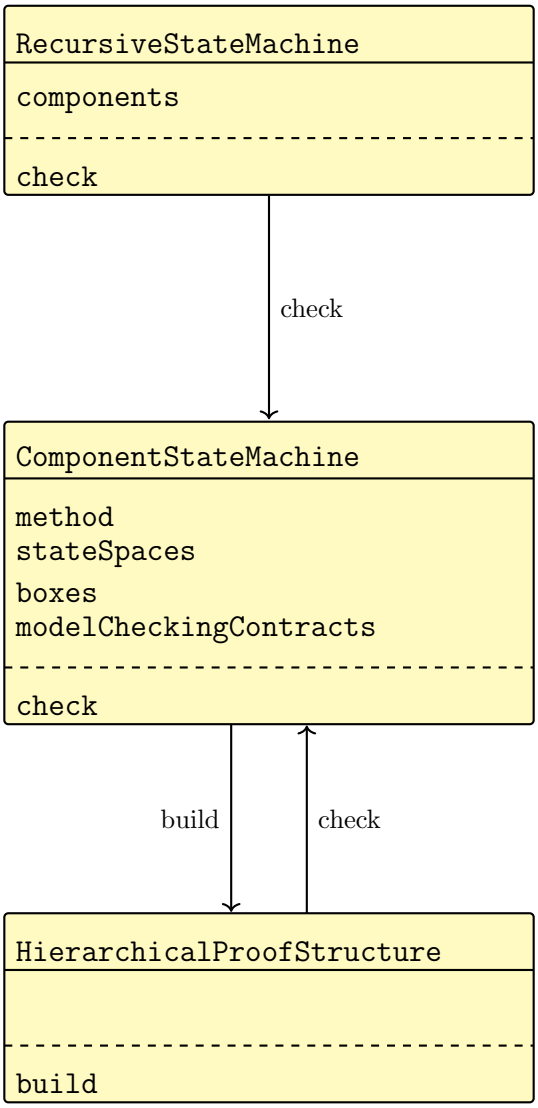


Figure 5.2: UML for hierarchical model checking with RSMs.

ment to store the state spaces and the model checking results. Hence, the memory usage of the algorithm increases with the number of distinct procedures called during program execution. Furthermore, generation of the state spaces prior to model checking requires that the complete state space is generated. Therefore, early termination of state space generation in case only a subset of states is sufficient for tableaux construction is not enabled.

In order to evaluate the algorithm in practice, Chapter 6 presents benchmarks on the RSM-based model checking algorithm and compares the approach to the performance of the previously introduced on-the-fly hierarchical model checking and ATTESTOR's top level model checking procedure.

Chapter 6

Benchmarks

We presented two algorithms for model checking hierarchical state spaces that result from analyzing input programs with procedure calls. The on-the-fly hierarchical model checking algorithm uses the tableaux construction by Grumberg et al. [4] to build a proof structure for checking a property while generating the underlying state space on-the-fly. The RSM-based hierarchical model checking algorithm models the hierarchical state space as a recursive state machine such that procedure state spaces are depicted. Based on the collection of state spaces, the tableaux construction is used to verify the property under consideration. In this chapter, we compare the performances of the algorithms with the model checking algorithm implemented in Attestor that uses the tableaux construction to check the top level state space. In this context, we evaluate the execution time and the number of generated states of the individual algorithms on 79 model checking instances.

6.1 Experimental Setup

The benchmarks are performed on an Intel Core i7-8650U CPU @ 1.90 GHz with the Java virtual machine (OpenJDK version 1.80_112). The benchmark instances are executed by the Attestor tool with the help of a benchmark helper class, that is based on the Attestor Benchmark Helper. For each model checking algorithm, i.e.

- Attestor's top level model checking,
- on-the-fly hierarchical model checking, and
- RSM-based hierarchical model checking,

we determine the following statistics:

- the number of generated states of the main state space and the total number of generated states,

- the execution time of the individual algorithm consisting of the time needed for state space generation and model checking in seconds.

A settings file specifies the options for the execution of a benchmark instance. Generally, the options include but are not limited to

- `--description` a description of the performed model checking task,
- `--classpath` the path to the `.java` file that contains the input program,
- `--class` the class to be analyzed,
- `--method` the method to be analyzed,
- `--predefined-grammar` the grammar to be employed during the analysis specifying which data structure is analyzed,
- `--initial` a file that specifies the heap of the initial state. If none is specified the heap is assumed to be initially empty.
- `--model-checking` the LTL formula to be checked,
- `--hierarchical-model-checking` an option that indicates that the RSM-based hierarchical model checking algorithm is executed instead of the top level model checking algorithm. The on-the-fly hierarchical model checking algorithm involves an own phase in ATTESTOR such that it is executed next to the tool's standard state space generation and model checking.

6.2 Instances

We compare the hierarchical model checking algorithms and ATTESTOR's top level model checking based on 79 benchmark instances. Each instance is a model checking task that verifies a property given as an LTL formula for an input program that operates on a specified data structure.

The input program contains methods working on data structures whose grammar is predefined in the ATTESTOR tool, i.e., singly-linked lists (`SLList`), doubly-linked lists (`DLList`), and trees (`Tree`). For each data structure, we implement several operations in JAVA code such as `SLList.traverse` or `DLList.reverse` that run through all elements of a singly-linked list or reverse the order of the contained elements in a doubly-linked list, respectively.

Each operation is verified for a number of properties specified as LTL formulae. We analyze the following properties during the model checking procedures:

- *Memory Safety* (M) is checked during state space generation. In this context, we analyze the LTL formula $\square$ `true` such that the on-the-fly hierarchical model checking generates the entire states space. Without a specified formula, the on-the-fly approach would not be executed as there is no specified formula to be verified.

- The *completeness property* (C) verifies whether all elements of the input are visited by a specific variable. For example, the formula

$$\square\Diamond\{\texttt{terminated}\} \rightarrow \Diamond\{\texttt{visited(current)}\}$$

  describes that all elements of the input are eventually visited by the variable `current` upon termination of the program.

- The *correctness property* (Co) summarizes properties that verify that an input program delivers correct results. Therefore, we consider formulae such as

$$\square\Diamond\{\texttt{terminated}\} \rightarrow \Diamond\{\texttt{(return == null) \& (terminated)}\}$$

  stating that the return value is `null` upon termination of the program. The converse property can be verified by using this formula if the model checking algorithm outputs that the property is violated.

- The *neighborhood preservance property* (N) proves whether the input data structure corresponds to the data structure upon termination of the program. An LTL formula to specify this property is

$$\square\Diamond\{\texttt{terminated}\} \rightarrow \Diamond\{\texttt{terminated \& identicNeighbours}\}.$$

- The *reachability property* (R) verifies whether a variable is reachable from another one via specified pointers upon termination. An example for an LTL formula specifying a reachability property is given by

$$\Diamond\{\bigcirc\{\texttt{terminated}\} \rightarrow \texttt{isReachable(head, tail, [next])}\}$$

  which states that the variable `tail` is reachable from the variable `head` via `next` pointers.

- The *shape property* (S) checks whether the heap is of a certain shape. For example, the formula

$$\square\Diamond\{\texttt{terminated}\} \rightarrow \square\Diamond\{\texttt{SLList}\}$$

  defines that the heap is of the shape of a singly-linked list.

For the completeness and neighborhood preservance properties it is required to track object identities during program execution. This is done by marking objects such that they are identified across different states in the state space [2, 7].

In order to analyze the effectiveness and efficiency of the algorithms in terms of hierarchical model checking, the instances include input programs with and without procedure calls as well as recursive methods. Furthermore, we introduced erroneous code snippets such that property violation (on procedure level) is examined.

An overview of all implemented methods and the verified properties is depicted in Table A.1. The table also shows the experimental results for executing each introduced model checking algorithm in this thesis for every instance.

6.3 Results

The executed benchmark instances give us a basis to compare the three model checking algorithms in terms of their *effectiveness* and *efficiency* in model checking hierarchical state spaces. We base the effectiveness of an algorithm on the correctness of the model checking result and its efficiency on the required number of generated states and time in order to perform a verification task.

6.3.1 Effectiveness

We executed 79 benchmark instances both on the hierarchical model checking algorithms and ATTESTOR's top level model checking algorithm. The model checking results for each algorithm is displayed in Table A.1. For most instances, the three algorithms yields the same result. However, within the benchmark instances six instances include code that introduce manipulations to the heap within a procedure call such that the property under consideration is violated within a procedure state space. Executing these examples shows that the top level model checking algorithm is not able to spot the violations within procedure state spaces, thus returns that the property is satisfied. However, the hierarchical model checking algorithms find failure traces within procedure states and output that the property under consideration is violated. Consider the example in Listing 1.2 from Chapter 3, where the method `SLList.traverseModifyList` implements a list traversal equipped with a `swap`-method that swaps out the current list with the `null` value back and forth, so that the `swap`-method has no lasting effect on the original list. Verifying the property that the initial list is never mutated, the top level model checking algorithm outputs that the property is verified. However, both hierarchical model checking algorithms indicate that the propery is violated within a procedure state space.

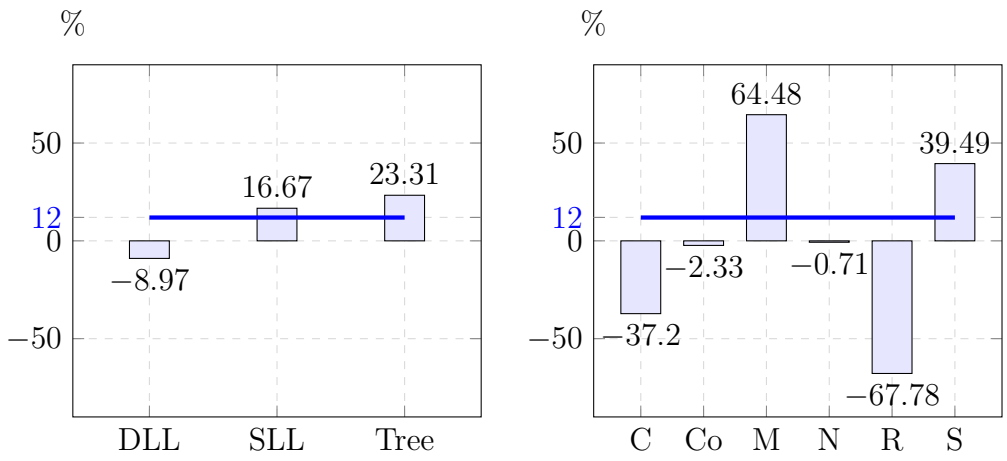
Consequently, in contrast to the top level model checking algorithm the hierarchical approaches verify hierarchical state spaces for each level such that erroneous behavior within procedure calls are detected.

6.3.2 Efficiency

The efficiency of the hierarchical model checking algorithms is evaluated based on the determined number of generated states required for model checking and the execution time. We compare these figures with the ones of the top level model checking algorithm that includes a preceding state space generation phase. We compare the algorithms' performances by computing the differences in percent. Grouped by different categories of benchmark instances, the differences in the number of generated states and execution times are depicted in Figures 6.1, 6.2, and 6.3, respectively.

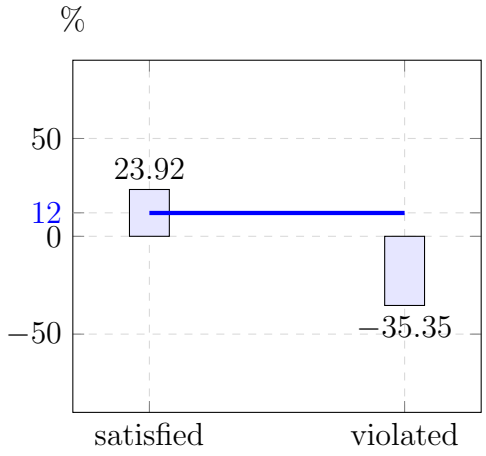
Figure 6.1 displays the difference in the number of generated states between the on-the-fly hierarchical model checking algorithm and the top level model checking algorithm. The overall average difference of 12% states that the on-the-fly hierarchical approach generally generates 12% more states than the top level approach. This is due to the fact that (procedure) state spaces are not stored such that states are possibly computed multiple times. However, there are also cases in which the on-the-fly hierarchical approach requires less states in order to conduct model checking. These include instances where properties are found to be violated as displayed in Figures 6.1c and 6.1f where up to 50% less states are generated compared to the top level model checking algorithm. This is due to the fact that violating states are found at a stage where not the complete state space has been generated yet, so that the state space generation is terminated. The RSM-based model checking algorithm employs ATTESTOR's state space generation such that there are no differences in the number of generated states compared to ATTESTOR's verification procedure.

Next to the number of generated states, the execution time of the model checking algorithms is an important factor to consider in evaluating their efficiency. Figures 6.2 and 6.3 show the difference in percent in the execution times compared to ATTESTOR's model checking algorithm including the state space generation for the on-the-fly hierarchical algorithm and the RSM-based hierarchical algorithm, respectively. Both algorithms require around 108% or 77% more time than the top level approach. This is due to the fact that the hierarchical algorithms generally verify more states than the top level algorithm as they also consider procedure states in the model checking procedure. It is striking that especially the on-the-fly hierarchical model checking algorithm requires considerably more time than the top level approach. This coincides with the greater amount of states generated during the on-the-fly state space generation. In this context, the only bars indicating that the on-the-fly approach is faster than ATTESTOR's implementation are the bars for in-

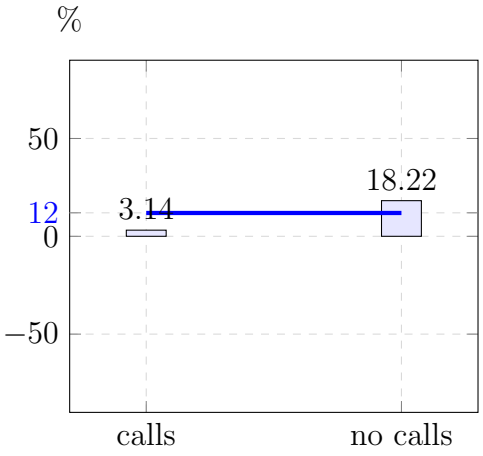


(a) Grouped by datastructure.

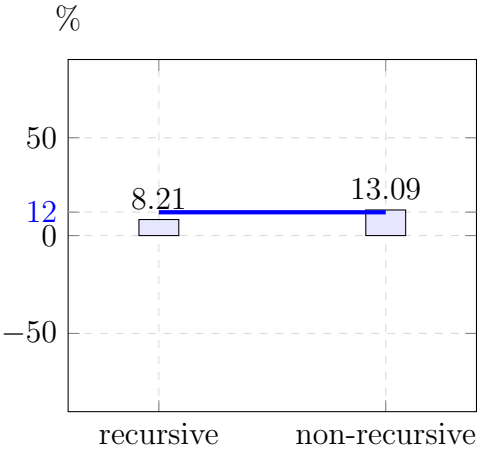
(b) Grouped by properties.



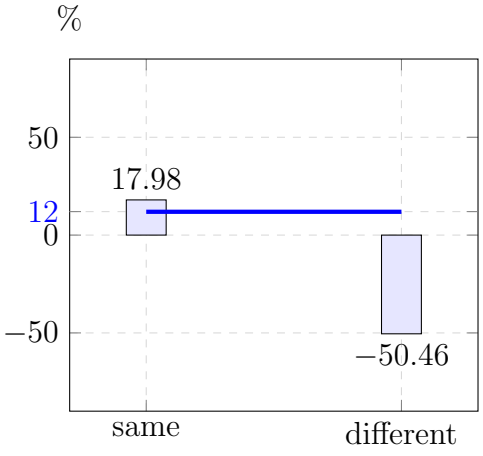
(c) Grouped by model checking results.



(d) Grouped by procedure calls.



(e) Grouped by recursion.

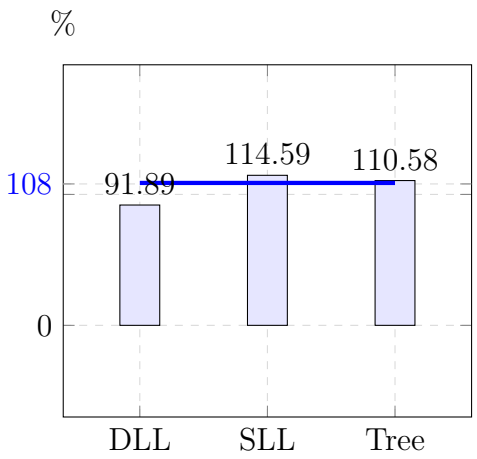


(f) Grouped by differing results.

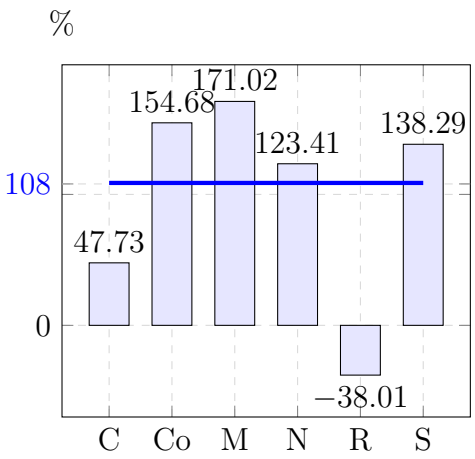
Figure 6.1: Difference in the number of generated states in % between the on-the-fly hierarchical model checking algorithm and the top level model checking grouped by different categories.

stances that verify reachability properties in Figure 6.2b and the instances for which the top level and the hierarchical algorithms deliver differing model checking results, which are also the categories for which the on-the-fly approach generates less states than ATTESTOR's state space generation.

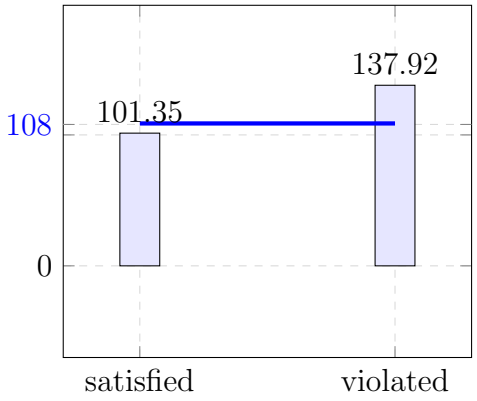
Comparing the on-the-fly and the RSM-based hierarchical model checking algorithms with each other, it becomes visible that the RSM-based algorithm is faster in average. As the number of generated states coincide with the figures of the top level algorithm, the RSM-based approach also generates less states in average than the on-the-fly procedure. Although the RSM-based approach is generally slower than the top level model checking algorithm, it comes with the advantage that procedure state spaces are also verified during model checking.



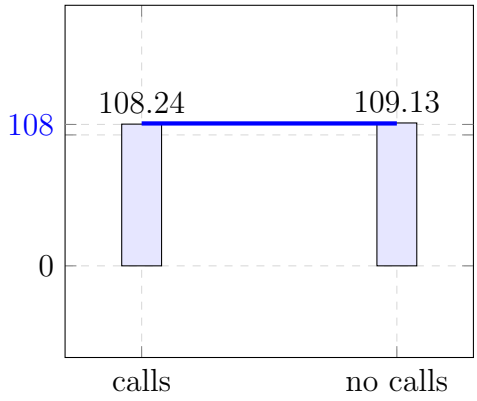
(a) Grouped by datastructure.



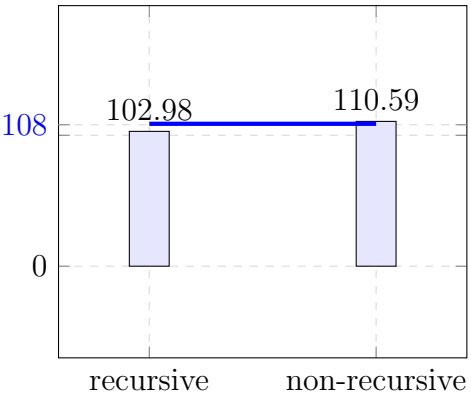
(b) Grouped by properties.



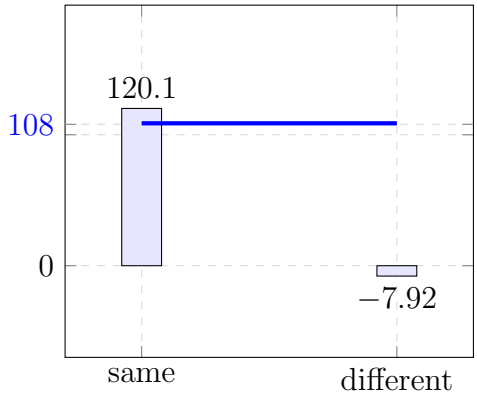
(c) Grouped by model checking results.



(d) Grouped by procedure calls.

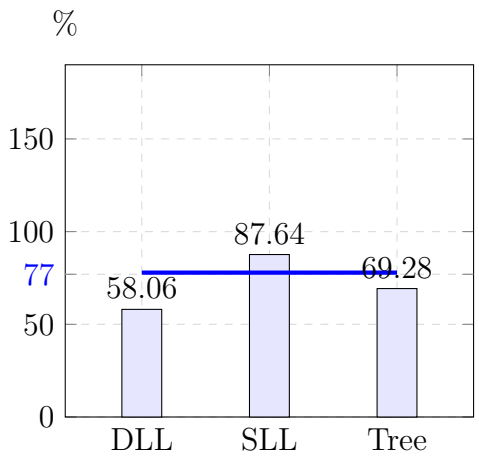


(e) Grouped by recursion.

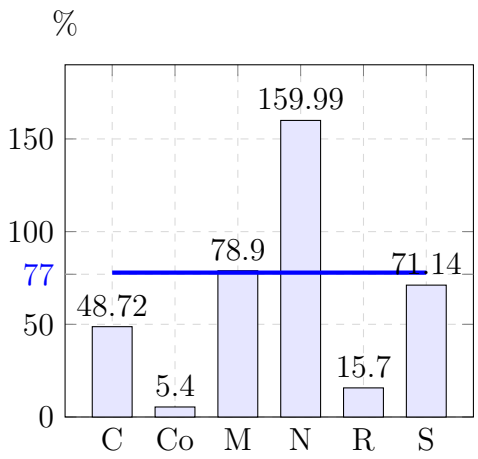


(f) Grouped by differing results.

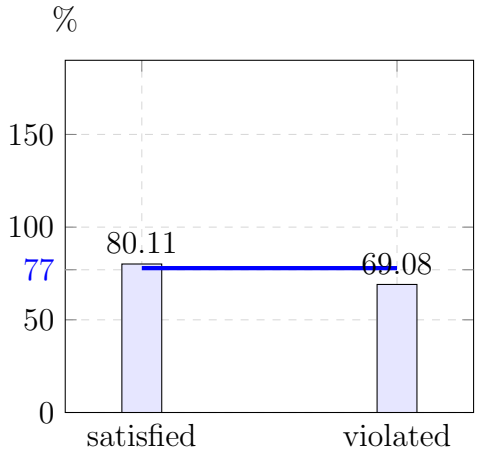
Figure 6.2: Difference in the execution time in % between the on-the-fly hierarchical model checking algorithm and the top level model checking grouped by different categories.



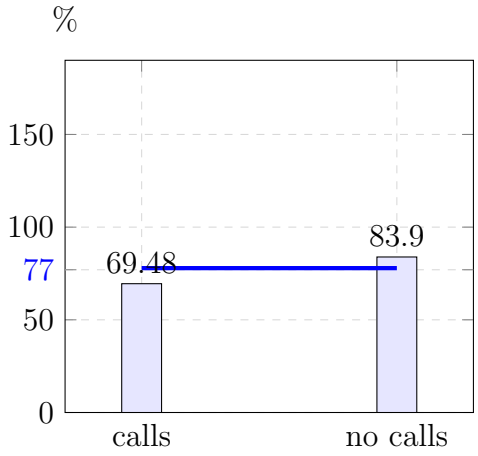
(a) Grouped by datastructure.



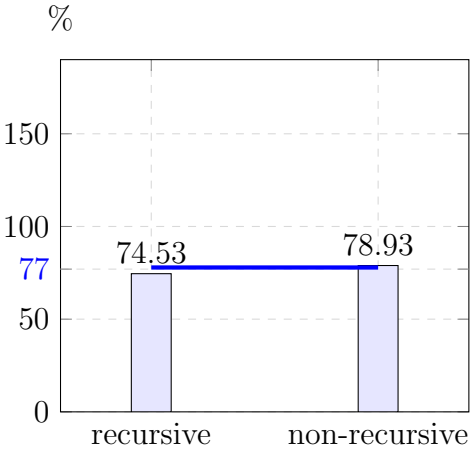
(b) Grouped by properties.



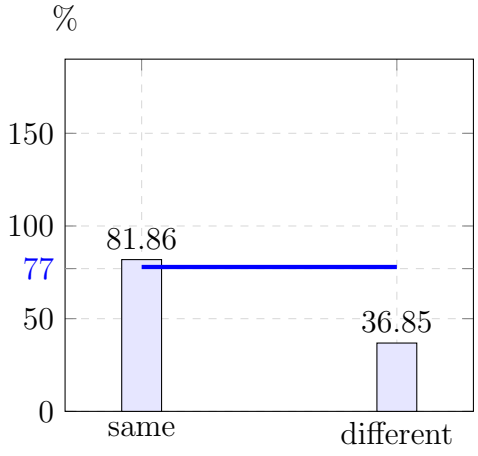
(c) Grouped by model checking results.



(d) Grouped by procedure calls.



(e) Grouped by recursion.



(f) Grouped by differing results.

Figure 6.3: Difference in the execution time in % between the RSM-based hierarchical model checking algorithm and the top level model checking grouped by different categories.

Chapter 7

Conclusion and Future Work

We introduced two hierarchical model checking algorithms, i.e., the on-the-fly and the RSM-based hierarchical model checking algorithms. We implemented the algorithms within the ATTESTOR tool in JAVA code and executed the algorithms on a set of 79 model checking tasks. Based on the performed benchmarks, we compared the two hierarchical model checking algorithms with ATTESTOR's model checking procedure that verifies a property under consideration for the main state space of an input program.

Although all three model checking algorithms presented in this thesis employ the tableaux construction Grumberg et al. [4], the algorithms demonstrate differing performance results in terms of effectiveness and efficiency. The differences are traced back to the underlying state space of the model checking task. While the state space is constructed on-the-fly (and terminates as soon as a statement can be made about the validity of the input formula) in the on-the-fly hierarchical model checking approach, ATTESTOR's top level analysis accesses an entirely generated the state space resulting from the preceding state space generation phase. As the tableaux construction does not always require the complete state space, the on-the-fly hierarchical model checking approach generates the states on demand during the tableaux construction. Furthermore, states in procedure state spaces are directly model checked, so that not only the top level state space is considered for model checking. Thus, verifying properties that are expected to not require a completely generated state space, i.e., properties that are expected to be violated, or properties referring to a limited set of states, is performed efficiently by the on-the-fly hierarchical model checking algorithm. The on-the-fly nature of the algorith, however, also entails the disadvantage that states might be generated multiple times as procedure state spaces are not stored. Model checking multiple LTL formulae for the same state space thus requires recomputation of the state space for every verification task. This problem is approached by the RSM-based model checking algorithm that stores all state spaces generated during ATTESTOR's state space generation phase such that procedure state spaces are available during model checking. The

RSM-based approach then structures the collection of state spaces within a recursive state machine and performs the tableaux construction for model checking based on the RSM. This approach has the advantage that states are only computed once and procedure state spaces can be checked. This is especially beneficial in cases where a hierarchical state space is checked for multiple different LTL formulae. However, storing all procedure state spaces infers a high demand of storage.

Consequently, the introduced hierarchical model checking algorithms show that top level model checking is not sufficient to trace failures that are introduced within procedure calls. Thus, including procedure state spaces in model checking reveals possibly erroneous behavior that is not visible from the context of the main program. Based on the handling of state space generation, different goals in model checking are approached; a combination of an on-the-fly state space generation with an on-the-fly tableaux construction delivers fast results for verifying possibly violated LTL formulae while RSM-based model checking is suitable to verify a set of properties on different levels of a method stack.

Further extensions in the area of hierarchical model checking include the generation of a counter example such that the input program can be tested and improved. In this context, the implemented hierarchical model checking algorithms introduced in this thesis deliver a failure trace that indicates a path within the state space that violates the property under consideration. It is left to implement the procedure of translating the failure trace into a counter example.

Furthermore, as both the on-the-fly and the RSM-based hierarchical model checking algorithm include advantages and disadvantages in model checking procedure state spaces, a possible solution is to create a hybrid method that generates procedure state spaces on-the-fly and storing them as an RSM. This approach allows us to benefit from the on-the-fly approach as well as from the RSM-based procedure. On the one hande, in case a model checking result is determined without requiring the entirely generated state space, we benefit from the early termination property of the on-the-fly approach. On the other hand, the RSM allows us to access previously (partly) generated state spaces. Hence, the disadvantages of both algorithms are mitigated by combining their advantages.

In a next step, our benchmark results are to be refined by including instances for additional datastructures and properties such that the algorithms are tested on a greater and more varied number of instances.

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Appendix A

Benchmark results

Method	P	Top Level MC						On-The-Fly HMC				RSM-based HMC					
		States			Time in s			States			Time in s	States			Time in s		
		t/f	Main	Σ	SSG	MC	Σ	t/f	Main	Σ	Σ	t/f	Main	Σ	SSG	MC	Σ
DLList.build	M	t	30	62	0.0578	0.0032	0.0609	t	30	62	0.1976	t	30	62	0.0553	0.0203	0.0756
DLList.build	S	f	30	62	0.0503	0.0015	0.0519	f	12	36	0.0621	f	30	62	0.0510	0.0072	0.0582
DLList.findLastCall	C	f	33	629	0.5845	0.0024	0.5869	t	17	236	0.5374	t	33	629	0.4778	0.0425	0.5202
DLList.reverse	C	t	572	572	0.0859	0.0067	0.0926	t	444	444	0.0897	t	572	572	0.0587	0.0580	0.1167
DLList.reverse	Co	f	82	82	0.0336	0.0062	0.0397	f	82	82	0.1528	f	82	82	0.0342	0.0109	0.0451
DLList.reverse	Co	f	86	86	0.0392	0.0047	0.0439	f	82	82	0.0813	f	86	86	0.0331	0.0096	0.0427
DLList.reverse	M	t	70	70	0.0041	0.0007	0.0048	t	133	133	0.0139	t	70	70	0.0062	0.0023	0.0085
DLList.reverse	N	f	1332	1332	0.5705	0.0310	0.6015	f	837	837	4.1222	f	1332	1332	0.7106	1.1568	1.8674
DLList.reverse	N	f	1332	1332	0.6196	0.0134	0.6330	f	817	817	0.8044	f	1332	1332	0.6230	0.3187	0.9417
DLList.reverse	R	t	100	100	0.0488	0.0003	0.0491	t	4	4	0.0067	t	100	100	0.0484	0.0004	0.0488
DLList.reverse	R	t	100	100	0.0363	0.0002	0.0365	t	4	4	0.0023	t	100	100	0.0335	0.0003	0.0338
DLList.reverse	S	t	70	70	0.0086	0.0009	0.0095	t	133	133	0.0222	t	70	70	0.0090	0.0025	0.0115
DLList.traverse	C	t	308	308	0.0474	0.0017	0.0491	t	228	228	0.0388	t	308	308	0.0353	0.0243	0.0596
DLList.traverse	M	t	38	38	0.0027	0.0002	0.0029	t	69	69	0.0065	t	38	38	0.0035	0.0008	0.0043
DLList.traverse	N	t	820	820	0.1068	0.0034	0.1103	t	1458	1458	0.3267	t	820	820	0.1308	0.2683	0.3991
DLList.traverse	N	t	820	820	0.1321	0.0175	0.1496	t	1458	1458	0.3805	t	820	820	0.1581	0.4518	0.6099
DLList.traverse	R	t	56	56	0.0174	0.0001	0.0176	t	4	4	0.0019	t	56	56	0.0140	0.0004	0.0144
DLList.traverse	S	f	38	38	0.0056	0.0001	0.0057	f	15	15	0.0017	f	38	38	0.0061	0.0003	0.0064
SLList.buildAndConcat	M	t	47	91	0.0205	0.0018	0.0222	t	87	150	0.0884	t	47	91	0.0221	0.0220	0.0441
SLList.buildAndConcat	S	t	47	91	0.0322	0.0046	0.0368	t	19	104	0.1314	t	47	91	0.0315	0.0617	0.0932
SLList.buildRecursive	M	t	13	92	0.0079	0.0004	0.0083	t	16	169	0.0343	t	13	92	0.0085	0.0059	0.0144
SLList.buildRecursive	S	f	13	92	0.0165	0.0007	0.0171	f	26	112	0.0598	f	13	92	0.0149	0.0123	0.0272
SLList.build	M	t	14	32	0.0029	0.0002	0.0032	t	24	48	0.0098	t	14	32	0.0024	0.0019	0.0042
SLList.build	S	t	14	32	0.0033	0.0003	0.0037	t	27	79	0.0171	t	14	32	0.0054	0.0047	0.0101
SLList.findMiddleFaultyCall	R	t	4	57	0.0132	0.0002	0.0134	f	2	29	0.0177	f	4	57	0.0197	0.0016	0.0213
SLList.findMiddleFaulty	M	t	53	53	0.0040	0.0019	0.0059	t	53	53	0.0116	t	53	53	0.0067	0.0020	0.0087
SLList.findMiddleFaulty	R	f	53	53	0.0139	0.0008	0.0147	f	27	27	0.0154	f	53	53	0.0138	0.0010	0.0147

SLList.findMiddle	C	t	384	384	0.0269	0.0064	0.0333	t	362	362	0.0450	t	384	384	0.0225	0.0277	0.0502
SLList.findMiddle	C	f	421	421	0.0489	0.0059	0.0548	f	211	211	0.1296	f	421	421	0.0419	0.1010	0.1429
SLList.findMiddle	M	t	84	84	0.0027	0.0006	0.0033	t	152	152	0.0079	t	84	84	0.0055	0.0017	0.0072
SLList.findMiddle	N	t	453	453	0.0356	0.0040	0.0396	t	843	843	0.1401	t	453	453	0.0336	0.0479	0.0816
SLList.findMiddle	R	t	84	84	0.0090	0.0008	0.0098	t	152	152	0.0300	t	84	84	0.0081	0.0024	0.0105
SLList.findMiddle	S	t	90	90	0.0055	0.0012	0.0068	t	162	162	0.0166	t	90	90	0.0064	0.0017	0.0081
SLList.find	C	t	80	80	0.0033	0.0007	0.0041	t	52	52	0.0043	t	80	80	0.0050	0.0019	0.0068
SLList.find	C	f	100	100	0.0082	0.0018	0.0100	f	52	52	0.0237	f	100	100	0.0069	0.0075	0.0144
SLList.find	M	t	29	29	0.0012	0.0003	0.0016	t	52	52	0.0036	t	29	29	0.0016	0.0012	0.0029
SLList.find	N	t	165	165	0.0104	0.0018	0.0122	t	293	293	0.0287	t	165	165	0.0117	0.0222	0.0340
SLList.find	S	t	31	31	0.0028	0.0005	0.0033	t	54	54	0.0076	t	31	31	0.0031	0.0013	0.0045
SLList.reverseRecursive	M	t	18	128	0.0029	0.0002	0.0031	t	32	313	0.0094	t	18	128	0.0049	0.0017	0.0066
SLList.reverseRecursive	N	f	132	2097	0.0416	0.0016	0.0433	f	117	567	0.0666	f	132	2097	0.0400	0.0222	0.0622
SLList.reverseRecursive	R	t	15	117	0.0136	0.0001	0.0137	t	5	5	0.0013	t	15	117	0.0121	0.0006	0.0127
SLList.reverseRecursive	S	t	18	128	0.0069	0.0002	0.0071	f	11	57	0.0099	f	18	128	0.0113	0.0018	0.0131
SLList.reverse	C	t	174	174	0.0085	0.0009	0.0094	t	82	82	0.0064	t	174	174	0.0095	0.0027	0.0122
SLList.reverse	M	t	37	37	0.0009	0.0003	0.0013	t	68	68	0.0028	t	37	37	0.0016	0.0011	0.0026
SLList.reverse	N	f	372	372	0.0326	0.0104	0.0430	f	263	263	0.2313	f	372	372	0.0497	0.1183	0.1681
SLList.reverse	N	f	372	372	0.0375	0.0026	0.0402	f	290	290	0.0507	f	372	372	0.0420	0.0502	0.0922
SLList.reverse	R	t	61	61	0.0103	0.0001	0.0104	t	5	5	0.0012	t	61	61	0.0116	0.0004	0.0120
SLList.reverse	S	t	40	40	0.0034	0.0013	0.0047	t	40	40	0.0303	t	40	40	0.0067	0.0096	0.0163
SLList.reverse	S	t	37	37	0.0028	0.0004	0.0033	t	68	68	0.0071	t	37	37	0.0042	0.0014	0.0056
SLList.traverseModifyHeap	S	t	4	16	0.0026	0.0001	0.0027	f	2	5	0.0015	f	4	16	0.0027	0.0005	0.0032
SLList.traverseModifyList	M	t	29	41	0.0023	0.0003	0.0025	t	49	65	0.0057	t	29	41	0.0027	0.0017	0.0043
SLList.traverseModifyList	N	t	184	220	0.0189	0.0016	0.0205	f	81	120	0.0171	f	184	220	0.0158	0.0218	0.0376
SLList.traverseRecursiveModifyHeap	S	t	15	68	0.0035	0.0001	0.0036	f	2	5	0.0012	f	15	68	0.0062	0.0005	0.0067
SLList.traverseRecursive	C	f	32	489	0.0091	0.0004	0.0094	f	45	214	0.0145	f	32	489	0.0129	0.0059	0.0187
SLList.traverseRecursive	N	t	90	1407	0.0152	0.0001	0.0153	t	11	11	0.0004	t	90	1407	0.0265	0.0009	0.0275
SLList.traverseRecursive	R	t	10	101	0.0068	0.0001	0.0069	t	3	3	0.0007	t	10	101	0.0116	0.0003	0.0119
SLList.traverseRecursive	S	t	10	45	0.0020	0.0002	0.0022	t	15	152	0.0102	t	10	45	0.0038	0.0022	0.0060
SLList.traverseRecursive	S	t	10	45	0.0028	0.0001	0.0029	t	15	152	0.0111	t	10	45	0.0050	0.0014	0.0064
SLList.traverse	C	t	47	47	0.0011	0.0002	0.0013	t	30	30	0.0017	t	47	47	0.0014	0.0005	0.0019
SLList.traverse	M	t	18	18	0.0004	0.0001	0.0005	t	32	32	0.0011	t	18	18	0.0008	0.0005	0.0013
SLList.traverse	N	t	95	95	0.0033	0.0001	0.0034	t	11	11	0.0004	t	95	95	0.0063	0.0005	0.0068
SLList.traverse	N	t	95	95	0.0031	0.0005	0.0036	t	168	168	0.0074	t	95	95	0.0042	0.0027	0.0069
SLList.traverse	R	t	34	34	0.0033	0.0001	0.0034	t	3	3	0.0005	t	34	34	0.0044	0.0002	0.0046
SLList.traverse	S	t	18	18	0.0010	0.0003	0.0013	t	32	32	0.0026	t	18	18	0.0011	0.0006	0.0018
SLList.traverse	S	t	18	18	0.0010	0.0001	0.0011	t	32	32	0.0023	t	18	18	0.0017	0.0004	0.0021
SLList.zipDummy	M	t	9	437	0.0200	0.0001	0.0201	t	9	591	0.0394	t	9	437	0.0229	0.0267	0.0496
Tree.build	M	t	472	480	0.1485	0.0138	0.1623	t	472	480	0.5840	t	472	480	0.1799	0.1086	0.2885
Tree.build	S	t	472	480	0.1979	0.0163	0.2142	t	10	26	0.0248	t	472	480	0.2029	0.1506	0.3535
Tree.getLeftmostChildModifyList	S	t	58	86	0.0181	0.0009	0.0190	f	35	65	0.0149	f	58	86	0.0195	0.0020	0.0215
Tree.getLeftmostChild	M	t	22	22	0.0015	0.0003	0.0019	t	46	46	0.0038	t	22	22	0.0015	0.0007	0.0022
Tree.getLeftmostChild	N	t	805	805	0.1383	0.0556	0.1939	t	1813	1813	0.5297	t	805	805	0.0962	0.8691	0.9653
Tree.getLeftmostChild	S	t	43	43	0.0037	0.0004	0.0040	t	91	91	0.0135	t	43	43	0.0042	0.0009	0.0051
Tree.getLeftmostChild	S	t	43	43	0.0059	0.0015	0.0074	t	91	91	0.0169	t	43	43	0.0071	0.0027	0.0097
Tree.traverseRecursiveModifyHeap	S	t	38	1037	0.0624	0.0005	0.0629	f	21	930	0.1070	f	38	1037	0.0561	0.0120	0.0681
Tree.traverseRecursive	C	f	220	27083	0.8981	0.0051	0.9032	f	52	23151	2.5851	f	220	27083	0.8303	0.0482	0.8785
Tree.traverseRecursive	M	t	34	972	0.0264	0.0006	0.0270	t	40	1525	0.0714	t	34	972	0.0272	0.0130	0.0402
Tree.traverseRecursive	N	t	1033	301704	11.0335	0.0296	11.0631	t	754	3614	0.4402	t	1033	301704	10.1003	8.6893	18.7896
Tree.traverseRecursive	S	t	34	972	0.0369	0.0005	0.0374	t	40	1525	0.1071	t	34	972	0.0540	0.0052	0.0593
Tree.traverseRecursive	S	t	34	972	0.0247	0.0005	0.0252	t	40	722	0.0594	t	34	972	0.0342	0.0136	0.0478

Table A.1: Benchmark results comparing model checking (MC) and hierarchical model checking (HMC) algorithms on the generated number of states during state space generation (SSG) and the execution times in seconds.